ИДЗ №2 (Линейная алгебра)

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1. $\begin{cases} ax - 4y = -4 \\ 6x + by - 9z = 1 \\ -5x - z = 1 \end{cases}$

Запишем в виде матрицы СЛУ: $\begin{pmatrix} a & -4 & 0 & | & -4 \\ 6 & b & -9 & | & 1 \\ -5 & 0 & -1 & | & 1 \end{pmatrix}$

$$A[0] = A[0]/a \Rightarrow \begin{pmatrix} 1 & \frac{-4}{a} & 0 & | & \frac{-4}{a} \\ 6 & b & -9 & | & 1 \\ -5 & 0 & -1 & | & 1 \end{pmatrix}$$

$$A[1] = A[1] - 6 * A[0] \Rightarrow \begin{pmatrix} 1 & \frac{-4}{a} & 0 & | & \frac{-4}{a} \\ 0 & b + \frac{24}{a} & -9 & | & \frac{a+24}{a} \\ -5 & 0 & -1 & | & 1 \end{pmatrix}$$

$$A[2] = A[2] + 5 * A[0] \Rightarrow \begin{pmatrix} 1 & \frac{-4}{a} & 0 & | & \frac{-4}{a} \\ 0 & b + \frac{24}{a} & -9 & | & \frac{a+24}{a} \\ 0 & -\frac{20}{a} & -1 & | & \frac{a-20}{a} \end{pmatrix}$$

$$A[1] = A[1]/(b + 24/a) \Rightarrow \begin{pmatrix} 1 & \frac{-4}{a} & 0 & | & \frac{-4}{a} \\ 0 & 1 & -\frac{9a}{ab+24} & | & \frac{a+24}{ab+24} \\ 0 & -\frac{20}{a} & -1 & | & \frac{a-20}{a} \end{pmatrix}$$

$$A[2] = A[2] - A[1] * (-20/a) \Rightarrow \begin{pmatrix} 1 & \frac{-4}{a} & 0 & | & \frac{-4}{a} \\ 0 & 1 & -\frac{9a}{ab+24} & | & \frac{a+24}{ab+24} \\ 0 & 0 & \frac{-ab-204}{ab+24} & | & \frac{ab-20b+44}{ab+24} \end{pmatrix}$$

$$A[2] = A[2]/((-a*b-204)/(a*b+24)) \Rightarrow \begin{pmatrix} 1 & \frac{-4}{a} & 0 & | & \frac{-4}{a} \\ 0 & 1 & -\frac{9a}{ab+24} & | & \frac{a+24}{ab+24} \\ 0 & 0 & 1 & | & \frac{-ab+20b-44}{ab+204} \end{pmatrix}$$

$$A[0] = A[0] - A[1] * (-4/a) \Rightarrow \begin{pmatrix} 1 & 0 & -\frac{36}{ab+24} & | & \frac{4(1-b)}{ab+24} \\ 0 & 1 & -\frac{9a}{ab+24} & | & \frac{a+24}{ab+24} \\ 0 & 0 & 1 & | & \frac{-ab+20b-44}{ab+204} \end{pmatrix}$$

$$A[0] = A[0]-A[2]*(-36/(a*b+24)) \Rightarrow \begin{pmatrix} 1 & 0 & 0 & | & \frac{4(-b-8)}{ab+204} \\ 0 & 1 & -\frac{9a}{ab+24} & | & \frac{a+24}{ab+24} \\ 0 & 0 & 1 & | & \frac{-ab+20b-44}{ab+204} \end{pmatrix}$$

$$A[1] = A[1]-A[2]*(-9*a/(a*b+24)) \Rightarrow \begin{pmatrix} 1 & 0 & 0 & | & \frac{4(-b-8)}{ab+204} \\ 0 & 1 & 0 & | & \frac{4(51-2a)}{ab+204} \\ 0 & 0 & 1 & | & \frac{-ab+20b-44}{ab+204} \end{pmatrix}$$

Система имеет 0 корней при:

$$ab + 204 = 0 \Rightarrow ab = -204$$

$x = y$ при:

$$\frac{4(-b-8)}{ab+204} = \frac{4(51-2a)}{ab+204} \Rightarrow a = \frac{b+59}{2}$$

$x = z$ при:

$$\frac{4(-b-8)}{ab+204} = \frac{-ab+20b-44}{ab+204} \Rightarrow a = 24 - \frac{12}{b}$$

$y = z$ при:

$$\frac{4(51-2a)}{ab+204} = \frac{-ab+20b-44}{ab+204} \Rightarrow a = \frac{20b-248}{b-8}$$

$x = y = z$ при:

$$\frac{b+59}{2} = 24 - \frac{12}{b} \Rightarrow \left[\begin{array}{l} b = -8 \Rightarrow a = \frac{51}{2} \\ b = -3 \Rightarrow a = 28 \end{array}\right.$$

Ответ:

- 0 решений при: $ab + 204 = 0$
- 1 решение при: $\left[\begin{array}{l} b = -8, \ a = \frac{51}{2} \\ b = -3, \ a = 28 \end{array}\right.$
- 2 решения при: $\left[\begin{array}{l} a = \frac{b+59}{2} \\ a = 24 - \frac{12}{b} \\ a = \frac{20b-248}{b-8} \end{array}\right.$

2. $A = \begin{pmatrix} 1 & 0 & -6 \\ 2 & -4 & 3 \\ 0 & 0 & 0 \end{pmatrix}$, $X = \begin{pmatrix} a & 0 & b \\ c & d & e \\ 0 & 0 & f \end{pmatrix}$

$$AX = \begin{pmatrix} a & 0 & b-6f \\ 2a-4c & -4d & 2b-4e+3f \\ 0 & 0 & 0 \end{pmatrix}, \; XA = \begin{pmatrix} a & 0 & -6a \\ c+2d & -4d & -6c+3d \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} a = a \\ b - 6f = -6a \Rightarrow b = 6f - 6a \\ 2a - 4c = c + 2d \Rightarrow a = \frac{5}{2}c + d \\ -4d = -4d \\ 2b - 4e + 3f = -6c + 3d \Rightarrow e = \frac{3}{2}c - \frac{3}{4}d + \frac{1}{2}b + \frac{3}{4}f \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} b = 6f - 6(\frac{5}{2}c + d) = 6f - 15c - 6d \\ a = \frac{5}{2}c + d \\ e = \frac{3}{2}c - \frac{3}{4}d + \frac{1}{2}(6f - 15c - 6d) + \frac{3}{4}f = -6c - \frac{15}{4}d + \frac{15}{4}f \end{cases}$$

Ответ: $\begin{cases} b = 6f - 15c - 6d \\ a = \frac{5}{2}c + d \\ e = -6c - \frac{15}{4}d + \frac{15}{4}f \end{cases} \quad f, c, d \in \mathbb{R}$

3. $A = \begin{pmatrix} 26 & 8 & -5 & 1 \\ -15 & -2 & 3 & 0 \\ -5 & -1 & 1 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 44 & -44 & 32 \\ -33 & 32 & -14 \\ -10 & 10 & -5 \end{pmatrix}$

$$AX = B \Rightarrow \left(\begin{array}{cccc|ccc} 26 & 8 & -5 & 1 & 44 & -44 & 32 \\ -15 & -2 & 3 & 0 & -33 & 32 & -14 \\ -5 & -1 & 1 & 0 & -10 & 10 & -5 \end{array}\right)$$

$$A[0] = A[0]/26 \Rightarrow \left(\begin{array}{cccc|ccc} 1 & \frac{4}{13} & -\frac{5}{26} & \frac{1}{26} & \frac{22}{13} & -\frac{22}{13} & \frac{16}{13} \\ -15 & -2 & 3 & 0 & -33 & 32 & -14 \\ -5 & -1 & 1 & 0 & -10 & 10 & -5 \end{array}\right)$$

$$\begin{cases} A[1] = A[1] + 15 * A[0] \\ A[2] = A[2] + 5 * A[0] \end{cases} \Rightarrow \begin{pmatrix} 1 & \frac{4}{13} & -\frac{5}{26} & \frac{1}{26} & \frac{22}{13} & -\frac{22}{13} & \frac{16}{13} \\ 0 & \frac{34}{13} & \frac{3}{26} & \frac{15}{26} & -\frac{99}{13} & \frac{86}{13} & \frac{58}{13} \\ 0 & \frac{7}{13} & \frac{1}{26} & \frac{5}{26} & -\frac{20}{13} & \frac{20}{13} & \frac{15}{13} \end{pmatrix}$$

$$A[1] = A[1] * 13/34 \Rightarrow \begin{pmatrix} 1 & \frac{4}{13} & -\frac{5}{26} & \frac{1}{26} & \frac{22}{13} & -\frac{22}{13} & \frac{16}{13} \\ 0 & 1 & \frac{3}{68} & \frac{15}{68} & -\frac{99}{34} & \frac{43}{17} & \frac{29}{17} \\ 0 & \frac{7}{13} & \frac{1}{26} & \frac{5}{26} & -\frac{20}{13} & \frac{20}{13} & \frac{15}{13} \end{pmatrix}$$

$$A[2] = A[2] - A[1] * 7/13 \Rightarrow \begin{pmatrix} 1 & \frac{4}{13} & -\frac{5}{26} & \frac{1}{26} & \frac{22}{13} & -\frac{22}{13} & \frac{16}{13} \\ 0 & 1 & \frac{3}{68} & \frac{15}{68} & -\frac{99}{34} & \frac{43}{17} & \frac{29}{17} \\ 0 & 0 & \frac{1}{68} & \frac{5}{68} & \frac{1}{34} & \frac{3}{17} & \frac{4}{17} \end{pmatrix}$$

$$A[2] = A[2] * 68 \Rightarrow \begin{pmatrix} 1 & \frac{4}{13} & -\frac{5}{26} & \frac{1}{26} & \frac{22}{13} & -\frac{22}{13} & \frac{16}{13} \\ 0 & 1 & \frac{3}{68} & \frac{15}{68} & -\frac{99}{34} & \frac{43}{17} & \frac{29}{17} \\ 0 & 0 & 1 & 5 & 2 & 12 & 16 \end{pmatrix}$$

$$A[0] = A[0] - A[1] * 4/13 \Rightarrow \begin{pmatrix} 1 & 0 & -\frac{7}{34} & -\frac{1}{34} & \frac{44}{17} & -\frac{42}{17} & \frac{12}{17} \\ 0 & 1 & \frac{3}{68} & \frac{15}{68} & -\frac{99}{34} & \frac{43}{17} & \frac{29}{17} \\ 0 & 0 & 1 & 5 & 2 & 12 & 16 \end{pmatrix}$$

$$A[0] = A[0] - A[2] * -7/34 \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 & 3 & 0 & 4 \\ 0 & 1 & \frac{3}{68} & \frac{15}{68} & -\frac{99}{34} & \frac{43}{17} & \frac{29}{17} \\ 0 & 0 & 1 & 5 & 2 & 12 & 16 \end{pmatrix}$$

$$A[1] = A[1] - A[2]*3/68 \Rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 & | & 3 & 0 & 4 \\ 0 & 1 & 0 & 0 & | & -3 & 2 & 1 \\ 0 & 0 & 1 & 5 & | & 2 & 12 & 16 \end{pmatrix}$$

$$\begin{cases} x_1 = 3 - x_4 \\ x_2 = -3 \\ x_3 = 2 - 5x_4 \end{cases} \quad \begin{cases} y_1 = -y_4 \\ y_2 = 2 \\ y_3 = 12 - 5y_4 \end{cases} \quad \begin{cases} z_1 = 4 - z_4 \\ z_2 = 1 \\ z_3 = 16 - 5z_4 \end{cases}$$

Ответ: $\begin{pmatrix} 3 - x_4 & -y_4 & 4 - z_4 \\ -3 & 2 & 1 \\ 2 - 5x_4 & 12 - 5y_4 & 16 - 5z_4 \\ x_4 & y_4 & z_4 \end{pmatrix}$

4. $A = \begin{pmatrix} 4 & 2 & 1 & -4 & -7 \\ 1 & 1 & 1 & -3 & -2 \\ -2 & -1 & 1 & -1 & 5 \\ 3 & 2 & -1 & 0 & -8 \end{pmatrix}$

$$(A|E) \to (PA|P) = (A'|P)$$

$$(A|E) = \begin{pmatrix} 4 & 2 & 1 & -4 & -7 & | & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & -3 & -2 & | & 0 & 1 & 0 & 0 \\ -2 & -1 & 1 & -1 & 5 & | & 0 & 0 & 1 & 0 \\ 3 & 2 & -1 & 0 & -8 & | & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A[0] = A[0]/4 \Rightarrow \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{4} & -1 & -\frac{7}{4} & | & \frac{1}{4} & 0 & 0 & 0 \\ 1 & 1 & 1 & -3 & -2 & | & 0 & 1 & 0 & 0 \\ -2 & -1 & 1 & -1 & 5 & | & 0 & 0 & 1 & 0 \\ 3 & 2 & -1 & 0 & -8 & | & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A[1] = A[1] - A[0] \Rightarrow \left(\begin{array}{ccccc|cccc} 1 & \frac{1}{2} & \frac{1}{4} & -1 & -\frac{7}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{4} & -2 & -\frac{1}{4} & -\frac{1}{4} & 1 & 0 & 0 \\ -2 & -1 & 1 & -1 & 5 & 0 & 0 & 1 & 0 \\ 3 & 2 & -1 & 0 & -8 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$A[2] = A[2] + 2 * A[0] \Rightarrow \left(\begin{array}{ccccc|cccc} 1 & \frac{1}{2} & \frac{1}{4} & -1 & -\frac{7}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{4} & -2 & -\frac{1}{4} & -\frac{1}{4} & 1 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & -3 & \frac{3}{2} & \frac{1}{2} & 0 & 1 & 0 \\ 3 & 2 & -1 & 0 & -8 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$A[3] = A[3] - 3 * A[0] \Rightarrow \left(\begin{array}{ccccc|cccc} 1 & \frac{1}{2} & \frac{1}{4} & -1 & -\frac{7}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{4} & -2 & -\frac{1}{4} & -\frac{1}{4} & 1 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & -3 & \frac{3}{2} & \frac{1}{2} & 0 & 1 & 0 \\ 0 & \frac{1}{2} & -\frac{7}{4} & 3 & -\frac{11}{4} & -\frac{3}{4} & 0 & 0 & 1 \end{array}\right)$$

$$A[3] = A[3] - A[1] \Rightarrow \left(\begin{array}{ccccc|cccc} 1 & \frac{1}{2} & \frac{1}{4} & -1 & -\frac{7}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{3}{4} & -2 & -\frac{1}{4} & -\frac{1}{4} & 1 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & -3 & \frac{3}{2} & \frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & -\frac{5}{2} & 5 & -\frac{5}{2} & -\frac{1}{2} & -1 & 0 & 1 \end{array}\right)$$

$$A[1] = A[1] * 2 \Rightarrow \left(\begin{array}{ccccc|cccc} 1 & \frac{1}{2} & \frac{1}{4} & -1 & -\frac{7}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 1 & \frac{3}{2} & -4 & -\frac{1}{2} & -\frac{1}{2} & 2 & 0 & 0 \\ 0 & 0 & \frac{3}{2} & -3 & \frac{3}{2} & \frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & -\frac{5}{2} & 5 & -\frac{5}{2} & -\frac{1}{2} & -1 & 0 & 1 \end{array}\right)$$

$$A[2] = A[2] * 2/3 \Rightarrow \left(\begin{array}{ccccc|cccc} 1 & \frac{1}{2} & \frac{1}{4} & -1 & -\frac{7}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 1 & \frac{3}{2} & -4 & -\frac{1}{2} & -\frac{1}{2} & 2 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & -\frac{5}{2} & 5 & -\frac{5}{2} & -\frac{1}{2} & -1 & 0 & 1 \end{array}\right)$$

$$A[3] = A[3] + A[2] * 5/2 \Rightarrow \left(\begin{array}{ccccc|cccc} 1 & \frac{1}{2} & \frac{1}{4} & -1 & -\frac{7}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & 1 & \frac{3}{2} & -4 & -\frac{1}{2} & -\frac{1}{2} & 2 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -1 & \frac{5}{3} & 1 \end{array}\right)$$

$$0 = 1 \ \varnothing$$

Ответ: такой матрицы P не существует

5.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & 0 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -5 \\ 3 & 2 & -1 & -4 \end{pmatrix}, C = \begin{pmatrix} 1 & -2 & -2 & -1 \\ -2 & 5 & 4 & 1 \\ 2 & -6 & -3 & -1 \\ -2 & 3 & 5 & 3 \end{pmatrix},$$

$$D = \begin{pmatrix} 10 & 9 & 4 & 0 \\ 94 & 29 & 0 & 16 \\ -84 & -20 & 4 & -16 \\ 64 & 2 & -12 & 16 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -5 \\ 3 & 2 & -1 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -5 \\ 3 & 2 & -1 & -4 \\ -10 & -2 & 22 & -10 \end{pmatrix}$$

$$ABC^{-1} = \begin{pmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -5 \\ 3 & 2 & -1 & -4 \\ -10 & -2 & 22 & -10 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & 2 & -2 \\ -2 & 5 & -6 & 3 \\ -2 & 4 & -3 & 5 \\ -1 & 1 & -1 & 3 \end{pmatrix} =$$

$$= \begin{pmatrix} 5 & -12 & 9 & -11 \\ -5 & 16 & -13 & 8 \\ 5 & -4 & 1 & -17 \\ -40 & 88 & -64 & 94 \end{pmatrix}$$

$$ABC^{-1}x = 0 \Rightarrow \begin{pmatrix} 5 & -12 & 9 & -11 & | & 0 \\ -5 & 16 & -13 & 8 & | & 0 \\ 5 & -4 & 1 & -17 & | & 0 \\ -40 & 88 & -64 & 94 & | & 0 \end{pmatrix} = F$$

$$\begin{cases} F[0] = F[0]/5 \\ F[1] = F[1] + F[0]*5 \\ F[2] = F[2] - F[0]*5 \\ F[3] = F[3] + F[0]*40 \end{cases} \Rightarrow \begin{pmatrix} 1 & -\frac{12}{5} & \frac{9}{5} & -\frac{11}{5} & | & 0 \\ 0 & 4 & -4 & -3 & | & 0 \\ 0 & 8 & -8 & -6 & | & 0 \\ 0 & -8 & 8 & 6 & | & 0 \end{pmatrix}$$

$$\begin{cases} F[2] = F[2] - 2*F[1] \\ F[3] = F[3] + 2*F[1] \\ F[1] = F[1]/4 \end{cases} \Rightarrow \begin{pmatrix} 1 & -\frac{12}{5} & \frac{9}{5} & -\frac{11}{5} & | & 0 \\ 0 & 1 & -1 & -\frac{3}{4} & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$F[0] = F[0] + F[1]*12/5 \Rightarrow \begin{pmatrix} 1 & 0 & -\frac{3}{5} & -4 & | & 0 \\ 0 & 1 & -1 & -\frac{3}{4} & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} x_1 = \frac{3}{5}x_3 + 4x_4 \\ x_2 = x_3 + \frac{3}{4}x_4 \end{cases} \Rightarrow x = \begin{pmatrix} \frac{3}{5}x_3 + 4x_4 \\ x_3 + \frac{3}{4}x_4 \\ x_3 \\ x_4 \end{pmatrix},\ x_3, x_4 \in \mathbb{R}$$

$$Dy = 0 \Rightarrow \begin{pmatrix} 10 & 9 & 4 & 0 & | & 0 \\ 94 & 29 & 0 & 16 & | & 0 \\ -84 & -20 & 4 & -16 & | & 0 \\ 64 & 2 & -12 & 16 & | & 0 \end{pmatrix} = G$$

$$\begin{cases} G[0] = G[0]/10 \\ G[1] = G[1] - G[0] * 94 \\ G[2] = G[2] + G[0] * 84 \\ G[3] = G[3] - G[0] * 64 \end{cases} \Rightarrow \begin{pmatrix} 1 & \frac{9}{10} & \frac{2}{5} & 0 & | & 0 \\ 0 & -\frac{278}{5} & -\frac{188}{5} & 16 & | & 0 \\ 0 & \frac{278}{5} & \frac{188}{5} & -16 & | & 0 \\ 0 & -\frac{278}{5} & -\frac{188}{5} & 16 & | & 0 \end{pmatrix}$$

$$\begin{cases} G[1] = G[1] * -5/278 \\ G[2] = G[2] - G[1] * 278/5 \\ G[3] = G[3] + G[1] * 278/5 \end{cases} \Rightarrow \begin{pmatrix} 1 & \frac{9}{10} & \frac{2}{5} & 0 & | & 0 \\ 0 & 1 & \frac{94}{139} & -\frac{40}{139} & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$G[0] = G[0] - G[1] * 9/10 \Rightarrow \begin{pmatrix} 1 & 0 & -\frac{29}{139} & \frac{36}{139} & | & 0 \\ 0 & 1 & \frac{94}{139} & -\frac{40}{139} & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} y_1 = \frac{29}{139}y_3 - \frac{36}{139}y_4 \\ y_2 = \frac{40}{139}y_4 - \frac{94}{139}y_3 \end{cases} \Rightarrow y = \begin{pmatrix} \frac{29}{139}y_3 - \frac{36}{139}y_4 \\ \frac{40}{139}y_4 - \frac{94}{139}y_3 \\ y_3 \\ y_4 \end{pmatrix}, \ y_3, y_4 \in \mathbb{R}$$

Найдём, когда $x = y$:

$$\begin{pmatrix} \frac{3}{5}x_3 + 4x_4 \\ x_3 + \frac{3}{4}x_4 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \frac{29}{139}y_3 - \frac{36}{139}y_4 \\ \frac{40}{139}y_4 - \frac{94}{139}y_3 \\ y_3 \\ y_4 \end{pmatrix}$$

$$\begin{cases} \frac{3}{5}x_3 + 4x_4 = \frac{29}{139}y_3 - \frac{36}{139}y_4 \Rightarrow \frac{3}{5}x_3 + 4x_4 = \frac{29}{139}x_3 - \frac{36}{139}x_4 \\ x_3 + \frac{3}{4}x_4 = \frac{40}{139}y_4 - \frac{94}{139}y_3 \Rightarrow x_3 + \frac{3}{4}x_4 = \frac{40}{139}x_4 - \frac{94}{139}x_3 \\ x_3 = y_3 \\ x_4 = y_4 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x_3 = -\frac{185}{17}x_4 \\ x_3 = -\frac{257}{932}x_4 \end{cases} \Rightarrow x_4 = 0 \Rightarrow x_3 = 0 \Rightarrow x_2 = 0 \Rightarrow x_1 = 0$$

Ответ: уравнения не имеют одинакового множества решений, есть одно общее решение, это - $\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$