Assignment 1: Linear Programming Example: The Diet Problem Revisited (Fall 2024)

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Introduction

"The Diet Problem" is a classic constraint optimization problem used as an exercise for students to learn linear programming concepts. In this assignment, I am trying to determine how many servings of each food I should eat to hit nutritional goals while minimizing the overall cost of groceries. I must consider sodium, energy, protein, vitamin D, calcium, iron, and potassium intake. I will be using rice, salmon, eggs, applesauce, and frozen stir fry vegetables as my food items.

Method

For this assignment, I gathered items from my pantry, refrigerator and freezer that I commonly eat and have nutritional facts associated with them. I do most of my shopping at Whole Foods because I have many health conditions, food allergies, and dietary restrictions. Whole Foods is expensive but it is the only store where I can make one stop to get all of the things I need for the week. The photos of the nutritional facts are saved in the associated Github repository in the img directory. Figure 1 shows the cost per serving of foods I commonly eat including rice, eggs, smoked salmon, frozen vegetables, and applesauce.

Food Item	Total Cost ($)	Number of Servings	Price per Serving ($)
Applesauce	7.99	5	1.598
Salmon	8.99	2	4.495
Frozen Stir Fry Vegetables	3.19	5	0.638

Eggs	9.39	12	0.783
Rice	14.99	50	0.30

Figure 1

I will be referencing and using the USDA recommended daily allowances for my nutritional components (Mente, O'Donnell, and Yusuf 2021).

Component	**Max/Min**	**Daily Amount and Measure**
Sodium	Maximum	5,000 mg
Energy	Minimum	2,000 Calories
Protein	Minimum	50 grams
Vitamin D	Minimum	20 mcg
Calcium	Minimum	1300 mg
Iron	Minimum	18 mg
Potassium	Minimum	4,700 mg

Figure 2

The standard form of the linear programming problem is as follows:

The objective function is designed to minimize cost per serving: $1.598x1 + 4.495x2 + 0.783x3 + 0.30x4 + 0.638x5$. There is a non-negativity constraint for each of the variables: $x1, x2, x3, x4, x5 \geq 0$. The nutritional constraints are as follows:

- **Sodium**: $0x1 + 450x2 + 70x3 + 310x4 + 20x5 \leq 35,0000$

- **Energy**: *160x1 + 120x2 + 70x3 + 40x4 + 25x5 ≥ 14,000*
- **Protein**: *3x1 + 12x2 + 6x3 + 1x4 + 1x5 ≥ 3503*
- **Vitamin D**: *0x1 + 30x2 + 0.6x3 + 0x4 + 0x5 ≥ 1400*
- **Calcium**: *0x1 + 10x2 + 30x3 + 0x4 + 20x5 ≥ 9,1000*
- **Iron**: *0x1 + 0x2 + 0.4x3 + 0.36x4 + 0.4x5 ≥ 1260*
- **Potassium**: *0x1 + 210x2 + 0x3 + 210x4 + 0x5 ≥ 32,9000*

My solution will be implemented with the Python PuLP library, a library purpose-built to solve linear programming problems. The code and output is provided in the associated Github repository. Said plainly, I am trying to find the lowest-cost combination of food servings that meets or exceeds the nutritional goals, without exceeding the maximum sodium limit. In linear programming terms, my decision variables are the number of servings, my constraints are the nutritional mins or maxes, and my objective function is the math problem where I'm attempting to minimize costs.

Results

The original solution of the diet problem was a rather improbable one. I was supposed to eat 303.33 servings of eggs and 12 servings of rice. That's 43 eggs a day and roughly 2 servings of rice a day. That's a lot of suffering for the weekly cost of $241.40. I tweaked my code a bit and added a constraint requiring at least one serving of each food per week, which resulted in an only ever-so-slightly more appetizing solution. A single serving of applesauce, stir fry vegetables and smoked salmon per week, plus 302 eggs, and thirteen servings of rice at a cost of $247.35.

Further research

If I had more time, I had planned to implement the llama index report generator multi-agent team. I believe given the correct context they could code the solution for me and write the paper as well. You can find the example notebook at this location: https://github.com/run-llama/llama_parse/blob/main/examples/multimodal/multimodal_report_generation_agent.ipynb

References

Food and Drug Administration, Department of Health and Human Services. 2016. Food Labeling: Revision of the Nutrition and Supplemental Facts Labels. Available at https://s3.amazonaws.com/public-inspection.federalregister.gov/2016-11867.pdfLinks to an external site.

Mente, Andrew, Martin O'Donnell, and Salim Yusuf. 2021, September. "Sodium Intake and Health: What Should We Recommend Based on the Current Evidence?" Nutrients, 13(9): 3232. Available online at https://www.mdpi.com/2072-6643/13/9/3232Links to an external site. . This is part of the September 2021 special issue of Nutrients: "Towards Better Dietary Guidelines: New Approaches Based on Recent Science."