Sec 3.

1. a. $\left(\frac{1-q}{n}\right)^{c} \cdot q$

$\sum_{c=0}^{n} 10^{c} = r$ = upper

$\sum_{c=0}^{n-1} 10^{c} + 1 = r$ = lower bound

b. $\lim_{r \to \infty} \frac{\log\left(\left(\frac{1-q}{n}\right)^{\log_n(r)} \cdot q\right)}{\log(r)}$ Straight into wolfram

$\log_n(r) = c$

$\rightarrow \frac{\log\left(\frac{1-q}{n}\right)}{\log(n)}$ Change of base $= \log_n\left(\frac{1-q}{n}\right)$

C. we have a non-zero asymptote, so it is heavy tailed.

i.e. the ratio is non-zero for $\frac{ft}{r}$ b/c the ratio at lags is non-zero. on one end of the scale, it is a very steep curve as well. This means heavy tail.

The difference is this eq. is prob of a specific word of rank r being chosen, instead of any of length C.

3. a. $E[f_d]$ $\quad f_d = c \cdot d^{-\alpha}$,

n is on order of c.

$$\frac{1}{C}\sum_{d=1}^{C} d \cdot c d^{-\alpha} = \frac{1}{C} \cdot c \sum_{d=1}^{C} d^{1-\alpha}$$

$$= \frac{1}{C} \cdot C \cdot \sum_{d=1}^{C^{\frac{1}{\alpha}}} d^{1-\alpha}$$

$$= \frac{1}{C} \cdot \Theta(C^{\frac{2}{\alpha}})$$

$$= \Theta(C^{\frac{2}{\alpha}-1})$$

ii) $P(D \geq cu) \leq \frac{E[D]}{cu}$

$$1 - P(D \geq cu) \geq 1 - \frac{E[D]}{cu}$$

$P(D < cu) \geq 1 - \frac{E[D]}{cu}$ we have terms on similar Θ, for O, we need

$\geq 1 - \frac{k_1}{k_2}$, Setting $k_2 = 2k$, gives us the desired result.

iii) $\frac{1}{2}\sum_{d=C^{\frac{1}{\alpha}-\varepsilon}}^{C^{\frac{1}{\alpha}}} d \cdot c d^{-\alpha} = \frac{1}{2} C \sum_{d=C^{\frac{1}{\alpha}-\varepsilon}}^{C^{\frac{1}{\alpha}}} d^{1-\alpha} = C \sum_{d=1}^{C^{\frac{1}{\alpha}}} d^{1-\alpha} - C \sum_{d=1}^{C^{\frac{1}{\alpha}-\varepsilon}} d^{1-\alpha}$.

$\uparrow \Theta(C^{\frac{2}{\alpha}})$ $\qquad \nwarrow \sum_{d=1}^{t} d^{-\beta} = \Theta(t^{1-\beta})$

$t = C^{\frac{1}{\alpha}-\varepsilon}$

$\beta = \alpha - 1$

$\Theta(C \cdot C^{(\frac{1}{\alpha}-\varepsilon)(2-\alpha)})$

$\Theta(C_2 \cdot C^{\frac{2}{\alpha}-1-2\varepsilon+\alpha\varepsilon})$

$\Theta(C^{\frac{2}{\alpha}-\varepsilon(2-\alpha)})$

$= \Theta(C^{\frac{2}{\alpha}} - C^{\frac{2}{\alpha}-\varepsilon(2-\alpha)})$

↑ Smaller order term goes away in Θ notation.

$= \Theta(C^{\frac{2}{\alpha}}) \in \Theta(cn)$ ✓

iv) $P(L) = C \dfrac{\sum_{d=C^{\frac{1}{\alpha}-\varepsilon}}^{C} d^{1-\alpha}}{\sum_{d=1}^{C} d^{1-\alpha}}$ $\Theta(m)$ ↙

So $= \Theta(1)$

$m = \Theta(cn)$ ↑

$P(S) = 1 - P(L)$

$P(\ell) = 1 - P(S)^d$

So: $\frac{1}{n}\sum_{d=1}^{C} C d^{-\alpha} P(\ell)$ $\qquad \Theta(1)$ ↓

$\frac{1}{n}\sum_{d=1}^{C} C d^{-\alpha}\left(1 - \left(1 - P(L)\right)^d\right)$

$\Theta(n)$ ↑ $\qquad$ Smaller than 1.

Smaller than 1

v) by ii) and iv), we have $\quad \frac{\Theta(n)}{n}\Theta(1) \Rightarrow \Theta(1)$ ✓ order of $\Theta(1)$

$\frac{C^{\frac{1}{\alpha}-\varepsilon}}{C^{\frac{2}{\alpha}-1}} \to C^{1-\frac{1}{\alpha}-\varepsilon}$ as desired.

c is order of n.

so we have

$\Omega(n^{1-\frac{1}{\alpha}-\varepsilon})$ as desired!

5. a. $\ell < k$

$$p_r = \sum_{i=0}^{\ell-1} p^i \cdot (1-p)^{\ell-1}$$

$\ell = k$

$$p_r = \left(\sum_{i=0}^{\ell-1} p^i (1-p)^{\ell-1}\right) + (1-p)^{k+1} + (k+1)(1-p)^k p$$

Plugging into mathematica, mult by ℓ for Ev, we get

$$\frac{2 - p + (1-p)^k (-2 + p - kp)}{p}$$

b. Expected Avg Shortest path length:

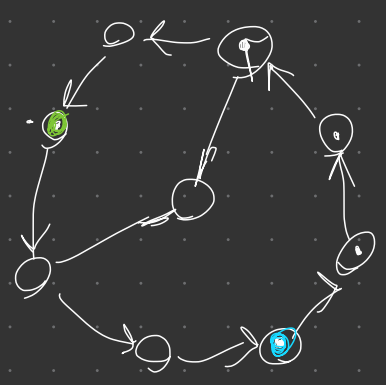
$$\frac{-3 + 2p + 2np - np^2 + (1-p)^n (3 + (-2+n)p)}{(-1+n)p^2}$$

Scales exponentially w/ n.

far no central node, $\frac{1}{n} \cdot n \sum_{i=1}^{n-1} k = \frac{(n-1)(n-2)}{2} = \frac{n-2}{2}$

Scales linearly

c.



Works in all but this case, in which case it will add 1 to the Shortest path

The algo would work in all but 1 class of cases. if B always routes to the closest node, we always want to take B, since in most cases, we skip at least 1 hop of length 1 if we take B. Thus, given a path through B or not through B, we would want to take B in most cases. The case we wouldn't want to is the one where if we take B, but the closest exit from B is the one we took, as above going from Blue to green. In cases like this, the algo takes B, but comes back where it entered and moves on, rather than skip B. in this case, we add 1 to the Shortest path, which is pretty close to the real Shortest path. An "accidental" traversal of B would only happen once, since if it happened twice, the first traversal would have come out at the Second traversal's point, a contradiction. Thus, This algo admits the Shortest path in all but 1 case, when we have to take B, but the edge closest to the destination is the one we took, adding 1 to the resulting path, which is very close to the real Shortest path.

6. a. $m_i(t+1) - m_i(t)$

$i = t+1$	$i \leq t$	$i > t$
indiv only, inc = 1	$\frac{m_i(t)}{2t-1}$ = inc.	inc = 0.

$$m_i(t+1) = \frac{m_i(t)}{2t-1} + m_i(t)$$

B. Using MV approx and cont. approx, we can see how
The eqn follows from newton's method w/ $h=1$.

C. DSolve:

$m_i(t) = 0$ if $t < i$
$= \sqrt{-1+2t}\, C_1$ if $t > i$
$= 1$ if $t = i$

$1 = m_i(i) = \sqrt{-1+2i}\cdot C_1$

$\log C = \frac{1}{\frac{1}{2}\log 2i}$

$C_1 = \frac{1}{\sqrt{2i}}$

D. $\frac{mp(t)}{\sum_{i=1}^{t} m_i(t)}$

Subbing in C and solving we get

$\frac{1}{\text{Harmonic\#}(t, \frac{1}{2})}$ which goes to 0 as time moves on.

Thus, popular products have less market share.