The Undecidable Universe and the Creative Act of Measurement: Topos, Decoherence and the Emergence of Time

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Résumé

Modern physics challenges the classical vision of a fixed and determined reality. Quantum mechanics suggests that the act of measurement is not a destruction, but an enrichment of the universe through the creation of logical information. Simultaneously, topos theory and Causal Fermion Systems (CFS) open a perspective where spacetime, gravity, and even the laws of nature emerge from local coherences and partial truths. This text merges two approaches : that of a fundamentally undecidable universe, and that of measurement understood as a creative act. The result is a vision where cosmology, human evolution, and future technologies all participate in a universal process of co-creation.

1 Introduction

Quantum physics shattered the illusion of a perfectly determined world, and relativity erased the idea of a universal time. But perhaps these theories only scratch the surface of a deeper truth : what if our observable universe were merely the thin decidable layer of an infinitely richer, essentially undecidable reality ?

From this perspective, the act of measurement plays a fundamental role : not as the collapse of a wave function, but as the creation of a new logical truth. To observe is to add a page to the book of the universe.

2 Plurality of Logics and Topoi

For a long time, classical binary logic (true/false) dominated our understanding of reality. However, the mathematics of the 20th century revealed other logics : intuitionistic, modal, temporal, categorical.

A *topos* is a logical universe where truth is local and context-dependent. A proposition can remain globally undecidable while becoming decidable in a sub-context. Thus, what we call laws or truths are local sections of a vaster reality.

Measurement, in this framework, is nothing other than a refinement of truth : an open undecidability becomes a decided value, enriching the logical fabric of the universe.

3 Measurement as a Creative Act

In quantum physics, measurement is often described as collapse. But it can be understood differently : before measurement, several outcomes are possible ; after, one reality is actualized.

— Before measurement : the universe contains open possibilities.

— After measurement : a local truth is fixed.

Measurement is therefore an act of **creation of logical information**. It is not a thermodynamic cost, but an enrichment of reality. Every decision, every observation adds a truth to the weave of the universe.

4 Time, Gravity and Decoherence

Quantum gravity (the Wheeler–DeWitt equation) suggests that there is no global time. The universe, fundamentally, is timeless.

Within the Topos + CFS framework, usual time emerges as a coherence parameter stabilizing local trajectories. The same holds for gravity : it is not a fixed given, but the harmonization between different contexts.

This can be compared to an orchestra without a conductor : harmony comes from mutual agreement rather than a single score. Thus, time and spacetime are not given, but emerge from decoherence understood as a correlational phase transition — a loss of access to distant correlations due to our cognitive and technological limitations.

5 Cosmology of the Undecidable

This vision illuminates certain great mysteries :

— **Dark matter** : effects of local truths visible through gravity but globally undecidable.

— **Dark energy** : weak coherence only appearing after the aggregation of contexts.

— **Black holes** : erasure of global decidability, with information remaining in the contextual core.

— **Big bang** : logical singularity, a gluing defect where globalization totally fails.

These enigmas then become signatures of the fundamental undecidability of reality.

6 Undecidability, Computation, and Feasibility

A radical way to reformulate the distinction between uncertainty and undecidability is to see the universe itself as a finite computer. The laws of physics are then nothing other than constraints ensuring that the internal processes of the universe remain *computable* within the effectively available computation time.

6.1 Reality as Feasible Computation

To say that a phenomenon "obeys the laws of the universe" is to say that its evolution corresponds to a computation that the universe can effectively execute. Fundamental constants (like c or $\hbar$) then appear as bounds of computational complexity : they guarantee that the computation remains feasible.

6.2 When Computation Becomes Impossible

When a physical question requires an infinite or non-terminating computation, it becomes *undecidable* for the universe itself. This is not a limit of knowledge (epistemic uncertainty), but an ontological limit : the universe cannot produce this truth. Here we find a parallel with Gödel's undecidability, but transposed into the physical domain. The difference is that the underlying "logical system" is not known : it can be thought of as an internal logic of the topos type, or even as a noncommutative space structure.

6.3 Link with Topos Logic and Non-Commutativity

In a topos, there is no absolute global truth : propositions are only valid relative to a context (a site). Likewise, if the universe is a computation, then certain global propositions make no sense because they would correspond to unrealizable computations. Undecidability thus reflects a structural impossibility rather than simple ignorance.

Analogously, a noncommutative space prohibits the simultaneous definition of certain observables : this is not an "uncertainty" in the probabilistic sense, but a constitutive limit of the logical structure of reality.

6.4 Philosophy of the Computational Limit

Obeying physical laws amounts to remaining within the domain of feasible computations. Singularities, black holes, or the big bang can then be interpreted as zones where the universe reaches its own computational limits : they become *zones of undecidability*.

In this framework, the act of measurement takes on a particular value : it forces the universe to actualize a feasible solution among the space of still open possibilities. Measurement is not merely observation : it is the choice of a feasible computation, and thus a creative act in the strong sense.

7 Computational Time : The Boundary Between the Decided and the Undecidable

In this framework, the future of the universe is not written in advance : it remains *undecidable* as long as it is not entangled with elements already decided. In other words, it is only when new correlations take root in the fixed history that certain truths become effective.

We can then define a notion of **computational time** :

> Computational time is the dynamic boundary between the domain of the "decided" (results already fixed by measurements, entanglements, effective choices) and the domain of the "undecidable" (still open possibilities, unexecuted computations).

Thus, the flow of time is not a prior given of the universe, but the very process of logical and computational decision. Each act of measurement, each interaction that fixes a truth, advances this boundary and creates time.

General relativity and quantum mechanics then appear as macroscopic descriptions of this boundary : they encode the coherence of decided truths, while undecidability remains the horizon of what has not yet been actualized.

8 Decidability Without a Referent : An Algebraic Characterization

8.1 Internal Logic and Modalities of Decision

Let H be a Heyting algebra (e.g., the classifier object Ω of a topos). A *modality of decision* is a nucleus $j : H \to H$ (monotone, inflationary, idempotent). We call *decided truths* the fixed points of j :

$$\mathrm{Fix}(j) \;=\; \{\, p \in H \mid j(p) = p \,\}.$$

Double negation $\neg\neg : H \to H$ induces booleanization ; $\mathrm{Fix}(\neg\neg)$ describes the classically decidable fragment. A family $\{j_t\}_{t\in T}$ of nuclei, increasing in t, models *computational time* :

$$t \le s \;\Rightarrow\; j_t \le j_s, \qquad \text{boundary at instant } t : \; \mathrm{Fix}(j_t) \subset H.$$

8.2 Local Version (Frames) and Lawvere–Tierney Topologies

Replace H by a frame L. A nucleus $j : L \to L$ (preserving $\wedge$, inflationary, idempotent) codes a *covering law* in the sense of Lawvere–Tierney. The j-closed elements are the *decided* statements. The choice $j = \neg\neg$ provides the booleanization of the frame, i.e., the part where $\varphi \vee \neg\varphi = \top$ is valid.

8.3 Referent-Free C^*-Algebraic Version

For a C^*-algebra (or von Neumann algebra) $\mathcal{A}$, denote by $\mathsf{P}(\mathcal{A})$ its lattice of projections (orthomodular) and $\mathrm{Ctx}(\mathcal{A})$ the set of maximal commutative subalgebras (contexts). A modality $\delta : \mathsf{P}(\mathcal{A}) \to \mathsf{P}(\mathcal{A})$ is called *contextually closing* if, for every $C \in \mathrm{Ctx}(\mathcal{A})$, the restriction $\delta|_{\mathsf{P}(C)}$ is a nucleus on the Boolean algebra $\mathsf{P}(C)$. We say that $p \in \mathsf{P}(\mathcal{A})$ is *decidable* relative to a history (set of constraints already fixed) if

$$p \in \bigcap_{C \in \mathcal{C}_{\mathrm{hist}}} \mathrm{Fix}\Big(\delta|_{\mathsf{P}(C)}\Big)\,.$$

In the commutative case $\mathcal{A} = C(X)$, we recover the topological description : $\mathsf{P}(\mathcal{A}) \simeq$ clopens of X, and δ corresponds to a nucleus j on the frame of opens.

8.4 Undecidable Future, Entanglement and Computational Time

The future remains undecidable as long as no entanglement anchors it to the decided past. Formally, if $\{j_t\}$ encodes computational time, the decision of a truth p occurs when $p \in \mathrm{Fix}(j_t)$. The flow of time is read as the monotonic enlargement of the decided fragment $t \mapsto \mathrm{Fix}(j_t)$, the boundary between decided and undecidable being the very dynamics of logical decision.

9 Humanity, Artificial Intelligence and Co-Creation

If the universe is made of contexts and emergent truths, then human evolution can be understood within this framework. Every human measurement, every technological choice is a creation of truth.

The future could be the constitution of a noosphere, a superconsciousness arising from the integration of human and artificial intelligences. In the same way that isolated cells gave rise to multicellular organisms, humanity connected to its AIs could represent a civilizational mutation.

We are not mere spectators : we are co-creators with the universe.

10 Conclusion

The observable universe is the thin decidable part of an undecidable whole. Every measurement is a creative act, every observation enriches the logical weave of reality.

Thus, the human quest for meaning joins the cosmic dynamic of creation. Our sciences, our philosophies, and our future civilizations are but the nets we cast to tame the infinite ocean of the undecidable.