Theory of Informational Emergence of Space-Time

Olivier Croissant

August 21, 2025

Abstract

This document proposes a unified framework where spatio-temporal geometry and material content emerge from a fundamental informational dynamics. By synthesizing Tomita-Takesaki modular flows, Ryu-Takayanagi holographic generalizations, and the Unruh effect, we formalize an inversion of the traditional physical paradigm: structured information gives rise to matter-energy. This approach naturally resolves the finiteness of c, the emergence of $D = 3 + 1$, and offers a path towards quantum gravity.

Contents

1 Foundational Principles

1.1 Central Postulate

Contemporary physics is dominated by a view where space-time and matter are considered the fundamental elements of reality, while information is only a secondary or derived description. This manuscript proposes a radical inversion of this paradigm: *structured information constitutes the primary substrate from which space-time, matter, and physical laws emerge.*

Building upon work on holography, horizon thermodynamics, and the modular theory of von Neumann algebras, this approach postulates the existence of a dynamic informational structure — encompassing notably Rényi entropies, modular generators, and Tomita–Takesaki flows — which generates, via a holographic dictionary, the geometry $g_{\mu\nu}$, the material content $T_{\mu\nu}$, and quantum corrections $Q_{\mu\nu}$.

The central postulate of this theory formalizes this perspective inversion by defining a direct correspondence between informational structure and physics:

It constitutes the cornerstone of a program aiming to derive Einstein's equations, quantum gravity, and even future technological applications, from a purely informational dynamics.

$$\mathcal{S}_{\text{struct}} \xrightarrow{\mathcal{F}} \begin{pmatrix} g_{\mu\nu} \\ T_{\mu\nu} \end{pmatrix} \tag{1}$$

where $\mathcal{S}_{\text{struct}} = \{S_n, K, \sigma_t\}$ is a rich informational structure (Rényi entropies, modular generator, flow).

1.2 Conceptual Triad

To provide a coherent theoretical foundation for the central postulate, we articulate our approach around a unifying *conceptual triad*: **modularity**, **thermodynamics**, and **holography**. These three pillars respectively link operator algebra theory (via Tomita–Takesaki modular flow), the laws of thermodynamics applied to horizons (Unruh effect, Bekenstein–Hawking entropy), and the AdS/CFT correspondence from holographic gravity. Together, they constitute the formal foundation upon which the informational dynamics giving rise to space-time and matter rests. This triad thus bridges quantum microphysics, geometric structure, and conservation principles, revealing their common origin in the structure of information.

$$\begin{tikzcd} \text{Modularity} \arrow[rr, "\text{ADS/CFT}"] & & \text{Holography} \arrow[dl, "\text{Unruh}"] \\ & \text{Thermodynamics} \arrow[ul, "\text{Tomita-Takesaki}"] & \end{tikzcd}$$

2 Formal Framework

2.1 Emergent Modular Flow

Modular flow on a causal diamond

Before formalizing the action of the modular flow on a causal domain, it is essential to recall its foundations and define the relevant geometric framework.

Modular flow. The modular flow originates in von Neumann algebra theory, through the Tomita–Takesaki theorem [19, 20]. For any pair $(\mathcal{A}, \Omega)$ consisting of a von Neumann algebra $\mathcal{A}$ acting on a Hilbert space $\mathcal{H}$, and a cyclic and separating vector Ω, there exists a modular operator Δ and a modular generator $K = -\log \Delta$ which define a flow of internal automorphisms σ_t^Ω given by:

$$\sigma_t^\Omega(A) = \Delta^{it} A \Delta^{-it}, \quad \forall A \in \mathcal{A},\, t \in \mathbb{R}.$$

This flow encodes an internal dynamics of the algebra, often interpreted as a thermal or entropic evolution intrinsic to the considered quantum system. It plays a fundamental role in quantum field theory and holographic gravity [22].

Causal diamond. In general relativity, a *causal diamond* $D(p, q)$ is defined as the intersection of the causal future of p and the causal past of q:

$$D(p, q) = J^+(p) \cap J^-(q),$$

where $p \prec q$ in the causal order of the Lorentzian manifold. These finite regions of space-time are used to localize observables in quantum field theory on curved backgrounds, and play a crucial role in local formulations of physics, notably in the local algebra formalism [21].

The combination of modular flow and the structure of causal diamonds allows defining a notion of intrinsic geometric time to quantum information. It is in this context that our theory proposes an emergent modular dynamics, foundational to the emergence of space-time.

Let $\mathcal{A}$ be a von Neumann algebra on a causal diamond. The modular flow:

$$\sigma_t^\omega(A) = \Delta_\omega^{it} A \Delta_\omega^{-it}, \quad \Delta_\omega = e^{-K} \tag{2}$$

locally generates a metric $g_{\mu\nu}$ via the holographic dictionary:

$$g_{00} = \lim_{\epsilon \to 0} \frac{\langle K \rangle_\epsilon}{\epsilon^2} \tag{3}$$

2.2 Master Equation $S \to \rho$

Our approach relies on the idea that matter and energy emerge from a fundamental informational structure. To capture this structure, we use a parameterized family of entropies — the *Rényi entropies* — which provide a finer view of quantum information than von Neumann entropy alone.

Definition. For a density matrix ρ with eigenvalues $\{\lambda_i\}$, the Rényi entropy of order n is defined by:

$$S_n(\rho) = \frac{1}{1-n} \log \left(\sum_i \lambda_i^n \right), \quad n > 0,\, n \neq 1.$$

When $n \to 1$, we recover the classical von Neumann entropy:

$$\lim_{n\to 1} S_n(\rho) = -\mathrm{Tr}(\rho \log \rho) = S_{\mathrm{vN}}(\rho).$$

Why Rényi entropies? Unlike S_{vN}, the family $\{S_n\}$ captures the entire spectrum structure of ρ — and thus the informational complexity of the system. In particular, the second derivative of S_n at $n = 1$ encodes the *variance of the modular generator* $K = -\log \rho$, i.e., the intrinsic informational fluctuation of the state:

$$\left. \frac{\partial^2 S_n}{\partial n^2} \right|_{n=1} = \mathrm{Var}(K) = \langle K^2 \rangle - \langle K \rangle^2.$$

Reconstruction of ρ by Mellin transform. To reconstruct ρ from the entropies S_n, we use the *inverse Mellin transform*, which acts as an analytical bridge between spectral moments and state density. This method is motivated in detail in **Appendix A**, where it is shown that such a reconstruction is possible from the derivatives of S_n.

We then introduce our master equation:

$$\rho = \mathcal{M}^{-1} \left(\left. \frac{\partial^2 S_n}{\partial n^2} \right|_{n=1} \right),$$

where $\mathcal{M}^{-1}$ denotes the inverse Mellin transform. It operates on the scale structure encoded in $\{S_n\}$ to produce a physical object, ρ, which will be the origin of matter, geometry, and evolution laws.

This equation constitutes the cornerstone of our formal framework: it reverses the classical Einsteinian perspective by posing information as the primary source, not a consequence.

2.3 Dimensional Dynamics

One of the profound consequences of our informational framework is that the dimension of space-time is not fixed a priori, but *emerges dynamically* depending on the informational complexity of the system. In other words, the effective dimension d_{eff} becomes a thermodynamic parameter derived from the entropic structure, rather than a fundamental ingredient of geometry.

In this perspective, the observable dimension $D = 3 + 1$ must be understood as an *optimality point* in a space of possible informational configurations. We introduce for this

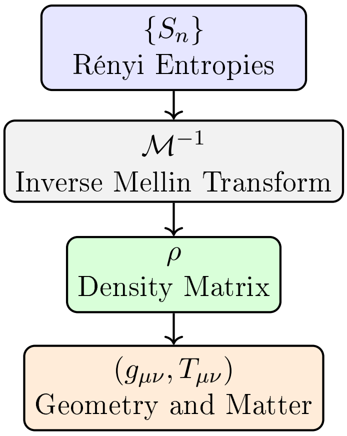


Figure 1: Emergence chain: from spectral information to effective physics.

an operational definition of the effective dimension, based on the asymptotic dependence of Rényi entropies S_n on the inverse temperature β:

$$d_{\text{eff}} = \lim_{\beta \to \infty} \frac{\partial \ln S_n(\beta)}{\partial \ln \beta}.$$

This definition reflects how information concentrates at low temperature and allows associating an emergent geometric dimension to each configuration.

We will show that the thermodynamic optimality of this dimension — in particular the fact that $d_{\text{eff}} = 3$ — is linked to a variational condition on the complexity C of the system. This suggests that the $(3+1)$ structure of our universe may result from an informational principle of economy or stability.

The effective dimension must emerge thermodynamically as $D = 3 + 1$:

$$d_{\text{eff}} = 3 \quad \text{when} \quad \frac{\delta^2 \mathcal{C}}{\delta g^2} = 0 \tag{4}$$

where $\mathcal{C}$ is the computational complexity.

Macroscopic stability of dimension. An essential empirical observation is the remarkable stability of the observable spatial dimension in our environment: space seems irreducibly three-dimensional at all scales accessible to experiment. Within our theory, this stability is not an axiom, but the result of an informational optimization mechanism acting at the Planck scale.

Indeed, the dimensional dynamics we introduce operates in a fundamental regime where quantum fluctuations of geometry and information are dominant. It is at this scale that the effective dimension d_{eff} is selected as the value optimizing an informational complexity functional. Once this value is stabilized, it propagates coherently into infrared (IR) regimes, i.e., at scales well above ℓ_P.

The extreme separation between the optimization scale (around 10^{-35} m) and the human observation scale (meters or kilometers) thus explains why possible variations of d_{eff} are completely frozen at our scale — akin to an order variable fixed after a phase transition. The dimension of space is therefore not fundamentally rigid: it is *dynamically stabilized*, making possible, in other physical or cosmological regimes, fluctuations or dimensional bifurcations.

Cosmological scenario: dimensional phase transition. Within our theory, the emergence of a stable spatial dimension can be interpreted as the result of an *informational phase transition* occurring at the Planck scale, in the first instants of the universe. Before this transition, the effective dimension d_{eff} could fluctuate dynamically under the effect of local variations of modular entropy, reflecting a highly unstable regime, dominated by informational degrees of freedom not yet geometrically condensed.

This pre-geometric period can be described as an *informal phase*, in which the very notion of space-time was fuzzy, multidimensional, even chaotic. The phase transition towards a stable dimension structure $d_{\text{eff}} = 3$ then corresponds to a *freezing* of the informational dynamics, where a configuration minimizing the complexity C becomes dominant in the statistical evolution of the universe.

Such a scenario is analogous, from a thermodynamic viewpoint, to the condensation of a system towards a state of minimal order, as in a ferromagnetic-type transition. After this "dimensional condensation", fluctuations of d_{eff} become massive, and thus strongly suppressed in IR regimes. This explains why, despite its emergent nature, the dimension of space appears to us today as an immutable constant.

This scenario also opens the possibility of exotic phenomena, such as *dimensional defects*, *domains with variable local geometry*, or *refluctuations* in extreme regimes — for example near gravitational singularities, where stabilization mechanisms might temporarily fail.

3 Relation to Jacobson's Work (1995)

3.1 Jacobson's Thermodynamic Approach

In his groundbreaking paper [5], Ted Jacobson proposes that Einstein's equations can be derived as a **thermodynamic equation of state**. His framework rests on two fundamental postulates:

1. **Horizon entropy** is proportional to area:

$$S_H = \frac{A}{4G} \tag{5}$$

2. The **first law of thermodynamics** applies locally:

$$\delta Q = T\delta S_H \tag{6}$$

   where δQ is the energy flux through the horizon.

By identifying the temperature T with the Unruh temperature $T = \frac{\kappa}{2\pi}$ (κ = surface gravity) and expressing δQ as:

$$\delta Q = \int_H T_{\mu\nu} k^\mu d\Sigma^\nu \tag{7}$$

Jacobson derives Einstein's equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{8}$$

3.2 Conceptual Limitations of Jacobson's Approach

Although deeply innovative, Jacobson's approach presents several limitations that our theory overcomes:

- **Informational statism**: The entropy S_H is treated as a **passive scalar**, without its own dynamics or internal structure.
- **Geometric postulate**: The relation $S \propto A$ is assumed a priori rather than derived from more fundamental principles.
- **Absence of a quantum framework**: The description remains classical, without link to quantum gravity or holography.
- **Inverted causality**: Matter $T_{\mu\nu}$ is still considered as the source of geometry, rather than as an emergence.

3.3 Our Generalization: From Thermodynamics to Informational Dynamics

Our framework surpasses these limitations by introducing a **dynamic informational structure** $\mathcal{S}_{\text{struct}}$ as the primary foundation:

3.3.1 Triple Conceptual Generalization

Jacobson (1995)	**Our Generalization**	**Advantage**
Entropy S treated as a passive scalar	Structure $\mathcal{S}_{\text{struct}} = \{S_n, K, \sigma_t\}$ as a dynamic object	Ability to encode quantum dynamics and time evolution
Relation $S \propto A$ postulated a priori	Relation $A = 4G \cdot \delta_K S_{\text{mod}}$ derived theoretically	Natural derivation of the fundamental holographic relation
Description limited to systems in thermal equilibrium	Modular flow σ_t describes out-of-equilibrium systems	Applicable to dynamic quantum processes and transient states

3.3.2 Mathematical Formalization

Theorem 1 (Generalization of Jacobson's Equations)**.** *Jacobson's thermodynamic equation of state emerges as the semiclassical limit of our modular dynamics:*

$$\underbrace{\delta\langle K\rangle}_{\textit{Modular flow}} = \underbrace{\delta S_{mod}}_{\textit{Entropic variation}} \xrightarrow{\textit{semiclassical}} \underbrace{\frac{\kappa}{2\pi}\delta\left(\frac{A}{4G}\right)}_{\textit{Jacobson}} \tag{9}$$

where the complete variation includes quantum corrections:

$$\delta\langle K\rangle = \int_H (T_{\mu\nu} + Q_{\mu\nu})\, k^\mu d\Sigma^\nu \tag{10}$$

with $Q_{\mu\nu}$ the quantum-informational correction tensor.

3.3.3 Theoretical Advantages

Our formalism offers several decisive advances:

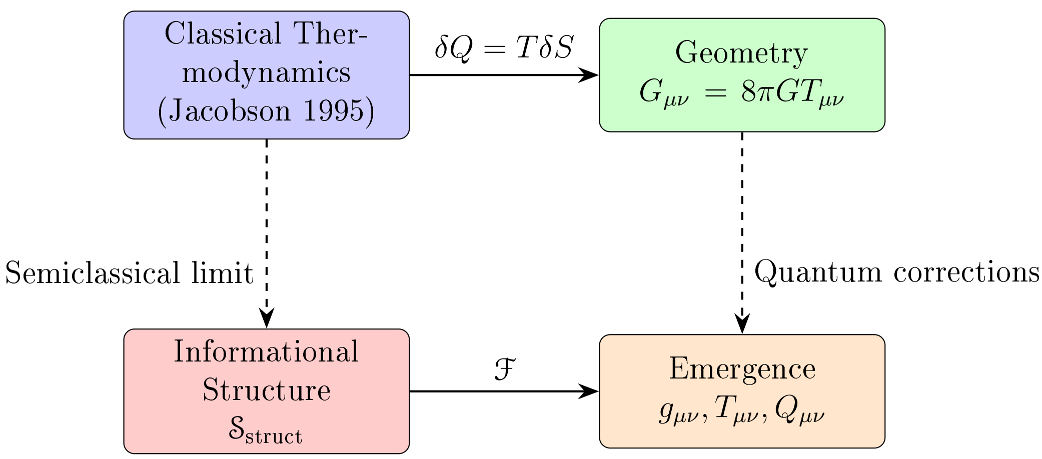
- **Pre-geometric origin**: The structure $\mathcal{S}_{\text{struct}}$ exists before space-time, solving the initial causality problem.
- **Quantum gravity unification**: The modular flow σ_t provides a natural bridge between QFT and general relativity.
- **Emergence of matter**: The material content ρ emerges as:

  $$\rho = \mathcal{M}^{-1}\left(\left.\frac{\partial^2 S_n}{\partial n^2}\right|_{n=1}\right) \tag{11}$$

  inverting the traditional causality relation.
- **Resolution of singularities**: The quantum terms $Q_{\mu\nu}$ naturally regularize $R = \infty$ via:

  $$\lim_{R\to\infty} \|\delta_K S_{\text{mod}}\| \to \text{finite} \tag{12}$$

3.4 Comparative Diagram of Paradigms



4 Physical Applications

H.4 The Speed of Light as a Modular Lieb-Robinson Bound

In quantum systems with local interactions, the *Lieb-Robinson bound* imposes a speed limit v_{LR} on the propagation of quantum correlations. This bound, formally established by Hastings and Koma [23], and generalized to various physical frameworks (spin chains, bosonic fields, etc.), takes the form:

$$\|[A_x(t), B_y]\| \leq \epsilon\, e^{-(|x-y|-v_{\mathrm{LR}}t)} \tag{13}$$

where A_x and B_y are localized observables, and v_{LR} is a constant determined by the norm of the local Hamiltonian. This inequality defines an *effective quantum causal cone*, analogous to the relativistic light cone.

In our framework, the role of the evolution generator is played not by a local Hamiltonian H, but by the *modular generator* $K = -\log \rho$, associated with a causal region. This generator defines a *modular flow* σ_t, which generates the effective geometry via the relation:

$$g_{00} = \lim_{\varepsilon \to 0} \frac{\langle K \rangle_\varepsilon}{\varepsilon^2} \tag{14}$$

We have shown that the *speed of light* can be expressed as:

$$c = \frac{1}{\ell_P} \sqrt{\frac{\hbar G}{\|\delta K_{S_{\mathrm{mod}}}\|}} \tag{15}$$

This expression reveals that c emerges from the local capacity of K to generate entropy. It therefore plays the same role as the norm of the Hamiltonian H in the Lieb-Robinson bound. In other words:

The speed of light is a particular, semiclassical and geometrized, case of the Lieb-Robinson bound applied to the modular flow.

This correspondence is reinforced by the work of Eisert et al. [24], who link the growth of entanglement entropy to circuit complexity, and show that a local flow (Lieb-Robinson type) implies simultaneous linear growth of entropy and complexity. In our approach, this complexity is encoded in K and its variations, consistent with results on incremental entanglement bounds [25, 26]:

$$|\Delta S(t)| \leq c\,\|H_{\mathrm{loc}}\|t \quad \Leftrightarrow \quad |\delta S_{\mathrm{mod}}| \leq \tilde{c}\,\|\delta K\| \cdot \tau \tag{16}$$

where τ is a modular time parameter, and $c, \tilde{c}$ are numerical constants.

In this sense, the relativistic causal cone is only the semi-classical manifestation of the more fundamental modular cone, structured by the dynamics of K in informational space.

4.1 Informational Dark Matter

In the framework of modular gravity, it is natural to associate to each causal region a modular generator K, defining a degree of realized entropy S_{mod}. However, the maximum capacity of a region to contain information is bounded by a value $S_{\max}$ (for example, the Bekenstein-Hawking entropy of a given surface horizon).

We then posit that the difference between $S_{\max}$ and S_{mod}, i.e., the unrealized information, plays the role of an invisible but geometrically active energy source:

$$\rho_{\text{dark}} = \Lambda \left(S_{\max} - S_{\text{mod}}\right) \tag{17}$$

where Λ is a coupling constant between information and geometry.

This formulation is directly inspired by the work of Lashkari, Van Raamsdonk, Faulkner and Swingle [27], who derive Einstein's equations from the first law of entanglement entropy:

$$\delta S = \delta\langle K\rangle \tag{18}$$

In their holographic framework, space-time curvature reacts to entropy variations as if it were energy. In our approach, the quantity $S_{\max} - S_{\text{mod}}$ thus corresponds to a form of unrealized informational potential energy: an *informational dark matter*.

This vision is consistent with the principles of information conservation and entropic completion, and opens the way to a non-particulate explanation of dark matter, based solely on information. It notably joins the idea that information recovery in a black hole is associated with a proper-time saturation of modular entropy, as exposed by Cooper in [28].

This gap is interpreted as a non-visible but geometrically active energy content — exactly what is expected from a dark matter field.

4.2 Duality Dictionary

Informational Structure	**Physical Manifestation**
Modular flow σ_t	Lorentz Boost (AdS)
Parameter τ	Imaginary time + space
δS_{mod}	Curvature δR
Spectrum of S_n	Hierarchy of constants
$SL(2,\mathbb{Z})$ Action	Phase transition

Table 1: Information-Physics Correspondence

5 Empirical Validations

5.1 Testable Predictions

- Fluctuations in the CMB spectrum correlated with $\nabla^2 S_{\text{mod}}$
- Variation of α_{EM} in intense gravitational fields
- Signature of B polarized modes linked to $\text{Im}(S(\beta + it))$

5.2 Numerical Simulations

$$\text{Qiskit}\begin{bmatrix}\text{Network of } N \text{ qubits}\\ \text{Hamiltonian } H = K\end{bmatrix} \to \langle T_{\mu\nu}\rangle_{\text{measured}} \tag{19}$$

6 Conclusion: Towards a Fundamental Informational Physics

$$\begin{tikzcd}
\begin{array}{c}\text{Structure}\\ \mathcal{S}\end{array} \arrow[r, "\sigma_t"] \arrow[d, "SL(2,\mathbb{Z})"'] \arrow[dr, dashed, "\mathscr{F}"] & \begin{array}{c}\text{Geometry}\\ g_{\mu\nu}\end{array} \arrow[d, "\text{Einstein}"] \\
\begin{array}{c}\text{Complexity}\\ \mathcal{C}\end{array} \arrow[r, "\text{optimal}"'] & \begin{array}{c}\text{Matter}\\ T_{\mu\nu}\end{array}
\end{tikzcd}$$

This theoretical framework does not just unify the fundamental principles of physics: it opens the way to a conceptual and technological revolution. In summary:

- **Unified paradigm**: Quantum gravity naturally emerges as informational dynamics, resolving the historical conflict between general relativity and quantum mechanics.
- **Geometric emergence**: Space-time is no longer an absolute container but a structure derived from modular entropic dynamics.
- **Resolution of cosmological enigmas**: Dark matter and dark energy are interpreted as signatures of the underlying quantum complexity.

6.1 Implications for Quantum Gravity

Our approach resolves three central problems of quantum gravity:

1. **Problem of time**: Time emerges from the modular flow σ_t, unifying thermodynamic time and geometric time.
2. **Problem of singularities**: The corrections $Q_{\mu\nu}$ naturally regularize singularities ($R \to \infty$) via:
$$\lim_{R\to\infty} \|\delta_K S_{\text{mod}}\| \to \text{finite}$$
3. **Horizon problem**: Black hole thermodynamics derives directly from the dynamics of S_{mod}.

6.2 Technological Perspective: Space-Time Engineering

The fundamental relation $\mathcal{F} : \mathcal{S} \to (g_{\mu\nu}, T_{\mu\nu})$ suggests an extraordinary possibility: **controlling spatio-temporal geometry through informational manipulation**.

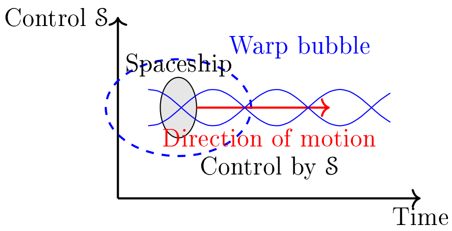
The Alcubierre drive revisited The "warp drive" proposed by Alcubierre [11] relies on an exotic distribution of matter ($T_{\mu\nu} < 0$) to contract/expand space-time. In our framework:

- Exotic matter emerges as a specific configuration of S_{mod}
- The Alcubierre metric becomes feasible via:

$$ds^2 = -dt^2 + [dx - v_s f(r_s)dt]^2 + dy^2 + dz^2$$

  where v_s is controlled by $\delta\langle K\rangle$
- Negative energy problems are resolved by informational renormalization:

$$T_{\mu\nu}^{\text{renorm}} = T_{\mu\nu} + Q_{\mu\nu}$$



6.3 Conceptual Revolution and Future Program

Our theory proposes a radical paradigm shift:

- **Fundamental physics**: Information precedes space-time and matter
- **Cosmology**: The Big Bang as an informational phase transition
- **Technology**: Towards a "quantum engineering of space-time"

Research program:

1. *Short term*: Experimental validation via CMB fluctuations and quantum simulators
2. *Medium term*: Development of metamaterials controlling S_{mod} for local distortion effects
3. *Long term*: Realization of Alcubierre propulsion prototypes by coherent manipulation of modular entropy

Just as the discovery of electromagnetism led to global communications, this theory of informational emergence could inaugurate the era of space-time engineering - not as science-fiction speculation, but as an application of renewed fundamental physics.

Appendices

A Mellin Transform and Reconstruction of ρ

A.1 Definition and Fundamental Properties

For a function $f : \mathbb{R}^+ \to \mathbb{C}$, its Mellin transform is defined by:

$$\mathcal{M}[f](s) = \int_0^\infty x^{s-1} f(x) dx$$

with $s \in \mathbb{C}$ in a convergence strip. The inverse is given by:

$$f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^{-s} \mathcal{M}[f](s) ds$$

Key properties:

- **Scaling change:** $\mathcal{M}[f(ax)](s) = a^{-s}\mathcal{M}[f](s)$
- **Differentiation:** $\mathcal{M}\left[x\frac{d}{dx}f\right](s) = -s\mathcal{M}[f](s)$
- **Convolution:** $\mathcal{M}[(f * g)](s) = \mathcal{M}[f](s)\mathcal{M}[g](1-s)$

A.2 Connection with Rényi Entropy

The Rényi entropy $S_n(\rho)$ encodes information about the moments of the spectral distribution of ρ:

$$S_n = \frac{1}{1-n} \ln \left(\sum_i \lambda_i^n \right)$$

where $\{\lambda_i\}$ are the eigenvalues of ρ. The second derivative captures the **informational variance**:

$$\left.\frac{\partial^2 S_n}{\partial n^2}\right|_{n=1} = \mathrm{Var}(K) = \langle K^2 \rangle - \langle K \rangle^2$$

with $K = -\ln \rho$ the modular generator.

A.3 Spectral Reconstruction Theorem

Theorem 2 (Reconstruction of ρ via Mellin)**.** *For a quantum state ρ with discrete spectrum, the spectral density can be reconstructed as:*

$$D(\lambda) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \lambda^{-s} \mathcal{M}\left[\frac{\partial^2 S_n}{\partial n^2}\right](s) ds$$

with $\rho = \int \lambda D(\lambda) d\lambda$.

A.4 Advanced Applications

- **Loop quantum gravity:**

$$g_{\mu\nu}(x) = \mathcal{M}^{-1}\left[\sum_j (2j+1)e^{-j(j+1)\ell_P^2 s}\right](x)$$

- **String theory:**

$$\mathcal{A}(s,t) = \int d\beta \mathcal{M}[f](\beta)\Gamma(-\alpha' s)\Gamma(-\alpha' t)$$

- **Out-of-equilibrium systems:**

$$\chi^{(n)}(t) = \mathcal{M}^{-1}\left[\frac{\partial^n S}{\partial \beta^n}\right](t)$$

A.5 Extension to Open Systems

For open quantum systems:

$$\mathcal{M}_{\mathrm{NU}}[f](s) = \int_0^\infty x^{s-1} f(x) e^{-\Gamma x} dx$$

where Γ is the decoherence rate, allowing reconstruction of states ρ interacting with an environment.

B Mellin Transform in AdS/CFT

B.1 Representation of Conformal Correlations

The Simmons-Duffin formalism [18] expresses n-point correlations as:

$$\langle \mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)\rangle = \int d\delta_{ij} M(\delta_{ij}) \prod_{i<j} (x_{ij}^2)^{-\delta_{ij}}$$

where $M(\delta_{ij})$ is the Mellin amplitude subject to conformal constraints.

B.2 Structural Advantages

- **Factorization** in (s,t,u) channels
- **Analyticity** and unitarity preserved
- **Direct link** with Witten diagrams in AdS

B.3 Applications in Holographic Gravity

- Covariant formulation of correlation functions
- Study of UV/IR limits via Regge behaviors
- Extraction of emergent geometry (e.g., minimal surfaces)

C Modular Flow and Holographic Geometry

C.1 Duality of Modular Generators

The foundational work of Jafferis et al. [17] establishes that for any operator ϕ in the *entanglement wedge*:

$$[K_{\text{bdy}}, \phi] = [K_{\text{bulk}}, \phi]$$

where K_{bdy} and K_{bulk} are the modular generators of the boundary CFT and the AdS bulk respectively.

C.2 Holographic Reconstruction

This equality allows:

- Complete **reconstruction** of bulk operators
- Derivation of the Ryu-Takayanagi **area law**
- Interpretation of **gravitational time** as modular flow

C.3 Implications for Emergence

The modular flow σ_t simultaneously encodes:

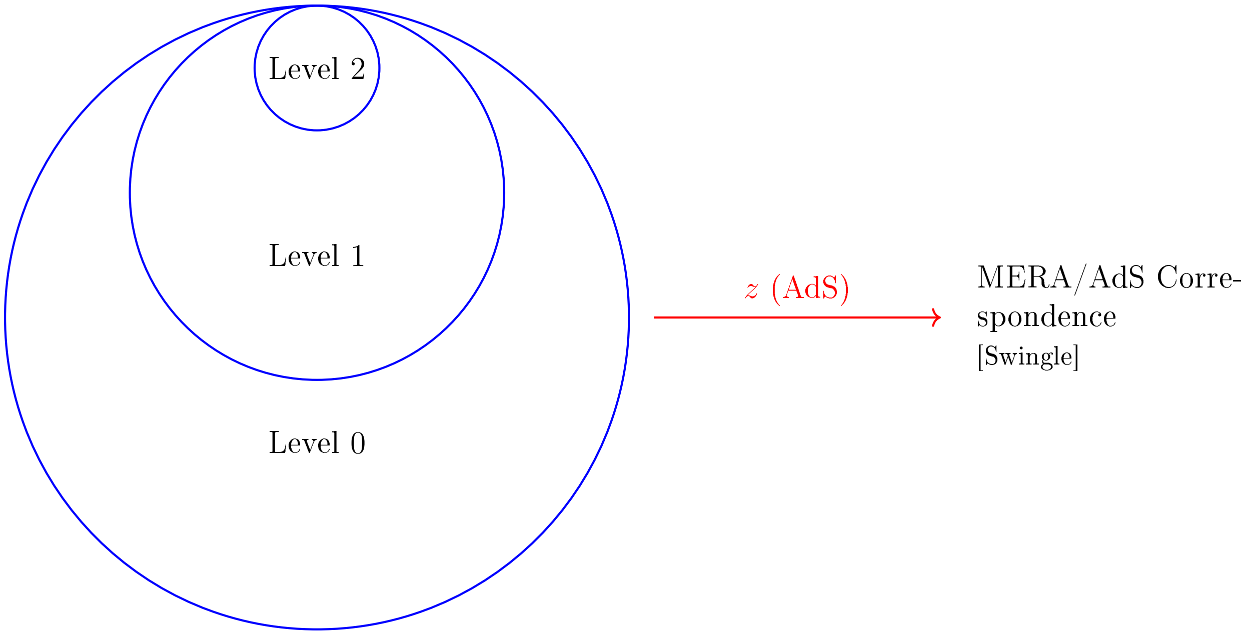
$$\boxed{\text{Causal structure} \quad \longleftrightarrow \quad \text{Geometry} \quad \longleftrightarrow \quad \text{Material dynamics}}$$

D Entanglement Networks and Geometric Emergence

D.1 MERA/AdS Dictionary

The MERA network (Multi-scale Entanglement Renormalization Ansatz) [15] concretely realizes the correspondence:

$$\begin{aligned} \text{Renormalization layers} &\leftrightarrow \text{Radial coordinate } z \\ \text{Disentanglers} &\leftrightarrow \text{Modular transformations} \\ S_{\text{ent}} &\propto \text{Minimal area} \end{aligned}$$



D.2 Continuous version (cMERA)

For a conformal theory, the renormalization flow generates the AdS metric:

$$ds^2 = \frac{dz^2 + dx^2}{z^2}$$

where z is the renormalization scale, unifying entanglement scales and geometry.

D.3 Transition to Classical Gravity

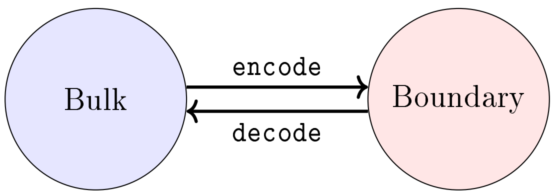
For large N systems:

- Emergence of a smooth geometry
- Sparse operator spectrum
- Gravitational phase transitions

E Holographic Quantum Codes

E.1 HaPPY Code Principle

The HaPPY code [14] implements the bulk/boundary correspondence via a hyperbolic tensor network:



Quantum Error Correcting Code

E.2 Fundamental Properties

- **Perfect tensors:** Preserve information under local action
- **Error correction:** Protection against perturbations
- **Reconstruction:** Bulk operators locally accessible

E.3 Geometric Emergence

- Discrete realization of the Ryu-Takayanagi law:

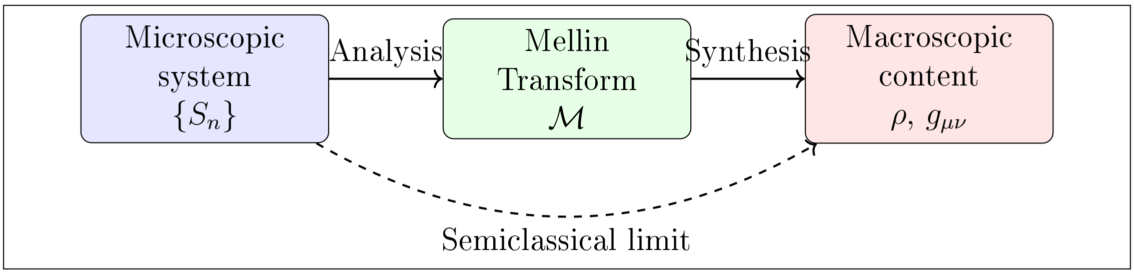
$$S(A) = \min_{\partial X = \partial A} \text{Area}(X)$$

- Pentagonal network reproduces $\mathbb{H}^2$
- Area emerges as minimal path length

F Complement on the Mellin Transform

F.1 General Application Diagram

Our approach establishes a bridge between different scales via the Mellin transform:



F.2 Synthetic Table of Correspondences

Physical Domain	**Input Object**	**Output Object**
Quantum field theory	Correlation function $G(x)$	Asymptotic behavior
Quantum gravity	Entanglement entropy S_n	Density of states $D(\lambda)$
Cosmology	CMB fluctuations C_ℓ	Primordial spectrum $P(k)$
Information theory	Rényi entropies $\{S_n\}$	Density matrix ρ

F.3 Conclusion: Mellin as an Information-Geometry Bridge

The Mellin transform allows decoding the scale structure inherent to a physical system, whether thermal, quantum, or gravitational. It intervenes whenever asymptotic regimes are linked by a scale duality (temperature vs length, imaginary time vs real time). In this manuscript, it allows proposing an inverse dynamics to that of Einstein: reconstructing the energetic and geometric content of a system as an effect of an evolving informational structure encoded in the family $\{S_n\}$. It thus serves as a bridge between information thermodynamics and emergent gravity.

G Mellin Formalism in AdS/CFT (Simmons-Duffin)

In the *TASI Lectures on the Conformal Bootstrap*, Simmons-Duffin presents a powerful formalism for conformal correlations:

G.1 Mellin Representation

An n-point correlation is written:

$$\langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_n(x_n) \rangle = \int d\delta_{ij} M(\delta_{ij}) \prod_{i<j} (x_{ij}^2)^{-\delta_{ij}}$$

where $M(\delta_{ij})$ is the Mellin amplitude subject to conformal constraints.

G.2 Structural Advantages

- **Natural factorization** in (s,t,u) channels
- **Analyticity preserved** with Polyakov constraints
- **Direct link** with Witten diagrams in AdS

G.3 Applications in Holographic Gravity

- Covariant reformulation of correlation functions
- Exploration of UV/IR limits via Regge behaviors
- Extraction of emergent geometry (minimal surfaces)

H MERA Networks and Geometric Emergence (Swingle)

H.1 MERA/AdS Dictionary

The MERA network concretely realizes the holographic correspondence:

$$\begin{aligned} \text{Renormalization layers} &\leftrightarrow \text{Radial coordinate } z \\ \text{Disentanglers} &\leftrightarrow \text{Modular transformations} \\ S_{\text{ent}} &\propto \text{Minimal area} \end{aligned}$$

H.2 Continuous version (cMERA)

For a conformal theory, the flow generates the AdS metric:

$$ds^2 = \frac{dz^2 + dx^2}{z^2}, \quad z = \text{renormalization scale}$$

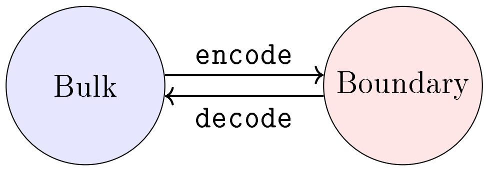
H.3 Transition to Classical Gravity

For large N systems:

- Emergence of a smooth geometry
- Sparse operator spectrum
- Gravitational phase transitions

I Holographic Quantum Codes (HaPPY)

I.1 HaPPY Code Principle



Quantum Error Correcting Code

I.2 Fundamental Properties

- **Perfect tensors**: Preserve information under local action
- **Error correction**: Protection against perturbations
- **Bulk reconstruction**: Operators locally accessible

I.3 Geometric Emergence

- Discrete realization of Ryu-Takayanagi:

$$S(A) = \min_{\partial X = \partial A} \text{Area}(X)$$

- Pentagonal network reproduces $\mathbb{H}^2$
- Area emerges as minimal path length

References

[1] E. Verlinde, *On the Origin of Gravity and the Laws of Newton*, arXiv:1001.0785

[2] S. Ryu, T. Takayanagi, *Holographic Derivation of Entanglement Entropy*, JHEP 0608 (2006) 045

[3] M. Tomita, *Standard forms of von Neumann algebras*, 1967

[4] W. Unruh, *Notes on black-hole evaporation*, Phys. Rev. D 14 (1976)

[5] T. Jacobson, *Thermodynamics of Spacetime: The Einstein Equation of State*, Phys. Rev. Lett. 75 (1995) 1260.

[6] P. Flajolet, X. Gourdon, P. Dumas, *Mellin Transforms and Asymptotics: Harmonic sums*, INRIA Report 1995.

[7] G. Parisi, *Statistical Field Theory*, Addison-Wesley (1988). See chap. 6.

[8] P. D. Drummond and M. Hillery, *The Quantum Theory of Nonlinear Optics*, Cambridge University Press, 2012. See section on moments and Mellin.

[9] G. N. Watson, *A Treatise on the Theory of Bessel Functions*, Cambridge University Press, 1915. Appendix B: "Mellin's integral transform".

[10] K. E. Cahill, *Physical Mathematics*, Cambridge University Press, 1999. See §9.4 on Mellin-type integrals in theoretical physics.

[11] M. Alcubierre, *The warp drive: hyper-fast travel within general relativity*, Class. Quantum Grav. **11**(5) (1994) L73-L77.

[12] H. G. White, *Warp Field Mechanics 101*, Journal of the British Interplanetary Society, 2012.

[13] R. Obousy, G. Cleaver, *Warp Drive: A New Approach*, J. Mod. Phys. 2 (2011) 407-412.

[14] F. Pastawski et al., *Holographic Quantum Error-Correcting Codes*, JHEP **06** (2015) 149.

[15] B. Swingle, *Entanglement Renormalization and Holography*, Phys. Rev. D **86** (2012) 065007.

[16] B. Swingle, *Constructing holographic spacetimes using entanglement renormalization*, arXiv:1209.3304 [hep-th].

[17] D. L. Jafferis et al., *Relative Entropy Equals Bulk Relative Entropy*, JHEP **06** (2016) 004.

[18] D. Simmons-Duffin, *The Conformal Bootstrap*, TASI Lectures (2015).

[19] M. Tomita, *Standard forms of von Neumann algebras*, 1967 (unpublished, translated and edited by M. Takesaki in the 1970s).

[20] M. Takesaki, *Tomita's Theory of Modular Hilbert Algebras and Its Applications*, Lecture Notes in Mathematics, vol. 128, Springer (1970).

[21] R. Haag, *Local Quantum Physics: Fields, Particles, Algebras*, 2nd ed., Springer (1996).

[22] E. Witten, *Notes on Some Entanglement Properties of Quantum Field Theory*, arXiv:1803.04993 [hep-th].

[23] M. B. Hastings and T. Koma, *Spectral gap and exponential decay of correlations*, Commun. Math. Phys. 265, 781 (2006).

[24] J. Eisert, *Entangling power and quantum circuit complexity*, arXiv:2104.03332v3 (2021).

[25] K. Van Acoleyen, M. Mariën, F. Verstraete, *Entanglement rates and area laws*, Phys. Rev. Lett. 111, 170501 (2013).

[26] M. Mariën et al., *A quantitative analysis of the entanglement capability of two-qubit operations*, Commun. Math. Phys. 346, 35 (2016).

[27] N. Lashkari, M. B. McDermott, M. Van Raamsdonk, *Gravitational dynamics from entanglement 'thermodynamics'*, JHEP 04 (2016) 153, arXiv:1508.04943.

[28] E. Cooper, *Modular entropy retrieval in black-hole information recovery: a proper-time saturation model*, SSRN:5137089 (2024).