Set 5.

1. $p(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$$E(X) = \sum_{x=0}^{\infty} x\frac{e^{-\lambda}\lambda^x}{x!} = \sum_{x=1}^{\infty} x\frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{x!}$$

$$= \lambda e^{-\lambda} e^{\lambda}$$

$$= \lambda \checkmark$$

2. $p(x=x|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$

A. $L(\lambda|x) = p(x=x_1|\lambda)\,p(x=x_2|\lambda)\cdots p(x=x_n|\lambda)$

$$= \frac{e^{-\lambda}\lambda^{x_1}}{x_1!} \cdot \cdots \cdot \frac{e^{-\lambda}\lambda^{x_n}}{x_n!} = \frac{e^{-n\lambda}\lambda^{x_1+x_2+\cdots+x_n}}{x_1!\,x_2!\,x_3!}$$

$$= \frac{e^{-n\lambda}\lambda^{\sum x_i}}{\prod_{i=1}^{n} x_i!}$$

$$\ln L(\lambda|x) = -n\lambda + \sum_{i=1}^{n} x_i \ln\lambda - \ln\left(\prod_{i=1}^{n} x_i!\right)$$

B. $$\frac{d\ln L(\lambda|x)}{d\lambda} = -n + \frac{\sum x_i}{\lambda} = 0$$

$$\hat{\lambda} = \frac{\sum_{i=1}^{n} x_i}{n}$$

3. $X, Y \sim Geom(p)$

$P(X=k) = (1-p)^{k-1}p$, $k = 1, 2, 3, \ldots$

a. Show $P(X+Y=k) = (k-1)(1-p)^{k-2}p^2$ for $k = 2, 3, \ldots$

$$\begin{aligned} P(X+Y=k) &= \sum_{i=1}^{k-1} P(X=i, X=k-i) \\ &= \sum_{i=1}^{k-1} P(X=i)P(X=k-i) \\ &= \sum_{i=1}^{k-1} p(1-p)^{i-1} p(1-p)^{k-i-1} \\ &= p^2(1-p)^{k-2} \sum_{i=1}^{k-1} 1 \\ &= p^2(1-p)^{k-2}(k-1) \end{aligned}$$ ✓

b. Z is negative binomial.

$$P(Z=z) = \binom{z-1}{r-1} p^r (1-p)^{z-r}, \quad z = r, r+1 \ldots$$

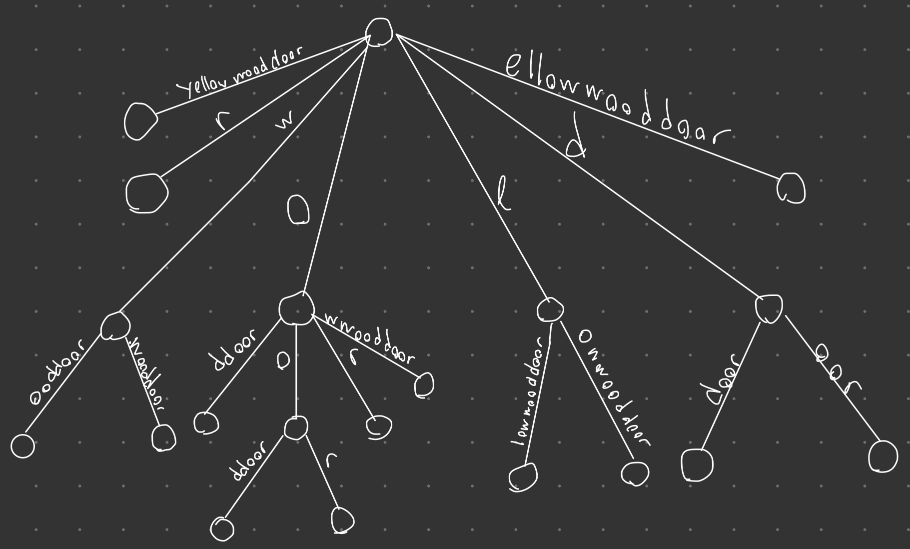
Show $X+Y \sim negbinom(2, p)$

$$P(Z=z) \overset{r=2}{=} \binom{z-1}{1} p^2 (1-p)^{z-2}$$

$$= (z-1)p^2(1-p)^{z-2} \quad \text{for } z = 2, 3, \ldots$$

$$= (k-1)(1-p)^{k-2}p^2 \quad \text{for } k = 2, 3, \ldots$$ ✓

4.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