Comparaison avec MonteCarlo

$dF_1 = \alpha_1\, C_1[F_1]\, dW_1;$

$d\alpha_1 = \nu_1\, \alpha_1\, dW_{v1};$

$dW_1\, dW_{v1} = \rho_1\, dt;$

$dF_2 = \alpha_2\, C_2[F_2]\, dW_2;$

$d\alpha_2 = \nu_2\, \alpha_2\, dW_{v2};$

$dW_2\, dW_{v2} = \rho_2\, dt;$

plus the links

$dW_1\, dW_2 = \rho_s\, dt;\ dW_1\, dW_{v2} = \rho_{c12}\, dt;\ dW_2\, dW_{v1} = \rho_{c21}\, dt;\ dW_{v1}\, dW_{v2} = \rho_v\, dt;$

We Take the log, to keep variables positive

$$d\,(\mathrm{Log}[F_1]) = \frac{\alpha_1\, C_1[F_1]}{F_1}\, dW_1 - \frac{(\alpha_1\, C_1[F_1])^2}{2\, F_1^2}\, dt;$$

$$d\,(\mathrm{Log}[\alpha_1]) = \nu_1\, dW_{v1} - \frac{\nu_1^2}{2}\, dt;$$

$dW_1\, dW_{v1} = \rho_1\, dt;$

$$d\,(\mathrm{Log}[F_2]) = \frac{\alpha_2\, C_2[F_2]}{F_2}\, dW_2 - \frac{(\alpha_2\, C_2[F_2])^2}{2\, F_2^2}\, dt;$$

$$d\,(\mathrm{Log}[\alpha_2]) = \nu_2\, dW_{v2} - \frac{\nu_2^2}{2}\, dt;$$

$dW_2\, dW_{v2} = \rho_2\, dt;$

plus the links

```
Clear[dt]
```

```
dt {{1, ρ1, ρs, ρc12}, {ρ1, 1, ρc21, ρv}, {ρs, ρc21, 1, ρ2}, {ρc12, ρv, ρ2, 1}} //
 MatrixForm
```

$$\begin{pmatrix} dt & dt\,\rho 1 & dt\,\rho s & dt\,\rho c12 \\ dt\,\rho 1 & dt & dt\,\rho c21 & dt\,\rho v \\ dt\,\rho s & dt\,\rho c21 & dt & dt\,\rho 2 \\ dt\,\rho c12 & dt\,\rho v & dt\,\rho 2 & dt \end{pmatrix}$$

$dW_1\, dW_2 = \rho_s\, dt;\ dW_1\, dW_{v2} = \rho_{c12}\, dt;\ dW_2\, dW_{v1} = \rho_{c21}\, dt;\ dW_{v1}\, dW_{v2} = \rho_v\, dt;$

```
BiSABRGeneratePath[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_,
  ν2_, T_, ρs_, ρv_, ρc12_, ρc21_, TimeStepsNb_, dt_, printflag_] :=
 Module[{i, j, Fn1 = F1, αn1 = α1, Fn2 = F2, αn2 = α2, W1, Wv1, W2, Wv2, C, sqdt = √dt},
  C = dt {{1, ρ1, ρs, ρc12}, {ρ1, 1, ρc21, ρv}, {ρs, ρc21, 1, ρ2}, {ρc12, ρv, ρ2, 1}};
  If[printflag > 1, Print["C=", C // MatrixForm]];
  Do[alea = Random[MultinormalDistribution[{0, 0, 0, 0}, C]];
   If[printflag > 2, Print["alea=", alea // MatrixForm]];
   {W1, Wv1, W2, Wv2} = alea;
   Fn1 *= Exp[α1 Fn1^(β1-1) W1 - (α1^2 Fn1^(2 β1-2))/2 dt];
   αn1 *= Exp[ν1 Wv1 - ν1^2/2 dt];
   Fn2 *= Exp[α2 Fn2^(β2-1) W2 - (α2^2 Fn2^(2 β2-2))/2 dt];
   αn2 *= Exp[ν2 Wv2 - ν2^2/2 dt];
   , {i, 1, TimeStepsNb}];
  {Fn1, αn1, Fn2, αn2}]
```

$$Fn1 \mathrel{*}= \mathrm{Exp}\left[\alpha1\, Fn1^{\beta1-1}\, W1 - \frac{\alpha1^2\, Fn1^{2\,\beta1-2}}{2}\, dt\right];$$

$$\alpha n1 \mathrel{*}= \mathrm{Exp}\left[\nu1\, Wv1 - \frac{\nu1^2}{2}\, dt\right];$$

$$Fn2 \mathrel{*}= \mathrm{Exp}\left[\alpha2\, Fn2^{\beta2-1}\, W2 - \frac{\alpha2^2\, Fn2^{2\,\beta2-2}}{2}\, dt\right];$$

$$\alpha n2 \mathrel{*}= \mathrm{Exp}\left[\nu2\, Wv2 - \frac{\nu2^2}{2}\, dt\right];$$

```
General::spell1 : Possible spelling error: new symbol name "αn1" is similar to existing symbol "α1".
   Plus…
General::spell1 : Possible spelling error: new symbol name "αn2" is similar to existing symbol "α2".
   Plus…
```

```
BiSABRGenerateAntitheticPath[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_,
   ρ2_, ν2_, T_, ρs_, ρv_, ρc12_, ρc21_, TimeStepsNb_, dt_, printflag_] :=
  Module[{i, j, Fn1 = F1, αn1 = α1, Fn2 = F2, αn2 = α2, Fn1a = F1,
    αn1a = α1, Fn2a = F2, αn2a = α2, W1, Wv1, W2, Wv2, C, sqdt = √dt},
   C = dt {{1, ρ1, ρs, ρc12}, {ρ1, 1, ρc21, ρv}, {ρs, ρc21, 1, ρ2}, {ρc12, ρv, ρ2, 1}};
   If[printflag > 1, Print["C=", C // MatrixForm]];
   Do[alea = Random[MultinormalDistribution[{0, 0, 0, 0}, C]];
    If[printflag > 2, Print["alea=", alea // MatrixForm]];
    {W1, Wv1, W2, Wv2} = alea;
```

$$Fn1 \mathrel{*}= \mathrm{Exp}\left[\alpha1\, Fn1^{\beta1-1}\, W1 - \frac{\alpha1^2\, Fn1^{2\beta1-2}}{2}\, dt\right];$$

$$\alpha n1 \mathrel{*}= \mathrm{Exp}\left[\nu1\, Wv1 - \frac{\nu1^2}{2}\, dt\right];$$

$$Fn2 \mathrel{*}= \mathrm{Exp}\left[\alpha2\, Fn2^{\beta2-1}\, W2 - \frac{\alpha2^2\, Fn2^{2\beta2-2}}{2}\, dt\right];$$

$$\alpha n2 \mathrel{*}= \mathrm{Exp}\left[\nu2\, Wv2 - \frac{\nu2^2}{2}\, dt\right];$$

$$Fn1a \mathrel{*}= \mathrm{Exp}\left[-\alpha1\, Fn1^{\beta1-1}\, W1 - \frac{\alpha1^2\, Fn1^{2\beta1-2}}{2}\, dt\right];$$

$$\alpha n1a \mathrel{*}= \mathrm{Exp}\left[-\nu1\, Wv1 - \frac{\nu1^2}{2}\, dt\right];$$

$$Fn2a \mathrel{*}= \mathrm{Exp}\left[-\alpha2\, Fn2^{\beta2-1}\, W2 - \frac{\alpha2^2\, Fn2^{2\beta2-2}}{2}\, dt\right];$$

$$\alpha n2a \mathrel{*}= \mathrm{Exp}\left[-\nu2\, Wv2 - \frac{\nu2^2}{2}\, dt\right];$$

```
    , {i, 1, TimeStepsNb}];
   {{Fn1, αn1, Fn2, αn2}, {Fn1a, αn1a, Fn2a, αn2a}}]
```

General::spell1 : Possible spelling error: new symbol name "Fn1a" is similar to existing symbol "Fn1". Plus…

General::spell1 : Possible spelling error: new symbol name "αn1a" is similar to existing symbol "αn1". Plus…

General::spell1 : Possible spelling error: new symbol name "Fn2a" is similar to existing symbol "Fn2". Plus…

General::stop : Further output of General::spell1 will be suppressed during this calculation. Plus…

```
BiSABRGenerateSample[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_, ν2_,
   T_, ρs_, ρv_, ρc12_, ρc21_, TimeStepsNb_, dt_, nbSample_, printflag_] :=
  Module[{k}, Table[BiSABRGeneratePath[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
     T, ρs, ρv, ρc12, ρc21, TimeStepsNb, dt, printflag], {k, 1, nbSample}]]
```

```
Module[{
  F1 = 0.0418,
  α1 = 0.0435,
  β1 = 0.6,
  ρ1 = -0.1819,
  ν1 = 0.3798,
  F2 = 0.0363,
  α2 = 0.0671,
  β2 = 0.7,
  ρ2 = -0.1136,
  ν2 = 0.3797,
  T = 5,
  ρs = 0.8,
  ρv = 0.5,
  ρc12 = -0.5,
  ρc21 = -0.,
  TimeStepsNb = 100,
  dt, printflag = 2
 },
 dt = T / TimeStepsNb;
 BiSABRGenerateAntitheticPath[F1, α1, β1, ρ1, ν1, F2,
  α2, β2, ρ2, ν2, T, ρs, ρv, ρc12, ρc21, TimeStepsNb, dt, printflag]]
```

$$C=\begin{pmatrix} \frac{1}{20} & -0.009095 & 0.04 & -0.025 \\ -0.009095 & \frac{1}{20} & 0. & 0.025 \\ 0.04 & 0. & \frac{1}{20} & -0.00568 \\ -0.025 & 0.025 & -0.00568 & \frac{1}{20} \end{pmatrix}$$

{{0.0461916, 0.032089, 0.0464056, 0.014218}, {0.0349947, 0.0286676, 0.0252865, 0.154007}}

```
Module[{
  F1 = 0.0418,
  α1 = 0.0435,
  β1 = 0.6,
  ρ1 = -0.1819,
  ν1 = 0.3798,
  F2 = 0.0363,
  α2 = 0.0671,
  β2 = 0.7,
  ρ2 = -0.1136,
  ν2 = 0.3797,
  T = 5,
  ρs = 0.8,
  ρv = 0.5,
  ρc12 = -0.5,
  ρc21 = -0.,
  TimeStepsNb = 100,
  nbSample = 2,
  dt, printflag = 0
 },
 dt = T / TimeStepsNb;
 BiSABRGenerateAntitheticSample[F1, α1, β1, ρ1, ν1, F2, α2, β2,
  ρ2, ν2, T, ρs, ρv, ρc12, ρc21, TimeStepsNb, dt, nbSample, printflag]]
```

```
{{0.0323098, 0.108202, 0.0207838, 0.0407681},
 {0.0495623, 0.00850183, 0.0557351, 0.0537104},
 {0.0271556, 0.00691219, 0.0156935, 0.0112271},
 {0.0579937, 0.133086, 0.0705171, 0.195035}}
```

```
BiSABRGenerateAntitheticSample[F1_, α1_, β1_,
  ρ1_, ν1_, F2_, α2_, β2_, ρ2_, ν2_, T_, ρs_, ρv_, ρc12_, ρc21_,
  TimeStepsNb_, dt_, nbSample_, printflag_] := Module[{k, samples},
  samples = Table[BiSABRGenerateAntitheticPath[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2,
     ν2, T, ρs, ρv, ρc12, ρc21, TimeStepsNb, dt, printflag], {k, 1, nbSample}];
  Flatten[samples,
   1]]
```

```
BiSABRMonteCarloOption[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_, ν2_, K_, T_, ρs_,
  ρv_, ρc12_, ρc21_, TimeStepsNb_, dt_, nbSample_, printflag_] := Module[{samples},
  samples = BiSABRGenerateAntitheticSample[F1, α1, β1, ρ1, ν1, F2, α2, β2,
    ρ2, ν2, T, ρs, ρv, ρc12, ρc21, TimeStepsNb, dt, nbSample, printflag];
  Sum[If[samples[[i, 1]] - samples[[i, 3]] - K ≥ 0, samples[[i, 1]] - samples[[i, 3]] - K, 0],
    {i, 1, Length[samples]}] / Length[samples]]
```

```
BiSABRMonteCarloSmile[F1_, α1_, β1_, ρ1_, ν1_, F2_,
  α2_, β2_, ρ2_, ν2_, StrikeList_, T_, ρs_, ρv_, ρc12_, ρc21_,
  TimeStepsNb_, dt_, nbSample_, printflag_] := Module[{samples, i, k},
  samples = BiSABRGenerateAntitheticSample[F1, α1, β1, ρ1, ν1, F2, α2, β2,
    ρ2, ν2, T, ρs, ρv, ρc12, ρc21, TimeStepsNb, dt, nbSample, printflag];
  Table[Sum[If[samples⟦i, 1⟧ - samples⟦i, 3⟧ - StrikeList⟦k⟧ ≥ 0,
      samples⟦i, 1⟧ - samples⟦i, 3⟧ - StrikeList⟦k⟧, 0], {i, 1, Length[samples]}] /
    Length[samples], {k, 1, Length[StrikeList]}]]
```

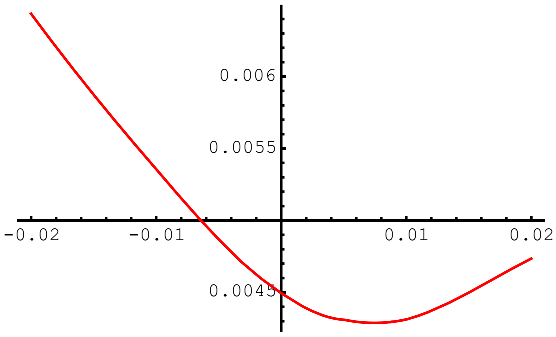
```
Module[{
  F1 = 0.0418,
  α1 = 0.0435,
  β1 = 0.6,
  ρ1 = -0.1819,
  ν1 = 0.3798,
  F2 = 0.0363,
  α2 = 0.0671,
  β2 = 0.7,
  ρ2 = -0.1136,
  ν2 = 0.3797,
  K = 0.001,
  T = 5,
  ρs = 0.8,
  ρv = 0.5,
  ρc12 = -0.5,
  ρc21 = -0.,
  TimeStepsNb = 100,
  nbSample = 10,
  dt, printflag = 0
 },
 dt = T / TimeStepsNb;
 BiSABRMonteCarloOption[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2,
  ν2, K, T, ρs, ρv, ρc12, ρc21, TimeStepsNb, dt, nbSample, printflag]]
```

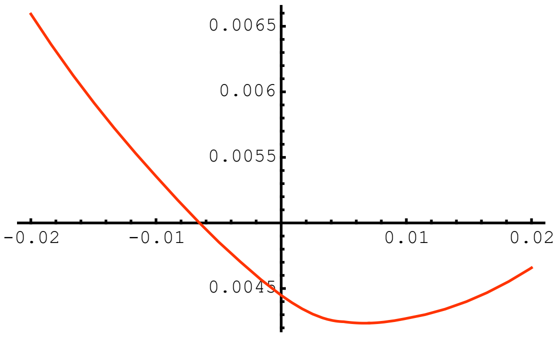
```
0.0062367
```

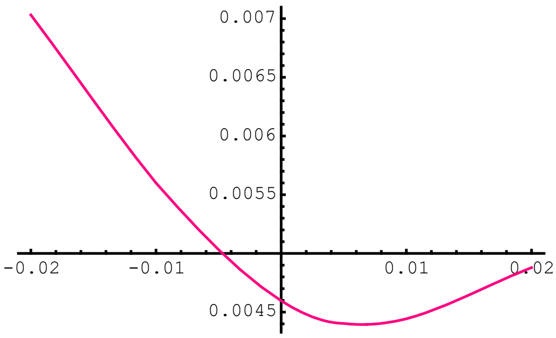
```
Timing[Module[{
   F1 = 0.0418,
   α1 = 0.0435,
   β1 = 0.6,
   ρ1 = -0.1819,
   ν1 = 0.3798,
   F2 = 0.0363,
   α2 = 0.0671,
   β2 = 0.7,
   ρ2 = -0.1136,
   ν2 = 0.3797,
   StrikeList =
    {-0.02, -0.015, -0.01, -0.005, -0.001, 0, 0.001, 0.0050, 0.01, 0.015, 0.02},
   T = 5,
   ρs = 0.8,
```

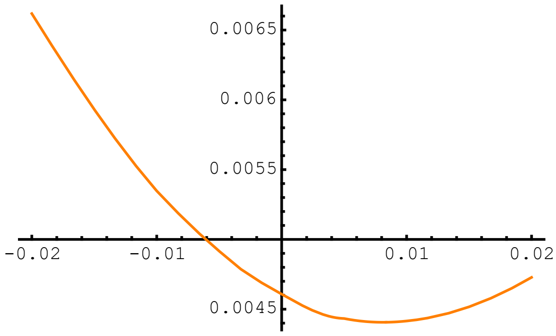
```
 ρv = 0.,
 ρc12 = -0.2,
 ρc21 = -0.,
 TimeStepsNb = 200,
 nbSample = 1000,
 dt, callvalues, implicitNormalvols, printflag = 0
},
dt = T / TimeStepsNb;
MCcallvalues = BiSABRMonteCarloSmile[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
  StrikeList, T, ρs, ρv, ρc12, ρc21, TimeStepsNb, dt, nbSample, printflag];
MCimplicitNormalvols = Table[NormalImplicitVol[F1 - F2, StrikeList[[i]],
   T, MCcallvalues[[i]]], {i, 1, Length[StrikeList]}];
MCInterpolation = Interpolation[Transpose[{StrikeList, MCimplicitNormalvols}]];
MCgraph = Plot[MCInterpolation[x], {x, Min[StrikeList], Max[StrikeList]},
  PlotRange → All, PlotStyle → {{Thickness[0.005], RGBColor[1, 0, 0]}}];

MCcallvalues2 = BiSABRMonteCarloSmile[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
  StrikeList, T, ρs, ρv, ρc12, ρc21, TimeStepsNb, dt, nbSample, printflag];
MCimplicitNormalvols2 = Table[NormalImplicitVol[F1 - F2, StrikeList[[i]],
   T, MCcallvalues2[[i]]], {i, 1, Length[StrikeList]}];
MCInterpolation2 = Interpolation[Transpose[{StrikeList, MCimplicitNormalvols2}]];
MCgraph = Plot[MCInterpolation2[x], {x, Min[StrikeList], Max[StrikeList]},
  PlotRange → All, PlotStyle → {{Thickness[0.005], RGBColor[1, 0.2, 0]}}];

MCcallvalues3 = BiSABRMonteCarloSmile[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
  StrikeList, T, ρs, ρv, ρc12, ρc21, TimeStepsNb, dt, nbSample, printflag];
MCimplicitNormalvols3 = Table[NormalImplicitVol[F1 - F2, StrikeList[[i]],
   T, MCcallvalues3[[i]]], {i, 1, Length[StrikeList]}];
MCInterpolation3 = Interpolation[Transpose[{StrikeList, MCimplicitNormalvols3}]];
MCgraph3 = Plot[MCInterpolation3[x], {x, Min[StrikeList], Max[StrikeList]},
  PlotRange → All, PlotStyle → {{Thickness[0.005], RGBColor[1, 0, 0.5]}}];

MCcallvalues4 = BiSABRMonteCarloSmile[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
  StrikeList, T, ρs, ρv, ρc12, ρc21, TimeStepsNb, dt, nbSample, printflag];
MCimplicitNormalvols4 = Table[NormalImplicitVol[F1 - F2, StrikeList[[i]],
   T, MCcallvalues4[[i]]], {i, 1, Length[StrikeList]}];
MCInterpolation4 = Interpolation[Transpose[{StrikeList, MCimplicitNormalvols4}]];
MCgraph4 = Plot[MCInterpolation4[x], {x, Min[StrikeList], Max[StrikeList]},
  PlotRange → All, PlotStyle → {{Thickness[0.005], RGBColor[1, 0.5, 0]}}];

Analyticcallvalues = Table[BiSABRSpreadOption[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
   StrikeList[[i]], T, ρs, ρv, ρc12, ρc21, 0], {i, 1, Length[StrikeList]}];
AnalyticalimplicitNormalvols = Table[NormalImplicitVol[F1 - F2,
   StrikeList[[i]], T, Analyticcallvalues[[i]]], {i, 1, Length[StrikeList]}];
AnalyticalInterpolation = Interpolation[
  Transpose[{StrikeList, AnalyticalimplicitNormalvols}]];
Analyticalgraph = Plot[AnalyticalInterpolation[x],
  {x, Min[StrikeList], Max[StrikeList]}, PlotRange → All,
  PlotStyle → {{Thickness[0.005], RGBColor[0, 1, 0]}}];
```

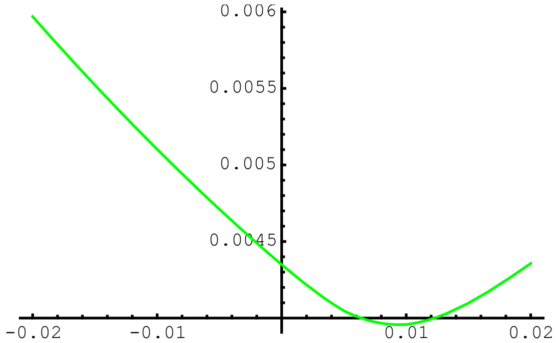
```
Analyticcallvalues2 = Table[BiSABRSpreadOptionCorrected[F1, α1, β1, ρ1, ν1, F2, α2, β2,
    ρ2, ν2, StrikeList[[i]], T, ρs, ρv, ρc12, ρc21, 0], {i, 1, Length[StrikeList]}];
AnalyticalimplicitNormalvols2 = Table[NormalImplicitVol[F1 - F2,
    StrikeList[[i]], T, Analyticcallvalues2[[i]]], {i, 1, Length[StrikeList]}];
AnalyticalInterpolation2 = Interpolation[
   Transpose[{StrikeList, AnalyticalimplicitNormalvols2}]];
Analyticalgraph2 = Plot[AnalyticalInterpolation2[x],
   {x, Min[StrikeList], Max[StrikeList]}, PlotRange → All,
   PlotStyle → {{Thickness[0.005], RGBColor[0, 0, 1]}}];

Plot[{MCInterpolation[x], MCInterpolation2[x], MCInterpolation3[x],
   MCInterpolation4[x], AnalyticalInterpolation[x], AnalyticalInterpolation2[x]},
  {x, Min[StrikeList], Max[StrikeList]}, PlotRange → All, PlotStyle →
   {{Thickness[0.005], RGBColor[1, 0, 0]}, {Thickness[0.005], RGBColor[1, 0.3, 0]},
    {Thickness[0.005], RGBColor[1, 0, 0.5]}, {Thickness[0.005], RGBColor[1, 0.6, 0]},
    {Thickness[0.005], RGBColor[0, 1, 0]}, {Thickness[0.005], RGBColor[0, 0, 1]}},
  PlotLegend → {"MC 1", "MC 2", "MC 3", "MC 4", "BiSABR", "Corrected BiSABR"}]
]]
```

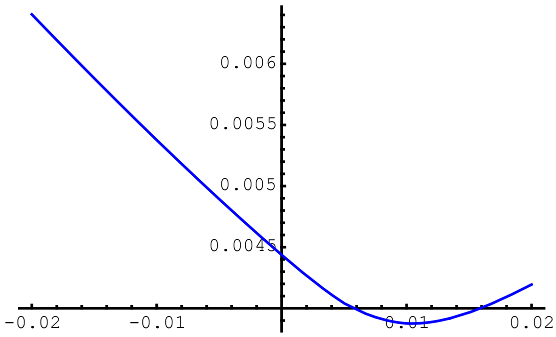


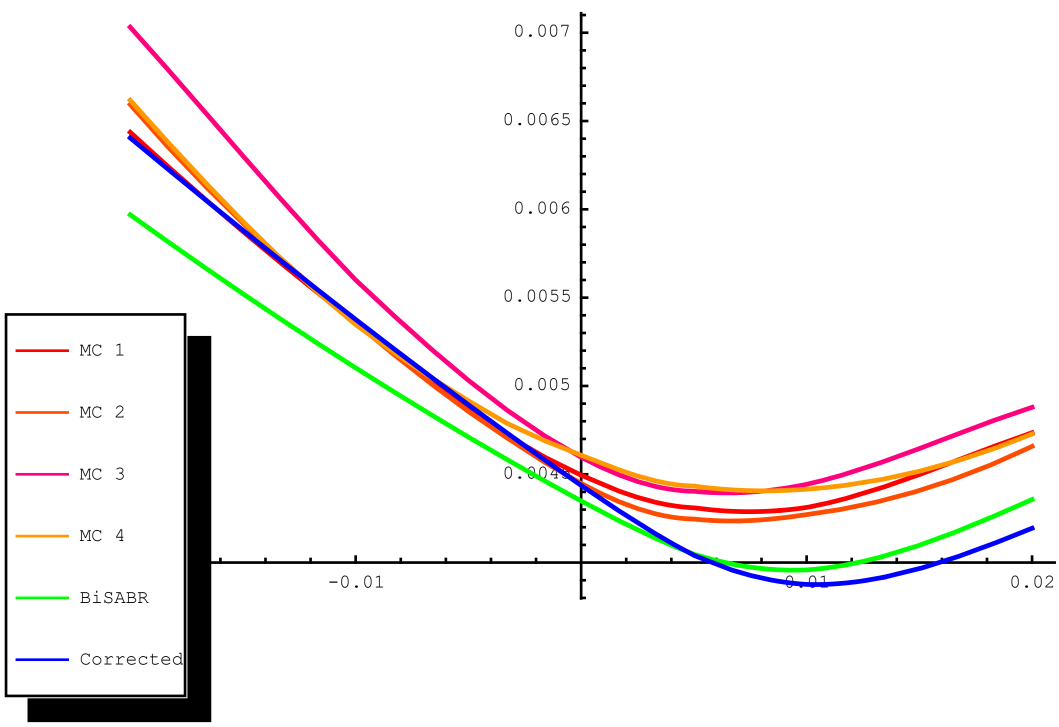




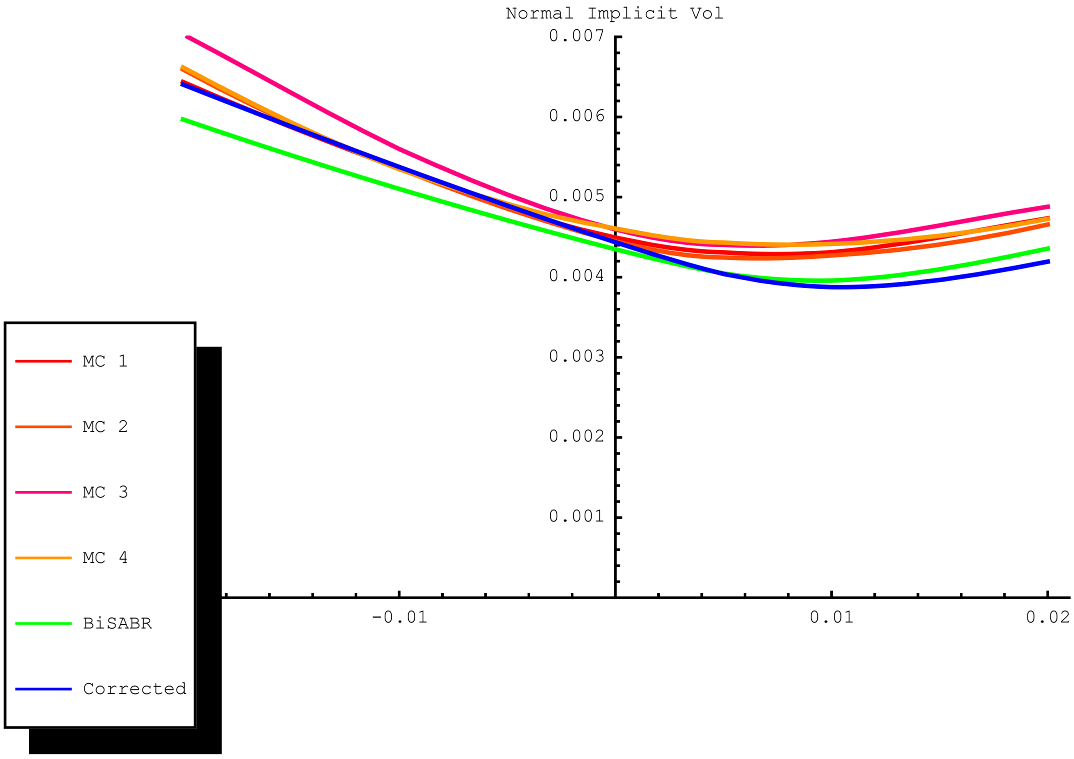

0.0065
0.006
0.0055
0.0045
-0.02
-0.01
0.01
0.02


0.006
0.0055
0.005
0.0045
-0.02
-0.01
0.01
0.02


0.006
0.0055
0.005
0.0045
-0.02
-0.01
0.01
0.02


0.007
0.0065
0.006
0.0055
0.005
0.0045
-0.01
0.01
0.02
MC 1
MC 2
MC 3
MC 4
BiSABR
Corrected

```
{705.437 Second, - Graphics -}

Module[{StrikeList =
    {-0.02, -0.015, -0.01, -0.005, -0.001, 0, 0.001, 0.0050, 0.01, 0.015, 0.02}},
 Plot[{MCInterpolation[x], MCInterpolation2[x], MCInterpolation3[x],
    MCInterpolation4[x], AnalyticalInterpolation[x], AnalyticalInterpolation2[x]},
   {x, Min[StrikeList], Max[StrikeList]}, PlotLegend →
    {"MC 1", "MC 2", "MC 3", "MC 4", "BiSABR", "Corrected BiSABR"}, PlotStyle →
    {{Thickness[0.005], RGBColor[1, 0, 0]}, {Thickness[0.005], RGBColor[1, 0.3, 0]},
     {Thickness[0.005], RGBColor[1, 0, 0.5]}, {Thickness[0.005], RGBColor[1, 0.6, 0]},
     {Thickness[0.005], RGBColor[0, 1, 0]}, {Thickness[0.005], RGBColor[0, 0, 1]}},
   PlotRange → {0, 0.007}, PlotLabel → "Normal Implicit Vol"]]
```




```
- Graphics -
```