Topos of Creative Measurement (measurement as creation)

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Guiding Idea

A quantum measurement is **not** a destructive "collapse" of information, but an **act of creation** : it **enriches** the universe of available truths by refining the logical context — **without** presupposing an external time. Formally, a measurement is modeled by a *geometric morphism* that transforms the topos of states *before* the measurement into a topos *after* the measurement, where the outcome becomes **decidable** (Boolean) in the relevant context. This operation is **atemporal** (no time parameter) and aligns with a reading of the Wheeler–DeWitt equation.

1 Starting Framework

- Quantum topos $\mathcal{E}$ (e.g. contravariant presheaves on $V(A)$) with :
  - spectral presheaf Σ,
  - classifier Ω (Heyting),
  - internal valuation $\boldsymbol{\mu} : \mathrm{Sub}(\Sigma) \to [0,1]^{\leftrightarrow}$ (internalized Born rule).
- A *measurement event* is represented by a *subobject* $U \hookrightarrow \Sigma$ (the proposition "the outcome belongs to U").

2 Measurement = localization + slicing (creation of decidability)

One associates to U two *canonical* steps :

1) Slicing (internal conditioning)

Pass to the *slice topos* $\mathcal{E}/U$, which internalizes that the universe is considered *under the condition* U.

- The valuation is *conditioned* : $\boldsymbol{\mu} \rightsquigarrow \boldsymbol{\mu}_{|U}$ on $\mathrm{Sub}(\Sigma)_{|U}$.
- Truths become *contextual relative* to U.

2) Logical localization (sheafification via Lawvere–Tierney)

Choose an internal topology $j_U : \Omega \to \Omega$ making the proposition U *decidable* (stable/closed).

- Form the *subtopos* $\mathrm{Sh}_{j_U}(\mathcal{E}/U)$ with *sheafification* functor $a_{j_U} : \mathcal{E}/U \to \mathrm{Sh}_{j_U}(\mathcal{E}/U)$ (left exact).
- In $\mathrm{Sh}_{j_U}(\mathcal{E}/U)$, the proposition "$U$" is **Boolean** (we have created decidability of the outcome).

Definition (Creative measurement). A *creative measurement* is the composite geometric morphism

$$\mathcal{E} \xrightarrow{/U} \mathcal{E}/U \xrightarrow{a_{j_U}} \mathcal{E}_U^{\text{meas}} := \mathrm{Sh}_{j_U}(\mathcal{E}/U),$$

where j_U is chosen so that U becomes **decidable** in $\mathcal{E}_U^{\text{meas}}$.

Expected properties.

- **(CM1) Monotonicity.** If $V \subseteq U$, there is a canonical comparison morphism $\mathcal{E}_V^{\text{meas}} \to \mathcal{E}_U^{\text{meas}}$ compatible with forgetting ; creation of decidability is monotone.
- **(CM2) Logical information created.** A measurement increases information : $\Delta\mathcal{H}(U) = H(\boldsymbol{\mu}) - H(\boldsymbol{\mu}_{|U}) \geq 0$; for $V \subseteq U$, $\Delta\mathcal{I}(U) \geq \Delta\mathcal{I}(V)$.
- **(CM3) Internal Born compatibility.** $\boldsymbol{\mu}_{|U}(X) = \boldsymbol{\mu}(X \wedge U)/\boldsymbol{\mu}(U)$.
- **(CM4) Contextual locality.** The decidability created by j_U is *local* to the slice $\mathcal{E}/U$; it does not produce a global point of Σ in $\mathcal{E}$.
- **(CM5) Naturalness (RG covariance).** Under change of frame (diffeomorphisms, region refinement, change of abelian context), the construction is *pseudonatural* (functorial) : it does not depend on a temporal background.

3 Creation of Information (independent of entropy)

Logical information created. The choice of an outcome U refines the internal Heyting algebra : we pass from an *open* truth value ("possible") to a *decidable* value (yes/no) in $\mathcal{E}_U^{\text{meas}}$. We can quantify this gain (à la Shannon/algorithmic) by

$$\Delta\mathcal{I}(U) := -\log \boldsymbol{\mu}(U)$$

(in bits, internally via $[0,1]^{\leftrightarrow}$). It is a *semantic* gain (refinement of truth), not a thermodynamic cost.

Independence with respect to entropy. The operation $\mathcal{E} \to \mathcal{E}_U^{\text{meas}}$ is logical/categorical. By itself, it does not entail any variation of the von Neumann entropy of a closed physical state.

4 Timeless reading and Wheeler–DeWitt

In a theory where states satisfy a *global constraint*

$$\widehat{\mathcal{H}}\,\Psi = 0,$$

(*Wheeler–DeWitt*), "evolution" is not temporal but an *order of refinement* of truths :

- The internal universe of solutions is an object $\mathcal{S} = \ker(\widehat{\mathcal{H}})$ in $\mathcal{E}$.
- A creative measurement selects a *decidable subobject* $\mathcal{S}_U \hookrightarrow \mathcal{S}$ via $\mathcal{E} \to \mathcal{E}_U^{\text{meas}}$.

Moral. The meaning of Wheeler–DeWitt is preserved : the fundamental dynamics is *timeless* ; what we call "becoming" is the ascent in the lattice of contexts (creative measurements) that increase the available logical information.

5 Interface with CFS

In CFS, ρ and the *closed chains* A_{xy} encode causality. With the creative measurement :

- The outcome U (a proposition about spectra/invariants) becomes **decidable** in $\mathcal{E}_U^{\text{meas}}$.
- Causal types (time-/space-/light-like) are *internal predicates* which, once localized, evaluate *unambiguously* for the measured context.
- The internal causal action $\mathbf{S}[\boldsymbol{\mu}]$ is evaluated *conditionally* and may be *re-optimized* in $\mathcal{E}_U^{\text{meas}}$ (reading : informational *back-reaction*).

6 Minimal example (qubit, σ_z)

- $\mathcal{E}$: presheaves on the contexts $\{\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle\}$.
- Outcome $U = \{\sigma_z = +1\} \hookrightarrow \Sigma$.
- Slice $\mathcal{E}/U$: we condition all propositions by U.
- j_U : internal topology making U **decidable**.
- $\mathcal{E}_U^{\mathrm{meas}}$: topos where "$\sigma_z = +1$" is **Boolean** (decidable), without manufacturing a global truth for σ_x, σ_y.
- Information created :

$$\Delta\mathcal{I}(U) = -\log \boldsymbol{\mu}(U).$$

Summary

A *creative measurement* is a geometric morphism $\mathcal{E} \to \mathcal{E}_U^{\mathrm{meas}}$ (slicing + localization) that **makes the outcome decidable** in the appropriate context, **increases logical information** (without presupposing entropy) and **respects atemporality** as expected in a constrained Wheeler–DeWitt-type theory.