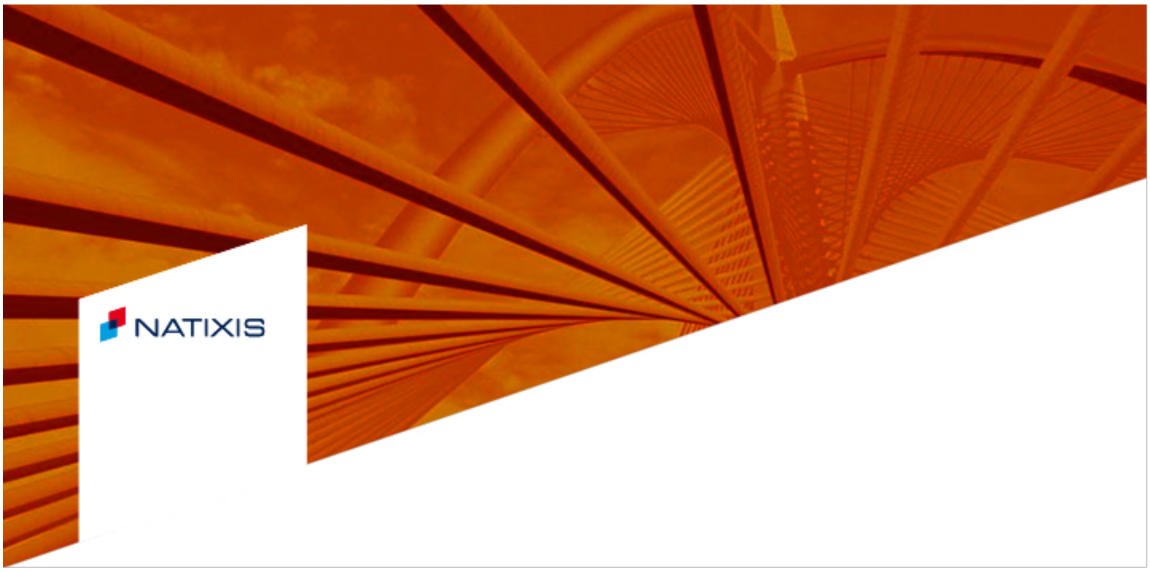

Global Alpha Model For Spreadoptions

By Olivier Croissant

The Requirements

-Anti - blackbox model

-Modular

-Simple to understand the principle

-Simple to modify and to amend

-Flexible enough for e few smiles or a large number of smiles

A Simple Heston Model

$$dS = \alpha \sqrt{v}\ dW_1;$$

$$dv = \lambda(\theta - v)\,dt + \nu\sqrt{v}\left(\rho\,dW_1 + \sqrt{1-\rho^2}\ dW_2\right);$$

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Integrate and apply Ito

$$S(t) = \int_0^t \alpha \sqrt{v(t)}\, dW_{1,s}$$

$$v(t) = \lambda \int_0^t (\theta - v(t))\, dt + \nu \int_0^t \sqrt{v(t)} \left(\rho\, dW_{1,s} + \sqrt{1-\rho^2}\, dW_{2,s}\right)$$

et en appliqua Ito

$$\sqrt{v(t)} = \sqrt{v(0)} + \lambda \int_0^t \left(\frac{(\theta - v(s))}{2\sqrt{v(s)}} - \frac{\nu^2}{4\sqrt{v(s)}} \right) ds +$$

$$\nu \int_0^t \frac{1}{2\sqrt{v(s)}} \left(\rho\, dW_{1,s} + \sqrt{1-\rho^2}\, dW_{2,s}\right)$$

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S(t) at the second order in t

$$S^{(2)}(t) =$$

$$\int_0^t \alpha\left(\sqrt{v(0)}\right) dW_{1,u} + \int_0^t \alpha\left(\nu \int_0^u \frac{1}{2\sqrt{v(s)}}\left(\rho\, dW_{1,s} + \sqrt{1-\rho^2}\, dW_{2,s}\right)\right) dW_{1,u} +$$

$$\int_0^t \alpha\left(\lambda \int_0^u \left(\frac{(\theta - v(s))}{2\sqrt{v(s)}} - \frac{\nu^2}{4\sqrt{v(s)}}\right) ds\right) dW_{1,u}$$

we reapply Ito and neglect everything beyond order 0 for $\frac{1}{\sqrt{v(s)}}$

After Simplification, the Matingale part of S at the second order

$$\text{Martingale}\left[S^{(2)}(t)\right](t) = \int_0^t \alpha\left(\sqrt{v(0)}\right) dW_{1,u} +$$

$$\frac{\alpha\,\rho\,\nu}{2\sqrt{v(0)}} \int_0^t \int_0^u dW_{1,s}\, dW_{1,u} + \frac{\alpha\sqrt{1-\rho^2}\,\nu}{2\sqrt{v(0)}} \int_0^t \int_0^u dW_{2,s}\, dW_{1,u}$$

Intrinseque formulation of the second order martingale part

let $\quad S_g(t) = \int_0^t \alpha\left(\sqrt{v(0)}\right) dW_{1,u}$

In the Wiener space, $\int_0^t \int_0^u dW_{1,s}\, dW_{1,u}$ and $\int_0^t \int_0^u dW_{2,s}\, dW_{1,u}$ are orthogonal

$$\left\|\mathrm{Martingale}\left[S^{(2)}(t)\right] - S_g(t)\right\|^2 =$$

$$\frac{\alpha\ \nu}{2\sqrt{v(0)}} \left\| \rho \int_0^t \int_0^u dW_{1,s}\, dW_{1,u} + \sqrt{1-\rho^2} \int_0^t \int_0^u dW_{2,s}\, dW_{1,u} \right\|^2 = \frac{\alpha^2\ \nu^2}{4\, v(0)}$$

let $\quad H(t) = \int_0^t \int_0^u dW_{1,s}\, dW_{1,u}$ $\quad$ it is a Chi2

$$\mathrm{Martingale}\left[S^{(2)}(t)\right].H(t) = \frac{\alpha\, \rho\, \nu}{2\sqrt{v(0)}}$$

Local formulation of the Intrinseque second order martingale approximation

$$\left\|\text{Martingale}\left[S^{(2)}(t)\right] - S_g(t)\right\|^2 = \frac{\text{var}(\text{vol}(S(t)))}{2}$$

$$\text{Martingale}\left[S^{(2)}(t)\right].H(t) = \frac{\text{covar}(S(t),\ v(t))}{2\,v(0)^2}$$

So a process having the same triplet {instantaneous volatility, variance of the volatility and instantaneous correlation Under - vol} is the same process in the sense of Watanabe at the second order in t

Gobal Alpha model with 4 assets and 4 main sources of noise + residuals

$$dS_1 = \alpha_{11}\sqrt{v_1}\ S_1\, dW_1 + \alpha_{12}\sqrt{v_2}\ S_1\, dW_2 + \alpha_{13}\sqrt{v_3}\ S_1\, dW_3 + \alpha_{14}\sqrt{v_4}\ S_1\, dW_4 + \sqrt{v_5}\ S_1\, dW_5;$$

$$dS_2 = \alpha_{21}\sqrt{v_1}\ S_2\, dW_1 + \alpha_{22}\sqrt{v_2}\ S_2\, dW_2 + \alpha_{23}\sqrt{v_3}\ S_2\, dW_3 + \alpha_{24}\sqrt{v_4}\ S_2\, dW_4 + \sqrt{v_6}\ S_2\, dW_6;$$

$$dS_3 = \alpha_{31}\sqrt{v_1}\ S_3\, dW_1 + \alpha_{32}\sqrt{v_2}\ S_3\, dW_2 + \alpha_{33}\sqrt{v_3}\ S_3\, dW_3 + \alpha_{34}\sqrt{v_4}\ S_3\, dW_4 + \sqrt{v_7}\ S_3\, dW_7;$$

$$dS_4 = \alpha_{41}\sqrt{v_1}\ S_4\, dW_1 + \alpha_{42}\sqrt{v_2}\ S_4\, dW_2 + \alpha_{43}\sqrt{v_3}\ S_4\, dW_3 + \alpha_{44}\sqrt{v_4}\ S_4\, dW_4 + \sqrt{v_8}\ S_4\, dW_8;$$

$$dv_1 = \lambda_1 (\theta_1 - v_1)\, dt + \nu_1 \sqrt{v_1}\ dW_9;$$

$$dv_2 = \lambda_2 (\theta_2 - v_2)\, dt + \nu_2 \sqrt{v_2}\ dW_{10};$$

$$dv_3 = \lambda_3 (\theta_3 - v_3)\, dt + \nu_3 \sqrt{v_3}\ dW_{11};$$

$$dv_4 = \lambda_4 (\theta_4 - v_4)\, dt + \nu_4 \sqrt{v_4}\ dW_{12};$$

$$dv_5 = \lambda_5 (\theta_5 - v_5)\, dt + \nu_5 \sqrt{v_5}\ dW_{13};$$

$$dv_6 = \lambda_6 (\theta_6 - v_6)\, dt + \nu_6 \sqrt{v_6}\ dW_{14};$$

$$dv_7 = \lambda_7 (\theta_7 - v_7)\, dt + \nu_7 \sqrt{v_7}\ dW_{15};$$

$$dv_8 = \lambda_8 (\theta_8 - v_8)\, dt + \nu_8 \sqrt{v_8}\ dW_{16};$$

$$dW_1\, dW_9 = \rho_1\, dt;$$
$$dW_2\, dW_{10} = \rho_2\, dt;$$

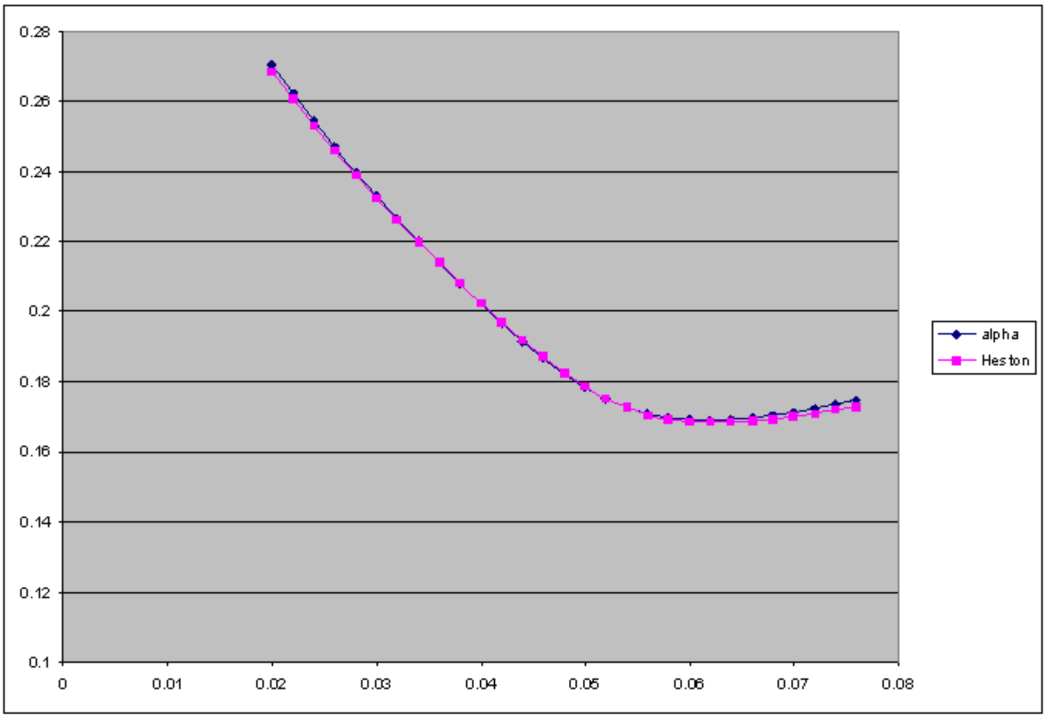
$$dW_3\, dW_{11} = \rho_3\, dt;$$
$$dW_4\, dW_{12} = \rho_4\, dt;$$
$$dW_5\, dW_{13} = \rho_5\, dt;$$
$$dW_6\, dW_{14} = \rho_6\, dt;$$
$$dW_7\, dW_{15} = \rho_7\, dt;$$
$$dW_8\, dW_{16} = \rho_8\, dt;$$
$$\text{other} = 0$$

The Observables for the 4+4 global alpha

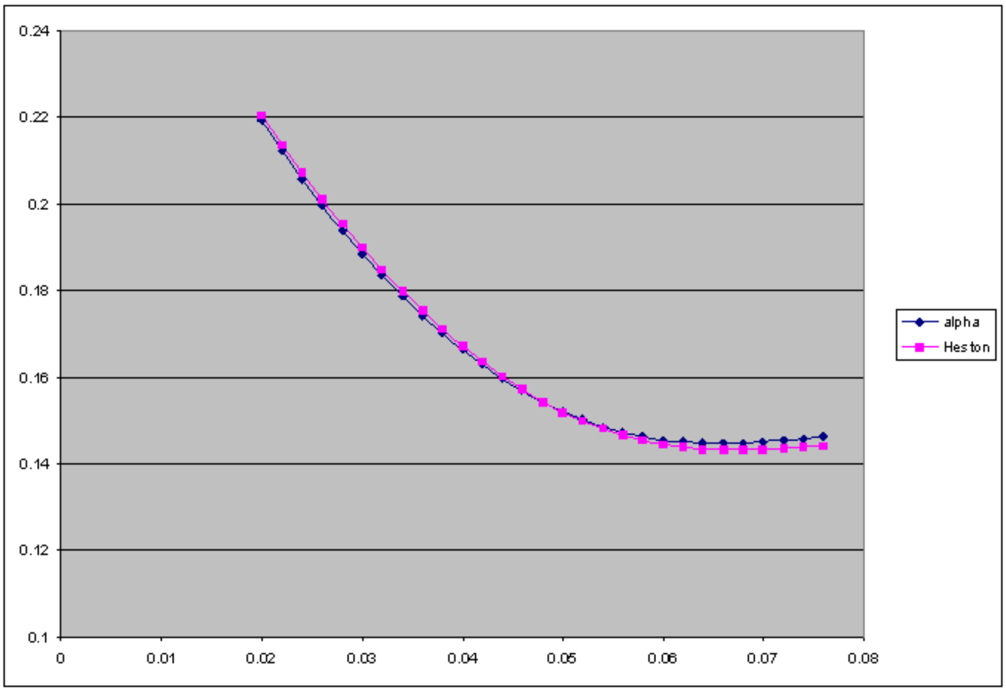
Alpha

Heston

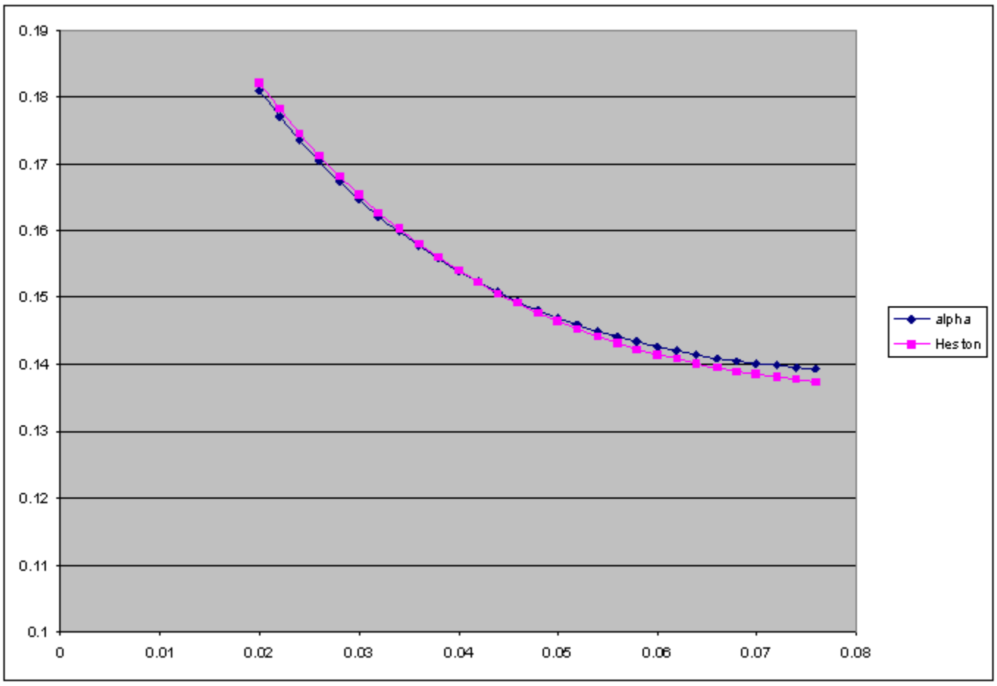
Example Implied volatility T=1 years



Example Implied volatility T=10 years



Example Implied volatility T=30 years



Explication for the high level of accuracy

- (1) The original process and the approximated process have the same mean reversion

⇒ Long term vol of vol the same

- (2) At any time   limit (vol of vol -> 0) exact

Spreadoptions

Same level of accuracy with Normal Heston

Applying it to Spreadoption with Normal Heston

Need to check condition (2)

First idea :

$$\text{observable}_N\,(\text{vol of vol}) = \text{observable}_R\,(\text{vol of vol}) - \text{observable}_R\,(\text{vol of vol} = 0) + \text{observable}_{Bilog}\,(\text{vol of vol} = 0)$$

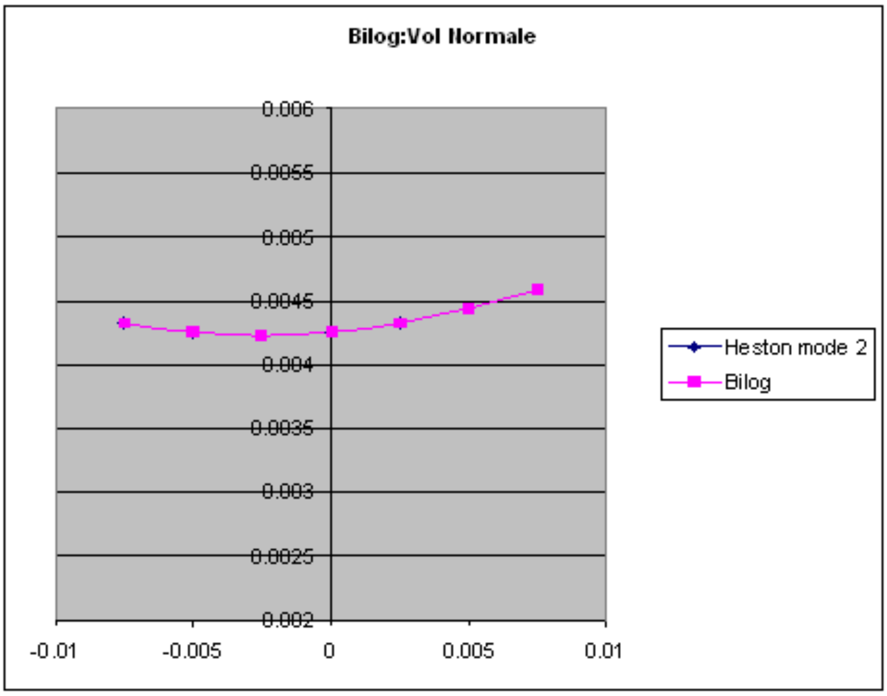
– but does not work very well → bias for long maturities and resulting negative observable_N (vol of vol) from time to time.

Better Idea :

$$\text{effective observable}_{Bilog} = \text{Calibrate}_{Bilog}\,(V0,\ \text{rho},\ \text{nu})$$

$$\text{observable}_N\,(\text{vol of vol}) = \text{observable}_R\,(\text{vol of vol}) - \text{observable}_R\,(\text{vol of vol} = 0) + \text{effective observable}_{Bilog}$$

(2) verified by construction


Bilog:Vol Normale
0.006
0.0055
0.005
0.0045
0.004
0.0035
0.003
0.0025
0.002
-0.01
-0.005
0
0.005
0.01
Heston mode 2
Bilog

Pricing of Spreadoption

- (1) Computation of the observables
- (2) Correction of the observables to get the right (volvol->0) behaviour)
- (3) Normal Heston

Schema of the calibration

$$\mathrm{Min}\Big[\sum_{i\in\{\text{option},\text{spreadoption}\}}^{n} \alpha^{i}{}_{\mathrm{V0}} \left(\mathrm{V0}^{i}{}_{\mathrm{market}} - \mathrm{V0}^{i}{}_{\mathrm{model}}\right)^{2} +$$

$$\alpha^{i}{}_{\mathrm{VarVar}} \left(\mathrm{VarVar}^{i}{}_{\mathrm{market}} - \mathrm{VarVar}^{i}{}_{\mathrm{model}}\right)^{2} + \alpha^{i}{}_{\mathrm{CoVar}} \left(\mathrm{CoVar}_{\mathrm{market}} - \mathrm{CoVar}^{i}{}_{\mathrm{model}}\right)^{2}\Big]$$

α^{i} are typically determined by :

$$\alpha^{i}{}_{\mathrm{V0}} = \frac{1}{\left(\mathrm{V0}^{i}{}_{\mathrm{model}}\right)^{2}}$$

$$\alpha^{i}{}_{\mathrm{VarVar}} = \frac{1}{\left(\mathrm{VarVar}^{i}{}_{\mathrm{model}}\right)^{2}}$$

$$\alpha^{i}{}_{\mathrm{CoVar}} = \frac{1}{\left(\mathrm{Abs}\left[\mathrm{CoVar}^{i}{}_{\mathrm{model}}\right]_{\mathrm{average}}\right)^{2}}$$

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Generalization of the alpha model to smiled basket and signed smiled basket

Smiled PCA approach

Conclusion