ИДЗ (Линейная алгебра)

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$$A = \begin{pmatrix} -1 & -4 & -1 \\ -1 & 0 & 6 \end{pmatrix},\ B = \begin{pmatrix} 7 & -1 & 6 \\ 3 & 0 & 1 \end{pmatrix},\ C = \begin{pmatrix} 6 & 1 \\ 0 & 6 \end{pmatrix},\ D = \begin{pmatrix} 13 & 13 \\ 13 & 13 \end{pmatrix}$$

$$tr(B^TB)DAA^T + tr((3BA^T + 4AB^T)D + D(2BA^T - 3AB^T))(B+A)(B^T - A^T) +$$
$$+9C^2 - 6CD + D^2$$

1)

$$tr(B^TB) = \begin{pmatrix} 7 & 3 \\ -1 & 0 \\ 6 & 1 \end{pmatrix} \cdot \begin{pmatrix} 7 & -1 & 6 \\ 3 & 0 & 1 \end{pmatrix} = tr(\begin{pmatrix} 58 & -7 & 45 \\ -7 & 1 & -6 \\ 45 & -6 & 37 \end{pmatrix}) =$$
$$= 58 + 1 + 37 = 96$$

$$DA = \begin{pmatrix} 13 & 13 \\ 13 & 13 \end{pmatrix} \cdot \begin{pmatrix} -1 & -4 & -1 \\ -1 & 0 & 6 \end{pmatrix} = \begin{pmatrix} -26 & -52 & 65 \\ -26 & -52 & 65 \end{pmatrix} \Rightarrow$$

$$\Rightarrow DAA^T = \begin{pmatrix} -26 & -52 & 65 \\ -26 & -52 & 65 \end{pmatrix} \cdot \begin{pmatrix} -1 & -1 \\ -4 & 0 \\ -1 & 6 \end{pmatrix} = \begin{pmatrix} 169 & 416 \\ 169 & 416 \end{pmatrix}$$

$$tr(B^TB)DAA^T = 96 \begin{pmatrix} 169 & 416 \\ 169 & 416 \end{pmatrix} = \begin{pmatrix} 16224 & 39936 \\ 16224 & 39936 \end{pmatrix}$$

2)

$$D^T = D$$

$$tr((3BA^T + 4AB^T)D + D(2BA^T - 3AB^T)) =$$

$$= tr((3BA^T + 4AB^T)D) + tr(D(2BA^T - 3AB^T)) =$$

$$= tr((3BA^T + 4AB^T)D) + tr((2AB^T - 3BA^T)D^T) =$$

$$= tr((3BA^T + 4AB^T)D) + tr((2AB^T - 3BA^T)D) =$$

$$= tr((3BA^T + 4AB^T)D + (2AB^T - 3BA^T)D) =$$

$$= tr(6AB^TD)$$

$$AB^T = \begin{pmatrix} -1 & -4 & -1 \\ -1 & 0 & 6 \end{pmatrix} \cdot \begin{pmatrix} 7 & 3 \\ -1 & 0 \\ 6 & 1 \end{pmatrix} = \begin{pmatrix} -9 & -4 \\ 29 & 3 \end{pmatrix}$$

$$AB^TD = \begin{pmatrix} -9 & -4 \\ 29 & 3 \end{pmatrix} \cdot \begin{pmatrix} 13 & 13 \\ 13 & 13 \end{pmatrix} = \begin{pmatrix} -169 & -169 \\ 416 & 416 \end{pmatrix}$$

$$6AB^TD = 6\begin{pmatrix} -169 & -169 \\ 416 & 416 \end{pmatrix} = \begin{pmatrix} -1014 & -1014 \\ 2496 & 2496 \end{pmatrix}$$

$$tr(6AB^TD) = -1014 + 2496 = 1482$$

3)

$$(B + A)(B^T - A^T) = (B + A)(B - A)^T$$

$$B + A = \begin{pmatrix} 7 & -1 & 6 \\ 3 & 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & -4 & -1 \\ -1 & 0 & 6 \end{pmatrix} = \begin{pmatrix} 6 & -5 & 5 \\ 2 & 0 & 7 \end{pmatrix}$$

$$B - A = \begin{pmatrix} 7 & -1 & 6 \\ 3 & 0 & 1 \end{pmatrix} - \begin{pmatrix} -1 & -4 & -1 \\ -1 & 0 & 6 \end{pmatrix} = \begin{pmatrix} 8 & 3 & 7 \\ 4 & 0 & -5 \end{pmatrix}$$

$$(B + A)(B - A)^T = \begin{pmatrix} 6 & -5 & 5 \\ 2 & 0 & 7 \end{pmatrix} \cdot \begin{pmatrix} 8 & 4 \\ 3 & 0 \\ 7 & -5 \end{pmatrix} = \begin{pmatrix} 68 & -1 \\ 65 & -27 \end{pmatrix}$$

$$tr((3BA^T + 4AB^T)D + D(2BA^T - 3AB^T))(B + A)(B^T - A^T) =$$

$$= 1482 \begin{pmatrix} 68 & -1 \\ 65 & -27 \end{pmatrix} = \begin{pmatrix} 100776 & -1482 \\ 96330 & -40014 \end{pmatrix}$$

4)

$$C^2 = \begin{pmatrix} 6 & 1 \\ 0 & 6 \end{pmatrix}^2 = \begin{pmatrix} 36 & 12 \\ 0 & 36 \end{pmatrix}$$

$$9C^2 = 9 \begin{pmatrix} 36 & 12 \\ 0 & 36 \end{pmatrix} = \begin{pmatrix} 324 & 108 \\ 0 & 324 \end{pmatrix}$$

$$CD = \begin{pmatrix} 6 & 1 \\ 0 & 6 \end{pmatrix} \cdot \begin{pmatrix} 13 & 13 \\ 13 & 13 \end{pmatrix} = \begin{pmatrix} 91 & 91 \\ 78 & 78 \end{pmatrix}$$

$$-6CD = -6 \begin{pmatrix} 91 & 91 \\ 78 & 78 \end{pmatrix} = \begin{pmatrix} -546 & -546 \\ -468 & -468 \end{pmatrix}$$

$$D^2 = \begin{pmatrix} 13 & 13 \\ 13 & 13 \end{pmatrix}^2 = \begin{pmatrix} 338 & 338 \\ 338 & 338 \end{pmatrix}$$

$$9C^2 - 6CD + D^2 = \begin{pmatrix} 324 & 108 \\ 0 & 324 \end{pmatrix} + \begin{pmatrix} -546 & -546 \\ -468 & -468 \end{pmatrix} + \begin{pmatrix} 338 & 338 \\ 338 & 338 \end{pmatrix} =$$

$$= \begin{pmatrix} 116 & -100 \\ -130 & 194 \end{pmatrix}$$

5)

$$tr(B^TB)DAA^T + tr((3BA^T + 4AB^T)D + D(2BA^T - 3AB^T))(B+A)(B^T - A^T) +$$

$$+9C^2 - 6CD + D^2 = \begin{pmatrix} 16224 & 39936 \\ 16224 & 39936 \end{pmatrix} + \begin{pmatrix} 100776 & -1482 \\ 96330 & -40014 \end{pmatrix} + \begin{pmatrix} 116 & -100 \\ -130 & 194 \end{pmatrix} =$$

$$= \begin{pmatrix} 117116 & 38354 \\ 112424 & 116 \end{pmatrix}$$

Ответ: $\begin{pmatrix} 117116 & 38354 \\ 112424 & 116 \end{pmatrix}$

2.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{pmatrix}, \; B = \begin{pmatrix} 0 & b_{12} & b_{13} & b_{14} \\ -b_{12} & 0 & b_{23} & b_{24} \\ -b_{13} & -b_{23} & 0 & b_{34} \\ -b_{14} & -b_{24} & -b_{34} & 0 \end{pmatrix}$$

$$A + B = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{pmatrix} + \begin{pmatrix} 0 & b_{12} & b_{13} & b_{14} \\ -b_{12} & 0 & b_{23} & b_{24} \\ -b_{13} & -b_{23} & 0 & b_{34} \\ -b_{14} & -b_{24} & -b_{34} & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} -18 & 42 & 28 & 8 \\ 20 & 4 & 22 & 38 \\ -12 & -18 & -22 & 6 \\ 0 & 0 & 52 & 38 \end{pmatrix}$$

$$\begin{cases} a_{11} = -18 \\ a_{12} = 42 - b_{12} = b_{12} + 20 \\ a_{13} = 28 - b_{13} = b_{13} - 12 \\ a_{14} = 8 - b_{14} = b_{14} \\ a_{22} = 4 \\ a_{23} = 22 - b_{23} = b_{23} - 18 \\ a_{24} = 38 - b_{24} = b_{24} \\ a_{33} = -22 \\ a_{34} = 6 - b_{34} = b_{34} + 52 \\ a_{44} = 38 \end{cases}$$

$$\begin{cases} a_{11} = -18 \\ a_{12} = 31, b_{12} = 11 \\ a_{13} = 8, b_{13} = 20 \\ a_{14} = 4, b_{14} = 4 \\ a_{22} = 4 \\ a_{23} = 2, b_{23} = 20 \\ a_{24} = 19, b_{24} = 19 \\ a_{33} = -22 \\ a_{34} = 29, b_{34} = -23 \\ a_{44} = 38 \end{cases}$$

$$A = \begin{pmatrix} -18 & 31 & 8 & 4 \\ 31 & 4 & 2 & 19 \\ 8 & 2 & -22 & 29 \\ 4 & 19 & 29 & 38 \end{pmatrix}, \ B = \begin{pmatrix} 0 & 11 & 20 & 4 \\ -11 & 0 & 20 & 19 \\ -20 & -20 & 0 & -23 \\ -4 & -19 & 23 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} -18 & 31 & 8 & 4 \\ 31 & 4 & 2 & 19 \\ 8 & 2 & -22 & 29 \\ 4 & 19 & 29 & 38 \end{pmatrix} \cdot \begin{pmatrix} 0 & 11 & 20 & 4 \\ -11 & 0 & 20 & 19 \\ -20 & -20 & 0 & -23 \\ -4 & -19 & 23 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} -517 & -434 & 352 & 333 \\ -160 & -60 & 1137 & 154 \\ 302 & -23 & 867 & 576 \\ -941 & -1258 & 1334 & -290 \end{pmatrix}$$

Ответ: $\begin{pmatrix} -517 & -434 & 352 & 333 \\ -160 & -60 & 1137 & 154 \\ 302 & -23 & 867 & 576 \\ -941 & -1258 & 1334 & -290 \end{pmatrix}$

3.

$$C = \begin{pmatrix} 1 & -3 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \quad J = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 3 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$DC = \begin{pmatrix} 1 & 3 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -3 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = E$$

$$A^{2021} = CJDCJDCJD \ldots CJD = CJEJEJE \ldots EJD = CJ^{2021}D$$

$$S = E + C(J + \cdots + J^{2021})D$$

$$a_1 = J^1 = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

$$a_2 = J^1 + J^2 = \begin{pmatrix} 0 & -1 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$a_3 = J^1 + J^2 + J^3 = \begin{pmatrix} -1 & 2 & -2 \\ 0 & -1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\ldots$$

Доказать через мат. индукцию:

$$a_k = \begin{pmatrix} -1 & \frac{k+1}{2} & -\frac{k-1}{2}(\frac{k-1}{2}+1) \\ 0 & -1 & \frac{k+1}{2} \\ 0 & 0 & -1 \end{pmatrix} \text{ для всех нечётных } k \Rightarrow$$

$$\Rightarrow a_{2021} = J^1 + \dots + J^{2021} = \begin{pmatrix} -1 & 1011 & -1021110 \\ 0 & -1 & 1011 \\ 0 & 0 & -1 \end{pmatrix} \Rightarrow$$

$$C \cdot a_{2021} \cdot D =$$

$$= \begin{pmatrix} 1 & -3 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1011 & -1021110 \\ 0 & -1 & 1011 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 & 1014 & -1024138 \\ 0 & -1 & 1009 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -1 & 1011 & -1026165 \\ 0 & -1 & 1011 \\ 0 & 0 & -1 \end{pmatrix} \Rightarrow$$

$$\Rightarrow S = E + \begin{pmatrix} -1 & 1011 & -1026165 \\ 0 & -1 & 1011 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1011 & -1026165 \\ 0 & 0 & 1011 \\ 0 & 0 & 0 \end{pmatrix}$$

Ответ: $\begin{pmatrix} 0 & 1011 & -1026165 \\ 0 & 0 & 1011 \\ 0 & 0 & 0 \end{pmatrix}$

4.

$$S = \begin{pmatrix} -15 & -30 & 30 \\ -9 & -18 & 18 \\ -18 & -36 & 36 \end{pmatrix}$$

$$u = \begin{pmatrix} x_1, y_1, z_1 \end{pmatrix}, \ v^T = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

$$uv^T = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \cdot \begin{pmatrix} x_2 & y_2 & z_2 \end{pmatrix} = \begin{pmatrix} x_1x_2 & x_1y_2 & x_1z_2 \\ y_1x_2 & y_1y_2 & y_1z_2 \\ z_1x_2 & z_1y_2 & z_1z_2 \end{pmatrix}$$

$$\begin{cases} x_1x_2 = -15 \\ x_1y_2 = -30 \\ x_1z_2 = 30 \\ y_1x_2 = -9 \\ y_1y_2 = -18 \\ y_1z_2 = 18 \\ z_1x_2 = -18 \\ z_1y_2 = -36 \\ z_1z_2 = 36 \end{cases} \Rightarrow u = \begin{pmatrix} 15 \\ 9 \\ 18 \end{pmatrix}, \ v = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} \ \text{(частное решение)}$$

$$S = \begin{pmatrix} 15 \\ 9 \\ 18 \end{pmatrix} \cdot \begin{pmatrix} -1 & -2 & 2 \end{pmatrix} = uv^T$$

$$v^T u = \begin{pmatrix} -1 & -2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 15 \\ 9 \\ 18 \end{pmatrix} = \begin{pmatrix} 3 \end{pmatrix}$$

$$S^{10} = u(3)(3)(3)(3)(3)(3)(3)(3)(3)v^T = u(19683)v^T =$$

$$= 19683 \begin{pmatrix} 15 \\ 9 \\ 18 \end{pmatrix} \cdot \begin{pmatrix} -1 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 295245 \\ 177147 \\ 354294 \end{pmatrix} \cdot \begin{pmatrix} -1 & -2 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} -295245 & -590490 & 590490 \\ -177147 & -354294 & 354294 \\ -354294 & -708588 & 708588 \end{pmatrix}$$

Ответ: $\begin{pmatrix} -295245 & -590490 & 590490 \\ -177147 & -354294 & 354294 \\ -354294 & -708588 & 708588 \end{pmatrix}$

5. (a)

$$\begin{cases} 4x_1 - 7x_2 - 10x_3 + x_4 = -8, \\ 8x_1 - 4x_2 + 12x_4 = -6, \\ 6x_1 - 5x_2 - 4x_3 + 7x_4 = 1, \\ -5x_1 + x_2 - 3x_3 - 9x_4 = 9. \end{cases}$$

Запишем в виде расширенной матрицы СЛУ:

$$A = \left(\begin{array}{cccc|c} 4 & -7 & -10 & 1 & -8 \\ 8 & -4 & 0 & 12 & -6 \\ 6 & -5 & -4 & 7 & 1 \\ -5 & 1 & -3 & -9 & 9 \end{array}\right)$$

$$A[0] = A[0]/4$$

$$\left(\begin{array}{cccc|c} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & -2 \\ 8 & -4 & 0 & 12 & -6 \\ 6 & -5 & -4 & 7 & 1 \\ -5 & 1 & -3 & -9 & 9 \end{array}\right)$$

$$A[1] = A[1] - 8 * A[0]$$

$$\left(\begin{array}{cccc|c} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & -2 \\ 0 & 10 & 20 & 10 & 10 \\ 6 & -5 & -4 & 7 & 1 \\ -5 & 1 & -3 & -9 & 9 \end{array}\right)$$

$$A[1] = A[1]/10$$

$$\left(\begin{array}{cccc|c} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & -2 \\ 0 & 1 & 2 & 1 & 1 \\ 6 & -5 & -4 & 7 & 1 \\ -5 & 1 & -3 & -9 & 9 \end{array}\right)$$

$$A[2] = A[2] - 6 * A[0]$$

$$\begin{pmatrix} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & | & -2 \\ 0 & 1 & 2 & 1 & | & 1 \\ 0 & \frac{11}{2} & 11 & \frac{11}{2} & | & 13 \\ -5 & 1 & -3 & -9 & | & 9 \end{pmatrix}$$

$$A[2] = A[2] * 2/11$$

$$\begin{pmatrix} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & | & -2 \\ 0 & 1 & 2 & 1 & | & 1 \\ 0 & 1 & 2 & 1 & | & \frac{26}{11} \\ -5 & 1 & -3 & -9 & | & 9 \end{pmatrix}$$

$$A[2] = A[2] - A[1]$$

$$\begin{pmatrix} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & | & -2 \\ 0 & 1 & 2 & 1 & | & 1 \\ 0 & 0 & 0 & 0 & | & \frac{15}{11} \\ -5 & 1 & -3 & -9 & | & 9 \end{pmatrix}$$

$$0 = \frac{15}{11} \text{ - } \varnothing \Rightarrow \text{ решений нет}$$

Ответ: $\varnothing$

(б)

$$\begin{cases} 4x_1 - 7x_2 - 10x_3 + x_4 & = -51, \\ 8x_1 - 4x_2 + 12x_4 & = -52, \\ 6x_1 - 5x_2 - 4x_3 + 7x_4 & = -49, \\ -5x_1 + x_2 - 3x_3 - 9x_4 & = 25. \end{cases}$$

Запишем в виде расширенной матрицы СЛУ:

$$\left(\begin{array}{cccc|c} 4 & -7 & -10 & 1 & -51 \\ 8 & -4 & 0 & 12 & -52 \\ 6 & -5 & -4 & 7 & -49 \\ -5 & 1 & -3 & -9 & 25 \end{array}\right)$$

$$A[0] = A[0]/4$$

$$\left(\begin{array}{cccc|c} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & -\frac{51}{4} \\ 8 & -4 & 0 & 12 & -52 \\ 6 & -5 & -4 & 7 & -49 \\ -5 & 1 & -3 & -9 & 25 \end{array}\right)$$

$$A[1] = A[1] - 8 * A[0]$$

$$\left(\begin{array}{cccc|c} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & -\frac{51}{4} \\ 0 & 10 & 20 & 10 & 50 \\ 6 & -5 & -4 & 7 & -49 \\ -5 & 1 & -3 & -9 & 25 \end{array}\right)$$

$$A[1] = A[1]/10$$

$$\left(\begin{array}{cccc|c} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & -\frac{51}{4} \\ 0 & 1 & 2 & 1 & 5 \\ 6 & -5 & -4 & 7 & -49 \\ -5 & 1 & -3 & -9 & 25 \end{array}\right)$$

$$A[2] = A[2] - 6 * A[0]$$

$$\begin{pmatrix} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & | & -\frac{51}{4} \\ 0 & 1 & 2 & 1 & | & 5 \\ 0 & \frac{11}{2} & 11 & \frac{11}{2} & | & \frac{55}{2} \\ -5 & 1 & -3 & -9 & | & 25 \end{pmatrix}$$

$$A[2] = A[2] - A[1] * 11/2$$

$$\begin{pmatrix} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & | & -\frac{51}{4} \\ 0 & 1 & 2 & 1 & | & 5 \\ 0 & 0 & 0 & 0 & | & 0 \\ -5 & 1 & -3 & -9 & | & 25 \end{pmatrix}$$

$$A[2], A[3] = A[3], A[2]$$

$$A[2] = A[2] + A[0] * 5$$

$$\begin{pmatrix} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & | & -\frac{51}{4} \\ 0 & 1 & 2 & 1 & | & 5 \\ 0 & -\frac{31}{4} & -\frac{31}{2} & -\frac{31}{4} & | & -\frac{155}{4} \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$A[2] = A[2] + A[1] * 31/4$$

$$\begin{pmatrix} 1 & -\frac{7}{4} & -\frac{5}{2} & \frac{1}{4} & | & -\frac{51}{4} \\ 0 & 1 & 2 & 1 & | & 5 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$A[0] = A[0] + A[1] * 7/4$$

$$\begin{pmatrix} 1 & 0 & 1 & 2 & | & -4 \\ 0 & 1 & 2 & 1 & | & 5 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$\begin{cases} x_1 = -4 - x_3 - 2x_4 \\ x_2 = 5 - 2x_3 - x_4 \end{cases} \quad \forall x_3, x_4 \in \mathbb{R}$$

$$x_3 = 1, x_4 = 1 \Rightarrow x_1 = -7, x_2 = 2$$

Ответ: $\begin{cases} x_1 = -4 - x_3 - 2x_4 \\ x_2 = 5 - 2x_3 - x_4 \end{cases} \quad \forall x_3, x_4 \in \mathbb{R}$ - общее решение,
$x_1 = -7, x_2 = 2, x_3 = 1, x_4 = 1$ - частное