(1) $$\frac{d^2 f(x_i)}{dx^2} = \frac{f(x_{i+2}) - 2f(x_i) + f(x_{i-2})}{4h^2}$$

Dem:

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$\frac{d}{dx}\left(f'(x)\right) = \frac{d}{dx}\left(\frac{f(x+h) - f(x-h)}{2h}\right)$$

$$f''(x) = \cancel{2h}\,\frac{1}{2h}\cdot\left(\frac{f(x+2h) - f(x)}{2h} - \frac{f(x) - f(x-2h)}{2h}\right)$$

$$f''(x) = \frac{1}{2h}\left(\frac{f(x+2h) - 2f(x) + f(x-2h)}{2h}\right)$$

$$f''(x) = \frac{f(x+2h) - 2f(x) + f(x-2h)}{4h^2}$$

(5)

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}f^{(4)}(x) + \frac{h^5}{5!}f^{(5)}(x) + \frac{h^6}{6!}f^{(6)}(x)$$

$+$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{3!}f'''(x) + \frac{h^4}{4!}f^{(4)}(x) - \frac{h^5}{5!}f^{(5)}(x) + \frac{h^6}{6!}f^{(6)}(x)$$

$$f(x+h) + f(x-h) = 2f(x) + \frac{2h^2}{2}f''(x) + \frac{2h^4}{4!}f^{(4)}(x) + \frac{2h^6}{6!}f^{(6)}(x)$$

$$\frac{f(x+h) + f(x-h) - 2f(x) - h^2f''(x)}{h^4} - \frac{2h^{\cancel{6}}}{\cancel{h^4}6!}f^{(6)}(x) = \frac{2f^{(4)}(x)}{4!}$$

$$\frac{f(x+h) + f(x-h) - 2f(x) - h^2f''(x)}{h^4} - O(h^2) = \frac{2f^{(4)}(x)}{4!}$$

$$\frac{f(x+h) + f(x-h) - 2f(x) - h^2\left(\frac{f(x+2h) - 2f(x) + f(x-2h)}{4\cancel{h^2}}\right)}{h^4} = \frac{2f^{(4)}(x)}{4!}$$

$$\frac{4f(x+h) + 4f(x-h) - \cancel{8}f(x) - f(x+2h) + 2f(x) - f(x-2h)}{4h^4} = \frac{2f^{(4)}(x)}{4!}$$

$$\frac{-3f(x+2h) + 12f(x+h) - \cancel{24}18f(x) + 12f(x-h) - 3f(x-2h)}{h^4} = f^{(4)}(x)$$

$$\frac{\cancel{-3}(f(x+2h) - 4f(x+h) + 6f(x) - 4f(x-h) + f(x-2h))}{h^4} \approx f^{(4)}(x)$$

R/ El orden de h para esta operación es $O(h^2)$