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Trading Potential in Limits Optimisation

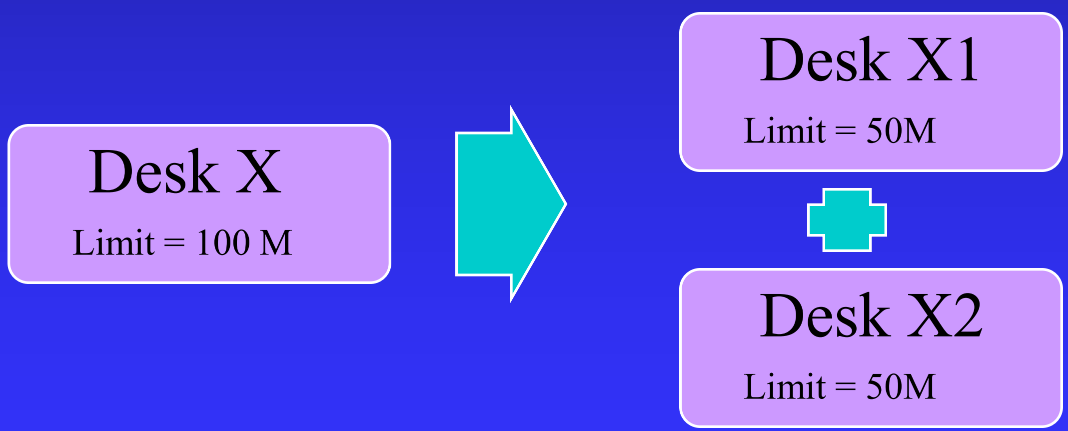
Olivier Croissant
Director, Financial Engineering
Renaissance Software, Inc.

Overview

- A new notion **The Trading Potential** is defined
- It complements the quantitative information of risk
- We will see how it can be used to analyse and optimise systems of trading limits

Loss of Trading Potential : Motivation

- In a risk-neutral world, all strategies have the same expectation. Outside a risk-neutral world, all strategies have an expectation limited by their risk limit
- Within a risk limit, we cannot optimise the profit
- Increasing the number of limits increases the level of determinism and statistically decreases the effective risk



But In case of statistical independence, Computed Risk= 71 M

Something has been lost ...

Limits Structure

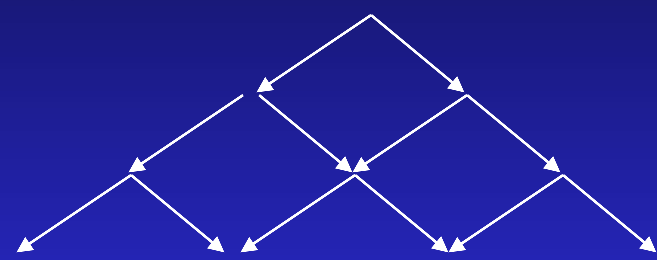
- Limits don't add up but can generally be presented in hierarchical parallel trees:

Trading Floor (London,…)

Department

Desk

Portfolios



Catastrophic Risks (scenario)

VaR Risks

Delta Risks

Counterparty Risk

By Product

By Market

By Counterparty

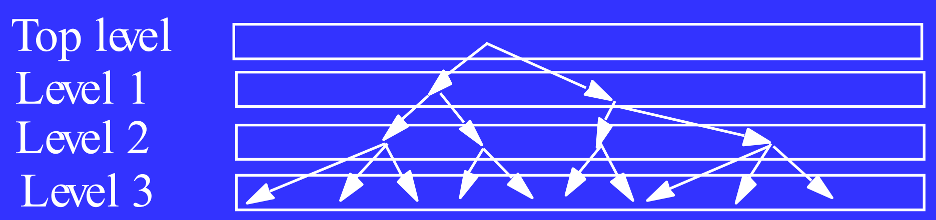
- The finer the tree is (i.e. the more levels it has), the more predictable the portfolio is, and the less trading opportunities it will offer. This will be measured by a **Trading Potential Loss**

The Duality Trading Potential / Risk

- Every pricing is done in the Arbitrage free approach
  - Every strategy on the market has an expectation equal to the application of the risk free rate
  - We can not expect to be able to optimise any position or any strategy at constant risk by optimising the profit.
- The profit comes from the exploitation of opportunities.
  - The number and quality of opportunities come from the diversity of the possible positions on the markets.
  - We want to optimise the diversity of the possible positions on the market, leaving the computation of the profit outside the model. *The expected profit is proportional to the number of possible strategies on the market*.
  - We construct a measure called "Trading Potential " that we oppose to the Risk, in order to do optimisations

Properties of the Trading Potential Loss

- The trading potential increases, when we suppress a limit
- Additivity of the Trading Potential loss across levels



Potential loss [L1+L2+L3]=Potential Loss[L1]+Potential Loss[L2]+Potential Loss[L3]

Trading Potential Loss (Definition)

- It measures the number of positions that can not be taken inside the sub-portfolios due to the limit splits and the structure of these limits.
- It is computed as a difference between two values called trading potentials Π
- The problem is now to give a sensible definition of Π

$$\Lambda[\{L_1, L_2, \ldots, L_n\} \rightarrow \{L_1, L_2, \ldots, L_p\}]$$
$$= (\Pi[\{L_1, L_2, \ldots, L_n\}] - \Pi[\{L_1, L_2, \ldots, L_p\}])$$

Trading Potential (Heuristics)

- Trading Potential (desk) = Independent of the way you partition the desk into portfolios
- Increases with the number of dimensions of the risk calculation.
- Includes information concerning the interest of the dimension , from the trading point of view.
- A heuristic form is :
- $\mu[\text{dim}]$ represents the interest in trading along the dimension dim

$$\text{Trading Potential} = \sum_{\text{independant dim.}} \text{Max}[\Delta_{\text{dim}}] \times \mu(\text{dim})$$

Trading Potential : (conceptual definition)

- Computes the number of different portfolios we can make within a given set of trading limits
- Summing over all possible portfolios is challenging.
- The Limts R[p]<= L can of course be multidimensional
- The log insures the summation for independent portfolios,

$$\text{Trading Potential} = \text{Log}\left[\int_{R[p] \leq L} \mu(p) \right]$$

$$\text{Trading Potential}\left[\sum_i P_i\right] = \sum_i \text{Trading Potential}\ [P_i]$$

Example of Factors

- Historical Simulation
  - The set of shifts in PV if we apply historical shifts can be a risk measure if we take the absolute value or the positive part of it
  - Functions of these shifts can also be considered : Percentiles of the implied empiric distribution are also risk measures
- Catastrophic scenarios
  - Synthetic shifts and weighted compounding of them can give rise to risk measures.
- Sensitivities
  - The sensitivities to every factors can be used as risk mesures.
  - Any functions of the sensitivities can also be used . Example : Hedge Ratio on the long term future market.

Credit Risk Handling

- For every counterparty k , we add a variable which value =1 at t=0 and which has a probability to drop to ρ_k with a poissonian probability of $\lambda_k dt$ between t and t+dt.
- We use the “counterparty exchange rate paradigm “.
- We can make the independence assumption
- Two risk measures can be used :
  - a maximal loss measure
  - a differential loss measure
- The limits on these risk factors are quadratic and can be simply expressed as

$$Q[D_k] = \frac{D_k^2}{\lambda_k^2} \leq L_k^2 \qquad \text{or} \qquad Q[M_k] = D_k^2 \leq L_k^2$$

Linear Case : Introduction

- A set of determining factors
  - (zero rates , Fx, bond prices, ..)
- Let's assume a linear behaviour of the portfolios with respect to these factors.
- Specifying a portfolio is equivalent to specifying a vector of numbers called the deltas
- Limits of trading are given by sub-additive functions of these deltas
- An interesting case is when these limits are quadratic polynomial of these factors (limits on projected variance)
  - Risk Metrics Approach, Second order (gammas) handling

Linear Case with Quadratic limits

- One quadratic limit -> analytical formula
- N limits on independent set of factors -> analytical formula

$$\int_{X^* Q_V X \le L^2} dX_V = \frac{\pi^{\frac{\mathrm{Dim}[V]}{2}}}{\Gamma\left[\frac{\mathrm{Dim}[V]}{2} + 1\right]} \frac{L^{\mathrm{Dim}[V]}}{\sqrt{\mathrm{Det}[Q_V]}}$$

Application of the Analytical Case

- 2 dimensions, one quadratic risk :
  - It shows that, within the same limit of risk , two correlated instruments offer more trading potential than two uncorrelated instruments. This allows to do bigger deals , therefore to receive larger commissions.
- N dimensions , N independent quadratic limits
  - It shows that, the potential sums over independent portfolios. We can put the different signature risk into different portfolios with the accompanying limits without changing the trading potential.

$$\Pi = \log\left[\frac{\pi L^2}{\sqrt{\sigma_1 \sigma_2 (1 - \rho_{1,2})}}\right]$$

$$\Pi = \log\left[\pi^n \frac{L_1 L_2 .. L_n}{\lambda_1 \lambda_2 .. \lambda_n}\right]$$

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The Trad. Potential of a Set of Portfolios

- We can play on partial hedging in portfolios to hold a large position (to avoid artificial potential creation)
- This will allow us to compare different sets A and B of sub-portfolio definitions, provided that they respect a monotonic constraint

Large instrument

$$X = X - H1 + H1 - H2 + H2 - H3 + H3 - H4$$

$\in$ Port-1 $\in$ Port-2 $\in$ Port-3 $\in$ Port-4

Ex :

A = B + additional limits

Limits on the Set A => Limits on the set B

The potential Loss [A->B] is defined

Trading Potential on Multiple Portfolios
The Constraints

- 1) Potential sum over independent portfolios concerning independent activity (splitting of credit risk into counterparties):P[A+B]=P[A]+P[B]
  - Sum of dimensions (definition)
- 2) Potential log-sum over portfolios concerning the same set of factors : P[A+B]=Log[Exp[P[A]]+Exp[P[B]]]
  - Sum of allowed delta =>Sum of volume of configurations
- What about portfolios with dependency on a common subset of factors (Interest rates,..)?

Trading Potential on Multiple Portfolios Hints

- We have 2 portfolios A and B
- Let assume the limits are naturally split

$$\Pi[A] = \log\left[\int_{Q_a[x,y]<L} dx\,dy\right] = \log\left[\int_{Q_1[x]<L_1} dx \int_{Q_2[x]<L_2} dy\right]$$

$$\Pi[B] = \log\left[\int_{Q_b[y,z]<L} dy\,dz\right] = \log\left[\int_{Q_3[y]<L_3} dy \int_{Q_4[z]<L_4} dz\right]$$

Then the potential sums over independent markets:

$$\Pi[A+B] = \log\left[\int_{Q_1[x]<L_1} dx \left(\int_{Q_2[x]<L_2} dy + \int_{Q_3[y]<L_3} dy\right) \int_{Q_4[z]<L_4} dz\right]$$

But limits are not always split...

Trading Potential on Multiple Portfolios
Final Definition

- We define a potential which is naturally a sum over independent dimensions (log of a product).
- We define rules to compound the potential across sets of portfolios

$$\Pi[Q[x_1, x_2, \ldots, x_n], L] = \mathrm{Log}[\langle x_1\rangle\langle x_2\rangle\ldots\langle x_n\rangle]$$

where the mean square allowance per dimension is :

$$\langle x_i\rangle^2 = \frac{\displaystyle\int_{Q([x_1, x_2, \ldots, x_n] \le L)} (x_i - \bar{x}_i)^2 dx_1 \ldots dx_n}{\displaystyle\int_{Q([x_1, x_2, \ldots, x_n] \le L)} dx_1 \ldots dx_n}$$

in the linear case...

How to Compute the Trading Potential

- Outside the analytical case and/or beyond the quadratic hypothesis for the limits :
- Monte carlo calculation :
  - X_k uniformly generated in W
- We can adjust W, using quasi-random generators, to increase the efficiency of monte-carlo calculations.

$$\left[\int_{\substack{\left(\|X\|_{Q_j}\big|_V \le L_j,\ j \in \{1, \dots, p\}\right) \\ X \in W}} f[X_V]\,dX_V\right] = \frac{\sum_{\{X_k\}} \mathrm{IF}\left[\|X_k\|_{Q_j}\big|_V \le L_j,\ j \in \{1, \dots, p\},\ f[X_k]\right]}{\mathrm{Cardinal}[\{X_k\}]}\,\mathrm{Volume}[W]$$

Generalisation

- Similar Analysis can be obtained with gammas . In addition we can separate a trading potential for delta positions and a trading potential for gamma positions. This allow us to qualify a portfolio for directional trading or for delta neutral positions
- We can compare the trading potential associated with sensitivities or monte-carlo Risks with the trading potential associated with historical simulation. This notion bring new insights in the holes potentially created with the historical approach.

Understanding the Profitability

- The profitability ratio

$$\frac{\text{Profit}}{\text{Risk}} = \frac{\text{Profit}}{\text{Trading Potential}} \times \frac{\text{Trading Potential}}{\text{Risk}}$$

Efficiency in portfolio management

Efficiency of the financial structure

Conclusion

- The trading potential deserves a close attention when designing or modifying the limits of a system of portfolios
- It can be adapted to the context in the same way than the risk measure
- It requires the same complexity of computation as a risk measure
- It has a limited analogy with an "entropy" measure and the optimisation of it resembles a maximum of likehood computation. Such computation should be considered as a profit maximisation.
- Criticism and comments welcomed...

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