Forces Acting on Dislocations

- Peach - Koehler equation
- External force - constant Peach - Koehler force
- Forces acting between dislocations
- Interactions between dislocations
- Energy of dislocation configurations
- Climb and chemical forces
- Image force
- Self-force (or line tension)

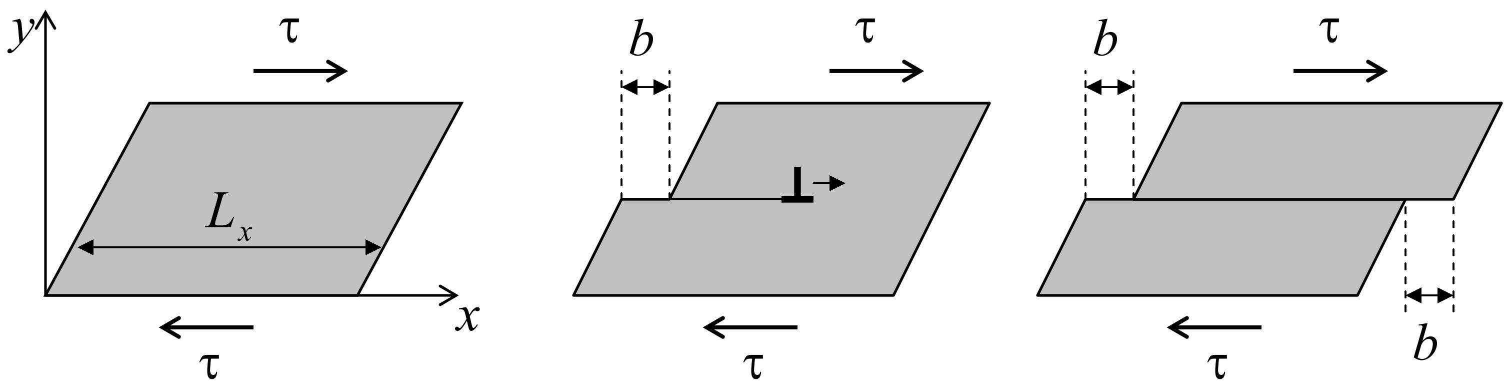
References:

Hull and Bacon, Ch. 4.5-4.8

Kelly and Knowles, Ch. 8

Force acting on dislocations: Peach - Koehler equation

Application of external stresses and stresses generated by other crystal defects may cause movement of a dislocation on its glide plane. As a result, work may be done by the applied stresses. Let's consider dislocation motion in a sample with dimensions $L_x \times L_y \times L_z$ due to the shear stress τ :



The work done by the shear stress τ in changing the system from the initial to the final state is equal to $\tau Sb = \tau L_x L_z b$. If we consider the process as movement of a dislocation under the action of force F acting on a unit length of the dislocation, the same work can also be written as
$(\,force\ FL_z\,) \times (\,distance\ L_x) = FL_zL_x$

Thus $\boxed{F = \tau b}$

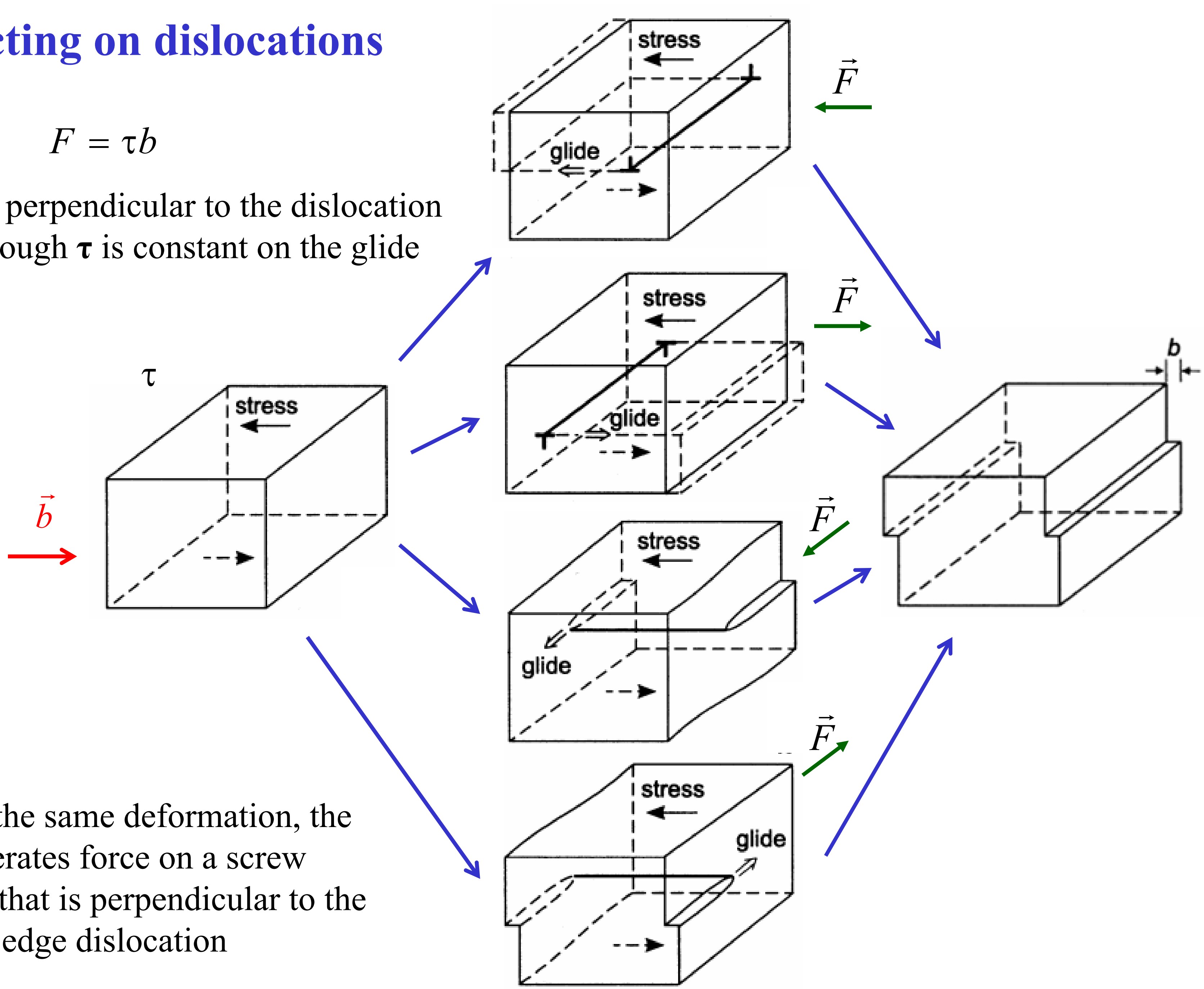
The force acting on a dislocation line is not a physical force (like mechanical force of a spring or electrostatic force acting on a charged particle) but a way to describe the tendency of dislocation to move through the crystal when external or internal stresses are present.

This work done at the slip plane is dissipated into heat (similar to work done by friction forcers)

Force acting on dislocations

$$F = \tau b$$

F is always perpendicular to the dislocation line even though **τ** is constant on the glide plane



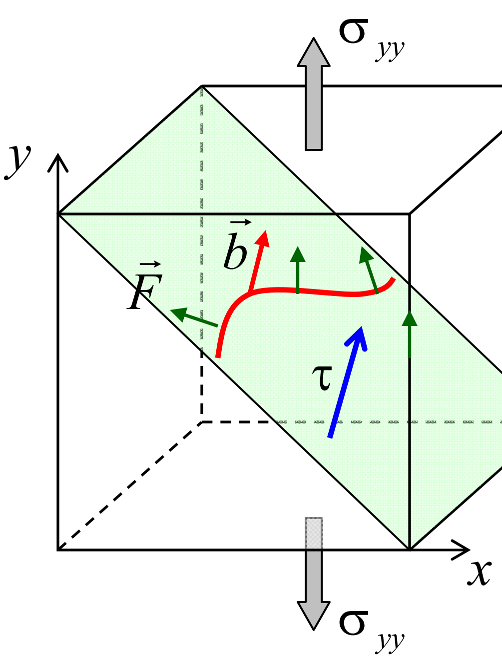
to produce the same deformation, the same τ generates force on a screw dislocation that is perpendicular to the force on an edge dislocation

Force acting on dislocations: Peach - Koehler equation

$F = \tau b$ τ is the shear stress in the glide plane resolved in the direction of ***b*** and ***F*** acts normal to the dislocation line

Since the dislocation moves on its glide plane, we only need to consider the shear stress on this plane. Stress components normal to the glide plane do not contribute to the dislocation movement

Moreover, only the shear stress components in the direction of ***b*** (called the *resolved shear stress τ*) are contribution to the movement of the dislocation.



F is perpendicular to the dislocation line
τ is constant on the glide plane

In general: $\vec{F} = \left(\sigma \cdot \vec{b}\right) \times \vec{l}$ Peach-Koehler equation

$\vec{F}$ - force per unit length at an arbitrary point P along the dislocation line

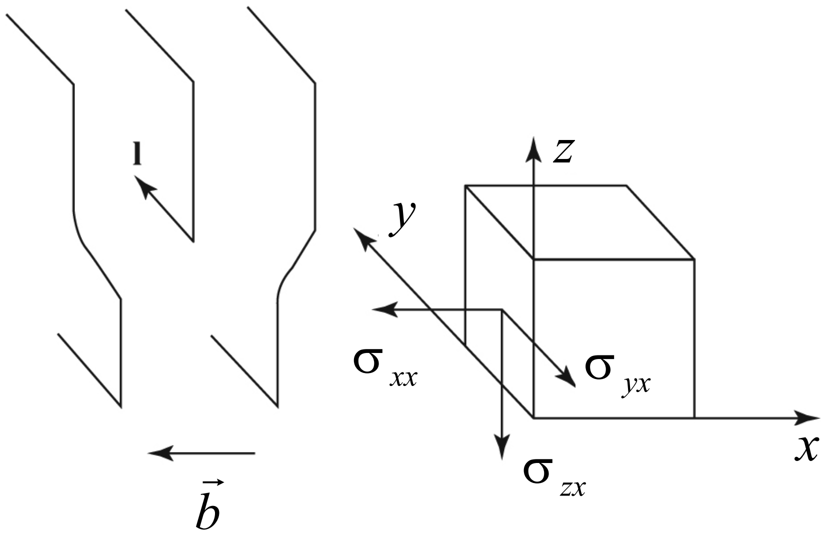
σ - local stress field

$\vec{l}$ - local line tangent direction at point P

$\sigma \cdot \vec{b}$ - local force per unit length acting on a plane (of area b) normal to the Burgers vector

Force acting on dislocations: Peach - Koehler equation

Example: edge dislocation runs along y-axis and Burgers vector is in the negative direction of x-axis



glide plane is (001) $\qquad \vec{b} = (b_x, b_y, b_z) = (-b, 0, 0)$

$$\vec{l} = (l_x, l_y, l_z) = (010)$$

$$\vec{F} = \underbrace{\left(\sigma \cdot \vec{b}\right)}_{\vec{g} = (g_x, g_y, g_z)} \times \vec{l}$$

$$g_x = \sigma_{xx} b_x + \sigma_{xy} b_y + \sigma_{xz} b_z = -\sigma_{xx} b$$

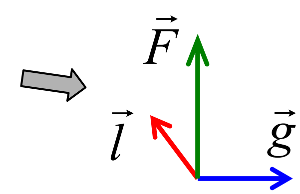
$$g_y = \sigma_{yx} b_x + \sigma_{yy} b_y + \sigma_{yz} b_z = -\sigma_{yx} b$$

$$g_z = \sigma_{zx} b_x + \sigma_{zy} b_y + \sigma_{zz} b_z = -\sigma_{zx} b$$

traction on the surface normal to ***b*** is defined by stress components σ_{xx}, σ_{yx}, σ_{zx}

$g_x = -\sigma_{xx} b$ is $\perp$ to ***l*** $\Rightarrow$ σ_{xx} produces $|\boldsymbol{F}| = \sigma_{xx} b$ downwards, perpendicular to the glide plane

if $\sigma_{xx} < 0$ (compression) $\Rightarrow$ ***F*** is directed upwards $\Rightarrow$ *climb force*

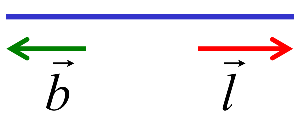


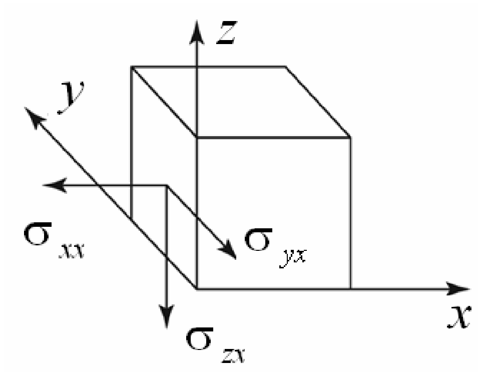
$g_y = -\sigma_{yx} b$ is || to ***l*** $\Rightarrow$ σ_{yx} makes no contribution to ***F***

$g_z = -\sigma_{zx} b$ is $\perp$ to ***l*** $\Rightarrow$ σ_{zx} produces $|\boldsymbol{F}| = \sigma_{zx} b$ along the x-axis $\Rightarrow$ *glide force*

Force acting on dislocations: Peach - Koehler equation

Example: screw dislocation runs along x-axis and Burgers vector is in the negative direction of x-axis





glide plane is (001) $\quad \vec{b} = (b_x, b_y, b_z) = (-b,0,0)$

$$\vec{l} = (l_x, l_y, l_z) = (100)$$

$$\vec{F} = \underbrace{\left(\sigma \cdot \vec{b}\right)}_{\vec{g} = (g_x, g_y, g_z)} \times \vec{l}$$

$$g_x = \sigma_{xx} b_x + \sigma_{xy} b_y + \sigma_{xz} b_z = -\sigma_{xx} b$$
$$g_y = \sigma_{yx} b_x + \sigma_{yy} b_y + \sigma_{yz} b_z = -\sigma_{yx} b$$
$$g_z = \sigma_{zx} b_x + \sigma_{zy} b_y + \sigma_{zz} b_z = -\sigma_{zx} b$$

traction on the surface normal to ***b*** is defined by stress components σ_{xx}, σ_{yx}, σ_{zx}

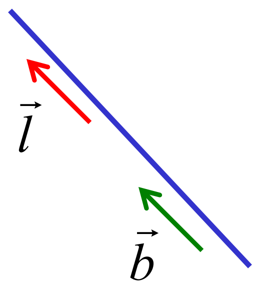
$g_x = -\sigma_{xx} b$ is || to ***l*** $\Rightarrow$ σ_{xx} makes no contribution to ***F***

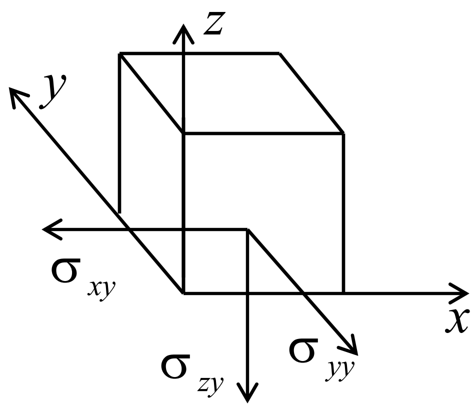
$g_y = -\sigma_{yx} b$ is $\perp$ to ***l*** $\Rightarrow$ σ_{yx} produces $|\boldsymbol{F}| = \sigma_{yx} b$ up- or down-wards (depending on the sign of σ_{yx}), perpendicular to the xy slip plane - can induce *cross-slip*

$g_z = -\sigma_{zx} b$ is $\perp$ to ***l*** $\Rightarrow$ σ_{zx} produces $|\boldsymbol{F}| = \sigma_{zx} b$ along the y-axis $\Rightarrow$ *glide force* along xy slip plane

Force acting on dislocations: Peach - Koehler equation

Example: screw dislocation runs along *y*-axis and Burgers vector is in the positive direction of *y*-axis





glide plane is (001) $\quad \vec{b} = (b_x, b_y, b_z) = (0, b, 0)$

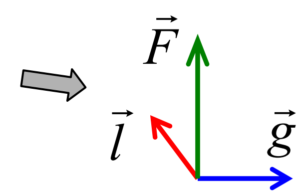
$$\vec{l} = (l_x, l_y, l_z) = (010)$$

$$\vec{F} = \underbrace{\left(\sigma \cdot \vec{b}\right)}_{\vec{g} = (g_x, g_y, g_z)} \times \vec{l}$$

$$g_x = \sigma_{xx} b_x + \sigma_{xy} b_y + \sigma_{xz} b_z = \sigma_{xy} b$$
$$g_y = \sigma_{yx} b_x + \sigma_{yy} b_y + \sigma_{yz} b_z = \sigma_{yy} b$$
$$g_z = \sigma_{zx} b_x + \sigma_{zy} b_y + \sigma_{zz} b_z = \sigma_{zy} b$$

traction on the surface normal to ***b*** is defined by stress components σ_{xy}, σ_{yy}, σ_{zy}

$g_x = \sigma_{xy} b$ is $\perp$ to ***l*** $\Rightarrow$ σ_{xy} produces $|\boldsymbol{F}| = \sigma_{xy} b$ up- or down-wards (depending on the sign of σ_{xy}), perpendicular to the *xy* slip plane - can induce *cross-slip*



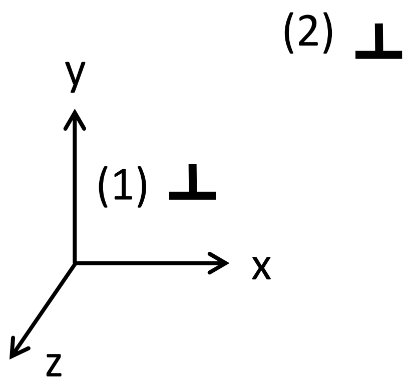
$g_y = \sigma_{yy} b$ is || to ***l*** $\Rightarrow$ σ_{yy} makes no contribution to ***F***

$g_z = \sigma_{zy} b$ is $\perp$ to ***l*** $\Rightarrow$ σ_{zy} produces $|\boldsymbol{F}| = \sigma_{zy} b$ along the *x*-axis $\Rightarrow$ *glide force* along *xy* slip plane

Forces between dislocations

Force due to the interaction with other dislocations = sum of all Peach–Koehler forces between the segments of all other dislocations in the system

Example: interaction between two parallel straight edge dislocations



$$\vec{F} = \underbrace{\left(\overset{(1)}{\sigma} \cdot \overset{(2)}{\vec{b}}\right)}_{\vec{g} = (g_x, g_y, g_z)} \times \overset{(2)}{\vec{l}}$$

$$\left.\begin{aligned} \vec{b} &= (b_x, b_y, b_z) \\ \vec{l} &= (l_x, l_y, l_z) \end{aligned}\right\} \text{disl. (2)}$$

$$\begin{aligned} g_x &= \sigma_{xx} b_x + \sigma_{xy} b_y + \sigma_{xz} b_z \\ g_y &= \sigma_{yx} b_x + \sigma_{yy} b_y + \sigma_{yz} b_z \\ g_z &= \sigma_{zx} b_x + \sigma_{zy} b_y + \sigma_{zz} b_z \end{aligned}$$

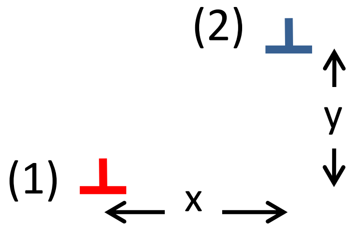
Edge dislocation (1) produces a stress field that dislocation (2) responds to

Peach–Koehler gives force acting on dislocation (2) due to the presence of dislocation (1)

Must use consistent convention to describe dislocations (direction of the Burgers circuit, line direction into page, start-finish)

Forces between dislocations

Example: interaction between two parallel straight edge dislocations



(1) $\vec{b} = (b_1, 0, 0)$

$\vec{l}_1 = (0,0,1)$

(2) $\vec{b} = (b_2, 0, 0)$

$\vec{l}_2 = (0,0,1)$

(1) (2)

$$\vec{F} = \underbrace{\left(\sigma \cdot \vec{b}_2\right)}_{\vec{g}} \times \vec{l}_2$$

Step 1:

$$g_x = \sigma_{xx} b_x + \sigma_{xy} b_y + \sigma_{xz} b_z = \sigma_{xx} b_2$$

$$g_y = \sigma_{yx} b_x + \sigma_{yy} b_y + \sigma_{yz} b_z = \sigma_{yx} b_2$$

$$g_z = \sigma_{zx} b_x + \sigma_{zy} b_y + \sigma_{zz} b_z = 0$$

$\vec{b} = (b_2, 0, 0)$ for disl. (2)

σ from disl. (1)

$$\vec{g} = (\sigma_{xx} b_2, \sigma_{yx} b_2, 0)$$

Step 2:

$$\vec{F} = \vec{g} \times \vec{l}_2 = (\sigma_{yx} b_2, -\sigma_{xx} b_2, 0)$$

Step 3:

use expressions for stresses generated by dislocation (1):

$$\sigma_{xx} = -\frac{G b_1}{2\pi(1-\nu)} \frac{y\left(3x^2 + y^2\right)}{\left(x^2 + y^2\right)^2}$$

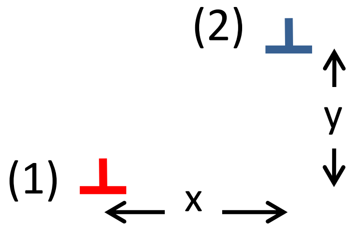
$$\sigma_{xy} = \frac{G b_1}{2\pi(1-\nu)} \frac{x\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}$$

$$\vec{F} = \frac{G b_1 b_2}{2\pi(1-\nu)} \frac{x\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2} \hat{x} + \frac{G b_1 b_2}{2\pi(1-\nu)} \frac{y\left(3x^2 + y^2\right)}{\left(x^2 + y^2\right)^2} \hat{y}$$

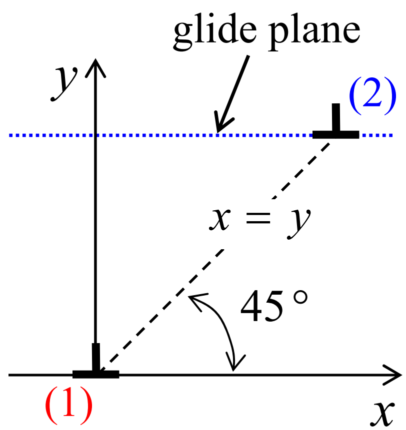
$$\mathbf{a} \times \mathbf{b} = \mathbf{c} = c_1 \boldsymbol{i} + c_2 \boldsymbol{j} + c_3 \boldsymbol{k} \qquad \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}.$$

Forces between dislocations

Example: interaction between two parallel straight edge dislocations



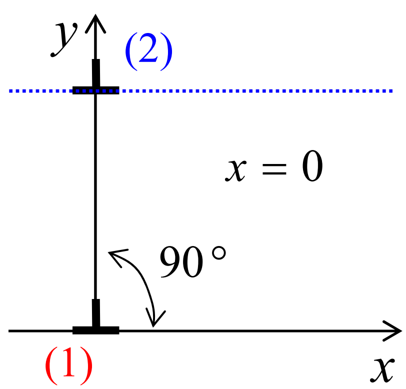
$$\vec{F} = \frac{Gb_1b_2}{2\pi(1-\nu)}\frac{x(x^2-y^2)}{(x^2+y^2)^2}\hat{x} + \frac{Gb_1b_2}{2\pi(1-\nu)}\frac{y(3x^2+y^2)}{(x^2+y^2)^2}\hat{y}$$



attraction   repulsion

$\vec{F}_x = 0$ - unstable equilibrium ($F_x < 0$ when $x < y$ and $F_x > 0$ when $x > y$)

$$\vec{F}_y = \frac{Gb_1b_2}{2\pi(1-\nu)}\frac{1}{y}$$ - climb force (need diffusion)



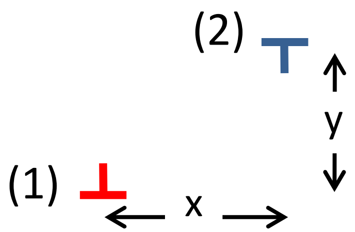
attraction   attraction

$\vec{F}_x = 0$ - stable equilibrium ($F_x > 0$ when $x < 0$ and $F_x < 0$ when $x > 0$)

$$\vec{F}_y = \frac{Gb_1b_2}{2\pi(1-\nu)}\frac{1}{y}$$ - climb force (need diffusion)

Forces between dislocations

Example: interaction between two parallel straight edge dislocations



(1) $\vec{b} = (b_1, 0, 0)$ (2) $\vec{b} = (-b_2, 0, 0)$

$\vec{l}_1 = (0,0,1)$ $\vec{l}_2 = (0,0,1)$

same as before, but opposite sign of (2)

$$\vec{F} = \underbrace{\left(\overset{(1)}{\sigma} \cdot \overset{(2)}{\vec{b}_2}\right)}_{\vec{g}} \times \overset{(2)}{\vec{l}_2}$$

Step 1:

$$g_x = \sigma_{xx} b_x + \sigma_{xy} b_y + \sigma_{xz} b_z = -\sigma_{xx} b_2$$

$$g_y = \sigma_{yx} b_x + \sigma_{yy} b_y + \sigma_{yz} b_z = -\sigma_{yx} b_2$$

$$g_z = \sigma_{zx} b_x + \sigma_{zy} b_y + \sigma_{zz} b_z = 0$$

$\vec{b} = (-b_2, 0, 0)$ for disl. (2)

σ from disl. (1)

$$\boxed{\vec{g} = (-\sigma_{xx} b_2, -\sigma_{yx} b_2, 0)}$$

Step 2:

$$\boxed{\vec{F} = \vec{g} \times \vec{l}_2 = (-\sigma_{yx} b_2, \sigma_{xx} b_2, 0)}$$

Step 3:

use expressions for stresses generated by dislocation (1):

$$\sigma_{xx} = -\frac{Gb_1}{2\pi(1-\nu)} \frac{y\left(3x^2 + y^2\right)}{\left(x^2 + y^2\right)^2}$$

$$\sigma_{xy} = \frac{Gb_1}{2\pi(1-\nu)} \frac{x\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2}$$

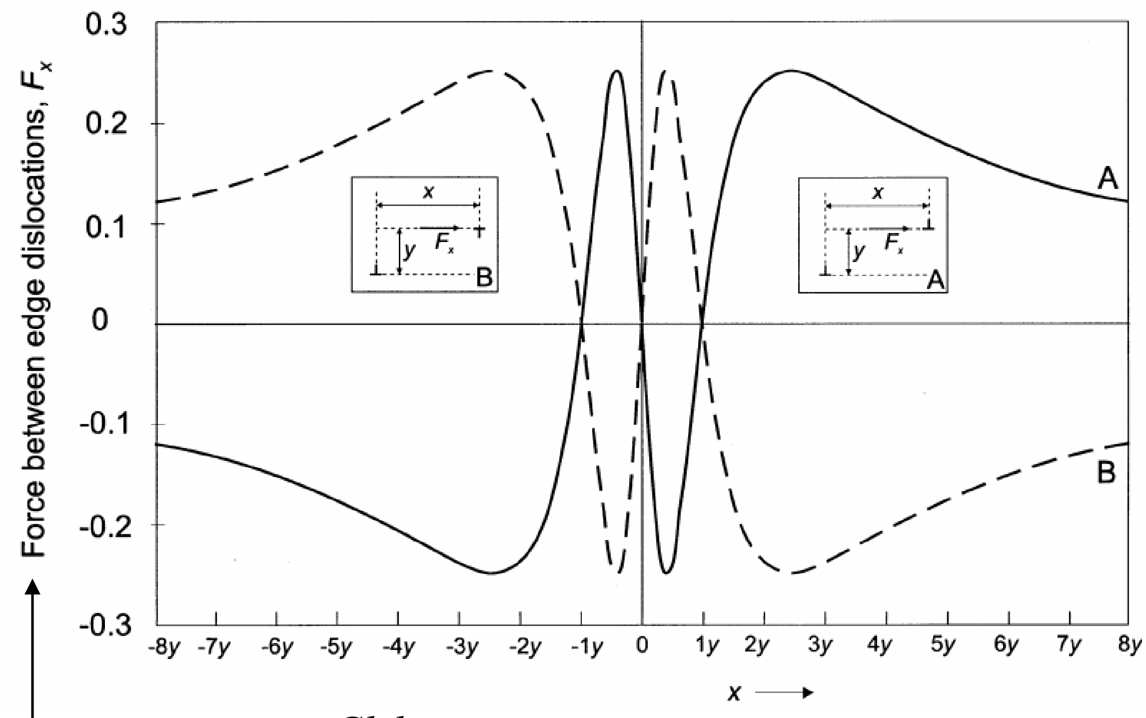
$$\boxed{\vec{F} = -\frac{Gb_1 b_2}{2\pi(1-\nu)} \frac{x\left(x^2 - y^2\right)}{\left(x^2 + y^2\right)^2} \hat{x} - \frac{Gb_1 b_2}{2\pi(1-\nu)} \frac{y\left(3x^2 + y^2\right)}{\left(x^2 + y^2\right)^2} \hat{y}}$$

the sign is reversed when the dislocations are of opposite sign

Forces between dislocations

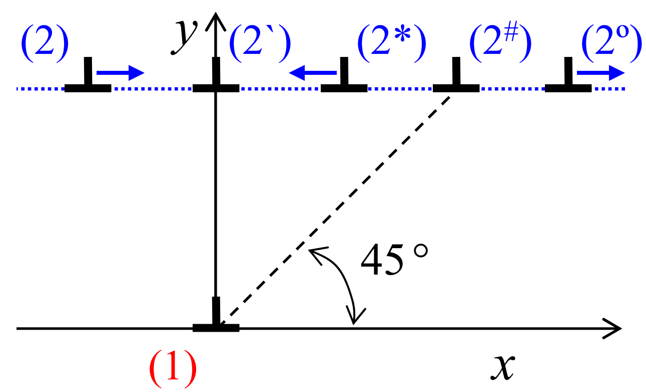
Example: interaction between two parallel straight edge dislocations

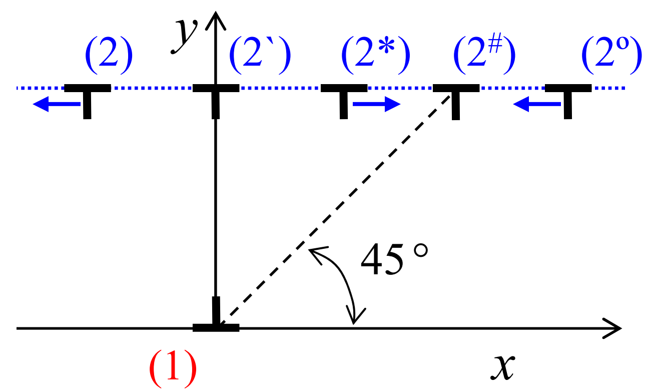
$$\vec{F} = \pm \frac{Gb_1b_2}{2\pi(1-\nu)} \frac{x(x^2 - y^2)}{(x^2+y^2)^2}\hat{x} \pm \frac{Gb_1b_2}{2\pi(1-\nu)} \frac{y(3x^2 + y^2)}{(x^2+y^2)^2}\hat{y}$$



force in units of $\frac{Gb_1b_2}{2\pi(1-\nu)y}$

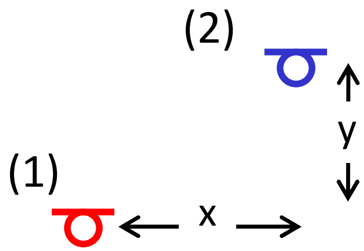
from Hull and Bacon





Forces between dislocations

Example: interaction between two parallel straight screw dislocations



(1) $\vec{b} = (0,0,b_1)$
$\vec{l}_1 = (0,0,1)$

(2) $\vec{b} = (0,0,b_2)$
$\vec{l}_2 = (0,0,1)$

$$\vec{F} = \underbrace{\left(\overset{(1)}{\sigma}\cdot \overset{(2)}{\vec{b}_2}\right)}_{\vec{g}}\times \overset{(2)}{\vec{l}_2}$$

Step 1:

$$g_x = \sigma_{xx} b_x + \sigma_{xy} b_y + \sigma_{xz} b_z = \sigma_{xz} b_2$$
$$g_y = \sigma_{yx} b_x + \sigma_{yy} b_y + \sigma_{yz} b_z = \sigma_{yz} b_2$$
$$g_z = \sigma_{zx} b_x + \sigma_{zy} b_y + \sigma_{zz} b_z = 0$$

$\vec{b} = (0,0,b_2)$ for disl. (2)

σ from disl. (1)

$$\boxed{\vec{g} = (\sigma_{xz} b_2, \sigma_{yz} b_2, 0)}$$

Step 2:

$$\boxed{\vec{F} = \vec{g}\times\vec{l}_2 = (\sigma_{yz} b_2, -\sigma_{xz} b_2, 0)}$$

Step 3:

use expressions for stresses generated by dislocation (1):

$$\sigma_{xz} = \sigma_{zx} = -\frac{Gb}{2\pi}\frac{y}{x^2+y^2} = -\frac{Gb}{2\pi}\frac{\sin\theta}{r}$$

$$\sigma_{yz} = \sigma_{zy} = \frac{Gb}{2\pi}\frac{x}{x^2+y^2} = \frac{Gb}{2\pi}\frac{\cos\theta}{r}$$

$$\boxed{\vec{F} = \frac{Gb_1 b_2}{2\pi}\frac{x}{x^2+y^2}\hat{x} + \frac{Gb_1 b_2}{2\pi}\frac{y}{x^2+y^2}\hat{y}}$$

$$\boxed{\vec{F} = \frac{Gb_1 b_2}{2\pi r}\left(\cos\theta\,\hat{x} + \sin\theta\,\hat{y}\right)}$$

repulsive for screws of the same sign and attractive for screws of opposite sign

Summary on interactions between dislocations

General basic rules:

The superposition of the stress fields of two dislocations as they move towards each other can result in

(1) *larger combined stress field* as compared to a single dislocation (e.g., overlap of the regions of compressive or tensile stresses from the two dislocations) ⇒ increase in the energy of the configuration ⇒ *repulsion* between dislocations.

(2) *lower combined stress field* as compared to a single dislocation (e.g., overlap of regions of compressive stress from one dislocation with regions of tensile stress from the other dislocation) ⇒ *attraction* between dislocations.

Arbitrarily (curved) dislocations on the same glide plane:

Dislocations with opposite ***b*** will attract each other and annihilate

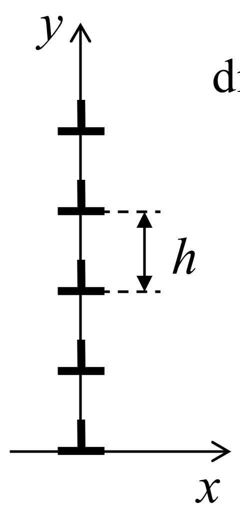
Dislocations with the same ***b*** will always repel each other

Edge dislocations with identical or opposite Burgers vector ***b*** on neighboring glide planes may attract or repulse each other, depending on the precise geometry.

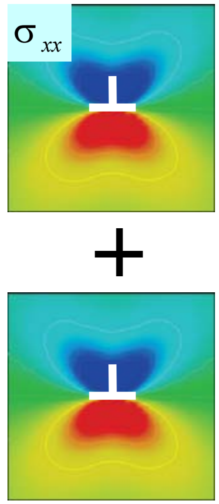
The force between screw dislocations is repulsive for dislocations of the same sign and attractive for dislocations of opposite sign.

Energy of dislocation configurations

stable configuration for parallel straight edge dislocations



dislocation wall



this configuration has strong long-range stress field

superposition of compressive and tensile stresses of dislocations in the wall ⇒ screening radius $R \sim h$

$$W_{disl} = W_{el} + W_{core} = \frac{Gb^2 L}{4\pi K}\left(\ln\frac{R}{r_0} + Z\right)$$

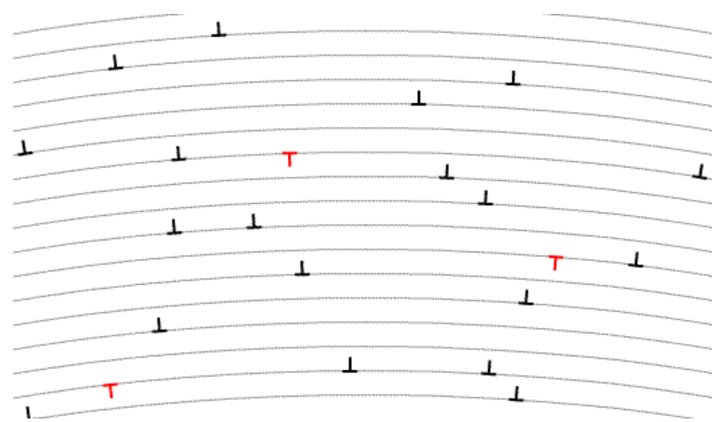
$$\text{for } h = 75b,\ \frac{h}{r_0} \approx 50,\ \text{ and }\ \ln\frac{h}{r_0} \approx 4$$

$$\ln\frac{R_{\max}}{r_0} \approx 16$$

energy of dislocation in the wall is up to 4 times lower than energy of an individual dislocation

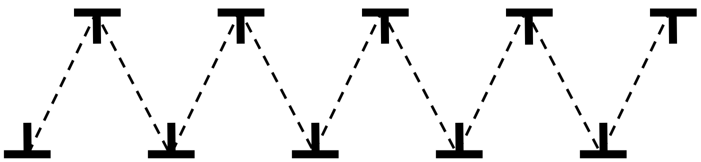
dislocation walls form during recovery, when stored internal energy accumulated during plastic deformation decreases as the dislocations form low-energy (stable) configurations

a wall of edge dislocations corresponds to a low-angle tilt grain boundary ⇒ the rearrangement of dislocations into low-angle grain boundaries can lead to the formation of cellular sub-grain structure



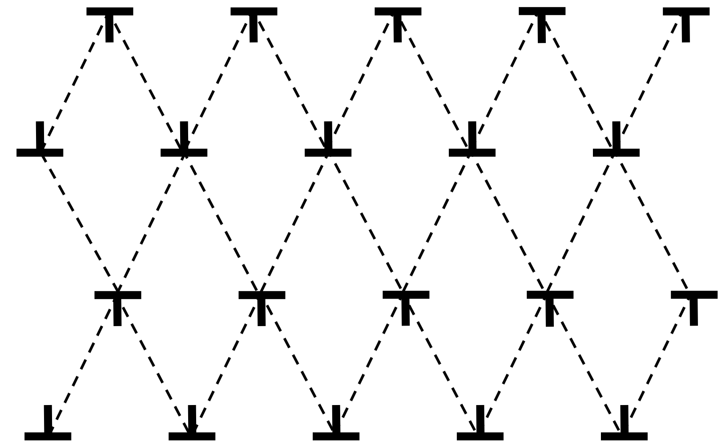
Energy of dislocation configurations

stable configuration for parallel straight edge dislocations



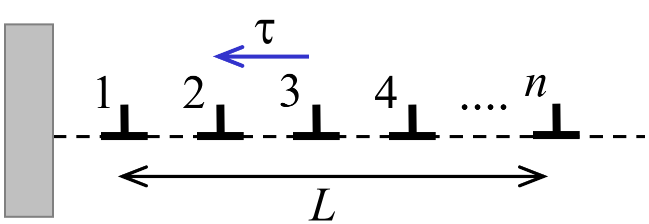
pile-ups of dislocations of opposite sign

dislocation dipoles



chessboard structure (Taylor lattice)

these configurations have weak long-range stress field



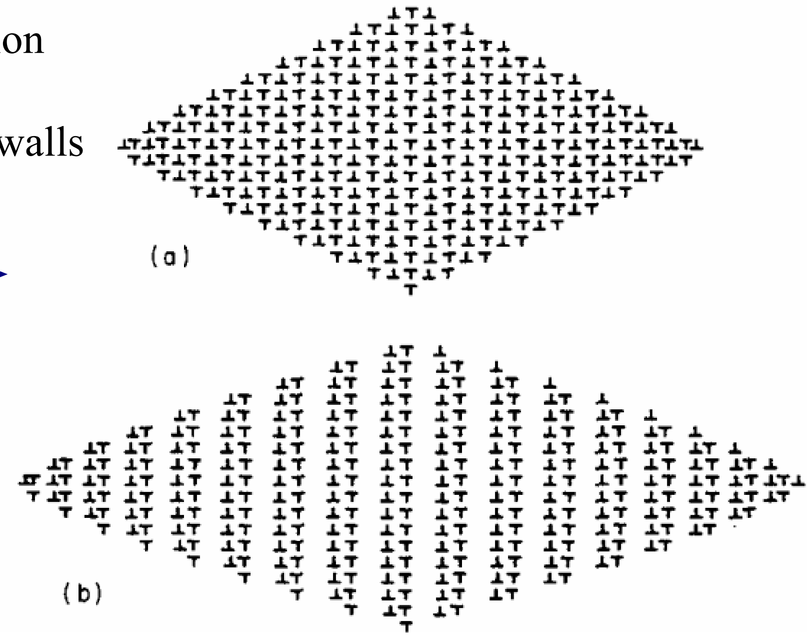
dislocation pile ups can be generated at the initial stage of plastic deformation (unstable configuration)

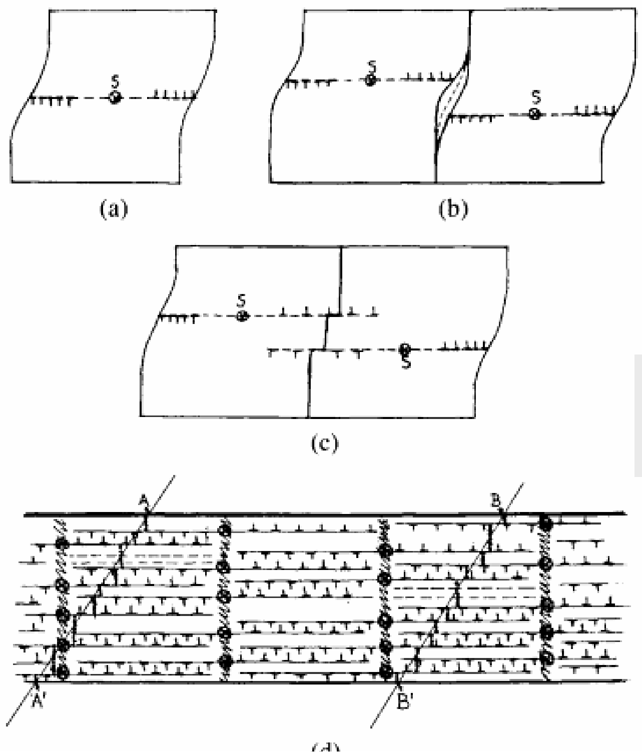
at large distances from pile-up ($r >> L$), the stress field created by the pile up is analogous to a super-dislocation with $\boldsymbol{B} = n\boldsymbol{b}$

Low-energy of dislocation configurations

2D computer simulation of a low-energy dislocation configuration (“Taylor lattice”): (a) relaxed quadrupole configuration and (b) configuration under critical stress for disintegration into dipole walls

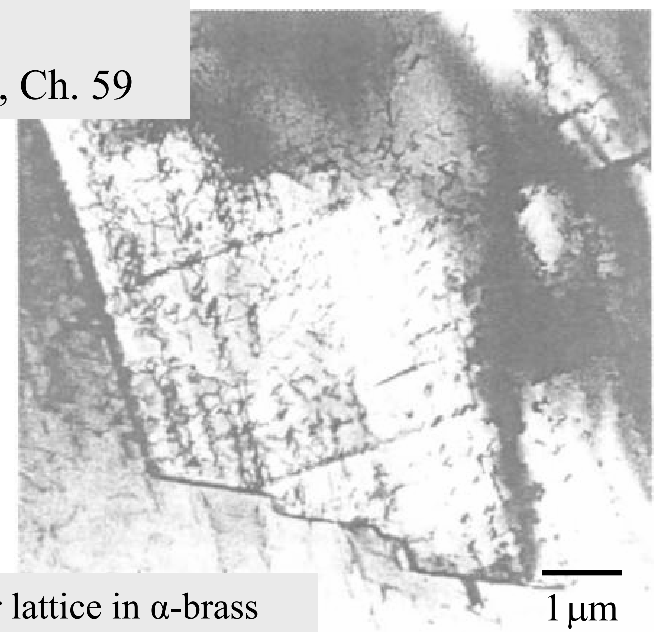
P. Neumann, Mater. Sci. Eng. **81**, 465, 1986





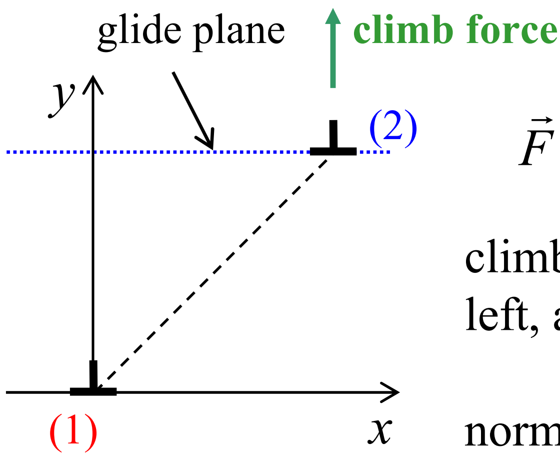
Model for formation of a Taylor lattice: (a) activation of a dislocation source in a volume element, (b) coordinated shape change of the volume elements as a similar sequence of dislocations arrives from a neighboring element, (c) formation dipoles and (d) formation of a Taylor lattice.

Doris Kuhlmann-Wilsdorf, in *Dislocations in Solids*, Vol. 11, Ch. 59



Formation of Taylor lattice in α-brass

Climb force



$$\vec{F} = (\sigma_{yx} b_2, -\sigma_{xx} b_2, 0)$$

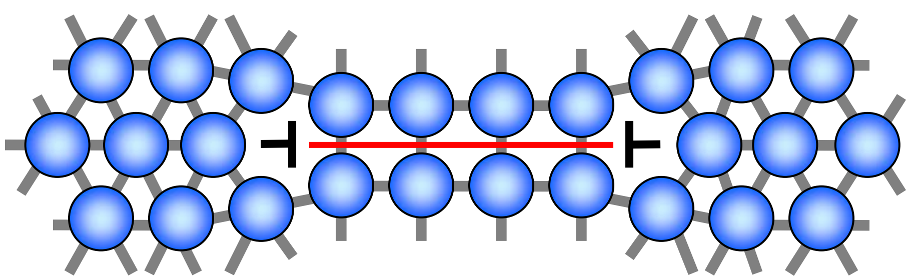
climb force originates from normal stress, e.g., σ_{xx} in the figure to the left, acting to squeeze the extra half plane of dislocation from the crystal

normal stress can also be created by external forces, line tension, etc.

edge or mixed dislocation can move away from its slip plane only with the help of point defects (vacancies or interstitials) - such motion is called *non-conservative motion* or *climb* (in contrast to the *conservative motion* within the slip plane)

motion of a screw dislocation is always conservative (never involves point defects)

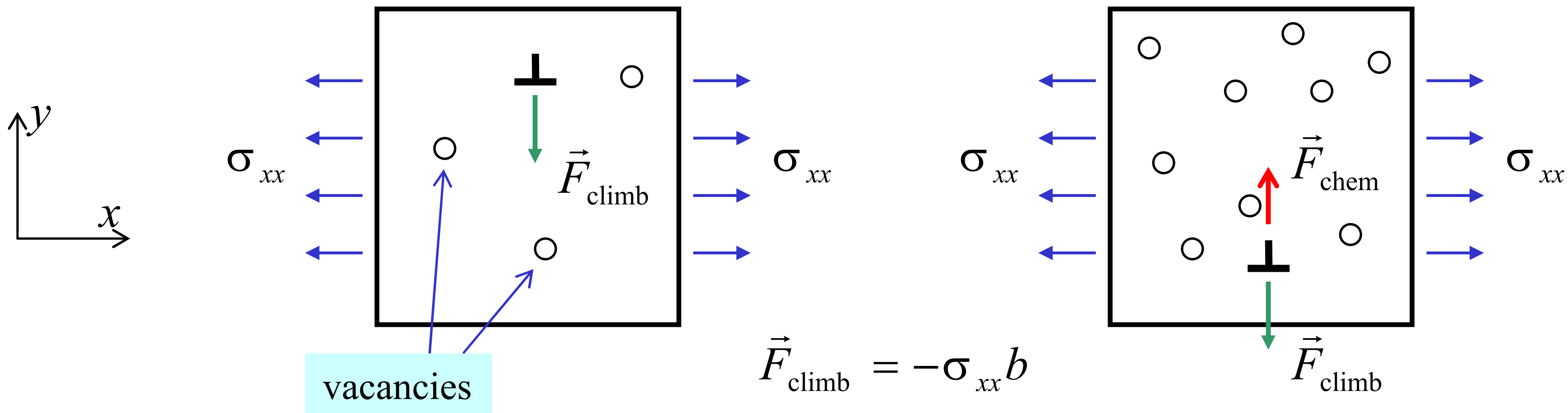
prismatic loop of partial dislocation



"conservative climb" is also possible for small prismatic loops of edge dislocations. The loop can move in its plane without shrinking or expanding at low T, when bulk diffusion of point defects is negligible.

the motion is due to the pipe diffusion of vacancies produced at one side of the loop and moving along the dislocation core to another side.

Chemical force



$$\vec{F}_{\text{climb}} = -\sigma_{xx} b$$

let's consider a volume of material where dislocation is the only sink and source of vacancies

without external stress, the equilibrium concentration of vacancies is maintained by absorption or birth of vacancies on the dislocation

when σ_{xx} is applied, $|\boldsymbol{F}_{\text{climb}}| = \sigma_{xx} b$ acts downwards and dislocation moves by emitting vacancies

vacancy concentration increases above the equilibrium concentration c_0 and it is increasingly difficult to create new vacancies

eventually, at vacancy concentration c, the movement stops; we can consider a *chemical force* acting against the climb force and opposite in direction, *i.e.*,

$$\vec{F}_{\text{chem}} = -\vec{F}_{\text{climb}} \quad \text{at} \quad c > c_0$$

Chemical force

the work done when a segment l climbs distance s in response to F_{climb} is $W_{\text{climb}} = F_{\text{climb}} ls$ and the number of vacancies emitted is $n_{\text{climb}} = bls/\Omega_a$, where Ω_a is volume per atom

the effective change in the vacancy formation energy is $W_{\text{climb}}/n_{\text{climb}} = F_{\text{climb}}\Omega_a/b$

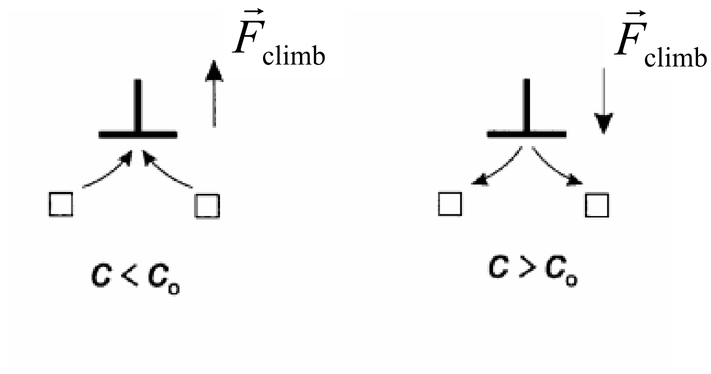
$$c = \exp\left(\frac{\Delta s_v}{k_B}\right)\exp\left(-\frac{\Delta h_f}{k_B T}\right)\exp\left(-\frac{F_{\text{climb}}\Omega_a/b}{k_B T}\right) = c_0 \exp\left(-\frac{F_{\text{climb}}\Omega_a/b}{k_B T}\right)$$

since $\vec{F}_{\text{chem}} = -\vec{F}_{\text{climb}}$

$$F_{chem} = \frac{bk_B T}{\Omega_a}\ln\left(\frac{c}{c_0}\right)$$

negative climb ($F_{\text{climb}} < 0$) $\Rightarrow$ vacancy emission $\Rightarrow c > c_0$

positive climb ($F_{\text{climb}} > 0$) $\Rightarrow$ vacancy absorption $\Rightarrow c < c_0$



supersaturation of vacancies $\Rightarrow$ chemical force $\Rightarrow$ dislocation climb until F_{chem} is not compensated by F_{climb} due to external/internal stresses or line tension

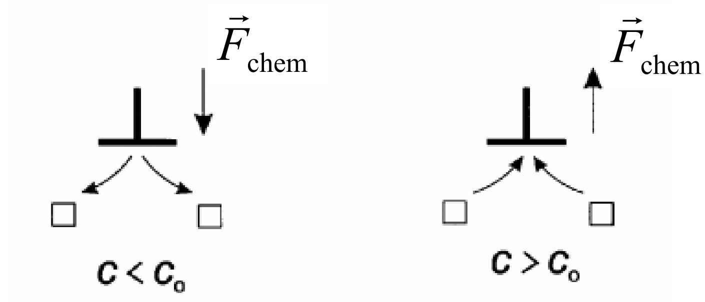
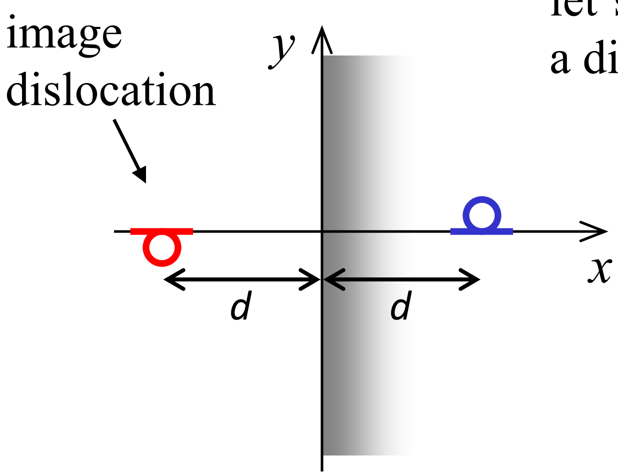


Image forces

The stress field generated by a dislocation is modified near a free surface, leading to extra forces acting on the dislocation (dislocation-surface interaction).

The normal and shear stress at a free surface are zero (there is "nothing" on one side of the boundary to provide reaction forces ⇒ there must be no normal or shear stress on the inside)



let's consider a straight screw dislocation parallel to z axes located at a distance d from a free surface at $x = 0$

to satisfy the condition of zero traction on plane $x = 0$, *i.e.*, $\sigma_{xx} = \sigma_{yx} = \sigma_{zx} = 0$, we can add stress field of an imaginary screw dislocation of opposite sign at $x = -d$

the stress field inside the body ($x > 0$) is then

$$\sigma_{zx} = \frac{Gb}{2\pi}\left[\frac{y}{(x+d)^2 + y^2} - \frac{y}{(x-d)^2 + y^2}\right]$$

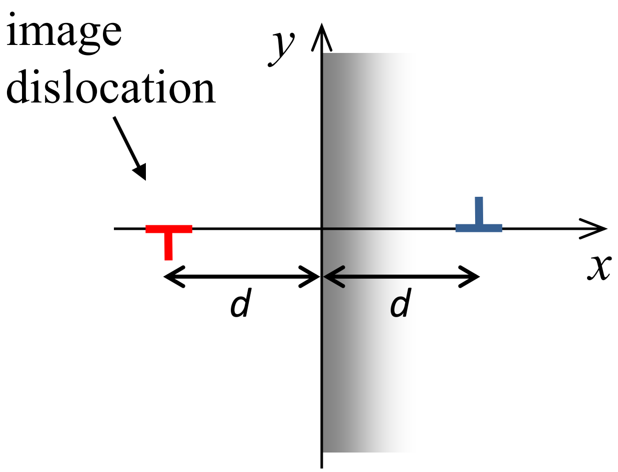
$$\sigma_{zy} = -\frac{Gb}{2\pi}\left[\underbrace{\frac{x+d}{(x+d)^2 + y^2}} - \underbrace{\frac{x-d}{(x-d)^2 + y^2}}\right]$$

force acting on the screw dislocation from the surface = force acting from the image dislocation:

$$F_x = \sigma_{zy}(x = d, y = 0)b = -\frac{Gb^2}{4\pi d}$$

only stress due to the image dislocation is accounted for

Image forces



For edge dislocation, adding an image dislocation at $x = -d$ cancels σ_{xx} and x = 0, but not σ_{yx}. Thus, an extra term should be added to match the boundary condition.

The shear stress is then:

$$\sigma_{yx} = -\frac{Gb}{2\pi(1-\nu)}\left[\underbrace{(x+d)\frac{(x+d)^2 - y^2}{\left((x+d)^2 + y^2\right)^2}}_{\top} - \underbrace{(x-d)\frac{(x-d)^2 - y^2}{\left((x-d)^2 + y^2\right)^2}}_{\perp} + \underbrace{\frac{2d(x-d)(x+d)^3 - 6x(x+d)y^2 + y^4}{\left((x+d)^2 + y^2\right)^3}}_{\text{extra term to ensure } \sigma_{yx} = 0 \text{ at } x = 0}\right]$$

force acting on the edge dislocation from the surface = force due to the 1st and 3rd terms

$$F_x = \sigma_{yx}(x = d, y = 0)b = -\frac{Gb^2}{4\pi(1-\upsilon)d}$$

(contribution from the 3rd term is zero)

dislocations are attracted to the surface ⇒ image forces can remove dislocations from the surface regions given that slip planes are oriented at large angles to the surface

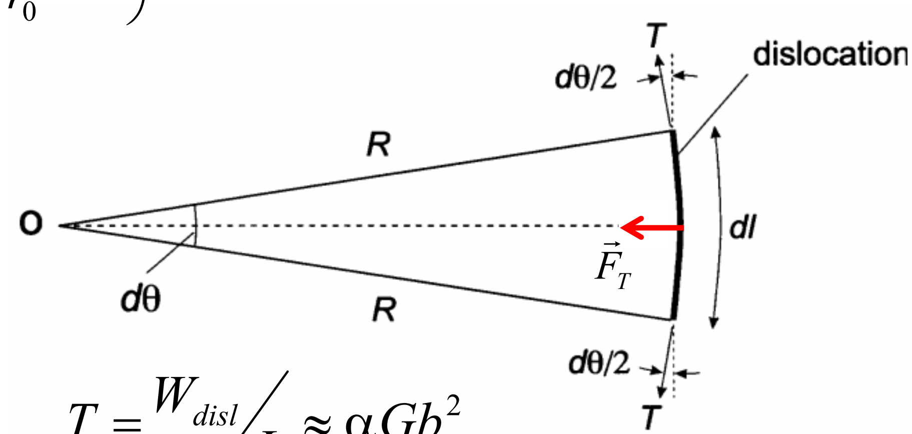
Interactions of curved dislocations, dislocation loops and dipoles, etc. can result in very complex stress fields that are often difficult to evaluate analytically

Line tension

Recall the result of our analysis of the dislocation energy: $W_{disl} = W_{el} + W_{core} = \frac{Gb^2 L}{4\pi K}\left(\ln\frac{R}{r_0} + Z\right)$

or, for energy per unit length, $W_{disl}/L = \frac{Gb^2}{4\pi K}\left(\ln\frac{R}{r_0} + Z\right) \approx \alpha G b^2$ $\quad \alpha \approx 0.5 - 1.5$

The line energy (energy per length) has the same dimension as a force and corresponds to line tension, *i.e.*, a force in the direction of the line vector which tries to shorten the dislocation



$$T = W_{disl}/L \approx \alpha G b^2$$

The resulting force $\boldsymbol{F_T}$ acting on an element *dl* of a dislocation line is related to the line tension $\boldsymbol{T}$ at the ends of the element and is perpendicular to the dislocation line $\left|\vec{F}_T\right| = 2T\sin(d\theta/2) \approx Td\theta$ $\quad d\theta \approx dl/R$

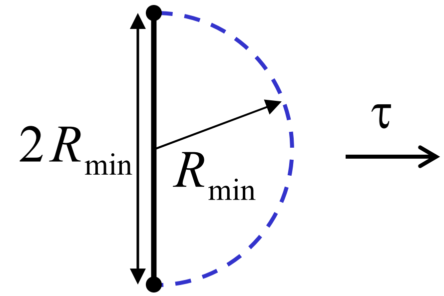
for small $d\theta$

force $\boldsymbol{F_\tau}$ acting on the same element *dl* due to the external shear stress is $\left|\vec{F}_\tau\right| = \tau b dl$

balance of forces to maintain radius *R* of the curved dislocation: $Td\theta = \tau b dl \Longrightarrow \tau = \frac{Td\theta}{bdl} = \frac{T}{bR} = \frac{\alpha G b}{R}$

Line tension

What happens when $\tau > \alpha Gb/R_{min}$?



dislocation segment pinned at its ends

How the dependence of the energy of dislocation on its type (edge vs. screw) affects the shape of the dislocation loop?