# Process de base du modele

$dF_1 = \alpha_1 C_1[F_1] dW_1;$

$d\alpha_1 = \nu_1 \alpha_1 dW_{v1};$

$dW_1 dW_{v1} = \rho_1 dt;$

$dF_2 = \alpha_2 C_2[F_2] dW_2;$

$d\alpha_2 = \nu_2 \alpha_2 c;$
$dW_2 dW_{v2} = \rho_2 dt;$

plus the links

$dW_1 dW_2 = \rho_s dt;\ dW_1 dW_{v2} = \rho_{c12} dt;\ dW_2 dW_{v1} = \rho_{c21} dt;\ dW_{v1} dW_{v2} = \rho_v dt;$

Les browniens sont donc : $\{dW_1, dW_{v1}, dW_2, dW_{v2}\}$

les variables sont { S1, alpha1, S2, alpha2}

# Implementation

```
Corr = {
    {1, rho1, rhos, rhoc12},
    {rho1, 1, rhoc21, rhov},
    {rhos, rhoc21, 1, rho2},
    {rhoc12, rhov, rho2, 1}
   };
```

```
Diff[x_] :=
 D[x, S1] {alpha1 S1^beta1, 0, 0, 0} +
  D[x, S2] {0, 0, alpha2 S2^beta2, 0} +
  D[x, alpha1] {0, alpha1 nu1, 0, 0} +
  D[x, alpha2] {0, 0, 0, alpha2 nu2}
```

```
zeroRule = {alpha1 → 0, alpha2 → 0};
```

```
CompuputePureFunctionBiSABRSpreadObservable[spread_] :=
  Module[{VS, VarVS, VarVS0, CoVarVS, CoVarVS0},
   VS = Simplify[Diff[spread].(Corr.Diff[spread])];
   VarVS = Simplify[Diff[VS].(Corr.Diff[VS])];
   VarVS0 = VarVS /. zeroRule;
   CoVarVS = Simplify[Diff[VS].(Corr.Diff[spread])];
   CoVarVS0 = CoVarVS /. zeroRule;
   Apply[Function, {{S1, alpha1, S2, alpha2},
     {VS, Simplify[VarVS - VarVS0],
      Simplify[CoVarVS - CoVarVS0], Simplify[VarVS0], Simplify[CoVarVS0]}}]];
```

```
CompuputePureFunctionBiSABRUnderlyingObservable[Under_] :=
  Module[{VS, VarVS, CoVarVS},
   VS = Simplify[Diff[Under].(Corr.Diff[Under])] / Under^2;
   VarVS = Simplify[Diff[VS].(Corr.Diff[VS])];
   CoVarVS = Simplify[Diff[VS].(Corr.Diff[Under])] / Under;
   Apply[Function, {{S1, alpha1, S2, alpha2},
     {VS, Simplify[VarVS], Simplify[CoVarVS]}}]];
```

# Underlyings

## Underlying 1

```
BiSABRUnderlyingObservable1z = CompuputePureFunctionBiSABRUnderlyingObservable[S1];
```

```
VSUnderlying1z[S1_, alpha1_, beta1_, rho1_, nu1_, S2_,
  alpha2_, beta2_, rho2_, nu2_, rhos_, rhov_, rhoc12_, rhoc21_] :=
 BiSABRUnderlyingObservable1z[S1, alpha1, S2, alpha2][[1]]
```

```
VarVSUnderlying1z[S1_, alpha1_, beta1_, rho1_, nu1_, S2_,
  alpha2_, beta2_, rho2_, nu2_, rhos_, rhov_, rhoc12_, rhoc21_] :=
 BiSABRUnderlyingObservable1z[S1, alpha1, S2, alpha2][[2]]
```

```
CoVarVSUnderlying1z[S1_, alpha1_, beta1_, rho1_, nu1_, S2_,
  alpha2_, beta2_, rho2_, nu2_, rhos_, rhov_, rhoc12_, rhoc21_] :=
 BiSABRUnderlyingObservable1z[S1, alpha1, S2, alpha2][[3]]
```

## Underlying 2

```
BiSABRUnderlyingObservable2z = CompuputePureFunctionBiSABRUnderlyingObservable[S2];
```

```
VSUnderlying2z[S1_, alpha1_, beta1_, rho1_, nu1_, S2_,
  alpha2_, beta2_, rho2_, nu2_, rhos_, rhov_, rhoc12_, rhoc21_] :=
 BiSABRUnderlyingObservable2z[S1, alpha1, S2, alpha2][[1]]
```

```
VarVSUnderlying2z[S1_, alpha1_, beta1_, rho1_, nu1_, S2_,
  alpha2_, beta2_, rho2_, nu2_, rhos_, rhov_, rhoc12_, rhoc21_] :=
 BiSABRUnderlyingObservable2z[S1, alpha1, S2, alpha2][[2]]
```

```
CoVarVSUnderlying2z[S1_, alpha1_, beta1_, rho1_, nu1_, S2_,
  alpha2_, beta2_, rho2_, nu2_, rhos_, rhov_, rhoc12_, rhoc21_] :=
 BiSABRUnderlyingObservable2z[S1, alpha1, S2, alpha2][[3]]
```

## Spreadoptions

```
BiSABRSpreadObservablez = CompuputePureFunctionBiSABRSpreadObservable[(S2 - S1)];
```

```
BiSABRVSSpreadz[S1_, alpha1_, beta1_, rho1_, nu1_, S2_,
  alpha2_, beta2_, rho2_, nu2_, rhos_, rhov_, rhoc12_, rhoc21_] :=
 BiSABRSpreadObservablez[S1, alpha1, S2, alpha2][[1]]
```

```
BiSABRVarVSSpreadz[S1_, alpha1_, beta1_, rho1_, nu1_, S2_,
  alpha2_, beta2_, rho2_, nu2_, rhos_, rhov_, rhoc12_, rhoc21_] :=
 BiSABRSpreadObservablez[S1, alpha1, S2, alpha2][[2]]
```

```
BiSABRCoVarVSSpreadz[S1_, alpha1_, beta1_, rho1_, nu1_, S2_,
  alpha2_, beta2_, rho2_, nu2_, rhos_, rhov_, rhoc12_, rhoc21_] :=
 BiSABRSpreadObservablez[S1, alpha1, S2, alpha2][[3]]
```

```
BiSABRVarVSSpreadz0[S1_, alpha1_, beta1_, rho1_, nu1_, S2_,
  alpha2_, beta2_, rho2_, nu2_, rhos_, rhov_, rhoc12_, rhoc21_] :=
 BiSABRSpreadObservablez[S1, alpha1, S2, alpha2][[2]] +
  BiSABRSpreadObservablez[S1, alpha1, S2, alpha2][[4]]
```

```
BiSABRCoVarVSSpreadz0[S1_, alpha1_, beta1_, rho1_, nu1_, S2_,
  alpha2_, beta2_, rho2_, nu2_, rhos_, rhov_, rhoc12_, rhoc21_] :=
 BiSABRSpreadObservablez[S1, alpha1, S2, alpha2][[3]] +
  BiSABRSpreadObservablez[S1, alpha1, S2, alpha2][[5]]
```

```
ColumnForm[BiSABRSpreadObservablez[S1, alpha1, S2, alpha2]]
```

$$\text{alpha1}^2\ \text{S1}^{2\,\text{beta1}} - 2\,\text{alpha1}\,\text{alpha2}\,\text{rhos}\,\text{S1}^{\text{beta1}}\,\text{S2}^{\text{beta2}} + \text{alpha2}^2\,\text{S2}^{2\,\text{beta2}}$$

$$\frac{4\left(\text{alpha1}^6\,\text{beta1}^2\,\text{S1}^{6\,\text{beta1}}\,\text{S2}^2 - 2\,\text{alpha1}^5\,\text{beta1}\,\text{S1}^{5\,\text{beta1}}\,\text{S2}^2\left(-\text{nu1}\,\text{rho1}\,\text{S1} + \text{alpha2}\,\text{beta1}\,\text{rhos}\,\text{S2}^{\text{beta2}}\right) + \text{alpha2}^4\,\text{S1}^2\,\text{S2}^{4\,\text{beta2}}\left(\text{nu2}^2\,\text{S2}^2 + \text{alpha2}^2\right.\right.}{}$$

$$\left.\left. - \frac{2\left(\text{alpha1}^4\,\text{beta1}\,\text{S1}^{4\,\text{beta1}}\,\text{S2} - \text{alpha2}^3\,\text{S1}\,\text{S2}^{3\,\text{beta2}}\left(\text{nu2}\,\text{rho2}\,\text{S2} + \text{alpha2}\,\text{beta2}\,\text{S2}^{\text{beta2}}\right) + \text{alpha1}^3\,\text{S1}^{3\,\text{beta1}}\,\text{S2}\left(\text{nu1}\,\text{rho1}\,\text{S1} - 2\,\text{alpha2}\,\text{beta1}\,\text{rhos}\,\text{S}\right.\right.\right.}{}$$

0

0

```
CForm[BiSABRSpreadObservablez[S1, alpha1, S2, alpha2][[3]]]
```

```
(-2*(Power(alpha1,4)*beta1*Power(S1,4*beta1)*S2 - Power(alpha2,3)*S1*Power(S2,3*beta2)
      alpha1*Power(alpha2,2)*Power(S1,1 + beta1)*Power(S2,2*beta2)*(nu2*(rhoc12 + rho2
      Power(alpha1,2)*alpha2*Power(S1,2*beta1)*Power(S2,beta2)*(nu1*(rhoc21 + rho1*rhc
```