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1 $P \rightarrow (Q \leftrightarrow \neg R)$

2 $P \vee \neg S$

3 $R \rightarrow S$

4 $\neg Q \rightarrow \neg R$

Conclusion $\neg R$.

5 $\neg S \vee P$ # 32, (2)

6 $S \rightarrow P$ # 21, (5)

7 $\neg S \vee P \rightarrow (Q \leftrightarrow \neg R) \vee \neg S$. # 15, (1)

8 $(S \rightarrow P) \rightarrow (Q \leftrightarrow \neg R) \vee \neg S$ # 21, (7)

9 $(Q \leftrightarrow \neg R) \vee \neg S$ # 10 modus ponens, (6), (8)

10 $\neg S \vee (Q \leftrightarrow \neg R)$ # 32, (9)

11 $S \rightarrow (Q \leftrightarrow \neg R)$ # 21, (10)

12 $S \rightarrow ((Q \rightarrow \neg R) \wedge (\neg R \rightarrow Q))$ # 22, (11)

13 $(S \rightarrow (Q \rightarrow \neg R)) \wedge (S \rightarrow (\neg R \rightarrow Q))$ # 25, (12)

14 $(S \rightarrow (\neg Q \vee \neg R)) \wedge (S \rightarrow (R \vee Q))$ # 21, (13)

15 $(\neg S \vee (\neg Q \vee \neg R)) \wedge (\neg S \vee (R \vee Q))$ # 21, (14)

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16 $\neg R \vee R \rightarrow S \vee \neg R$ #15, (3)

17 $P \vee \neg P$ #1 Law of the excluded middle.

18 $S \vee \neg R$ #10, (16), (17)

19 $((S \vee \neg Q) \vee \neg R) \wedge ((S \vee R) \vee Q)$ #33, (15)

20 $((S \vee \neg R) \vee \neg Q)$ #4, (19) Basis for simplification

21 $(S \vee \neg R) \rightarrow \neg Q$ # 21, (20)

22 $\neg Q$ # 10 modus ponens, (18), (22)

23 $\neg R$ #10 modus ponens, (4) applied to (22)

qed.

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Q2 There are two cases that arise, n is odd and n is even.

Case 1: n is odd.

$n = 2k + 1$

$(2k+1)^2 - 3(2k+1)$

$4k^2 + 4k + 1 - 6k - 3$

$4k^2 - 2k - 2$

$2(2k^2 - k - 1)$

Any number multiplied by 2 gives an even number, ~~so n is~~ This contradicts our assumption that n is odd.

Case 2: n is even

$n = 2k$

$(2k)^2 - 3(2k)$

$4k^2 - 6k$

$2(2k^2 - 3k)$

Both cases resulted in even values so if n is any integer, then $n^2 - 3n$ must be even.

qed.

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Q3 To Prove: $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$

To Show: $A \cap (B \setminus C) \subseteq (A \cap B) \setminus (A \cap C)$

$\forall x \in A \cap (B \setminus C) \Rightarrow x \in A$ and $x \in (B \setminus C)$

By definition $(B \setminus C) = B \cap C^c$

$x \in B \cap C^c \Rightarrow x \in B$ and $x \in C^c \Rightarrow x \notin C$

$x \in A$ and $x \in B$ so by definition
$x \in (A \cap B)$

$(A \cap B) \setminus (A \cap C) \Rightarrow (A \cap B) \cap (A \cap C)^c$

$x \in (A \cap B)$ and $x \in (A \cap C)^c$

$(A \cap C)^c$ (using De Morgan's) $\Rightarrow (A^c \cup C^c)$

$x \in A^c$ or $x \in C^c$

Since x cannot be in both A and A^c at the same time, we conclude $x \in C^c$

What we have shown:

$\forall x (x \in A \cap (B \setminus C) \Rightarrow x \in (A \cap B) \setminus (A \cap C)$

So $A \cap (B \setminus C) \subseteq (A \cap B) \setminus (A \cap C)$

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Q3. To show: $(A \cap B) \setminus (A \cap C) \subseteq A \cap (B \setminus C)$

$\forall x \in (A \cap B) \setminus (A \cap C) \Rightarrow x \in (A \cap B) \cap (A \cap C)^c$

(By the def. $X \setminus Y = X \cap Y^c$)

$x \in A \cap B$ and $x \in (A \cap C)^c$

$(A \cap C)^c \Rightarrow (A^c \cup C^c)$ (DeMorgan's)

$x \in A \cap B \Rightarrow x \in A$ and $x \in B$

$x \in A^c$ or $x \in C^c$

x cannot be both A and A^c so we conclude $x \in C^c$.

$x \in A$ and $x \in B \setminus C$

$(B \setminus C) \Rightarrow (B \cap C^c)$ (De Morgan's)

$x \in B$ and $x \in C^c$

so $x \in A \cap (B \setminus C)$

What we have shown.

$\forall x \, (x \in (A \cap B) \setminus (A \cap C) \Rightarrow x \in A \cap (B \setminus C))$

So $(A \cap B) \setminus (A \cap C) \subseteq A \cap (B \setminus C)$

So $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$

qed.

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Q4 $\forall A,B \in P(N \times N)$ $A \sim B$ iff $(A \setminus B) \cup (B \setminus A) = C$ and C is a finite set.

a) Reflexive iff $\forall A \in P(N \times N)$, $A \sim A$

$(A \setminus A) \cup (A \setminus A) = \phi$ empty set.

ϕ is a finite set $\Rightarrow A \sim A \rightarrow$ Reflexive

b) Symmetric iff $\forall A, B \in A$, $A \sim B \rightarrow B \sim A$.

Proof tha union is commutative.

$\forall x \in A \cup B$, by definition of union, $x \in A$ or $x \in B$. If $x \in A$ or $x \in B$ then by definition of union $x \in B \cup A$. Therefore $A \cup B \subseteq B \cup A$.

$\forall x \in B \cup A$, by definition of union, $x \in A$ or $x \in B$. This implies $x \in A$ or $x \in B$ so $x \in A \cup B$. Therefore $B \cup A \subseteq A \cup B$

Hence, $A \cup B = B \cup A$ so union is commutative.

So $(A \setminus B) \cup (B \setminus A) = (B \setminus A) \cup (A \setminus B) = C$ and C is finite $\Rightarrow B \sim A \rightarrow$ Symmetric

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C Transitive iff $\forall A, B, C \in P(\mathbb{N}\times\mathbb{N}), (A\sim B) \wedge (B\sim C) \Rightarrow A\sim C$

$A\sim B \Rightarrow (A\setminus B) \cup (B\setminus A) = D$ and D is finite.

$B\sim C \Rightarrow (B\setminus C) \cup (C\setminus B) = E$ and E is finite.

To show: $A\sim C$

meaning $(A\setminus C) \cup (C\setminus A) = F$ and F is finite.

$\forall x \in D,\ x \in (A\setminus B) \cup (B\setminus A)$

$(A\setminus B) = (A \cap B^c)$ by definition.

$x \in (A\cap B^c)$ or $x \in (B\cap A^c)$

$x \in A$ and $x \in B^c$ or $x \in B$ and $x \in A^c$

So $x \in A$ or $x \in B$ but it can't be both because of the $\cap$.

if $x \in B$, so $x \notin A$

if $x \in E$, $x \in (B\setminus C) \cup (C\setminus B)$

$x \in B\cap C^c$ or $x \in C\cap B^c$

$x \in B$ and $x \in C^c$ or $x \in C$ and $x \in B^c$

But $x \in B$ so $x \notin C$

This means $x \notin F$ becaus for $x \in B$,

$x \notin A$. Also $x \notin C$ so $x \notin (A\setminus C)\cup(C\setminus A)$.

if $x \in A$, so $x \notin B$

if $x \in E$, $x \in (B\setminus C) \cup (C\setminus B)$

so as shown above $x \in C$ and $x \in B^c$

or $x \in B$ and $x \in C^c$

but $x \notin B$ so $x \in C$.

But if $x \in C$ and $x \in A$

then $x \notin F$ because $x \notin (A \setminus C) \cup (C \setminus A)$.

Therefore $(A \sim B) \cap (B \sim C) \rightarrow (A \sim C)$

is not true.

So it is not transitive.

$\forall A, B \in P(N \times N)$, $(A \sim B)$ iff $(A \setminus B) \cup (B \setminus A) = C$

and C is a finite set. $A \sim B$ is not an equivalence relation because it's not reflexive, symmetric and transitiv. It is only reflexive and symmetric.

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