1. Find the eigenvalues and eignevectors of the matrices

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} -3 & 1 & -3 \\ 20 & 3 & 10 \\ 2 & -2 & 4 \end{bmatrix}$$

2. Determine if either of the following matrices is diagonalizable

$$\boldsymbol{A} = \begin{bmatrix} -1 & -1 & -2 \\ 8 & -11 & -8 \\ -10 & 11 & 7 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & -4 & -4 \\ 8 & -11 & -8 \\ -8 & 8 & 5 \end{bmatrix}$$

3. Estimate the eigenvalues of $\boldsymbol{A} = \begin{bmatrix} 5 & 1 & 1 \\ 0 & 6 & 1 \\ 1 & 0 & -5 \end{bmatrix}$

4. Determine the induced norm $\|A\|_2$ as well as $\|A^{-1}\|_2$ for the nonsingular matrix

$$\boldsymbol{A} = \frac{1}{\sqrt{3}} \begin{bmatrix} 3 & -1 \\ 0 & \sqrt{8} \end{bmatrix}$$

5. Let $A \in \mathbb{C}^{m\times n}$. Please prove the following statements. (a) $\|A\|_p = \max_{\|x\|_p=1} \|Ax\|_p$ is a norm for any $p \geq 1$ (b) $\|\boldsymbol{Ax}\|_p \leq \|\boldsymbol{A}\|_p \|\boldsymbol{x}\|_p$ holds for any $p \geq 1$ (c) $\|AB\|_p \leq \|A\|_p \|B\|_p$ holds for any $p \geq 1$ (d) $\|\boldsymbol{QAU}\|_F = \|\boldsymbol{A}\|_F$ holds for any unitary matrices $\boldsymbol{Q} \in \mathbb{C}^{m\times m}$ and $\boldsymbol{U} \in \mathbb{C}^{n\times n}$ (e) $\|QAU\|_2 = \|A\|_2$ holds for any unitary matrices $Q \in \mathbb{C}^{m\times m}$ and $U \in \mathbb{C}^{n\times n}$

6. Using the induced matrix norm, prove that if A is nonsingular, then

$$\min_{\|x\|=1} \|\boldsymbol{Ax}\| = \frac{1}{\|\boldsymbol{A}^{-1}\|}$$

7. Let $A \in \mathbb{C}^{m\times n}$ is a Vandemonde matrix with distinct roots. Please verify that any collection of r columns of $\boldsymbol{A}$, with $r \leq m$, is linearly independent.