Ground truth - cisc 330 - A1

Thursday, September 19, 2024 9:47 AM

① example $N=1$ (generate 1 unit vector)

generate vector $[x,y,z]$

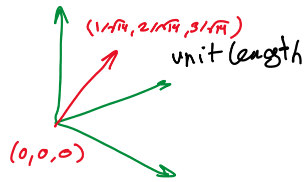
random $[1,2,3]$

$mag = \sqrt{1^2+2^2+3^2}$

$= \sqrt{1+4+9}$

$= \sqrt{14}$

$(1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14})$



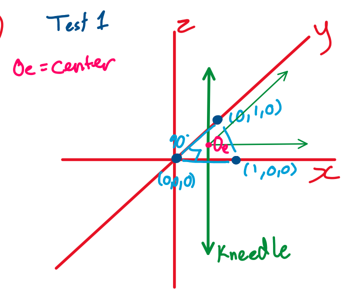
make a unit length sphere around (0, 0,0) and fit the vectors in it from (0,0,0) in a random direction

② Question 2 explanation of approach:

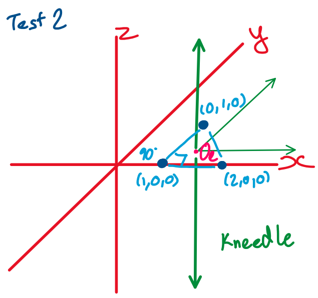
Generate points on a unit sphere and ensure the points lie on the surface (these are ground truth for testing). Generate points by adding 10% error. Reconstruct the inner sphere and use least squares to fit the sphere by minimizing the distance between the points. Then calculate the RMS error. If the error is above a threshold, remove outliers and reconstruct the fit iteratively. When the error is within the threshold, calculate the new radius using the center point and return C and R.

③ Test 1

O_e = center



Test 2



This is the same output as my figure output in the MATLAB code. Therefore, my program produces the expected output.

④ 3 ground truth cases

a) $\underbrace{75°}_{\text{angle}}$ $\underbrace{'x'}_{\text{axis}}$ $\underbrace{[2,3,4]}_{\text{point (P)}}$

homogeneous_transformation_matrix = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(135) & -\sin(135) & 0 \\ 0 & \sin(135) & \cos(135) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.707 & -0.707 & 0 \\ 0 & 0.707 & -0.707 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

matrix * P = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.707 & -0.707 & 0 \\ 0 & 0.707 & -0.707 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.000 \\ 4.3164 \\ 2.5237 \\ 1.000 \end{bmatrix}$

b) $90°$ 'y' $[2,1,1]$

homogeneous_transformation_matrix = $\begin{bmatrix} \cos(90) & 0 & \sin(90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(90) & 0 & \cos(90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

matrix * P = $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.0022 \\ 1.0000 \\ -2.2361 \\ 1.000 \end{bmatrix}$

c) $-45°$ 'z' $[1,3,0]$

homogeneous_transformation_matrix = $\begin{bmatrix} \cos(-45) & -\sin(-45) & 0 & 0 \\ \sin(-45) & \cos(-45) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.707 & 0.707 & 0 & 0 \\ -0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

matrix * P = $\begin{bmatrix} 0.707 & 0.707 & 0 & 0 \\ -0.707 & 0.707 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.0780 \\ 0.7251 \\ 0.0000 \\ 1.000 \end{bmatrix}$

This is the same output as my figure output in the MATLAB code. Therefore, my program produces the expected output.

⑤ $T_{V+oh} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$ $R = 3\times3$

$t = 3\times1$

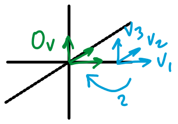
3 ground truth cases

a) pure translation

Test ⓐ $O_v = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ $V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $V_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $V_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

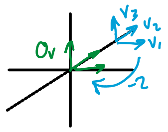
a) pure translation

Test (a) $O_v = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$



$$T_{v \to h} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

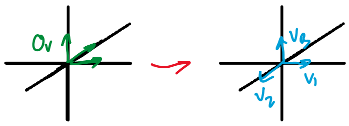
Test (b) $O_v = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$ $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$



$$T_{v \to h} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

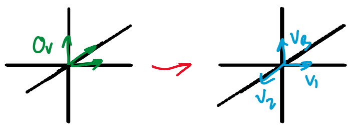
b) pure rotation

Test (a) $O_v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$



$$T_{v \to h} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

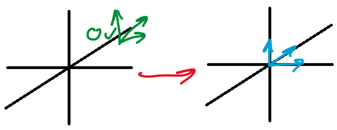
Test (b) $O_v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$



$$T_{v \to h} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

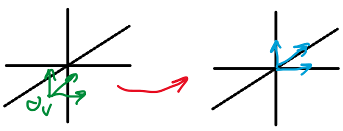
c) Both

Test (a) $O_v = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $v_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$



$$T_{h \leftarrow v} = \begin{bmatrix} 1 & 4 & 7 & 18 \\ 2 & 5 & 8 & 24 \\ 3 & 6 & 9 & 30 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Test (b) $O_v = \begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}$ $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $v_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$



$$T_{h \leftarrow v} = \begin{bmatrix} 1 & 4 & 7 & -18 \\ 2 & 5 & 8 & -21 \\ 3 & 6 & 9 & -24 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

∴ my code runs and produces the same output as shown here

6 3 ground truth cases

a) pure translation

Input: $M_{1}\_tool = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$M_{2}\_tool = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$M_{3}\text{-}tool = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$Ptool = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

output: $P_{ct} : \begin{bmatrix} 1.5380 \\ 0.1238 \\ -0.3938 \end{bmatrix}$

$V_{ct} : \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$M_3\text{-tool} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\qquad V_{ct} : \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$Ptool = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$vtool = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

b) pure rotation

Input: $M_1\_tool = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $M_2\_tool = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $M_3\text{-tool} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $Ptool = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $vtool = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

output:

$P_{ct} : \begin{bmatrix} 0.5286 \\ -0.5286 \\ 1.0000 \end{bmatrix}$

$V_{ct} : \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

rotation : 90°

$$R_{rot} = \begin{bmatrix} \cos(90) & -\sin(90) & 0 \\ \sin(90) & \cos(90) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c) Both

Input: $M_1\_tool = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $M_2\_tool = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $M_3\text{-tool} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $Ptool = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $vtool = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

output:

$P_{ct} : \begin{bmatrix} 1.0666 \\ -1.4048 \\ -0.3938 \end{bmatrix}$

$V_{ct} : \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$R_{rot} = \begin{bmatrix} \cos(90) & -\sin(90) & 0 \\ \sin(90) & \cos(90) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Translation : $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

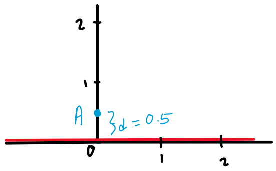
The expected outcome is that for pure translation, only the tool tip moves, while the axis remains unchanged.
For pure rotation, the tool tip stays fixed, but the axis rotates.
With both transformations, the tool tip and axis should change accordingly. Successful results confirm accurate transformation and alignment between frames.

7 Point to line

① point = [0, 0.5]

line: $P = [0, 0]$, $V = [1, 0]$

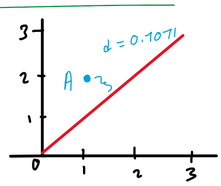
$$\text{distance} = \frac{|PA \cdot n|}{\|V\|} = \frac{|(0-0, 0-0.5)\cdot(0,1)|}{\sqrt{1^2+0^2}} = \frac{|(0,-0.5)\cdot(0,1)|}{\sqrt{1^2}} = \frac{|0\times 0 + -0.5\times 1|}{1} = \frac{|-0.5|}{1} = 0.5 \checkmark$$



② point = [1, 2]

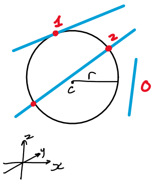
line: $P = [0, 0]$, $V = [3, 3]$

$$\text{distance} = \frac{|PA \cdot n|}{\|V\|} = \frac{|(1-0, 2-0)\cdot(-3,3)|}{\|(3-0, 3-0)\|} = \frac{|(1,2)\cdot(-3,3)|}{\|(3,3)\|} = \frac{|1\times -3 + 2\times 3|}{\sqrt{3^2+3^2}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0.7071 \checkmark$$



8 Ground Truth cases

sphere
C = (a,b,c) = (0,0,0) R = 5
$(x-a)^2+(y-b)^2+(z-c)^2=r^2$
$x^2+y^2+z^2=25$



0: a=3 b=6 c=-1 discriminant = b^2-4ac = 48

discriminant > 0 so num Intersections = 2
intersection points: [1.1547 1.1547 1.1547]
[-1.1547 -1.1547 -1.1547]

1: a=2 b=0 c=0 discriminant = b^2-4ac = 0

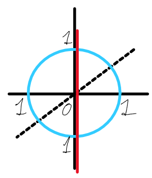
discriminant = 0 so num Intersections = 1
intersection point = [0 0 2]

2: a=2 b=0 c=5 discriminant = b^2-4ac = -40

discriminant < 0 so num Intersections = 0
no intersection point!

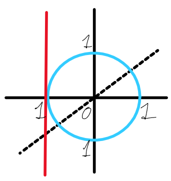
9 Ground Truth cases

① needle intersects the tumor



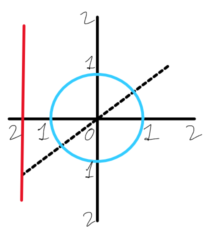
Targeting error: 0
Max Length of tissue Core: 2
needle insertion depth: 1

② needle touches the tumor



Targeting error: 1
Max Length of tissue Core: 0
needle insertion depth: 2

③ needle misses



Targeting error: 2
Max Length of tissue Core: 0
needle insertion depth: Nan

10

Pass rate = 100% - maximum allowable marker localization error
Pass rate < 100% - target error exceeds the clinical threshold (3). Beyond the allowable marker localization error
The targeting error refers to the distance between the needle's tool axis and the target point.
Marker localization error is the simulated displacement of the markers due to measurement inaccuracies.
The graphs show how the average targeting error changes as the magnitude of the marker localization error increases.
Up to 2mm of marker localization error, the targeting error remains consistent and relatively low.
Beyond 2mm, the targeting error begins to oscillate significantly.
The pass rate refers to the percentage of trials where the targeting error stays within clinical requirements, such as not exceeding a maximum allowed error.
In the graphs, the pass rate is consistently 0 for trials where the marker localization error is greater than 4 or 5mm.

Here are 3 examples of the output graphs.
Observe that the average targeting error in mm stays consistent below 2mm of marker localization error magnitude and then sclates quite significantly. For example, in the first screenshot, it is observed that the maximum average targeting error is 6.3 and the minimum is about 5.85.

The pass rate however stays consistently 0 until the marker localization error goes over 4 or 5 as seen in the screenshots.