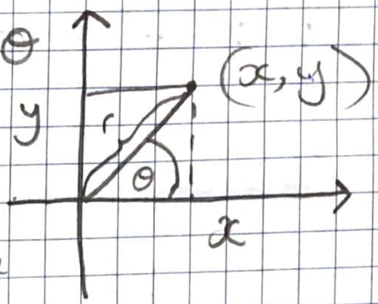
1. We have shown in class that $\mathbb{R}$ was uncontably infinite and $\mathbb{R} \subset \mathbb{C}$. $\mathbb{C}$ has an uncountably infinite subset so $\mathbb{C}$ must be uncountably infinite.

The set $\{1\} \times \{1\} \times \{1\} \times \mathbb{C} \subset A$, but $\{1\} \times \{1\} \times \{1\} \times \mathbb{C} \sim \mathbb{C}$, which is uncountably infinite. A has an uncountably infinite subset, which means it must itself be $\boxed{\text{uncountably infinite.}}$

2. A is the set of points in $\mathbb{R}^2$ whose polar coordinates (r, θ) satisfy $r^2 = (\sin\theta - 1)^2$

$x = r\cos\theta$



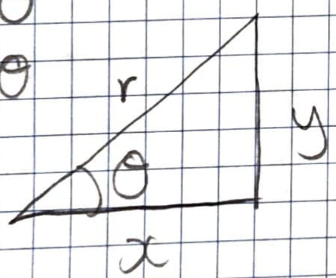
r distance between (x,y) and the origin $(0,0)$

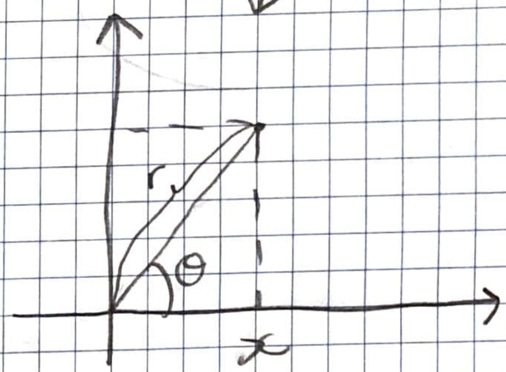
$r = \sqrt{(x-0)^2 + (y-0)^2}$

$= \sqrt{x^2 + y^2}$

$x = r\cos\theta$

$y = r\sin\theta$





$(x,y) \longleftrightarrow (r, \theta)$

Cartisian Coordinates

polar Coordinates.

$r^2 = (\sin\theta - 1)^2$

$\left(\sqrt{x^2+y^2}\right)^2 = \left(\frac{y}{r} - 1\right)^2$

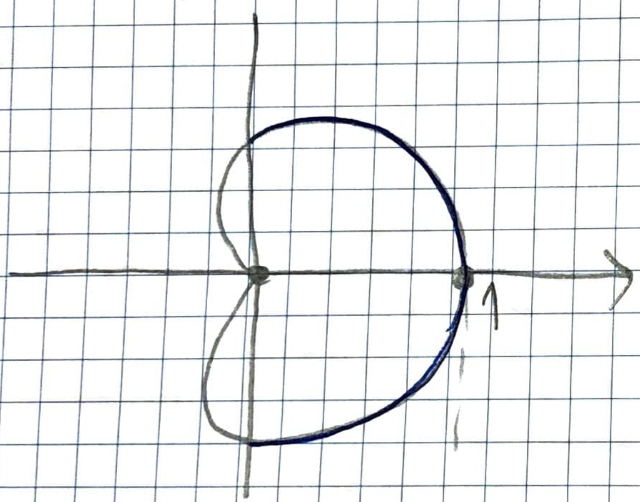
$= \left(\frac{y}{\sqrt{x^2+y^2}} - 1\right)^2$

$x^2 + y^2 = \frac{\left(y - \sqrt{x^2+y^2}\right)^2}{\left(\sqrt{x^2+y^2}\right)^2} = \frac{\left(y - \sqrt{x^2+y^2}\right)^2}{x^2+y^2} =$

$$\left(x^2+y^2\right)^2 = \left(y - \sqrt{x^2+y^2}\right)^2$$

$$= y^2 + x^2 + y^2 - 2y\sqrt{x^2+y^2}$$

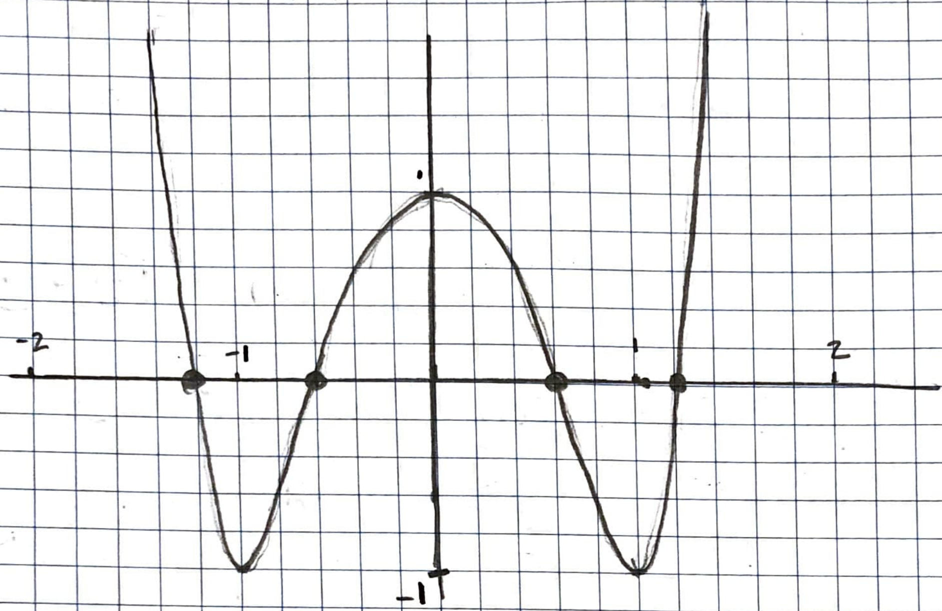
expression in terms of x and y.



2 pieces of the curve given by $r^2 = (\sin\theta - 1)^2$ that are in one to one correspondence with the interval $(0,1)$

So the set A is uncountably infinite as we proved in class that $(0,1)$ was uncountably infinite.

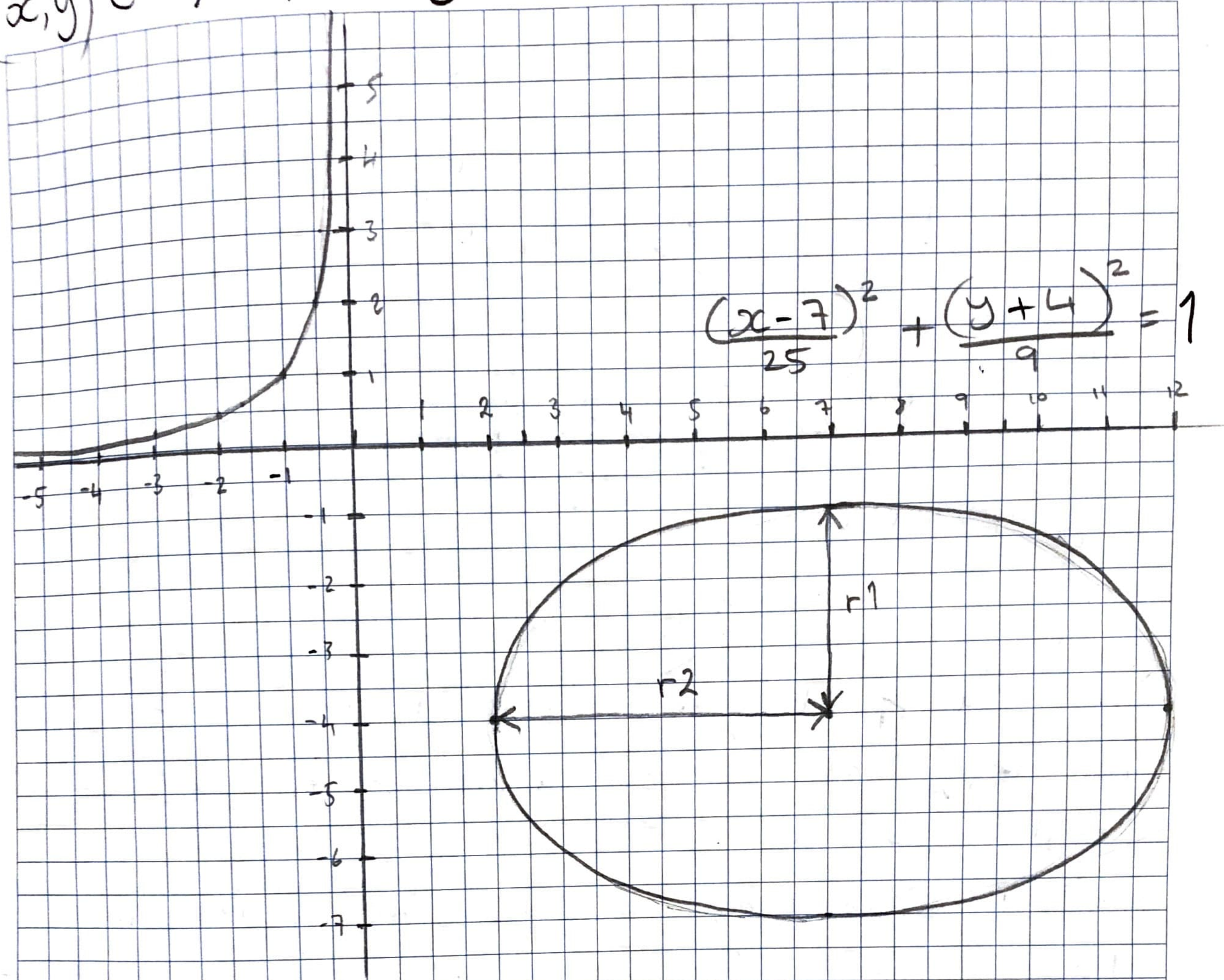
3. $x^6 - 3x^2 + 1 = 0$



x is a finite set with 6 elements, 4 real and 2 complex. However there's no condition for y and in Q1 we proved that $\mathbb{C}$ is uncountably infinite. The set $\{1.2377\} \times \mathbb{C} \subset A$ but $\{1.2377\} \times \mathbb{C} \sim \mathbb{C}$, which is uncountably infinite. A has an uncountably infinite subset so it itself must be uncountably infinite.

$\{(x,y) \in R, R^+ \mid 1+xy=0\}$

4



$$\frac{(x-7)^2}{25}+\frac{(y+4)^2}{9}=1$$

center = $(7,-4)$

$r2 = \sqrt{25} = 5$

$r1 = \sqrt{9} = 3$

The graphs $\{(x,y) \in R, R^+ \mid 1+xy=0\}$ and $\{(x,y) \in R^2 \mid \frac{(x-7)^2}{25}+\frac{(y+4)^2}{9}=1\}$ do not intersect. A dose not have any elements.

$P(A) = P(\emptyset) = \{\emptyset\}$. $P(A)$ is a $\boxed{\text{finite set}}$ with 1 element.

5) $\det |A| = ad - bc$

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$ad - bc = 1$

let $b = 0$, $c = 0$ and $d = \frac{1}{a}$

$a\frac{1}{a} - bc = 1$

$(a, b, c, d) \in \mathbb{R}^4$

The set $\{a\} \times \{0\} \times \{0\} \times \{\frac{1}{a}\} \subset A$.

$a \sim \mathbb{R}$ and in class we proved that $\mathbb{R}$ is uncountably infinite, therefore the subset $\{a\} \times \{0\} \times \{0\} \times \{\frac{1}{a}\}$ is uncountably infinite. A contains an uncountably infinite subset so A must be $\boxed{\text{uncountably infinite}}$.

6) We have two planes. They are not parallel because $\frac{1}{3} \neq \frac{2}{-1} \neq \frac{3}{2}$, so they intersect. let $z = a$.

$$x + 2y + 3a = 0$$
$$2(3x - y + 2a = 0)$$
$$6x - 2y + 4a = 0$$

$$7x + 7a = 0$$

$$x = \frac{-\cancel{7}a}{\cancel{7}}$$

$$\boxed{x = -a}$$

$$-6(x + 2y + 3a = 0)$$
$$-6x - 12y - 18a = 0$$
$$6x - 2y + 4a = 0$$

$$-14y - 14a = 0$$

$$\boxed{y = -a}$$

$a \in \mathbb{R}$ and $\mathbb{R}$ is uncountably infinite as we proved in class. The subset $\{a\} \times \{a\} \times \{a\} \subset A$ and $\{a\} \times \{a\} \times \{a\} \sim \mathbb{R}^3$. A contains a subset that is uncountably infinite so A must be $\boxed{\text{uncountably infinite}}$

7) $\left[(0 \cup \epsilon)^* \circ (1 \cup \epsilon)\right] \cap (A \circ A)^*$

$= \left(\left(0(00)^*\right) \circ 1\right) \cup \epsilon \cup (00)^*$

$(00)^*$ is countrably infinite so the given set is $\boxed{\text{countrablly infinite.}}$

(my justification is similar to the one in Q3 tutorial 19)

8) A^* is ~~the~~ a union of countrably infinite sets. We learnt in class that the union of 2 countrably infinite sets yeilds a countrably infinite set. So therefore A^* is $\boxed{\text{countrably infinite.}}$

9) The size of the smallest string that L can have is 4, 3 odd and 1 even or odd.
The number of strings with length 4 is $3 \times 3 \times 3 \times 6 = 162$
The number of strings with length 6 is $162 \times 6^2 = 5832$
The numbe of strings with length 8 is $162 \times 6^4 = 209,952$
and so on....
We get for strings with length $2n$, $n \geq 2$ we have 162×6^{2n} possible strings.

$f(n) = 162 \times 6^{2n}$ for $n \geq 2$, $n \in \mathbb{N}$

$f(n) \to \mathbb{N}$. We showed in class that $\mathbb{N}$ is countably infinite. We also learnt in class that the union of two countably infinite set yeilds a countably infinite set. L is the union of the countably infinite sets, $f(n)$ so L must be itself countabley infinite.

10. A language is a regular language if and only if some finite state machine recognizes it, for example $L = \{ w \in \{a, b\}^* : w = a^n b^{2n}$ for some $n \geq 0 \}$. in <u>not</u> regular.

A sequence of languages over a finite alphabet A such that x_i is not a regular language $\forall i \geq 1$ is the sequence $x_i \in \{a, b\}^*$: $x_i = a^n b^{in}$, for some $n \geq 1$

I asked in the Q and A session if an example was sufficient and the professor said it was ok.