# Optimizations in VaR Computations

Olivier Croissant
Director, Financial Engineering
Renaissance Software, Inc.

Miami, Oct 7th 1997

# Overview

- Risk calculation process
  - **Optimizing a MC VaR computation**
  - **Optimizing a Historical VaR computation**
  - **Optimizing a Delta-Gamma VaR computation**
- Integration
- Conclusion

re
nais
sance

# Characteristics of Operational Risk Calculations

- 1- Include in the handled risk dimensions, all the variable determinant parameters of the PV
- 2- The arbitragefreeness of the models is assumed, but it will be destroyed most of the time by the handling
- 3- Perform for all necessary computations within the time frame compatible with the targeted risk management.

**Factors** → **F(Factors)**

t

Pricing : static

Risk : Dynamic

Convention : - no theta effect

- theta effect

Step 1 — Fulfill a minimal 1,2,3

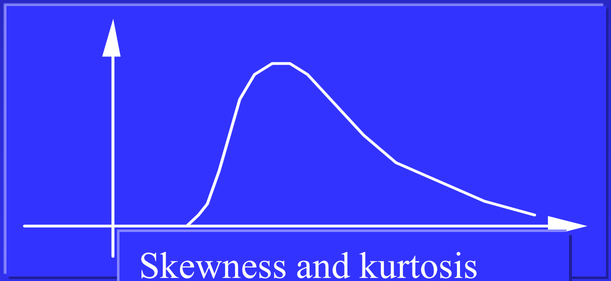
Step 2 — Improve the performances

# Risk Methodologies

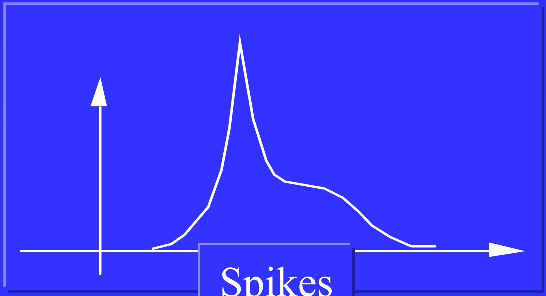
- Probabilistic methods
  - **Monte Carlo calculation**
  - **Historical simulation -> no covariance matrix**
  - **First order sensitivities based risk computations**
- Non probabilistic methods
  - **Worst case analysis**
  - **Preferred (customized) scenarios based analysis**
- Improved Methods
  - **Randomized quasi Monte-Carlo : improves the convergence**
  - **Enhanced historical methods : increases the number of points**
  - **Second order sensitivities based risk computations**

# Difficulties Associated with Real Portfolios Distributions

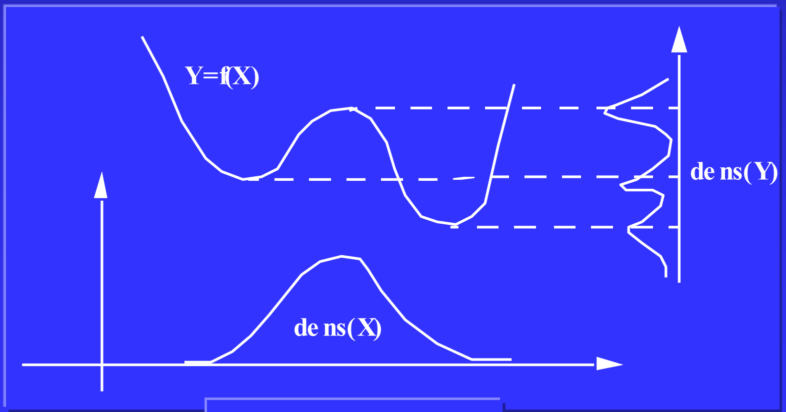
- Skewness and kurtosis
  - **Risk of buying and option # Risk of selling an option**
- Spikes in the density
  - **High gamma without corresponding delta : infinite density**
- Multi Modality
  - **Non Local effects (pile of option spreads)**



Skewness and kurtosis



Spikes




multi-modality

# Optimization of VaR Computation

- Monte Carlo VaR MC
- Historical VaR H
- Sensitivities Based VaR SB

# Link Between MC Risk and MC Price

- MC price = Expectation of P&L[X]
- MC risk = Expectation of $(\text{P\&L}[X])^n$ -> Distribution

MC Risk at the level of 1 %

15 001 samples

$\text{P\&L}[X_1], \text{P\&L}[X_2], \ldots \text{P\&L}[X_n]$

M-R= 151 th

M= 7501 th

MC

# Do you believe in your risk calculation ?

- Observe a convergence ? When it will happen ?
- Expect a convergence. Because of a theoretical result.

Two types of theoretical results :

Probabilistic : Variance[ S - I ] < epsilon [ N ]

Absolute : | S - I | < epsilon [ N ]

I : Real value
S : Estimator

| | | | | | |
|---|---|---|---|---|---|
| N= | 1000 | 2000 | 3000 | 5000 | .... |
| S= | 45.3 | 48.3 | 47.3 | 52.3 | |

re nais sance

MC

# Two Approaches

| S - I | < epsilon [ N ]

## Old Style

Std Dev [ 10 000 sample ] = 1%

Very often practical
good convergence

No more than 12 dimensions

10 000 samples

The same pseudo-generator
for all cases

## New Style

MC Discrepancy = 0.003

Structured RQMC is performed
on a class of integrand

25 dimensions among 360

10 000 samples

The structure of the sample
set is adapted to the effective
dimensionality

Information

re
nais
sance

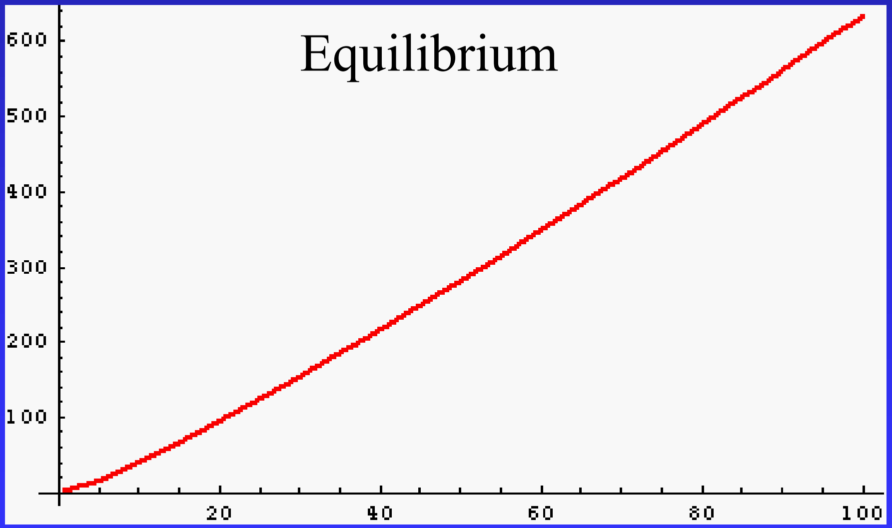
MC

# High dimension MC calculation

- 20 - 2000 dimensions MC : only a probabilistic answer
  - **Some dimensions can be overlooked : hidden risk**
- Low Discrepancies of Quasi-Monte-Carlo are no help.

Discrepancy for QMC → $\dfrac{(\mathrm{Log}[N])^{\mathbf{Dim}}}{N} < \dfrac{1}{\sqrt{N}}$ ← Discrepancy for MC

Power of 10



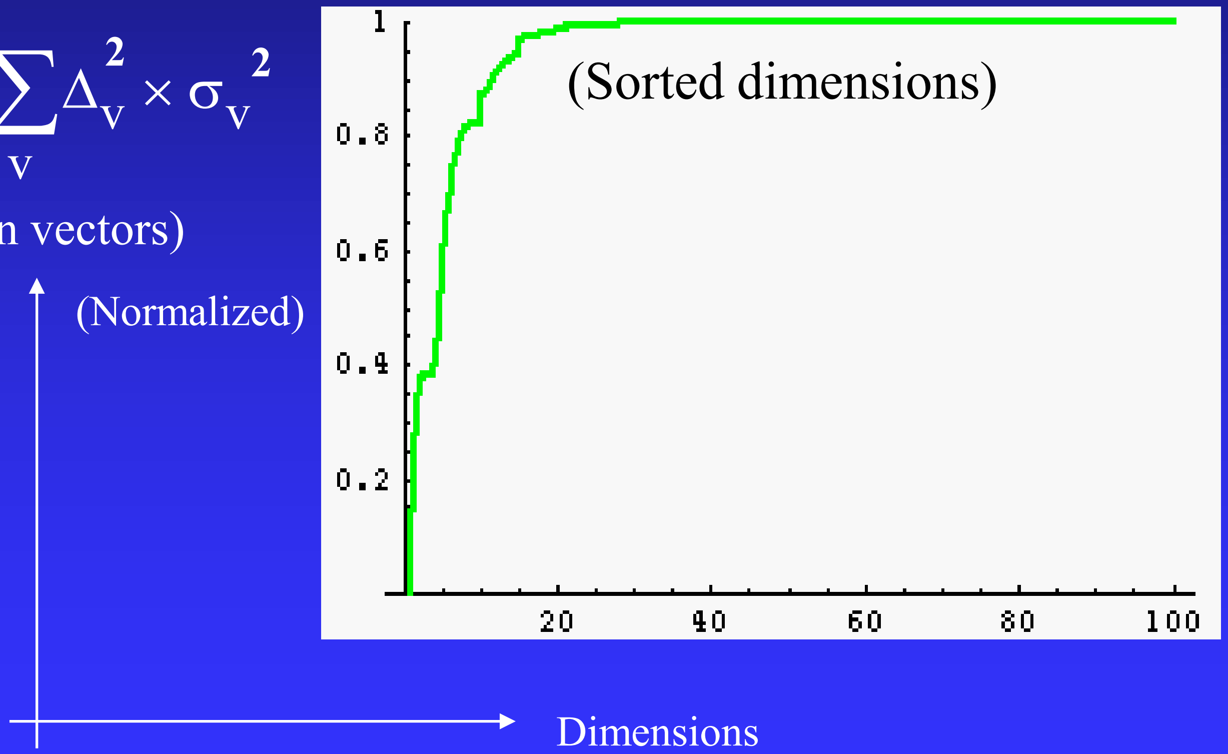

Dimension

# Real Dimensionality (effective)

- Analysis of a typical portfolio in 121 dimensions, with 5 currencies, 14 maturities, bonds, swaps, caps and floors

$$\sum_{v} \Delta_v^2 \times \sigma_v^2$$

(eigen vectors)

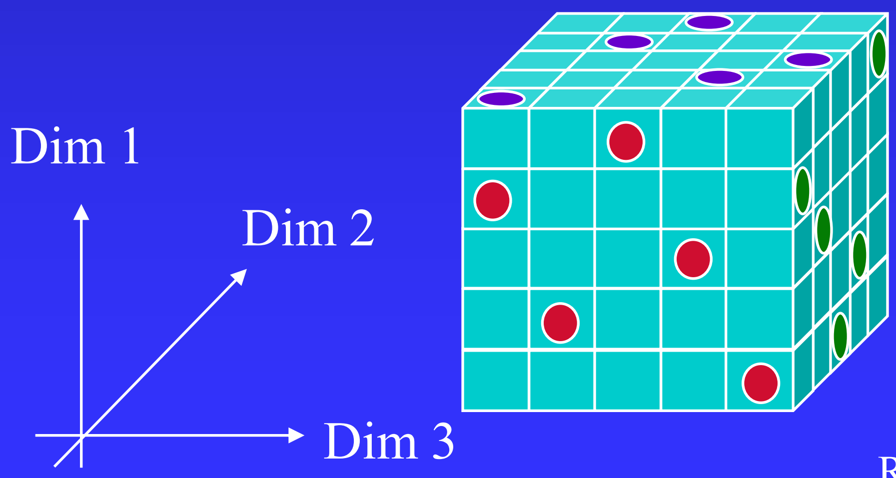


# How to exploit the real dimensionality

- Adapt the sample generator
  - **Some dimensions are stably more important than the others**
    - Analysis in principal components of the interest rates

First component : General level of rates
Second component : Slope of the curve
Third component : Torsion of the curve

**Stratification of the Samples : <u>Latin Hypercube</u>**




Projections of 5 points

H5

| |
|---|
| -2.0202 |
| -0.9586 |
| 0 |
| +0.9586 |
| +2.0202 |

<u>N Dim SuperCube</u>

H

# Historical Simulation

- Use of Historical series of market prices
  - **Factors / Market Prices**
- Empirical Distribution
  - **Simplicity**
  - **Leptokurtic distribution**
  - **real correlations with non linear behavior**
- Limited Historic Series
  - **One year = 250 Points**
  - **Synchronicity of sampling , liquidity of the markets, emerging markets**
  - **Smoothing of the Empirical distribution to get a risk (parameterization)**

re nais sance

# Increasing the resolution of a historical VaR

- We assume a market prices set of 1000 points
- We want a 10 000 samples
  - **To have a finer "resolution" of our VaR measure**
- We want to keep the mean and covariance structure

Dimension =1 **Data set =** $(-1,\ 1,\ 2)$

**Multipliers** $= \dfrac{-3-\sqrt{21}}{6}\ ,\ \dfrac{-3+\sqrt{21}}{6}$

**Enhanced Dat set**

{-1.52 , -1 , -0.26 , 0.73 , 1 , 1.26 , 1.52 , 2 , 2.26}

**N Dimensions**

# Engineering the Market Data Set (I)

- Keeping **E[XY] and E[X]**

$$(\vec{x}_j)_{1 \le j \le N}$$

$$\overrightarrow{y_{i,j}} = \lambda_i \vec{x}_j + \vec{x}_0$$

Multipliers

Conditions

$$\sum_i \left( \lambda_i^2 + \frac{x_0}{x^2}(2\bar{x} - x_0) - 1 \right) = 0$$

$$\sum_i \left( \lambda_i + \frac{x_0}{\bar{x}} - 1 \right) = 0$$

- Extension of the resolution associated with the samples

Cost -> Perturbation of the higher orders

Skewness
Kurtosis

Third order correlation information

$E[X^2Y]$ $E[XY^2]$ $E[XYZ]$

- Extend our analysis to include higher moments
- Control the samples distribution for important subspaces
- Consider linear combinations of market points

H

MC

# Engineering the Market Data Set (II)

- It is possible to decompose the factors into domains

| | |
|---|---|
| Interest rates and volatilities | = 20 X (14 + 6) = 400 dim |
| Equity indexes and volatilities | = 20 X (1 + 6) = 140 dim |
| FX Rates and volatilities | = 19 X (1 + 3) = 76 dim |

**300 Samples**

300 X 300 X 300 = 27 000 000 Samples

(Quasi) Random Selection

30 among 300
X 30 among 300
X 30 among 300

27 000 Samples

Sampling on
Empirical Distribution

re
nais
sance

# The Quadratic VaR

- Distribution of a quadratic polynomial in normal variables

$$PV[X_1, X_2, \ldots, X_s] = PV_0 + \Theta + \sum_{1 \le i \le s} \Delta_i X_i + \frac{1}{2} \sum_{1 \le i, j \le s} \Gamma_{i,j} X_i X_j$$

$\Gamma_{i,j}$

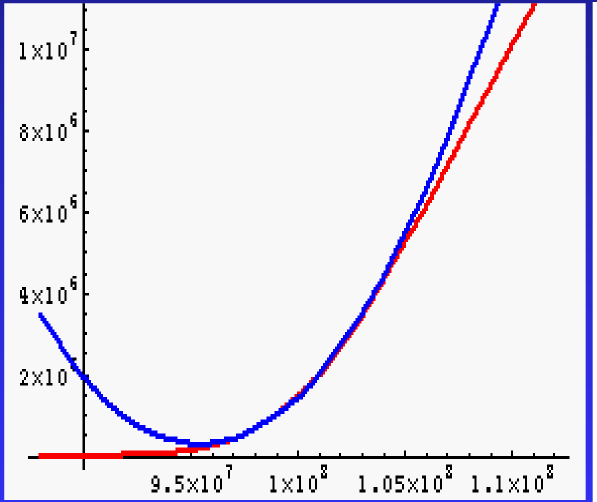
- Real gamma matrix (Derivative)
  - Diagonal gammas
  - Non diagonal gammas
- Pseudo gamma matrix

Quadratic VaR[pseudo gamma] ~True VaR

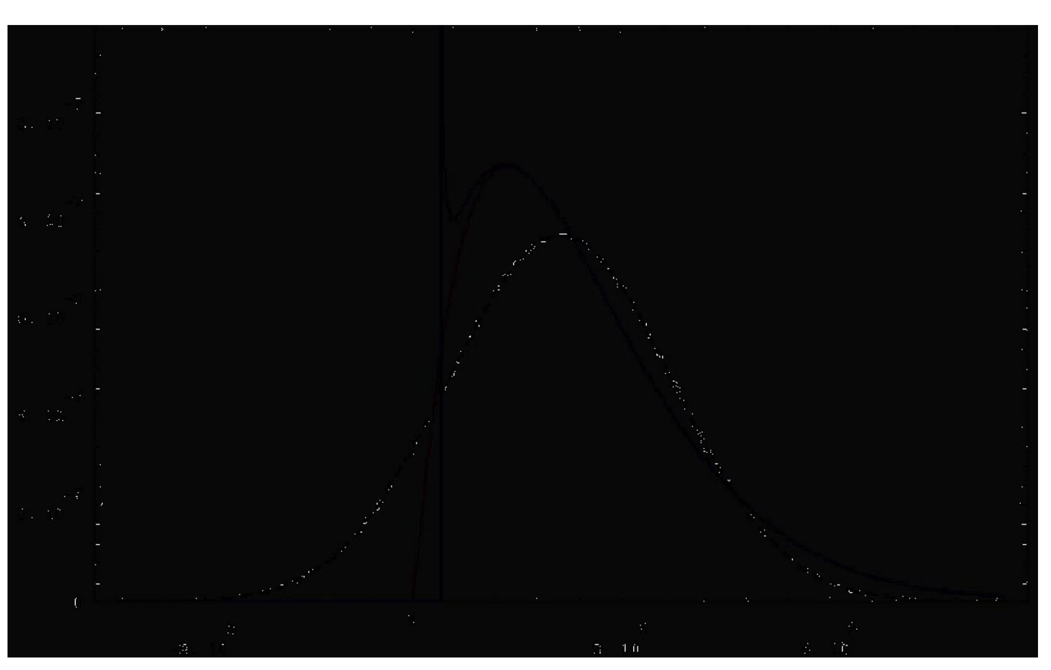
# Quadratic Approximation of a Call

- An at the money call
  - **two weeks**
  - **notional=100m**

- 1% risk=1.27 M
- 1% Quad=1.14 M
- 1% Lin = 2.47 M

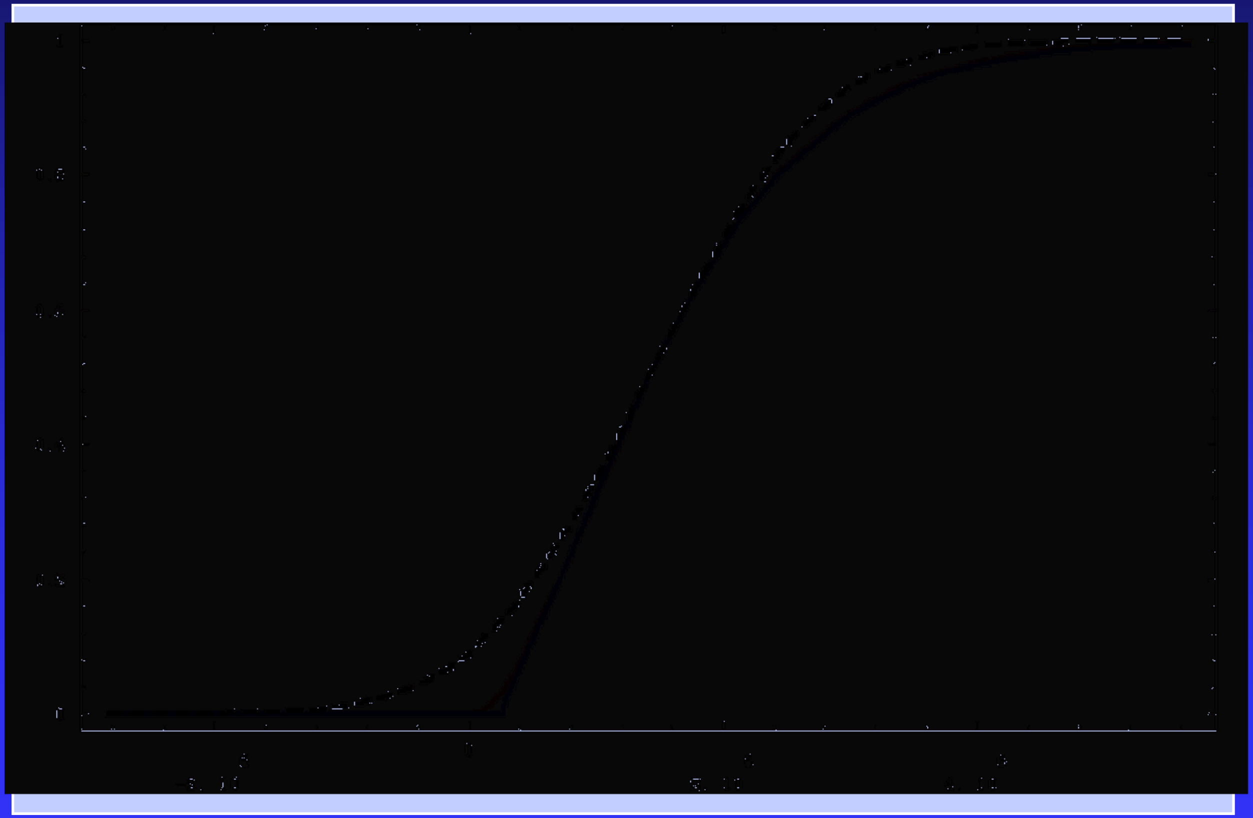



Infinite Density



# Quadratic Approximation (II)

- Risk Curve : Distribution for a call k=1.05, T=1 week



# Quadratic Approximation (III)

- Compact formula for the Computation of the 4 first cumulants

$$\mathrm{var}[Y] = {}^{t}\Delta\Sigma\Delta + \frac{1}{2}\mathrm{Trace}[(\Gamma\Sigma)^2]$$

$$\mathrm{skew}[Y] = 3\,{}^{t}\Delta\Sigma\Gamma\Sigma\Delta + \mathrm{Trace}[(\Gamma\Sigma)^3]$$

$$\mathrm{kurtosis}[Y] = 12\,{}^{t}\Delta\Sigma(\Gamma\Sigma)^2\Delta + \mathrm{Trace}[3(\Gamma\Sigma)^4]$$

Var
Skewness
kurtosis

Parameterization

Pearson,
Johnson,
EdgeWorth,
customized...

Risk 1%

SB

# Mapping of the Sensitivities

- First order mapping
- Second order mapping

$$\left(\frac{\partial}{\partial r_{18m}}\right) \Rightarrow \alpha_{1y}\left(\frac{\partial}{\partial r_{1y}}\right) + \alpha_{2y}\left(\frac{\partial}{\partial r_{2y}}\right)$$

Interpolation

$$r_{18m} = \frac{r_{1y} + r_{2y}}{2}$$

For any deal dependent on the 18 m rate

$$PV \approx PV_0 + \delta \cdot (r_{18m} - r_{0,\,18m}) + \frac{1}{2}\gamma \cdot (r_{18m} - r_{0,\,18m})^2$$

Therefore by replacing

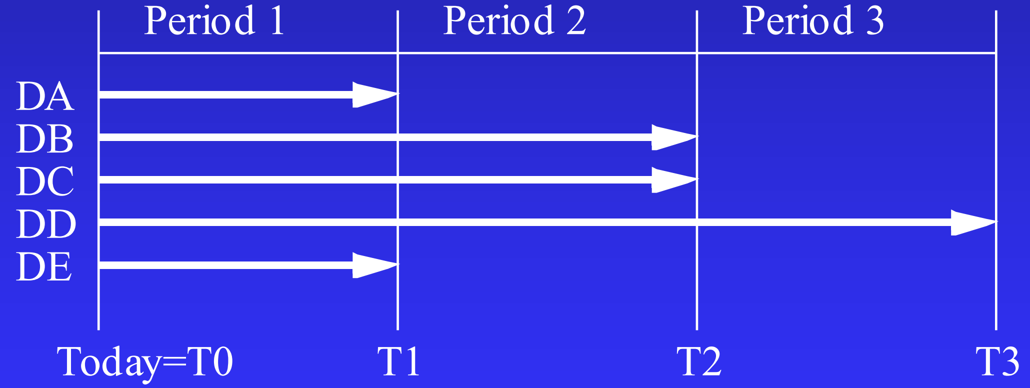
$$PV \approx PV_0 + \Delta \cdot \begin{pmatrix} dr_{1y} \\ dr_{2y} \end{pmatrix} + \frac{1}{2} {}^t\!\begin{pmatrix} dr_{1y} \\ dr_{2y} \end{pmatrix} \Gamma \begin{pmatrix} dr_{1y} \\ dr_{2y} \end{pmatrix} \qquad \Delta = \delta \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \qquad \Gamma = \gamma \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Does the cash flow mapping derive from an interpolation ?

re nais sance

# Handling Liquidity Risk

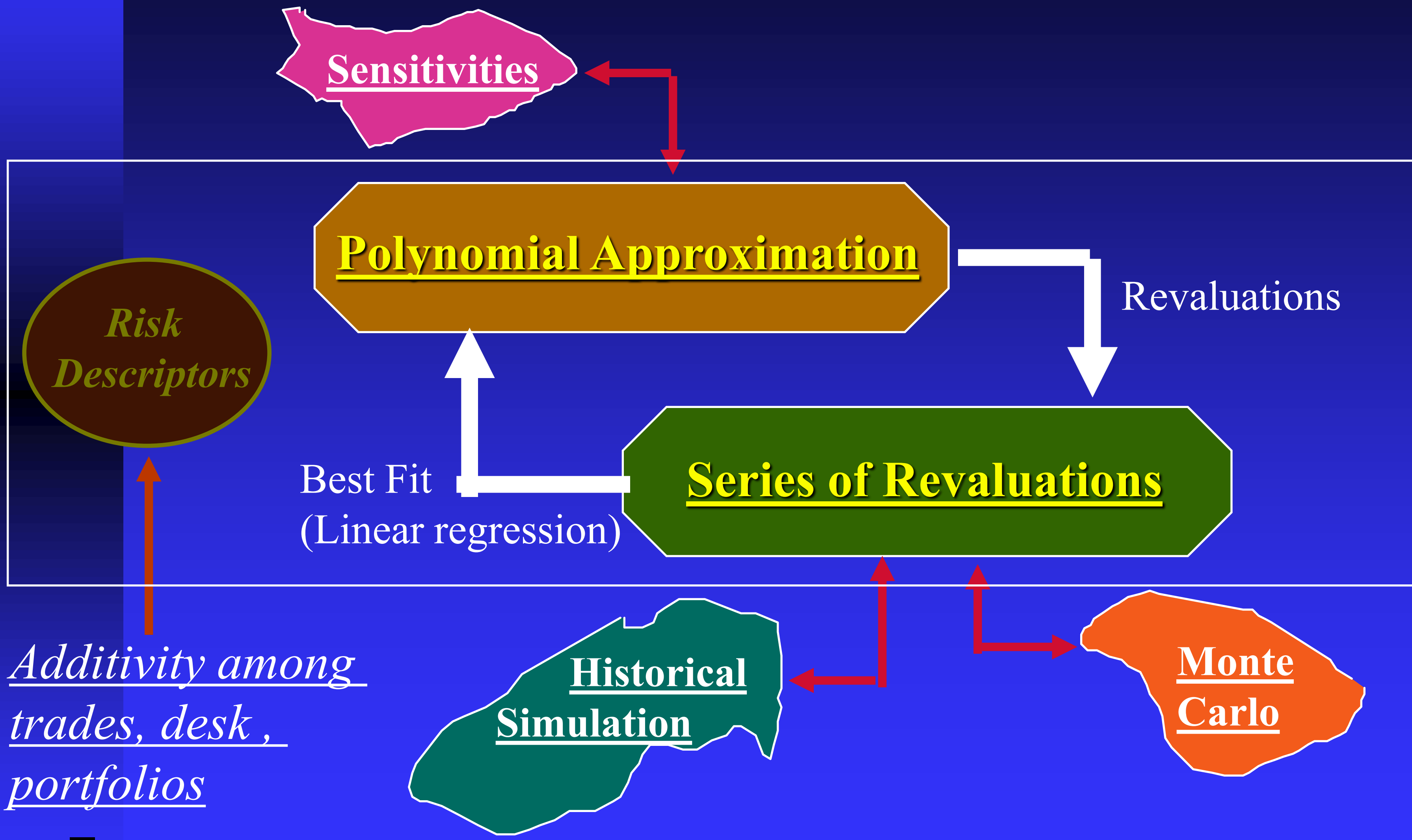
- DEaR # VaR
- The sensitivity framework is the only one which can simply take into account the liquidity risk (unwinding period)
- Cumulants (variance, skewness, kurtosis,..) cumul over independent period of time.




PL(3)-PL(0)={PL(3)-PL(2)} + {PL(2)-PL(1)} + {PL(1)-PL(0)}

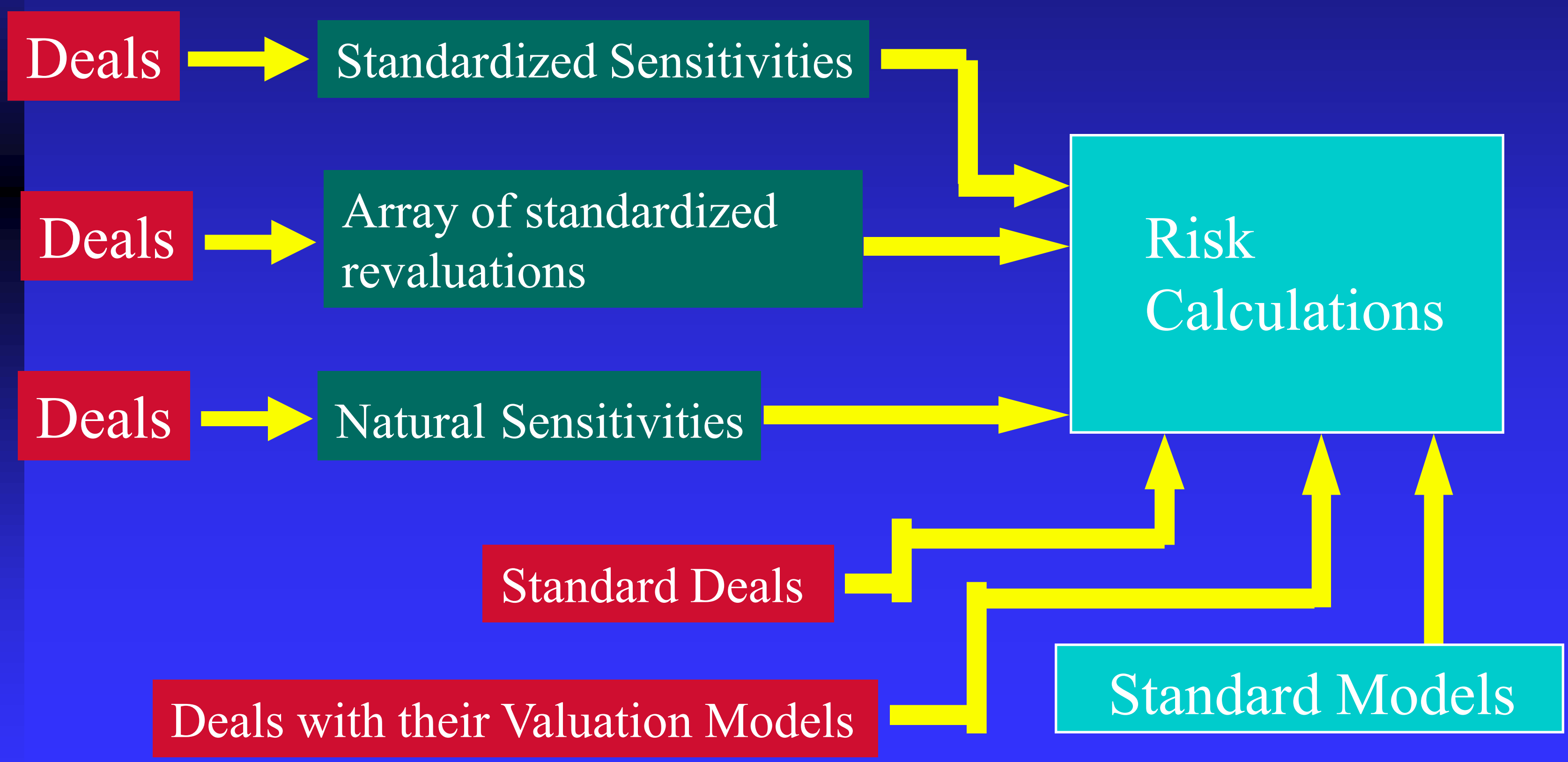
$$Var[PL_3 - PL_0] = Var[PL_3 - PL_2] + Var[PL_2 - PL_1] + Var[PL_1 - PL_0]$$

# Merging the Three VaR Types



# Integrated Handling of Risk Descriptors

- The usefulness of risk calculation depends on the complete integration of risks.



# Conclusion

- No method is better than the others, each one bring a light on an aspect of the risk.
- Integration is the only way for a worldwide risk management
- Flexible design allows optimization
- Smart use of statistics can beat the combinatorial problem