Equity Based Credit Pricing

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Plan

- Structured Models : the Merton Approach
- The Algo Approach
- Credit Spread Behaviour
- The JP Morgan /Chase /CreditGrades Approach
- Use of this Pricing Model

The Merton Approach

- The Firm's assets follow

$$V_t = V_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma W_t}$$

- We observe a default when

$$V_t \geq L$$

- The the default probability is given by

$$p_T = N(-d_2(T)) \qquad d_2(T) = \frac{Log\left[\frac{V_0}{L}\right] + \left(\mu - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

- d_2 is called the distance to default

The Merton Framework

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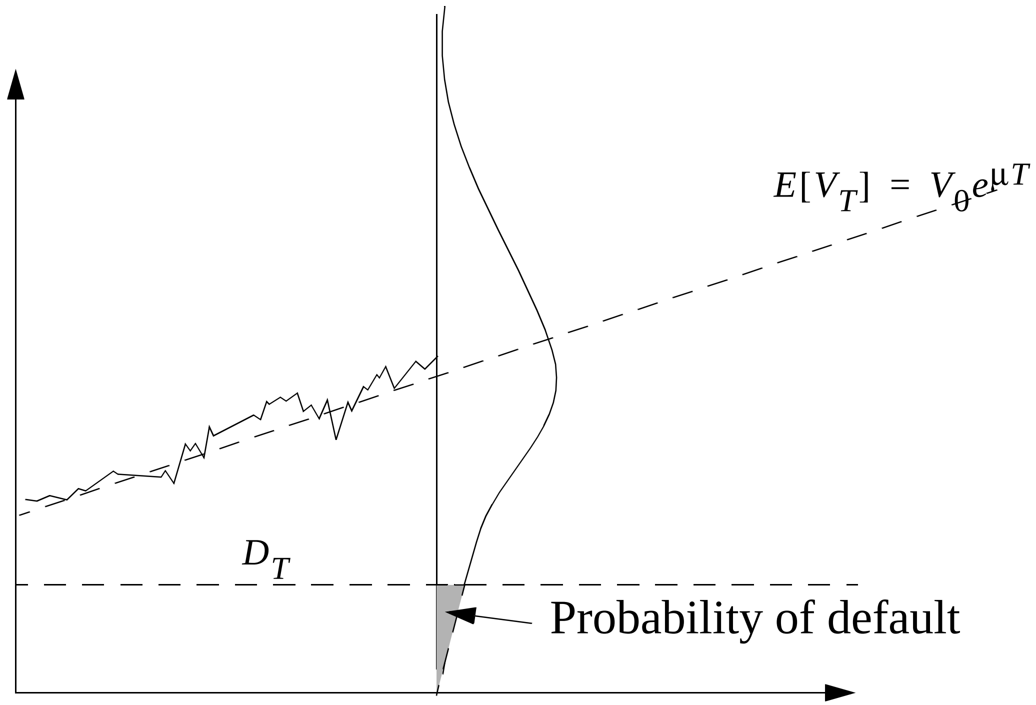
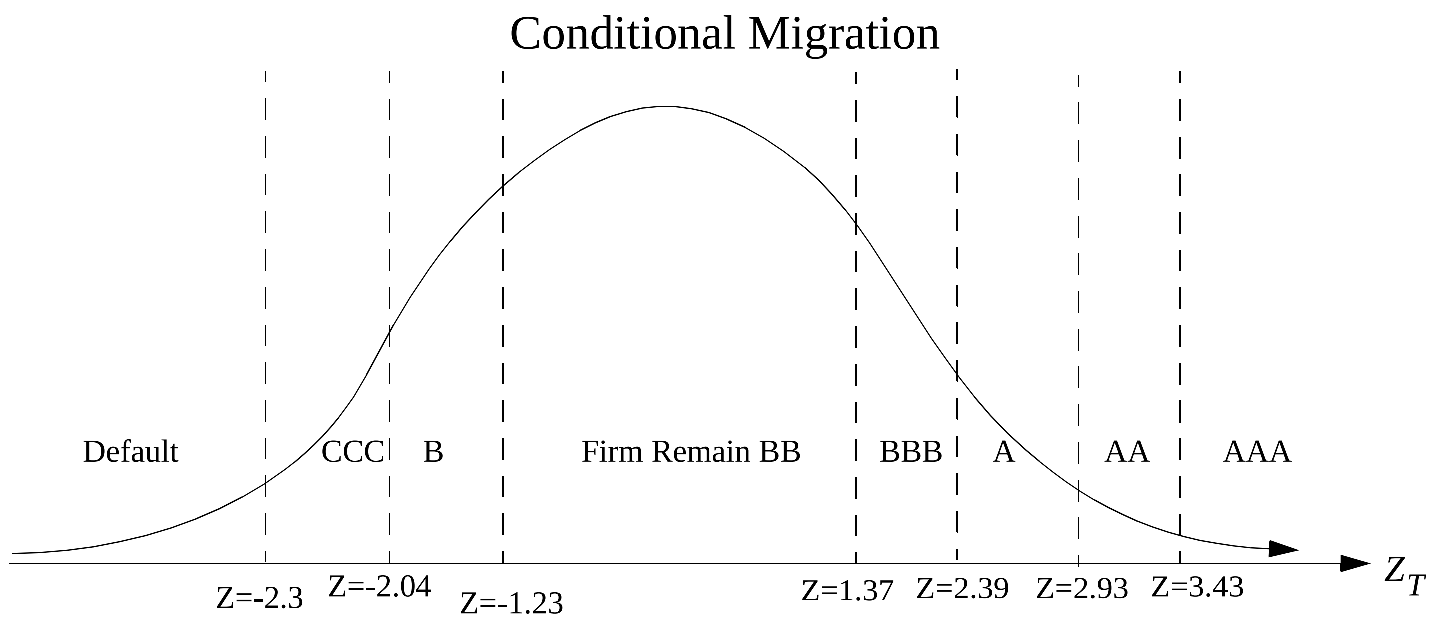


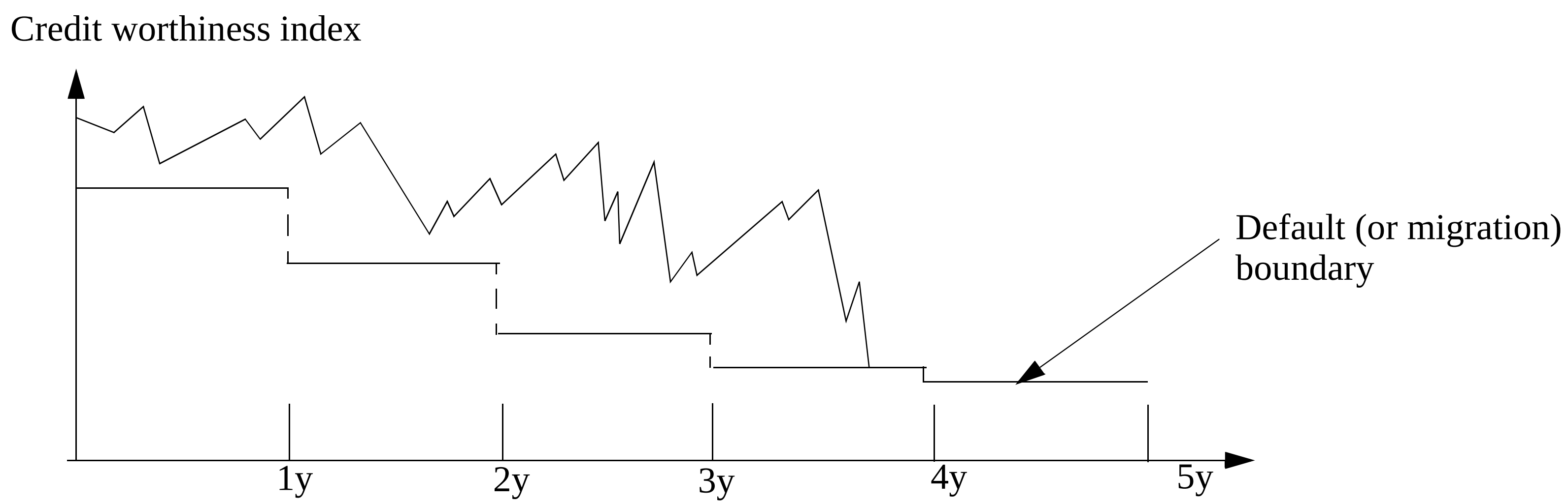
Generalization : CreditMetrics

- To Include rating migrations



MultiStep Credit Spead Model

- A Boundary is associated with every period



- We can extract the levels of the barrier from the default probabilities (res. the migration conditional probabilities)

- we need to have a multiperiod framework to match realistic curves.(behaviour of non zero implicit default probability for very short term default swaps)

Alternative Model

- We can use a jump diffusion process for the credit worthiness/asset :

$$\frac{dZ_t}{Z_t} = \mu dt + \sigma dW_t + J dQ_t$$

- we can tune the parameters to accomodate nonzero default probabilities at time=0 even with a constant default boundary.

- we need at least 2 additional parameters :
  - Average size of the jumps
  - Intensity of the jump process (frequency)

The JP /Chase/Creditgrades Model

- Introduction with a uncertainty associated with the level of the default boundary

- ---> implies a non zero default probability at time =0

- Asset process: $\frac{dV_t}{V_t} = \mu dt + \sigma dW_t$ and the recovery rate L is lognormal with

$$Var[Log[L]] = \lambda \qquad E[L] = L_0$$

- So the default happens when $V_0 e^{\sigma W_t - \sigma^2 \frac{t}{2}} > L_0 D e^{\lambda Z - \frac{\lambda^2}{2}}$,

- So the survival probability in this model is given by : $A_t = \sqrt{\sigma^2 t + \lambda^2}$ is a "total volatility"

$$p_t = \Phi_2\left(-\frac{\lambda}{2} + \frac{Log[d]}{\lambda}, -\frac{A_t}{2} + \frac{Log[d]}{\lambda}, \frac{\lambda}{A_t}\right) - d\Phi_2\left(\frac{\lambda}{2} + \frac{Log[d]}{\lambda}, -\frac{A_t}{2} - \frac{Log[d]}{\lambda}, -\frac{\lambda}{A_t}\right)$$

- that can be approximated to : $p_t = \Phi\left(-\frac{A_t}{2} + \frac{Log[d]}{A_t}\right) - d\Phi\left(-\frac{A_t}{2} - \frac{Log[d]}{A_t}\right)$

Volatility Stories

- Equity Volatility : σ_s function of σ : asset volatility

- We can assume $\sigma_s = \sigma\frac{V\partial S}{S\partial V}$ (Eq1))

- Distance to Default Definition : $\eta = \frac{1}{\sigma}Log\left[\frac{V}{LD}\right]$ (Eq2)

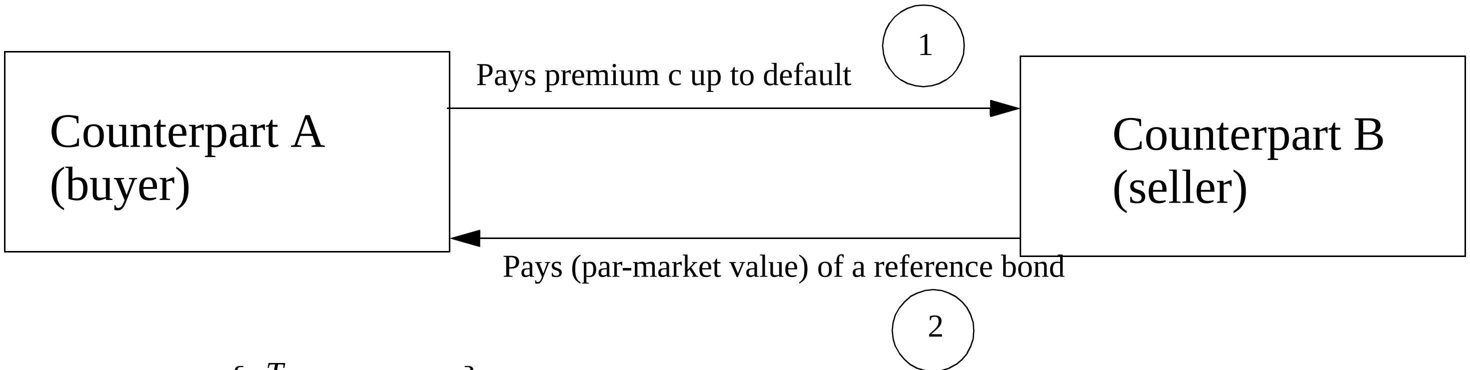
$S \to 0$ $V\vert_{S \to 0} = LD$ First order : $V \approx LD + \frac{\partial V}{\partial S}S$ Eq1 + Eq2 => $\eta \approx \frac{1}{\sigma_S}$ Eq3	$S \to \infty$ $\frac{S}{V} \to 1$ Eq1 + Eq2 => $\eta \approx \frac{1}{\sigma_S}Log\left(\frac{S}{LD}\right)$ Eq4

- A simple functional form that satisfy the boundary conditions (Eq1,Eq2,Eq3,Eq4)

$$V = S + LD$$

$$\eta = \frac{S + LD}{\sigma_S}Log\left(\frac{S + LD}{LD}\right) ==> V_0 = S_0 + L_0 D ==> \sigma = \sigma_S\frac{S}{S + L_0 D}$$

Credit Default Swaps



Value of side 1 : $c\left\{\int_0^T P(s)e^{-rs}ds\right\}N$

probability of event at time 0

density of event at time t

Value of side 2 : $(1-R)\left[(1-P(0)) + \int_0^T \left(-\frac{dP}{ds}\right)e^{-rs}(ds)\right]N$

$$\boxed{\text{CDS spread } = (1-R)\frac{(1-P(0)) + \int_0^T \left(-\frac{dP}{ds}\right)e^{-rs}(ds)}{\int_0^T P(s)e^{-rs}ds}}$$

Pricing Formula

- Using the gaussian approximation (Rubinstein and Reiner)

Bond Specific recovery

$$\text{CDS spread (T)} = \frac{r(1-R)(1-(P(0)+H(T)))}{P(0)-P(T)e^{-rt}-H(T)}$$

$$\begin{cases} H(t) = e^{r\frac{\lambda^2}{\sigma^2}}\left\{G\left(t+\frac{\lambda^2}{\sigma^2}\right)-G\left(\frac{\lambda^2}{\sigma^2}\right)\right\} \\ G(t) = d^{z+\frac{1}{2}}\Phi\left(-\frac{z\sigma\sqrt{t}}{2}+\frac{Log[d]}{\sigma\sqrt{t}}\right)+d^{-z+\frac{1}{2}}\Phi\left(-\frac{z\sigma\sqrt{t}}{2}-\frac{Log[d]}{\sigma\sqrt{t}}\right) \\ P(t) = \Phi\left(-\frac{A_t}{2}+\frac{Log[d]}{A_t}\right)-d\Phi\left(-\frac{A_t}{2}-\frac{Log[d]}{A_t}\right) \\ z = \sqrt{\frac{1}{4}+2\frac{r}{\sigma^2}} \qquad d = \frac{S_0+L_0D}{L_0D}e^{\lambda^2} \qquad A_t = \sqrt{\sigma^2 t+\lambda^2} \\ \sigma = \sigma_S\frac{S}{(S+L_0D)} \end{cases}$$

Average Name Recovery

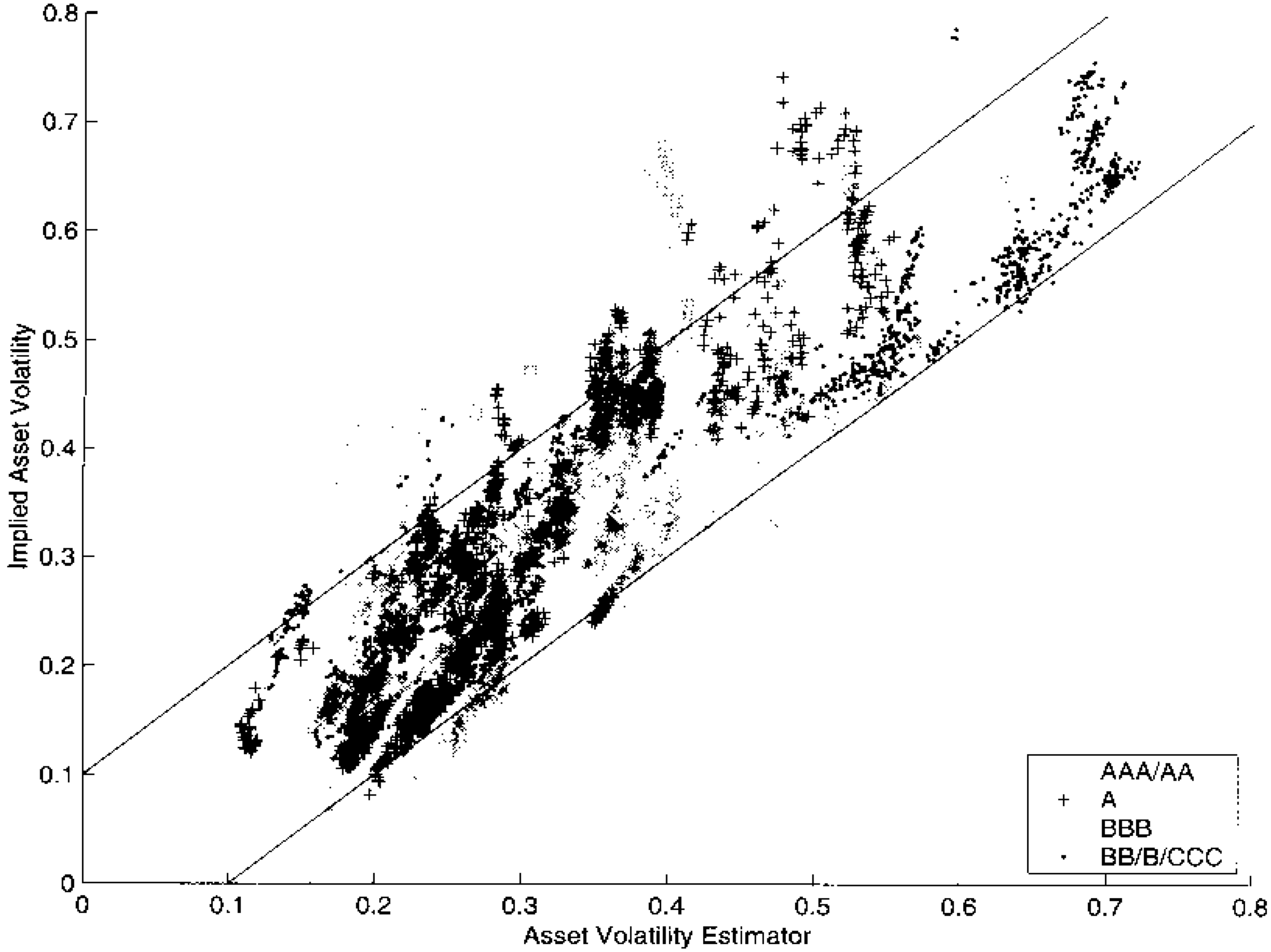
- So we have a pricing function :

$$\text{CDS spread (T)} = Function(S_0, \sigma_{S,T}, \lambda)$$

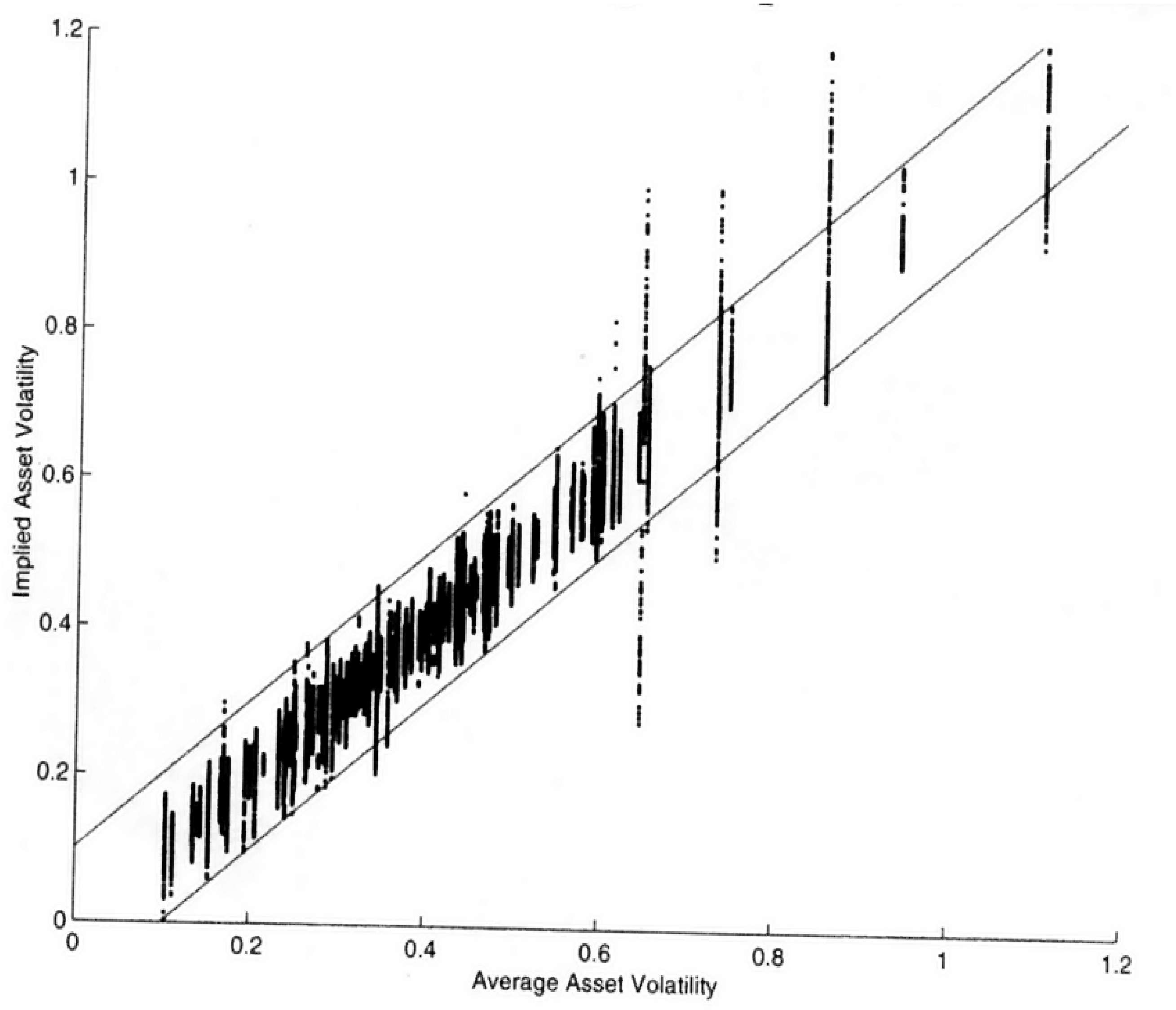
- The CDS become an equity derivative (~ put option on the name) with a specific market price given by λ and an implicit volatility like any other equity derivative

Implied Asset Volatility / Empirical

Figure 3.3: Historical versus Implied Volatility by Credit Quality

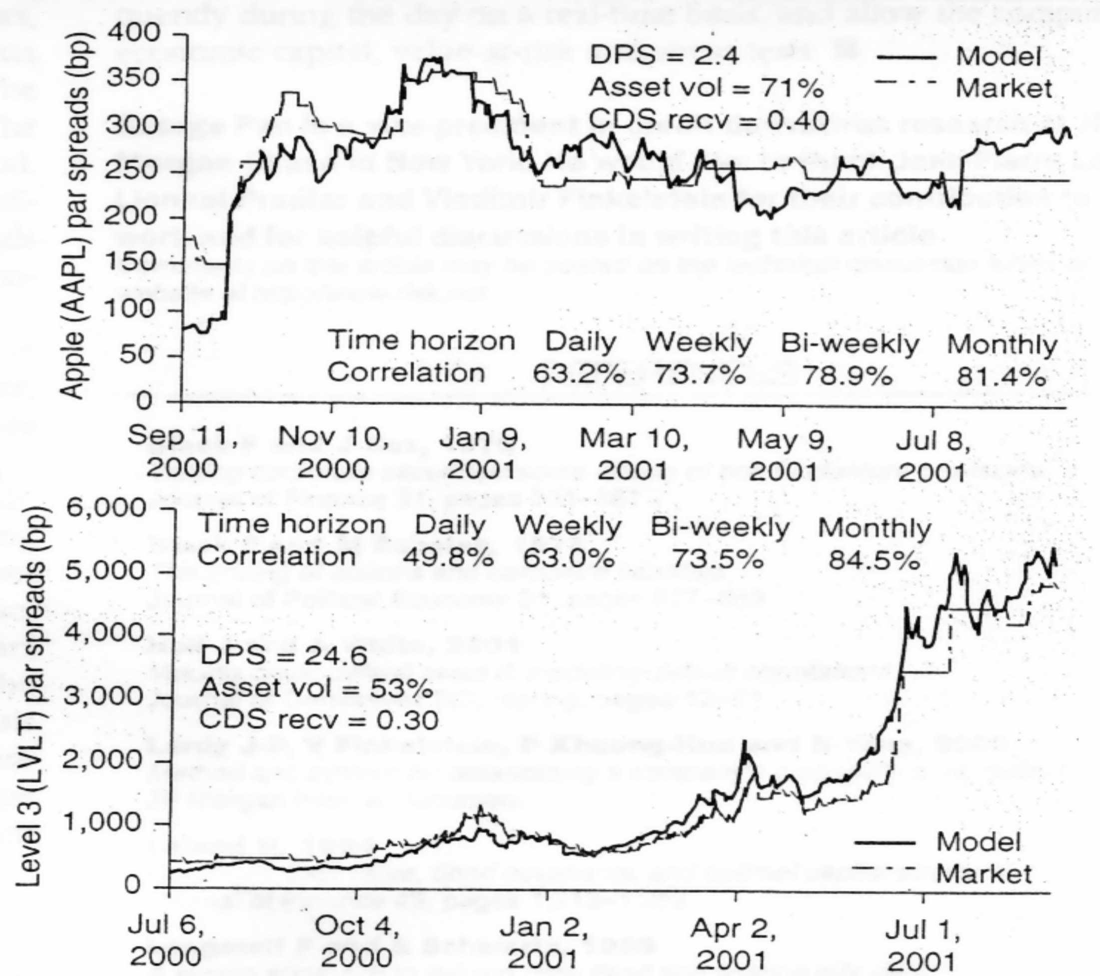


Asset Volatility Universality



Pricing Tool : Spreads Term Structure

3. Comparison of the five-year CDS market par spreads with the model par spreads



Advantage of the Pricing Formula

- Provides an early warning for the credit rating migration likelihood, as shown by studies made by the Riskmetrics group

- Unify Risk Management , by extracting from the credit derivative the dependency with respect to the equity factors .

Conclusion

- We are seeing an emerging standard formula to price the basic credit derivative .
- The link is made with equity instruments that allow a unified risk management at the name level.
- The Credit Specific Risks are statistically limited, as schown by the Riskmetrics group study
- This methodology open the door to generalizations to more sophisticated credit derivatives.

Reading Advices

- Equity to credit pricing, by Georges Pan, Risk Magazine,November, 2001,p99-102
- Creditgrades technical document, by RiskMetrics Group
- Valuing Credit Default Swap I: by J. hull and A. White, The Journal of Derivatives, Fall 2000, pp 29-40