

The goal is to calculate the number of ways to navigate from the top-left corner of a square grid to the bottom-right corner in a taxicab geometry. We take the size of the grid to be  $2N \times 2N$ .

The problem is equivalent to asking how many ways there are to permute a set of  $2N$  objects when it is comprised of two distinct sets of  $N$  identical objects. We represent the first type of object as  $A$  and the second as  $B$ . For example, take  $N = 2$ , so that the six types of lattice path are represented by

$$\{AABB\}, \{ABBA\}, \{BBAA\}, \{BAAB\}, \{ABAB\}, \{BABA\}. \quad (1)$$

If we were dealing with a set of  $2N$  distinct objects, the number of permutations would be  $(2N)!$ . Since there are two subsets of  $N$  identical objects, and there are  $N!$  ways to arrange each subset, then the number of lattice paths is given by

$$\frac{(2N)!}{N!N!}. \quad (2)$$

Therefore, setting  $N = 20$  gives us the answer

$$\frac{40!}{20!20!} = 137846528820. \quad (3)$$