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③ Hacer pasos intermedios para encontrar la regla de simpson simple

$$\int_a^b f(x)dx \cong \int_a^b P_2(x)dx = \frac{h}{3}(f(a) + 4f(x_m) + f(b))$$

$$f(x) \cong P_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)$$

$$x_m = \frac{a+b}{2} \qquad h = \frac{b-a}{2}$$

\# Dividir en tres integrales

① $$\int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a)\, dx$$

$$\cancel{\int_a^b} \frac{f(a)}{(a-b)(a-x_m)} \int_a^b (x-b)(x-x_m)\, dx$$

$$\frac{f(a)}{(a-b)\left(\frac{2a}{2} - \left(\frac{a+b}{2}\right)\right)} \int_a^b x^2 - x_m x - bx + bx_m\, dx$$

$$\frac{2f(a)}{(a-b)(a-b)} \cdot \left(\frac{x^3}{3} - \frac{x_m x^2}{2} - \frac{bx^2}{2} + bx_m x \Big|_a^b\right)$$

$$\frac{2f(a)}{(a-b)^2}\cdot\left(\frac{b^3}{3}-\frac{x_m b^2}{2}-\frac{b^3}{2}+b^2x_m-\left(\frac{a^3}{3}-\frac{a^2x_m}{2}-\frac{ba^2}{2}+abx_m\right)\right)$$

$$\frac{2f(a)}{(a-b)^2}\cdot\left(-\frac{b^3}{6}+\frac{b^2x_m}{2}-\frac{a^3}{3}+\frac{a^2x_m}{2}+\frac{a^2b}{2}-abx_m\right)$$

$$\frac{2f(a)}{(a-b)^2}\cdot\left(\frac{-b^3+3b^2x_m-2a^3+3a^2x_m+3a^2b-6abx_m}{6}\right)$$

$$\frac{2f(a)}{\cancel{(a-b)^2}}\cdot\frac{\cancel{(a-b)^2}(3x_m-2a-b)}{6}$$

$$\frac{2f(a)}{6}\cdot\left(3\left(\frac{a+b}{2}\right)-\frac{2\cdot 2a}{2}-\frac{2b}{2}\right)$$

$$\frac{2f(a)}{6}\cdot\left(\frac{-a+b}{2}\right)=\frac{b-a}{2}=h$$

$$f(a)\cdot\frac{h}{3}$$

② $\displaystyle\int_a^b \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}f(x_m)\,dx$

$$\frac{f(x_m)}{(x_m-a)(x_m-b)}\int_a^b (x-a)(x-b)\,dx$$

$$\frac{f(x_m)}{\left(\frac{a+b}{2}-\frac{2a}{2}\right)\left(\frac{a+b}{2}-\frac{2b}{2}\right)}\int_a^b x^2-bx-ax+ab\,dx$$

$$\frac{4f(x_m)}{(b-a)(a-b)}\left(\frac{x^3}{3}-\frac{bx^2}{2}-\frac{ax^2}{2}+abx\Big|_a^b\right)$$

$$\frac{4f(x_m)}{ba-b^2-a^2+ab}\left(\frac{b^3}{3}-\frac{b^3}{2}-\frac{ab^2}{2}+ab^2-\left(\frac{a^3}{3}-\frac{a^2b}{2}-\frac{a^3}{2}+a^2b\right)\right)$$

$$\frac{4f(x_m)}{-(a+b)^2}\left(\frac{-b^3}{6}+\frac{ab^2}{2}+\frac{a^3}{6}-\frac{a^2b}{2}\right)$$

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$$-\frac{4f(x_m)}{(a-b)^2}\left(\frac{a^3-3a^2b+3ab^2-b^3}{6}\right)$$

$$-\frac{4f(x_m)}{(a-b)^2}\cdot\frac{(a-b)^3}{6} = -\frac{4}{3}f(x_m)\frac{(a-b)}{2}$$

$$= \frac{4}{3}f(x_m)\cdot\frac{(b-a)}{2} = \frac{4h}{3}f(x_m)$$

③ $\displaystyle\int_a^b \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)\,dx$

$$\frac{f(b)}{(b-a)(b-x_m)}\int_a^b x^2 - x_m x - ax + a x_m\,dx$$

$$\frac{f(b)}{(b-a)\left(\frac{2b}{2}-\left(\frac{a+b}{2}\right)\right)}\cdot\left(\frac{x^3}{3}-\frac{x_m x^2}{2}-\frac{ax^2}{2}+a x_m x\Big|_a^b\right)$$

$$\frac{2f(b)}{(b-a)^2}\cdot\left(\frac{b^3}{3}-\frac{x_m b^2}{2}-\frac{ab^2}{2}+ab x_m-\left(\frac{a^3}{3}-\frac{a^2x_m}{2}-\frac{a^3}{2}+a^2x_m\right)\right)$$

$$\frac{2f(b)}{(b-a)^2}\cdot\left(\frac{b^3}{3}-\frac{b^2x_m}{2}-\frac{ab^2}{2}+abx_m+\frac{a^3}{6}-\frac{a^2x_m}{2}\right)$$

$$\frac{2f(b)}{(b-a)^2}\cdot\left(\frac{b^3}{3}-\frac{b^2}{2}\cdot\left(\frac{a+b}{2}\right)-\frac{ab^2}{2}+ab\left(\frac{a+b}{2}\right)+\frac{a^3}{6}-\frac{a^2}{2}\left(\frac{a+b}{2}\right)\right)$$

$$\frac{2f(b)}{(b-a)^2}\cdot\left(\frac{4b^3-3ab^2-3b^3-6ab^2+6a^2b+6ab^2+2a^3-3a^3-3a^2b}{12}\right)$$

$$\frac{2f(b)}{(b-a)^2}\cdot\left(\frac{b^3-3ab^2+3a^2b-a^3}{12}\right)$$

$$\frac{2f(b)}{(b-a)^2}\cdot\frac{(b-a)^3}{12} = \frac{f(b)}{3}\cdot\frac{(b-a)}{2} = h$$

$$\frac{h}{3}f(b)$$

con ①, ② y ③

$$\int_a^b f(x)\,dx \cong \int_a^b P_2(x)\,dx = \frac{h}{3}\left(f(a)+4f(x_m)+f(b)\right)$$