Norwegian School of Economics

Project 3 Report

BAN402: Decision Modelling in Business

Candidates 28, 139

Fall 2024

Contents

Part A:

Task A1

Model formulation:

Sets:

I: Set of seven days of the week

Parameters:

D_i: Intercept from demand estimation from historical data, $\forall i \in I$

m_i: Slope from demand estimation from historical data, $\forall i \in I$

$MaxCap$: The maximum number of tickets the theater can sell on a given day

$MinSales$: The minimum number of tickets the theater should sell on a given day

Variables:

p_i: Price per ticket on day i, $\forall i \in I$

Q_i: Demand (number of tickets sold) on day i, $\forall i \in I$

Objective:

Maximize revenue over the week:

$$\max \sum_{i \in I} p_i Q_i$$

Subject to:

Max capacity constraint:

$$Q_i \leq MaxCap, \qquad \forall i \in I \tag{1}$$

Minimum 100 tickets sold:

$$Q_i \geq MinSales, \qquad \forall i \in I \tag{2}$$

Q_i is a function of p_i, D_i and m_i:

$$Q_i = D_i + m_i p_i + \sum_{j \in I} 2(p_j - p_i), \qquad \forall i \in I \tag{3}$$

Non-negativity:

$$p_i \geq 0, \qquad \forall i \in I \tag{4}$$

Result:

Solving the above model using *minos* in AMPL yields the following optimal prices, tickets sold, revenues and percentage of capacity filled for each day:

	Price	Tickets Sold	Revenue	% Capacity
Monday	15.6368	705.528	11,032	88%
Tuesday	21.995	800	17,596	100%
Wednesday	32.6618	800	26,129	100%
Thursday	25.6627	800	20,530	100%
Friday	40.5272	800	32,422	100%
Saturday	44.7609	800	35,809	100%
Sunday	40.6248	800	32,500	100%

The total revenue over the week is **176,018**.

Task A2

To incorporate the two prices, we modify the model as follows:

We add a subset I^W:

$I^W \subset I$: Subset of weekend days from set I:

Add a new constraint stating that the weekday prices must be equal:

$$p_i - p_j = 0, \qquad \forall i, j \in I \backslash \{I^W\}, \qquad i \neq j \tag{3}$$

Add a new constraint stating that the weekend prices must be equal:

$$p_i - p_j = 0, \qquad \forall i, j \in I^W, \qquad i \neq j \tag{4}$$

Results:

Running the model twice, once with a dataset where Friday is included in I^W and once without yields the following results:

Without Friday included as a weekend day:

	Price	Tickets Sold	Revenue	% Capacity
Monday	25.1915	101.149	2,548	13%
Tuesday	25.1915	602.681	15,182	75%
Wednesday	25.1915	800	20,153	100%
Thursday	25.1915	800	20,153	100%
Friday	25.1915	800	20,153	100%
Saturday	40.1805	800	32,144	100%
Sunday	40.1805	800	32,144	100%

Total revenue: **142,479**

With Friday included as a weekend day:

	Price	Tickets Sold	Revenue	% Capacity
Monday	25.7642	104.585	2,695	13%
Tuesday	25.7642	610.699	15,734	76%
Wednesday	25.7642	800	20,611	100%
Thursday	25.7642	800	20,611	100%
Friday	40.7205	794.323	32,345	99%
Saturday	40.7205	800	32,576	100%
Sunday	40.7205	800	32,576	100%

Total revenue: **157,149**

We recommend having Friday included in the weekend price as the resulting total revenue is higher.

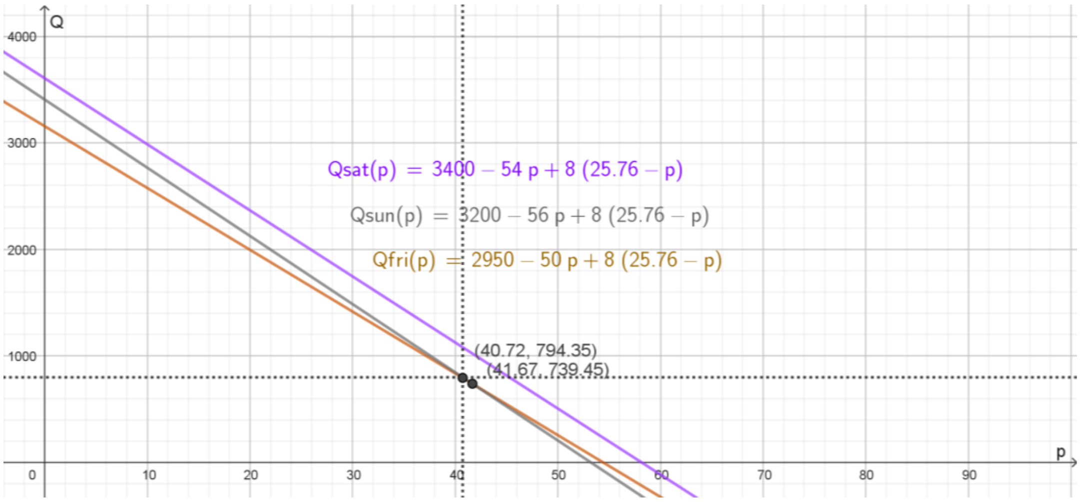
Comparing the *% Capacity* to the result from Task 1, we notice that not only Monday but also Tuesday and (Friday when it is included in the weekend price) has vacant capacity. This can be understood by examining the D_i parameter where Monday has the lowest base demand, followed by Tuesday. Friday also has a lower base demand than Saturday and Sunday.

We can explain why Friday has excess capacity when it is included in the weekend prices by plotting its demand as a function of $P_{weekend}$.

$$Q_{fri} = D_{fri} + m_{fri} p_{Weekend} + 8 * (p_{Weekday} - p_{Weekend})$$

$$= 2950 - 50 * P_{weekend} + 8(25.7642 - p_{Weekend})$$

Plotting Q_{fri} together with Q_{Sat} and Q_{sun} we see that Friday will always have the lowest demand for $P_{weekend} \leq 41.67$ and the demand is lower than the capacity at the optimal price $Q^*_{Weekend} = 40.72$.

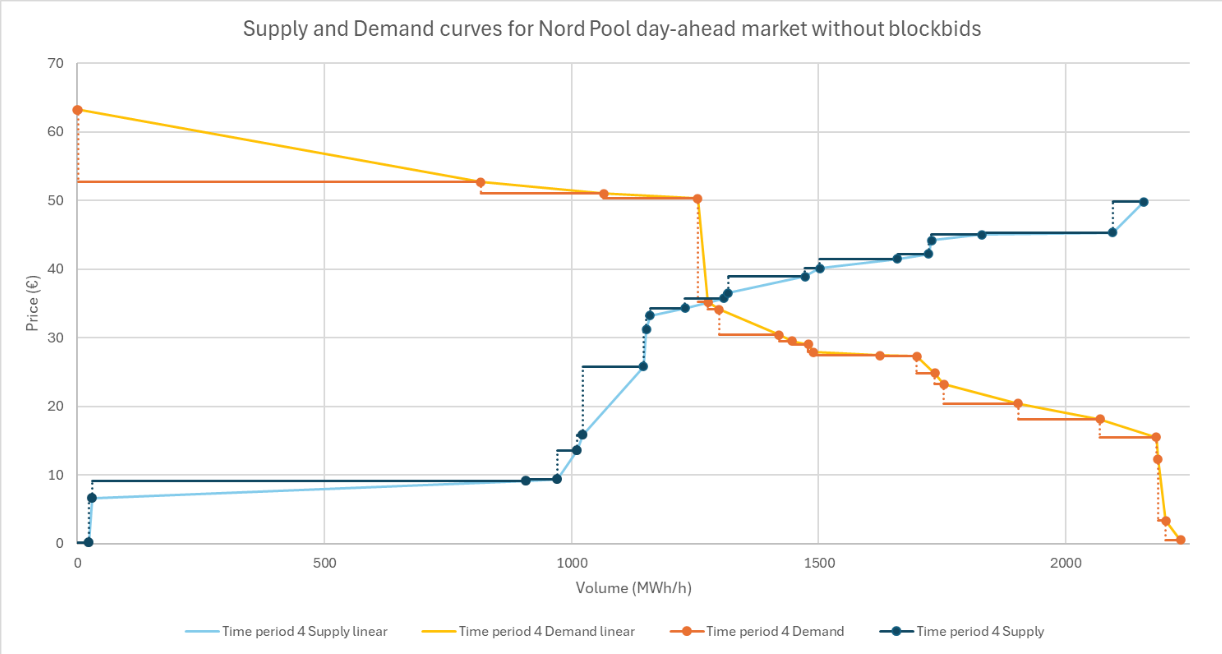


Part B:

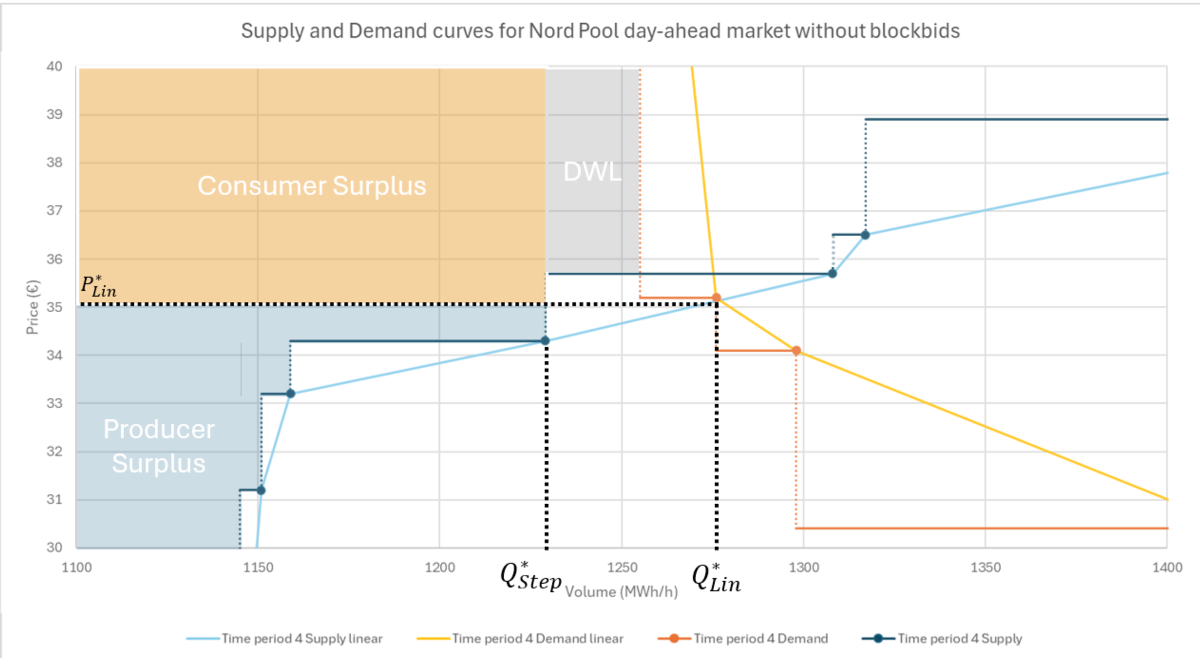
Task B1:

Because our candidate numbers are 28 and 139, we will consider period 4.

Plotting the step-wise and linearized supply and demand curves:



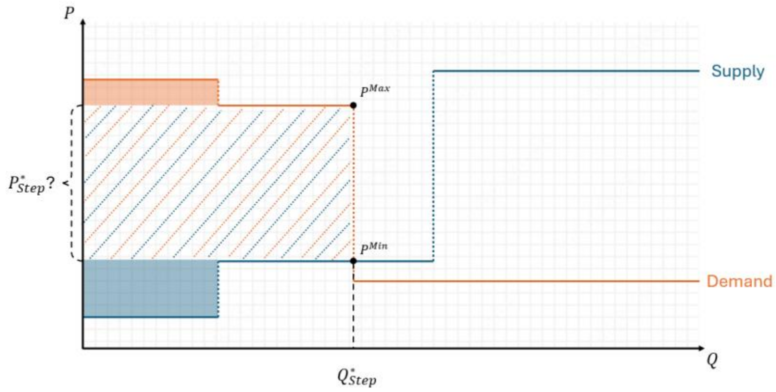
Adjusting the axes to examine the equilibrium more closely:



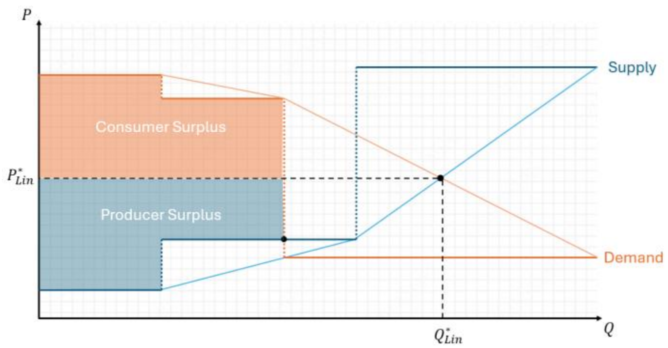
The market clears (the linearized supply and demand curves cross) just under the bid at 35.2. Thus, we propose an equilibrium system price $P^*$of approximately 35.15 €/MWh. At this price point, the producers are only willing to produce Q^*_{Step}.

The Demand bids that are accepted are: 63.2, 57.7, 51.0 and 50.3. The Supply Bids that are accepted are 0.1, 6.6, 9.1, 9.3, 13.5, 15.8, 25.8, 31.2, 33.2 and 34.3.

The advantage of using linear approximations instead of step functions can be illustrated with the following simplified example:

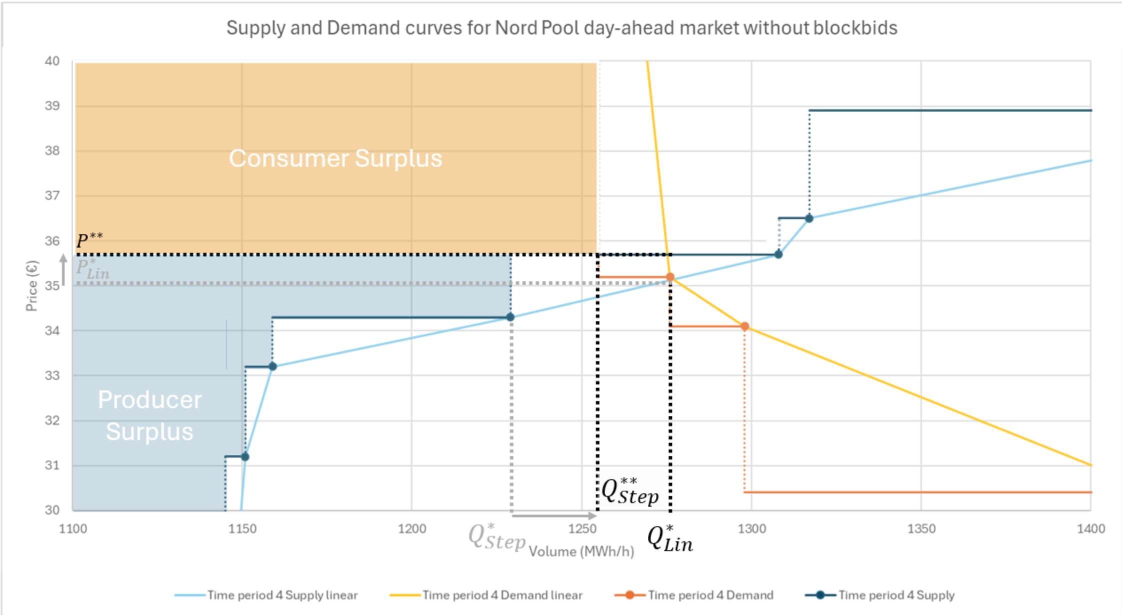


From the step functions, which reflect the actual bids, we know that the volume produced will be Q^*_{Step} as no consumers are willing to pay the price required by suppliers for producing electricity beyond this volume. However, the equilibrium price P^*_{Step} can be anywhere between P^{Min} and P^{Max}. Adding a linear approximation solves this “ambiguity” and gives the model a distinct point to clear the market and solve the maximization problem:



In this simplified example, which price between P^{Max} and P^{Min} is chosen makes no difference with respect to total social value. However, from the perspective of the consumer or producer, the chosen price will significantly change their respective surpluses.

A disadvantage of using linear approximation to find the equilibrium is that it sometimes may lead to dead weight losses. In the case of Period 4; If instead of using P^*_{lin}, the price would have been set equal to the supply bid at 35.7 €/MWh, the realized quantity produced would have increased from Q^*_{Step} to Q^{**}_{step} and the dead weight loss would have been removed.



Task B2:

Scenario a)

To find the most profitable bid strategy we first needed to figure out a methodology to find the optimal prices for a given volume. Assuming that our *accepted volume* is equal to *s* if the *bid price* exactly matches *PS*, or the entire *bid volume* if the $bid\ price$ is below PS, we can calculate profit as follows:

$$\pi = Accepted\ volume * (System\ price -\ marginal\ cost)$$

Because the time periods are independent, we can find the most profitable prices for a given bid volume by experimenting with different prices and comparing their resulting profit. In most cases, this was the price that gave an *accepted volume* closest to the *bid volume,* as the gain in volume sold compensates for the loss in system price due to higher total supply.

To determine the optimal bid volume, we found it natural to start at the capacity of 1200. Optimizing the prices yielded the following bid:

Hour	Bid Price	Bid Volume	System Price	PS	s	Accepted Vol.	Revenue	Production Cost	Profit
1	19	1200	20.13	19	1200	1,200	24,156	13,200	10,956
2	12	1200	11.92	11.4	13	0	0	0	0
3	27	1200	27.35	27.1	9	1,200	32,820	13,200	19,620
4	13	1200	13.17	13	1200	1,200	15,804	13,200	2,604
5	23	1200	25.75	23	1195	1,195	30,771	13,145	17,626
								Total profit:	**50,806**

However, as we did not win any bids in hour 2, we next found the highest possible volume, where we could win that hour with a positive contribution margin (price =12), which was 1036. Optimizing the prices for all hours at Q=1036 gives:

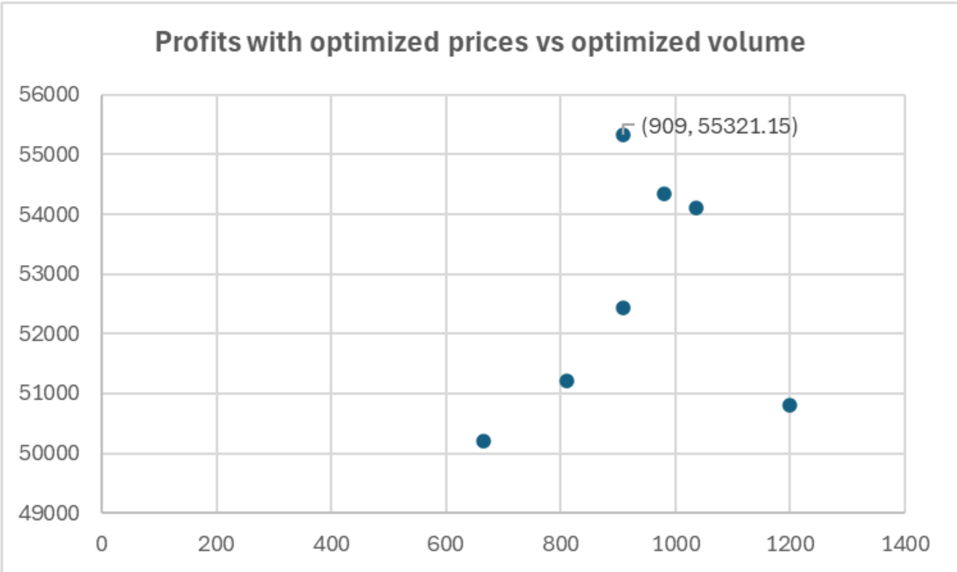
Hour	Bid Price	Bid Volume	System Price	PS	s	Accepted Vol.	Revenue	Production Cost	Profit
1	21	1036	21.39	21.3	81	1,036	22,160	11,396	10,764
2	12	1036	12	12	1036	1,036	12,432	11,396	1,036
3	27	1036	27.49	27.1	9	1,036	28,480	11,396	17,084
4	17	1036	18.01	17	1036	1,036	18,658	11,396	7,262
5	29	1036	29.28	29	982	982	28,753	10,802	17,951
								Total profit:	**54,097**

Since the profit increased, it is natural to experiment further with a lower volume. Because hour 4 had the highest difference between our bid price and the system price, we suspect that this hour has the lowest price elasticity. Our next goal was then to find what volume gave us the highest profit for hour four, and then optimize the remaining prices for this volume. Our result was:

Hour	Bid Price	Bid Volume	System Price	PS	s	Accepted Vol.	Revenue	Production Cost	Profit
1	22	909	22.2	22	909	909	20,180	9,999	10,181
2	14	909	14.34	14	909	909	13,035	9,999	3,036
3	27	909	27.6	27.1	9	909	25,088	9,999	15,089
4	20	909	20.01	20	881	881	17,629	9,691	7,938
5	32	909	32.01	32	908	908	29,065	9,988	19,077
								Total profit:	**55,321**

After further experimentation with small changes in prices and volumes around this solution, we were not able to find any improvements to the profit. Thus, we suspect that this is at least a local optimum.

To further explore whether the above bid is a global optimum, we experimented with much lower volumes to find their optimal prices, and then the optimal volume for those prices at three different volumes. Our results, presented in the below table, strengthens our hypothesis that the bid volume of 909 is the most profitable:

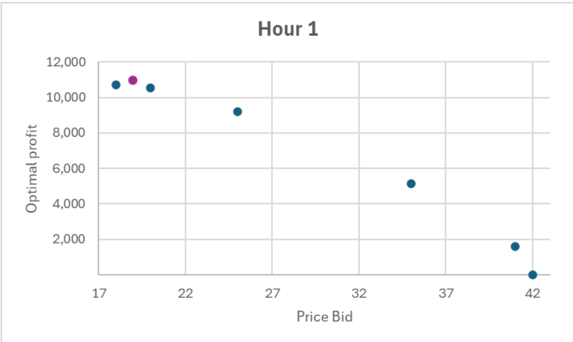


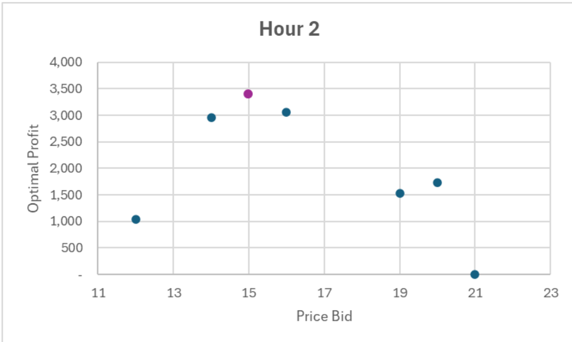
Scenario b)

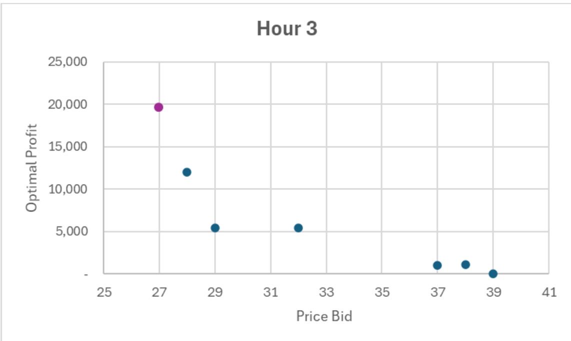
Without the constraint that the volume bid had to be equal across time periods, our new methodology for finding an optimal bid is to go through each time period and find the optimal volume for each bid price and compare the resulting profit.

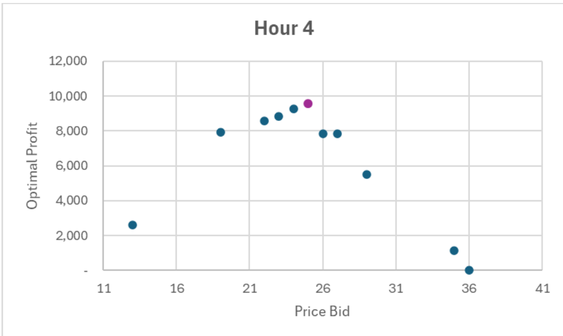
To reduce the number of different prices to evaluate, we use the price from our optimal bid at capacity ($Bid\ Volume = 1200$) as a natural lower bound on the price for each time period. We also use the original system price without our bid as a preliminary upper bound, as a price above the equilibrium usually would not give any accepted volume except in cases with a very high demand (only Hour 3, in our case)

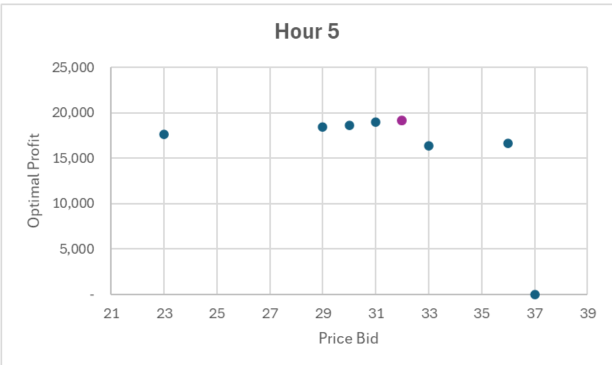
Our results were as follows:











Because we did not examine every price, we cannot be 100% certain that our results are global optima. However, we think there is a decent likelihood that the following is profit-maximizing bid:

Hour	Bid Price	Bid Volume	System Price	PS	s	Accepted Vol.	Revenue	Production Cost	Profit
1	19	1200	20.13	19	1200	1,200	24,156	13,200	10,956
2	15	840	15.05	15	840	840	12,642	9,240	3,402
3	27	1200	27.35	27.1	9	1,200	32,820	13,200	19,620
4	25	680	25.17	25	675	675	16,990	7,425	9,565
5	32	910	32.01	32	908	908	29,065	9,988	19,077
								Total profit:	**62,620**

We see that this bid yields a higher total profit than the optimal bid from Scenario a, which is as expected as the problem was less constrained.

Task B3:

We decided on an approach where we would marginally underbid each of the previously accepted block bids, and then try to optimize the profit by fixing these prices and changing the volumes. First off, we simulated the data without us bidding to see what block bids were accepted. This turned out to be:

Bid with id 3: $16.5 for 18 MW from hour 2 to 4

Bid with id 4: $22.0 for 85 MW from hour 2 to 5

Bid with id 6: $34.7 for 20 MW form hour 3 to 5

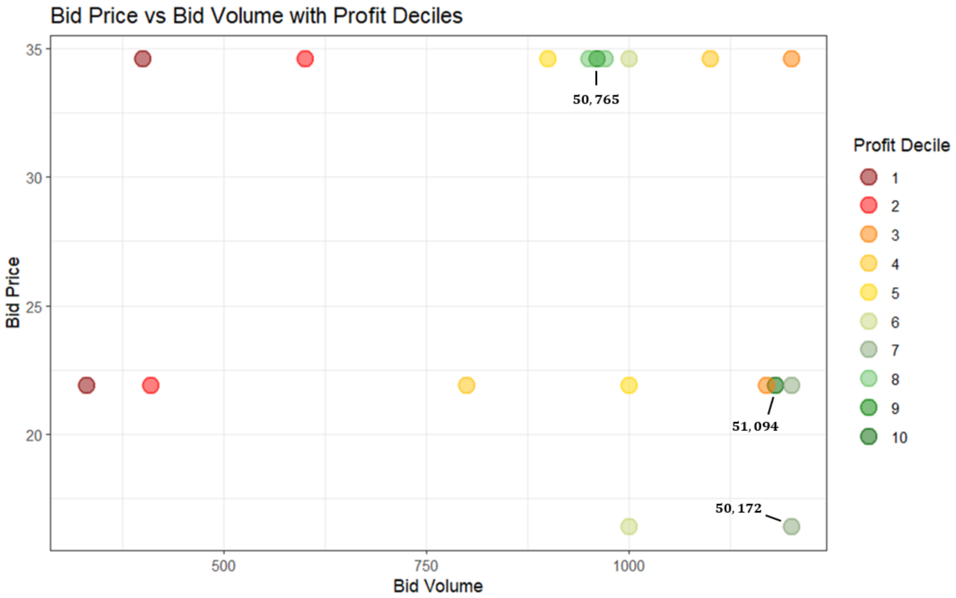
Because both the system price and volume for hour 1 was significantly lower than the other periods, we decided that our bid should be from hour 2 to 5. To undercut the accepted bids, the prices we wanted to optimize the volume for was $16.4, $21.9 and $34.6.

Starting with a fixed bid price of $16.4 we got our entire production capacity accepted and were not able to improve profit by offering a smaller bid volume. The optimal profit was $50,172

For a fixed bid price of $21.9 the optimal bid volume was 1180 with a profit of $51,094. This was the lowest bid volume that caused the block bid with id 4 to be rejected, and thus keeping the price as high as possible.

For a fixed bid price of $34.6 the optimal bid volume was 960, which we found by evaluating profits for volumes in the range from 400 to 1200.

The resulting profit for different combinations of bid price and bid volume from hour 2 to 5, as well as the optimal profit undercutting each of the previously accepted bids, can be seen in the figure below:



Evaluating different combinations of prices and volumes near the proposed optimal solutions for each initial price point, it appears that (960, 34.6) and (1200, 16.4) are local optima, whilst (1180, 21.9) is more likely to be the global optimum for the problem. To determine where the global optimum is with more confidence, there are still many combinations yet to be explored.

It is worth noting that the total profit from our proposed bid is considerably lower with a block bid compared to specifying separate bids for each hour. Some suppliers might prefer to submit block bids if they value operational stability over maximal profit.

Part C:

Task C1:

To address the new scenario, we add the following to the model:

We assume that set T includes a first element 0, where charging is not allowed.

New parameter:

n_k: Minimum charge amount for vehicle k if it charges in a given period, where $(0 < n_k < m_k)$

Two new binary variables:

$$\delta_{k,t} = \begin{cases} 1 \text{ if vehicle } k \text{ charges at time } t \ (x_{k,t} \text{ is positive}) \\ 0 \text{ otherwise} \end{cases}$$

$$SC_{k,t} = \begin{cases} 1 \text{ if vehicle } k \text{ starts charging at time } t \\ 0 \text{ otherwise} \end{cases}$$

Linking δ_t to x_t:

$$x_{k,t} \leq M * \delta_{k,t}, \quad \begin{matrix} \forall k \in K \\ \forall t \in T \end{matrix} \tag{6}$$

$$x_{k,t} \geq \delta_{k,t}, \quad \begin{matrix} \forall k \in K \\ \forall t \in T \end{matrix} \tag{7}$$

(6) ensures that if x_t is greater than zero, δ_t must be one
(7) ensures that if x_t is zero, δ_t must be zero

Linking SC_t to δ_t:

$$SC_{k,t} \leq \delta_{k,t}, \quad \begin{matrix} \forall k \in K \\ \forall t \in T \end{matrix} \tag{8}$$

$$SC_{k,t} \leq 1 - \delta_{k,t-1}, \quad \begin{matrix} \forall k \in K \\ \forall t \in T: t > 0 \end{matrix} \tag{9}$$

$$SC_{k,t} \geq \delta_{k,t} + \left(1 - \delta_{k,t-1}\right) - 1, \quad \begin{matrix} \forall k \in K \\ \forall t \in T: t > 0 \end{matrix} \tag{10}$$

(8) ensures that if there is no charging in period t, it cannot be a start time

(9) ensures that SC_t can only take value 1, if there was no charging in the previous period

(10) ensures that SC_t must take value 1, if $x_{k,t}$ is positive and $x_{k,t-1}$ is not)

Ensure there is at most one start period per car:

$$\sum_{t \in T} SC_{k,t} \leq 1, \quad \forall k \in K \tag{11}$$

If a car is charging in a given period, then the charging amount must be at least n_k:

$$x_{k,t} \geq \delta_{k,t} * n_k, \qquad \begin{matrix} \forall k \in K \\ \forall t \in T \end{matrix} \tag{12}$$

Ensure there is no charging in hour zero:

$$x_{k,0} = 0, \qquad \forall k \in K \tag{13}$$

Add new binary constraints:

$$SC_{k,t} \in \{0,1\}, \qquad \begin{matrix} \forall k \in K \\ \forall t \in T \end{matrix} \tag{14}$$

$$\delta_{k,t} \in \{0,1\}, \qquad \begin{matrix} \forall k \in K \\ \forall t \in T \end{matrix} \tag{15}$$

If we attempt to solve the model again for the new scenario, we expect the objective value to be higher than in the original scenario because we are considering a minimization problem, and the model was further restricted.

Part D:

Task D1:

Model formulation:

(To make it easier to check for linearity, decision variables are colored red)

Sets:

L: Set of candidate locations.

I: Set of inhabitants in the city

Parameters:

F_l: Fixed cost for opening a charging station at location l, $\forall l \in L$

C_l: Variable cost per charging device installed at location l, $\forall l \in L$

$D_{i,l}$: Distance in km from inhabitant i's house to location l, $\forall i \in I, \forall L \in L$

$MinD_i$: The lowest distance in km from inhabitant i's house to any location, $\forall i \in I$

T: Threshold distance for an inhabitant to be classified as a potential user for any location.

H: Installation cost per high-speed charging device

N: Installation cost per normal-speed charging device

B: Budget

Variables:

n_l: Number of normal-speed charging devices to install at location l

h_l: Number of high-speed charging devices to install at location l

x_l: $\begin{cases} 1 \text{ if a charging station is opened at location } l \\ 0 \text{ otherwise} \end{cases}$

$u_{i,l}$: $\begin{cases} 1 \text{ if inhabitant } i \text{ is a potential user of location } l \\ 0 \text{ otherwise} \end{cases}$

Objective:

Maximize the number of potential users of the network

$$\max \sum_{l \in L} \sum_{i \in I} u_{i,l}$$

Subject to:

For an inhabitant, i, to be a potential user of location l, it must be closer than the threshold T:

$$u_{i,l} * MinD_i \leq T, \qquad \begin{matrix} \forall i \in I \\ \forall l \in L \end{matrix} \tag{1}$$

For an inhabitant, i, to be a potential user of location l, l must be their closest station:

$$u_{i,l} * D_{i,l} \leq MinD_i, \qquad \begin{matrix} \forall i \in I \\ \forall l \in L \end{matrix} \tag{2}$$

Each potential user should be assigned at most one station:

$$\sum_{l \in L} u_{i,l} \leq 1\,, \qquad \forall i \in I \tag{3}$$

Each locations' number of installed devices must be at least 1% of its potential users:

$$h_l + n_l \geq 1\% * \sum_{i \in I} u_{i,l}\,, \qquad \forall l \in L \tag{4}$$

If a station is not opened, it cannot have any potential users:

$$\sum_{i \in I} u_{i,l} \leq M * x_l, \qquad \forall l \in L \tag{5}$$

Where M is a sufficiently large number.

If a station is opened, it must have 2 high-speed chargers installed:

$$h_l = 2 * x_l, \qquad \forall l \in L \tag{6}$$

If a station is not opened, it cannot have any normal chargers installed:

$$n_l \leq M * x_l, \qquad \forall l \in L \tag{7}$$

Where M is a sufficiently large number.

Total cost cannot exceed budget:

$$TC \leq B \tag{8}$$

Where the total cost TC is defined as:

$$\sum_{l \in L} F_l * x_l$$ (Sum of fixed costs for opened stations)

$$+ \sum_{l \in L} C_l (h_l + n_l)$$ (Sum of variable installation costs per device * number of devices installed for all opened stations)

$+ H * \sum_{l \in L} h_l$	(Installation costs for high speed charging devices)
$+ N * \sum_{l \in L} n_l$	(Installation costs for normal speed charging devices)
$= TC$	

Non negativity and binary constraints:

$$n_l \geq 0 \tag{9.a}$$

$h_l \in \{0, 2\}$ (derives its value directly from x_l) $(9.b)$

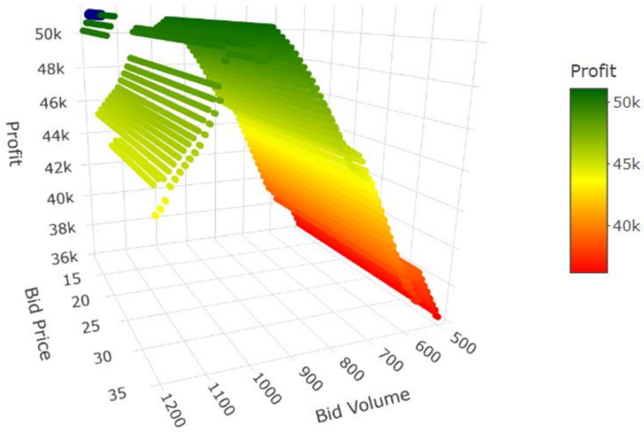
$$x_l \in \{0,1\} \tag{9.c}$$

$$u_{i,l} \in \{0,1\} \tag{9.d}$$

Appendix:

Task B3: Empirical approach

Although the task stated that we were not expected to run simulations, our curiosity got the best of us:



Solving the model for bid volumes between 500 and 1200 in increments of 10 and bid prices between 11 to 40 in increments of 0.1, we find that the optimal profit (marked in blue) can be achieved with a bid volume of 1180 and bid prices below 16.5. These bid prices yield a slightly higher profit (an increase of €0.05) compared to our proposed solution from B3 ($Q = 1080, P = 21.9$).

A key insight from the simulation is that bid volume has a substantially greater influence on the producer's optimal profit than bid price. This is intuitive, as bid volume directly impacts profit, while bid price affects it only indirectly through its influence on the system price and bid acceptance.