Topos of Creative Measurement
(measurement as creation)

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Core Idea

A quantum measurement is not a destructive "collapse" of information, but an act of creation: it enriches the universe of available truths by refining the logical context—without presupposing an external time. Formally, a measurement is modeled by a geometric morphism that transforms the topos of states before the measurement into a topos after the measurement, where the result becomes decidable (Boolean) in the relevant context. This operation is atemporal (no time parameter) and aligns with an interpretation of the Wheeler–DeWitt equation.

1 Starting Framework

- Quantum topos $\mathcal{E}$ (e.g., contravariant presheaves on $V(A)$) with:
  - spectral presheaf Σ,
  - subobject classifier Ω (Heyting algebra),
  - internal valuation $\mu : \mathrm{Sub}(\Sigma) \to [0,1]^{\leftrightarrow}$ (internalized Born rule).
- A measurement event is represented by a subobject $U \leftrightarrow \Sigma$ (the proposition "the result belongs to U").

2 Measurement = Localization + Slicing (Creation of Decidability)

Two canonical steps are associated with U:

2.1 1) Slicing (Internal Conditioning)

We pass to the slice topos $\mathcal{E}/U$, which internalizes the fact that the universe is considered under the condition U.

- The valuation is conditioned: $\mu \leftrightarrow \mu_{|U}$ on $\mathrm{Sub}(\Sigma)_{|U}$.
- Truths become contextual relative to U.

2.2 2) Logical Localization (Sheafification via Lawvere–Tierney)

We choose an internal topology $j_U : \Omega \to \Omega$ that makes the proposition U decidable (stable/closed).

- We form the sub-topos $\mathrm{Sh}_{j_U}(\mathcal{E}/U)$ with sheafification functor $a_{j_U} : \mathcal{E}/U \to \mathrm{Sh}_{j_U}(\mathcal{E}/U)$ (left exact).
- In $\mathrm{Sh}_{j_U}(\mathcal{E}/U)$, the proposition "$U$" is Boolean (we have created the decidability of the result).

Definition (Creative Measurement)

A creative measurement is the composed geometric morphism

$$\mathcal{E} \xrightarrow{[f_U]} \mathcal{E}/U \xrightarrow{a_{j_U}} \mathcal{E}_U^{\mathrm{meas}} := \mathrm{Sh}_{j_U}(\mathcal{E}/U),$$

where j_U is chosen so that U becomes decidable in $\mathcal{E}_U^{\mathrm{meas}}$.
Intuition. The passage $\mathcal{E} \to \mathcal{E}_U^{\mathrm{meas}}$ creates a new logical universe where the result is decided in the right context—without violating Kochen-Specker (we are not manufacturing a global section of Σ in $\mathcal{E}$; we work in $\mathcal{E}/U$ and then locally Booleanize).

3 Creation of Information (Independent of Entropy)

We distinguish contextual logical information from thermodynamic entropy.

Logical Information Created

The choice of a result U refines the internal Heyting algebra: we go from an open truth value ("possible") to a decidable value (yes/no) in $\mathcal{E}_U^{\mathrm{meas}}$. We can quantify this gain (à la Shannon/algorithmic) by

$$\Delta\mathcal{I}(U) := -\log \mu(U) \quad \text{(in bits, internal via } [0,1]^{\leftrightarrow}\text{)}.$$

This is a semantic gain (refinement of truth), not a thermodynamic cost.

Independence from Entropy

The operation $\mathcal{E} \to \mathcal{E}_U^{\mathrm{meas}}$ is logical/categorical. It does not, in itself, imply a variation of the von Neumann entropy of a closed physical state.

4 Axioms (CM) for Creative Measurement

- (CM1) **Atemporality.** The functor $\mathcal{E} \to \mathcal{E}_U^{\mathrm{meas}}$ does not depend on any external temporal parameter.
- (CM2) **Information Monotonicity.** If $U \le V$ (refinement), then $\Delta\mathcal{I}(U) \ge \Delta\mathcal{I}(V)$.
- (CM3) **Internal Born Compatibility.** $\mu_{|U}(X) = \mu(X \wedge U)/\mu(U)$.
- (CM4) **Contextual Locality.** The decidability created by j_U is local to the slice $\mathcal{E}/U$; it does not generate a global point of Σ in $\mathcal{E}$.
- (CM5) **Naturalism (GR Covariance).** Under changes of frame (diffeomorphisms, refinement of region, change of abelian context), the construction is *pseudo-natural* (functorial): it does not depend on a temporal background.

5 "Timeless" Reading and Wheeler–DeWitt

In a theory where states satisfy a global constraint

$$\hat{H}\psi = 0,$$

(Wheeler–DeWitt), the "evolution" is not temporal but an order of refinement of truths:

- The internal universe of solutions is an object $S = \ker(\hat{H})$ in $\mathcal{E}$.
- A creative measurement selects a decidable subobject $S_U \leftrightarrow S$ via $\mathcal{E} \to \mathcal{E}_U^{\mathrm{meas}}$.
- This passage does not evolve ψ in time; it refines the description in an atemporal manner: we condition the truth of propositions about S.

Moral. The meaning of Wheeler–DeWitt is preserved: the fundamental dynamics is timeless; what we call "becoming" is the ascent through the lattice of contexts (creative measurements) that increase the available logical information.

6 Interface with CFS

In CFS, ρ and the *closed chains* A_{xy} encode causality. With creative measurement:

- The result U (proposition on spectra/invariants) becomes **decidable** in $\mathcal{E}_U^{\text{meas}}$.
- The causal types (time-/space-/light-like) are *internal predicates* which, once localized, evaluate *unambiguously* for the measured context.
- The internal causal action $\mathbf{S}[\mu]$ is evaluated *conditionally* and can be *re-optimized* in $\mathcal{E}_U^{\text{meas}}$ (interpretation: informational *back-reaction*).

7 Minimal Example (Qubit, σ_z

- $\mathcal{E}$: presheaves on contexts $\{\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle\}$
- Result $U = \{\sigma_z = +1\} \leftrightarrow \Sigma$
- Slice $\mathcal{E}/U$: all propositions are conditioned by U
- j_U: internal topology making U **decidable**.
- $\mathcal{E}_U^{\text{meas}}$: topos where "$\sigma_z = +1$" is **Boolean** (locally), without manufacturing a global truth for σ_x, σ_y
- Information created:

$$\Delta\mathcal{I}(U) = -\log\mu(U).$$

Summary

A *creative measurement* is a geometric morphism $\mathcal{E} \to \mathcal{E}_U^{\text{meas}}$ (slicing + localization) which **makes the result decidable** in the appropriate context, **increases logical information** (without prejudging entropy), and **respects the atemporality** expected from a constrained theory like Wheeler-DeWitt.