Optimization in Finance

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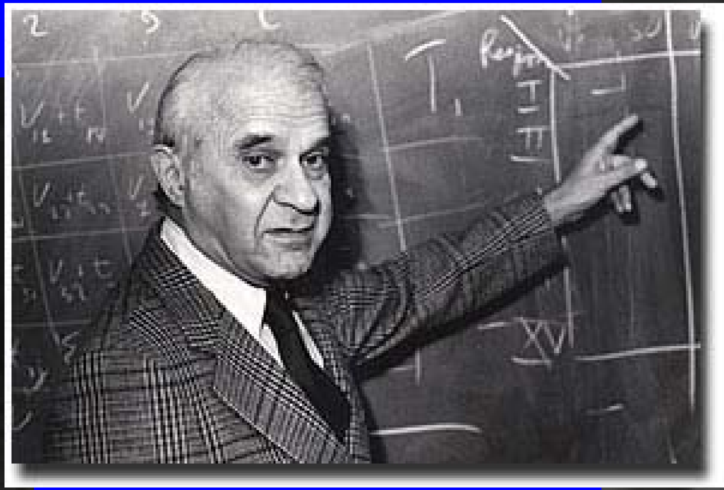
Kantorovich, Leontief, Dantzig.



Plan

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Introduction

Understanding Duality

Use of Optimization in ALM and AM

Solver Evolution

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Version	Year	Time (CPU seconds)
1.0	1988	57,840
3.0	1994	4,555
5.0	1996	3,835
6.5	1999	165
7.0	2000	161

Table 1: Performance Improvement on One Linear Program

A 20 000 variables typical problem

Hardware Evolution

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Machine/Chip	Year	Time (CPU seconds)
Sun 3/150	1985	44,064.0
Intel Pentium (60 MHz)	1992	222.6
IBM RS/6000 Model 590	1993	65.0
SGI Power Challenge R8000 75 MHz	1994	44.8
Intel Pentium III (550 MHz)	1999	31.2
AMD Athlon (650 MHz)	1999	22.2
Dell Pentium IV (1.7 GHz)	2001	6.1

Table 2: Time for dual simplex to solve pilot on various machines

Total technological Gain

We can estimate the technological gain in 15 years :

7200 (hard) X 360 (soft) = 2 500 000 .

Exemple of a problem solved using modern tools:

69,418 constraints, 612,608 variables, et 1,722,112 matrix elements which are different from 0

in 119 seconds (source ILOG)

Understanding Duality

The Dual Lagrange Function

$$\left[\begin{array}{l} Min\{f(x)\} \\ g(x) = 0 \\ h(x) \leq 0 \end{array}\right.$$

$$\left[\begin{array}{l} L(\lambda, \mu) = Inf_{x \in D}\{f(x) + \lambda g(x) + \mu h(x)\} \\ \mu \geq 0 \end{array}\right.$$

Check the feasible x

Very important : $L(\lambda, \mu) \leq f(x\ \ )$

Dual Optimal Pb

$$Sup_{\lambda,\mu \geq 0}\{L(\lambda,\mu)\}$$

Weak Duality

$$\underbrace{L(\lambda^*,\mu^*) \leq f(x^*)}_{\text{Duality Gap}}$$

Strong Duality

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If the **function f,g,h are convex** and if there is a **feasible point Inside the relative interior of the feasible set,** (Slater's Conditions) then :

Duality Gap =0

$$\Leftrightarrow \boxed{L(\lambda^*, \mu^*) = f(x^*)}$$

Lagrange Dual of a Standard LP

$$\begin{cases} Min\{c.x\} \\ Ax = b \\ x \geq 0 \end{cases}$$

$$L(\lambda, \mu) = Inf_x \{cx - \lambda x + \mu(Ax - b)\}$$

$$= -\mu b + Inf_x \{(c + {}^t A \mu - \lambda) x\}$$

$$= \begin{cases} -\mu b \\ -\infty \end{cases}$$ if $$c + {}^t A \mu - \lambda = 0$$

Dual Pb of a Standard LP

$$L(\lambda, \mu) = \begin{cases} -\mu b \\ -\infty \end{cases}$$ if $$c + {}^t A \mu - \lambda = 0$$

$$Max_{\lambda \geq 0, \mu} \{ L(\lambda, \mu) \}$$

Removing the slack variable: $\lambda \geq 0$

$$\begin{cases} Max \{ -b\mu \} \\ {}^t A \mu \geq -c \end{cases}$$

Lagrangian and Saddle Point (1)

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$$\left[\begin{array}{l} L(\lambda,\mu) = Inf_{x\in D}\{f(x)+\lambda g(x)+\mu h(x)\} \\ \qquad \mu \ge 0 \end{array}\right.$$

Dual function

$$\left[\begin{array}{l} H(x) = Sup_{\lambda,\mu\ge 0}\{f(x)+\lambda g(x)+\mu h(x)\} \\ \qquad x \in D \end{array}\right.$$

$$= \left[\begin{array}{ll} f(x) & \text{if } g(x)=0 \text{ and } h(x)\le 0 \\ +\infty & \text{otherwise} \end{array}\right.$$

So $Inf_{x\in D} H(x) = f(x^*)$

Lagrangian and Saddle Point (2)

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Lagrangian

So we found a function $l(x,\lambda,\mu) = f(x) + \lambda g(x) + \mu h(x)$

Such $Inf_{x \in D}\{Sup_{\lambda,\mu \geq 0}\{l(x,\lambda,\mu)\}\} = f(x^*)$

and

$Sup_{\lambda,\mu \geq 0}\{Inf_{x \in D}\{l(x,\lambda,\mu)\}\} = L(\lambda^*,\mu^*)$

Weak Duality $\Leftrightarrow$ $Sup_x\{Inf_y\{s(x,y)\}\} \leq Inf_y\{Sup_x\{s\{x,y\}\}$

Always True

Strong Duality $\Leftrightarrow$ $Sup_x\{Inf_y\{s(x,y)\}\} = Inf_y\{Sup_x\{s\{x,y\}\}$

Saddle point Property

Sup[Inf] < = Inf[Sup]

10	1	3	5	10
6	6	1	6	2
7	5	1	7	2
Max=				
Min=	1	1	5	2

A Global Inequality

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A perturbated problem
$$\begin{bmatrix} Min\{f(x)\} \\ g(x) - u = 0 \\ h(x) - v \le 0 \end{bmatrix}$$

$$L_{u=0,v=0}(\lambda^*, \mu^*) \le f(x) + \lambda^* g(x) + \mu^* h(x)$$

X feasible =>
$$L_{u=0,v=0}(\lambda^*, \mu^*) \le f(x) + \lambda^* u + \mu^* v$$

Minimizing over x
And
Strong duality =>
$$L_{u=0,v=0}(\lambda^*, \mu^*) \le L_{u,v}(\lambda^*, \mu^*) + \lambda^* u + \mu^* v$$

Sensitivity of the Optimum

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We have $$p^*_{u,v} \geq p^*_{0,0} - \lambda^* u - \mu^* v$$

Right Taylor Development of $p^*_{u,v}$

$$p^*_{0,0} + \frac{\partial p^*_{0,0}}{\partial u} u + \frac{\partial p^*_{0,0}}{\partial v} v + \varepsilon \geq p^*_{0,0} - \lambda^* u - \mu^* v$$

and

Left Taylor Development of $p^*_{u,v}$

$$p^*_{0,0} - \frac{\partial p^*_{0,0}}{\partial u} u - \frac{\partial p^*_{0,0}}{\partial v} v + \varepsilon \geq p^*_{0,0} + \lambda^* u + \mu^* v$$

$$\Rightarrow \quad \frac{\partial p^*_{0,0}}{\partial u} = -\lambda^* \qquad \frac{\partial p^*_{0,0}}{\partial v} = -\mu^*$$

Complementary Slackness

Strong Duality

$$f(x^*) = L(\lambda^*, \mu^*) \le f(x^*) + \lambda^* g(x^*) + \mu^* h(x^*)$$

$$\Rightarrow \quad 0 \le \lambda^* g(x^*)$$

But we know that $g(x^*) \le 0$ and $\lambda \ge 0$

$$\Rightarrow \quad \lambda^* g(x^*) = 0$$

KKT Conditions

If there is a minimum to $f(x\ ) + \lambda^{*} g(x\ ) + \mu^{*} h(x\ )$

Differentiability $\Rightarrow$ $\nabla f(x\ ) + \lambda^{*} \nabla g(x\ ) + \mu^{*} \nabla h(x\ ) = 0$

$(x^{*}, \lambda^{*}, \mu^{*})$

optimum

$$\Leftrightarrow \left[\begin{array}{c} \nabla f(x\ ) + \lambda^{*} \nabla g(x\ ) + \mu^{*} \nabla h(x\ ) = 0 \\ \lambda g(x\ ) = 0 \\ g(x^{*}) \le 0, h(x^{*}) = 0, \lambda^{*} \ge 0 \end{array}\right.$$

Karush-Kuhn-Tucker Conditions

Linear PG : | X |

$$Min\{y^+ + y^-\}$$

Subject to

$$\begin{cases} x = y^+ - y^- \\ y^+ \geq 0 \\ y^- \geq 0 \end{cases}$$

Linear PG : (Ax + b)/(Cx + d)

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$$Min\{\frac{Cx+d}{Ex+f}\}$$

Subject to

$$\begin{cases} Ex+f \geq 0 \\ Gx \leq h \\ Ax = b \end{cases}$$

Is equivalent to

$$Min\{cy+dz\}$$

Subject to

$$\begin{cases} Gy - hz \leq 0 \\ Ay - bz = 0 \\ Ey + fz = 1 \\ z \geq 0 \end{cases}$$

That we see by doing :

$$y = \frac{x}{Ex+f} \qquad z = \frac{1}{Ex+f}$$

Transaction Costs

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Ask Prices : A

Bid Prices : B

The Position Shift is decomposed into $\theta_i = \theta_i^+ - \theta_i^-$

with $\begin{cases} \theta_i^+ \geq 0 \\ \theta_i^- \geq 0 \end{cases}$

And the cost in the objective is written as $A\theta^+ - B\theta^-$

Replication Problem :introduction

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	today		maturity		
Target portfolio	c	$\longrightarrow$	τ_i	probabilities	p_i
Tradable instruments	q_j	$\longrightarrow$	$D_{j,i}$	Positions	x

Tracking error $Dx - \tau$

Return Difference $(D-q)x-(\tau-c)$

Risk $p([Dx-\tau]_{-})$

Return Constraint (to describe the efficient frontier) $p(Dx-\tau)-qx+c \geq K$

Budget Constraint $qx - c = 0$

Replication Problem : the Primal

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$$Min\{py_-\}$$

s.t.

$$y_+ - y_- = Dx - \tau$$

$$p(Dx-\tau) - qx + c \geq K$$

with

$$y_+ \geq 0$$

$$y_- \geq 0$$

$$Min\{py_-\}$$

s.t.

$$y_+ - y_- = Dx - \tau$$

$$p(Dx-\tau) - qx + c = K + \varepsilon$$

with

$$y_+ \geq 0$$

$$y_- \geq 0$$

$$\varepsilon \geq 0$$

Replication Problem : the dual

$$L(\mu,\lambda,\nu,\alpha,\beta) = Inf_{x,y_+,y_-,\varepsilon}\{py_- - \nu\varepsilon - \alpha y_+ - \beta y_- + \mu(y_+ - y_- - Dx + \tau) + \lambda(py_+ - py_- - qx + c - K - \varepsilon)\}$$

$$L(\mu,\lambda,\nu,\alpha,\beta) = \mu\tau + \lambda(c-K) + Inf_{x,y_+,y_-,\varepsilon}\{(-\mu D - \lambda q)x + + (p - \beta - \mu - \lambda p)y_- + (-\alpha + \mu + \lambda p)y_+ + (-\nu - \lambda)\varepsilon\}$$

So the dual is :

$$Max_{\mu,\lambda,\nu,\alpha,\beta}\{\mu\tau + \lambda(c-K)\}$$

s.t.

$$\mu D + \lambda q = 0$$

$$p - \beta - \mu - \lambda p = 0$$

$$-\alpha + \mu + \lambda p = 0$$

$$-\nu - \lambda = 0$$

$$\nu \geq 0, \alpha \geq 0, \beta \geq 0$$

$\Leftrightarrow$

$$Max_{\mu,\lambda,\nu,\alpha,\beta}\{\mu\tau + \lambda(K-c)\}$$

s.t.

$$\mu D = \lambda q$$

$$0 \leq \mu - \lambda p \leq p$$

$$\lambda \geq 0$$

State Price Vector

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$$Max_{\mu,\lambda,\nu,\alpha,\beta}\{\mu\tau + \lambda(K-c)\}$$

s.t.

$$\mu D = \lambda q$$

$$0 \leq \mu - \lambda p \leq p$$

$$\lambda \geq 0$$

$$^{t}D\frac{\mu}{\lambda} = q$$

Any tradable can be priced using a measure on Scenarios and the outcome in these scenarios of the tradable

$$\sum_{i \in Scenarios} D_{i,j}\left(\frac{\mu_i}{\lambda_i}\right) = q_j$$

State Price Vector

Arbitrage Freeness rediscovered

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$$L(\mu,\lambda,\nu,\alpha,\beta) = \mu\tau + \lambda(c-K) + Inf_{x,y_+,y_-,\varepsilon}\{(-\mu D - \lambda q)x + \\ + (p-\beta-\mu-\lambda p)y_- + (-\alpha+\mu+\lambda p)y_+ + (-\nu-\lambda)\varepsilon\}$$

Dependency of the solution in p

Complementary slackness says that

Independency of the solution / p ⇔ $y_- = y_+ = 0$ $\Leftrightarrow Dx^* = \tau$

To have an preference free price, we have to be able to replicate the derivative

Issuance of a New Security

Strong Duality : optimum(dual)=optimum(primal)

$$\tau\mu + (K - c)\lambda = py_{-} = Risk[K]$$

$$c = r\left(\frac{\mu}{\lambda}\right) + \left(K - Risk[K] \;/\; \lambda\right)$$

Fair price

Premium associated with the risk

Optimization in ALM and AM

Classical Asset Allocation

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Strategic Allocation

Country Allocation

Sector Allocation

. . .

Pbs Classical M-V Analysis

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For which period is the Optimal portfolio is determined ? 6 months ? one year? 5 years?

The Underlying assets are supposed to be normal over the analysis period. What about using derivatives? What about Fat Tails?

Solution

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Use multistage stochastic analysis

Modeling/Calibration
of the
underlying processes

Modeling of
-the usable instruments
- the existing assets
- the liabilities and business constraints
-objective function

Simulation and Optimization

Post Optimal Analysis

MultiStage Stochastic Programming

- Flexibility with modeling of the underlying processes, including insights, forecasts,..
- Includes handling of derivatives and liquidity problems
- Allow Corporate, legal and policy constraints to be represented
- The model makes you diversify your real risks

What can we optimize ?

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-The financial duration of an investment

-the optimal mix in tactical asset allocation

-the level of a titrization

-the frequency of rebalancement of an optimal mix

Optimization builds an optimal frontier

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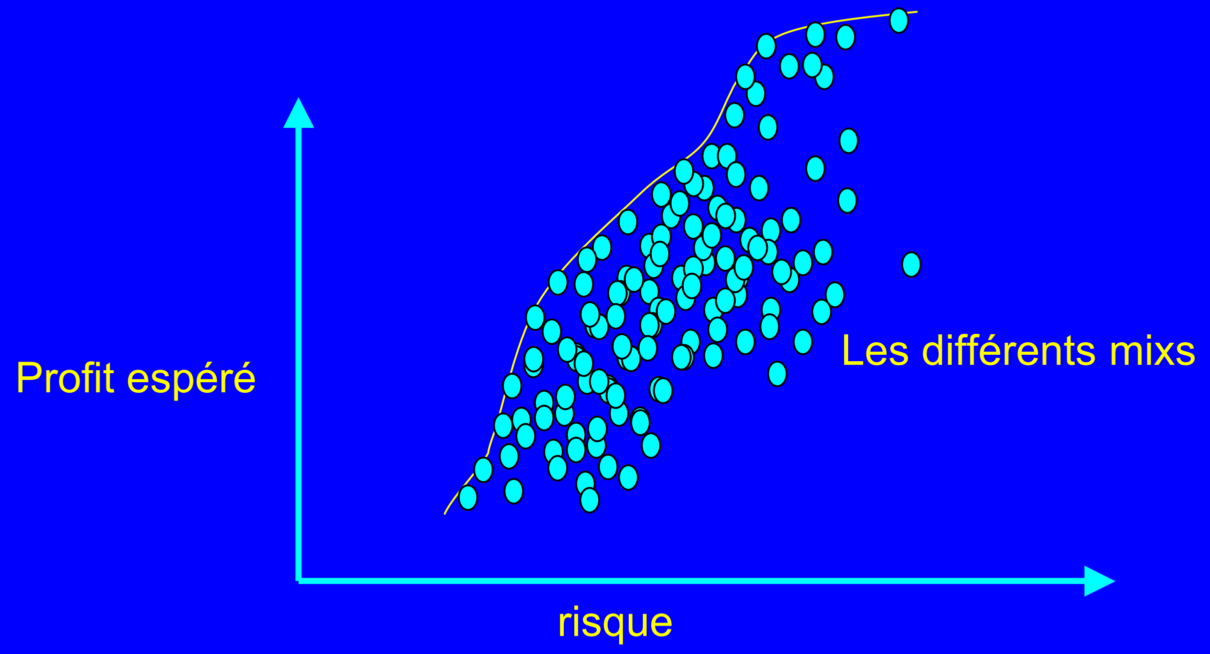

Note: multistage stochastic optimization can build non smooth frontier

Differents level of optimization

- Hedging of a specific risk component by using specific financial instruments. (hedge)

- Hedging of a finite set of risk components by using several hedge. (geometric hedge)

- Determination of a hedge policy that minimize a risk or maximize an expected profit or both. (simple optimisation)

- Determination of an optimal policy that assume opimal rebalancement in the future. (multistage stochastic optimisation)

Simple hedge ratio

Option on bond → Black&Sholes(BondPrice, other param.)

$$Hedge\ \ Ratio = \frac{\partial Black\&Sholes\ \ (\ \ Bond,\ \ other\ \ params)}{\partial\ \ Bond}$$

At short term behaves like

Option on bond $\approx Hedge\ \ Ratio\ \ \times\ \ Bond$

Geometric Hedge Ratio

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Several dimensions , only one hedge

$$\nabla Option = \begin{pmatrix} \frac{\partial Option}{\partial r_1} \\ \frac{\partial Option}{\partial r_2} \\ \frac{\partial Option}{\partial v} \\ \ldots \\ \frac{\partial Option}{\partial \rho} \end{pmatrix}$$

$$\nabla Hedge = \begin{pmatrix} \frac{\partial Hedge}{\partial r_1} \\ \frac{\partial Hedge}{\partial r_2} \\ \frac{\partial Hedge}{\partial v} \\ \ldots \\ \frac{\partial Hedge}{\partial \rho} \end{pmatrix}$$

$$Hedge \;\; Ratio = \frac{\nabla Option \;.\; \nabla Hedge}{\|\nabla Hedge\|^2}$$

Best Geometric Hedge

We choose the scalar product defined by the covariance matrix C of the factors,

$$X \, . \, Y = \sum_{i,j} x_i \, C_{i,j} \, y_j$$

$$\|X\|^2 = \sum_{i,j} x_i \, C_{i,j} \, x_j = Risque \; \{X\}$$

$$\|Option - hedge \;\; ratio \times Hedge\|^2 = \min_h \|Option - h \times Hedge\|^2$$

$$= \min_h \; Risque \; \{Option - h \times Hedge\}$$

Hedge Ratio = risk minimizer

Hedge = portfolio replication

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We can use a multidimensional hedge

P is a potfolio

$$\begin{pmatrix} H_1 & H_2 & H_3 & \ldots & H_n \end{pmatrix}$$

Hedge Vector

$$\min_{\{x_1, x_2, \ldots, x_n\}} Risk \left\{ P - \sum_i x_i \, H_i \right\}$$

Build the best hedge or the best replication portfolio

Minimiser un risque avec des simulations

If the underlying factors are $y_i(T)$ **horizon =T**

$$\min_{\{x_1,x_2,\ldots,x_n\}} \quad Risk \ \{\sum_i (P(y_i,T) \ - \ \sum_i x_j \ H_j(y_i,T))w_i\}$$

$$risk \ \{ \ X \ \} \ = \ X^2$$

We can also optimize at several dates in the same time

$$\min_{\{x_1,x_2,\ldots,x_n\}} \quad \sum_k \lambda_k \ Risk \ \{\sum_i (P(y_i,T_k) \ - \ \sum_i x_j \ H_j(y_i,T_k))w_i\}$$

The different symetrical risks

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$$\sum_{i,j} \Delta_i C_{i,j} \Delta_j$$

Variance

$$\sum_s p_s (P_s - B_s)^2$$

Risk L^2

$$\sum_s p_s \left| P_s - B_s \right|$$

Risk L^1

$$\max_s \left| P_s - B_s \right|$$

Risk L^∞

Linear Program

Other non symetrical risks

$$\begin{cases} \sum_s p_s(P_s - \bar{P})^2 & si \quad P_s - \bar{P} \geqslant 0 \\ 0 \;\; si & P_s - \bar{P} < 0 \end{cases}$$

semi-variance

$$-\min_s \{P_s\}$$

Maximal loss

(Linear Programme)

A coherent risk measure : expected shortfall

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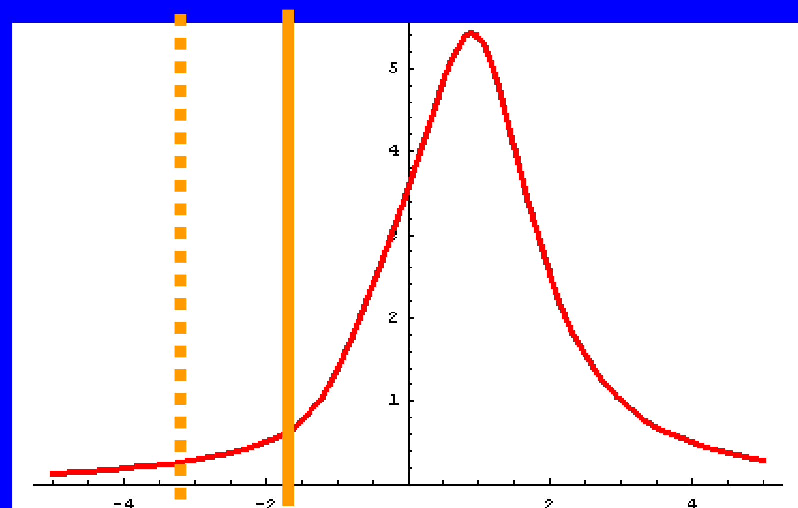
$$\boxed{ESF(0.95) = E[Loss \mid Loss > Var(0.95)]}$$



Expected ShortFall

VaR

Confidence level =0.95

Coherence : risk(a+b) <= risk(a) + risk(b)

Expected ShortFall as a LN PG

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N Simulations of the Factors y_i

The Confidence interval x

The Profit of the Portfolio as $f(y_i)$

=> The expected shortfall is equal to

$$\underset{\alpha > 0}{Min} \quad \{\alpha + \frac{1}{(1-x)N} \sum_{i=1}^{N} z_i\}$$

Under the constraints

$$\begin{cases} z_i \geq 0 \\ z_i \geq -f(y_i) - \alpha \end{cases}$$

And we get for free the VaR of level x **As the optimal value** α^*

Multistage simple optimization

$$\min_{\substack{\{x_1(t_1), x_2(t_1), ..., x_n(t_1)\} \\ ... \\ \{x_1(t_q), x_2(t_q), ..., x_n(t_q)\}}} Risque \ \{\sum_i (P(y_i, T) \quad - \quad \sum_i x_j(t_q) \ \ H_j(y_i, T))w_i\}$$

Avec les contraintes d'autofinancement

$$\sum_i (P(y_i, t_q) \quad - \quad \sum_i x_j(t_{q-1}) \ \ H_j(y_i, t_q) = \sum_i (P(y_i, t_q) \quad - \quad \sum_i x_j(t_q) \ \ H_j(y_i, t_q)$$

Stochastic Minimization

Determinist Minimization

$$\min_{\{x_1, x_2, \ldots, x_n\}} \quad Risk \ \{\sum_i (P(y_i, T) \ - \ \sum_i x_j \ H_j(y_i, T))w_i\}$$

Stochastic Minimization

$$x(t_q) = \ x(t_q, \ \{past \ and \ present \ stochastic \ state\})$$

We rebalance at intermediary dates the postions in order to stay optimal, using the available information

Portfolio Replication

Simple

Generalization

Delta hedging using a model

Minimization of VaR (RiskMetrics type)

Minimization based on simulations with a horizon T

Minimization based on Simulations with several horizons T(I)

Stochastic Multistage Minimization

Minimize $$\sum_i Risk_{BenchMark}[T_i] \times \lambda_i$$

ALM

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Interest Rate Gap Management : Duration (1 dim)
Convexity +Duration (2 dim)
Liquidity Gap Management : determinist universe

Interest Rate Gap Management : curve risk (n dim)
Liquidity Gap Management : determinist universe

Interest Rate Gap Management : curve risk (n dim)
Liquidity Gap Management : stochastic Cash-flows
determinist optimization

Interest Rate Gap Management : risque de courbe (n dim)
Liquidity Gap Management : stochastic Cash-flows
stochastic optimization

Maximise

$$E[Wealth[T]]$$
$$LiquidityGap[T_k] \leq 0$$
$$Risk[T_i] \leq K$$

ALM/Réplication G-Duality

Maximize

$$Wealth[T]$$
$$LiquidityGap[T_k] \leq 0$$
$$Risk_{Benchmark}[T_i] \leq K_i$$
$$Policy[\alpha]$$

Parameters

$$K_i, \alpha$$

Generalized Duality : the 2 problems Describe the same efficient frontier

Minimize

$$\sum_i Risk_{BenchMark}[T_i] \times \lambda_i$$
$$LiquidityGap[T_k] \leq 0$$
$$Wealth[T] \geq M$$
$$Policy[\alpha]$$

Parameters

$$M, \lambda_i, \alpha$$

Exploitation of the G-duality

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1) Instead of maximizing the expected profit, we can minimize the risk and use the risk information that may be better known .

2) Risk minimization can be addressed with a variety of technique from simple hedge to multistage stochastic optimization.

Order Statistics of the Minimum

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We sort the samples

$$\{S_1, S_2, ..., S_n\}$$

$$\{S_{1:n} = Inf(S_i), S_{2:n}, ..., S_{n:n}\}$$

$$S_i \sim N(\sigma, \mu)$$

$$S_{1:n} \approx \mu + \sigma \Phi^{-1}\left(\frac{1}{n+1}\right)$$

Minimize σ^2

Minimize $\frac{1}{\Phi^{-1}\left(\frac{1}{n+1}\right)}\left(S_{1:n} - \frac{1}{N}\sum_{i=1}^{N} S_i\right)$

In the normal case ,

minimizing variance is equivalent to minimizing maximum loss

Probabilistic Liquidity Constraint

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Determinist Environnement t

$$LiquidityGap[T] \geqslant 0$$

stochastic Environnement

$$Pr(LiquidityGap[T] < 0) \leqslant \alpha$$

$$LiquidityGap[T] \backsim N(\sigma, \mu)$$

$$\sigma\Phi^{-1}(\alpha) + \mu \geqslant 0$$

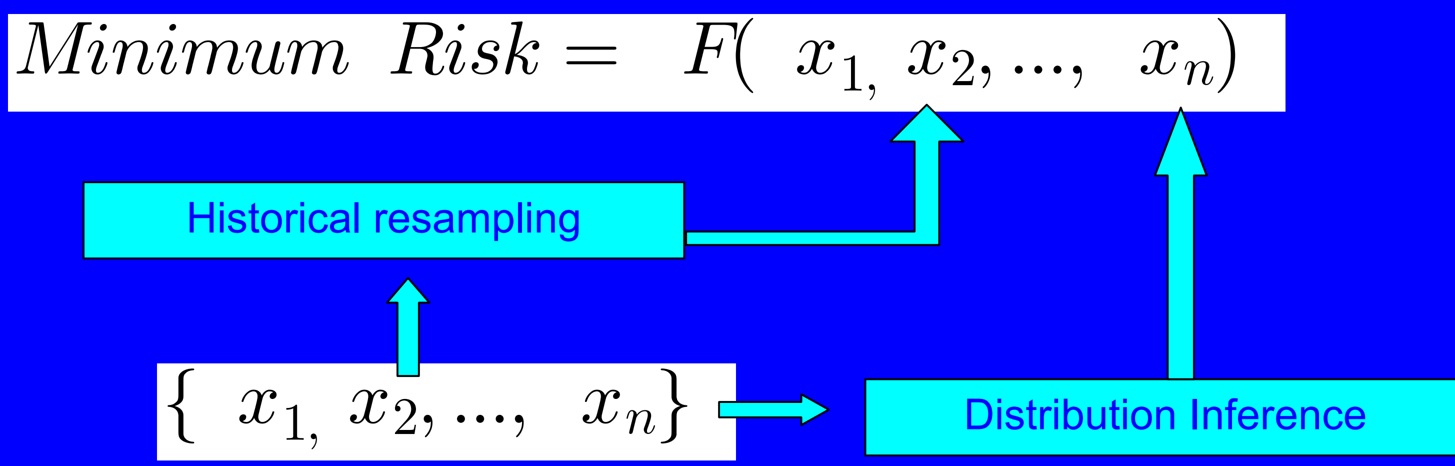
Linear Constraint

$$\frac{\Phi^{-1}(\alpha)}{\Phi^{-1}\left(\frac{1}{n+1}\right)}\left(S_{1:n} - \frac{1}{N}\sum_{i=1}^{N} S_i\right) + \frac{1}{N}\sum_{i=1}^{N} S_i \geqslant 0$$

Robustness of an optimal solution

- The difficulty to appreciate the robustness of optimal solution the main reason for not using optimisation in asset management and ALM.

- We can address the problem using a technique called Resampling:



Minimum Risk is a random variable

Optimal Portfolio Statistics

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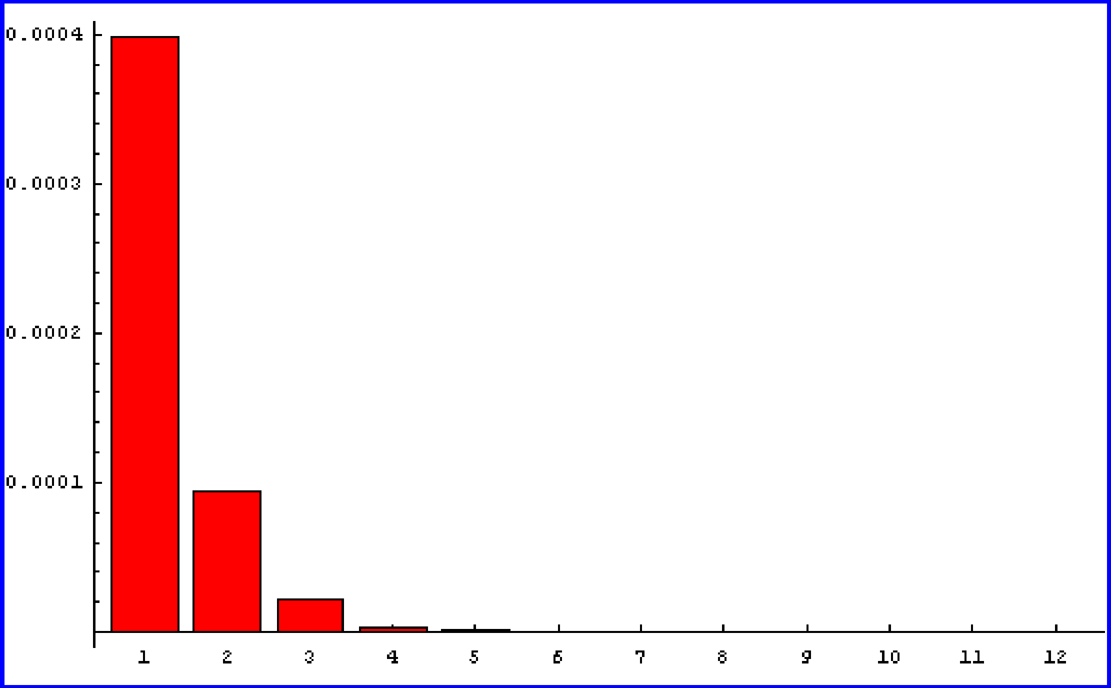
The Optimal portfolio isassociated with a covariance matrix.



Eigen values of the covariance matrix

Key Factors for a Successful Optimization

1) Selection of the important factors for the risk or the profit to optimize.
2) Choice of a good geometry for the optimization problem (time scale and weigthing of the dates).
3) Arbitrage Freeness of the underlying stochastic factors.
4) Consisten estimation of the parameters of the stochastic evolution (Stein Estimators, Ledoit Estimators,…)
5) Robustness Study of the optimization (resampling, principal components, projections of robust constraints, …)

Determinist/Stochastic Efficient Frontier in Asset Management

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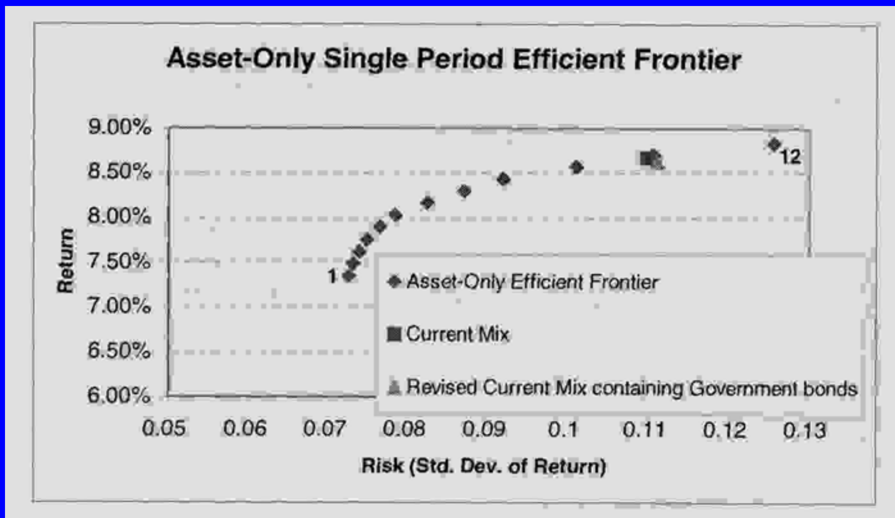
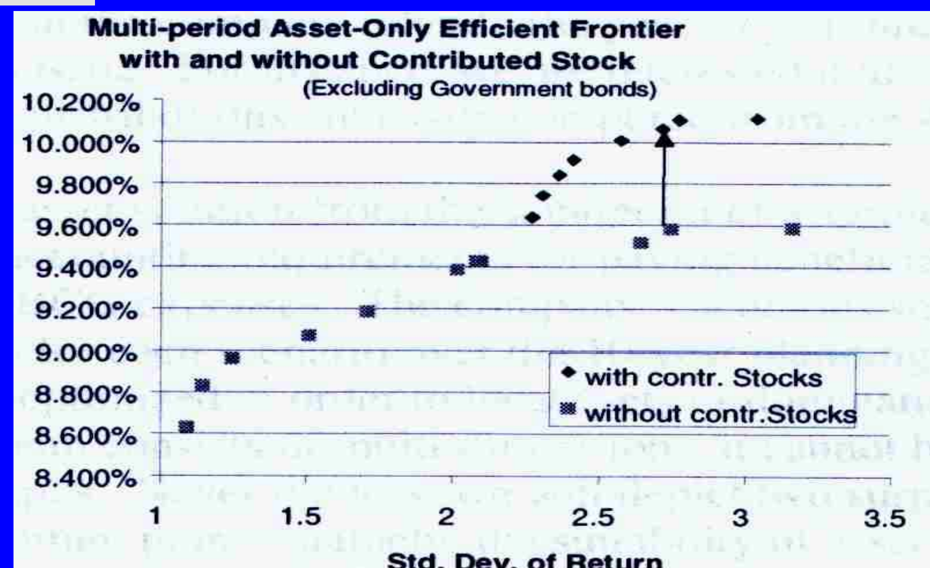


Optimal Mix

determined once

Optimal Mix

Regularly rebalanced



Understanding an Optimization

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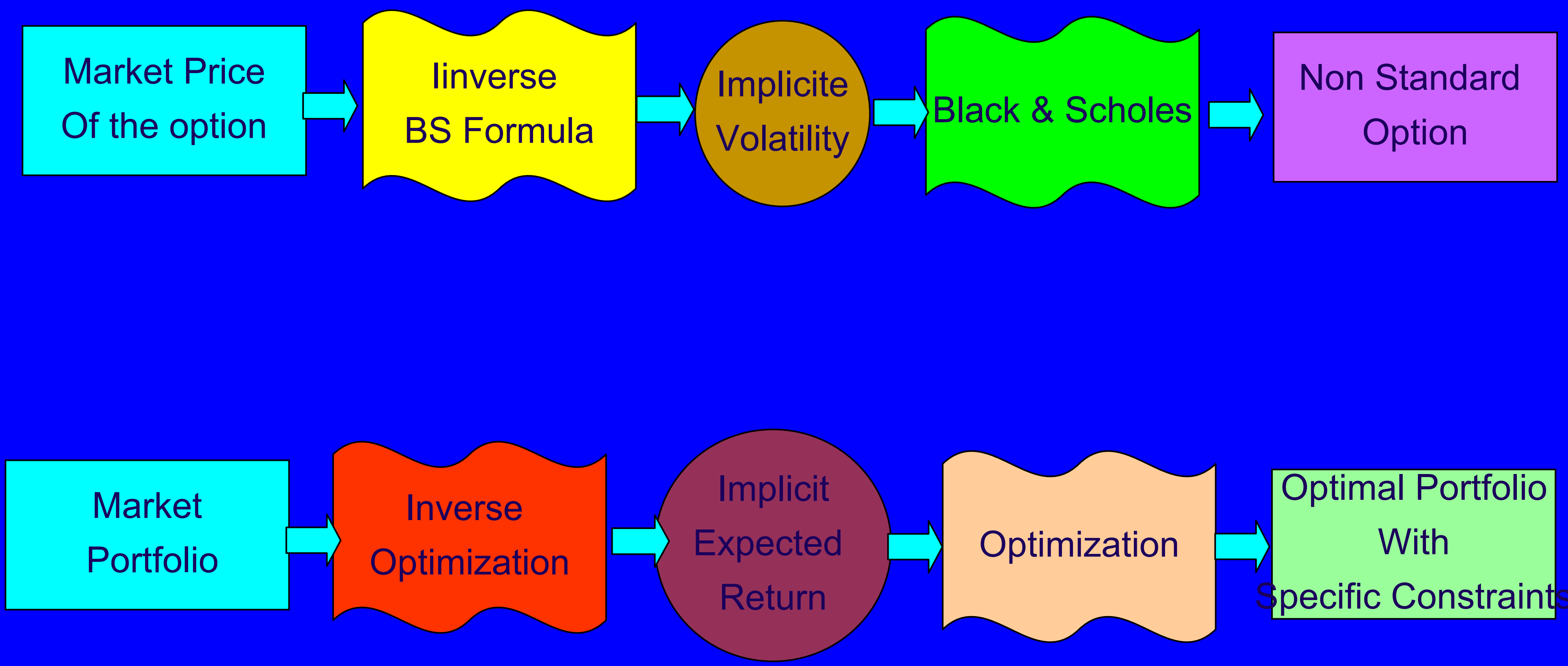
The most important result is the optimized portfolio.

It is important to understand the source of the additional performance (Is the research producing unrealistic forecasts?).

The Optimal holdings should also be analyzed and understood.

Inverse optimization



Attention to Specific problems of inverse optimization :

-Implicit Expected returns defined up to a multiplicative constant

- The market portfolio is assumed to be efficient

- Stocks not in the market portfolio will not have an implicit return

Conclusion

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- Convergence of Risk management techniques and Optimization techniques give us new tools to improve efficiency and auditability in ALM and asset management .
- Robustness of the solution can be addressed with resampling techniques.
- Liquidity risk should be represented by a probabilisitc constraint.
- Implicit Expected Returns should be used in optimal asset allocation