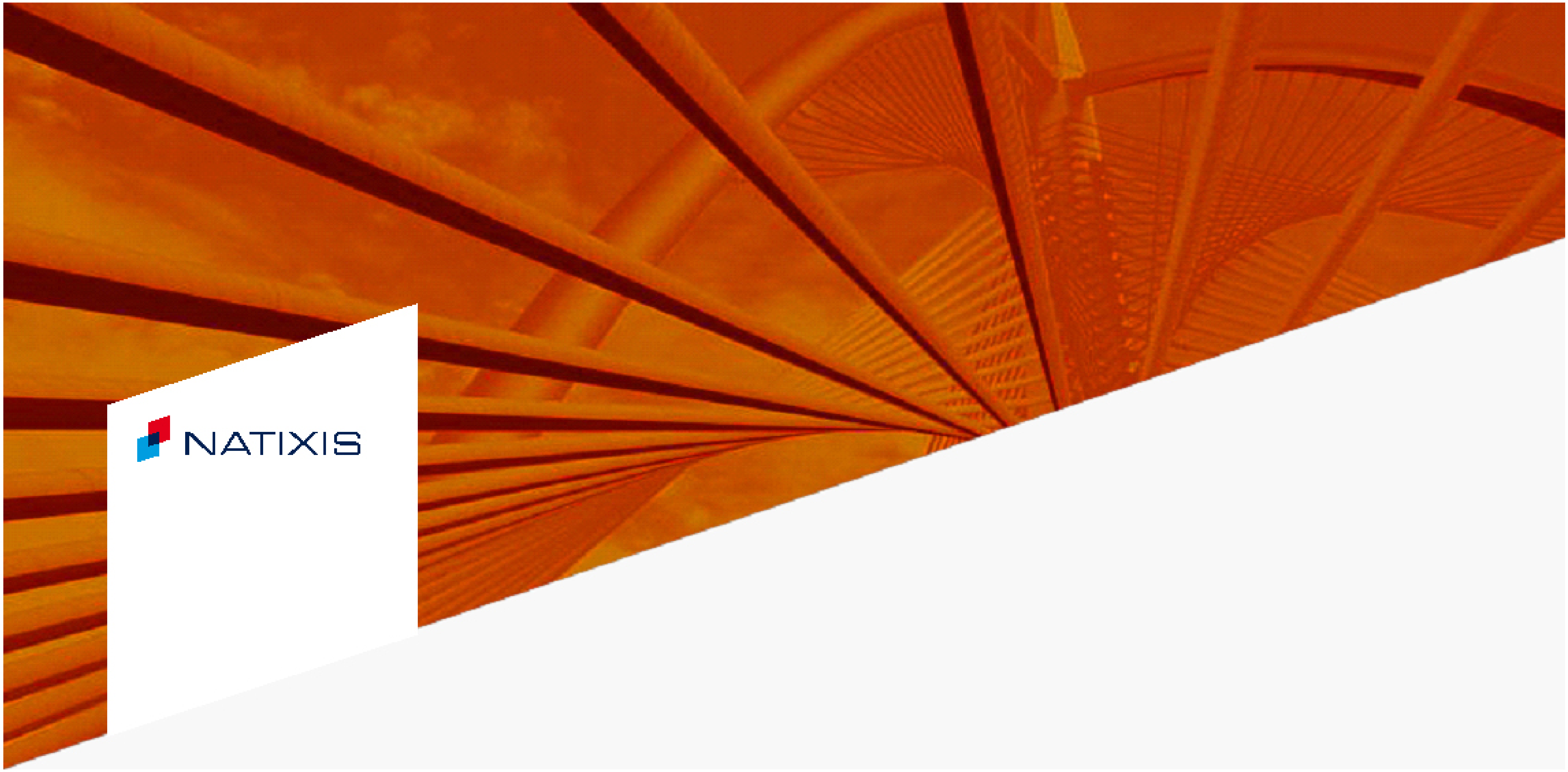


A Model For Spreadoptions

By Olivier Croissant

The Requirements

BiSabr, NormalHeston are common ways to represent a spread risk.The main problem is to have a consistent way to :

- 1) - Take into account the smile of the underlying smiles inside an arbitrage free framework
- 2) - Have additional flexibility to introduce the notion of correlation smile that can be adjusted to market data.
- 3) - Be useable up to 40 years maturity options.

In addition of these requirements some "nice to have" feature can also be considered:

4) - The ability to fit several maturities for the options, so allowing to have a real diffusion model, that can be used to price american deals

5) - The ability to do fit several matiurities in an autonomous way, that mean without using ime dependant coefficients, so improving forward smile behaviour, which is important for american deals

6) - The ability to have closed form formula for other type of options : Digitals on the underlyings, Min - Max options, Double condition options,

7) - The ability to have additional flexibility to represent smile behaviour of the tail of the underlying beyond the usual 4 parameters framework

8) - The ability to fit several maturities for the options, so allowing to have a real diffusion model, that can be used to price american deals

The final test of acceptance will be of course the stability of the hedge ratios

Asset Returns

$$\mathrm{Vect}\left[\frac{dS_{i,t}}{S_{i,t}}\right] = \sqrt{\Sigma_t}\, dZ_t + \mu\, \Sigma_t\, dt$$

μ here is useful for the normalisation process where we have E[S] with a certain process = $E[S^{\alpha}]$ with *a* different process and *a* drift

Volatility Process

$$d\Sigma_t = (M(\Sigma_t - \Sigma_\infty) + (\Sigma_t - \Sigma_\infty) M^*)\, dt + \sqrt{\Sigma_t}\, dW_t\, Q + Q^*\, dW_t^* \left(\sqrt{\Sigma_t}\right)^*$$

That we can rewrite as

$$d\Sigma_t = \left[M(\Sigma_t - \Sigma_\infty)\right]_{\mathrm{Symetric}} dt + \left[\sqrt{\Sigma_t}\, dW_t\, Q\right]_{\mathrm{Symetric}}$$

where $\sqrt{\Sigma_t} = \sigma$ means $\sigma^t\sigma = \Sigma_t$ and $[A]_{\mathrm{Symetric}} = A + {}^tA$

$\beta\, Q^*\, Q = -M\,\Sigma_\infty - \Sigma_\infty\, M^*$ where

β Gindikin index $\in]2,+\infty[$

Bru shows the β>1 is enough to insure non explosion of the model

Hypothesis done for simplification $Q <-> \Sigma_\infty$

$\Omega\Omega^* = -M\,\Sigma_\infty - \Sigma_\infty\, M^*$

$$\Sigma_t = \begin{pmatrix} (\Sigma^{11})_t & (\Sigma^{12})_t \\ (\Sigma^{12})_t & (\Sigma^{22})_t \end{pmatrix}$$ <u>3 process of variance</u>

we note :

$$\sqrt{\Sigma_t} = \sigma_t = \begin{pmatrix} (\sigma^{11})_t & (\sigma^{12})_t \\ (\sigma^{21})_t & (\sigma^{22})_t \end{pmatrix} \text{ such that } \Sigma_t = \sigma_t\, \sigma_t^*$$

In the 1 dim case, we compute the vol of vol by : $\nu = \sqrt{\frac{-2\, M\, \Sigma_\infty}{\beta}}$

Variance Process Parameters

Initial value of the variance : $\Sigma_0 = \begin{pmatrix} (\Sigma^{11})_0 & (\Sigma^{12})_0 \\ (\Sigma^{21})_0 & (\Sigma^{22})_0 \end{pmatrix}$

Vol of vol : $Q = \begin{pmatrix} \nu_{11} & \nu_{12} \\ \nu_{21} & \nu_{22} \end{pmatrix}$

Mean reverting speed : $M = \begin{pmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{pmatrix}$

Long term variance : $\Sigma_\infty = \begin{pmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{pmatrix}$

M definite negative
Q inversible
Σ_0 definite positive
Σ_∞ definite positive

Covariance asset - asset volatility

We introduce correlation via : $Z_{i,t} = \mathrm{Tr}[W_t R_i^*] + B_{i,t}\sqrt{1 - \mathrm{Tr}[R_i R_i^*]}$ $i \in \{1, 2\}$

such that $\|R\| \leq 1$ with W_t et B_t independant

Affinity of the process implies that $\exists \{\rho_1, \rho_2\}$

such that $R_1 = \begin{pmatrix} \rho_1 & \rho_2 \\ 0 & 0 \end{pmatrix}$ et $R_2 = \begin{pmatrix} 0 & 0 \\ \rho_1 & \rho_2 \end{pmatrix}$

Underlying 1 Variance

$\mathrm{Var}[Y_1]\,dt = \Sigma_{11}\,dt$, $\mathrm{Covar}[Y_1, Y_2]\,dt = \Sigma_{12}\,dt$,

Variance of Underlying 1 Variance :

$\mathrm{Var}[\Sigma_{11}]\,dt = 4\,\Sigma_{11}\left(Q_{11}^2 + Q_{12}^2\right)dt$

Covariance l' underlying i - vol of underlying 1 :

$\mathrm{Covar}[Y_i, \Sigma_{11}]\,dt = 2\,\Sigma_{1i}\,(Q_{11}\,\rho_1 + Q_{21}\,\rho_2)\,dt$

NATIXIS

Correspondance Heston / BiHeston (martingale part: exact)

$$\frac{\nu_1^2}{4} = Q_{11}^2 + Q_{21}^2$$

$$\frac{\nu_2^2}{4} = Q_{12}^2 + Q_{22}^2$$

$$\rho_{s1}\,\nu_1 = \frac{Q_{11}\,\rho_1 + Q_{21}\,\rho_2}{2}$$

$$\rho_{s2}\,\nu_2 = \frac{Q_{12}\,\rho_1 + Q_{22}\,\rho_2}{2}$$

Correspondance Heston / BiHeston (Drift part : freeze)

Drift of $d\Sigma_{11}$: $\left(\beta\left(Q_{11}^2 + Q_{21}^2\right) + 2\,(M_{11}\,\Sigma_{11} + M_{21}\,\Sigma_{12})\right)dt$

Drift of $d\Sigma_1$ est $\lambda_1\,(\Sigma_{\infty 1} - \Sigma_1)\,dt$

So we get $\begin{cases} \beta\,(Q_{11}^2 + Q_{21}^2) + 2\,M_{21}\,\Sigma_{12} = \lambda_1\,\Sigma_{\infty 1} \\ 2\,M_{11} = -\lambda_1 \end{cases}$ and similar equations for the other asset

therefore $$M = \begin{pmatrix} \frac{-\lambda_1}{2} & \frac{\lambda_2\,\Sigma_{\infty 2} - \beta\left(Q_{12}^2 + Q_{22}^2\right)}{2\,\Sigma_{12}} \\ \frac{\lambda_1\,\Sigma_{\infty 1} - \beta\left(Q_{21}^2 + Q_{11}^2\right)}{2\,\Sigma_{12}} & \frac{-\lambda_2}{2} \end{pmatrix}$$

Normalisation

Equation followed by $L_1 = S_1^{\alpha}$ et $L_2 = S_2^{\beta}$

$$\text{Vect}\left[\frac{dL_i}{L_i}\right] = \sqrt{\overline{\Sigma_t}}\, dZ' + \mu' \,\overline{\Sigma_t}\, dt$$

$$\mu' = \mu + \begin{pmatrix} \frac{\alpha-1}{2\alpha} \\ \frac{\beta-1}{2\beta} \end{pmatrix}$$

$$\overline{\Sigma_t} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \Sigma_t \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$$

$$d\,\overline{\Sigma_t} = \text{Sym}[\bar{M}\,(\overline{\Sigma_t} - \overline{\Sigma_\infty})]\, dt + \text{Sym}\left[\sqrt{\overline{\Sigma_t}}\, dW_t\, \overline{Q}\right]$$

$$\overline{\Sigma_\infty}\,|_0 = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \Sigma_\infty\,|_0 \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}, \quad \bar{M} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} M \begin{pmatrix} \alpha^{-1} & 0 \\ 0 & \beta^{-1} \end{pmatrix}, \quad \overline{Q} = Q \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix}$$

Symetry :

$$E_{\Sigma_t}[(S_1 - S_2 - K)^+] = E_{\overline{\Sigma_t}}\left[\left(S_1^{1/\alpha} - S_2^{1/\beta} - K\right)^+\right]$$

NATIXIS

Differences with the BiSABR Framework

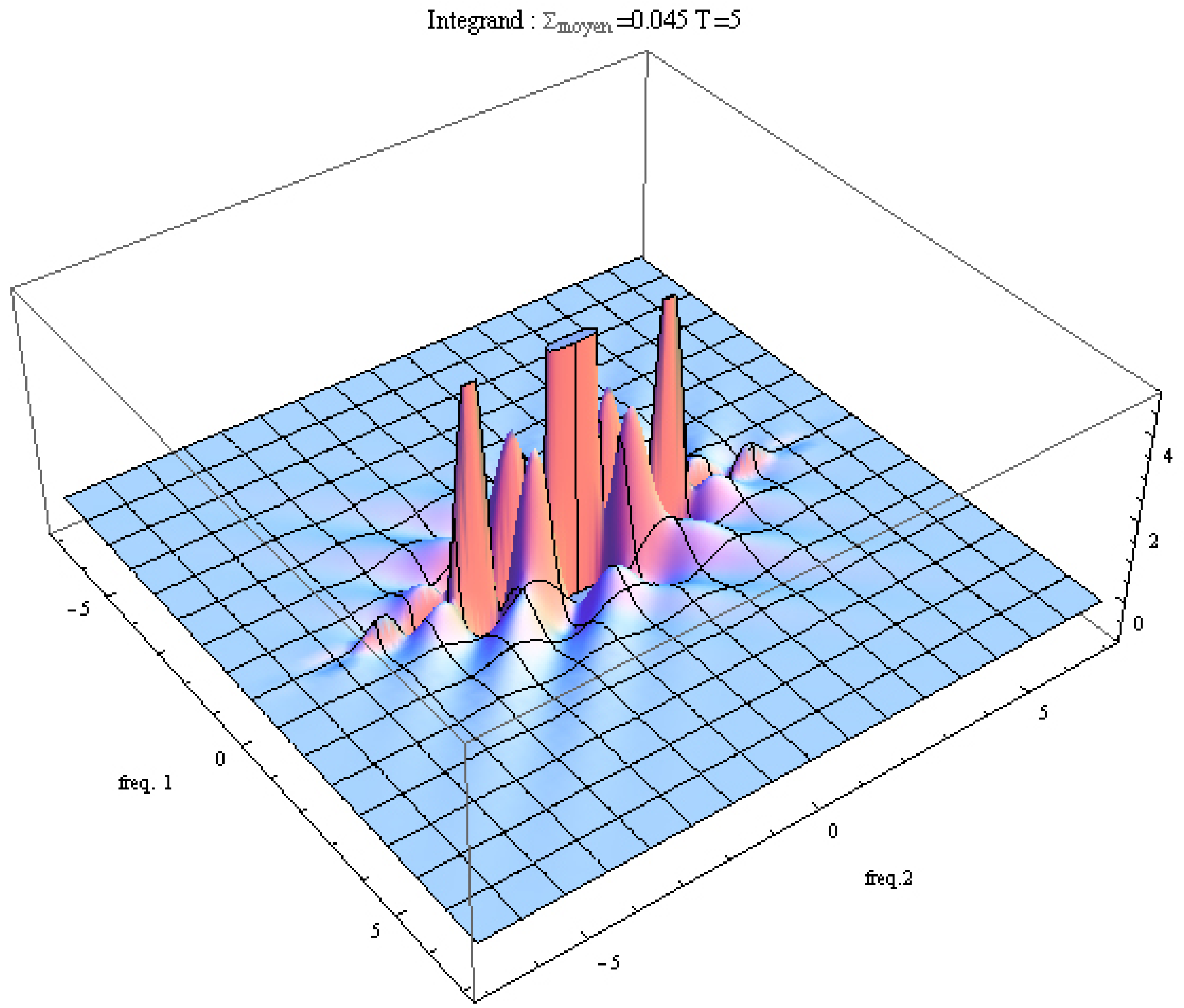
Feature	BiSABR	Splash
Formula Spreadoption	Approximation (T < 25 years)	Exact
Formula Digital paying float	Approximation (T < 5 years)	Exact
Degenerecence Underlying	Difficult	Yes
Comparaison Monte Carlo	ATM (like SABR)	Yes
Formula Power Option	No	Yes
Formula Max Option	No	Yes
Formula Double Condition	No	Yes
High Maturity T > 30 years	No	Yes
More flexibility Underlying Smile	No	yes (super heston)
Small Maturity	Yes (T -> 0)	No (T > 2 years)
Adding Jumps to underlying ?	No	Yes
Adding Jumps to Vol	No	Yes
Delicate Numerical Convergence	No	Yes
Address the term structure problem	No	Yes (more wishart factors)
□	□	□
□	□	□
□	□	□
□	□	□
□	□	□

Exemple of execution with specification of a global convexity β

```
In[7]:= << "C:\\Documents and Settings\\ocroissant\\My Documents\\BiHeston\\BiHeston_Process.m"

In[8]:= << "C:\\Documents and Settings\\ocroissant\\My Documents\\BiHeston\\BiHeston_VanillaInstruments.m"

In[11]:= Timing[Module[{S1 = 0.05, S2 = 0.05, KList = {0.02, 0.03, 0.04, 0.05, 0.06}, M1 = -0.01,
    M2 = -0.02, θ1 = 0.03, θ2 = 0.041, ρs = 0.6, ρsinf = 0.8, ρm1 = 0.3, ρm2 = -0.3, ρ1 = 0.5,
    ρ2 = 0.8, Σ1 = 0.04, Σ2 = 0.05, β = 5, τ = 5, λ1 = 1.1, λ2 = 1.2, ν1 = 0.01, ν2 = 0.01, M, Σinf,
    Σ, Q, βm, S, ρ, printflag = {0}, μ = {0, 0}},
   M = ( M1                ρm1 √(M1 M2) ); Σinf = ( θ1                  √(θ1 θ2) ρsinf ); Σ = ( Σ1              √(Σ1 Σ2) ρs );
       ( ρm2 √(M1 M2)      M2           )          ( √(θ1 θ2) ρsinf      θ2             )       ( √(Σ1 Σ2) ρs     Σ2          )
   S = {S1, S2}; ρ = {ρ1, ρ2};
   SmileBiHestonVanilla[S, 1, 1, 1, 1, KList, τ, Σ, M, Σinf, β, ρ, μ, printflag]]]

Out[11]= {2.766, {0.00301551, 0.00196807, 0.00147071, 0.00122959, 0.00110903}}
```

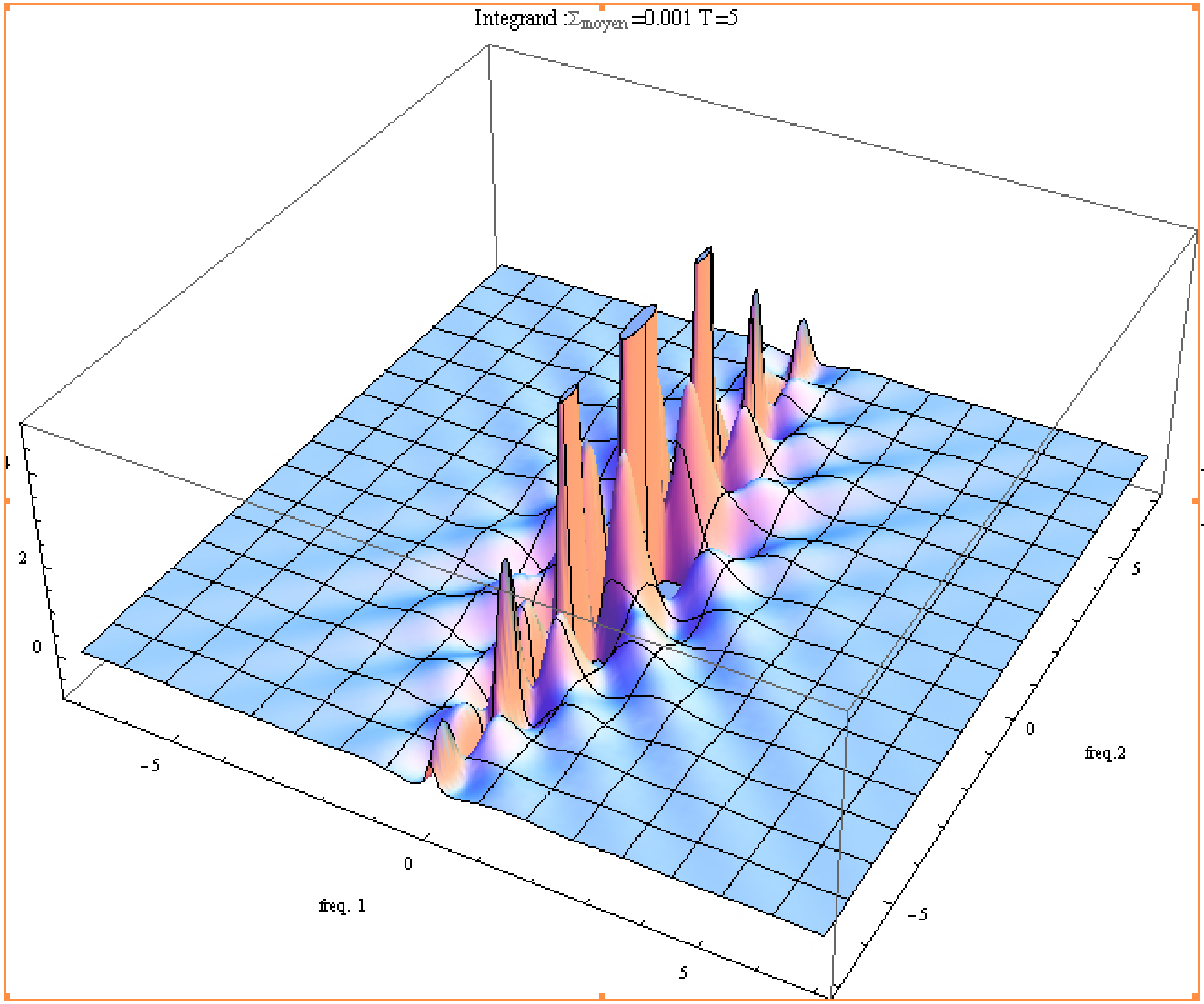
Integrand for the Bidim Inverse Fourier

$\{55.828,$



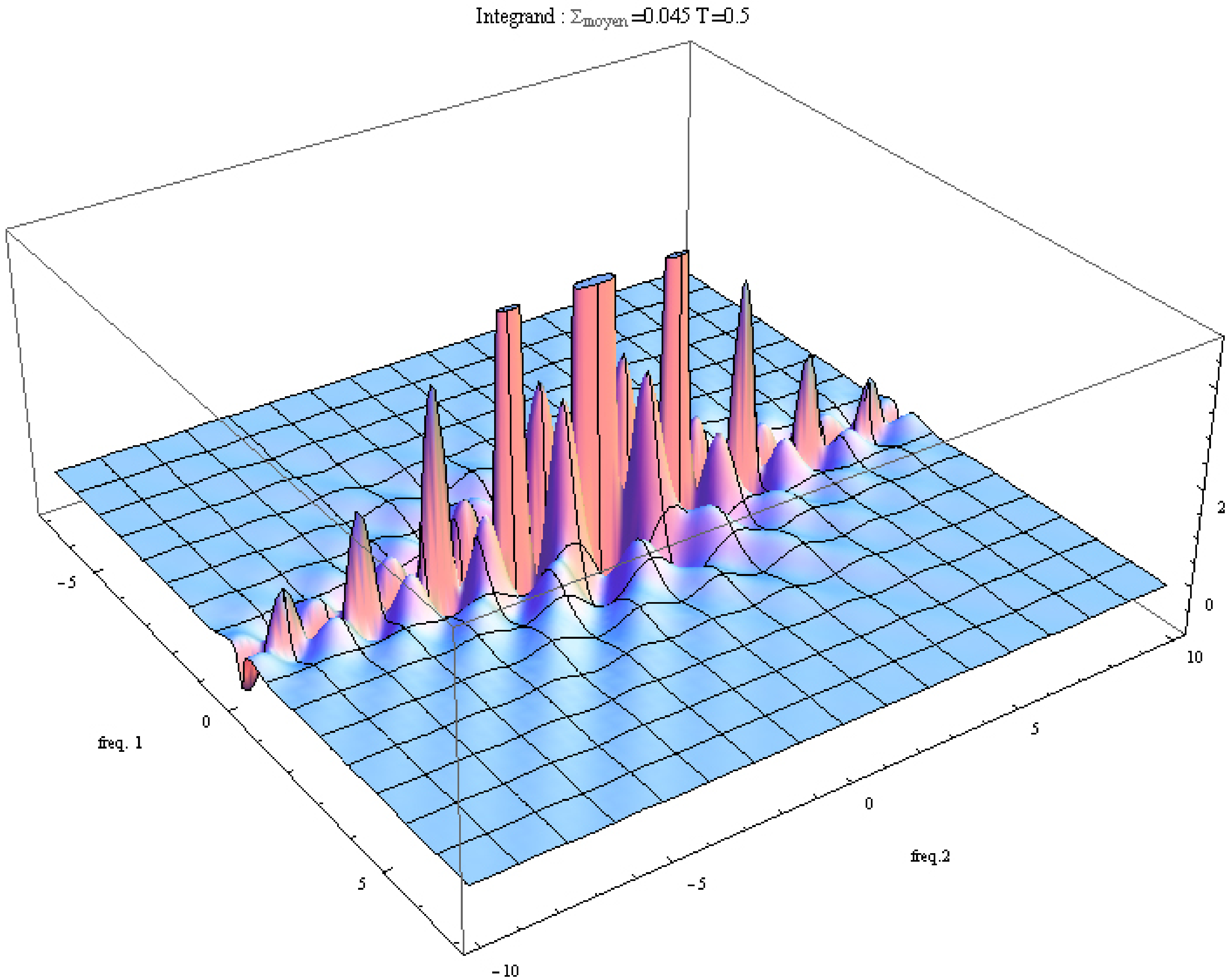
$\}$

Integrand for the Bidim Inverse Fourier small volatility

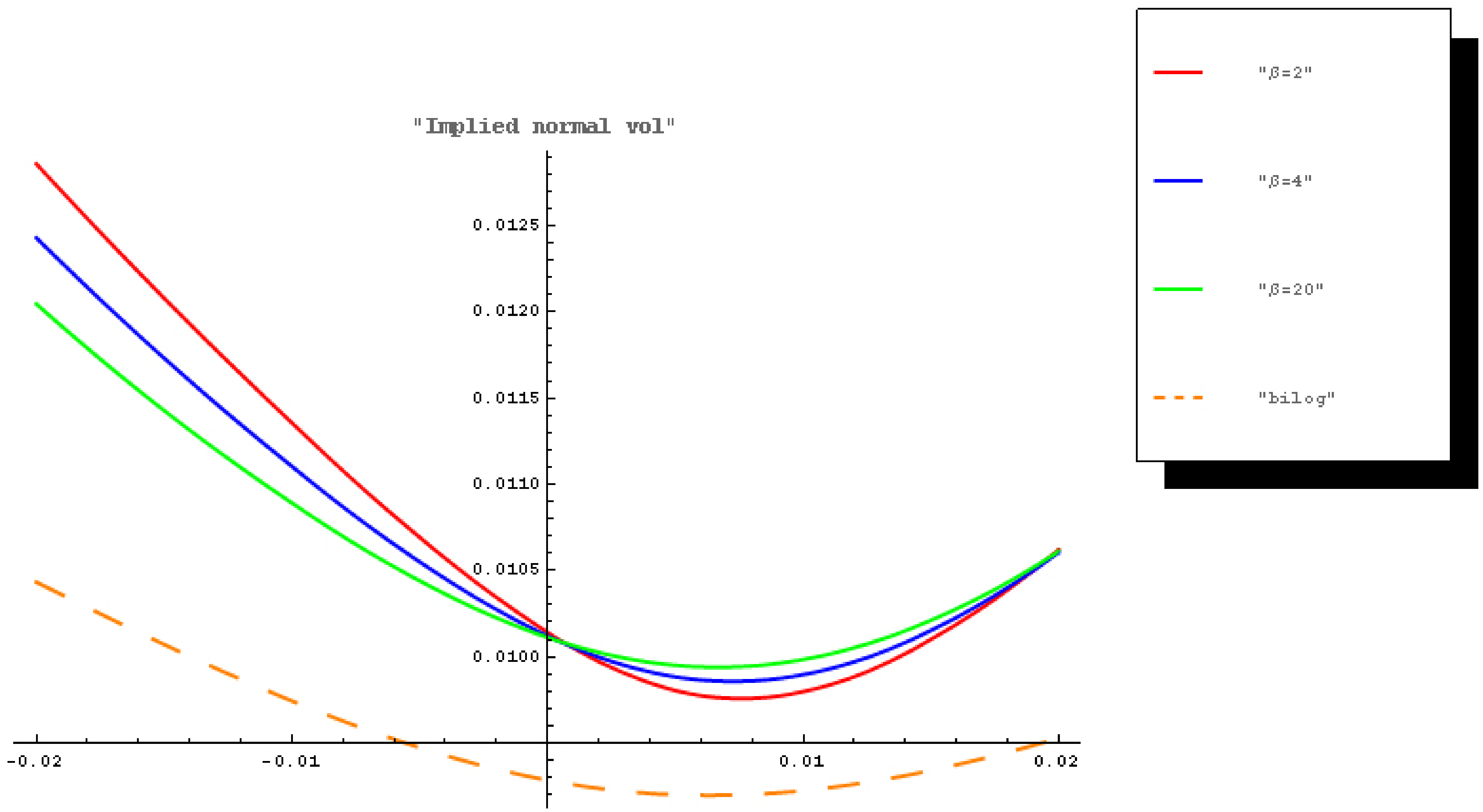
{62.937, 

}

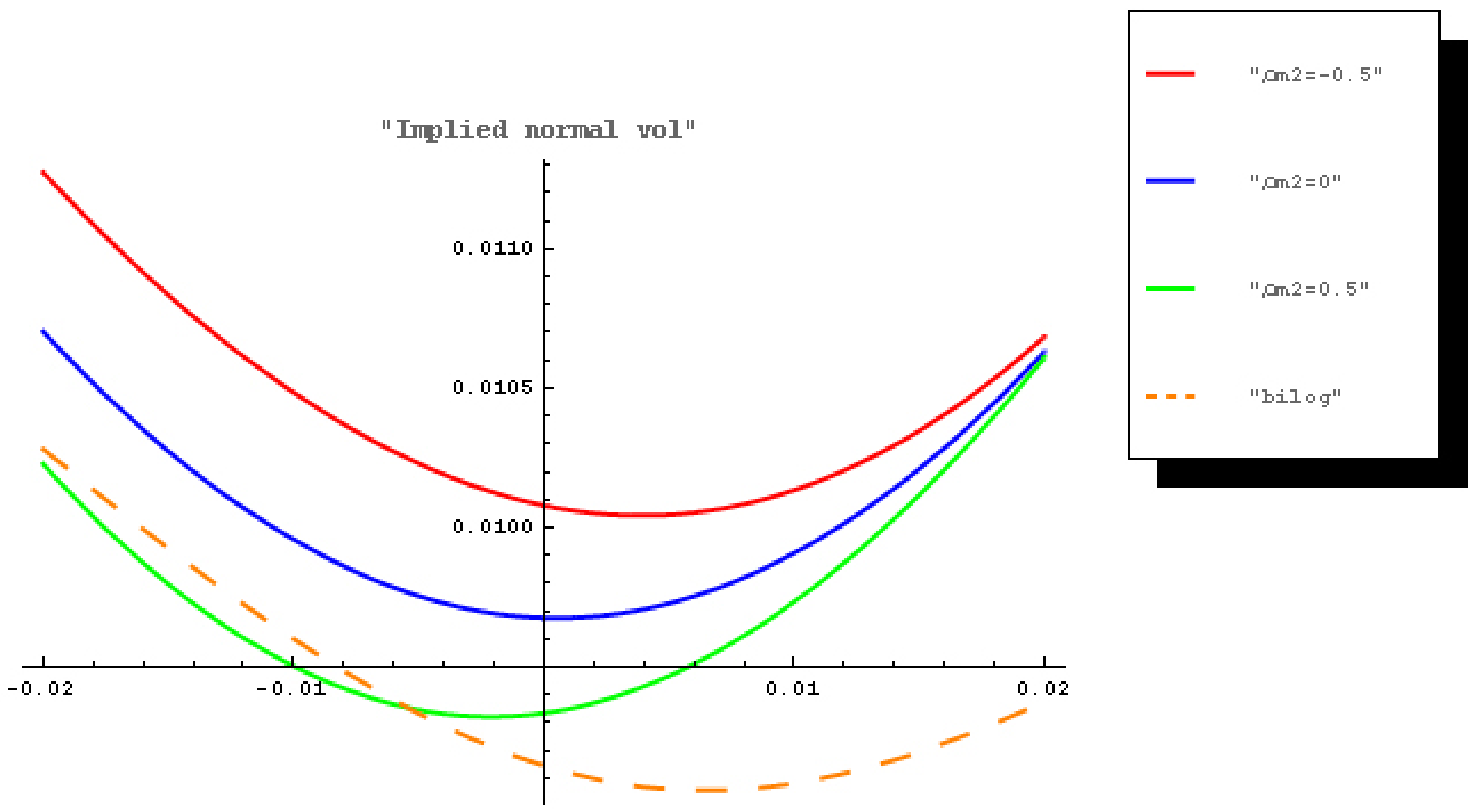
Integrand for the Bidim Inverse Fourier small maturity

Out[6]= {61.969,  }

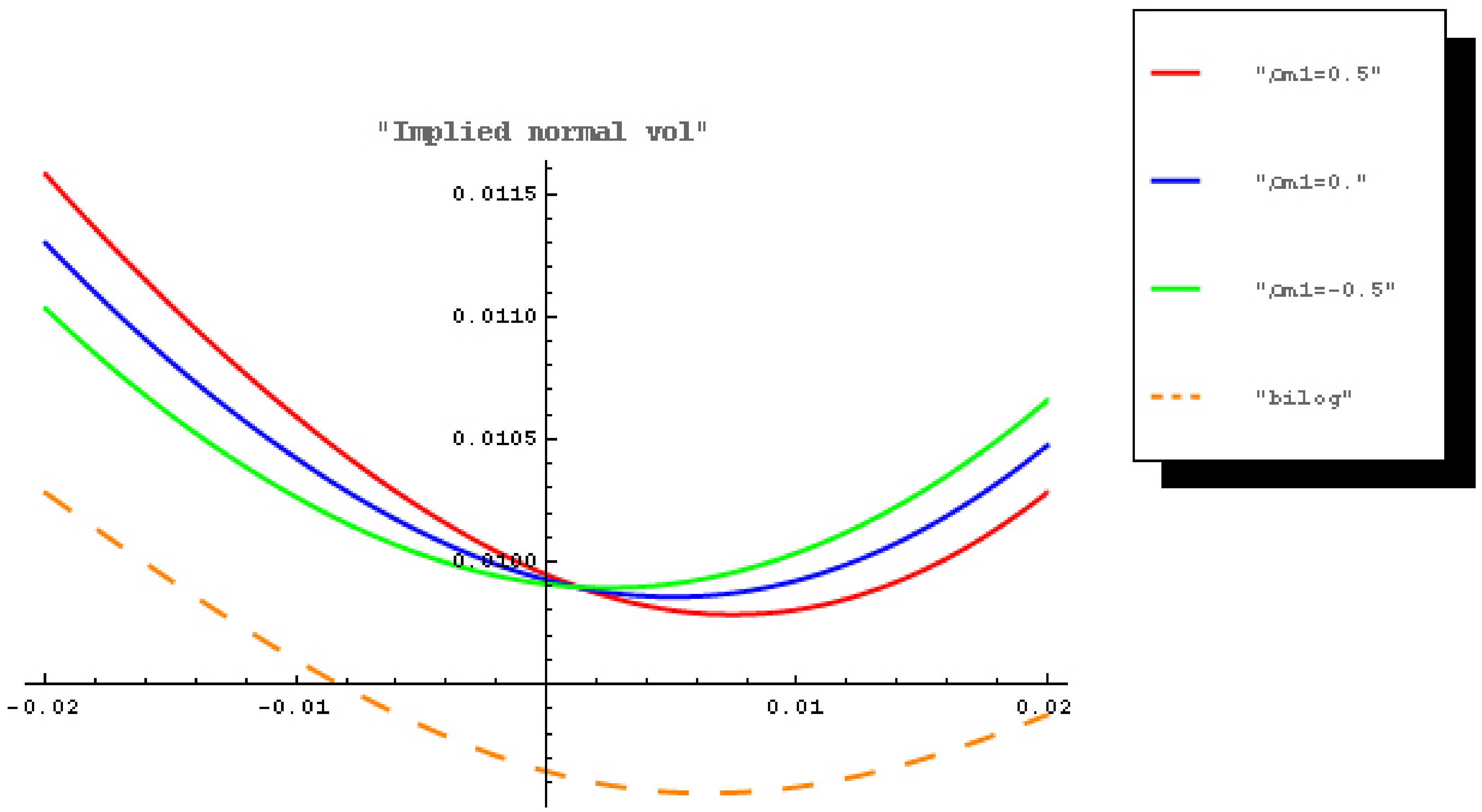
Global Convexity at work



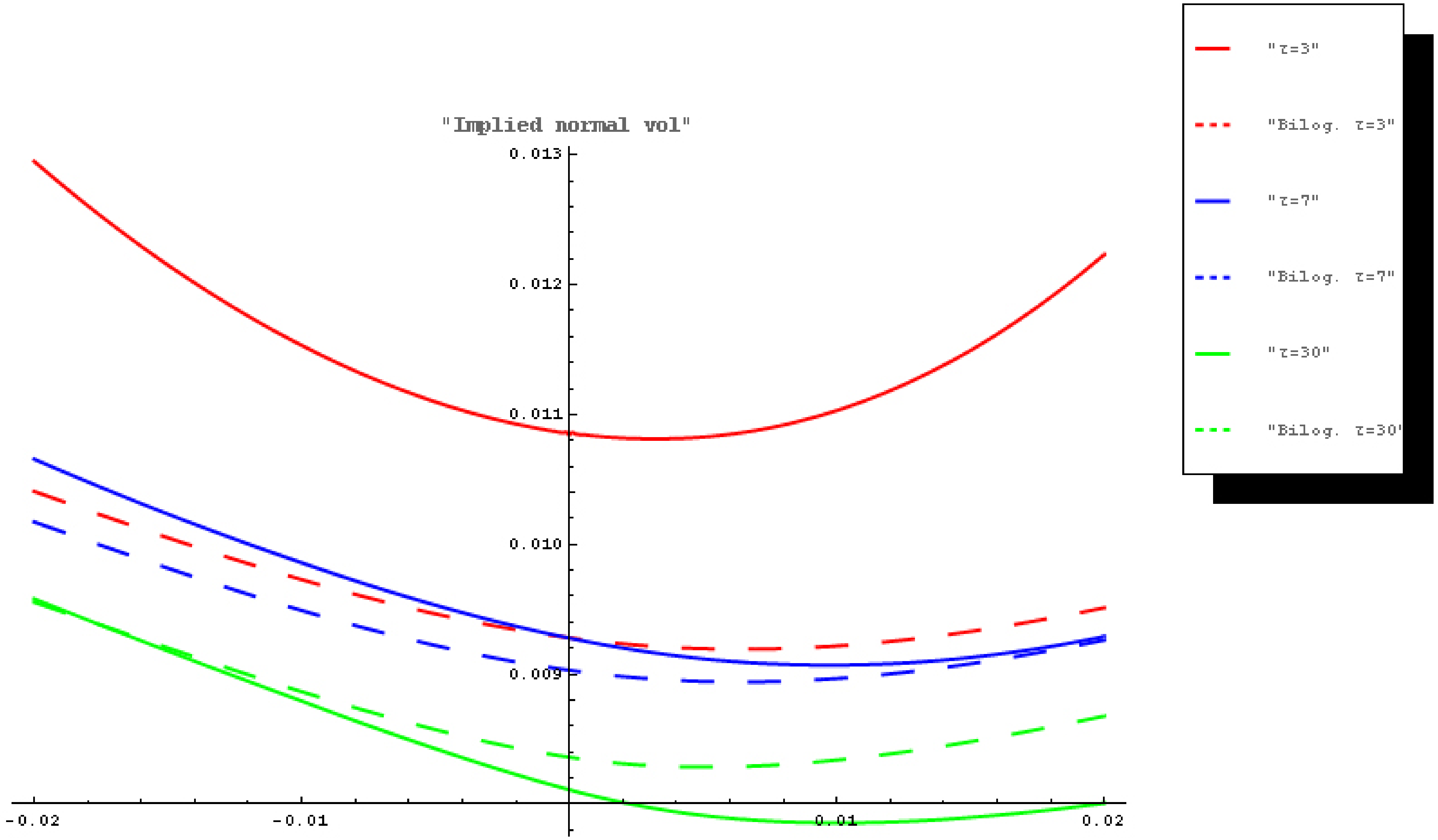
Cross Mean reversion at work (1)



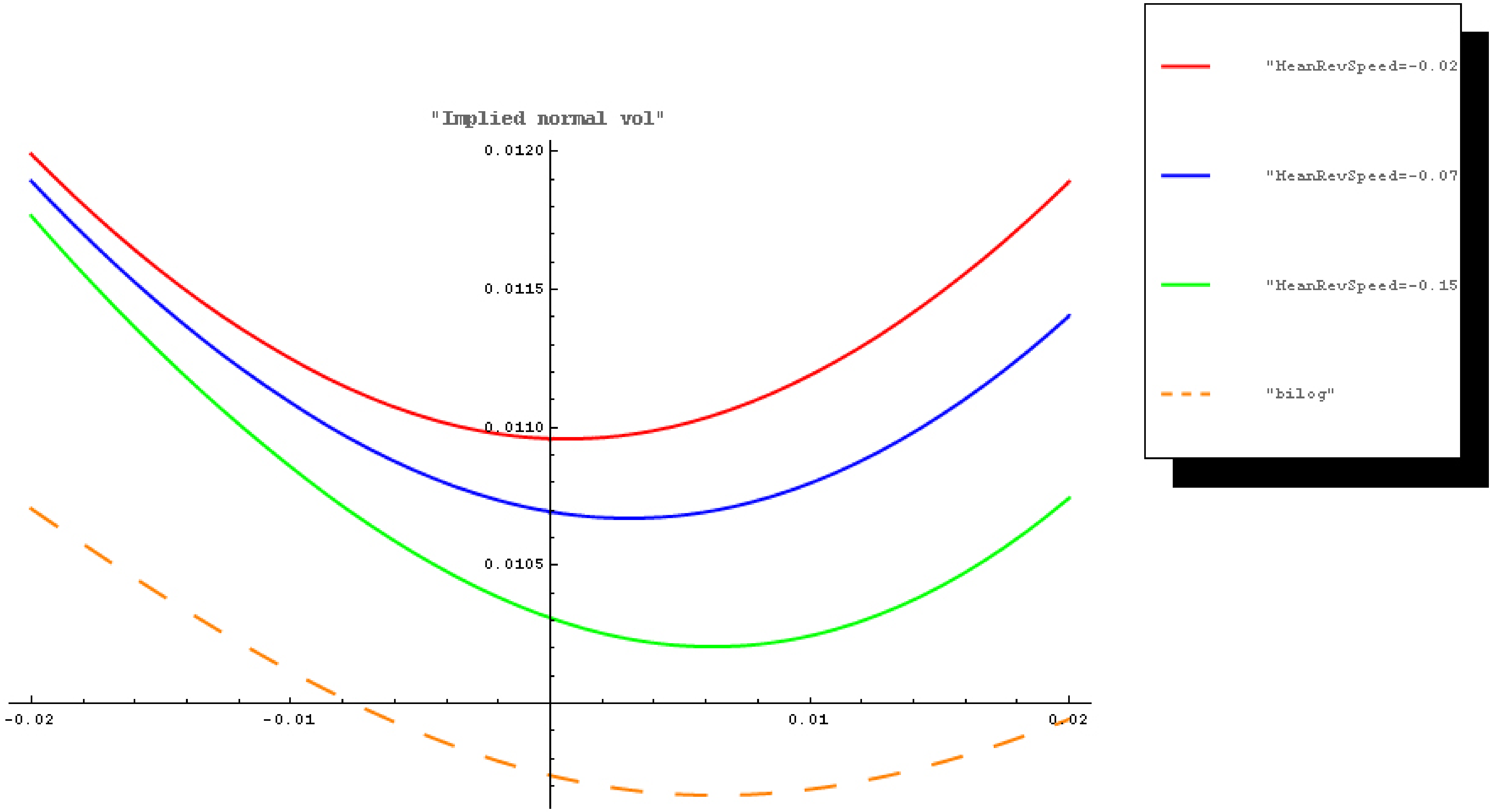
Cross Mean reversion at work (2)



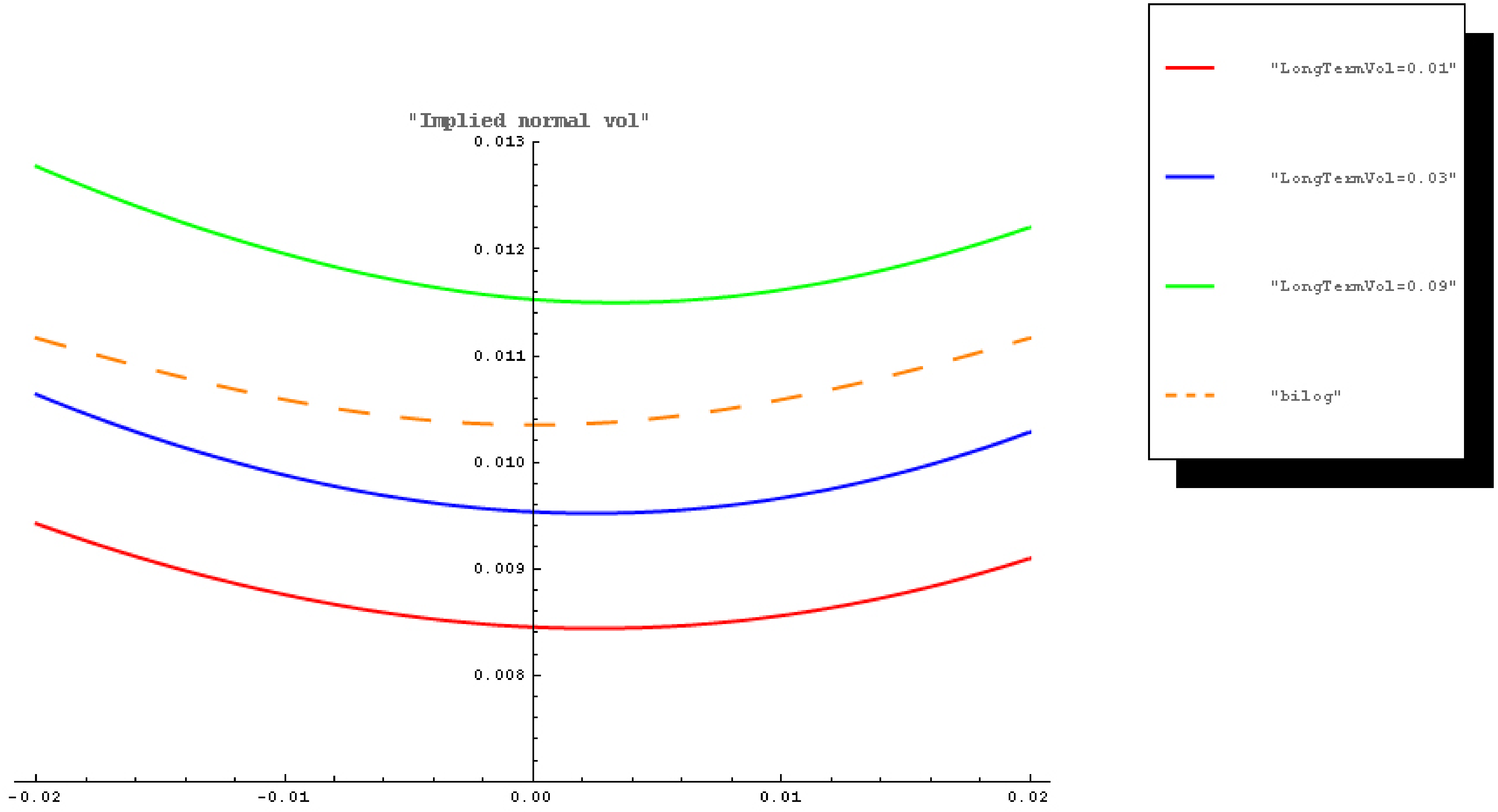
Aging at work



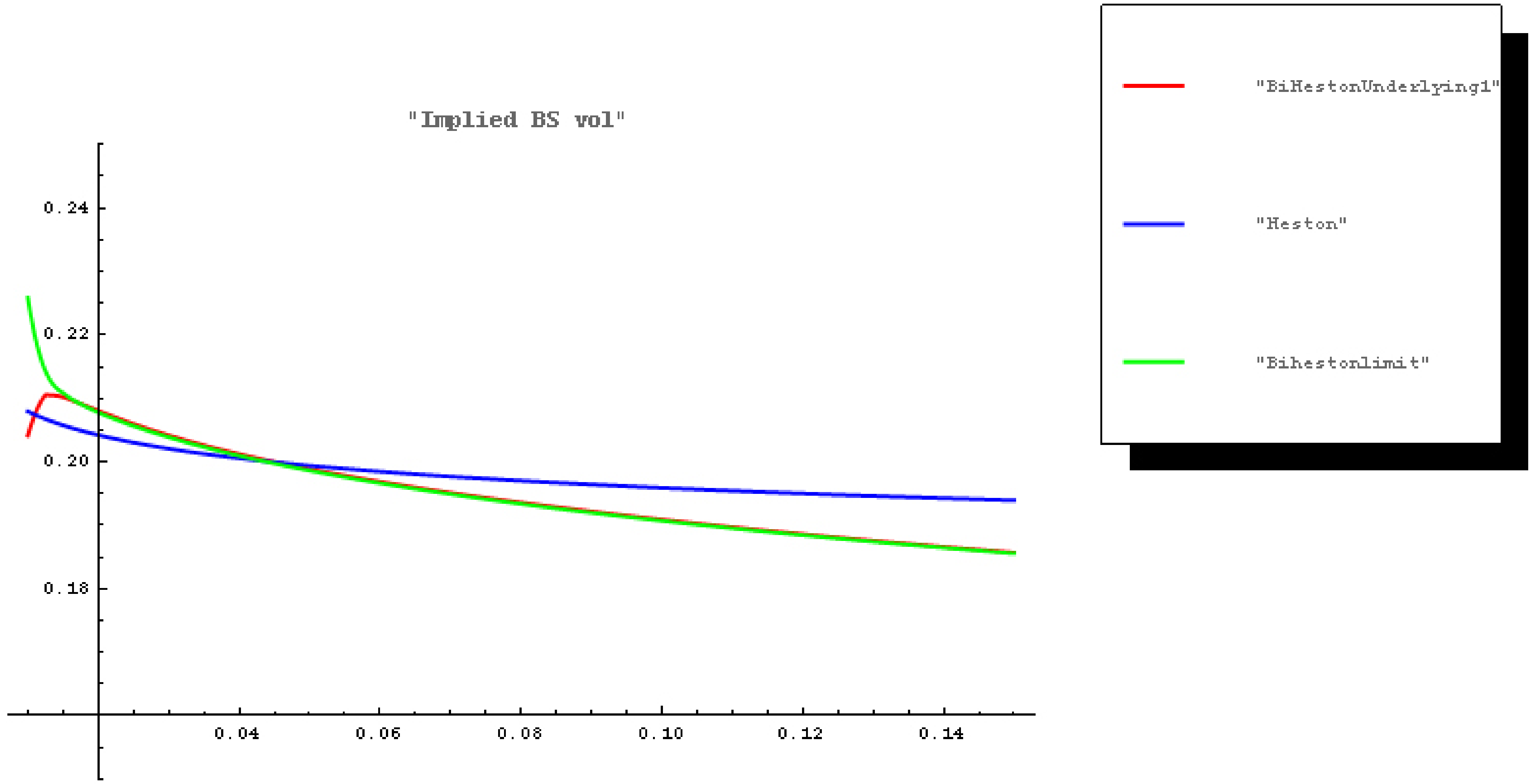
Mean Reversion at work



Long term Variance at work

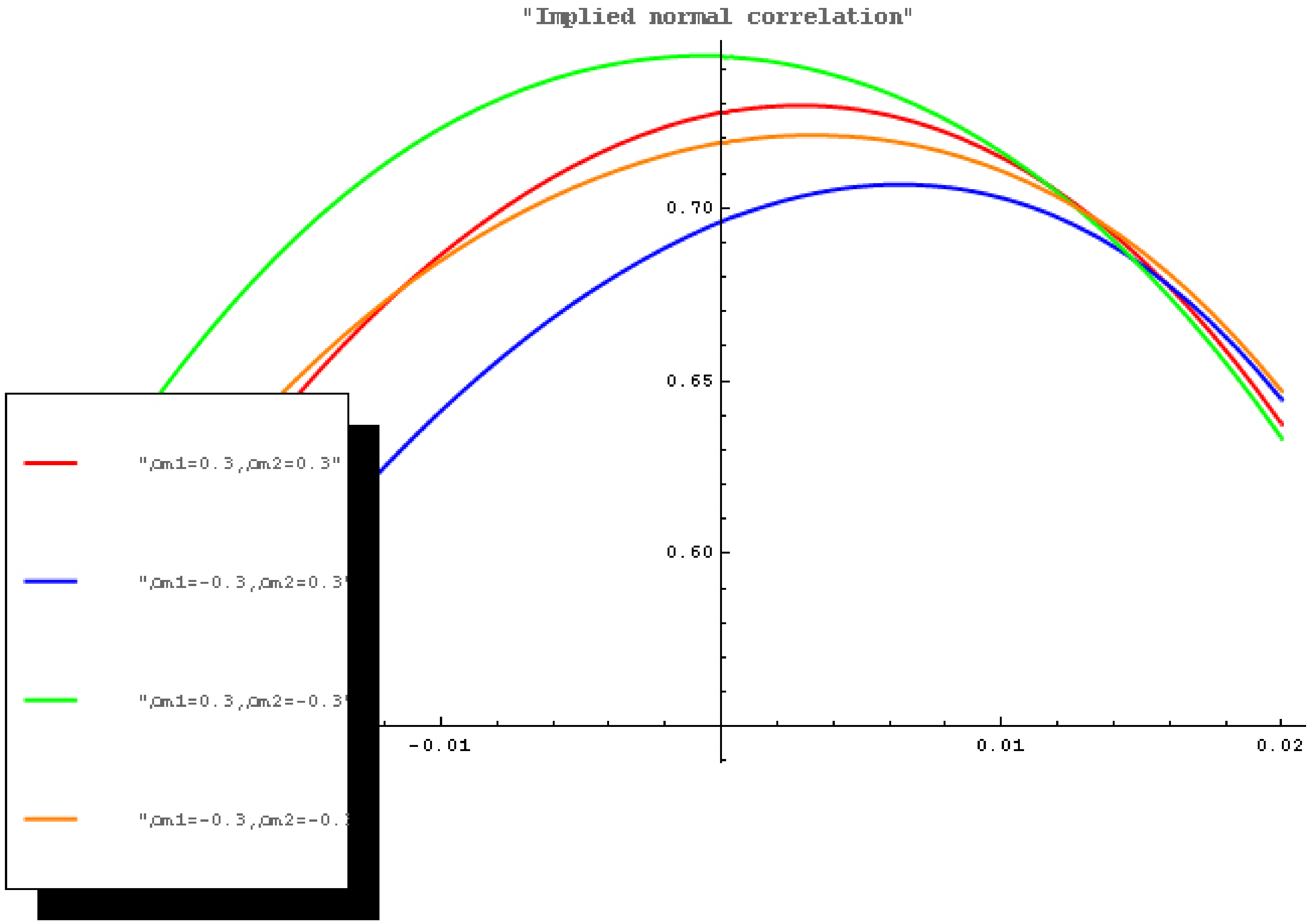


Degenerescence, Convergence toward the underlying (S1 = 0.05, S2 = 0.0001)



Correlation Smile slope business

Normal Vol ATM = Heston Implied Vol * S



Volatility Leverage effect in Heston

Heston Model :

$$\frac{dS_t}{S_t} = \sqrt{\Sigma_t}\, dZ_t\,,\ d\Sigma_t = \kappa\,(\Sigma_\infty - \Sigma_t)\, dt + \nu \sqrt{\Sigma_t}\, dW_t\,,\quad \langle dZ_t\,,\ dW_t \rangle = \rho\, dt$$

Leverage effect

$$\langle dS_t\,,\ d\Sigma_t \rangle = \nu\, S_t\, \Sigma_t\, \rho\, dt$$

Correlation Leverage effect in BiHeston

$$\frac{dS_t}{S_t} = \sqrt{\Sigma_t}\, dZ_t\,,\ d\Sigma_t = [M\,(\Sigma_t - \Sigma_\infty)]_S\, dt + \left[\sqrt{\Sigma_t}\, dW_t\, Q\right]_S\,,\quad \langle dZ_t\,,\ dW_t \rangle = R$$

$$\text{let } \rho^{12} = \frac{\Sigma^{12}}{\sqrt{\Sigma^{11}\,\Sigma^{22}}}$$

$$\left\langle \frac{dS^1{}_t}{S^1{}_t}\,,\ d\rho^{12}{}_t \right\rangle = \sqrt{\frac{\Sigma^{11}{}_t}{\Sigma^{22}{}_t}}\ \left(1 - \left(\rho^{12}{}_t\right)^2\right) \mathrm{Trace}[R_1\, Q]\, dt$$

$$\left\langle d\left(S^1{}_t - S^2{}_t\right),\ d\rho^{12}{}_t \right\rangle = \sqrt{\frac{\Sigma^{11}{}_t}{\Sigma^{22}{}_t}}\ \left(1 - \rho^{12}{}_t\right)\ \left(S^1{}_t\, \mathrm{Trace}[R_1\, Q] - S^2{}_t\, \mathrm{Trace}[R_2\, Q]\right) dt$$

So in a stationary hypothesis the Correlation Leverage effect is linked to the averaging of $\left(1 - \rho^{12}{}_t\right)$, i.e. controlled by the the cross mean reversion coefficients

Derivation of Option Formulas

1) Feynman Kac

the security price verifies :

$$\begin{cases} -\frac{\partial p}{\partial t} = L\,p \\ p[0] = (S_1 e^{Y_1} - S_2 e^{Y_2} - K)^+ \end{cases}$$

$$\text{where } L = \mathrm{Tr}\left[(\Omega\,\Omega^* + M\,\Sigma + \Sigma\,M)\,D + \frac{\Sigma\,D\,Q^*\,Q\,D}{2} \right] + \frac{1}{2} \nabla_Y \Sigma\ \nabla_Y{}^* + \mathrm{Tr}[D\,Q^*\,\rho\,\nabla_Y\,\Sigma] - \frac{1}{2}\,\mathrm{Vec}[\Sigma_{ii}]\,\nabla_Y$$

$$\text{where } D_{ij} \equiv \frac{\partial}{\partial \Sigma_{ij}}$$

we can assume for non explosion purposes (see Bru 1987)

$$Q\,Q^* = \frac{-2\,M\,\Sigma\mathrm{inf}}{\beta^2}$$

NATIXIS

Derivation of Option Formulas (2)

2) Fourier Transform ---> Riccati

$$E\left[\left(S_1 e^{Y_{1,T}} - S_2 e^{Y_{2,T}} - K\right)^+\right] = \frac{S_1 S_2}{(2\pi)^2} \int_{i\gamma-\infty}^{i\gamma+\infty} e^{-2 i (Y_1 \gamma_1 + Y_2 \gamma_2)} q[-i\gamma_1, -i\gamma_2]\, d\gamma_1\, d\gamma_2$$

$$q[\gamma_1, \gamma_2] == E[\exp[<\gamma, Y>]] == \exp\left[\mathrm{Tr}[A.\Sigma] + \gamma^T.Y + c\right]$$

The laplace transform of the probability transition is affine (Duffie and al) with coefficients A and c

we can show (see Fonseca 2007)

$$\begin{cases} \frac{\partial c[\tau]}{\partial \tau} = M\theta A[\tau] \\ \frac{\partial A[\tau]}{\partial \tau} = \frac{Q Q^*}{2} A[\tau]^2 + A[\tau] M + M^* A[\tau] + \gamma \rho^* Q A[\tau] + A[\tau] Q^* \rho \gamma^* + \frac{\gamma \gamma^*}{2} \end{cases}$$

Derivation of Option Formulas (3)

3) Solution of the riccati : General computation of A

We want to solve : $dA = (A\,a2\,A + a1\,A + A\,a1s + a0)\,dt$

where

let $A[\tau] = F^{-1} G$
we plug and ..

it is sufficient to have

$(G\,a1s + F\,a0)\,dt = dG$
$(G\,a2 + F\,a1)\,dt + dF = 0$

then $dF = (-F\,a1 - G\,a2)\,dt$ et $dG = (G\,a1s + F\,a0)\,dt$

therefore $d(F, G) = (F, G) . \begin{pmatrix} -a1 & a0 \\ -a2 & a1s \end{pmatrix} dt$

General computation of c

we show that

$$c = 2\,\mathrm{Tr}\left[\left(\mathrm{Log}[F] + \left(\frac{M^* + \gamma R Q}{2}\right)\tau\right)(M\,\Sigma\mathrm{inf} + \Sigma\mathrm{inf}\,M)\,(Q^* Q)^{-1}\right]$$

Fourier transform of the payoff $(\alpha\, S_1{}^a - \beta\, S_2{}^b - K)^+$ (Vanilla option):

x2 > 0 and Im[k1] > a, Im[k2] > 0

$$\mathrm{FourierTransform}\left[\left(\alpha\, e^{a\,x1} - \beta\, e^{b\,x2} - K\right) 1_{x2>0}\right] ==$$

$$-\frac{\mathrm{i}\, a\, \left(\frac{\beta}{\alpha}\right)^{\frac{\mathrm{i}\,k1}{a}}}{(a + \mathrm{i}\,k1)\, k1\, b} \left(\beta \left(\frac{\mathrm{Hypergeometric2F1}\left[-\frac{\mathrm{i}\,k1}{a},\, -1 - \frac{\mathrm{i}\,k1}{a} - \frac{\mathrm{i}\,k2}{b},\, -\frac{\mathrm{i}\,k1}{a} - \frac{\mathrm{i}\,k2}{b},\, -\frac{e^{-b\,x2}\,K}{\beta}\right]}{\left(1 + \frac{\mathrm{i}\,k1}{a} + \frac{\mathrm{i}\,k2}{b}\right)}\right) + K \left(\frac{\mathrm{Hypergeometric2F1}\left[-\frac{\mathrm{i}\,k1}{a},\, -\frac{\mathrm{i}\,k1}{a} - \frac{\mathrm{i}\,k2}{b},\, 1 - \frac{\mathrm{i}\,k1}{a} - \frac{\mathrm{i}\,k2}{b},\, -\frac{e^{-b\,x2}\,K}{\beta}\right]}{\left(\frac{\mathrm{i}\,k1}{a} + \frac{\mathrm{i}\,k2}{b}\right)}\right)\right)$$

x2 < 0 and Im[k1] > a, Im[k2] < 0

$$\mathrm{FourierTransform}\left[\left(\alpha\, e^{a\,x1} - \beta\, e^{b\,x2} - K\right) 1_{x2<0}\right] ==$$

$$-\frac{1}{(a + \mathrm{i}\,k1)\, k1\, k2\, (-\mathrm{i}\,b + k2)}\, a \left(\frac{K}{\alpha}\right)^{\frac{\mathrm{i}\,k1}{a}} \left(k2\, \beta \left(\frac{\left(\frac{\beta}{K}\right)^{-1-\frac{\mathrm{i}\,k2}{b}} \mathrm{Gamma}\left[-1 - \frac{\mathrm{i}\,k1}{a} - \frac{\mathrm{i}\,k2}{b}\right] \mathrm{Gamma}\left[2 + \frac{\mathrm{i}\,k2}{b}\right]}{\mathrm{Gamma}\left[-\frac{\mathrm{i}\,k1}{a}\right]} + \frac{\left(\frac{\beta}{K}\right)^{\frac{\mathrm{i}\,k1}{a}} \left(1 + \frac{\mathrm{i}\,k2}{b}\right) \mathrm{Hypergeometric2F1}\left[-\frac{\mathrm{i}\,k1}{a},\, -1 - \frac{\mathrm{i}\,k1}{a} - \frac{\mathrm{i}\,k2}{b},\, -\frac{\mathrm{i}\,k1}{a} - \frac{\mathrm{i}\,k2}{b},\, -\frac{K}{\beta}\right]}{\left(1 + \frac{\mathrm{i}\,k1}{a} + \frac{\mathrm{i}\,k2}{b}\right)}\right) + K\, (-\mathrm{i}\,b + k2) \left(\frac{\left(\frac{\beta}{K}\right)^{-\frac{\mathrm{i}\,k2}{b}} \mathrm{Gamma}\left[-\frac{\mathrm{i}\,k1}{a} - \frac{\mathrm{i}\,k2}{b}\right] \mathrm{Gamma}\left[1 + \frac{\mathrm{i}\,k2}{b}\right]}{\mathrm{Gamma}\left[-\frac{\mathrm{i}\,k1}{a}\right]} + \frac{\left(\frac{\beta}{K}\right)^{\frac{\mathrm{i}\,k1}{a}} \left(\frac{\mathrm{i}\,k2}{b}\right) \mathrm{Hypergeometric2F1}\left[-\frac{\mathrm{i}\,k1}{a},\, -\frac{\mathrm{i}\,k1}{a} - \frac{\mathrm{i}\,k2}{b},\, 1 - \frac{\mathrm{i}\,k1}{a} - \frac{\mathrm{i}\,k2}{b},\, -\frac{K}{\beta}\right]}{\left(\frac{\mathrm{i}\,k1}{a} + \frac{\mathrm{i}\,k2}{b}\right)}\right)\right)$$

Special case : K = 0

$$\mathrm{FourierTransform}\left[\left(\alpha\, e^{a\,x1} - \beta\, e^{b\,x2} - K\right) 1_{x2<0}\right] == \frac{a^2\, e^{\left(b + \frac{\mathrm{i}\,b\,k1}{a} + \mathrm{i}\,k2\right) x2}\, \beta\, \left(\frac{\beta}{\alpha}\right)^{\frac{\mathrm{i}\,k1}{a}}}{(\mathrm{i}\,a - k1)\, k1\, (a\,b + \mathrm{i}\,b\,k1 + \mathrm{i}\,a\,k2)}$$

$$\mathrm{FourierTransform}\left[\left(\alpha\, e^{a\,x1} - \beta\, e^{b\,x2} - K\right) 1_{x2>0}\right] == \frac{-a^2\, e^{\left(b + \frac{\mathrm{i}\,b\,k1}{a} + \mathrm{i}\,k2\right) x2}\, \beta\, \left(\frac{\beta}{\alpha}\right)^{\frac{\mathrm{i}\,k1}{a}}}{(\mathrm{i}\,a - k1)\, k1\, (a\,b + \mathrm{i}\,b\,k1 + \mathrm{i}\,a\,k2)}$$

Fourier transform of the payoff $\left(\mathrm{Max}\left[\alpha S_1{}^{a}, \beta S_2{}^{b}\right] - K\right)^{+}$ (Max Option):

b x2- a x1 > Log[α] - Log[β] and $\mathrm{Re}[a] < \mathrm{Im}\left[\frac{b\,k1}{a}\right] + \mathrm{Im}[k2]\ \&\&\ \mathrm{Im}\left[\frac{b\,k1}{a} + k2\right] > 0\ \&\&\ \mathrm{Im}[k1] < 0$

$$\mathrm{FourierTransform}\left[\left(\mathrm{Max}\left[\alpha\, S_1{}^{a},\ \beta\, S_2{}^{b}\right] - K\right)^{+} 1_{b\,x2>a\,x1+\mathrm{Log}[\alpha]-\mathrm{Log}[\beta]}\right] == \frac{a\,K^{i\left(\frac{k1}{a}+\frac{k2}{b}\right)}\,\alpha^{-\frac{i\,k1}{a}}\,\beta^{-\frac{a+i\,k2}{b}}\left(i\,K^{a/b}\,(b\,k1 + a\,k2)\,\beta^{2} - K\,(a^{2} + i\,b\,k1 + i\,a\,k2)\,\beta^{a/b}\right)}{k1\,(a^{2} + i\,b\,k1 + i\,a\,k2)\,(b\,k1 + a\,k2)}$$

b x2- a x1 < Log[α] - Log[β] and $\mathrm{Re}[a] < \mathrm{Im}[k1] + \mathrm{Im}\left[\frac{a\,k2}{b}\right]\ \&\&\ \mathrm{Im}\left[k1 + \frac{a\,k2}{b}\right] > 0\ \&\&\ \mathrm{Im}[k2] < 0$

$$\mathrm{FourierTransform}\left[\left(\mathrm{Max}\left[\alpha\, S_1{}^{a},\ \beta\, S_2{}^{b}\right] - K\right)^{+} 1_{b\,x2<a\,x1+\mathrm{Log}[\alpha]-\mathrm{Log}[\beta]}\right] == -\frac{b\,K^{1+\frac{i\,k1}{a}+\frac{i\,k2}{b}}\left(a\,(b - i\,k2\,(-1+\alpha)) - i\,b\,k1\,(-1+\alpha)\right)\alpha^{-\frac{i\,k1}{a}}\,\beta^{-\frac{i\,k2}{b}}}{(i\,b\,k1 + a\,(b + i\,k2))\,k2\,(b\,k1 + a\,k2)}$$

Fourier transform of the payoff Underlying Vanilla Option: $(\alpha\, S_1^{a} - K)^{+}$

$x2 > 0$ and $Im[k1] > a$, $Im[k2] > 0$

$$\text{FourierTransform}\left[\left(\alpha\, e^{a\,x1} - K\right) 1_{x2\geq 0}\right] == \frac{a\,K\,\left(\frac{K}{\alpha}\right)^{\frac{i\,k1}{a}}}{a\,k1\,k2 + i\,k1^2\,k2}$$

$x2 < 0$ and $Im[k1] > a$, $Im[k2] < 0$

$$\text{FourierTransform}\left[\left(\alpha\, e^{a\,x1} - K\right) 1_{x2<0}\right] == \frac{-a\,K\,\left(\frac{K}{\alpha}\right)^{\frac{i\,k1}{a}}}{a\,k1\,k2 + i\,k1^2\,k2}$$

The Heston Case :

$$\text{FourierTransform}\left[\left(e^{x} - K\right)\right] == \frac{-K\,(K)^{i\,k1}}{k + i\,k^2}$$

What Remains to be done before the first roll out of the ARM add-ins

1) Robustification of the vanilla underlying option for calibration purposes

2) Calibration by hand to the CMS 2 - 10 market

What will to be done as the second step

1) Digital that pays float

2) double conditions options

What will to be done as the Third step

1) Other methods to compute vanilla options to speed up the calibration (Cumulant expansion or Singleton - Umantsev type of approximation)

2) 3 Assets cases

3) Super heston underlying (sum of 2 wishart volatility to replicate a term structure)