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SABR and BiSABR

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Why do we need a SABR Model?

- We need to represent Smiles (cap/swaption/CMS swaptions)

  - Parameter acting on the general level of the volatility : alpha
  - Parameters acting on the smile slope : beta, rho.
  - Parameters acting on the convexity of the smile : nu
  - Euro markets : beta ~ 0.5-0.7
  - Yen : beta ~ 0.5
  - Calibrating the smile ⇔ determining (alpha, rho, nu)

- The SABR Model has an European option Formula (Hagan approximation)

SABR : Equations

Equations

$$\Delta S_t = \alpha_t S_t^{\beta} \Delta W_{1,t}$$

$$\Delta \alpha_t = \nu \alpha_t \Delta W_{2,t}$$

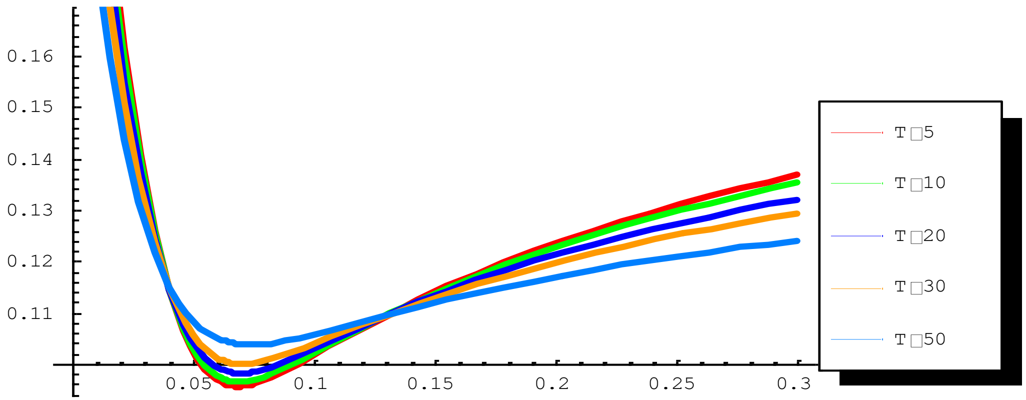
$$\Delta W_{1,t} \cdot \Delta W_{2,t} = \rho \Delta t$$

Parameters

$$\alpha_0, \beta, \rho, \nu$$

Stochastic Volatility Models

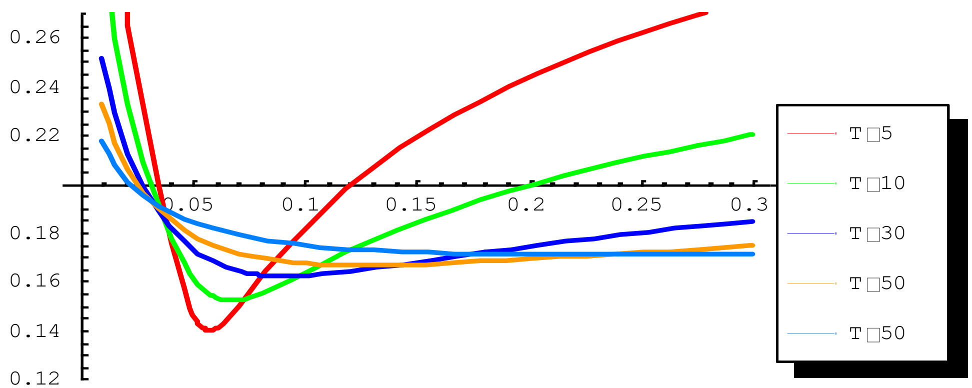
- Volatility surfaces for SABR



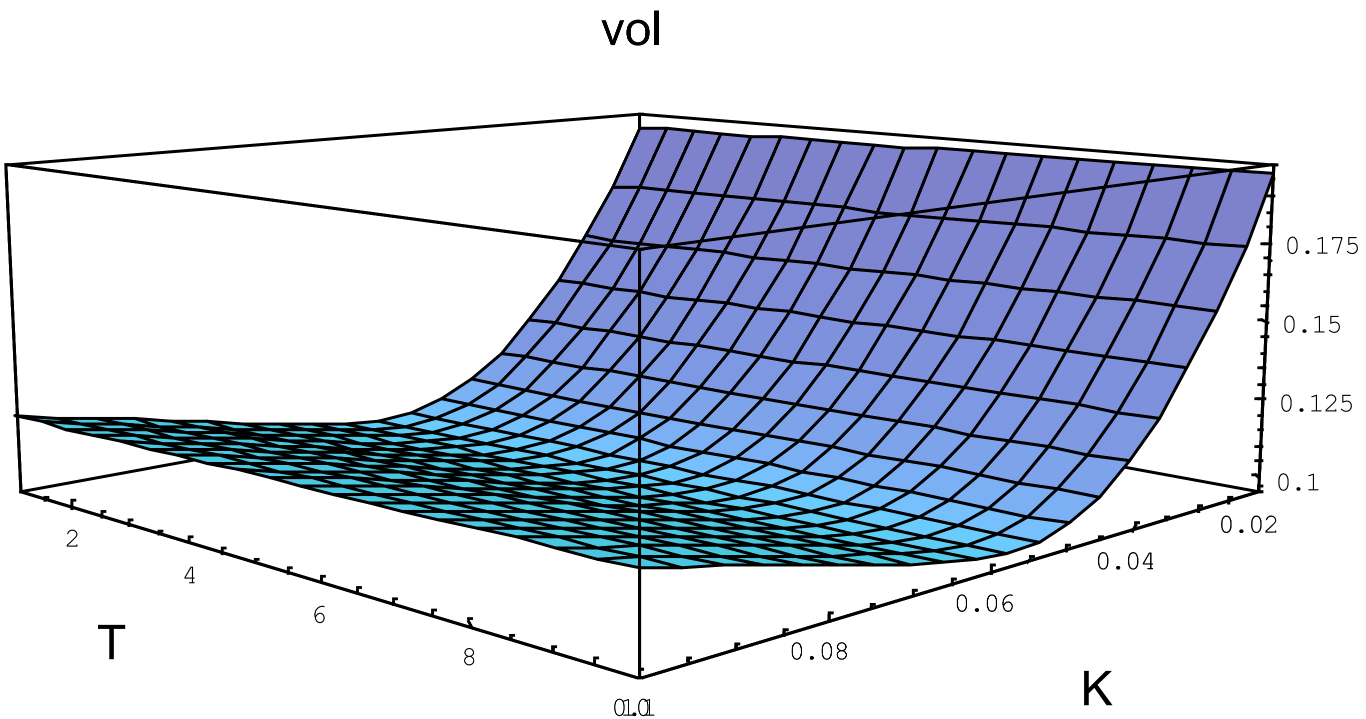
SABR smile persists longer
Than Heston SABR

Easier to control

- Compared with a Heston model

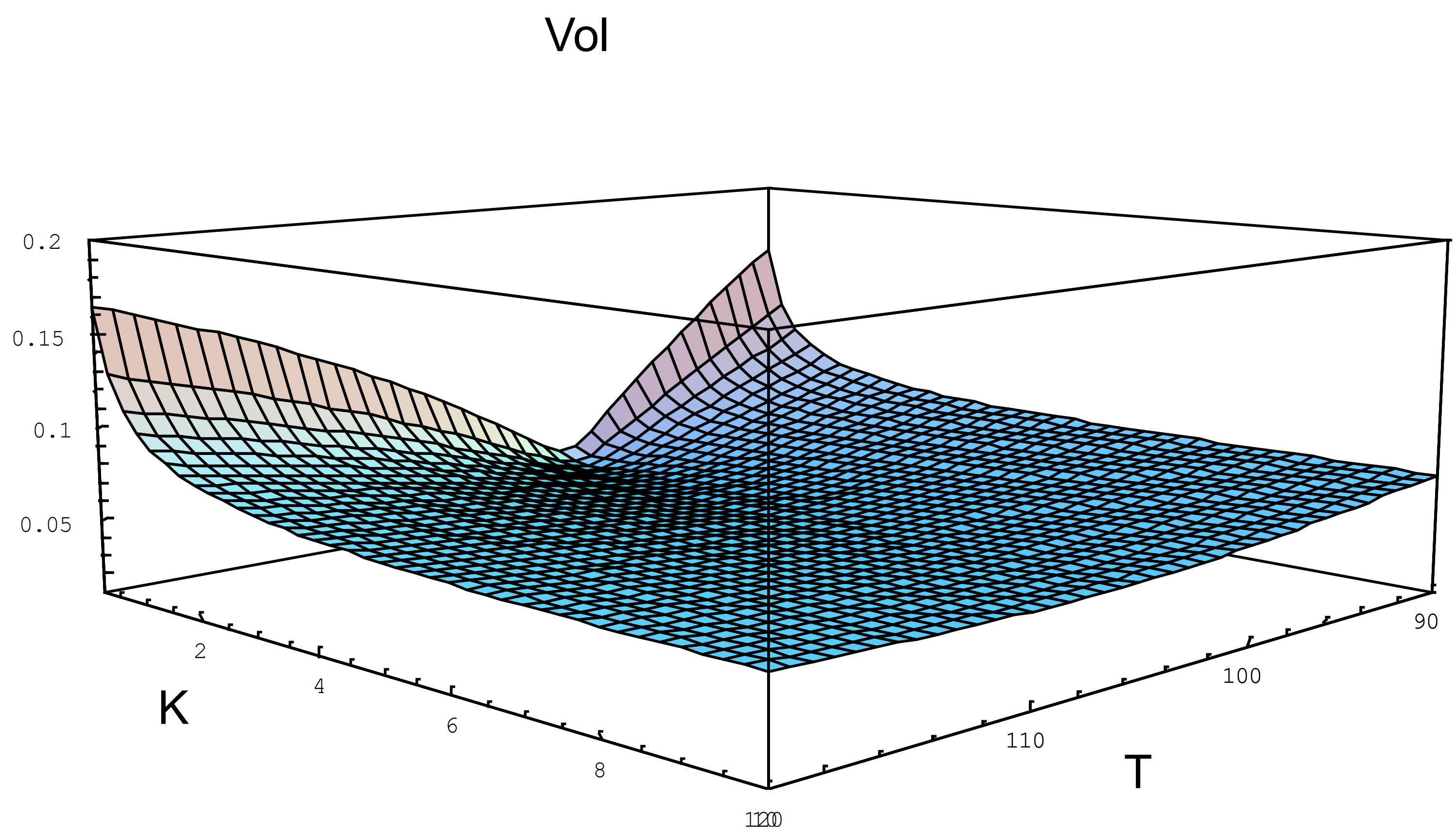


SABR Vol surface

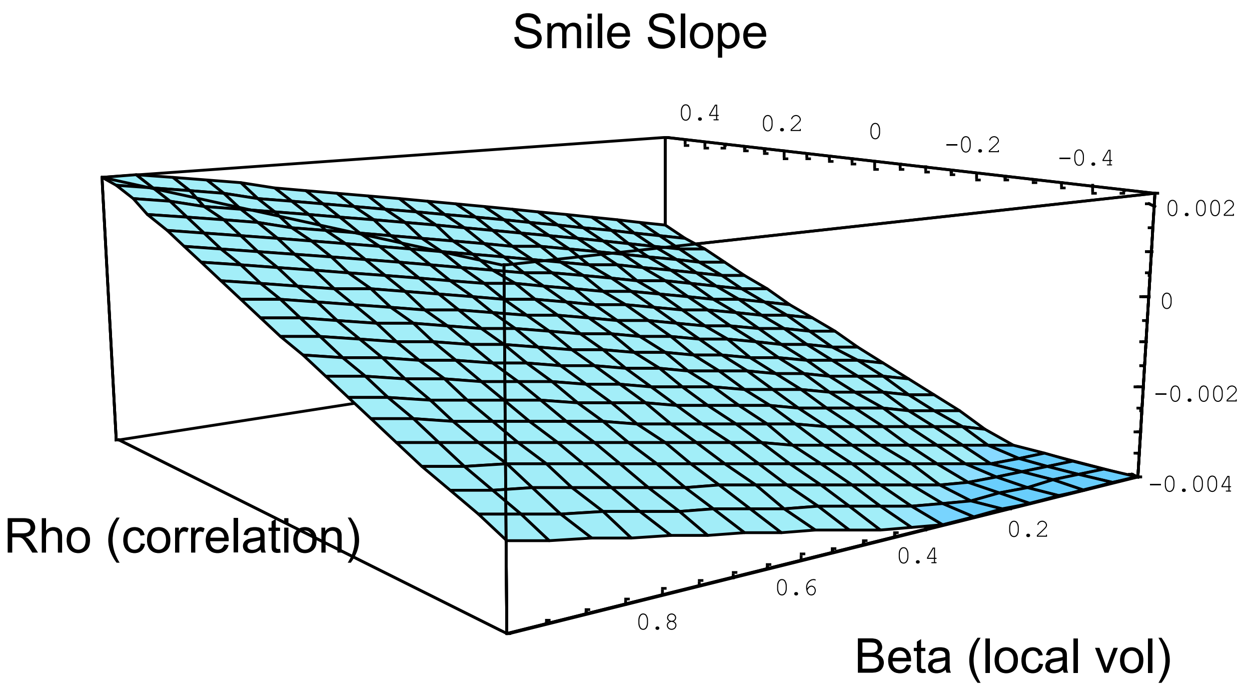


The short term smile is not captured by the hagan formula

Heston Vol Surface



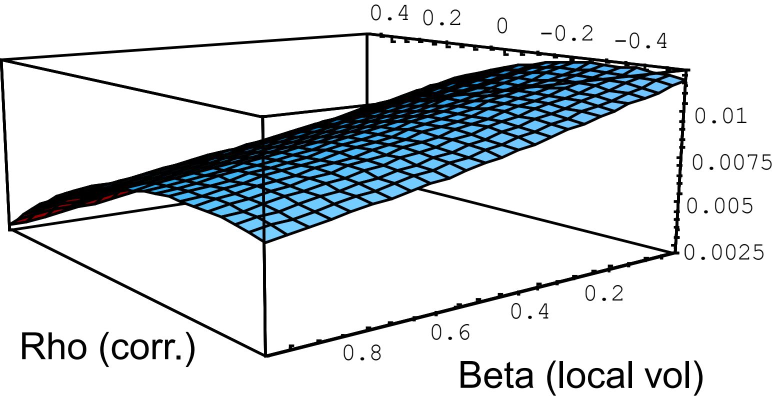
Beta , Rho & Smile Slope

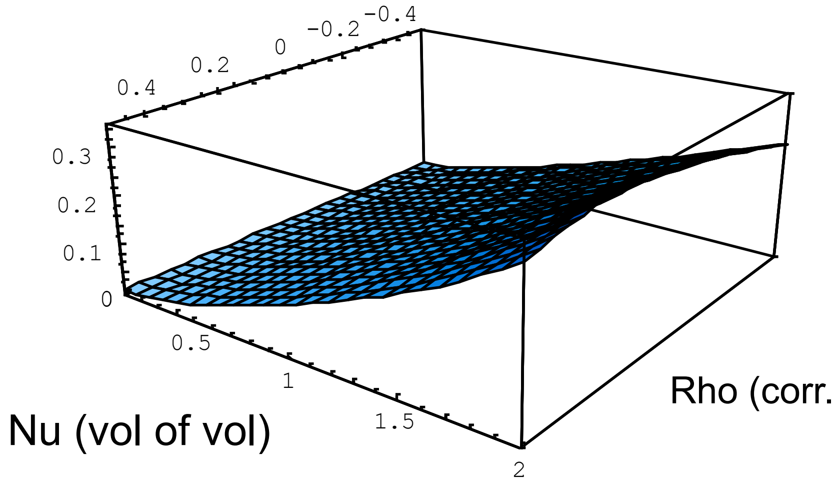


So low beta ⇔ low rho

The convexity is given by nu (vol - vol)

Smile Convexity





Change of PL due to Recalibration

- In general

	In The Money	Out Of The Money
Rho ↑	Price ↓	Price ↑
Nu ↑	Price ↑	

Why do we need BiSABR

- We need to represent spreadoption prices, Given the smiles of the underlyings
- Notion of implied normal correlation: given a spreadoption implied normal volatility => Correlation smile
- The BiSABR Formula is similar to the Hagan formula for Vanilla options.
- 4 correlations to represent the correlation smile

  one correlation between underlyings

  two cross-correlations

  one correlation between volatilities

BiSABR : Equations

Index 1

$$\Delta S_{1,t} = \alpha_{1,t} S_{1,t}^{\beta_1} \Delta W_{1,t}$$
$$\Delta \alpha_{1,t} = \nu_1 \alpha_{1,t} \Delta W_{2,t}$$
$$\Delta W_{1,t} \cdot \Delta W_{2,t} = \rho_1 \Delta t$$

Index 2

$$\Delta S_{2,t} = \alpha_{2,t} S_{2,t}^{\beta_2} \Delta W_{3,t}$$
$$\Delta \alpha_{2,t} = \nu_2 \alpha_{2,t} \Delta W_{4,t}$$
$$\Delta W_{3,t} \cdot \Delta W_{4,t} = \rho_2 \Delta t$$

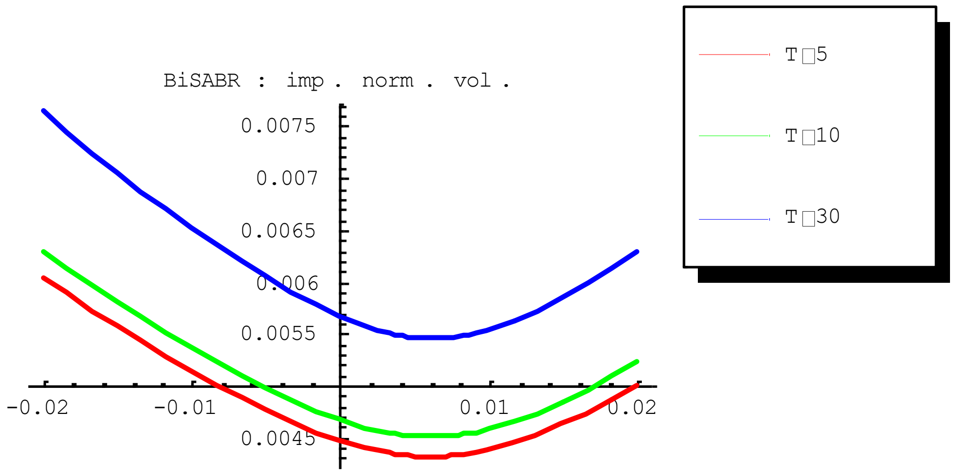
Correlations

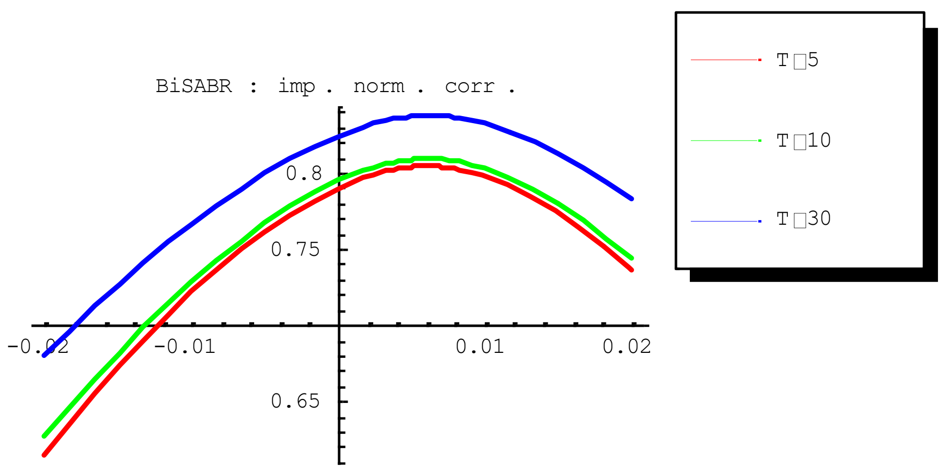
$$\Delta W_{1,t} \cdot \Delta W_{3,t} = \rho_s \Delta t$$
$$\Delta W_{2,t} \cdot \Delta W_{4,t} = \rho_v \Delta t$$
$$\Delta W_{1,t} \cdot \Delta W_{4,t} = \rho_{c12} \Delta t$$
$$\Delta W_{2,t} \cdot \Delta W_{3,t} = \rho_{c21} \Delta t$$

Parameters

$$\alpha_{1,0}, \beta_1, \rho_1, \nu_1$$
$$\alpha_{2,0}, \beta_2, \rho_2, \nu_2$$
$$\rho_s, \rho_v, \rho_{c12}, \rho_{c21}$$

BiSABR for the Correlation Smile

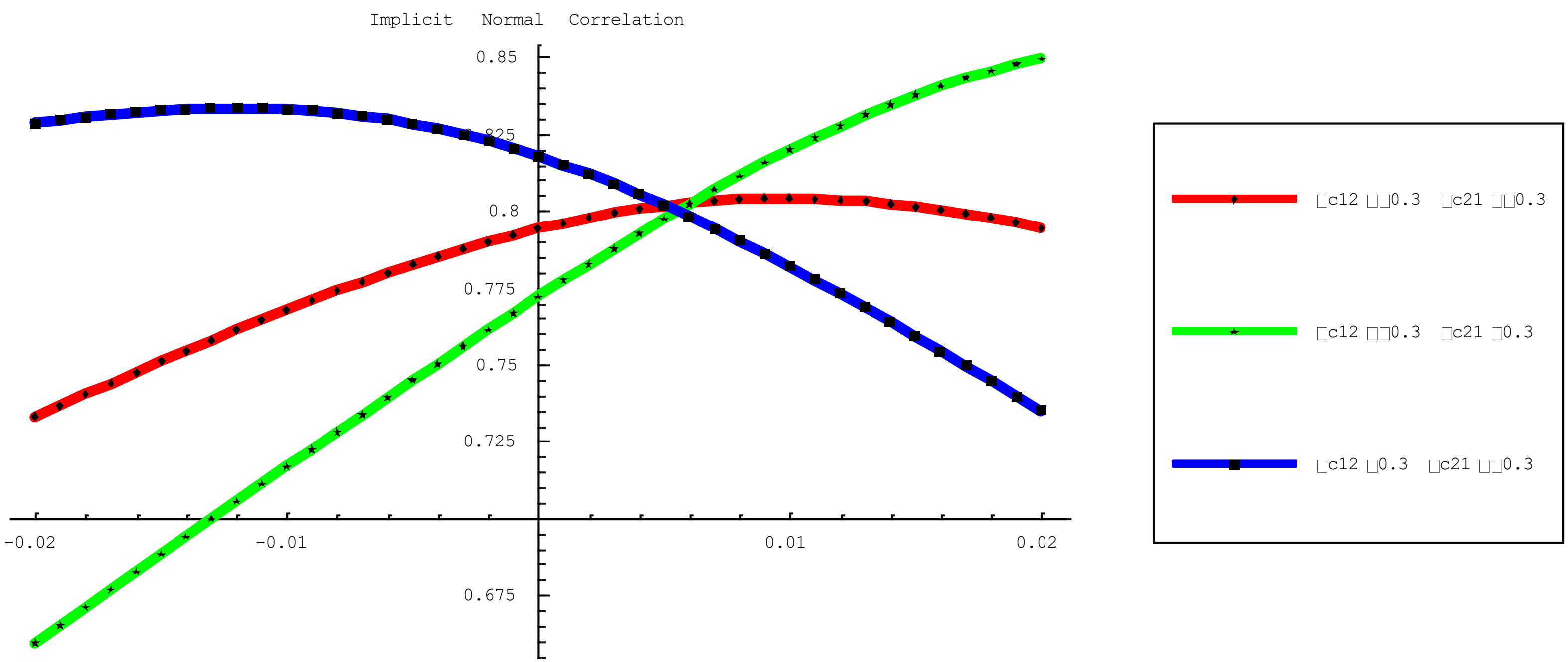




A natural effect : rise of the implied correlation with maturity

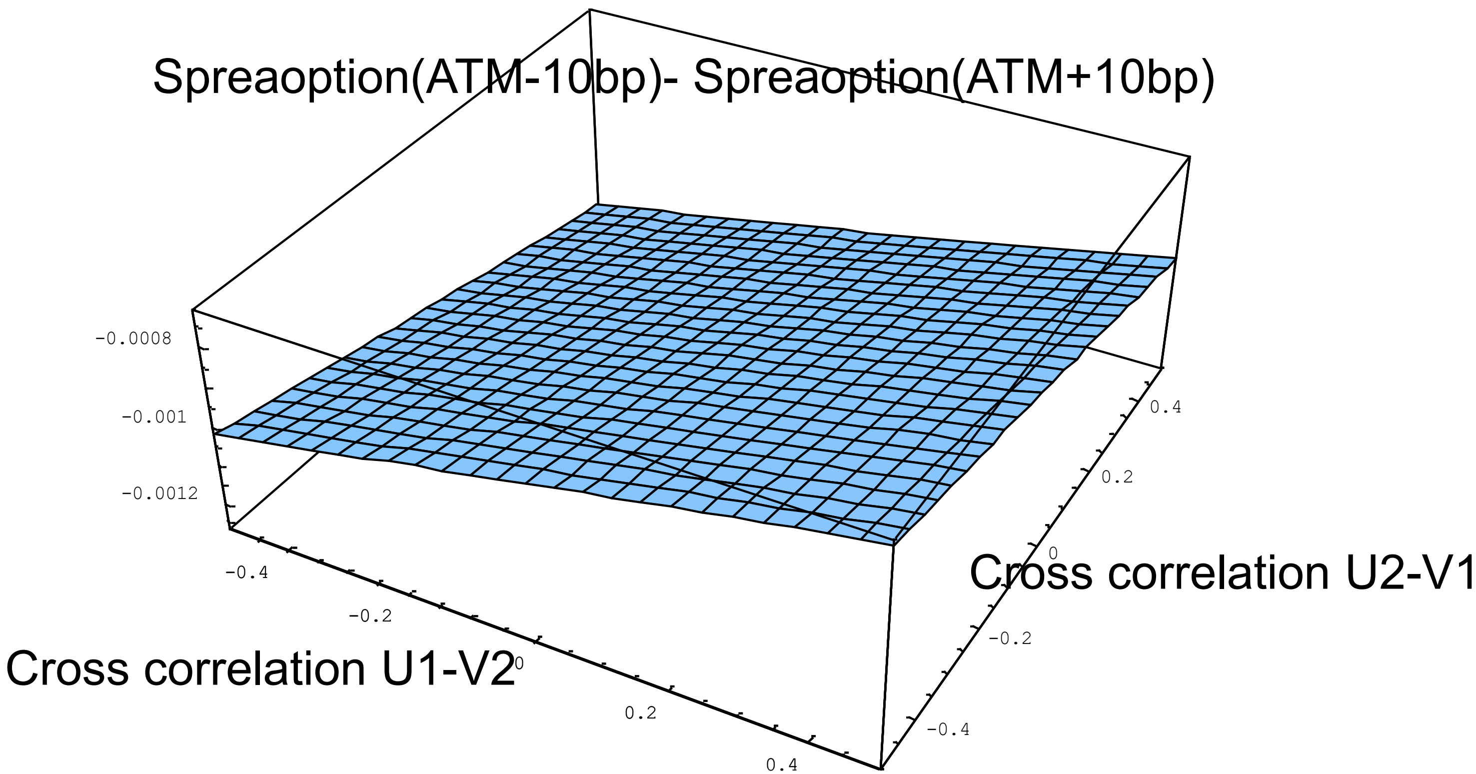
BiSABR : Cross Correlations

- Calibrating the Correlation Smile Slope



Calibration of the Cross Correlations

- Only the difference between the cross correlations matters



Closed Forms For Float Paying Digitals

- Digital that Pays 1 , S1 or S2 if S1-S2>K

  Non gaussian distribution