Домашнее задание на 2.10.2024 (Линейная алгебра)

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1.1.1.

$$X \cdot \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix} \cdot X^T = \begin{pmatrix} -1 & -5 \\ 2 & 6 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & 5 & | & -1 & -5 \\ -2 & -4 & | & 2 & 6 \end{pmatrix}$$

$$A[0] = A[0]/3$$

$$\begin{pmatrix} 1 & \frac{5}{3} & | & \frac{-1}{3} & \frac{-5}{3} \\ -2 & -4 & | & 2 & 6 \end{pmatrix}$$

$$A[1] = A[1] + 2A[0]$$

$$\begin{pmatrix} 1 & \frac{5}{3} & | & \frac{-1}{3} & \frac{-5}{3} \\ 0 & -\frac{2}{3} & | & \frac{4}{3} & \frac{8}{3} \end{pmatrix}$$

$$A[1] = A[1] \cdot \frac{-3}{2}$$

$$\begin{pmatrix} 1 & \frac{5}{3} & | & \frac{-1}{3} & \frac{-5}{3} \\ 0 & 1 & | & -2 & -4 \end{pmatrix}$$

$$A[0] = A[0] - A[1] \cdot \frac{5}{3}$$

$$\begin{pmatrix} 1 & 0 & | & 3 & 5 \\ 0 & 1 & | & -2 & -4 \end{pmatrix}$$

$$\begin{cases} x_1 = 3 \\ x_2 = -2 \\ y_1 = 5 \\ y_2 = -4 \end{cases} \Rightarrow X^T = \begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix} \Rightarrow X = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$$

1.2.

$$\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix} X \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix}$$

$$Y = X \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix} Y = \begin{pmatrix} 14 & 16 \\ 9 & 10 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & -1 & | & 14 & 16 \\ 5 & -2 & | & 9 & 10 \end{pmatrix}$$

$$A[0] = A[0]/3$$

$$\begin{pmatrix} 1 & -\frac{1}{3} & | & \frac{14}{3} & \frac{16}{3} \\ 5 & -2 & | & 9 & 10 \end{pmatrix}$$

$$A[1] = A[1] - 5A[0]$$

$$\begin{pmatrix} 1 & -\frac{1}{3} & | & \frac{14}{3} & \frac{16}{3} \\ 0 & -\frac{1}{3} & | & -\frac{43}{3} & -\frac{50}{3} \end{pmatrix}$$

$$A[0] = A[0] - A[1]$$

$$\begin{pmatrix} 1 & 0 & | & 19 & 22 \\ 0 & -\frac{1}{3} & | & -\frac{43}{3} & -\frac{50}{3} \end{pmatrix}$$

$$A[1] = A[1] \cdot (-3)$$

$$\begin{pmatrix} 1 & 0 & | & 19 & 22 \\ 0 & 1 & | & 43 & 50 \end{pmatrix} \Rightarrow Y = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix} = X \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix} X^T = \begin{pmatrix} 19 & 43 \\ 22 & 50 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 7 & | & 19 & 43 \\ 6 & 8 & | & 22 & 50 \end{pmatrix}$$

$$A[0] = A[0]/5$$

$$\begin{pmatrix} 1 & \frac{7}{5} & | & \frac{19}{5} & \frac{43}{5} \\ 6 & 8 & | & 22 & 50 \end{pmatrix}$$

$$A[1] = A[1] - 6A[0]$$

$$\begin{pmatrix} 1 & \frac{7}{5} & | & \frac{19}{5} & \frac{43}{5} \\ 0 & -\frac{2}{5} & | & -\frac{4}{5} & -\frac{8}{5} \end{pmatrix}$$

$$A[1] = -5 * A[1]/2$$

$$\begin{pmatrix} 1 & \frac{7}{5} & | & \frac{19}{5} & \frac{43}{5} \\ 0 & 1 & | & 2 & 4 \end{pmatrix}$$

$$A[0] = A[0] - 7 * A[1]/5$$

$$\begin{pmatrix} 1 & 0 & | & 1 & 3 \\ 0 & 1 & | & 2 & 4 \end{pmatrix} \Rightarrow X^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \Rightarrow X = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

1.3

$$\begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix} X = \begin{pmatrix} 3 & 7 \\ 1 & 7 \\ 7 & 7 \end{pmatrix}$$

$$A = \begin{pmatrix} 3 & -1 & 2 & | & 3 & 7 \\ 4 & -3 & 3 & | & 1 & 7 \\ 1 & 3 & 0 & | & 7 & 7 \end{pmatrix}$$

$$A[0] = A[0]/3$$

$$\begin{pmatrix} 1 & -\frac{1}{3} & \frac{2}{3} & | & 1 & \frac{7}{3} \\ 4 & -3 & 3 & | & 1 & 7 \\ 1 & 3 & 0 & | & 7 & 7 \end{pmatrix}$$

$$A[1] = A[1] - 4 * A[0]$$

$$\begin{pmatrix} 1 & -\frac{1}{3} & \frac{2}{3} & | & 1 & \frac{7}{3} \\ 0 & -\frac{5}{3} & \frac{1}{3} & | & -3 & -\frac{7}{3} \\ 1 & 3 & 0 & | & 7 & 7 \end{pmatrix}$$

$$A[1] = A[1] * (-3)/5$$

$$\begin{pmatrix} 1 & -\frac{1}{3} & \frac{2}{3} & | & 1 & \frac{7}{3} \\ 0 & 1 & -\frac{1}{5} & | & \frac{9}{5} & \frac{7}{5} \\ 1 & 3 & 0 & | & 7 & 7 \end{pmatrix}$$

$$A[2] = A[2] - A[0]$$

$$\begin{pmatrix} 1 & -\frac{1}{3} & \frac{2}{3} & | & 1 & \frac{7}{3} \\ 0 & 1 & -\frac{1}{5} & | & \frac{9}{5} & \frac{7}{5} \\ 0 & \frac{10}{3} & -\frac{2}{3} & | & 6 & \frac{14}{3} \end{pmatrix}$$

$$A[2] = A[2] * 3/10$$

$$\begin{pmatrix} 1 & -\frac{1}{3} & \frac{2}{3} & | & 1 & \frac{7}{3} \\ 0 & 1 & -\frac{1}{5} & | & \frac{9}{5} & \frac{7}{5} \\ 0 & 1 & -\frac{1}{5} & | & \frac{9}{5} & \frac{7}{5} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{3} & \frac{2}{3} & | & 1 & \frac{7}{3} \\ 0 & 1 & -\frac{1}{5} & | & \frac{9}{5} & \frac{7}{5} \end{pmatrix}$$

$$A[0] = A[0] + A[1]/3$$

$$\begin{pmatrix} 1 & 0 & \frac{3}{5} & | & \frac{8}{5} & \frac{14}{5} \\ 0 & 1 & -\frac{1}{5} & | & \frac{9}{5} & \frac{7}{5} \end{pmatrix}$$

$$\begin{cases} x_1 + \frac{3}{5}x_3 = \frac{8}{5} \\ x_2 - \frac{1}{5}x_3 = \frac{9}{5} \\ y_1 + \frac{3}{5}y_3 = \frac{14}{5} \\ y_2 - \frac{1}{5}y_3 = \frac{7}{5} \end{cases} \Rightarrow X = \begin{pmatrix} \frac{8}{5} - \frac{3}{5}x_3 & \frac{14}{5} - \frac{3}{5}y_3 \\ \frac{9}{5} + \frac{1}{5}x_3 & \frac{7}{5} + \frac{1}{5}y_3 \\ x_3 & y_3 \end{pmatrix}, \forall x_3, y_3 \in \mathbb{R}$$

1.4

$$A^T X + X = B,\ A = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 & 3 \\ 4 & 0 & 6 \end{pmatrix}$$

$$(A^T + E)X = B$$

$$(\begin{pmatrix} 0 & -1 \\ 2 & -3 \end{pmatrix} + E)X = \begin{pmatrix} 2 & 0 & 3 \\ 4 & 0 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix} X = \begin{pmatrix} 2 & 0 & 3 \\ 4 & 0 & 6 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & -1 & | & 2 & 0 & 3 \\ 2 & -2 & | & 4 & 0 & 6 \end{pmatrix}$$

$$C[1] = C[1] - 2 * C[0]$$

$$\begin{pmatrix} 1 & -1 & | & 2 & 0 & 3 \\ 0 & 0 & | & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 & | & 2 & 0 & 3 \end{pmatrix}$$

$$\begin{cases} x_1 - x_2 = 2 \\ y_1 - y_2 = 0 \\ z_1 - z_2 = 3 \end{cases} \Rightarrow X = \begin{pmatrix} 2 + x_2 & y_2 & 3 + z_2 \\ x_2 & y_2 & z_2 \end{pmatrix} \ \forall x_2, y_2, z_2 \in \mathbb{R}$$

2.

$$\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 1 \end{pmatrix} X = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ 0 & 1 & 2 & \dots & n-1 \\ 0 & 0 & 1 & \ddots & n-2 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 & 1 & 2 & 3 & \dots & n \\ 0 & 1 & 1 & \dots & 1 & 0 & 1 & 2 & \dots & n-1 \\ 0 & 0 & 1 & \dots & 1 & 0 & 0 & 1 & \ddots & n-2 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

$$A[0] = A[0] - A[1]$$

$$A[1] = A[1] - A[2]$$

$$\dots$$

$$A[n-1] = A[n-1] - A[n]$$

$$\begin{pmatrix} 1 & 0 & 0 & \dots & 0 & 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 0 & \dots & 0 & 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 0 & 0 & 0 & 1 & \ddots & 1 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 1 & 0 & 0 & \dots & 0 & 1 \end{pmatrix} \Rightarrow X = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

Ответ: $X = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \dots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 1 \end{pmatrix}$

3.3.1. $A = \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$ A^{-1}–?

$$\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 2 & | & 1 & 0 \\ 4 & 5 & | & 0 & 1 \end{pmatrix}$$

$$C[1] = C[1] - 4C[0]$$

$$\begin{pmatrix} 1 & 2 & | & 1 & 0 \\ 0 & -3 & | & -4 & 1 \end{pmatrix}$$

$$C[1] = C[1]/-3$$

$$\begin{pmatrix} 1 & 2 & | & 1 & 0 \\ 0 & 1 & | & \frac{4}{3} & -\frac{1}{3} \end{pmatrix}$$

$$C[0] = C[0] - 2 * C[1]$$

$$\begin{pmatrix} 1 & 0 & | & -\frac{5}{3} & \frac{2}{3} \\ 0 & 1 & | & \frac{4}{3} & -\frac{1}{3} \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} -\frac{5}{3} & \frac{2}{3} \\ \frac{4}{3} & -\frac{1}{3} \end{pmatrix}$$

Ответ: $A^{-1} = \begin{pmatrix} -\frac{5}{3} & \frac{2}{3} \\ \frac{4}{3} & -\frac{1}{3} \end{pmatrix}$

3.2. $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ A^{-1}–?

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 4 & 5 & 6 & | & 0 & 1 & 0 \\ 7 & 8 & 9 & | & 0 & 0 & 1 \end{pmatrix}$$

$$C[1] = C[1] - 4 * C[0]$$

$$\begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & -3 & -6 & | & -4 & 1 & 0 \\ 7 & 8 & 9 & | & 0 & 0 & 1 \end{pmatrix}$$

$$C[1] = C[1] / -3$$

$$\begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & \frac{4}{3} & -\frac{1}{3} & 0 \\ 7 & 8 & 9 & | & 0 & 0 & 1 \end{pmatrix}$$

$$C[2] = C[2] - 7 * C[0]$$

$$\begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & \frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & -6 & -12 & | & -7 & 0 & 1 \end{pmatrix}$$

$$C[2] = C[2] + 6 * C[1]$$

$$\begin{pmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & \frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & | & 1 & -2 & 1 \end{pmatrix}$$

$$0 = 1 \text{ - } \varnothing$$

Ответ: нет

3.3. $A = \begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix} \quad A^{-1} - ?$

$$\begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix} A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 2 & 7 & 3 & | & 1 & 0 & 0 \\ 3 & 9 & 4 & | & 0 & 1 & 0 \\ 1 & 5 & 3 & | & 0 & 0 & 1 \end{pmatrix}$$

$$C[0] = C[0]/2$$

$$\begin{pmatrix} 1 & \frac{7}{2} & \frac{3}{2} & | & \frac{1}{2} & 0 & 0 \\ 3 & 9 & 4 & | & 0 & 1 & 0 \\ 1 & 5 & 3 & | & 0 & 0 & 1 \end{pmatrix}$$

$$C[1] = C[1] - 3 * C[0]$$

$$\begin{pmatrix} 1 & \frac{7}{2} & \frac{3}{2} & | & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{3}{2} & -\frac{1}{2} & | & -\frac{3}{2} & 1 & 0 \\ 1 & 5 & 3 & | & 0 & 0 & 1 \end{pmatrix}$$

$$C[1] = C[1] * -2/3$$

$$\begin{pmatrix} 1 & \frac{7}{2} & \frac{3}{2} & | & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & | & 1 & -\frac{2}{3} & 0 \\ 1 & 5 & 3 & | & 0 & 0 & 1 \end{pmatrix}$$

$$C[2] = C[2] - C[0]$$

$$\begin{pmatrix} 1 & \frac{7}{2} & \frac{3}{2} & | & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & | & 1 & -\frac{2}{3} & 0 \\ 0 & \frac{3}{2} & \frac{3}{2} & | & -\frac{1}{2} & 0 & 1 \end{pmatrix}$$

$$C[2] = C[2] - C[1] * 3/2$$

$$\begin{pmatrix} 1 & \frac{7}{2} & \frac{3}{2} & | & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & | & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 1 & | & -2 & 1 & 1 \end{pmatrix}$$

$$C[0] = C[0] - C[1] * 7/2$$

$$\begin{pmatrix} 1 & 0 & \frac{1}{3} & | & -3 & \frac{7}{3} & 0 \\ 0 & 1 & \frac{1}{3} & | & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 1 & | & -2 & 1 & 1 \end{pmatrix}$$

$$C[0] = C[0] - C[2]/3$$

$$\begin{pmatrix} 1 & 0 & 0 & | & -\frac{7}{3} & 2 & -\frac{1}{3} \\ 0 & 1 & \frac{1}{3} & | & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 1 & | & -2 & 1 & 1 \end{pmatrix}$$

$$C[1] = C[1] - C[2]/3$$

$$\begin{pmatrix} 1 & 0 & 0 & | & -\frac{7}{3} & 2 & -\frac{1}{3} \\ 0 & 1 & 0 & | & \frac{5}{3} & -1 & -\frac{1}{3} \\ 0 & 0 & 1 & | & -2 & 1 & 1 \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} -\frac{7}{3} & 2 & -\frac{1}{3} \\ \frac{5}{3} & -1 & -\frac{1}{3} \\ -2 & 1 & 1 \end{pmatrix}$$

Ответ: $A^{-1} = \begin{pmatrix} -\frac{7}{3} & 2 & -\frac{1}{3} \\ \frac{5}{3} & -1 & -\frac{1}{3} \\ -2 & 1 & 1 \end{pmatrix}$

4.

$$C = (A|E) = \begin{pmatrix} 2 & 10 & 3 & | & 1 & 0 \\ 1 & 5 & 4 & | & 0 & 1 \end{pmatrix}$$

$$C[0], C[1] = C[1], C[0]$$

$$\begin{pmatrix} 1 & 5 & 4 & | & 0 & 1 \\ 2 & 10 & 3 & | & 1 & 0 \end{pmatrix}$$

$$C[1] = C[1] - 2 * C[0]$$

$$\begin{pmatrix} 1 & 5 & 4 & | & 0 & 1 \\ 0 & 0 & -5 & | & 1 & -2 \end{pmatrix}$$

$$C[1] = C[1]/-5$$

$$\begin{pmatrix} 1 & 5 & 4 & | & 0 & 1 \\ 0 & 0 & 1 & | & -\frac{1}{5} & \frac{2}{5} \end{pmatrix}$$

$$C[0] = C[0] - 4 * C[1]$$

$$(A'|U) = \begin{pmatrix} 1 & 5 & 0 & | & \frac{4}{5} & -\frac{3}{5} \\ 0 & 0 & 1 & | & -\frac{1}{5} & \frac{2}{5} \end{pmatrix} \Rightarrow U = \begin{pmatrix} \frac{4}{5} & -\frac{3}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{pmatrix}$$

$$UA = \begin{pmatrix} \frac{4}{5} & -\frac{3}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{pmatrix} \cdot \begin{pmatrix} 2 & 10 & 3 \\ 1 & 5 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix} = A' \text{ - верно}$$

Ответ: $U = \begin{pmatrix} \frac{4}{5} & -\frac{3}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{pmatrix}$

5.

$$A = \begin{pmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & 0 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & 0 \end{pmatrix}$$

$$A \cdot A^{-1} = E$$

$$(A|E) = C = \begin{pmatrix} 0 & 1 & 1 & \dots & 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 1 & \dots & 1 & 0 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 1 & 1 & \dots & 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

$$C[1, 2, 3 \dots, n] = C[1, 2, 3, \dots, n] - C[0]$$

$$\begin{pmatrix} 0 & 1 & 1 & \dots & 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & -1 & 0 & \dots & 0 & -1 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots & -1 & -1 & 0 & 0 & \dots & 1 \end{pmatrix}$$

$$C[0] = C[1, 2, 3, \dots, n] + C[0]$$

$$\begin{pmatrix} n-1 & 0 & 0 & \dots & 0 & 2-n & 1 & 1 & \dots & 1 \\ 1 & -1 & 0 & \dots & 0 & -1 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots & -1 & -1 & 0 & 0 & \dots & 1 \end{pmatrix}$$

$$C[0] = C[0]/(n-1)$$

$$\begin{pmatrix} 1 & 0 & 0 & \dots & 0 & \frac{2-n}{n-1} & \frac{1}{n-1} & \frac{1}{n-1} & \dots & \frac{1}{n-1} \\ 1 & -1 & 0 & \dots & 0 & -1 & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & 0 & 0 & \dots & -1 & -1 & 0 & 0 & \dots & 1 \end{pmatrix}$$

$$C[1,2,3,\dots,n] = C[1,2,3,\dots,n] - C[0]$$

$$\begin{pmatrix} 1 & 0 & 0 & \dots & 0 & \frac{2-n}{n-1} & \frac{1}{n-1} & \frac{1}{n-1} & \dots & \frac{1}{n-1} \\ 0 & -1 & 0 & \dots & 0 & -1-\frac{2-n}{n-1} & 1-\frac{1}{n-1} & -\frac{1}{n-1} & \dots & -\frac{1}{n-1} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -1 & -1-\frac{2-n}{n-1} & -\frac{1}{n-1} & -\frac{1}{n-1} & \dots & 1-\frac{1}{n-1} \end{pmatrix}$$

$$C[1,2,3,\dots,n] = -C[1,2,3,\dots,n]$$

$$\begin{pmatrix} 1 & 0 & 0 & \dots & 0 & \frac{2-n}{n-1} & \frac{1}{n-1} & \frac{1}{n-1} & \dots & \frac{1}{n-1} \\ 0 & 1 & 0 & \dots & 0 & 1+\frac{2-n}{n-1} & \frac{1}{n-1}-1 & \frac{1}{n-1} & \dots & \frac{1}{n-1} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & 1+\frac{2-n}{n-1} & \frac{1}{n-1} & \frac{1}{n-1} & \dots & \frac{1}{n-1}-1 \end{pmatrix} \Rightarrow$$

$$A^{-1} = \begin{pmatrix} \frac{2-n}{n-1} & \frac{1}{n-1} & \frac{1}{n-1} & \dots & \frac{1}{n-1} \\ 1+\frac{2-n}{n-1} & \frac{1}{n-1}-1 & \frac{1}{n-1} & \dots & \frac{1}{n-1} \\ \dots & \dots & \dots & \dots & \dots \\ 1+\frac{2-n}{n-1} & \frac{1}{n-1} & \frac{1}{n-1} & \dots & \frac{1}{n-1}-1 \end{pmatrix}$$