Natural Sensitivities Algebra

By Olivier Croissant

1 Introduction

The First order and second order sensitivities verify the following compounding formula :if $f = f(x_1, x_2, ..., x_n)$ an every y_i is a function of other y_j;

$x_i = x_i(y_1, y_2, ..., y_p)$ then the formula to transform the sensitivities

expressed in x variables into sensitivities expressed in y variables are:

$$\frac{\partial}{\partial y_j} f = \sum_{i=1}^{n} \frac{\partial}{\partial x_i} f \cdot \frac{\partial x_i}{\partial y_j}$$

and

$$\frac{\partial^2}{\partial y_j \partial y_k} f = \sum_{k=1}^{n} \sum_{l=1}^{n} \frac{\partial^2}{\partial x_l \partial x_m} f \cdot \frac{\partial x_l}{\partial y_j} \cdot \frac{\partial x_m}{\partial y_k} + \sum_{l=1}^{n} \frac{\partial}{\partial x_l} f \cdot \frac{\partial^2}{\partial y_j \partial y_k} x_l$$

then the natural sensitivities

could be written as:

$$df = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} dx_i \qquad and \qquad d^2 f = \sum_{i,j=1}^{n} \frac{\partial^2 f}{\partial x_i \partial x_j} dx_i \otimes dx_j + \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} d^2 x_i$$

If we have different submodels , then we have :

$$df = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} dx_i \qquad d^2 f = \sum_{i,j=1}^{n} \frac{\partial^2 f}{\partial x_i \partial x_j} dx_i \otimes dx_j + \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} d^2 x_i$$

$$df = \sum_{i=1}^{n} \frac{\partial f}{\partial y_i} dy_i \qquad d^2 f = \sum_{i,j=1}^{n} \frac{\partial^2 f}{\partial x_i \partial x_j} dy_i \otimes dy_j + \sum_{i=1}^{n} \frac{\partial f}{\partial y_i} d^2 y_i$$

The model risk is represented by the way the first derivatives change when we go from one model to another