Simulating Electricity Forward Curves

by Olivier Croissant and Douglas Gardner

Abstract

A number of researchers have developed models for the evolution of electricity spot prices. In many applications, however, it is important to accurately model the entire electricity forward curve. In this paper, we propose a model for the evolution of the electricity forward curve that captures some key empirically-observed characteristics, namely prices spikes (with time-dependent intensities), mean reversion and nonlinear correlation between different forward maturities. The model is calibrated to forward curve data from the ECAR-CINERGY hub and found to fit both return and price distributions relatively accurately.

Introduction

The deregulation of electricity markets occurring around the world has dramatically changed the structure of these markets and the environment in which market players operate. One key impact of this innovation has been the rapid change in behavior of electricity prices, in particular, the extreme price volatility which seems to characterize most deregulated markets.

Understanding the behavior of electricity prices is critical for a number of reasons:

- pricing and hedging of physical and financial transactions that depend on electricity prices
- valuation of real assets (e.g., power generation, pumped storage, and transmission capacity) and determination of optimal operating policies (Johnson et al. 1999; Gardner and Zhuang 2000)
- estimation of the risk (and potential reward) in the market value of physical and financial transactions (Value-at-Risk)
- estimation of the risk of earnings shortfalls (Earnings-at-Risk)
- estimation of potential counterparty credit exposures (Aziz and Charupat 1998) for portfolios whose value depend on electricity prices.

Note that in many of these applications, modelling the entire electricity forward curve (as opposed to just spot electricity prices) is required. This includes any analysis in which projections of the future value of instruments is required, as in the Mark-to-Future risk framework (Dembo et al. 2000). For example, calculating value-at-risk over a specified time horizon will in general require the valuation of instruments that have not matured; this in turn requires forward electricity prices as seen at the horizon date. Modeling the entire forward curve is particularly important for accurately assessing the risk of instruments or portfolios that depend on multiple forward prices.

Johnson and Barz (1998) examine the behavior of electricity prices in a number of markets around the world and identify the following key properties:

- **Strong mean reversion**. Electricity prices are strongly mean reverting, tending to move toward the cost of production.
- **Seasonality**. The mean price of electricity varies by time of day, week and year.
- **Price-dependent volatility**. Price volatility increases with price level.
- **Price spikes**. Prices often jump, sometimes by an order of magnitude or more, and then just as suddenly, return to “normal” levels.

As one example, Figure 1 shows average daily spot electricity prices between June 2,1997 and September 3,1999 for the Comed market. While each market has its own unique characteristics, this series is not atypical. This series clearly demonstrates strong mean reversion, price spikes and seasonality. In particular, note that price spikes occur much more frequently in summer when electricity loads are high than in winter when loads are lower.

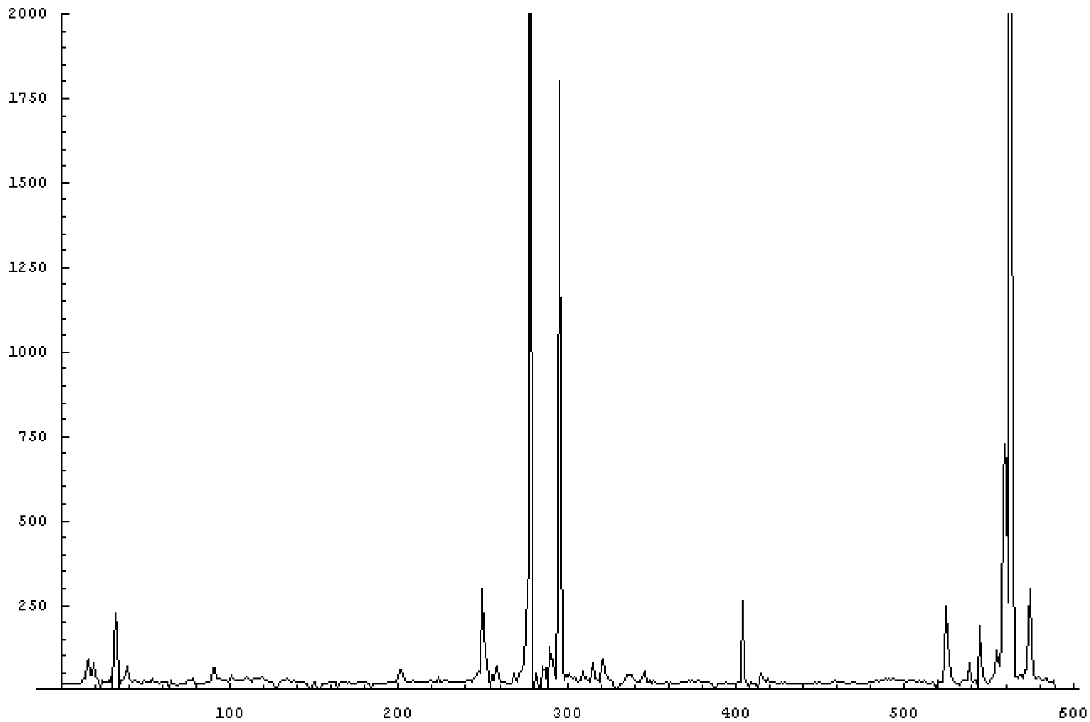

Figure 1: ELECTRICITY SPOT PRICES FOR COMED MARKET, JUNE 1997 TO SEPTEMBER 1999

While the approach taken here does not rely on a fundamental theory of electricity prices, it is still useful to consider the driving forces behind electricity prices. Electricity is produced by generation plants that convert other forms of energy (fossil fuel, hydraulic, nuclear, wind, etc.) to electricity. Given a set of available plants, the cost-minimizing strategy to meet a given level of demand is to use "merit-order" loading: load the plants in order of ascending operating cost until demand is met. Using this rule, it is possible to calculate the short-term marginal cost of production in a particular region.

Figure 2 shows a typical generation "stack". For low to medium levels of demand relative to total capacity, marginal running costs might be in the neighborhood of 10 to 20 dollars per megawatt-hour ($/MWh). Plants with marginal costs at this level might include nuclear, coal-fired, hydraulic and gas-combined cycle plants. These plants might be expected to run a large fraction of the year and hence are referred to as "baseload" plants. For high levels of demand relative to total capacity, marginal running costs increase rapidly, perhaps reaching levels in the neighborhood of 100 $/MWh. Plants with these level of costs include gas combustion turbines and old oil-fired plants. Note that these costs only include running expenses: the actual prices operators require to recover their capital costs could be significantly higher, particularly for peaking plants that might run for only a small number of hours during the year.

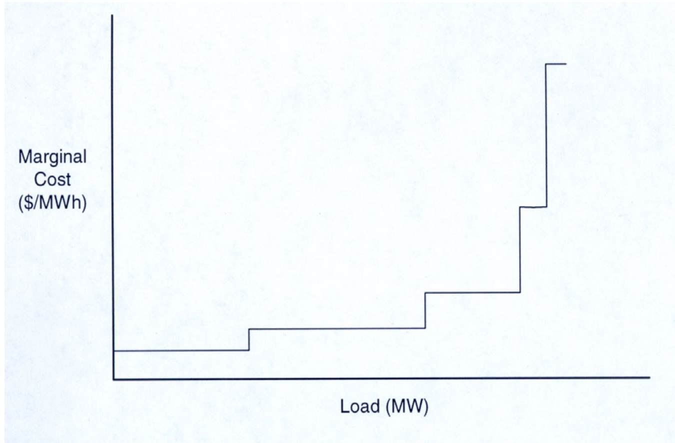

Figure 2: Generation stack

Electricity also has some unique physical characteristics that impacts its price behavior. A key characteristic is the difficulty (if not the impossibility) of storing it. As a result electricity must be produced as it is consumed, implying the market is segmented according to time of delivery. Electricity transport is costly and limited in capacity, leading to segmentation of the market by location.

These characteristics imply that as the load approaches the available capacity (and hence the "reserve" capacity approaches zero), the likelihood of price spikes dramatically increases. Price spikes are often precipitated by forced outages of plants that suddenly reduce reserve capacity. Price spikes are also exacerbated by the fact that in most markets, almost all consumers pay a flat rate for electricity regardless of the prevailing market price and hence the elasticity of demand is very low.

A number of models have been proposed to model electricity prices. Most of the models considered to date are spot price models. A common strategy is to incorporate both mean reversion and a jump component. For example, Clewlow and Strickland (2000) propose the following jump-diffusion model for spot electricity prices:

$$d \ln S = \alpha(\beta - \ln S)dt + \sigma dW + \theta dJ \tag{1}$$

where S is the spot price, α is the mean reversion rate, β is the long term reversion level, σ is the volatility, dW is an increment of a Brownian motion, θ is the proportional jump size (whose log is normally distributed) and dJ is a Poisson-distributed random variable which takes on a value of one when a jump event occurs and is zero otherwise.

Another method of capturing price spikes is regime switching models (Duan et al. 1999) that posit a set of market states (e.g., normal and high) with associated transition probabilities. Deng (1999) describes models that incorporate both jumps and regime switching. Other models incorporate information on loads and capacity into the spot price process based on the relationships described above (Davison et al. 2001).

Given any spot price model, forward prices for any maturity may be obtained by taking the expectation of the risk-neutralized spot process. Hence any spot model may be used to obtain a model for the evolution of the entire forward curve. In practice, however, the implied forward curve behavior is unlikely to be realistic for two reasons. First, spot models are usually calibrated based on spot price history only, ignoring the information present in historical movements of the forward curve. Second, while spot models may be capable of reproduce accurate spot price behavior, it should not be surprising that they would be ill-suited to mimic the richer behavior observed for full forward curve term structures. For example, using typical values for the mean-reversion rate α, the jump-diffusion model above does not capture the jumps in long-dated forward prices that are observed in practice.

The contribution of this paper is to propose a model that captures key empirically-observed features not just of spot prices but of the entire forward curve. We also describe a robust method by which the model may be calibrated, taking into account the movement of the entire electricity forward curve.

The plan for the paper is as follows. The next section describes the proposed model. The section following describes the methodology for calibrating the required parameters. This is then followed by an application to data from the ECAR-Cinergy trading hub in the US midwest, including a comparison with a simple mean-reverting model. The final section concludes.

Model description

Let $F(t, T)$ denote the instantaneous price of power delivered at time $t + T$ as seen at time t. We assume that this is a step function defined on a set of standard maturities (e.g., T = 1, 3, 5, 10, 30, 50, 100 days). The advantage of using a constant maturities (whereby T represents a fixed time ahead) as opposed to fixed maturities (whereby T represents a specific date) is the relative ease of calibration. The appendix describes the relationship between models defined using these two approaches.

To motivate our model, it is instructive to examine some historical data. Figure 3 shows forward prices for peak power at the ECAR-Cinergy hub over the period February 15 1999 to February 11, 2000. Each forward curve in this figure was bootstrapped using quoted forward contracts. Some important characteristics are evident:

- Spikes occur not just in spot prices but across a range of maturities.
- The magnitude of spikes decreases with maturity.
- The occurrence of spikes across different maturities is strongly correlated.
- Spikes are much more likely in summer than in winter.
- The spike-size distribution is highly non-normal.

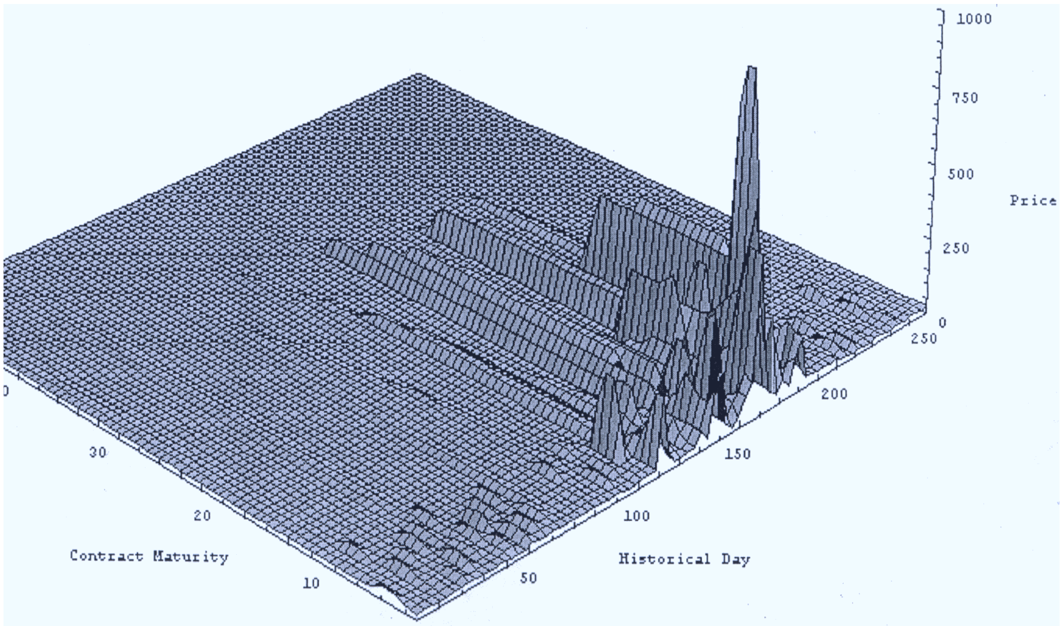

Figure 3: Forward prices for peak power (ECAR-Cinergy, February 15, 1999 to February 11, 2000)

These considerations motivate us to propose the following model:

$$dX_T = \alpha_T(\beta_T - X_T)dt + \sum_{i=1}^{n} \sigma_{Ti} dW_i + \Theta_T dJ, \qquad 1$$

where $X_T = \ln F(t, T)$, α_T is a mean reversion rate, β_T is a mean reversion level, σ_{Ti} is the volatility of maturity T associated with factor i, dW_i is an increment of a Brownian motion, dJ is a Poisson-distributed 0-1 random variable with intensity of $\lambda(t)$, and Θ_T is a random jump size, assumed to be independent of other random variables. We assume that Θ_T is independent of the other random variables but make no specific distributional assumptions.

The model may be viewed as an extension of the spot price model described in equation 1, in which we model multiple maturities instead of just the spot price. In this model, each maturity is affected by a set of Brownian motions together with a single time-dependent jump arrival variable common to all maturities. We employ the technique of normal rank correlations (Kendall and Gibbons 1990) to model the joint distribution of Θ_T across the set of maturities. The model hence allows nonlinear correlation between jump sizes of different maturities. Seasonality is handled via the time dependency of the jump arrivals. These features provide the necessary ingredients to capture the key features of observed forward curve price series.

Model calibration

Relative to spot price models, the model proposed here may have a large number of parameters depending on the number of maturities modelled. As a result, calibration of the model requires specialized techniques. The method of maximum likelihood estimation is one possibility. Our experience, however, is that it may perform poorly due to the nonconcave nature of the likelihood function leading to multiple local maxima. Maximum likelihood estimation may also produce unrealistically high jump intensities (Clewlow and Strickland 2000). Instead, we perform a structured calibration, by which we mean a staged approach. The steps involved are as follows (see Figure 4):

1. Estimate the mean reversion speed and level parameters α_T and β_T.
2. Using these parameters, compute the residuals, separating these into diffusion ("slow moves") and jump components, and estimate the jump arrival density.
3. Estimate the covariance structure of the diffusion component.
4. Estimate the structure of the jump component.
5. Fine tune the parameter estimates to fit the empirical price distribution.

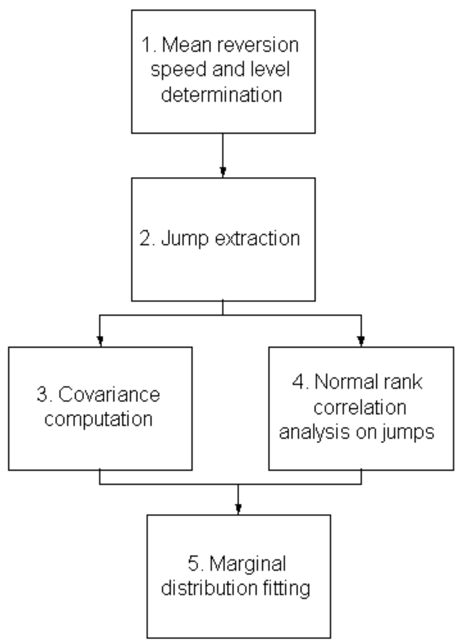

Figure 4: Structured calibration procedure

Each of these steps is discussed in detail.

Estimation of mean reversion parameters

We choose to focus on accurately estimating the mean reversion speeds α_T when prices are comparatively high, given that α_T may be a nonlinear function of X_T. The first step performed is thus to separate the data into high and low categories, where the associated boundary is chosen in order to provide a reasonably large number of "high" price data. Using data from the high category, we then employ an "envelope" technique on the set of points in the price-return plane to estimate the mean reversion speed parameters.[More details required - references?]

Since our focus is to use this model for risk management purposes, the mean reversion level parameters β_T are chosen so as to ensure that the expected prices produced by the model match the empirical mean, i.e., $E[F(t, T)] = \bar{F}_T$. Should the intent be to use this model for derivative pricing, the appropriate goal would be to ensure that forward prices are martingales, i.e., $E_t[F(t+\tau, T-\tau)] = F(t, T)$ for all T and $\tau \le T$.

Separation of the jumps and jump density extraction

Using the estimated drift parameters, one can then apply equation 1 to compute a set of residuals. These can then be classified as either diffusion or jump returns using a boundary that ensures that a reasonable number of returns are placed in each category for purposes of estimating the parameters of each component.

It is usually the case that jumps in prices occur more commonly during certain periods of the year, normally when load is highest, although price spikes may also occur in off-peak months since a higher proportion of capacity is normally off-line for planned maintenance. A number of different methods are available to estimate a time-dependent jump arrival frequency. One simple approach, employed here, is to simply make the jump arrival frequency proportional to the number of jumps observed historically in different times of the year (e.g., by month). Kernel estimation techniques are another possibility for estimating this function.

Computation of volatility functions

The diffusion returns are used to compute the volatility functions σ_{Ti} using principal components analysis. Principal components analysis is a standard approach for obtaining a reduced set of abstract factors that explain the maximum variation in the returns of a set of risk factors. Applications to energy forward curves are illustrated in Clewlow and Strickland (2000).

Normal rank correlation analysis on jump returns

The next step is to compute the parameters for the jump return distribution. Given the highly nonlinear nature of these returns, we employ the normal rank correlation technique, which is in fact a specific copula function. Normal rank correlation analysis requires that one first determine a transformation for each maturity f_T such that the associated return series is normally distributed. The assumption is then made that the set of transformed return series is multivariate normal. This technique allows us to model the highly nonlinear jump size distributions for each maturity and to accurately represent the relationship between the different maturities.

To estimate f_T, we first construct a quantile-quantile (Q-Q) plot (see Gilchrist 2000) of the jump returns. Briefly, for any random variable *x*, the quantile function *Q*(*p*) gives the value x_p such that $Pr(x \le x_p) = p$. Given a specific quantile function *Q*(*p*), *m* ordered returns $y_1 \le y_2 \le \ldots \le y_m$, and corresponding probabilities $p_r = (r - 0.5)/m$, the Q-Q plot is a plot of the points $(Q(p_r), y_r)$, $r = 1 \ldots m$. If the chosen quantile function fits the data, the Q-Q plot should be approximately a straight line at $45°$. As one example, figure 6 shows the Q-Q plot for the next-day (i.e., *T* = 1) jump returns for the ECAR-Cinergy data series, where we have employed the quantile function of the standardized normal distribution. It is clear from this plot that this series is far from normal.

The function f_T is chosen using nonlinear regression so as to best fit the Q-Q plot. In figure 6, the fitted function is shown as a solid line. Under this transformation, the Q-Q plot $(Q(p_r), f_T^{-1}(y_r)),\ r = 1\ldots m$ is approximately linear at $45°$, indicating the transformed data are approximately normal.

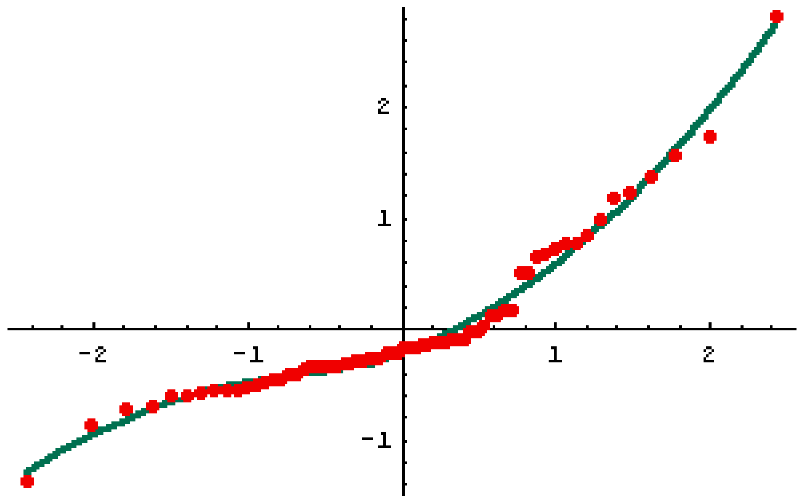

Figure 5: Q-Q plot and fitted function for next-day jump returns

Marginal distribution fitting

At this stage in the calibration procedure, estimates of all the parameter values for the model are available. Since the calibration of these parameters has focused on matching the return distribution, however, it is possible that the implied long-term price distribution may deviate in some respects from some target, which we take here to be the historical price distribution (see Pilipovic (1998) for a detailed discussion of this issue.)

In the marginal distribution fitting stage, therefore, we focus on making adjustments to the previously calibrated parameters to more accurately match both the return and price distributions. The method for doing this involves simulating the calibrated process and then comparing the resulting return and price distributions to the target distributions. Based on the deviations between these distributions, it is possible to determine the required modifications to the parameters to reduce the deviations. The process is repeated until the deviations decline to an acceptable level. Since the deviations are a function of both true differences between the distributions and differences due solely to sampling error of the simulated distribution, the efficiency of this process is enhanced by using the same random number seed in each iteration.

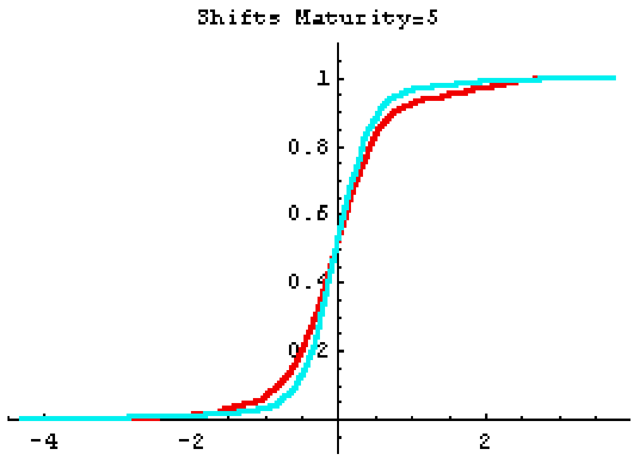

Figure 6: DISTRIBUTION OF THE RETURN (simulated=red,historical)

We can compute the difference between the two curves graphed in Figure 6.

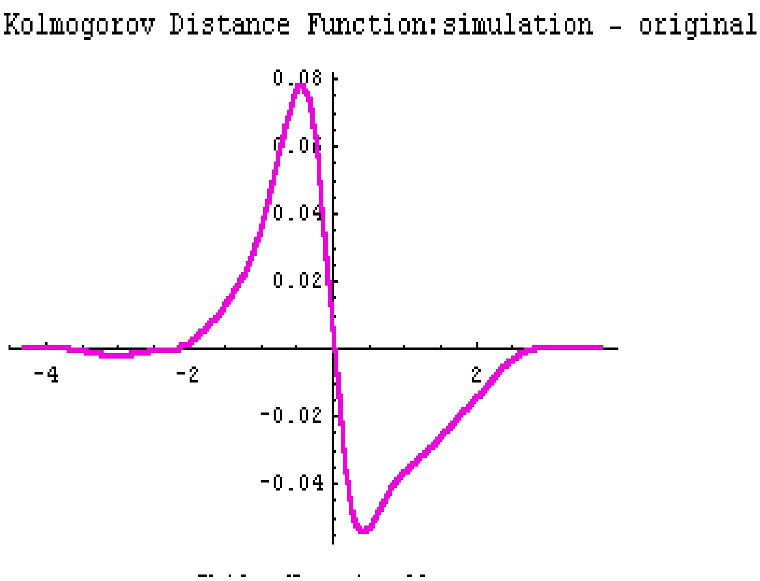

Figure 7: KOLMOGOROV FUNCTION ASSOCIATED WITH THE RETURNS

The shape of the curve illustrated in Figure 7 is typical of an error in the second moment of the distribution. This is why we use these curves to compute a tuning of the parameters. Such an adjustment can be very simply implemented through the multiplication of α_T, $\sigma_{T,i}$ and Θ_T by a common factor to adjust for the second moment.

Application

In this section we discuss the calibration of the model to data from the ECAR-Cinergy hub, illustrating the steps outlined above.

The first step in the analysis was to create elec

Electricity prices are commonly quoted for power delivered during both peak (normally 7 am -11 pm, Monday-Friday) and off-peak hours. In this study, the set of forward contracts for which prices were available include the following: next day (for power delivered during peak hours during the next peak day), balance of the week (next on-peak day until the following Friday), next week, balance of the month (for the remaining on-peak days in the month), and for each complete month afterwards. Given prices for each of these contracts, a complete electricity forward curve was constructed, using a technique similar to that used to bootstrap zero-coupon yield curves from quoted prices for bonds, swaps and futures.

Construction of the forward curve

We use one year of constant-maturity log prices estimated using interpolation techniques and implicit prices determined from OTC contract prices. The data used for the calibration are shown in Figure 3 as a 3D graphs of reconstructed forward curve (forward price of delivery of electricity between one day in the future and the next day). To perform this reconstruction, we used one year of OTC contracts from ECAR-Cinergy.

We have data every weekday, from FEbruary 15, 1999 to February 11, 2000 inclusive.

The data consist of forward contract prices that give prices for delivery of 1 megawatt of average power of electricity between two future prices

- **the next day contract :** provides power for one day the next business day (after the negociation)
- **balance of the week :** provides power between the next day and the end of the week (not the week-end)
- **next week** : provides power during the 5 business days of the next week following the week of the negociation.
- **balance of the month** : provides power between next day (included) and the last business day of the month
- **next month** : provides power between the first business day and the last business day of the next month following the month of the negociation.
- **N2 month** : provides power between the first business day and the last business day of the second next month following the month of the negociation.
- **N3 month** : provides power between the first business day and the last business day of the third next month following the month of the negociation.
- **N4 month** : provides power between the first business day and the last business day of the fourth next month following the month of the negociation.

Using a rule-based technique, we reconstructed a forward curve for every day of the study. Then, using a few relative maturity dates and some interpolation techniques, we constructed standardized maturities historical series .To understand the concept of rule based technique, we can compare it to an iterative algorithm that compute the price of every elementary forward contract based on the forward prices that are given. We call elementary forward contract for a given set of date $\{t_1, t_2, .., t_n\}$, a forward contract associated with the delivery of power between t_k and t_{k+1} . If we know the price of the forward $\{t_1, t_3\}$ and the price of the forward $\{t_2, t_3\}$ we can deduce the price of the forward $\{t_1, t_2\}$ by simple difference. If we know the price of the the forward$\{t_1, t_2\}$ and the price of the forward $\{t_3, t_4\}$ but we cannot deduce the price of the forward $\{t_1, t_4\}$ then we will interpolate the price of the forward $\{t_2, t_3\}$.

By accumulating enough rules of this type, we can build a logical system that can compute all elementary forward contracts. Then by linear interpolation we can price all contracts providing power between two arbitrary dates. So we choose a set of relative maturities : H={1 day, 3 days, 5 days, 10 days, 20 days ,30 days, 50 days} and recomputed the elementary forward prices associated with this system of relative dates for every date in the database. Then the calibration algorithm uses these relative maturity forward price to compute the dynamic. This dynamic has of course a seasonality feature that needs to be handled

Parameter estimation for the mean reversion

As it is always the case for the estimation of an ARMA model with non linear noise, we begin by estimatimating the parameters of the mean reversion. The mean reversion is mainly used to come back from jumps that send the price very high and this mean reversion is non existent when the price is low, therfore we will calibrate the speed only on moves that bring the prices from very high levels (called the high price “region”) to low prices levels (called the low price region). These moves are plotted on Figure 8 where we put in X axis the level of price originating the move and in Y axis the size of the downmove . A classical argument could lead us to compute the least square strait line that goes through these points to get, as a slope the mean reverting speed. But if we do that, then we will notice much more jumps that what we see in reality. The reason is that for down moves , an aditional jumps makes the complete down move unobservable. Therefore , if we assume a certain linearity between the down move and the height of the starting price, we should compute the envelope of all these points to find the mean reverting dependency (speed and long term limit). Here some robustness issue forbid us to do that. We have to leave some headroom for the statistical error. We built an iterative algorithm that start with the least square line, then converges toward the envelope when the number of itartion tend toward infinity. We choose to stop the iterations at 3. This number of iteration can thought as a parameter and 3 iterations give us the optimal fit between the distribution of the prices of the simulations and the distribution of the prices of the original price serie..

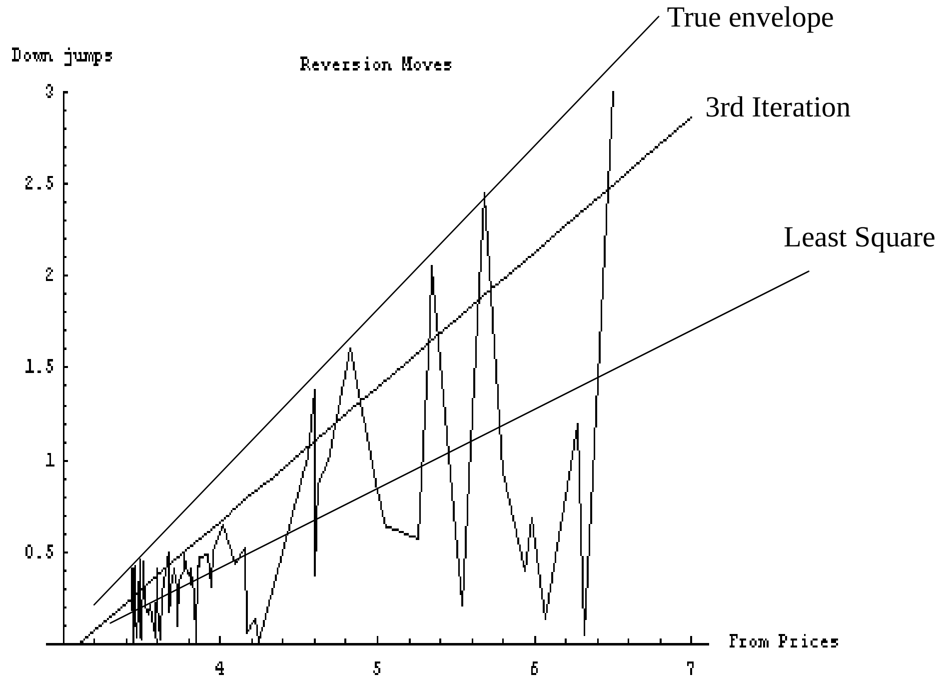

Figure 8: COMPUTATION OF THE WEAK ENVELOPE OF THE MEAN REVERSION

Estimation of the jumps

The next thing to do after the mean reverting effect has been filtered out, is to extract and calibrate the jumps.

Using the cyclicity of yearly data, we build the jump density the following way: for every successive couple of dates, we define the density as :

$$d(t) = \frac{k}{(t_2 - t_1)} \qquad t_1 < t \le t_2$$

The constant k is then determined by a normalization consideration. : here it is that the integral of the is function over one year equal the total number of jumps over one year.

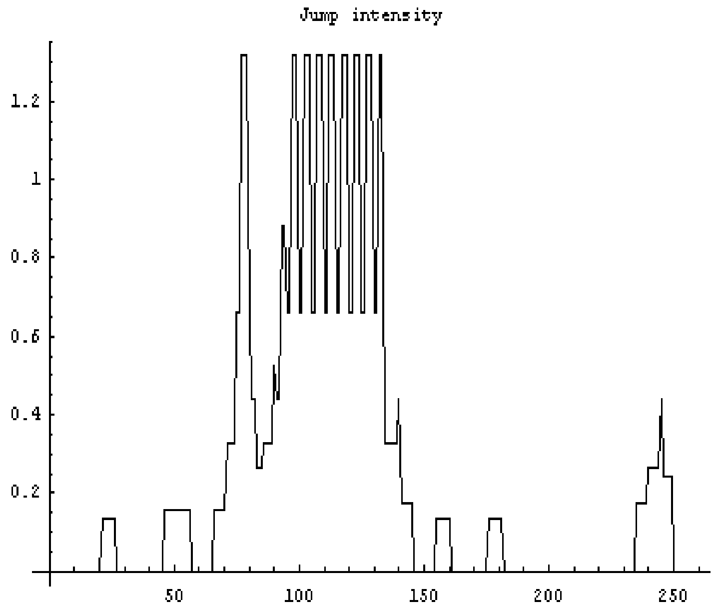

Figure 9: JUMP DENSITY FOR ONE YEAR OF DATA

Then we perform the jump analysis per maturity by parametrization of the QQ plots. THe QQ plots (Quantile-Quantile) are defined by plot of the function f(x) obtained by :

$$f(x) = \Phi^{-1}[\Psi[x]]$$

where Φ is the distribution function of the gaussian law, and Ψ is the distribution of the law to analyze.

The QQ plots are used to assess the nonlinearity of the transformation that make f(x) a linear function. If f[x] is a linear function then le law Ψ is gaussian. For all the maturity we observe a non linear function f[x] that we parametrize using a least square optimization on a family of piecewise quadratic polynomials with up to 3 pieces. We show in Figure 10 two maturities as an example: the one- to three-day contract and the five- to 10-day contract, which are the most meaningful for the jump analysis. (The three- to five-day contract has an intermediate behaviour).

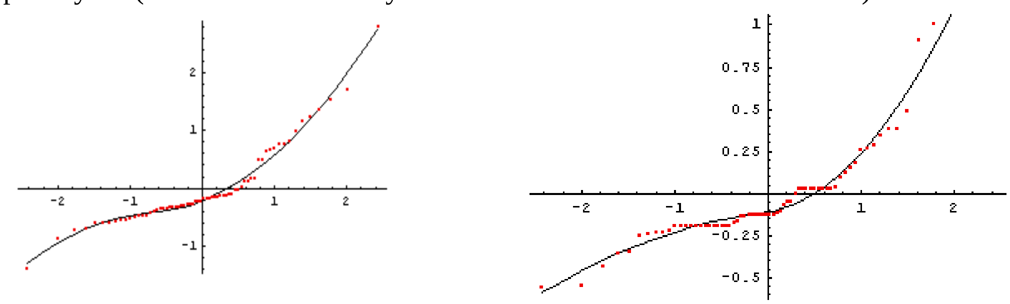

Figure 10: FITTING OF THE EMPIRCAL DISTRIBUTION BY THREE-PIECE QUADRATIC FUNCTIONS

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Kolmogorov analysis of the simulation

Having good result for the distribution of the returns does not imply good results for the distribution of the prices. To see that we take as example the case where we keep the parameters of mean reversion determined at the first step of our analysis without determining the long term mean reverting level.In this case, the distribution of the market returns $X_n - X_{n-1}$ (conditional distribution) is correct but the distribution of log prices X_n (unconditional distribution) is not. For example, the graphs of the conditional and unconditional distributions for the short term contract are illustrated in Figures 11 and 12, respectively.

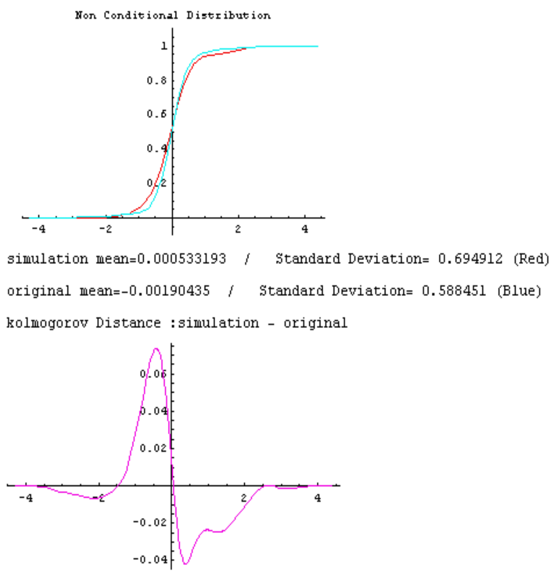


Figure 11: DISTRIBUTION OF THE RETURNS WITH MEAN DETERMINED BY LOCAL ANALYSIS (SIMULATION IN RED, HISTORICAL, AND KOLMOGOROFF FUNCTION)

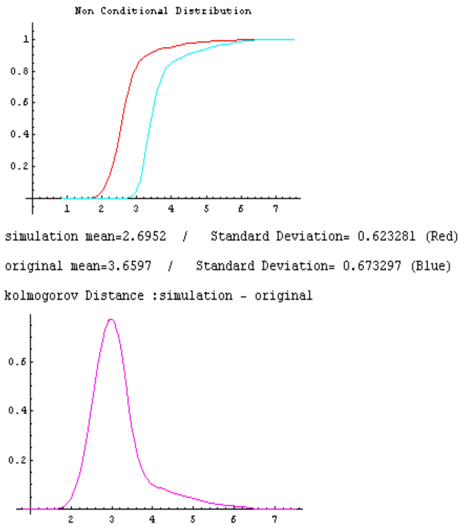


Figure 12: DISTRIBUTION OF THE LOGPRICES WITH MEAN DETERMINED BY LOCAL ANALYSIS (SIMULATION IN RED, HISTORICAL, AND KOLMOGOROFF FUNCTION)

If the mean-reverting level is determined independently by using the diffusion sector, both the conditional and unconditional distributions are more consistent as illustrated in Figure 13 and Figure 14, respectively. The diffusion sector is defined by the moves that are not considered as a jump or a reversion from high price following a jump.This sector is identified, when the jumps density is extracted.

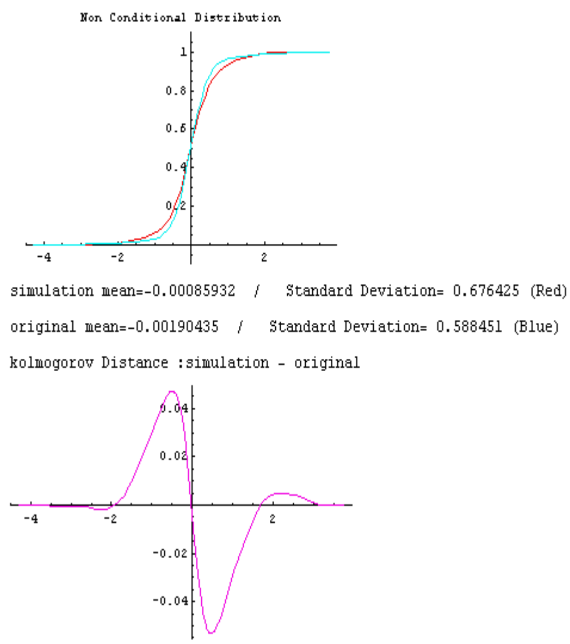


Figure 13: DISTRIBUTION OF THE RETURNS WITH MEAN DETERMINED BY GLOBAL ANALYSIS (SIMULATION IN RED, HISTORICAL, AND KOLMOGOROFF FUNCTION)

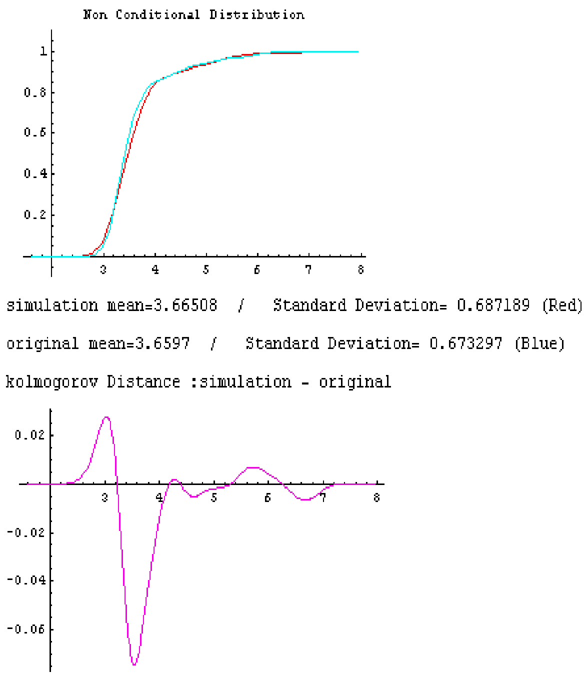
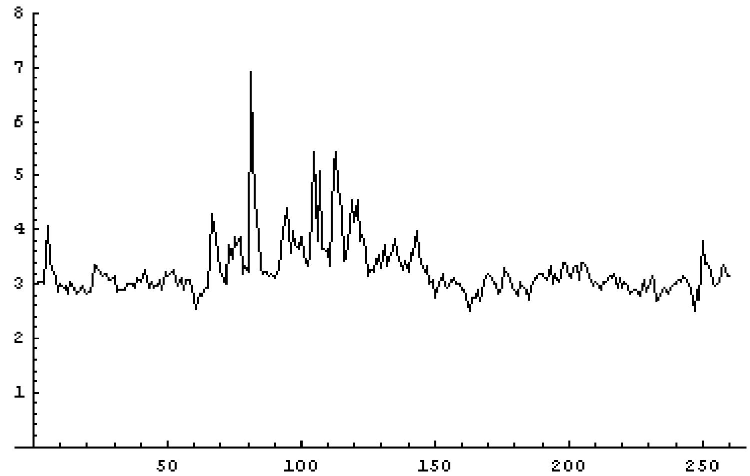


Figure 14: DISTRIBUTION OF THE LOGPRICES WITH MEAN DETERMINED BY GLOBAL ANALYSIS (SIMULATION IN RED, HISTORICAL, AND KOLMOGOROFF FUNCTION)

Example of simulation

The best to see if our framework is acceptable is to look visually at examples of simulations and compare it to original price series, from which the calibration has been obtained. This is done in graphs 15. We check that the prices reached during maximum jumps are in an acceptable price range (beetween $1000 and 10000 usually). We also check that the volatility between jumps still remain in acceptable zones. (not too small, that would indicate a mean reversion too strong, and not too high , that would indicate a too picky jump extraction job).

Simulation



Original
Time Series

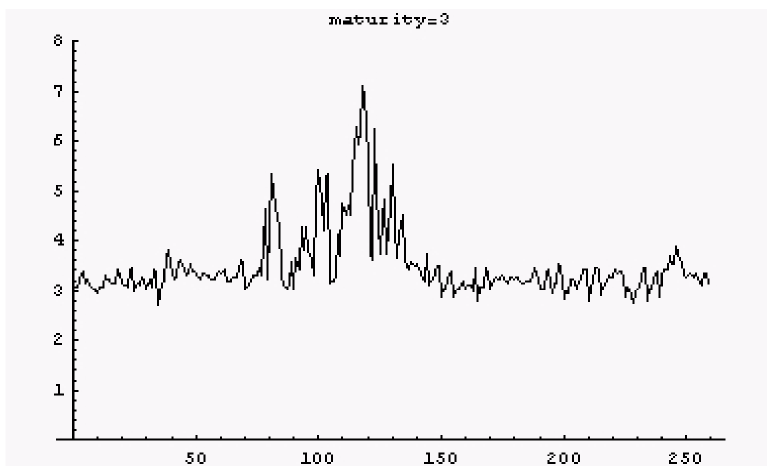


Figure 15: VISUAL COMPARAISON BETWEEN THE HISTORICAL LOGPRICES OF 3 DAYS MATURITY CONTRACT AND THE SIMULATION

Comparison with a Standard MR process

According to our philosophie to use kolmogoroff curves of the returns and the logprices to fit calibrate the model, Figures 13 to 15 show that the calibration procedure performed well. To better appreciate the benefits brought by the jump diffusion model, we try to calibrate a much simpler model with no jumps.

So we performed a calibration of a standard mean reverting brownian motion on the log of the prices, and then do simulation and compute the Kolmogorov function for the returns and the log prices. The results are presented in Figure 16,17 and 18.

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Simulation

Simple Mean Reverting

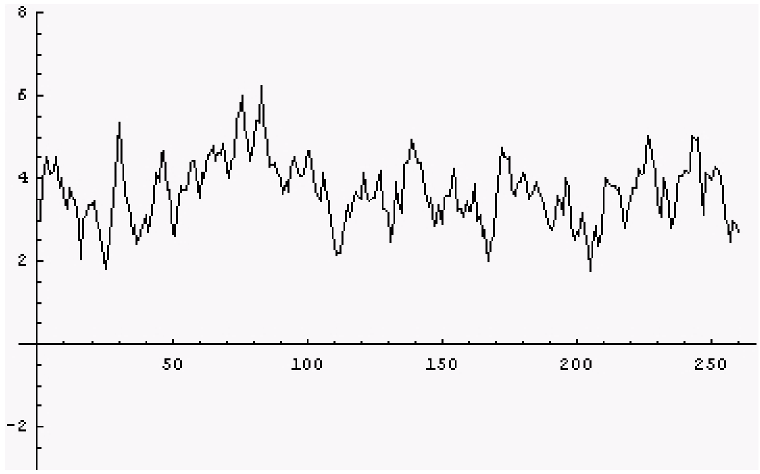

Figure 16: STANDARD MEAN REVERTING SIMULATION OF NEXT DAY LOGPRICE

in Figure 16, we see that the seasonality does not appear in the simulation, and we observe an every day volatility which is extremely high compared to the one observed outside crisis time.

This impression is corroborated by the computation of the distribution of the prices and its Kolmogorov function, as shown in Figure 17 and Figure 18, respectively,

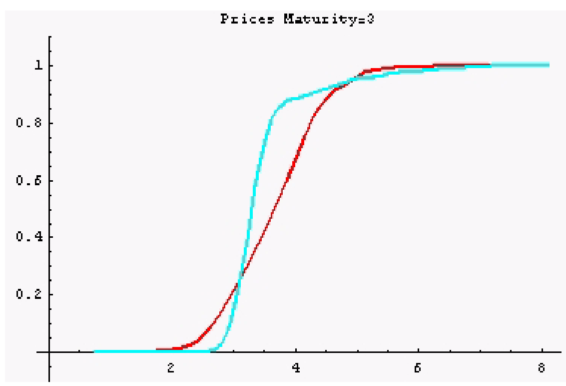

Figure 17: DISTRIBUTION OF THE RETURNS (SIMULATION OF STANDARD MEAN REVERTING OF NEXT DAY ,HISTORICAL)

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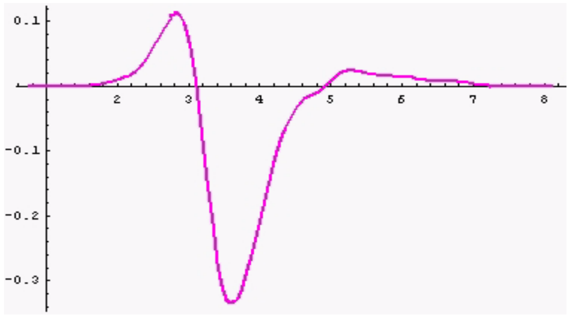

Figure 18: KOLMOGOROFF FUNCTION FOR THE RETURNS OF A SIMULATION OF MEAN REVERTING OF NEXT DAY PRICE

We observe that despite a common overall volatility (see Table 1 below), the mid-centre region is overemphasized in the simulation while the extreme are under estimated. The fact that both models exhibits the same volatility is associated with the fact that the integral of the left tail of the kolmogorof function is roughly equal to the integral of the rigth tail (on region where this function is positive). The skewness of the jumps is apparent in the dissymetry of the kolmogoroff curve.

The jumps are clearly not understood in the model, leading to a lack of high values for the prices.

The values of the Kolmogorov function give us an idea of how badly the MR model perform with respect to the jump diffusion model. The error in the cumulative distribution function reaches a maximum level of 30% over an extended region.

A good measure of the improvement is given by the improvement of the following measure

$$E = \int_{-\infty}^{\infty} |K(x)| dx = \int_{-\infty}^{\infty} \left| Cdf_{Empirical}(x) - Cdf_{Simulation}(x) \right| dx$$

This measure describes the way the distribution of probability of the simulation fit the distribution of probability of the historical serie. we perform these measure for the returns and the logprices

In this case we get an improvement of a factor 10 forthe log price distribution

Table 1:

	Classical Mean Reverting Process	**The suggested Model**
$E_{Returns}$	0.01261	0.01207
$E_{LogPrices}$	0.1423	0.01406

We have chosen here a model where we do not worry about seasonality of the jump. The reason is that doing the comparison with a jump diffusion on distribution criteria does not involve time. Distributions are totally insensitive to when the events happen, they are just sensitive to wether they happen or not.

Despite The fact that the calibration matches the volatility of the original process , we may have a bad matching of the long term distribution of the prices .As usual in the mean reverting problem the misfit between the return distribution and the price distribution come from a bad parametrization of the mean reversion. In the standard MR model it is underestimated. But even if we introduce the right value for the mean reversion as computed in the jump diffusion model, it does not improve the situation because the brownian motion is completely dampens the moves and we cannot see the " high prices" of the original serie". Having the volatility equal to the one of the original time series is not enough, only jumps can give to the process the needed energy to reach high prices.

Drawbacks and future developments

Using refinements of the preceding tuning ideas, it is possible to improve the fitting of the shifts $X_n - X_{n-1}$ and the log prices X_n. The main difficulty to get in the same time good kolmogorov measures for the shifts and for the log prices is the assumption of linearity of the mean reversion is too naive. In reality, when the prices get very high, they can stay one or more days at very high level. This corresponds to the points that are below the envelopes discussed in the first paragraph.A way out for this problem is to create a true regime switching mechanism in the process.

Another problem is the seasonality of the size of the jumps. This effects is visible in the historical series, but difficult to capture. With a single time series, we can extract a distribution, but our assumption is for this extraction is a stationarity of the process. If we try to extract an accurate time dependency, we loose the stochasticity of the phenomenon. A possible solution is to use the time series to extract a time dependency, but in the same time we have to keep some stochasticity. The balance is delicate and need more research to build robust estimators.

Conclusion

In the case of markets where very high volatility and seasonality are the problem, our model show flexibility and good distributional properties that classical models built only on brownian motion cannot match. Electricity markets show these characteristics, and due to the fundamental non storability of the good, in a context where the demand is not very elastic, we think that it will remain the case (imposing a political cap to prices can lead to disasters that the recent californian example is here to illustrate). It is therefore expected that rational risk management need accurate models. Despite it is improvable, this electricity model can be used for risk management and pricing problems that are not too seasonal. Its calibration is robust and fast.

Appendix A : relationship between Fix Maturity and Constant Maturity

Let $F_{Fixed}(t, T)$ denote the instantaneous price of power delivered at time T as seen at time t.

$$F_{Cons}(t, T) = F_{Fixed}(t, T + t)$$

The prices are modelled with diffusive processes and jumps. This increases the number of parameters of the model but it is needed by the shape of observed time series of prices. Proof that the constant maturity approach is equivalent to the fixed maturity approach.

So the dynamic term are unchanged and the difference is just a shift that translates to a different long term limit β_T and a different long-term limit α_T.

$$
\begin{aligned}
d(F_{cons}(t,T)) &= (F_{cons}(t+dt,T) - F_{cons}(t,T)) \\
&= F_{fixed}(t+dt, t+dt+T) - F_{fixed}(t, t+T) \\
&= F_{fixed}(t+dt, t+dt+T) - F_{fixed}(t+dt, t+T) \\
&\quad + F_{fixed}(t+dt, t+T) - F_{fixed}(t, t+T) \\
&= \frac{\partial F_{fixed}}{\partial T}(t, t+T)dt + d(F_{fixed}(t, t+T))
\end{aligned}
$$

So we can use the constant maturity approach to calibrate the diffusion and the jumps even if we want to calibrate a fixed maturity approach.

Appendix B: Mean Reversion Level Determination and Mixed measure inference

During the preceding step a linear function has been determined as the linear superior envelope for the mean reversion. This gives us an indication of how to revert from high prices to lower prices, however this is not very precise regarding the slow-move sector. This indicates that we need to adjust the long term mean reversion independently from the mean-reverting analysis.

The second indication is that for markets that are mean reverting, the adjustment of the long-term mean reversion is performed in addition to the adjustment of the shifts due to the jumps and diffusion terms. In other words, it is possible to fit perfectly the diffusion terms and the jump diffusion terms without fitting the price distribution. The mean reversion adjustment can be used:

- as it is here, to fit the distribution of an empirical series of price; this approach is called a natural measure inference,
- or, to match derivative prices in a risk neutral manner: we will call this approach a mixed measure inference.

The mixed measure inference is justified by the following result:

If in addition to a historical series of prices, we are given a series of current forward prices F_T and a process X_T following Equation 2 and fitted to the historical series of forward curve,

$$dX_T(t) = \alpha_T(\beta_T(t) - X_T(t))dt + \sum_{1 \le i \le N} \sigma_{T,\,i} dW_i(t) + \Theta_T(t)dJ(t) \qquad \text{(equation 2)}$$

Then there exists a shift function ω_T and a process $\bar{X}_T$ following the equation 3

$$d\overline{X_T}(t) = \left(\alpha_T(\beta_T(t) + \omega_T - \overline{X_T}(t))dt + \sum_{1 \le i \le N} \sigma_{T,\,i} dW_i(t) + \Theta_T(t)dJ(t)\right) \qquad \text{(equation 3)}$$

such that $\bar{X}_T$ reprice exactly the forward prices F_T and also fit the historical series of prices in the same sens that X_T

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Despite its “Girsanov theorem” appearance, It is just an application of the implicit function theorem. This shows that the pricing of derivatives on mean reverting markets crucially depends on the drift structure. This drift has a different origin than the one for arbitraged markets. (i.e. martingale measure including a deterministic bond structure). It is just a reparametrization of the forward curve.

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