HW 2 **MATH 156** **Spring 2024**

Name:……

1. (§2.1) Decide whether or not the following is a statement. If it is a statement, say if it is true or false, if possible. Explain your thinking: The derivative of cos(x) is sin(x).

This is an open statement. It is either definitively true or false, but it is only true when

$$x = n\left(\frac{\pi}{4}\right), n \in Z$$

2. (§2.1) Decide whether or not the following is a statement. In the case of a statement, say if it is true:

$(\mathbb{R} \times \mathbb{N}) \cap (\mathbb{N} \times \mathbb{R}) = \mathbb{N} \times \mathbb{N}$.

This is a statement. It is either definitively true or false. It is also true, as for an ordered pair (a, b) = (c,d), with (a,b) in RxN and (c,d) in NxR, then a must equal c, and therefore a is in N, and b = d, and therefore d is in N. Since a,b,c,d are in N, (a,b) and (c,d) are in N as well.

3.(§2.2) Express the open statement in one of the forms $P \wedge Q$, $P \vee Q$, or $\sim P$:

At least one of the numbers x or y equals 0.

$$x = 0 \vee y = 0$$

4. Convert the following sentence into a sentence having the form "If P, then Q":

Whenever a surface has only one side, it is non-orientable.

If a surface has only one side, then it is non-orientable

5. Find the converse and contrapositive of the statement:

$f(x) = f(y) \Rightarrow x =$ y.

Converse

$$(x = y) \Rightarrow (f(x) = f(y))$$

contrapositive

$$\neg(x = y) \Rightarrow \neg(f(x) = f(y))$$

6. Without changing the meaning, convert the following sentence into a sentence having the form "If P, then Q."

A function is rational if it is a polynomial.

If a function is polynomial, then it is rational

7. Without changing the meaning, convert the following sentence into a sentence of the form "P if and only if Q."

If $x-a\,/y-b = 0$ then $x = a$ and $y \neq b$, and conversely.

$$\frac{x-a}{y-b} = 0 \text{ If and only if } (x = a) \cap (y \neq b)$$

8. Use a truth table to help decide whether the following pairs of statements are logically equivalent: ~$(P \Longrightarrow Q)$ and $P \wedge$ ~Q.

P	Q	$P \Rightarrow Q$	$\neg(P \Rightarrow Q)$	$\neg Q$	$P \wedge \neg Q$
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

The statements are equivalent.