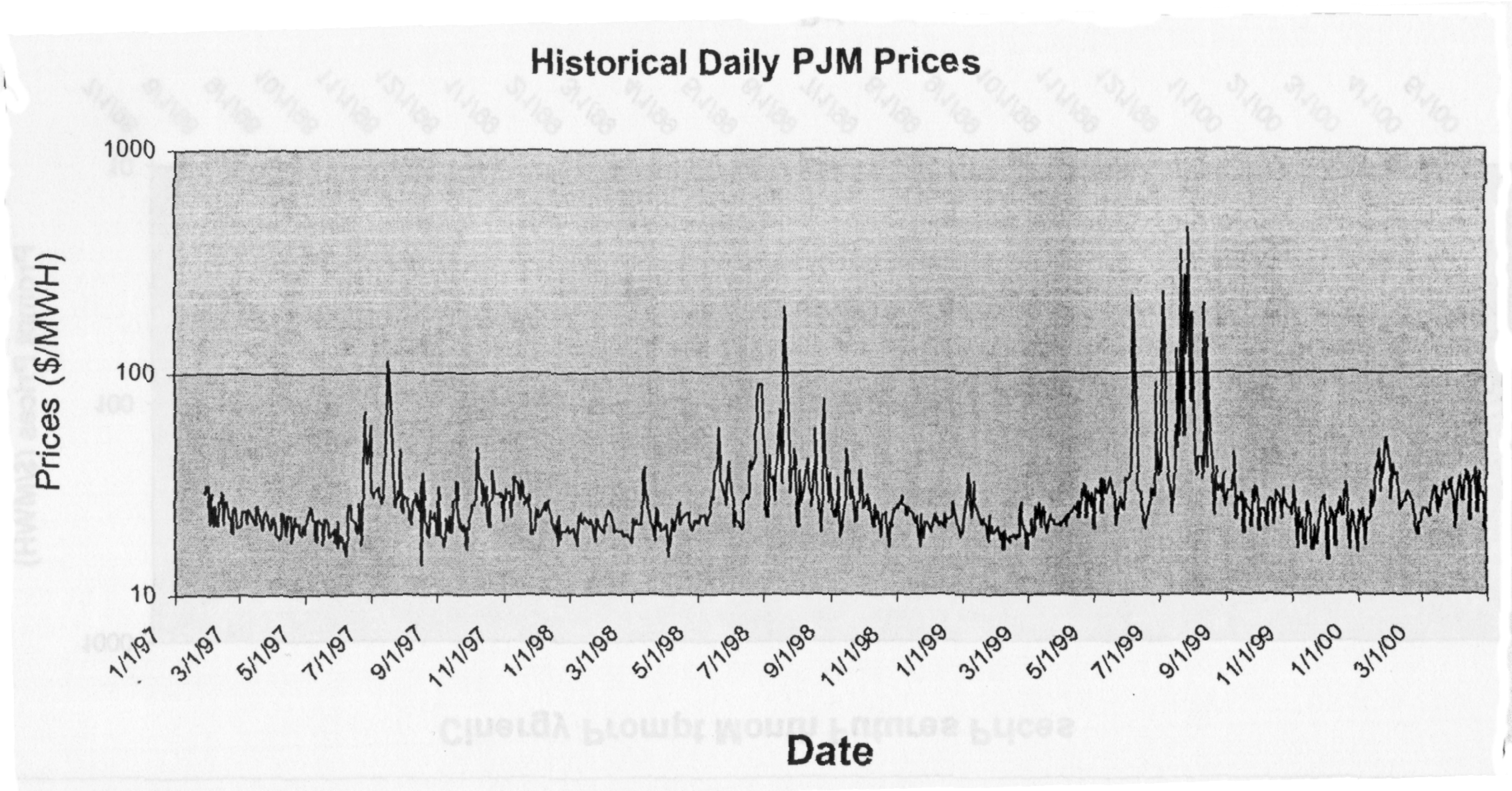
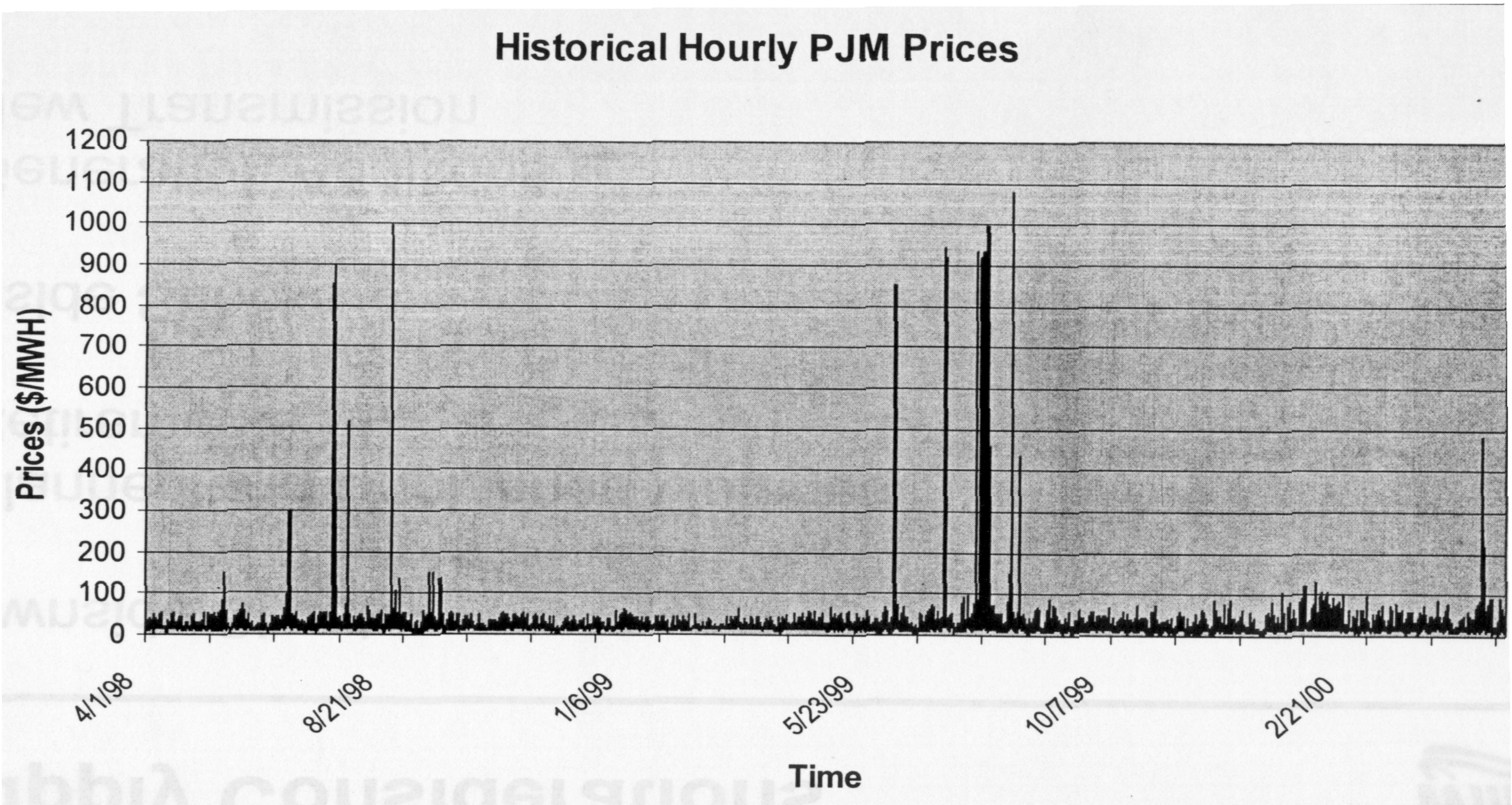
Introduction to Electricity Markets
Pricing Theory

by Olivier Croissant

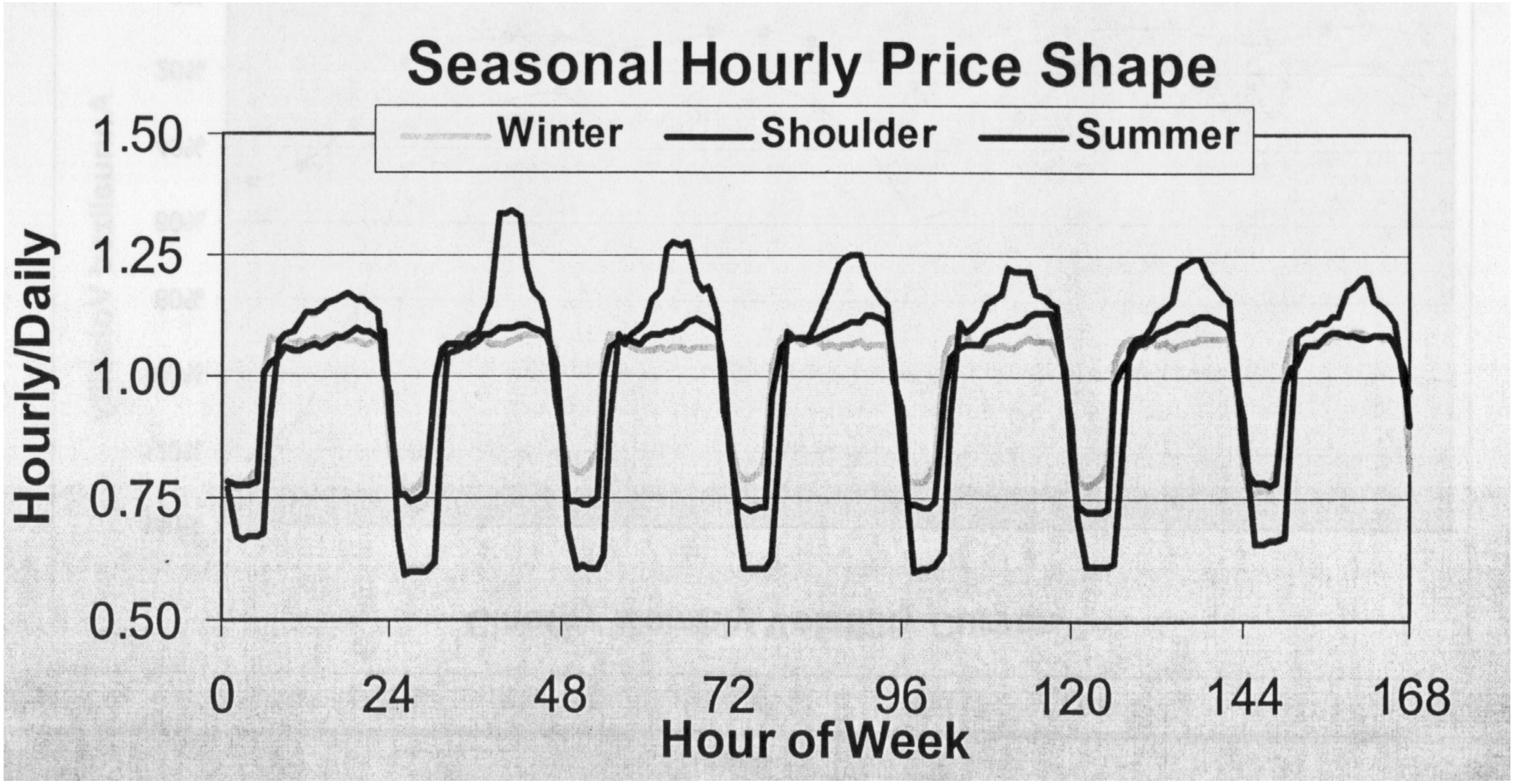
Daily Prices



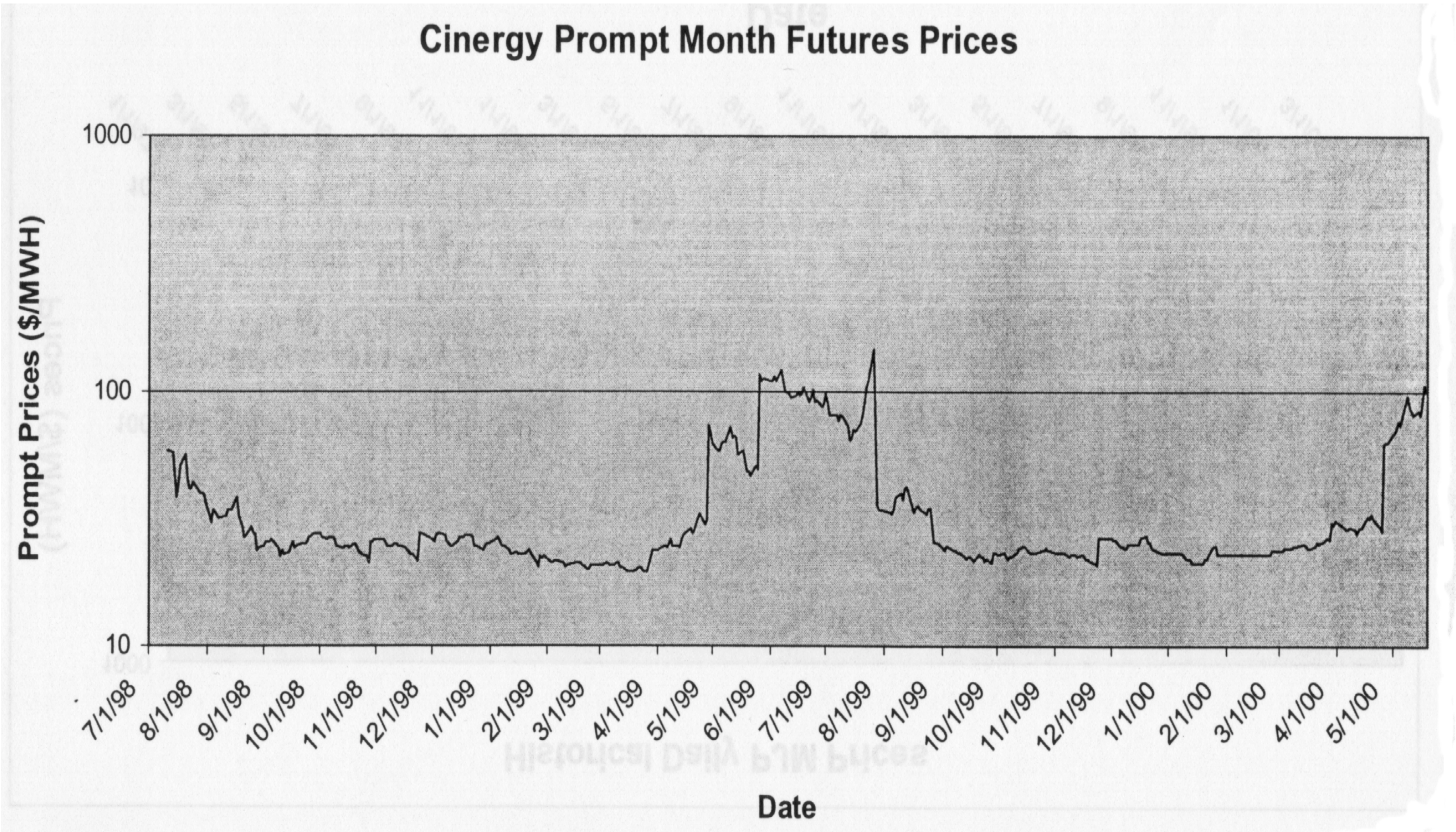
Hourly Prices



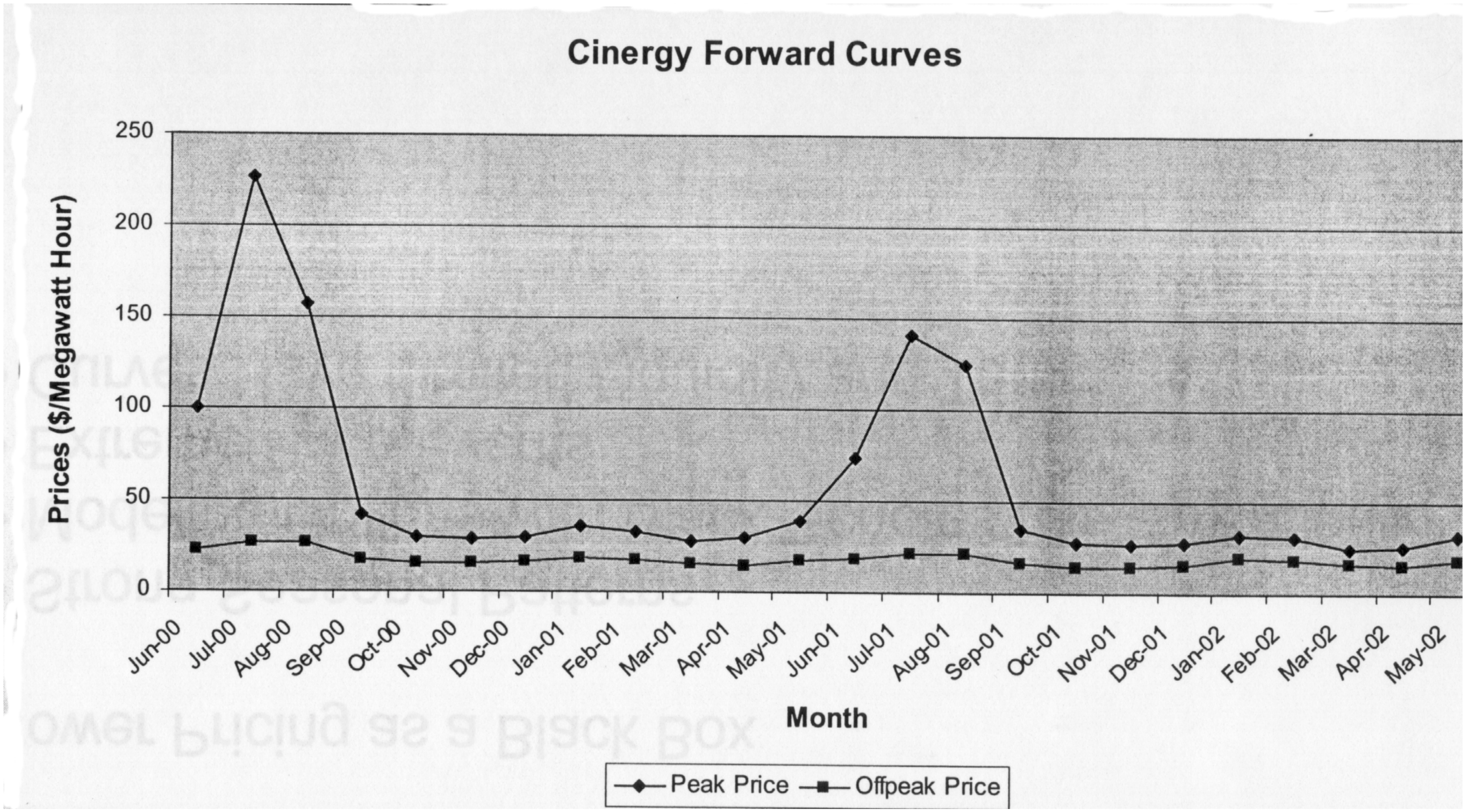
Hourly Seasonality



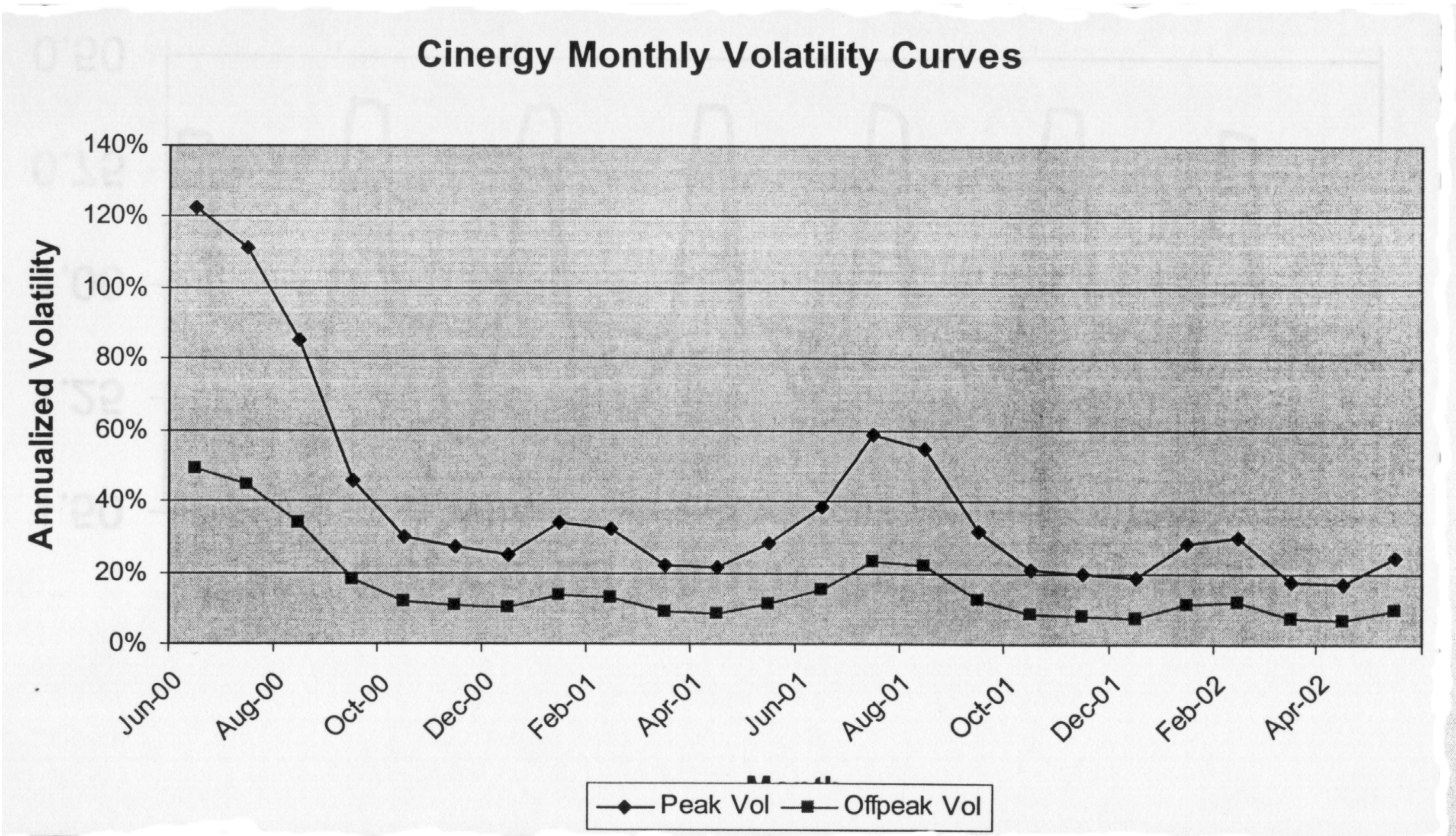
Prompt Month



Forward Curve



Historical Volatility of The next 1 month Contract



The Mathematical Opportunity

- Given a jump diffusion process: $$dX_t = (\mu_0 + \mu_1 \cdot X_t)dt + (\sigma(X_t) \cdot X_t) \cdot dW_t + \sum_k J_k dq_k$$

$$\sigma^2(x) = \Sigma_{0,t} + \Sigma_{1,t} \cdot x \qquad \nu_k(X) \qquad (\lambda_{k,0} + \lambda_{k,1} \cdot X_t)$$

- Then the generalized transform : $$\phi[u, X_t, t] = E_t\left[e^{\left(-\int_t^T (R_0 + R_1 \cdot X_s)ds\right)} e^{u \cdot X_T}\right]$$ can be computed solving ordinary differential equations: $\phi[u, x, t] = e^{\alpha(t) + \beta(t) \cdot x}$

  where $\frac{d\beta}{dt} = R_1 - \mu_1\beta - \frac{1}{2}\beta\Sigma_1\beta - \lambda_1(\theta(\beta(t)) - 1)$, $\frac{d\alpha}{dt} = R_0 - \mu_0\alpha - \frac{1}{2}\beta\Sigma_0\beta - \lambda_0(\theta(\beta(t)) - 1)$, $\beta(T) = u$, $\alpha(T) = 0$ $\theta(c) = \int e^{cz} d\nu_k(z)$

- The optional return can be computed using fourier transform :

$$G_{a,b}(y) = E\left[e^{\left(-\int_t^T (R_0 + R_1 \cdot X_s)ds\right)} e^{a \cdot X_T} 1_{b \cdot X_T \leq y}\right] = \frac{\phi[a, X_0, 0]}{2} - \frac{1}{\pi}\int_0^\infty Im[\phi[a + ivb, X_0, 0]e^{-ivy}]dv$$ and

$$E\left[e^{\left(-\int_t^T (R_0 + R_1 \cdot X_s)ds\right)} \left(e^{d \cdot X_T} - c\right)^+\right] = G_{d,-d}(-Log[c], X_0, T) - cG_{0,-d}(-Log[c], X_0, T)$$

Extension of the preceding Framework

- We can add more terms to the expectation, to compute more options :

$$\phi[u, v, w, X_t, t] = E_t\left[e^{\left(-\int_t^T (R_0 + R_1 \cdot X_s)ds\right)} e^{u \cdot X_T}(v + w \cdot X_T)\right] \quad ,$$

- We define the following security :

$$\tilde{G}_{a,b,d}(y) = E\left[e^{\left(-\int_t^T (R_0 + R_1 \cdot X_s)ds\right)} e^{d \cdot X_T} a \cdot X_T 1_{b \cdot X_T \le y}\right] = \frac{\Psi(a, d, x, 0)}{2} - \frac{1}{\pi}\int_0^{\infty} Im[\Psi(a, d + ibv, X_0, 0)e^{-ivy}](dv)$$

- where $\Psi(v, u, x, t) = \phi[u, x, t](A(t) + B(t) \cdot x)$ and $\begin{cases} -\frac{dB}{dt} = \mu_1 B + \frac{1}{2} B \Sigma_1 B + \lambda_1 \nabla\theta(\beta(t))B \\ -\frac{dA}{dt} = \mu_0 B + \frac{1}{2} B \Sigma_0 B + \lambda_0 \nabla\theta(\beta(t))B \end{cases}$ with $\begin{cases} B(T) = v \\ A(T) = 0 \end{cases}$