

- 1A
- (1) If  $\sqcup$  is in current cell, then REJECT
  - (2) If  $a$  is in current cell, then erase and move right, goto (5)
  - (3) If  $l$  is in current cell, then erase and move right, goto (9)
  - (4) If  $p$  is in current cell, then erase and move right goto (1).
  - (5) If  $l$  is in current cell, then ACCEPT.
  - (6) If  $\sqcup$  is in current cell, then REJECT.
  - (7) If  $p$  is in current cell, then erase and go right.
  - (8) If  $a$  is in current cell, then erase and go right, goto (5)
  - (9) If  $a$  is in current cell, then ACCEPT.
  - (10) If  $\sqcup$  is in current cell, then REJECT.
  - (11) If  $p$  is in current cell, then erase and go right.
  - (12) If  $p$  is in current cell, then erase and go right.

1a)  $P \rightarrow \_ \rightarrow \text{REJECT}$

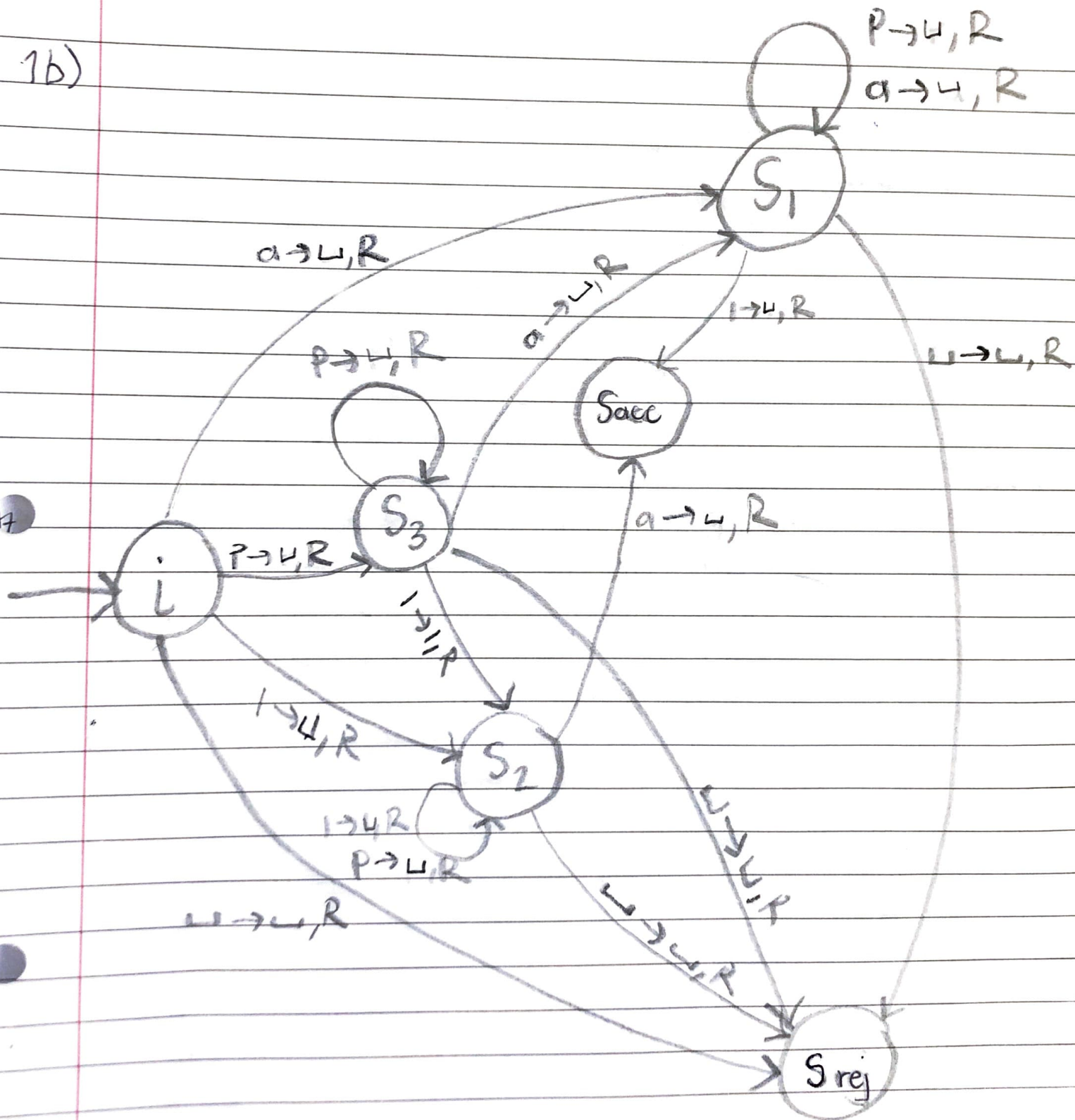
$al \rightarrow \_al \rightarrow \text{ACCEPT}$

$pap \rightarrow \_ap \rightarrow \_up \rightarrow \_up \rightarrow \text{REJECT}$

$pla \rightarrow \_la \rightarrow \_la \rightarrow \text{ACCEPT}$

$aapppla \rightarrow \_aapppla \rightarrow \_uapppla \rightarrow \_uuppla$   
 $\rightarrow \_uuuppla \rightarrow \_uuuuuila \rightarrow \text{ACCEPT.}$

1b)





$$2 \quad A^* = \{0, 1, 2, 3, 4, \dots\}$$

$\begin{matrix} \text{"} & \text{"} & \text{"} & \text{"} & \text{"} \\ w_1 & w_2 & w_3 & w_4 & w_5 \end{matrix}$

$$m=0 \quad 3(0)+1 = 1 \quad (\text{not even}) \quad \times$$

$$m=1 \quad 3(1)+1 = 4 \quad (\text{even}) \quad \checkmark$$

$$m=2 \quad 3(2)+1 = 7 \quad (\text{not even}) \quad \times$$

$$m=3 \quad 3(3)+1 = 10 \quad (\text{even}) \quad \checkmark$$

.....

$$m=5 \quad 3(5)+1 = 16 \quad \checkmark$$

$$\begin{array}{ccccccccc}
 m=1 & & m=3 & & m=5 & & m=7 & & m=9 \\
 4 & & 10 & & 16 & & 22 & & 28 \\
 \hline
 & 6 & & 6 & & 6 & & 6 & 
 \end{array}$$

$$w_i = i-1 \quad \text{i.e.} \quad w_2 = 2-1 = 1$$

$$\text{want } i-1 = 6m+4$$

$$\text{want } i = 6m+5$$

multiplying by an even number always gives an even number and adding one to an even number always gives an odd number. odd numbers  $\notin L$ . Only when  $m$  is an odd number then it's  $\in L$ .

2.  $E =$  Given the input tape  $A^* = \{w_1, w_2, \dots\}$

1. Repeat the following for  $i = 1, 2, 3, \dots$

2. At step  $i$ , if  $i = 6m+5$  for some  $m \in \mathbb{N}$ , then print out  $w_i$ .

3 a)  $E_{PSG} = \{ \langle G \rangle \mid G \text{ is a phrase structure grammar and } L(G) = \emptyset \}$

We define the Turing machine as  
 $M =$  on input  $\langle G \rangle$ , where  $G$  is a CFG.

1. Mark all terminal symbols in  $G$
2. Repeat until no new variable get marked.
3. A production rule for a phrase structure grammar on the left can have any string of terminals and nonterminals provided  $\exists$  at least one nonterminal. Mark any nonterminal on the left if ~~it~~ <sup>each</sup> ~~contains~~ <sup>symbol</sup> (terminal or nonterminal) on the right has already been marked.
4. If the start symbol  $\langle S \rangle$  is not marked, then accept; otherwise, reject.

### 3(b) The language

$$E_{PSG} = \{ \langle G \rangle \mid G \text{ is a phase structure grammar and } L(G) \neq \emptyset \}$$

consists of two conditions,

→  $G$  is a PSG

→  $L(G) \neq \emptyset$

Let  $C$  be the set of all Turing recognizable languages over a given finite alphabet  $A$ . As proven, the set of all Turing machines is countably infinite, so  $C$  must also be countably infinite. Clearly  $E_{PSG} \subseteq C$  as the condition  $L(B) \neq \emptyset$  shrinks the size of the set.  $E_{PSG}$  is thus a subset of a countably infinite set, so it could be finite or countably infinite. In tutorial 22, we prove that the language  $E_{DFA} = \{ \langle B \rangle \mid B \text{ is a DFA and } L(B) \neq \emptyset \}$  is countably infinite.  $E_{DFA} \subseteq E_{PSG}$ . We have shown  $E_{PSG}$  has a countably infinite subset. We conclude  $E_{DFA}$  must be countably infinite.