An Improved SVD Algorithm Based on Virtual Matrix

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Abstract— **In this paper, an improved singular value decomposition (SVD) algorithm for high-resolution direction of arrival (DOA) estimation is proposed, which is based on virtual matrix, the SVD-VM algorithm for short. The virtual matrix is employed as the preprocessor for the uniform linear array (ULA), and then the rotational matrix in ESPRIT is used to estimate the directions of the coherent sources. The simulation results show that the SVD-VM algorithm provides higher resolution and robustness performance for coherent signals estimation than conventional singular value decomposition.**

Keywords— singular value decomposition (SVD), virtual matrix, direction of arrival (DOA), low SNR, coherent signal

I. INTRODUCTION

In the array signal processing, DOA estimation has been widespread concerned, and extensively applied in the field of radar, sonar, communication, astronautics, aeronautics, etc. Many algorithms, such as multiple signal classification (MUSIC)[1], estimation of signal parameters via rotation invariance techniques (ESPRIT)[2] have been developed over years and provide excellent performance. However, in real environments, those high-resolution methods may fail to work when there are highly correlated or coherent signals because of multipath propagation.

Many methods have been proposed to solve the coherency problem, such as the spatial smoothing technique, singular value decomposition (SVD)，Toeplitz algorithms, etc. One of the most noteworthy is spatial smoothing technique[3],[4]. Their solution is based on a preprocessing scheme that groups the total array of sensors into overlapping subarrays and then averages the subarray output covariance matrices to form. In order to reduce the number of the sensors, an improved smoothing scheme is proposed, which referred to as the forward/backward spatial smoothing (FBSS) technique [5]. The main idea of the matrix pencil (MP) is used in [6],[7] based on the spatial samples of the data, the MP method can find the high-resolution direction of arrival (DOA) easily in the presence of coherent signals instead of additional processing of spatial smoothing. But MP has a poor performance at low SNR. In [8], SVD is proposed, the main idea of SVD is to perform eigenvalue decomposition on the DOA matrix, and reconstruct the matrix by the eigenvector corresponding to the largest eigenvalue, then perform eigenvalue decomposition on the new matrix again, then find out the noise subspace and signal subspace. But they have poor performance at low SNR and impose bias in the DOA estimation.

In this paper an improved SVD for high-resolution direction of arrival estimation is proposed. We use the data covariance matrix and its conjugation form to construct the virtual array, and get the data covariance matrix. Simulation Results show that the proposed algorithm can achieve good resolution performance at low SNR.

II. DATA MODEL

Suppose a uniform linear array (ULA) with M linear equispaced omni-directional sensors with interspacing $d = \lambda/2$, and N narrowband signals of the different direction of arrival $\theta_1, \theta_2 \dots \theta_N$ then, the steering vector is

$$\boldsymbol{a}(\theta_i) = [1, e^{j\frac{2\pi d}{\lambda}\sin\theta_i}, \dots, e^{j\frac{2\pi d}{\lambda}(M-1)\sin\theta_i}]^{\mathrm{T}} \quad i = 1, 2 \dots N \tag{1}$$

Where λ is the carrier wavelength of the signal, and $(\bullet)^{\mathrm{T}}$ is the transpose operator. The observed snapshot from the M array elements can be presented as

$$\begin{aligned} \boldsymbol{x}(t) &= \boldsymbol{A}(\theta)\boldsymbol{s}(t) + \boldsymbol{n}(t) \\ &= \left[\boldsymbol{x}_1(t), \boldsymbol{x}_1(t), \ \dots, \boldsymbol{x}_M(t)\right]^{\mathrm{T}} \end{aligned} \tag{2}$$

Where $x(t) \in C^{M\times 1}$, $x(t)$ is the array output vector; $\boldsymbol{A}(\theta) \in C^{M\times N}$, $\boldsymbol{A}(\theta) = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \dots, \boldsymbol{a}(\theta_N)]$ is the array manifold matrix; $\boldsymbol{s}(t) \in C^{N\times 1}$, $\boldsymbol{s}(t) = [\boldsymbol{s}_1(t), \boldsymbol{s}_2(t), \dots, \boldsymbol{s}_N(t)]^{\mathrm{T}}$ is the source vector; and $\boldsymbol{n}(t) \in C^{M\times 1}$, $\boldsymbol{n}(t) = [n_1(t), n_2(t), \dots, n_M(t)]^{\mathrm{T}}$ is the noise vector with the power of each entry equal to σ_n^2.

The covariance matrix of received data can be expressed as

$$\begin{aligned} \boldsymbol{R}_x &= E\left\{\boldsymbol{x}(t)\boldsymbol{x}(t)^{\mathrm{H}}\right\} \\ &= AE\left\{\boldsymbol{ss}^{\mathrm{H}}\right\}\boldsymbol{A} + \sigma^2\boldsymbol{I} \\ &= \boldsymbol{AR}_s\boldsymbol{A} + \boldsymbol{R}_N \end{aligned} \tag{3}$$

Where $\boldsymbol{R}_x \in C^{M\times M}$ is the covariance matrix .

978-0-7695-4077-1/10 $26.00 © 2010 IEEE
DOI 10.1109/ICICTA.2010.359

III. SVD-VM ALGORITHM

Consider N narrow band signal, M linear equispaced omni-directional sensors, the array manifold matrix A is a rank-K matrix, the rank of signal covariance matrix is $K(K\leqslant N)$. Suppose the noise covariance matrix $\boldsymbol{R}_N$ is a full rank matrix, it can be shown that

$$\boldsymbol{R}_N\boldsymbol{e}_k=\sum_{n=1}^{N}\alpha_k(n)\boldsymbol{a}(\theta_n) \tag{4}$$

Where $1\leqslant k\leqslant K$, $\boldsymbol{e}_k$ is the eigenvector, $a_k(n)$ is linear combination factor. If the noise covariance matrix is white noise, (4) is simplified

$$\boldsymbol{e}_k=\sum_{n=1}^{N}\alpha_1(n)\boldsymbol{a}(\theta_n) \qquad 1\le k\le K \tag{5}$$

It means that whether the source is coherent or not, the eigenvector corresponding to the largest eigenvalue is a linear combination of all signal sources' steering vector. When the signal is totally coherent, that is $K=1$, the left of (5) is only the eigenvector corresponding to the largest eigenvalue.

$$\boldsymbol{e}_1=\sum_{n=1}^{N}\alpha_1(n)\boldsymbol{a}(\theta_n) \tag{6}$$

This formula shows that the largest eigenvector of data covariance matrix includes all the signal information. We can use (6) to construct the following matrix

$$\boldsymbol{Y}=\begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1p} \\ e_{12} & e_{13} & \cdots & e_{1p+1} \\ \cdots & \cdots & \ddots & \cdots \\ e_{1m} & e_{1m+1} & \cdots & e_{1M} \end{bmatrix} \tag{7}$$

Where $p=M-m+1$, $m>N$, $p>N$

Equation (7) can estimate the coherent signal, but the resolution is not satisfied, especially in the condition of low SNR. In order to overcome the shortcoming, we proposed an improved singular value decomposition algorithm based on virtual matrix(SVD-VM), it doesn't carry out correlation processing with receiving data directly, but to construct K virtual matrix with receiving data and its conjugate information[9]. Suppose the first matrix is $\{\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3,\ldots,\boldsymbol{x}_M\}$, the second one is $\{\boldsymbol{x}^*_2, \boldsymbol{x}_1, \boldsymbol{x}_2,\ldots,\boldsymbol{x}_{M-1}\}$. By parity of reasoning, the Kth matrix is $\{\boldsymbol{x}^*_M, \boldsymbol{x}^*_{M-1}, \boldsymbol{x}^*_{M-2},\ldots, \boldsymbol{x}_1\}$, then the vector form of the Kth virtual matrix receiving data is

$$\boldsymbol{x}_k(t)=\boldsymbol{A}\Phi^{*(k-1)}\boldsymbol{s}(t)+\boldsymbol{n}_k(t) \tag{8}$$

$$\Phi^*=\mathrm{diag}\{e^{j\frac{2\pi}{\lambda}d\sin\theta_1},e^{j\frac{2\pi}{\lambda}d\sin\theta_2},\ldots,e^{j\frac{2\pi}{\lambda}d\sin\theta_N}\} \tag{9}$$

Where Φ^* is the conjugate form of rotational matrix in ESPRIT. The autocorrelation matrix of the Kth virtual matrix is

$$\boldsymbol{R}_k=\boldsymbol{A}\Phi^{*(k-1)}\boldsymbol{S}[\Phi^{*(k-1)}]^H\boldsymbol{A}^H+\sigma^2\boldsymbol{I} \tag{10}$$

So we define the spatial smoothing matrix of virtual matrix as

$$\boldsymbol{R}_V=\frac{1}{M}\sum_{k=1}^{M}\boldsymbol{R}_k \tag{11}$$

We performing singular value decomposition on the modified matrix, and can get modified signal subspace and noise subspace, then the signal direction of arrival can be estimated. That is the method proposed in this paper.

The SVD-VM algorithm includes the following steps:

- Construct M virtual matrix with the receiving data and its conjugate information according to (8).
- Calculate the covariance matrix of each virtual matrix according to (10) and (11)
- Perform eigenvalue decomposition on the constructed covariance matrix, and get the largest eigenvalue.
- Reconstruct the matrix according to (7).
- Estimate the direction of arrival of the signal by MUSIC algorithm.

IV. SIMULATION RESULTS

The performance of our method is illustrated in this section. In all the simulations a uniform linear array of $M=8$ omni-directional sensors with half wavelength spacing was used, the wavelength $\lambda=2d$, and $N=2$ narrowband signals with the different direction of arrival of 40° and 50°. 2000 snapshots were used to estimate the array covariance matrix. The noise is all white noise.

Simulation1: The performance of SVD-VM in condition of SNR=0dB and -5dB is investigated in this simulation. We compare SVD-VM with SVD in Fig 1. SNR=0dB. It can be found out that SVD-VM can distinguish the two signals obviously, but SVD can't. In Fig 2. SNR=-5dB, SVD-VM can also distinguish the two signals, but has less performance, however, SVD has already fail to work.

Simulation2: The probability of detection in different SNR is investigated in this simulation. We compare SVD-VM with SVD. Each probability at a particular SNR is computed from 500 Monte-Carlo runs. The probability of the resolution versus SNR is shown in Fig.3. Obviously, the SVD-VM algorithm has higher resolution capability than SVD.

Simulation3: The accuracy of the algorithm can be evaluated by examining the Root Mean Square Error

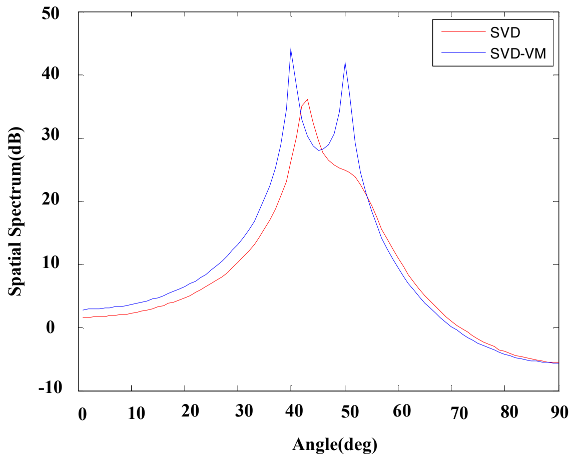

Figure 1. Performance of SVD-VM and SVD in condition of SNR=-5dB

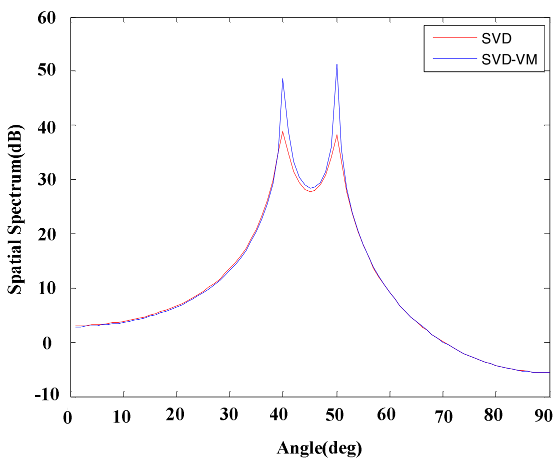

Figure 2. Performance of SVD-VM and SVD in condition of SNR=0dB

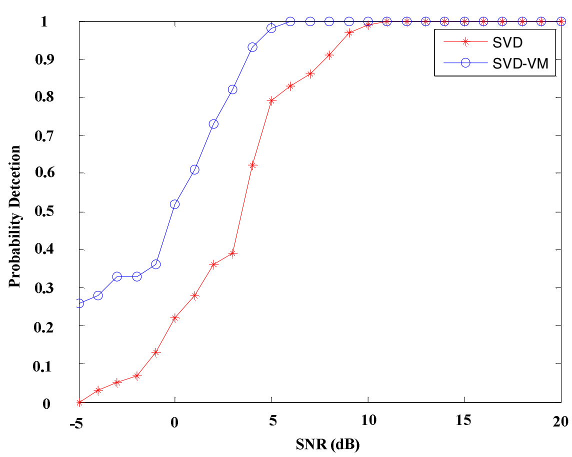

Figure 3. Probability of detection for different SNR

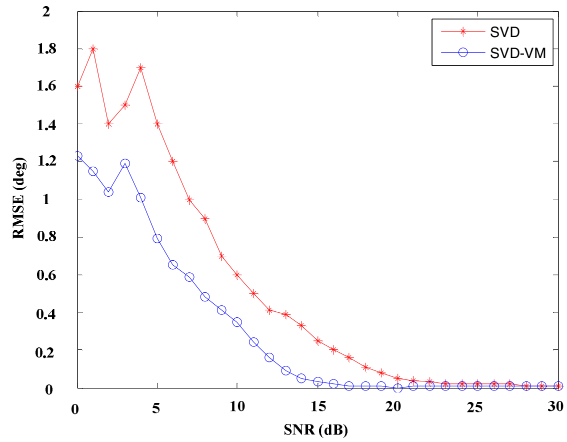

Figure 4. RMSE for different SNR

(RMSE)of the estimation. In this experiment, each RMSE at a particular SNR is computed from 500 Monte-Carlo runs. Consider two correlated sources with directions of arrival 40 ° and 50° . Fig.3 shows the RMSE of the estimated DOA at 40 ° versus SNR. Obviously, the proposed SVD-VM algorithm provides lower RMSE than the SVD. It's indicated that the SVD-VM algorithm has the higher robustness than SVD.

V. Conclusions

In this paper, we proposed an improved singular value decomposition (SVD) algorithm for high-resolution direction of arrival (DOA) estimation, which is based on virtual matrix. The virtual matrix is employed as the preprocessor for the united linear arrays. We perform eigenvalue decomposition on the constructed covariance matrix, and get the largest eigenvalue, then reconstruct the matrix by the eigenvector corresponding to the largest eigenvalue. That is the modified data covariance matrix. The simulation results show that the SVD-VM algorithm provides higher resolution and robustness performance for coherent signals estimation than conventional singular value decomposition.

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