Gamma Knife Radiosurgery Planning
Take-home exam for CISC/CMPE 330

Problem Statement

We will treat a small metastatic brain tumor with Gamma Knife radiosurgery (**Figure 1**). Gamma Knife radiosurgery is a type of radiation therapy used to treat tumors, vascular malformations and other abnormalities in the brain. Gamma Knife radiosurgery is an alternative to Linac radiosurgery and Cyber Knife radiosurgery. Gamma Knife uses $Cobalt^{60}$ sources. Essentially, the classic Gamma Knife does not contain electronic components, and if it needs to, it can be operated without even electricity, and as such it is suitable for harsh environments where even electric power is uncertain.

Fig. 1: Gamma Knife Radiosurgery

The Gamma Knife helmet provides $Cobalt^{60}$ pencil beams over a hemisphere (**Figure 2**). With some forgivable simplification of a real-life Gamma Knife helmet, our beams are separated by uniform polar angles. The centerlines of the pencil beams intersect in the center of the hemispherical helmet. This point is called the isocenter point or shortly isocenter. The shape of each pencil beam is considered as a cylinder of a given radius.

Fig. 2: Full hemispherical Gamma Knife helmet

The tumor was contoured in cross sectional images. The prescribed target volume (PTV) was contoured by leaving some margin around the tumor: the yellow contour in **Figure 3**. The organ at risk (OAR) was also contoured: the red contour in **Figure 3**. The PTV was reconstructed in 3D and approximated as a sphere, which in real life is often appropriate for small convex tumors, like many metastatic cancer lesions. Let us assume that the OAR was reconstructed in 3D and, for the purpose of the exercise, it was approximated as an ellipsoid.

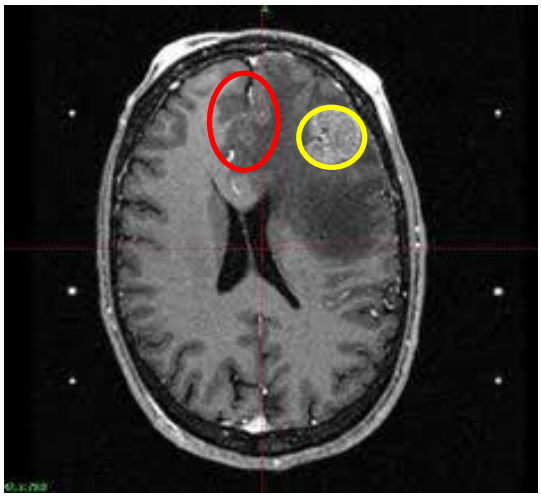


Fig. 3: The PTV (yellow) and OAR (red) contoured in a cross-sectional image.

We will irradiate the PTV with a helmet that provides pencil beams aimed at a common center, called isocenter. With the use of the head frame bolted into the skull, we will position the patient inside the helmet so that the isocenter coincides with the center of the PTV. For a small and spherical tumor, we plan to treat deliver the full prescribed dose without moving the head inside the helmet during the treatment. ***For the sake of this exercise, the treatment goal is to envelop the entire PTV in the prescribed therapeutic dose (D_{100}) as tightly as possible, while none of the OAR receives more than the maximum allotted safe dose (D_{OAR})***. This is a cautious approach with strong protection for OAR. In most clinical cases, we allow a certain maximum percentage of the OAR's volume to receive a higher dose.

To spare the OAR from toxic radiation, we plan to "plug" the beams that would directly hit the OAR. (Literally, we place metal plugs in the beam holes that we want to block out.) If we are not able to achieve sufficient dose for the PTV, we will unplug some of the "less dangerous" beams to increase dose coverage for the tumor.

Let us assume that we will turn on the beams for a nominal irradiation time (T_{nom}) and compute the radiation dose in the PTV and OAR. We use a uniform Cartesian grid of some suitable resolution inside these structures and compute the radiation dose in each node of the grid. We compute the coldest dose in the tumor and in the hottest dose in the organ at risk.

Next, we start the process known as "*Iterative Planning*". First, we determine if the treatment goal can be achieved by manipulation of the irradiation time and we compute the optimal irradiation time (T_{opt}), to ensure that we envelop the PTV entirely in the prescribed dose (D_{100}) without endangering the OAR. Unfortunately, it is possible that the treatment goal is not attainable by simply manipulating the irradiation time. If we are unable to give enough dose to the PTV, then we will unplug some of the "less dangerous" beams. If the treatment goal cannot be achieved because we exceed the toxic dose limit to the OAR, then we will plug additional beams. If we are still unable to reach the treatment goal, then we will deliver the prescribed dose in multiple fractions: we will apply different irradiation times with unplugging/plugging different subsets of the beams. If this still does not help enough, then we will slightly reposition the patient several times inside the helmet for each fraction of irradiation. If we are still unsuccessful, we will use different helmets with different beam diameters for the different irradiation fractions.

Once the treatment goal is achieved, we must evaluate how large portions of the PTV and OAR are overdosed or underdosed. The dose coverage is analyzed with cumulative dose volume histograms (DVH) such as in **Figure 4**),

showing what percentage of the structure's volume receives a certain amount of dose. DVH is crucial because all clinical dosimetry standards in radiation therapy are defined in terms of DVH metrics. The DVH in this case for PTV (red curve) and OAR (purple curve) should look like this below. The entire 100% of the PTV volume gets the D_{100} dose, then the curve drops – this means that the PTV is fully dosed without over-radiating. At the same time, no part of the OAR reaches the D_{oar} dose limit, so the OAR is safe

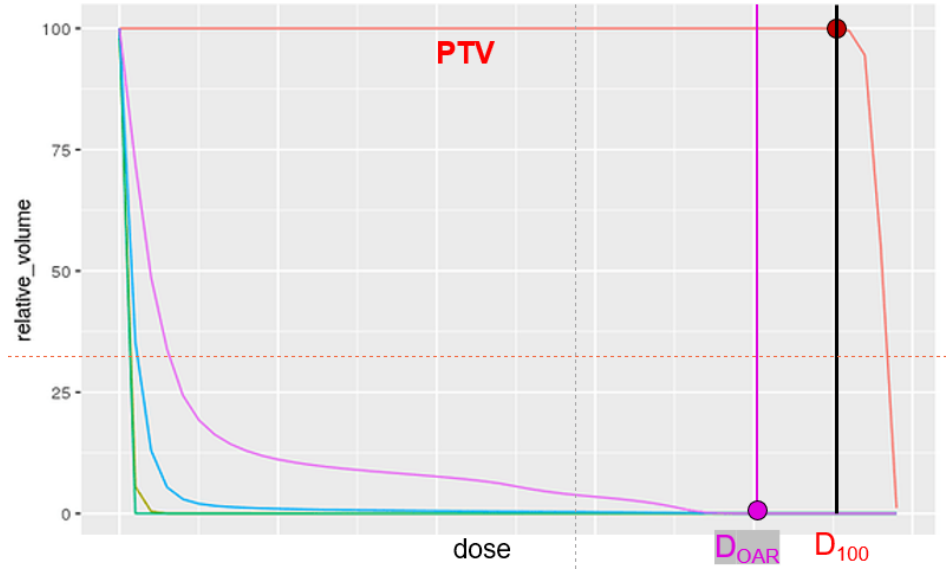


Fig. 4: Cumulative Dose Volume Histogram (DVH)

Treatment Parameters & Simplifying Assumptions

We will make a few simplifying assumptions that do not alter the essence of the clinical and computational problem at hand, and as such they are quite permissible for the sake of this exercise.

For treatment planning, the head is contoured in the cross-sectional magnetic resonance images (MRI) and reconstructed in 3D. For this exercise, the PTV is approximated as a sphere and the OAR is approximated as an ellipsoid. In many real-life patient cases, we can approximate the head as an ellipsoid. With clever pre-alignment of the patient in the MRI scanner, we can achieve that head ellipsoid lays in center of the MRI frame without rotation, so the principal axes of the ellipsoid are aligned with the axes of the MRI imaging coordinate system, such as shown in **Figure 5**, where a, b, and c mark the half length of the principal axes of the ellipsoid.

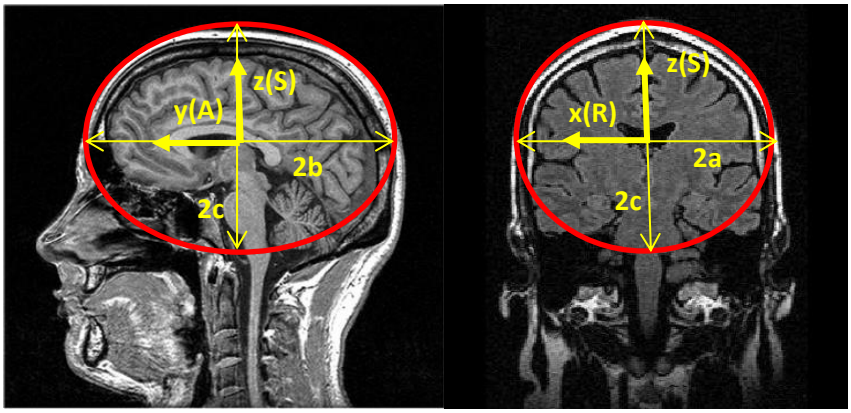


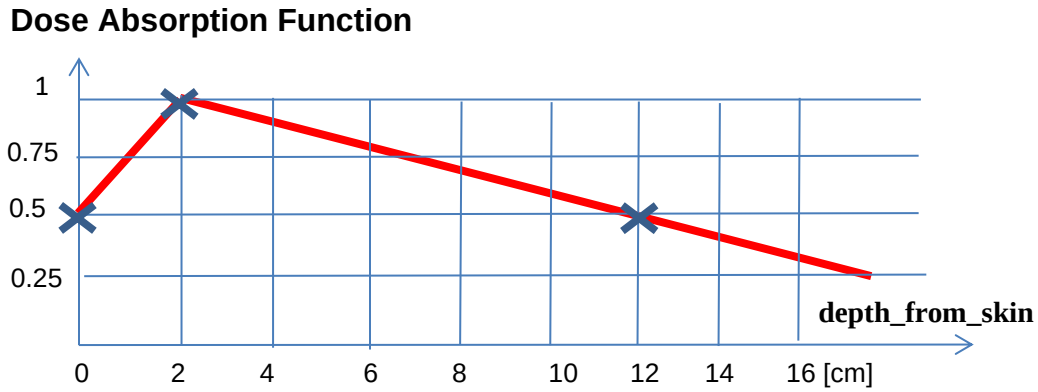
Fig. 5: Ellipsoid approximating the patient's head

['DU' means dose unit below]
D_0 = 0.10 DU / minute
T_{nom} = 10 minutes
D_{100} = 20 DU
D_{OAR} = 10 DU

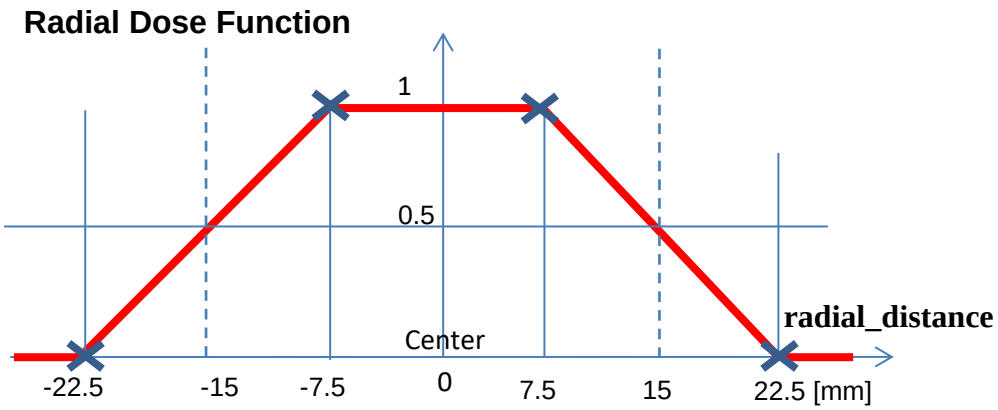
All in "mm" units:
Helmet: beam_separation_angles = 30 deg longitude, 30 deg latitude, beam_diameter = 30
PTV sphere: ptv_radius = 15, ptv_center = (30, 0, 15)
OAR ellipsoid: oar_a=15, oar_b=25, oar_c = 15, oar_center = (0, 30, 45), axes aligned with the MRI frame
Head ellipsoid: head_a = 80, head_b =100, head_c=80, head_center = (0, 0, 0), axes aligned with the MRI frame.

The OAR and Head ellipsoids are "elongated" in the y (anterior) direction. They are aligned with coordinate axes of the MRI frame with no rotation.

The Dose Absorption Function (DAF) and the Radial Dose Functions (RDF) were approximated as shown below. In the Dose Absorption Function (DAF), the "build-up" region is linear, the beam delivers a Do=1.0 unit of dose at d_{max} = 2.0 cm depth in the beam's centerline, then the absorbed dose attenuates linearly function according to the figure below:



For the Radial Dose Function, we assume that the beam is radially "leaking" some dose outside the collimator, approximated by linear functions as shown below:



Questions, Functions and Problems

To produce a viable radiosurgery treatment plan with adequate dosimetry analysis, you will need to produce comprehensive system design and step by step implementation. We will break down the process to small steps, modules and functions. You are in full control of the system and software design. Communicate your choices by generous comments, and/or diagrams, and/or sketches – anything you find suitable. No need to go overboard, this is not a software design course. Do not hardcode variables and parameters inside the functions. Instead, you should communicate parameters through input/output, or you can apply global variables or external .m files – whichever you find suitable. There is no need to design elaborate data structures, arrays will suffice.

1. Draw 3D Scene

Description: Draw the 3D scene with Head, PTV, OAR, isocenter, and the coordinate axes. Choose the appropriate surface representations, colors, etc. for meaningful visual presentation. The purpose of this step is to form some visual and spatial comprehension of the scene.
Output: None beyond the plots
Test: Run the function, and plot and save some representative views.
Hint: You could use MATLAB functions for sphere, ellipsoid, etc. as needed.

To speed up the dose calculation and analysis and to simplify our software design and workflow, we precompute constant parameters for the beams, head, PTV and OAR, thus we can avoid many costly recurring computations. You should save the "precomputed" results in appropriate arrays for later use.

2. Compute Dose Absorption Function Table

Description: Write a function to compute Dose Absorption Function according to the plot provided earlier. Write the results into a table with some sensible resolution (such as 1 mm increments). You will use this table for quick access during dose computation.
Test: Run the function, print the table, compare to the plot, ascertain correctness.

3. Compute Radial Dose Function Table

Description: Write a function to compute radial dose function value according to according to the plot provided earlier. Write the results into a table with some sensible resolution (such as 1 mm increments). You will use this table for quick access during dose computation.
Test: Run the function, print the table, compare to the plot, ascertain correctness.

4. Compute Beam Directions

Description: Write a function to compute the unit direction vector for each pencil beam's centerline.
Test: Draw a 3D scene of the beams and coordinate axes, ascertain correctness; save and submit some representative views.

5. Compute Skin Entry Points

Description: Write a function to compute the skin entry point for each beam's central line. Also compute the depth of the isocenter from the skin entry point.

Test: Plot the 3D scene with the head and centerline of each pencil beam. Add a marker to the computed skin entry points. Check visually whether the markers coincide with the beams intersecting the head. If the plot is too busy, randomly select fewer beams for testing; save and submit some representative views.
Hint: Write up the equation of the beam's central line and intersect it with the head's ellipse.

6. Compute Dose Grid for PTV

Description: Develop a function to generate a box around the PTV and a uniform cartesian grid in the box. For each grid point, compute its (x,y,z) position, compute a flag whether the grid point falls in/on/out of PTV, and set the dose values to zero. (You will populate the grid with dose values later.)
Test: Plot the box, grid and flags in 3D – adjust grid size to avoid crowding the plot. Inspect the outcome visually.
Hint: Make the grid size variable; you may like to use a coarser grid size for testing and debugging, and then use a finer grid size (such as 1.0 mm) for dosimetric analysis.

7. Compute Dose Grid for OAR

Repeat the same as above for the OAR.

8. Compute Beam Safety

Description: Write a function to compute the volume of interaction between each beam and the OAR. (As you should remember, a beam is generally unsafe when it has a direct hit on the OAR. In the initial treatment plan, the unsafe beams will be excluded so they will not contribute any dose. But if the treatment objective cannot be met with all safe beams, then we will unplug the "least unsafe" beams (i.e. beams with smallest intersection with the OAR.) Make a sketch and explain your approach in comments – as you see it best.
Test: Plot the 3D scene with the unsafe beams and OAR; inspect and interpret; save and submit some representative views.
Hint: While it is not simple to compute this analytically, you can discretize.

Next, you need to develop utility functions toward computing the "point dose", which is the dose delivered to a given point of interest by all allotted beams. (Remember, beams that we designated 'unsafe' will be excluded.) You will call these functions numerous times in loops; they will take the "point of interest" for input and return the appropriate result to the caller.

9. Compute Radial Distance

Description: Write a function to compute the radial distance between arbitrary point of interest and the beam central, for a given beam.
Input: point_of_interest, beam_index
Test: Construct ground truth tests with three "easy beams" along the x, y, and z axes and with a convenient setting of the "point_of_interest". Show that your module works correctly.
Hint: Refer to the lecture notes.

10. Compute Depth from Skin

Description: Write a function to compute the skin depth along the beam central belonging to an arbitrary point of interest, for a given pencil beam.
Input: point_of_interest, beam_index

Test: Run the function for the same ground truth tests as above. Show that your module works correctly.
Hint: Refer to the lecture notes.

11. Compute Point Dose by One Beam

Description: Write a function to compute the dose at a point of interest from a given beam.
Input: point_of_interest, beam_index
Test: Run the function for the same three "easy ground truth beams" as above. Show that your module works correctly.
Hint: Refer to the lecture notes

12. Compute Point Dose by All Beams

Description: Write a function to compute the so called "point dose", which is the dose delivered to a given point of interest by all allotted beams. (Remember, beams that we designated 'unsafe' will be excluded.).
Input: point_of_interest
Test: Full testing this function is not easy. For partial testing, set the "point_of_interest" in the center of the PTV.
Hint: Refer to the lecture notes.

Having developed the functions to compute the dose in an arbitrary point in the head, you proceed to compute the dose for the PTV and OAR…

13. Compute Dose for PTV

Description: Compute the dose delivered to the PTV over the dose grid. Compute the extreme doses.
Test: Add these to the 3D scene plot. Evaluate your findings with respect to treatment objective.
Hint: Refer to the lecture notes.

14. Compute Dose for OAR

Repeat the same for the OAR.

15. Compute Optimal Irradiation Time

Description: Examine the extreme doses in the PTV and OAR on the results from above and determine if the treatment objective can be attained with the current beam configuration. If yes, then compute the optimal irradiation time. Otherwise exit this function. Re-examine the 3D scene plot and the hot/cold spots in the PTV and OAR and decide which additional beams you should plug/unplug; recompute the hottest and coldest dose in the PTV and OAR and repeat this function.
Test: Recompute the dose inside the PTV and OAR and show that you achieved the treatment goal.

Having computed the optimal dose inside PTV and OAR, you need to analyze the spatial distribution of the dose in a cumulative dose volume histogram.

16. Compute Dose Volume Histogram

Description: Compute the common cumulative dose volume histogram for both PTV and OAR.
Test: Plot the DVH in the standard form (see Figure 4) with marking D_{OAR} and D_{100} and explain your findings with respect to the treatment goal.

Hint: You may like to use MATLAB's "histcounts (X,edges)" function, where X are the dose values and the bin edges can go from 0, by increments of 1% of D_{100} - which then should yield a smooth curve.

In many cases, we are concerned about excessive radiation dose delivered to the surface of the organ at risk. For example, in treating prostate cancer, the main organ at risk is the rectum with a thin mucosal layer that is exceedingly sensitive to radiation. To this end, we need to visualize the distribution of the dose on the surface of the organ at risk and analyze the dose distribution with a Dose Surface Histogram, showing what percentage of the organ's surface receives a certain amount of dose.

17. Compute Surface Dose for OAR

Description: Write a function to compute the dose on the surface of the OAR. Compute the hottest and coldest dose and locations.

Test: Plot the result as a colored surface with a 'colorbar' showing a color scale, mark the extremes on the surface plot. Analyze your findings with respect to dosimetric objectives.

Hint: Consider using the "ellipsoid" function to get surface points and patches on the ellipsoid. Consider "surf" or "mesh" with "colorbar" for plotting.

18. Compute Dose Surface Histogram for OAR

Description: Compute the cumulative Dose Surface Histogram (DSH) for the organ at risk.

Test: Plot the histogram in the standard form (refer to course notes) with marking D_{OAR} and explain your findings with respect to the treatment goal.

Hint: Having computed the dose for each patch on the OAR, you can assign the same dose value to each square mm in the patch, so you can we use the "histcounts" function.

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