$$\text{(* call=N}\left[\text{Max[f-K,0]}+\frac{\text{(f-K)}}{2\sqrt{2\pi}}\,\frac{E^{\theta}}{x}\int_0^T\frac{E^{-\frac{b^2}{2u}-a^2u}}{\sqrt{u}}\,du\right]\text{;*)}$$

```
<< "C:\\Documents and Settings\\ocroissant\\My Documents\\NumericalIntegration.m"
```

```
<< "C:\\Documents and Settings\\ocroissant\\My Documents\\BS.m"
```

```
<< "C:\\Documents and Settings\\ocroissant\\My Documents\\SABR.m"
```

```
<< "PlotLegends`"
```

```
<< "MultivariateStatistics`"
```

```
Off[General::"spell"]
```

Fonctions utiles

Performs the integral : $\int_0^x e^{-a^2 s^2-\frac{b^2}{s^2}}\,ds$ = GIntegralAnalytical[a, b, x]

$$
\begin{aligned}
&\text{GIntegralAnalytical[a1\_, b1\_, x1\_] := Module}\Big[\{a, b, x\},\\
&\quad x = \text{Abs[x1]};\\
&\quad b = \text{Abs[b1]};\\
&\quad a = \text{Abs[a1]};\\
&\quad \text{Re}\Big[\text{If}\Big[\text{Abs[a]} > 0.001,\\
&\qquad \text{Sign[x1]}\left(\frac{\sqrt{\pi}}{4\,a}\left(e^{-2\,a\,b}\left(1-e^{4\,a\,b}+e^{4\,a\,b}\,\text{Erf}\left[\frac{b}{x}+a\,x\right]-\text{Erf}\left[\frac{b}{x}-a\,x\right]\right)\right)\right),\\
&\qquad \text{Sign[x1]}\Bigg(\left(-b\sqrt{\pi}+e^{-\frac{b^2}{x^2}}\,x+b\sqrt{\pi}\,\text{Erf}\left[\frac{b}{x}\right]\right)+\\
&\qquad\quad \frac{1}{3}\left(-e^{-\frac{b^2}{x^2}}\left(2\,b^3\,e^{\frac{b^2}{x^2}}\sqrt{\pi}-2\,b^2\,x+x^3\right)+2\,b^3\sqrt{\pi}\,\text{Erf}\left[\frac{b}{x}\right]\right)a^2+\\
&\qquad\quad \frac{1}{30}\,e^{-\frac{b^2}{x^2}}\left(-4\,b^5\,e^{\frac{b^2}{x^2}}\sqrt{\pi}+4\,b^4\,x-2\,b^2\,x^3+3\,x^5+4\,b^5\,e^{\frac{b^2}{x^2}}\sqrt{\pi}\,\text{Erf}\left[\frac{b}{x}\right]\right)a^4\Bigg)\\
&\quad \Big]\Big]\Big]
\end{aligned}
$$

If we use the No function defined by No[x] := $\frac{1}{2}\,\text{Erf}\left[\frac{x}{\sqrt{2}}\right]+\frac{1}{2}$

the ATM is caracterized by : b1 = 0

Implementation of GIntegralAnalytical using NormalCDF function

```
GIntegralAnalytical1[a1_, b1_, x1_] := Module[{a, b, x},
  x = Abs[x1];
  b = Abs[b1];
  a = Abs[a1];
  Re[If[Abs[a] > 0.001, Sign[x1]
```

$$\frac{e^{-2ab}\sqrt{\pi}\left(1 - e^{4ab} - \text{NormalCDF}\left[\sqrt{2}\left(\frac{b}{x} - a\,x\right)\right] + e^{4ab}\,\text{NormalCDF}\left[\sqrt{2}\left(\frac{b}{x} + a\,x\right)\right]\right)}{2\,a},$$

$$\frac{\text{Sign}[x1]}{30}\left(e^{-\frac{b^2}{x^2}}\,x\left(30 + 10\,a^2\left(2\,b^2 - x^2\right) + a^4\left(4\,b^4 - 2\,b^2\,x^2 + 3\,x^4\right)\right) + 4\,b\left(15 + 10\,a^2\,b^2 + 2\,a^4\,b^4\right)\sqrt{\pi}\left(-1 + \text{NormalCDF}\left[\frac{\sqrt{2}\,b}{x}\right]\right)\right)$$

```
]]]
```

Computes $\int_A^B \frac{r^\gamma}{\sqrt{r^2 - r\,G + F}}\,dr$

```
BiSABRIntegral[A_, B_, G_, F_, γ_] := Module[{r},

  Print["A=", A, " B=", B, " G=", G,
   " F=", F, " γ=", γ, " roots=", NSolve[r^2 - r G + F == 0, r],
```

$$\text{" X1=", AppellF1}\left[1+\gamma, \frac{1}{2}, \frac{1}{2}, 2+\gamma, \frac{2\,A}{G - \sqrt{-4\,F + G^2}}, \frac{2\,A}{G + \sqrt{-4\,F + G^2}}\right],$$

$$\text{" X2=", AppellF1}\left[1+\gamma, \frac{1}{2}, \frac{1}{2}, 2+\gamma, \frac{2\,B}{G - \sqrt{-4\,F + G^2}}, \frac{2\,B}{G + \sqrt{-4\,F + G^2}}\right]];$$

$$\frac{1}{\sqrt{F}\,(1+\gamma)}\left(-A^{1+\gamma}\,\text{AppellF1}\left[1+\gamma, \frac{1}{2}, \frac{1}{2}, 2+\gamma, \frac{2\,A}{G - \sqrt{-4\,F + G^2}}, \frac{2\,A}{G + \sqrt{-4\,F + G^2}}\right] + B^{1+\gamma}\,\text{AppellF1}\left[1+\gamma, \frac{1}{2}, \frac{1}{2}, 2+\gamma, \frac{2\,B}{G - \sqrt{-4\,F + G^2}}, \frac{2\,B}{G + \sqrt{-4\,F + G^2}}\right]\right)]$$

```
BiSABRIntegral[A_, B_, G_, F_, γ_] := Module[
  {legendrelst = LegendreCoeffs[10]}, BiSABRIntegralN[A, B, G, F, γ, legendrelst]]
```

```
BiSABRIntegralN[A_, B_, G_, F_, γ_, legendrelst_] := Module[{},
  CoeffBasedIntegrate[(#^γ/Sqrt[#^2 - # G + F]) &,
   LegendreCoeffsFromLegendre[legendrelst, A, B]]
  ]
```

```
Module[{A = 1.2, B = 3.5, F = 3.4, G = 0.9, γ = 0.54, legendrelst = LegendreCoeffs[10]}, {
  BiSABRIntegralN[A, B, G, F, γ, legendrelst], BiSABRIntegral[A, B, G, F, γ]}]
```

{1.36681, 1.36681 + 0. i}

"A="0.1445206290517883`" B="0.14883523895735173`" G="0.24226344208464132`" F="0.022926396629179044`" γ="0.6666666666666667`

Computation of :

```
Module[{A =0.1445206290517883, B =0.14883523895735173, F =0.022926396629179044,
  G = 0.24226344208464132, γ = 0.66666666666666674, legendrelst = LegendreCoeffs[10]}, {
  BiSABRIntegralN[A, B, G, F, γ, legendrelst], BiSABRIntegral[A, B, G, F, γ]}]
```

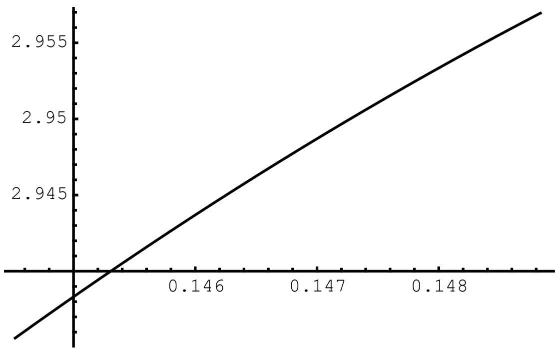
A=0.144521 B=0.148835 G=0.242263 F=0.0229264 γ=0.666667

roots={{r$4102 → 0.121132 - 0.0908488 i}, {r$4102 → 0.121132 + 0.0908488 i}}

X1=1.51264231592120 + $0.\times10^{-15}$ i X2=1.5170278437152 + $0.\times10^{-14}$ i

{0.0127145, 0.0127145 + 0. i}

```
Module[{A =0.1445206290517883, B =0.14883523895735173, F =0.022926396629179044,
  G = 0.24226344208464132, γ = 0.66666666666666674}, Plot[r^γ/Sqrt[r^2 - r G + F], {r, A, B}]]
```



- Graphics -

$$z = \frac{1}{\epsilon\,\alpha}\int_K^f \frac{1}{C[s]}\,ds\ ;\ b1 = C'[f];\ b2 = C''[f]\,C[f] + b1^2;\ B0B\alpha z = C[K]\times C[f]$$

with

$$C[f] = \frac{\sqrt{(f + F2)^{2\beta 1}\,\alpha 1^2 + F2^{2\beta 2}\,\alpha 2^2 - 2\,(f + F2)^{\beta 1}\,F2^{\beta 2}\,\alpha 1\,\alpha 2\,\rho s}}{\sqrt{F1^{2\beta 1}\,\alpha 1^2 + F2^{2\beta 2}\,\alpha 2^2 - 2\,F1^{\beta 1}\,F2^{\beta 2}\,\alpha 1\,\alpha 2\,\rho s}}$$

BiSABRIntegral[A_, B_, G_, F_, γ_] computes $\int_A^B \frac{r^\gamma}{\sqrt{r^2 - r\,G + F}}\, dr$ **; so ...**

```
BiSABRz[K_, F1_, F2_, α1_, α2_, ρs_, β1_, β2_] :=
```
$$\frac{1}{\alpha 1\, \beta 1}\,\text{BiSABRIntegral}\left[(K+F2)^{\beta 1},\ (F1)^{\beta 1},\ \frac{2\ F2^{\beta 2}\ \alpha 2\,\rho s}{\alpha 1},\ F2^{2\,\beta 2}\left(\frac{\alpha 2}{\alpha 1}\right)^2,\ \frac{1-\beta 1}{\beta 1}\right]$$

```
BiSABRz[K_, F1_, F2_, α1_, α2_, ρs_, β1_, β2_, legendrelst_] :=
```
$$\frac{1}{\alpha 1\, \beta 1}\,\text{BiSABRIntegralN}\left[(K+F2)^{\beta 1},\ (F1)^{\beta 1},\ \frac{2\ F2^{\beta 2}\ \alpha 2\,\rho s}{\alpha 1},\ F2^{2\,\beta 2}\left(\frac{\alpha 2}{\alpha 1}\right)^2,\ \frac{1-\beta 1}{\beta 1},\ \text{legendrelst}\right]$$

```
Module[{K = -0.1, F1 = 0.0418, F2 = 0.00728,
  α1 = 0.0435, α2 = 0.041402972188513, ρs = 0.808, β1 = 0.6, β2 = 0.7},
 BiSABRz[K, F1, F2, α1, α2, ρs, β1, β2]
]
```

0.150625 − 2.42599 ⅈ

```
Csabr[f_, F1_, α1_, β1_, F2_, α2_, β2_, ρs_] :=
```
$$\frac{\sqrt{(f+F2)^{2\,\beta 1}\,\alpha 1^2 + F2^{2\,\beta 2}\,\alpha 2^2 - 2\,(f+F2)^{\beta 1}\,F2^{\beta 2}\,\alpha 1\,\alpha 2\,\rho s}}{\sqrt{F1^{2\,\beta 1}\,\alpha 1^2 + F2^{2\,\beta 2}\,\alpha 2^2 - 2\,F1^{\beta 1}\,F2^{\beta 2}\,\alpha 1\,\alpha 2\,\rho s}}$$

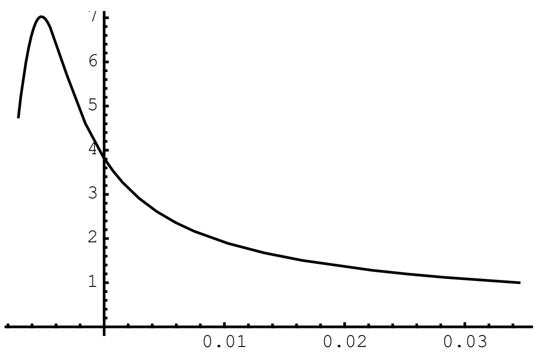
```
Module[{K = -0.1, F1 = 0.0418, F2 = 0.00728, α1 = 0.0435,
  α2 = 0.041402972188513, ρs = 0.808, β1 = 0.6, β2 = 0.7, f},
 f = F1 - F2;
 Plot[1 / Csabr[fx, F1, α1, β1, F2, α2, β2, ρs], {fx, K - 0.01, f}]
]
```

Plot::plnr : $\frac{1}{\text{Csabr[fx, F1\$3792295, α1\$3792295, β1\$3792295, F2\$3792295, α2\$3792295, β2\$3792295, ρs\$3792295]}}$ is not a machine-size real number at fx = -0.11. Plus…

Plot::plnr : $\frac{1}{\text{Csabr[fx, F1\$3792295, α1\$3792295, β1\$3792295, F2\$3792295, α2\$3792295, β2\$3792295, ρs\$3792295]}}$ is not a machine-size real number at fx = -0.104137. Plus…

Plot::plnr : $\frac{1}{\text{Csabr[fx, F1\$3792295, α1\$3792295, β1\$3792295, F2\$3792295, α2\$3792295, β2\$3792295, ρs\$3792295]}}$ is not a machine-size real number at fx = -0.0977434. Plus…

General::stop : Further output of Plot::plnr will be suppressed during this calculation. Plus…



- Graphics -

Definition of AppellF1

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{\text{Pochhammer}[a, k+l]\ \text{Pochhammer}[b_1, k]\ \text{Pochhammer}[b_2, l]\ z_1^k z_2^l}{\text{Pochhammer}[c, k+l]\ k!\ l!} \ /;\ \text{Abs}[z_1] < 1 \land \text{Abs}[z_2] < 1$$

low order development

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == 1 + \frac{a\, b_1\, z_1}{c} + \frac{a\,(1+a)\, b_1\,(1+b_1)\, z_1^2}{2\, c\,(1+c)} + \frac{a\, b_2\, z_2}{c} + \frac{a\,(1+a)\, b_1\, b_2\, z_1\, z_2}{c\,(1+c)} + \frac{a\,(1+a)\times(2+a)\, b_1\,(1+b_1)\, b_2\, z_1^2\, z_2}{2\, c\,(1+c)\times(2+c)} + \frac{a\,(1+a)\, b_2\,(1+b_2)\, z_2^2}{2\, c\,(1+c)} + \frac{a\,(1+a)\times(2+a)\, b_1\, b_2\,(1+b_2)\, z_1\, z_2^2}{2\, c\,(1+c)\times(2+c)} + \frac{a\,(1+a)\times(2+a)\times(3+a)\, b_1\,(1+b_1)\, b_2\,(1+b_2)\, z_1^2\, z_2^2}{4\, c\,(1+c)\times(2+c)\times(3+c)} + \ldots \ /;\ \text{Abs}[z_1] < 1 \land \text{Abs}[z_2] < 1$$

general development

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == \frac{\text{Gamma}[c]}{\text{Gamma}[a]\ \text{Gamma}[b_1]\ \text{Gamma}[b_2]} \sum_{k=0}^{\infty}\sum_{j=0}^{\infty} \text{Residue}\Big[\frac{\text{Gamma}[a-s-t]\ \text{Gamma}[s]\ \text{Gamma}[b_1-s]\ \text{Gamma}[t]\ \text{Gamma}[b_2-t]}{\text{Gamma}[c-s-t]} (-z_1)^{-s} (-z_2)^{-t}, \{s, -j\}, \{t, -k\}\Big]$$

Integral representation

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == \frac{\text{Gamma}[c]}{\text{Gamma}[a]\ \text{Gamma}[c-a]} \int_0^1 t^{a-1} (1-t)^{c-a-1} (1-t z_1)^{-b_1} (1-t z_2)^{-b_2} \, dt \ /;\ \text{Re}[a] > 0 \land \text{Re}[c-a] > 0$$

Differential equation followed

$$((1-z_1) z_1 \partial_{\{z_1,2\}} w[z_1, z_2] + (1-z_1) z_2 \partial_{z_1,z_2} w[z_1, z_2] + (c-(a+b_1+1) z_1) \partial_{z_1} w[z_1, z_2] - b_1 z_2 \partial_{z_2} w[z_1, z_2] - a b_1 w[z_1, z_2] == 0 \land (1-z_2) z_2 \partial_{\{z_2,2\}} w[z_1, z_2] + z_1 (1-z_2) \partial_{z_1,z_2} w[z_1, z_2] - b_2 z_1 \partial_{z_1} w[z_1, z_2] + (c-(a+b_2+1) z_2) \partial_{z_2} w[z_1, z_2] - a b_2 w[z_1, z_2] == 0)\ /;\ w[z_1, z_2] == \text{AppellF1}[a, b_1, b_2, c, z_1, z_2]$$

Integral and differential

$$\int \text{AppellF1}[a, b_1, b_2, c, a z_1, z_2] \, dz_1 == \frac{(c-1)}{a\ (a-1)\ (b_1-1)} \text{AppellF1}[a-1, b_1-1, b_2, c-1, a z_1, z_2]$$

$$\partial_{\{z_1,n\}} \text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == \frac{\text{Pochhammer}[a, n]\ \text{Pochhammer}[b_1, n]}{\text{Pochhammer}[c, n]} \text{AppellF1}[a+n, b_1+n, b_2, c+n, z_1, z_2]\ /;\ n \in \text{Integers} \land n > 0$$

Formule asymptotiques

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == \frac{\text{Gamma}[c]\ \text{Gamma}[b_1-a]}{\text{Gamma}[c-a]\ \text{Gamma}[b_1]} (-z_1)^{-a} \text{AppellF1}\Big[a, 1+a-c, b_2, 1+a-b_1, \frac{1}{z_1}, \frac{z_2}{z_1}\Big] + \frac{\text{Gamma}[c]}{\text{Gamma}[a]} (-z_1)^{-b_1} \sum_{k=0}^{\infty} \frac{\text{Gamma}[a-b_1+k]\ \text{Pochhammer}[b_2, k]}{k!\ \text{Gamma}[c-b_1+k]} \text{Hypergeometric2F1}\Big[b_1, 1-c+b_1-k, 1-a+b_1-k, \frac{1}{z_1}\Big] z_2^k$$

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == \frac{\text{Gamma}[c]\ \text{Gamma}[b_1-a]}{\text{Gamma}[c-a]\ \text{Gamma}[b_1]} (-z_1)^{-a} \text{AppellF1}\Big[a, b_2, 1+a-c, 1+a-b_1, \frac{z_2}{z_1}, \frac{1}{z_1}\Big] + \frac{\text{Gamma}[c]\ \text{Gamma}[a-b_1]}{\text{Gamma}[a]\ \text{Gamma}[c-b_1]} (-z_1)^{-b_1} \Bigg(\frac{(a-b_1)^2 b_2}{(c-b_1)^2} z_2\, \text{HypergeometricPFQ}\Big[\{\{a-b_1+1, b_2+1\}, \{b_1\}, \{a-b_1+1, 1\}\}, \{\{c-b_1+1, 2\}, \{\}, \{c-b_1+1\}\}, \frac{z_2}{z_1}, z_2\Big] + \text{HypergeometricPFQ}\Big[\{\{b_1\}, \{b_2, a-b_1\}, \{1-c+b_1, 1\}\}, \{\{1\}, \{c-b_1\}, \{1-a+b_1\}\}, \frac{z_2}{z_1}, \frac{1}{z_1}\Big]\Bigg)$$

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] \propto \frac{\text{Gamma}[c]\ \text{Gamma}[b_1-a]}{\text{Gamma}[c-a]\ \text{Gamma}[b_1]} (-z_1)^{-a} \Big(1 + O\Big[\frac{1}{z_1}\Big]\Big) + \frac{\text{Gamma}[c]\ \text{Gamma}[a-b_1]}{\text{Gamma}[c-b_1]\ \text{Gamma}[a]} (-z_1)^{-b_1} \Big(1 + O\Big[\frac{1}{z_1}\Big]\Big)\ /;\ (\text{Abs}[z_1] \to \infty) \land a \neq b$$

Implementation of AppellF1

```
HypergeometricAppellF1[a_, b1_, b2_, c_, x_, y_, Nb_] :=
 Module[{i, j, sumtotale, currentterm, firstTerm,
   amnfirst, nx, cmnfirst, my, amn, cmn, b1n, b2m},
  currentterm = 1; nx = 1; my = 1; b2m = b2; firstTerm = 1; b1n = b1;
  sumtotale = 0; amnfirst = a; cmnfirst = c;
  Do[
   currentterm = firstTerm;
   b2m = b2;
   amn = amnfirst;
   cmn = cmnfirst;
   my = 1;
   Do[sumtotale += currentterm; currentterm *= amn b2m y / cmn / my;
    (*
    Print["i=",i," j=",j," currentterm=",currentterm," sumtotal=",sumtotale];
    *)
    amn++; b2m++; cmn++; my++;, {j, 1, Nb}];
   firstTerm *= x amnfirst b1n / cmnfirst / nx;
   amnfirst++; cmnfirst++; b1n++; nx++;
   , {i, 1, Nb}];
  sumtotale]

HypergeometricAppellF1Version1[a_, b1_, b2_, c_, z1_, z2_, Nb_] := Module[{},
  (1 - z1) ^ (-b1) × (1 - z2) ^ (-b2)
   HypergeometricAppellF1[c - a, b1, b2, c, z1 / (z1 - 1), z2 / (z2 - 1), Nb]
 ]

HypergeometricAppellF1Version2[a_, b1_, b2_, c_, z1_, z2_, Nb_] := Module[{},
  (1 - z1) ^ (-a)
   HypergeometricAppellF1[a, c - b1 - b2, b2, c, z1 / (z1 - 1), (z1 - z2) / (z2 - 1), Nb]
 ]

HypergeometricAppellF1Version3[a_, b1_, b2_, c_, z1_, z2_, Nb_] := Module[{},
  (1 - z2) ^ (-a)
   HypergeometricAppellF1[a, c - b1 - b2, b2, c, (z2 - z1) / (z1 - 1), z2 / (z2 - 1), Nb]
 ]

HypergeometricAppellF1Version4[a_, b1_, b2_, c_, z1_, z2_, Nb_] := Module[{},
  (1 - z1) ^ (c - a - b1) × (1 - z2) ^ (-b2)
   HypergeometricAppellF1[c - a, c - b1 - b2, b2, c, z1, (z1 - z2) / (z2 - 1), Nb]
 ]

HypergeometricAppellF1Version5[a_, b1_, b2_, c_, z1_, z2_, Nb_] := Module[{},
  (1 - z1) ^ (-b1) × (1 - z2) ^ (c - a - b2)
   HypergeometricAppellF1[c - a, c - b1 - b2, b2, c, (z1 - z2) / (z1 - 1), z2, Nb]
 ]
```

```
Module[{a = 1.4285714285714, b1 = 0.5, b2 = 0.5,
   c = 2.4285714285714, z1 = 1.1737504870683 + i 0.88031286530120,
   z2 = 1.1737504870683 - i 0.88031286530120, Nb = 150},
  {HypergeometricAppellF1Version1[a, b1, b2, c, z1, z2, Nb],
   HypergeometricAppellF1Version2[a, b1, b2, c, z1, z2, Nb],
   HypergeometricAppellF1Version3[a, b1, b2, c, z1, z2, Nb],
   , HypergeometricAppellF1Version4[a, b1, b2, c, z1, z2, Nb],
   HypergeometricAppellF1Version5[a, b1, b2, c, z1, z2, Nb]}]
```

$$\{8.19125\times10^{57}+6.96898\times10^{41}\,i,\ 6.59849\times10^{72}+1.43861\times10^{71}\,i,\ 6.61825\times10^{72}-1.82686\times10^{71}\,i,\ \text{Null},\ -5.76809\times10^{63}-4.6784\times10^{64}\,i,\ -1.84154\times10^{64}-1.38627\times10^{65}\,i\}$$

```
Collect[HypergeometricAppellF1[a, b1, b2, c, x, y, 4], x]
```

$$1+\frac{a\,y\,b_2}{c}+\frac{a\,(1+a)\,y^2\,b_2\,(1+b_2)}{2\,c\,(1+c)}+\frac{a\,(1+a)\times(2+a)\,y^3\,b_2\,(1+b_2)\times(2+b_2)}{6\,c\,(1+c)\times(2+c)}+$$
$$x\left(\frac{a\,b_1}{c}+\frac{a\,(1+a)\,y\,b_1\,b_2}{c\,(1+c)}+\frac{a\,(1+a)\times(2+a)\,y^2\,b_1\,b_2\,(1+b_2)}{2\,c\,(1+c)\times(2+c)}+\frac{a\,(1+a)\times(2+a)\times(3+a)\,y^3\,b_1\,b_2\,(1+b_2)\times(2+b_2)}{6\,c\,(1+c)\times(2+c)\times(3+c)}\right)+$$
$$x^2\left(\frac{a\,(1+a)\,b_1\,(1+b_1)}{2\,c\,(1+c)}+\frac{a\,(1+a)\times(2+a)\,y\,b_1\,(1+b_1)\,b_2}{2\,c\,(1+c)\times(2+c)}+\frac{a\,(1+a)\times(2+a)\times(3+a)\,y^2\,b_1\,(1+b_1)\,b_2\,(1+b_2)}{4\,c\,(1+c)\times(2+c)\times(3+c)}+\frac{a\,(1+a)\times(2+a)\times(3+a)\times(4+a)\,y^3\,b_1\,(1+b_1)\,b_2\,(1+b_2)\times(2+b_2)}{12\,c\,(1+c)\times(2+c)\times(3+c)\times(4+c)}\right)+x^3$$
$$\left(\frac{a\,(1+a)\times(2+a)\,b_1\,(1+b_1)\times(2+b_1)}{6\,c\,(1+c)\times(2+c)}+\frac{a\,(1+a)\times(2+a)\times(3+a)\,y\,b_1\,(1+b_1)\times(2+b_1)\,b_2}{6\,c\,(1+c)\times(2+c)\times(3+c)}+\frac{a\,(1+a)\times(2+a)\times(3+a)\times(4+a)\,y^2\,b_1\,(1+b_1)\times(2+b_1)\,b_2\,(1+b_2)}{12\,c\,(1+c)\times(2+c)\times(3+c)\times(4+c)}+\frac{a\,(1+a)\times(2+a)\times(3+a)\times(4+a)\times(5+a)\,y^3\,b_1\,(1+b_1)\times(2+b_1)\,b_2\,(1+b_2)\times(2+b_2)}{36\,c\,(1+c)\times(2+c)\times(3+c)\times(4+c)\times(5+c)}\right)$$

```
HypergeometricAppellF1[1.4285714285714,0.50000000000000,0.50000000000000,2.4285714285714
1.1737504870683+i 0.88031286530120,1.1737504870683-i \
0.88031286530120,150]
```

$1.11846\times10^{45}+3.16913\times10^{30}\,i$

```
AppellF1[1.1,2.09,3.7,4.12,0.19-0.5 i,0.9]
```

$6.6165-5.87146\,i$

```
AppellF1[1.4285714285714, 0.50000000000000, 0.50000000000000, 2.4285714285714,
 1.1737504870683 + i 0.88031286530120, 1.1737504870683 - i 0.88031286530120]
```

$1.43017+0.\,i$

```
√(0.19098042207655^2 - 4 × 0.014247469381460)
0. + 0.143235 i

z1 = 1.1737504870683 + i 0.88031286530120;
z2 = 1.1737504870683 - i 0.88031286530120;

Abs[(z2 - z1) / (z2 - 1)]
1.96215

Abs[z2]
1.46719

Abs[z2 / (z2 - 1)]
1.63512

Power[(z2 - z1) / (z2 - 1), 10]
-312.101 - 786.207 i
```

We extend the definition of the appell function to x, y couple outside the real axis x > 1 or y > 1

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == (1-z_1)^{c-a-b_1}(1-z_2)^{-b_2}\,\text{AppellF1}\left[c-a, c-b_1-b_2, b_2, c, z_1, \frac{z_2-z_1}{z_2-1}\right]$$

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == (1-z_1)^{-b_1}(1-z_2)^{c-a-b_2}\,\text{AppellF1}\left[c-a, b_1, c-b_1-b_2, c, \frac{z_1-z_2}{z_1-1}, z_2\right]$$

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == (1-z_2)^{-a}\,\text{AppellF1}\left[a, b_1, c-b_1-b_2, c, \frac{z_2-z_1}{z_2-1}, \frac{z_2}{z_2-1}\right] \,/;\, \text{Not}[\text{IntervalMemberQ}[\text{Interval}[\{1, \infty\}], z_1]] \land \text{Not}[\text{IntervalMemberQ}[\text{Interval}[\{1, \infty\}], z_2]]$$

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == (1-z_1)^{-a}\,\text{AppellF1}\left[a, c-b_1-b_2, b_2, c, \frac{z_1}{z_1-1}, \frac{z_1-z_2}{z_1-1}\right] \,/;\, \text{Not}[\text{IntervalMemberQ}[\text{Interval}[\{1, \infty\}], z_1]] \land \text{Not}[\text{IntervalMemberQ}[\text{Interval}[\{1, \infty\}], z_2]]$$

$$\text{AppellF1}[a, b_1, b_2, c, z_1, z_2] == (1-z_1)^{-b_1}(1-z_2)^{-b_2}\,\text{AppellF1}\left[c-a, b_1, b_2, c, \frac{z_1}{z_1-1}, \frac{z_2}{z_2-1}\right] \,/;\, \text{Not}[\text{IntervalMemberQ}[\text{Interval}[\{1, \infty\}], z_1]] \land \text{Not}[\text{IntervalMemberQ}[\text{Interval}[\{1, \infty\}], z_2]]$$

Generic Algorithm For Generic SABR

```
(* C[f_]:=(f+A)^β          (* this determine z,b1,b2 *) *)
```

```
(* C[f]=B[ϵ α z] determines B, so : *)
```

$$(*\ z=\frac{1}{\epsilon\ \alpha}\ \int_K^f \frac{1}{C[s]}\,ds\quad \text{determines z}\ *)$$

$$(*\ b1=\frac{B'[z\ \alpha]}{B[z\ \alpha]}=C'[f]\ ,\ b2=\frac{B''[z\ \alpha]}{B[z\ \alpha]}\ =C''[f]C[f]+b1^2\ *)$$

$$(*\ \text{SqB0B}\alpha\text{z represente bien sur}\ \sqrt{B[z\ \alpha]B[0]}\quad *)$$

Regles pour la transcription en C++

```
rulesz = {α1 → alpha1, α2 → alpha2, β1 → beta1, β2 → beta2, ρ1 → rho1, ρ2 → rho2, ν1 → nu1,
   ν2 → nu2, ρs → rhos, ρv → rhov, ρc12 → rhoc12, ρc21 → rhoc21, αs → alphas};
ruleszz1 = {X1^6 → (X1SQ * X1SQ * X1SQ), X2^6 → (X2SQ * X2SQ * X2SQ), X1^5 → (X1 * X1SQ * X1SQ),
   X2^5 → (X2 * X2SQ * X2SQ), X1^4 → (X1SQ * X1SQ), X2^4 → (X2SQ * X2SQ),
   X1^3 → (X1 * X1SQ), X2^3 → (X2 * X2SQ), α1^6 → (alpha1SQ * alpha1SQ * alpha1SQ),
   α2^6 → (alpha2SQ * alpha2SQ * alpha2SQ), α1^5 → (alpha1 * alpha1SQ * alpha1SQ),
   α2^5 → (alpha2 * alpha2SQ * alpha2SQ), α1^4 → (alpha1SQ * alpha1SQ),
   α2^4 → (alpha2SQ * alpha2SQ), α1^3 → (alpha1 * alpha1SQ), α2^3 → (alpha2 * alpha2SQ)};
ruleszz2 = {α1^2 → alpha1SQ, α2^2 → alpha2SQ, β1^2 → beta1SQ, β2^2 → beta2SQ, ρ1^2 → rho1SQ,
   ρ2^2 → rho2SQ, ν1^2 → nu1SQ, ν2^2 → nu2SQ, ρs^2 → rhosSQ, ρv^2 → rhovSQ, ρc12^2 → rhoc12SQ,
   ρc21^2 → rhoc21SQ, αs^2 → alphasSQ, X1^2 → X1SQ, X2^2 → X2SQ, F1^2 → F1SQ, F2^2 → F2SQ};
```

```
CTranform[x_] := CForm[x /. ruleszz1 /. ruleszz2 /. rulesz ]
```

$$\text{CTranform}\Big[2\,X1^3\,X2\,\alpha1^3\,\alpha2\,\rho s\left(\left(\frac{X1}{F1}\right)^2\alpha1^2\,\beta1^2+\frac{X1}{F1}\,\alpha1\,\beta1\left(2\,\nu1\,\rho1+\nu2\,\rho c12+\frac{X2}{F2}\,\alpha2\,\beta2\,\rho s\right)+\nu1\left(\nu1+\frac{X2}{F2}\,\alpha2\,\beta2\,\rho c21+\nu2\,\rho v\right)\right)\Big]$$

```
2*Power(alpha1,3)*alpha2*rhos*Power(X1,3)*X2*
   ((alpha1SQ*beta1SQ*X1SQ)/Power(F1,2) + nu1*(nu1 + nu2*rhov + (alpha2*beta2*rhoc21*X
     (alpha1*beta1*X1*(2*nu1*rho1 + nu2*rhoc12 + (alpha2*beta2*rhos*X2)/F2))/F1)
```

Precalcul pour le spreadoption

```
νBiSABR1[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_, ν2_, ρs_,
  ρv_, ρc12_, ρc21_, X1_, X2_, αs_] := Module[{term1, term2, term3},
  term1 = X1^4 α1^4 ((X1/F1)^2 α1^2 β1^2 + ν1^2 + 2 X1/F1 α1 β1 ν1 ρ1);
  term2 = X2^4 α2^4 ((X2/F2)^2 α2^2 β2^2 + ν2^2 + 2 X2/F2 α2 β2 ν2 ρ2);
  term3 = X1^2 X2^2 α1^2 α2^2 ((X1/F1)^2 α1^2 β1^2 ρs^2 + (X2/F2)^2 α2^2 β2^2 ρs^2 +
      ν1^2 ρs^2 + ν2^2 ρs^2 + 2 X2/F2 α2 β2 (ν2 ρ2 ρs^2 + ν1 ρc21 (1 + ρs^2)) + 2 X1/F1 α1 β1
       (ν1 ρ1 ρs^2 + ν2 ρc12 (1 + ρs^2) + X2/F2 α2 β2 ρs (1 + ρs^2)) + 2 ν1 ν2 ρv + 2 ν1 ν2 ρs^2 ρv);
  term4 = 2 X1 X2^3 α1 α2^3 ρs (F2^(-2+2 β2) α2^2 β2^2 +
      (F2^(-1+β2) α2 β2 (2 F1 ν2 ρ2 + F1 ν1 ρc21 + F1^β1 α1 β1 ρs))/F1 +
      ν2 (ν2 + X1/F1 α1 β1 ρc12 + ν1 ρv));
  term5 = 2 X1^3 X2 α1^3 α2 ρs ((X1/F1)^2 α1^2 β1^2 + X1/F1 α1 β1 (2 ν1 ρ1 + ν2 ρc12 + X2/F2 α2 β2 ρs) +
      ν1 (ν1 + X2/F2 α2 β2 ρc21 + ν2 ρv));
  Sqrt[term1 + term2 + term3 - term4 - term5]/αs^2]
```

```
ρBiSABR1[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_,
  ν2_, ρs_, ρv_, ρc12_, ρc21_, X1_, X2_, αs_, νs_] := Module[{},
  (X1^3 α1^3 ((X1 α1 β1)/F1 + ν1 ρ1) - X2^3 α2^3 ((X2 α2 β2)/F2 + ν2 ρ2) -
     X1^2 X2 α1^2 α2 (ρs ((2 X1 α1 β1)/F1 + ν2 ρc12 + (X2 α2 β2 ρs)/F2) + ν1 (ρc21 + ρ1 ρs)) +
     X1 X2^2 α1 α2^2 (ρs ((2 X2 α2 β2)/F2 + ν1 ρc21 + (X1 α1 β1 ρs)/F1) + ν2 (ρc12 + ρ2 ρs))) / (νs αs^3)]
```

```
Module[{F1 = 0.040908314711852, alpha1 = 0.013242339206841,
  beta1 = 0.40000000000000, rho1 = 0.35242800000000, nu1 = 0.25283800000000,
  F2 = 0.044141630651630, alpha2 = 0.014300324948643, beta2 = 0.40000000000000,
  rho2 = 0.37737400000000, nu2 = 0.23093200000000,
  rhos = 0.90400000000000, rhov = 0.60000000000000, rhoc12 = -0.050000000000000,
  rhoc21 = -0.050000000000000, X1 = 0.27843551868880, X2 = 0.28703796939008,
  alphas = 0.0017550658859387, nus = 0.25040352068222},
 νBiSABR1[F1, alpha1, beta1, rho1, nu1, F2, alpha2,
  beta2, rho2, nu2, rhos, rhov, rhoc12, rhoc21, X1, X2, alphas]]
```

```
0.250404
```

```
Module[{F1 = 0.040908314711852, alpha1 = 0.013242339206841,
  beta1 = 0.40000000000000, rho1 = 0.35242800000000, nu1 = 0.25283800000000,
  F2 = 0.044141630651630, alpha2 = 0.014300324948643, beta2 = 0.40000000000000,
  rho2 = 0.37737400000000, nu2 = 0.23093200000000,
  rhos = 0.90400000000000, rhov = 0.60000000000000, rhoc12 = -0.050000000000000,
  rhoc21 = -0.050000000000000, X1 = 0.27843551868880, X2 = 0.28703796939008,
  alphas = 0.0017550658859387, nus = 0.25040352068222},
 ρBiSABR1[F1, alpha1, beta1, rho1, nu1, F2, alpha2,
  beta2, rho2, nu2, rhos, rhov, rhoc12, rhoc21, X1, X2, alphas, nus]]
```

```
-1.028
```

```
CTranform[X1^2 X2^2 F2 α1^2 α2^2 (X1^2 F2 α1^2 β1 (-1 + 2 β1 + (-2 + β1) ρs^2) +
    2 X1 F1 α1 (-F1 β2 ν2 ρ2 ρs + F2 β1 (ν2 ρc12 (-1 + ρs^2) + ν1 ρ1 (2 + ρs^2))) -
    F1^2 F2 (-1 + ρs^2) (ν1^2 + ν2^2 - 2 ν1 ν2 ρv))]
```

```
alpha1SQ*alpha2SQ*F2*X1SQ*(-(F1SQ*F2*(-1 + rhosSQ)*(nu1SQ + nu2SQ - 2*nu1*nu2*rhov)) +
    2*alpha1*F1*(-(beta2*F1*nu2*rho2*rhos) + beta1*F2*(nu2*rhoc12*(-1 + rhosSQ) + nu1
    alpha1SQ*beta1*F2*(-1 + 2*beta1 + (-2 + beta1)*rhosSQ)*X1SQ)*X2SQ
```

```
μBiSABR1[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_, ν2_, ρs_, ρv_,
  ρc12_, ρc21_, X1_, X2_, αs_] := Module[{term1, term2, term3, term4},
  term1 = F1^2 X2^6 α2^6 (-1 + β2) β2 + X1^5 F2^2 α1^5 β1 (X1 α1 (-1 + β1) + 2 F1 ν1 ρ1);
  term2 = -3 X1^4 X2 F2^2 α1^4 α2 β1 (X1 α1 (-1 + β1) + 2 F1 ν1 ρ1) ρs +
    F1^2 X2^5 α2^5 β2 (2 F2 ν2 ρ2 - 3 X1 α1 (-1 + β2) ρs) +
    X1 F1^2 X2^4 α1 α2^4 β2 (-6 F2 ν2 ρ2 ρs + X1 α1 (-1 + 2 β2 + (-2 + β2) ρs^2));
  term3 = X1^2 X2^3 α1^2 α2^3
    (X1 α1 ρs (-F2^2 (-1 + β1) β1 - F1^2 (-1 + β2) β2 + 2 F1 F2 β1 β2 (-1 + ρs^2)) +
      2 F1 F2 (-F2 β1 ν1 ρ1 ρs + F1 β2 (ν1 ρc21 (-1 + ρs^2) + ν2 ρ2 (2 + ρs^2))));
  term4 = X1^2 X2^2 F2 α1^2 α2^2 (X1^2 F2 α1^2 β1 (-1 + 2 β1 + (-2 + β1) ρs^2) +
      2 X1 F1 α1 (-F1 β2 ν2 ρ2 ρs + F2 β1 (ν2 ρc12 (-1 + ρs^2) + ν1 ρ1 (2 + ρs^2))) -
      F1^2 F2 (-1 + ρs^2) (ν1^2 + ν2^2 - 2 ν1 ν2 ρv));
  (term1 + term2 + term3 + term4) / (2 F1^2 F2^2 αs^3)]
```

$$\text{Simplify}\left[\text{Normal}\left[\text{Series}\left[\frac{\sqrt{(\text{s}+\text{F2})^{2\,\beta 1}\,\alpha 1^2+\text{X2}^2\;\alpha 2^2-2\,(\text{s}+\text{F2})^{\beta 1}\,\text{X2}\,\alpha 1\,\alpha 2\,\rho\text{s}}}{\alpha\text{s}},\{\text{s},\text{F2},2\}\right]\right]\right]$$

$$\Big(-2^{\beta 1}\,\text{F2}^{-1+\beta 1}\,(\text{F2}-\text{s})\,\alpha 1\,\beta 1\,\big(2^{\beta 1}\,\text{F2}^{\beta 1}\,\alpha 1-\text{X2}\,\alpha 2\,\rho\text{s}\big)+2\times\big(4^{\beta 1}\,\text{F2}^{2\,\beta 1}\,\alpha 1^2+\text{X2}^2\,\alpha 2^2-2^{1+\beta 1}\,\text{F2}^{\beta 1}\,\text{X2}\,\alpha 1\,\alpha 2\,\rho\text{s}\big)+\Big(2^{-2+\beta 1}\,\text{F2}^{-2+\beta 1}\,(\text{F2}-\text{s})^2\,\alpha 1\,\beta 1\,\big(8^{\beta 1}\,\text{F2}^{3\,\beta 1}\,\alpha 1^3\,(-1+\beta 1)-3\times 4^{\beta 1}\,\text{F2}^{2\,\beta 1}\,\text{X2}\,\alpha 1^2\,\alpha 2\,(-1+\beta 1)\,\rho\text{s}-\text{X2}^3\,\alpha 2^3\,(-1+\beta 1)\,\rho\text{s}+2^{\beta 1}\,\text{F2}^{\beta 1}\,\text{X2}^2\,\alpha 1\,\alpha 2^2\,\big(-1-2\,\rho\text{s}^2+\beta 1\,(2+\rho\text{s}^2)\big)\big)\Big)\Big/\big(4^{\beta 1}\,\text{F2}^{2\,\beta 1}\,\alpha 1^2+\text{X2}^2\,\alpha 2^2-2^{1+\beta 1}\,\text{F2}^{\beta 1}\,\text{X2}\,\alpha 1\,\alpha 2\,\rho\text{s}\big)\Big)\Big/\Big(2\,\alpha\text{s}\,\sqrt{4^{\beta 1}\,\text{F2}^{2\,\beta 1}\,\alpha 1^2+\text{X2}^2\,\alpha 2^2-2^{1+\beta 1}\,\text{F2}^{\beta 1}\,\text{X2}\,\alpha 1\,\alpha 2\,\rho\text{s}}\Big)$$

```
BiSABRspreadC[F1_, F2_, α1_, α2_, β1_, β2_, ρs_, X1_, X2_, αs_, s_] := Module[{},
   If[Abs[s + F2] < 2 × 10^(-3) F2,
    (-2^β1 F2^(-1+β1) (F2 - s) α1 β1 (2^β1 F2^β1 α1 - X2 α2 ρs) +
       2 × (4^β1 F2^(2 β1) α1^2 + X2^2 α2^2 - 2^(1+β1) F2^β1 X2 α1 α2 ρs) +
       (2^(-2+β1) F2^(-2+β1) (F2 - s)^2 α1 β1 (8^β1 F2^(3 β1) α1^3 (-1 + β1) - 3 × 4^β1 F2^(2 β1) X2 α1^2 α2 (-1 + β1)
             ρs - X2^3 α2^3 (-1 + β1) ρs + 2^β1 F2^β1 X2^2 α1 α2^2 (-1 - 2 ρs^2 + β1 (2 + ρs^2)))) /
        (4^β1 F2^(2 β1) α1^2 + X2^2 α2^2 - 2^(1+β1) F2^β1 X2 α1 α2 ρs)) /
     (2 αs Sqrt[4^β1 F2^(2 β1) α1^2 + X2^2 α2^2 - 2^(1+β1) F2^β1 X2 α1 α2 ρs]),
    Sqrt[(s + F2)^(2 β1) α1^2 + X2^2 α2^2 - 2 (s + F2)^β1 X2 α1 α2 ρs] / αs ]]
```

```
BiSABRspreadC[F1_, F2_, α1_, α2_, β1_, β2_, ρs_, X1_, X2_, αs_, s_] := Module[{},
  Sqrt[(s + F2)^(2 β1) α1^2 + X2^2 α2^2 - 2 (s + F2)^β1 X2 α1 α2 ρs] / αs ]
```

$$\text{Series}\left[\frac{(\text{s}+\text{F2})^{-1+2\,\beta 1}\,\alpha 1^2\,\beta 1-\text{X2}\,(\text{s}+\text{F2})^{-1+\beta 1}\,\alpha 1\,\alpha 2\,\beta 1\,\rho\text{s}}{\alpha\text{s}\,\sqrt{(\text{s}+\text{F2})^{2\,\beta 1}\,\alpha 1^2+\text{X2}^2\;\alpha 2^2-2\,(\text{s}+\text{F2})^{\beta 1}\,\text{X2}\,\alpha 1\,\alpha 2\,\rho\text{s}}},\{\text{s},-\text{F2},1\}\right]$$

$$\frac{(\text{F2}+\text{s})^{-1+2\,\beta 1}\,\alpha 1^2\,\beta 1-(\text{F2}+\text{s})^{-1+\beta 1}\,\text{X2}\,\alpha 1\,\alpha 2\,\beta 1\,\rho\text{s}}{\alpha\text{s}\,\sqrt{(\text{F2}+\text{s})^{2\,\beta 1}\,\alpha 1^2+\text{X2}^2\,\alpha 2^2-2\,(\text{F2}+\text{s})^{\beta 1}\,\text{X2}\,\alpha 1\,\alpha 2\,\rho\text{s}}}$$

```
BiSABRspreadCderivative[F1_, F2_, α1_,
   α2_, β1_, β2_, ρs_, X1_, X2_, αs_, s_] := Module[{},
   ((s + F2)^(-1+2 β1) α1^2 β1 - X2 (s + F2)^(-1+β1) α1 α2 β1 ρs) /
    (αs Sqrt[(s + F2)^(2 β1) α1^2 + X2^2 α2^2 - 2 (s + F2)^β1 X2 α1 α2 ρs]) ]
```

```
BiSABRspreadCderivative2[F1_, F2_, α1_,
  α2_, β1_, β2_, ρs_, X1_, X2_, αs_, s_] := Module[{},
```

$$\frac{\left(2\ (s+F2)^{-1+2\beta1}\ \alpha1^2\ \beta1 - 2\ X2\ (s+F2)^{-1+\beta1}\ \alpha1\ \alpha2\ \beta1\ \rho s\right)^2}{4\left(\sqrt{(s+F2)^{2\beta1}\ \alpha1^2 + X2^2\ \alpha2^2 - 2\ (s+F2)^{\beta1}\ X2\ \alpha1\ \alpha2\ \rho s}\right)^3} +$$

$$\left.\frac{2\ (s+F2)^{-2+2\beta1}\ \alpha1^2\ \beta1\ (-1+2\beta1) - 2\ F2^{\beta2}\ (s+F2)^{-2+\beta1}\ \alpha1\ \alpha2\ (-1+\beta1)\ \beta1\ \rho s}{2\sqrt{(s+F2)^{2\beta1}\ \alpha1^2 + X2^2\ \alpha2^2 - 2\ (s+F2)^{\beta1}\ X2\ \alpha1\ \alpha2\ \rho s}}\right]$$

Spreadoption

```
BiSABRSpreadOption[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_,
  ρ2_, ν2_, K_, T_, ρs_, ρv_, ρc12_, ρc21_, printflag_] := Module[
  {intrinsec, z, b1, b2, SqB0Bαz, favg, zavg, Ck, αsp, νsp, ρsp,
   μsp, X1, X2, Qs, Q1, Q2, Qb, C1, C2, C1p, C2p, Q, COV, COV1,
   COV0, ρsb, ρsα1, ρsα2, ρbα1, ρbα2, N1, N2, νspZ, ρspZ},
  X1 = F1^β1; X2 = F2^β2; αsp = Sqrt[X1^2 α1^2 + X2^2 α2^2 - 2 X1 X2 α1 α2 ρs];
  If[printflag > 0, Print["X1=", X1, " X2=", X2, " αsp=", αsp]];
  νsp =
   νBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2, ρs, ρv, ρc12, ρc21, X1, X2, αsp];
  ρsp = ρBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
    ρs, ρv, ρc12, ρc21, X1, X2, αsp, νsp];
  μsp = μBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
    ρs, ρv, ρc12, ρc21, X1, X2, αsp];
  z = Re[BiSABRz[K, F1, F2, α1, α2, ρs, β1, β2]];
  favg = (F1 - F2 + K) / 2;
  zavg = z;
  b1 = BiSABRspreadCderivative[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, favg];
  Csecond = BiSABRspreadCderivative2[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, favg];
  Cfavg = BiSABRspreadC[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, favg];
  Ck = BiSABRspreadC[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, K];
  b2 = Csecond Cfavg + b1^2;
  SqB0Bαz = Sqrt[Ck];
  intrinsec = Max[F1 - F2 - K, 0];
  If[printflag > 0, Print["BiSABRSpreadOption:K=", K, " b1:Cprime[favg]=",
    b1, " Csecond[favg]=", Csecond, " b2=", b2, " Cfavg=", Cfavg]];
  If[printflag > 0, Print["BiSABRSpreadOption:α=", αsp,
    " ν=", νsp, " ρ=", ρsp, "  μ=", μsp]];
  If[printflag > 0, Print["BiSABRSpreadOption:intrinsec Value=",
    intrinsec, "  z=", z, " SqB0Bαz=", SqB0Bαz]];
  If[printflag > 0,
   C1 = F1^β1; C2 = F2^β2; C1p = β1 F1^(β1 - 1); C2p = β2 F2^(β2 - 1);
   Q = Sqrt[α1^2 C1^2 + α2^2 C2^2 - 2 ρs α1 α2 C1 C2];
   Qs = α1^2 C1 C1p - α1 α2 ρs C2 C1p;
   Qb = α1^2 C1 C1p - α1 α2 ρs C2 C1p - α1 α2 ρs C1 C2p + α2^2 C2 C2p;
   Q1 = α1 C1^2 - α2 ρs C1 C2;
```

```
Q2 = -α1 ρs C1 C2 + α2 C2²;
ρsb = ( ρs α1 C1 -  α2 C2) / Q ;
ρsα1 = ( ρ1  α1 C1 - ρc21 α2 C2) / Q ;
ρsα2 = (ρc12 α1 C1 - ρ2 α2 C2) / Q ;
ρbα1 = ρc21;
ρbα2 = ρ2;
COV0 = {
  { α1² C1²,
   ρs α1 C1 α2  C2 ,          ρ1  α1 C1 ν1 α1,          ρc12 α1 C1 ν2 α2},
  { ρs α1 C1 α2  C2 ,          (α2  C2)²,
   ρc21 α2 C2 ν1 α1,          ρ2 α2 C2 ν2 α2},
  { ρ1  α1 C1 ν1 α1,          ρc21 α2 C2 ν1  α1,
   (ν1 α1)²,                  ρv ν1 α1 ν2 α2},
  { ρc12 α1 C1 ν2 α2,          ρ2 α2  C2 ν2 α2 ,
   ρv ν1 α1 ν2 α2,            (ν2 α2)²}
 };
Plot[Det[{{ α1² C1²,
    ρs α1 C1 α2  C2 ,          ρ1  α1 C1 ν1 α1,          rc12 α1 C1 ν2 α2},
   { ρs α1 C1 α2  C2 ,          (α2  C2)²,
    ρc21 α2 C2 ν1 α1,          ρ2 α2 C2 ν2 α2},
   { ρ1  α1 C1 ν1 α1,          ρc21 α2 C2 ν1  α1,
    (ν1 α1)²,                  ρv ν1 α1 ν2 α2},
   { rc12 α1 C1 ν2 α2,          ρ2 α2  C2 ν2 α2 ,
    ρv ν1 α1 ν2 α2,            (ν2 α2)²}}], {rc12, -1, +1}];
Plot[Det[{{ α1² C1²,                          ρs α1 C1 α2  C2 ,
    ρ1  α1 C1 ν1 α1,          ρc12 α1 C1 ν2 α2},
   { ρs α1 C1 α2  C2 ,          (α2  C2)²,
    rc21 α2 C2 ν1 α1,          ρ2 α2 C2 ν2 α2},
   { ρ1  α1 C1 ν1 α1,          rc21 α2 C2 ν1  α1,
    (ν1 α1)²,                  ρv ν1 α1 ν2 α2},
   { ρc12 α1 C1 ν2 α2,          ρ2 α2  C2 ν2 α2 ,
    ρv ν1 α1 ν2 α2,            (ν2 α2)²}}], {rc21, -1, +1}];
Print["Initial Eigenvalues=", Eigenvalues[COV0]];
Print["Initial Det=", Det[COV0]];
COV1 = {
  { α1² C1² + α2² C2² - 2 ρs α1 α2 C1 C2,      ( ρs α1 C1 -  α2 C2) α2  C2 ,
   ( ρ1  α1 C1 - ρc21 α2 C2) ν1 α1,      (ρc12 α1 C1 - ρ2 α2 C2) ν2 α2},
  {( ρs α1 C1 -  α2 C2) α2  C2,          (α2  C2)²,
   ρc21 α2 C2 ν1 α1,          ρ2 α2 C2 ν2 α2},
  {( ρ1  α1 C1 - ρc21 α2 C2) ν1  α1,          ρc21 α2 C2 ν1  α1,
   (ν1 α1)²,                  ρv ν1 α1 ν2 α2},
```

```
      {(ρc12 α1 C1 - ρ2 α2 C2) ν2 α2,          ρ2 α2  C2 ν2 α2 ,
       ρv ν1 α1 ν2 α2,                        (ν2 α2)^2}
    };
   COV = {
      {Q^2, ρsb Q α2  C2 , ρsα1 Q ν1 α1, ρsα2 Q ν2 α2},
      {ρsb Q α2  C2, (α2  C2)^2, ρbα1 α2 C2 ν1 α1, ρbα2 α2 C2 ν2 α2},
      {ρsα1 Q ν1  α1, ρbα1 α2 C2 ν1  α1, (ν1 α1)^2, ρv ν1 α1 ν2 α2},
      {ρsα2 Q ν2 α2, ρbα2 α2  C2 ν2 α2 , ρv ν1 α1 ν2 α2, (ν2 α2)^2}
    };
   Print[" COV1-COV=", COV1 - COV];
   N1 = {Q, 0, 0, 0}.(COV.{Q, 0, 0, 0});
   N2 = {Qs / Q, Qb / Q, Q1 / Q, Q2 / Q}.(COV.{Qs / Q, Qb / Q, Q1 / Q, Q2 / Q});
   νspZ = Sqrt[N2] / Q;
   ρspZ = ({Q, 0, 0, 0}.(COV.{Qs / Q, Qb / Q, Q1 / Q, Q2 / Q})) / Sqrt[N2 N1] ;
   Print["BiSABRSpreadOption:N1=", N1];
   Print["BiSABRSpreadOption:N2=", N2];
   Print["BiSABRSpreadOption:νspZ=", νspZ];
   Print["BiSABRSpreadOption:ρspZ=", ρspZ];
   Print["BiSABRSpreadOption:ds=", {Q, 0, 0, 0}];
   Print["BiSABRSpreadOption:dα11=", {Qs / Q, Qb / Q, Q1 / Q, Q2 / Q}];
   Print["BiSABRSpreadOption:COV=", COV];
   Print["BiSABRSpreadOption:eigenvalues=", Eigenvalues[COV]];
   Print["BiSABRSpreadOption: Det=", Det[COV]];
  ];
  Re[SABRgeneric5[intrinsec, z, b1,
    b2, zavg, SqB0Bαz, T, μsp, αsp, ρsp, νsp, printflag]]]
```

```
On[General::"spell"]
```

Utilise une integration numerique pour le calcul du z

```
BiSABRSpreadOption[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_, ν2_,
  K_, T_, ρs_, ρv_, ρc12_, ρc21_, legendrelst_, printflag_] := Module[
  {intrinsec, z, b1, b2, SqB0Bαz, favg, zavg, Ck, αsp, νsp, ρsp,
   μsp, X1, X2, Qs, Q1, Q2, Qb, C1, C2, C1p, C2p, Q, COV, COV1,
   COV0, ρsb, ρsα1, ρsα2, ρbα1, ρbα2, N1, N2, νspZ, ρspZ},
  X1 = F1^β1; X2 = F2^β2; αsp = Sqrt[X1^2 α1^2 + X2^2 α2^2 - 2 X1 X2 α1 α2 ρs];
  νsp =
   νBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2, ρs, ρv, ρc12, ρc21, X1, X2, αsp];
  ρsp = ρBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
    ρs, ρv, ρc12, ρc21, X1, X2, αsp, νsp];
  μsp = μBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
    ρs, ρv, ρc12, ρc21, X1, X2, αsp];
  z = BiSABRz[K, F1, F2, α1, α2, ρs, β1, β2, legendrelst];
  favg = (F1 - F2 + K) / 2;
  zavg = BiSABRz[favg, F1, F2, α1, α2, ρs, β1, β2, legendrelst];
  b1 = BiSABRspreadCderivative[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, favg];
  Csecond = BiSABRspreadCderivative2[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, favg];
  Cfavg = BiSABRspreadC[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, favg];
  Ck = BiSABRspreadC[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, K];
  b2 = Csecond Cfavg + b1^2;
  SqB0Bαz = Sqrt[Ck];
  intrinsec = Max[F1 - F2 - K, 0];
  If[printflag == 10, Print["BiSABRSpreadOption:X1=", X1, " X2=", X2, " z=", z]];
  If[printflag == 10,
   Print["BiSABRSpreadOption:b1=", b1, " b2=", b2, " SqB0Bαz=", SqB0Bαz]];
  If[printflag == 10, Print["BiSABRSpreadOption:α=",
    αsp, " ν=", νsp, " ρ=", ρsp, "  μ=", μsp]];
  res = SABRgeneric5[intrinsec, z, b1, b2, zavg, SqB0Bαz,
    T, μsp, αsp, ρsp, νsp, printflag];
  If[printflag == 10, Print["BiSABRSpreadOption:res=", res]];
  Re[res]]
```

```
BiSABRSpreadOptionVol[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_,
  ν2_, K_, T_, ρs_, ρv_, ρc12_, ρc21_, legendrelst_, printflag_] :=
 NormalImplicitVol[F1 - F2, K, T, BiSABRSpreadOption[F1, α1, β1, ρ1,
   ν1, F2, α2, β2, ρ2, ν2, K, T, ρs, ρv, ρc12, ρc21, legendrelst, 0]]
```

```
BiSABRSpreadOptionNormalCorrelation[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_,
  ρ2_, ν2_, K_, T_, ρs_, ρv_, ρc12_, ρc21_, legendrelst_, printflag_] :=
 Module[{method = "Directe", zetavmethod = "Exact", zmethod = "Exact",
   favmethod = "Arithmetic", σATMnormal1, σATMnormal2, implicitNormalvols},
  {σATMnormal1, σATMnormal2} = {F1 SABROption[F1, α1, β1, ρ1, ν1,
       {method, zmethod, zetavmethod, favmethod}, F1 1.0001, T][[1]],
    F2 SABROption[F2, α2, β2, ρ2, ν2, {method, zmethod, zetavmethod, favmethod},
       F2 1.0001, T][[1]]};
  implicitNormalvols = BiSABRSpreadOptionVol[F1, α1, β1, ρ1, ν1, F2,
    α2, β2, ρ2, ν2, K, T, ρs, ρv, ρc12, ρc21, legendrelst, 0];
  NormalImplicitCorrelation[σATMnormal1, σATMnormal2, implicitNormalvols]]
```

```
(* utilise SABRgeneric6 ! *)
```

```
BiSABRSpreadOption2[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_, ν2_,
  K_, T_, ρs_, ρv_, ρc12_, ρc21_, legendrelst_, printflag_] := Module[
  {intrinsec, z, b1, b2, SqB0Bαz, favg, zavg, Ck, αsp, νsp, ρsp,
   μsp, X1, X2, Qs, Q1, Q2, Qb, C1, C2, C1p, C2p, Q, COV, COV1,
   COV0, ρsb, ρsα1, ρsα2, ρbα1, ρbα2, N1, N2, νspZ, ρspZ},
  X1 = F1^β1; X2 = F2^β2; αsp = Sqrt[X1^2 α1^2 + X2^2 α2^2 - 2 X1 X2 α1 α2 ρs];
  νsp =
   νBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2, ρs, ρv, ρc12, ρc21, X1, X2, αsp];
  ρsp = ρBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
    ρs, ρv, ρc12, ρc21, X1, X2, αsp, νsp];
  μsp = μBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
    ρs, ρv, ρc12, ρc21, X1, X2, αsp];
  z = BiSABRz[K, F1, F2, α1, α2, ρs, β1, β2, legendrelst];
  favg = (F1 - F2 + K) / 2;
  zavg = BiSABRz[favg, F1, F2, α1, α2, ρs, β1, β2, legendrelst];
  b1 = BiSABRspreadCderivative[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, favg];
  Csecond = BiSABRspreadCderivative2[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, favg];
  Cfavg = BiSABRspreadC[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, favg];
  Ck = BiSABRspreadC[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, K];
  b2 = Csecond Cfavg + b1^2;
  SqB0Bαz = Sqrt[Ck];
  intrinsec = Max[F1 - F2 - K, 0];
  If[printflag == 10, Print["BiSABRSpreadOption2:X1=", X1, " X2=", X2, " z=", z]];
  If[printflag == 10,
   Print["BiSABRSpreadOption2:b1=", b1, " b2=", b2, " SqB0Bαz=", SqB0Bαz]];
  If[printflag == 10, Print["BiSABRSpreadOption2:α=",
    αsp, " ν=", νsp, " ρ=", ρsp, "   μ=", μsp]];
  res = SABRgeneric6[intrinsec, z, b1, b2, zavg, SqB0Bαz,
    T, μsp, αsp, ρsp, νsp, printflag];
  If[printflag == 10, Print["BiSABRSpreadOption2:res=", res]];
  Re[res]]
```

```
(* utilise SABRgenericOption ! *), et retourne une vol normale
```

```
BiSABRSpreadOptionNormalOption[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_,
  ν2_, K_, T_, ρs_, ρv_, ρc12_, ρc21_, legendrelst_, printflag_] := Module[
  {intrinsec, z, b1, b2, SqB0Bαz, favg, zavg, Ck, αsp, νsp, ρsp,
   μsp, X1, X2, Qs, Q1, Q2, Qb, C1, C2, C1p, C2p, Q, COV, COV1,
   COV0, ρsb, ρsα1, ρsα2, ρbα1, ρbα2, N1, N2, νspZ, ρspZ},
  X1 = F1^β1; X2 = F2^β2; αsp = Sqrt[X1^2 α1^2 + X2^2 α2^2 - 2 X1 X2 α1 α2 ρs];
  If[printflag > 0, Print["BiSABRSpreadOptionNormalOption: K=",
    K, " X1=", X1, " X2=", X2, " αsp=", αsp]];
  νsp = νBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
    ρs, ρv, ρc12, ρc21, X1, X2, αsp];
  ρsp = ρBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
    ρs, ρv, ρc12, ρc21, X1, X2, αsp, νsp];
  μsp = μBiSABR1[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
    ρs, ρv, ρc12, ρc21, X1, X2, αsp];
  z = BiSABRz[K, F1, F2, α1, α2, ρs, β1, β2, legendrelst];
  favg = (F1 - F2 + K) / 2;
  zavg = BiSABRz[favg, F1, F2, α1, α2, ρs, β1, β2, legendrelst];
  b1 = BiSABRspreadCderivative[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, favg];
  Csecond = BiSABRspreadCderivative2[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, favg];
  Cz = BiSABRspreadC[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, F1 - F2];
  Ck = BiSABRspreadC[F1, F2, α1, α2, β1, β2, ρs, X1, X2, αsp, K];
  b2 = Csecond Cfavg + b1^2;
  If[printflag > 0, Print["BiSABRSpreadOptionNormalOption:α=",
    αsp, " ν=", νsp, " ρ=", ρsp, "  μ=", μsp]];
  Re[SABRgenericOption[F1 - F2, K, z, Ck, b1, b2, Cz, T, μsp, αsp, ρsp, νsp, printflag]]]
```

Debiaisage (symetrisation du role S1 et S2)

```
BiSABRSpreadOptionCorrected[F1_, α1_, β1_, ρ1_, ν1_, F2_,
  α2_, β2_, ρ2_, ν2_, K_, T_, ρs_, ρv_, ρc12_, ρc21_, printflag_] :=
 (BiSABRSpreadOption[F1, α1, β1, ρ1, ν1, F2, α2, β2,
     ρ2, ν2, K, T, ρs, ρv, ρc12, ρc21, printflag] +
    BiSABRSpreadOption[F2, α2, β2, ρ2, ν2, F1, α1, β1, ρ1, ν1,
     -K, T, ρs, ρv, ρc21, ρc12, printflag] + F1 - F2 - K) / 2
```

Extended BISABR spreadoption (with coefficients)

```
(* compute E[(a1*S1-a2*S2-K)1_{a1*S1-a2*S2-K>0}] *)
```

```
GeneralizedBiSABRSpreadOption[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_,
  ν2_, K_, T_, ρs_, ρv_, ρc12_, ρc21_, a1_, a2_, printflag_] := Module[{},
  BiSABRSpreadOption[a1 F1, α1 a1^(1-β1), β1, ρ1, ν1, a2 F2,
   α2 a2^(1-β2), β2, ρ2, ν2, K, T, ρs, ρv, ρc12, ρc21, printflag]
 ]
```

```
GeneralizedBiSABRSpreadOption[F1_, α1_, β1_, ρ1_, ν1_, F2_, α2_, β2_, ρ2_, ν2_, K_, T_,
  ρs_, ρv_, ρc12_, ρc21_, a1_, a2_, legendrelst_, printflag_] := Module[{},
  BiSABRSpreadOption[a1 F1, α1 a1^(1-β1), β1, ρ1, ν1, a2 F2, α2 a2^(1-β2),
   β2, ρ2, ν2, K, T, ρs, ρv, ρc12, ρc21, legendrelst, printflag]
 ]
```

Application numerique

```
TestCorrelation[ρ1_, ρ2_, ρs_, ρv_, ρc12_, ρc21_] :=
  Module[{}, Print["Test Correlation : Eigenvalues=", Eigenvalues[
     {{1, ρ1, ρs, ρc12}, {ρ1, 1, ρc21, ρv}, {ρs, ρc21, 1, ρ2}, {ρc12, ρv, ρ2, 1}}]]];
```

Simple execution

```
Module[{
  F1 = 0.0418,
  α1 = 0.0435,
  β1 = 0.6,
  ρ1 = -0.1819,
  ν1 = 0.3798,
  F2 = 0.0363,
  α2 = 0.0671,
  β2 = 0.7,
  ρ2 = -0.1136,
  ν2 = 0.3797,
  T = 1,
  ρs = 0.808,
  ρv = 0.,
  ρc12 = -0.2,
  ρc21 = 0.,
  K = -0.002, legendrelst = LegendreCoeffs[10]
 },
 {BiSABRSpreadOption[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2,
   ν2, K, T, ρs, ρv, ρc12, ρc21, 0], BiSABRSpreadOptionCorrected[F1,
   α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2, K, T, ρs, ρv, ρc12, ρc21, 0],
  BiSABRSpreadOption[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2,
   ν2, K, T, ρs, ρv, ρc12, ρc21, legendrelst, 0],
  BiSABRSpreadOptionNormalOption[F1, α1, β1, ρ1, ν1, F2, α2,
   β2, ρ2, ν2, K, T, ρs, ρv, ρc12, ρc21, legendrelst, 0]}]
```

```
{0.00758326, 0.00759723, 0.00758326, 0.00721031}
```

```
Module[{
  F1 = 0.0418,
  α1 = 0.0435,
  β1 = 0.6,
  ρ1 = -0.1819,
  ν1 = 0.3798,
  F2 = 0.0363,
  α2 = 0.0671,
  β2 = 0.7,
  ρ2 = -0.1136,
  ν2 = 0.3797,
  T = 10,
  ρs = 0.8,
  ρv = 0.5,
  ρc12 = -0.5,
  ρc21 = -0.,
  K = 0.0035, legendrelst = LegendreCoeffs[10]
 },
 {BiSABRSpreadOptionNormalCorrelation[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2,
   K, T, ρs, ρv, ρc12, ρc21, legendrelst, 0], BiSABRSpreadOptionVol[F1, α1,
   β1, ρ1, ν1, F2, α2, β2, ρ2, ν2, K, T, ρs, ρv, ρc12, ρc21, legendrelst, 0]}]
```

```
{0.814766, 0.00446159}
```

Evaluation of $\frac{1}{2}\ \alpha_2{}^2\, C_2[b]^2\, \frac{\partial^2 P}{\partial b^2}$

```
Module[{
  F1 = 0.0418,
  α1 = 0.0435,
  β1 = 0.6,
  ρ1 = -0.1819,
  ν1 = 0.3798,
  F2 = 0.0363,
  α2 = 0.0671,
  β2 = 0.7,
  ρ2 = -0.1136,
  ν2 = 0.3797,
  T = 5,
  ρs = 0.8,
  ρv = 0.5,
  ρc12 = -0.5,
  ρc21 = -0.,
  K = 0.0035,
  shift = 0.001
 },
 1/2 α2^2 F2^(2 β2)
  (BiSABRSpreadOption[F1, α1, β1, ρ1, ν1, F2 + shift, α2, β2, ρ2, ν2, K, T, ρs, ρv,
      ρc12, ρc21, 0] + BiSABRSpreadOption[F1, α1, β1, ρ1, ν1, F2 - shift, α2,
      β2, ρ2, ν2, K, T, ρs, ρv, ρc12, ρc21, 0] - 2 BiSABRSpreadOption[F1, α1, β1,
      ρ1, ν1, F2, α2, β2, ρ2, ν2, K, T, ρs, ρv, ρc12, ρc21, 0]) / (shift^2)]
```

```
0.00103682
```

Smile de vol et de correl , en fonction de la maturité

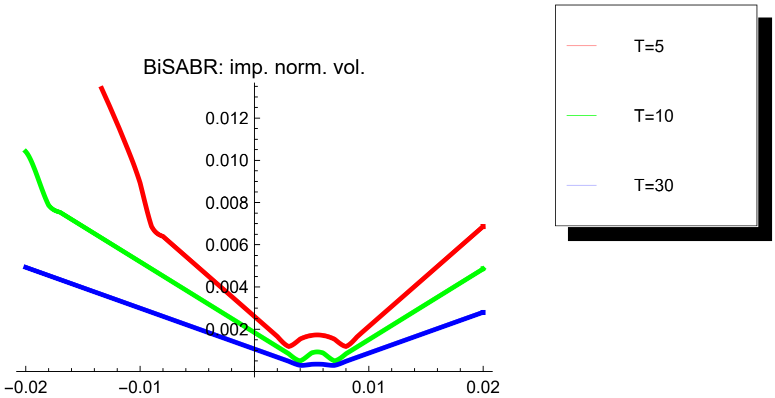
```
Timing[Module[{method = "Directe",
   zetavmethod = "Exact", zmethod = "Exact", favmethod  = "Arithmetic",
   F1 = 0.0418,
   α1 = 0.0435,
   β1 = 0.6,
   ρ1 = -0.1819,
   ν1 = 0.3798,
   F2 = 0.0363,
   α2 = 0.0671,
   β2 = 0.7,
   ρ2 = -0.1136,
   ν2 = 0.3797,
   T1 = 5, T2 = 10, T3 = 30,
   ρs = 0.8,
   ρv = 0.5,
   ρc12 = -0.3,
   ρc21 = -0.3, legendrelst = LegendreCoeffs[20]},
  Print["money=", F1 - F2];
  TestCorrelation[ρ1, ρ2, ρs, ρv, ρc12, ρc21];
```

```
strikevalues = Table[k, {k, -0.020, 0.02, 0.001}];
callvalues1 = Table[
  BiSABRSpreadOption[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2, strikevalues⟦i⟧,
   T1, ρs, ρv, ρc12, ρc21, legendrelst, 0], {i, 1, Length[strikevalues]}];
callvalues2 = Table[
  BiSABRSpreadOption[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2, strikevalues⟦i⟧,
   T2, ρs, ρv, ρc12, ρc21, legendrelst, 0], {i, 1, Length[strikevalues]}];
callvalues3 = Table[
  BiSABRSpreadOption[F1, α1, β1, ρ1, ν1, F2, α2, β2, ρ2, ν2, strikevalues⟦i⟧,
   T3, ρs, ρv, ρc12, ρc21, legendrelst, 0], {i, 1, Length[strikevalues]}];
implicitNormalvols1 = Table[NormalImplicitVol[F1 - F2, strikevalues⟦i⟧,
   T1, callvalues1⟦i⟧], {i, 1, Length[strikevalues]}];
implicitNormalvols2 = Table[NormalImplicitVol[F1 - F2, strikevalues⟦i⟧,
   T2, callvalues2⟦i⟧], {i, 1, Length[strikevalues]}];
implicitNormalvols3 = Table[NormalImplicitVol[F1 - F2, strikevalues⟦i⟧,
   T3, callvalues3⟦i⟧], {i, 1, Length[strikevalues]}];
g1 = Interpolation[Transpose[{strikevalues, implicitNormalvols1}]];
g2 = Interpolation[Transpose[{strikevalues, implicitNormalvols2}]];
g3 = Interpolation[Transpose[{strikevalues, implicitNormalvols3}]];
{σATMnormal1T1, σATMnormal2T1} =
 {F1 SABROptionAnalytic[F1, α1, β1, ρ1, ν1, F1 1.0001, T1]⟦1⟧,
  F2 SABROptionAnalytic[F2, α2, β2, ρ2, ν2, F2 1.0001, T1]⟦1⟧};
{σATMnormal1T2, σATMnormal2T2} =
 {F1 SABROptionAnalytic[F1, α1, β1, ρ1, ν1, F1 1.0001, T2]⟦1⟧,
  F2 SABROptionAnalytic[F2, α2, β2, ρ2, ν2, F2 1.0001, T2]⟦1⟧};
{σATMnormal1T3, σATMnormal2T3} =
 {F1 SABROptionAnalytic[F1, α1, β1, ρ1, ν1, F1 1.0001, T3]⟦1⟧,
  F2 SABROptionAnalytic[F2, α2, β2, ρ2, ν2, F2 1.0001, T3]⟦1⟧};
implicitcorrelations1 = Table[NormalImplicitCorrelation[σATMnormal1T1,
   σATMnormal2T1, implicitNormalvols1⟦i⟧], {i, 1 Length[strikevalues]}];
implicitcorrelations2 = Table[NormalImplicitCorrelation[σATMnormal1T2,
   σATMnormal2T2, implicitNormalvols2⟦i⟧], {i, 1 Length[strikevalues]}];
implicitcorrelations3 = Table[NormalImplicitCorrelation[σATMnormal1T3,
   σATMnormal2T3, implicitNormalvols3⟦i⟧], {i, 1 Length[strikevalues]}];
f1 = Interpolation[Transpose[{strikevalues, implicitcorrelations1}]];
f2 = Interpolation[Transpose[{strikevalues, implicitcorrelations2}]];
f3 = Interpolation[Transpose[{strikevalues, implicitcorrelations3}]];
{
 Plot[{g1[x], g2[x], g3[x]}, {x, -0.02, 0.02}, PlotStyle →
   {{Thickness[0.01], RGBColor[1, 0., 0]}, {Thickness[0.01], RGBColor[0., 1, 0.]},
    {Thickness[0.01], RGBColor[0, 0, 1]}, {Thickness[0.01], RGBColor[1, 0.6, 0]},
    {Thickness[0.01], RGBColor[0, 0.5, 1]}, {Thickness[0.01], RGBColor[0.5, 0, 1]}},
  PlotLegend → {"T=" <> ToString[T1], "T=" <> ToString[T2], "T=" <> ToString[T3]},
  LegendPosition → {1, 0.}, PlotLabel → "BiSABR: imp. norm. vol."],
 Plot[{f1[x], f2[x], f3[x]}, {x, -0.02, 0.02}, PlotStyle →
   {{Thickness[0.01], RGBColor[1, 0., 0]}, {Thickness[0.01], RGBColor[0., 1, 0.]},
    {Thickness[0.01], RGBColor[0, 0, 1]}, {Thickness[0.01], RGBColor[1, 0.6, 0]},
    {Thickness[0.01], RGBColor[0, 0.5, 1]}, {Thickness[0.01], RGBColor[0.5, 0, 1]}},
  PlotLegend → {"T=" <> ToString[T1], "T=" <> ToString[T2], "T=" <> ToString[T3]},
  LegendPosition → {1, 0.}, PlotLabel → "BiSABR: imp. norm. corr."]}]]
```

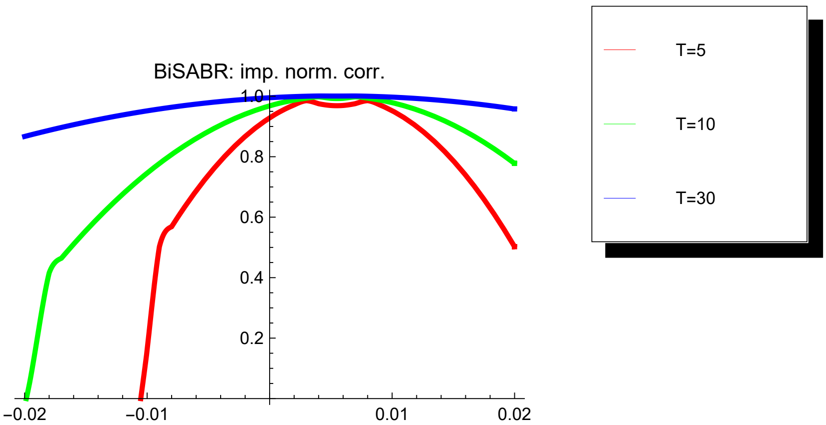
money=0.0055

Test Correlation : Eigenvalues={2.12284, 1.17947, 0.562149, 0.135538}

{2.485, {



,



}}