

Checkpoint 2

$$D = \begin{pmatrix} 4 & 8 & 12 \\ 6 & 10 & 15 \\ 5 & 7 & 9 \\ 3 & 6 & 9 \end{pmatrix}$$

Row = Different regions

Column = Specific product category (A, B, C)

Tasks

① Identify the data type of matrix elements

Type integer (or numeric)

② Calculate the mean, median, std for each column (Pdt A, B, & C)

- Mean Product A (First column, 4, 6, 5, 3)

$$\text{Mean} = 4 + 6 + 5 + 3$$

$$= 4.5,$$

- Median:

$$\text{Sorted data} = 3, 4, 5, 6$$

$$\text{Median} = 4 + 5$$

$$2 = 4.5,$$

- Standard deviation

$$\text{Variance} = (4 - 4.5)^2 + (6 - 4.5)^2 + (5 - 4.5)^2 + (3 - 4.5)^2$$

$$4$$

$$= 0.25 + 2.25 + 0.25 + 2.25 = .5$$

$$+ 4$$

$$\# 4$$

$$= 4.25,$$

$$\begin{aligned} \text{Std} &= \sqrt{\text{Variance}} \\ &= \sqrt{1.25} \\ &= 1.118 \end{aligned}$$

* Product B (second column, 8, 10, 7, 6)

- Mean

$$= 8 + 10 + 7 + 6$$

4

$$= 31$$

$$\frac{31}{4} = 7.75$$

- Median = ~~10 + 7~~ 6, 7, 8, 10

$$\frac{7+8}{2} =$$

$$\Rightarrow 7.5$$

$$\sigma = 7.5$$

- Standard deviation

$$\text{Variance} = \frac{\sum (x - \bar{x})^2}{n}$$

$$= (8 - 7.5)^2 + (10 - 7.5)^2 + (7 - 7.5)^2 + (6 - 7.5)^2$$

4

$$= \frac{0.0625 + 5.0625 + 0.5625 + 3.0625}{4} = 8.75$$

4

$$\frac{8.75}{4} = 2.1875$$

$$\text{Std} = \sqrt{\text{Variance}}$$

$$= \sqrt{2.1875} = 1.479,$$

Product C (Third column, 12, 15, 9, 9)

$$\text{Mean} = \frac{12+15+9+9}{4}$$

$$= 11.25$$

$$\text{H} = 11.25,$$

Median

Sorted data = 9, 9, 12, 15

$$\Rightarrow 9+12$$

$$2 = 10.5,$$

Standard deviation

$$\text{Variance} = \frac{(12-11.25)^2 + (15-11.25)^2 + (9-11.25)^2 + (9-11.25)^2}{4}$$

$$= 0.5625 + 14.0625 + 5.0625 + 5.0625$$

$$4$$

$$= 24.75$$

$$\text{H} = 6.1875,$$

$$\text{Std} = \sqrt{6.1875}$$

$$= 2.487,$$

③ For each column determine the range and variance

* Product A (First column 14, 6, 5, 3)

- Range = Max - Min

$$6 - 3 = 3,$$

- Variance

$$\text{Mean} = 4.5,$$

$$\text{Variance} = 1.25$$

* Product B (2nd column 8, 10, 7, 6)

Range = Max - Min

$$= 10 - 6 = 4,$$

$$\text{Variance} = 2.1875,$$

* Product C (3rd column 12, 15, 9, 9)

Range = Max - Min

$$= 15 - 9 = 6,$$

$$\text{Variance} = 6.1875,$$

Part 2: Linear Algebra

1 Compute the transpose of matrix

$$D = \begin{pmatrix} 4 & 6 & 5 & 3 \\ 8 & 10 & 7 & 6 \\ 12 & 15 & 9 & 9 \end{pmatrix}$$

2) Multiply the matrix D by a price vector

$$P \text{ where: } P = [5 \ 10 \ 15]$$

Solution

- First row

$$(4 \times 5) + (8 \times 10) + (12 \times 15)$$

$$20 + 80 + 180 = 280,$$

- Second row

$$(6 \times 5) + (10 \times 10) + (15 \times 15)$$

$$30 + 100 + 225 = 355,$$

- Third row

$$(5 \times 5) + (7 \times 10) + (9 \times 15)$$

$$25 + 70 + 135 = 230,$$

- Fourth row

$$(3 \times 5) + (6 \times 10) + (9 \times 15)$$

$$15 + 60 + 135 = 210,$$

$$\therefore D * P = \begin{pmatrix} 280 \\ 355 \\ 230 \\ 210 \end{pmatrix}$$

* this vector represents the total sales value for each region based on the given price vector P.

3) Determine which region generates the highest revenue

Region 2 with a total revenue of 355

- For each product, calculate the total revenue across all regions

* Product A (1st column 4, 6, 5, 3)

$$(4 \times 5) + (6 \times 5)$$

$$D = \begin{pmatrix} 4 & 8 & 12 \\ 6 & 10 & 15 \\ 5 & 7 & 9 \\ 3 & 6 & 9 \end{pmatrix}$$

$$P = \begin{pmatrix} 5 \\ 10 \\ 15 \end{pmatrix}$$

- Product A (1st column 4, 6, 5, 3)

$$(4 \times 5) + (6 \times 5) + (5 \times 5) + (3 \times 5)$$

$$20 + 30 + 25 + 15 = 90,$$

- Product B (2nd column 8, 10, 7, 6)

$$(8 \times 10) + (10 \times 10) + (7 \times 10) + (6 \times 10)$$

$$80 + 100 + 70 + 60 = 310,$$

- Product C (3rd column 12, 15, 9, 9)

$$(12 \times 15) + (15 \times 15) + (9 \times 15) + (9 \times 15)$$

$$180 + 225 + 135 + 135 = 675$$

Thus the overall total revenue across all regions for each product is

Product A: 90

Product B: 310

Product C: 675

Prob

Q4) What does the mean tell you about the typical sales for each product

- Product A

Mean = 4.5

This means that, on average each region

Q. How does the coefficient of variation help you understand sales consistency across products?

- The coefficient of variation (CV) is a standardized measure of dispersion that is calculated as the ratio of the standard deviation to the mean, often expressed as a percentage. It helps you understand the relative variability of sales for each product, which is useful for comparing consistency across products with different means.

Formula

$$CV = \left(\frac{\text{Standard deviation}}{\text{Mean}} \right) \times 100\%$$

- A lower CV indicates more consistent sales (less variability relative to the mean)
- A higher CV indicates less consistent sales (more variability relative to the mean)

Product A

$$\text{Mean} = 4.5$$

$$\text{Std} = \cancel{1.479}, 1.118$$

$$CV = \frac{1.118}{4.5} \times 100\% = 24.84\%$$

Product B

$$\text{Mean} = 7.75$$

$$\text{Std} = 1.479$$

$$CV = \frac{1.479}{7.75} \times 100\% = 19.08\%$$

Product C

$$\text{Mean} = 11.25$$

$$\text{Std} = 2.487$$

$$CV = \frac{2.487}{11.25} \times 100\% = 22.11\%$$

5) Discuss how matrix multiplication is used in real-world data science problems (e.g. revenue calculations, recommendation systems)

Answer

Matrix multiplication is a fundamental operation in data science and is widely used in various real-world applications.

1. Revenue calculations

Matrix multiplication is often used to calculate total revenue by multiplying a sales matrix by a price vector.

This approach is efficient for handling large datasets and can be easily extended to include additional dimensions such as discounts or taxes.

2. Natural Language Processing (NLP)

Matrix multiplication is used in various NLP tasks such as word embeddings, neural networks. This helps in capturing semantic relationships b/w words and improve the performance of NLP models.

3 Image Processing

In image processing, matrix multiplication is used in operations such as filtering and transformations. For example convolution (apply filters (kernels) to images for tasks like edge detection or blurring), affine transformations (perform operations like rotation, scaling and translation on images). These operations are essential for tasks like computer vision and image recognition.

4 Recommendation Systems

Matrix multiplication is crucial in collaborative filtering, a common technique in recommendation systems. e.g user-item matrix (rows represent users, columns represent items and entries represent ratings or interactions), matrix factorization, predicted ratings. This method helps in predicting user preferences and suggests relevant products or content.

3. Network analysis

In network analysis, matrix multiplication is used to study relationships and connections. E.g. adjacency matrix (represent connections b/w nodes in a network), power of matrix (compute the number of paths of a certain length b/w nodes). This is useful for analysing social networks, transportation systems, and other complex networks.

* Matrix multiplication is a versatile tool in data science, enabling efficient computations and transformations across various domains. Its applications range from simple revenue calculations to complex tasks like recommendation systems, image processing and network analysis.

Understands and leverages matrix multiplication can significantly enhance one's ability to analyse and interpret data in real-world scenarios.