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# Conceptualizing Surprise with the Jensen–Shannon distance\*

A Bayesian Information-Theoretic approach for the Social Sciences

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1 February 2026

## Abstract

Generally in the social sciences, results are informally deemed *surprising* if their associated  $p$ -value is sufficiently small. This implicit interpretation, however, is both conceptually and mathematically inappropriate, and such misuse of the  $p$ -value can lead to erroneous conclusions about the novelty of results. To solve that issue, this paper builds on Bayesian inference, Information theory, and considerations specific to the social sciences, to argue for the adoption of a more appropriate conceptualization of surprise as the *relative entropy between prior and posterior knowledge*. This formal conceptualization enables researchers to appropriately measure surprise as the Jensen-Shannon distance, for which the paper provides easily implementable software and a demonstration of its use in relation to empirical data.©<sup>1</sup>

**Keywords:** *Surprise; Novelty; Jensen-Shannon distance; JS distance; Jensen-Shannon divergence; JS divergence; Relative entropy; Kullback-Leibler divergence; KL divergence; Distance; Divergence; Dissimilarity; p-value; S-value; Information theory; Bayesian inference*

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\***Version:** 2026-02-01-13-58. **Citation:** Meyer-Hansen, E. N. (2026): ‘Conceptualizing Surprise with the Jensen–Shannon distance: A Bayesian Information-Theoretic Approach for the Social Sciences’, *Open Science Framework*, Working paper (v2026-02-01-13-58). DOI: 10.17605/OSF.IO/GQ6C8

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# 1 Introduction

Assessments of how surprising the results of a study are is an important yet theoretically vague concept within the social sciences. Generally, for a research paper to be deemed worthy of publication in a peer-reviewed scientific journal, besides adhering to standards on rigor and replicability (Christensen et al., 2019; Shaughnessey et al., 2015; Toshkov, 2016), its results should also be *surprising* (or novel) when compared to expectations based on intuition or theory (French and Murphy, 2023; Morgan, 2005; Wang et al., 2024). This preference for surprise/novelty could be attributed to the belief that scientific studies should serve to produce knowledge relevant for solving societal problems (Buunk and van Vugt, 2013; Lugg, 1979), which implies a limitation of current problem-solving capabilities and thus the need for a study to acquire *new* and improved knowledge. Similarly, from the perspective of editors, if the goal is to increase the readership of a journal, novelty may be preferred, since surprising results are more likely to attract attention and engage readers (cf. Itti and Baldi, 2009). To avoid their research collecting dust in the file-drawer (cf. Rosenthal, 1979), the surprise of results thus becomes an important factor for researchers to assess and emphasize in order to increase its perceived relevance and chances of publication.

Recognizing this preference, the scientific community has tried to standardize and quantify novelty to make it less subjective (e.g., Modirshanechi et al., 2021; Wang et al., 2024), but despite their best efforts, there is currently no broad consensus, leaving different fields to rely on different solutions (Wang et al., 2024). Within the social sciences, this has led to the development of informal practices for assessing novelty, with particularly the (in)famous *p*-value currently serving as a popular heuristic. Here, the *p*-value is effectively treated as a measure of surprise through the informal assessment of checking whether it is smaller than some threshold (e.g.,  $p \leq 0.05$ ), and if it is, the results are implicitly considered to be 'surprising'.

However, using the *p*-value as a measure of novelty can be shown to be inappropriate given its conceptualization and mathematical properties. Specifically, it can be made evident that almost any result becomes 'surprising' given a sufficiently large sample size, irrespective of how derivative the results actually are compared to the expectations of intuition or existing theory. Misleading conclusions about the novelty of results are thus abound to incur from this informal practice, and it therefore constitutes an issue for the social sciences that may lead to incorrect decisions in relation to assessments of relevance and publication.

This paper argues for the need to adopt a formal (re)conceptualization of surprise within the social sciences to solve this issue. Building on Bayesian inference and Information theory, this involves conceptualizing *knowledge* as (inter-)subjective beliefs, where the expected results given existing theory can be characterized as *prior* knowledge, while the actual results given the data can be characterized as *posterior* knowledge. This enables a theoretically-coherent, formal conceptualization of surprise as *the relative entropy between prior and posterior knowledge*. As demonstrated in relation to empirical data, surprise can then be appropriately measured as the *Jensen-Shannon distance*, which possess numerous advantages suited for the social sciences. By arguing for this conceptualization, the paper thus adds to the growing literature that calls for formally conceptualizing surprise in accordance with Bayesian inference and Information theory (e.g., Bencomo and Belaggoun, 2014; Burkhart et al., 2025; Burnham and Anderson, 2001; Itti and Baldi, 2009; Mello et al., 2025; Seehars et al., 2014).

To achieve this solution, the paper is structured as the following: Following this introduction (Section 1), the paper considers the current problem within the social sciences by reviewing the informal conceptualization

of surprise and the  $p$ -value to argue for the latter's (mis)use as an implicit indicator of surprise within the social science literature (Section 2). The paper then builds towards a solution by presenting concepts central to Bayesian inference and Information theory, incorporating them into a formal conceptualization of surprise as the relative entropy between prior and posterior knowledge, measurable as the Jensen-Shannon distance (Section 3). A practical implementation of the Jensen-Shannon distance for popular statistical software is presented, before a demonstration of its ease of use in relation to empirical data is provided (Section 4). The uses and limitations of this solution are discussed (Section 5) before the paper is concluded (Section 6).

## 2 Problem

To build towards a formal conceptualization of surprise as the relative entropy between prior and posterior knowledge, this section first covers the current, informal definition of surprise within the social sciences (Section 2.1), before reviewing the conceptualization and mathematical properties of the  $p$ -value that make it inappropriate as a measure of surprise (Section 2.2). Consistent with related literature (e.g., Itti and Baldi, 2009), novelty and surprise are treated as synonymous throughout the paper.

### 2.1 Surprise, informally

In layman's terms, *surprise* can generally be defined as a reaction 'caused by something unexpected happening' (Cambridge Dictionary, 2025). While this definition is consistent with researchers' application of the term (cf. Ivanova and Vaidyanathan, 2024; Itti and Baldi, 2009), including within the social sciences (e.g., Alford et al., 2011: 362; Goren and Chapp, 2017: 124; Petersen and Laustsen, 2019: 26), it can be argued to be too vague to constitute a formal scientific conceptualization. However, usage of this term by researchers can nevertheless be analyzed as a starting point to understand why the  $p$ -value is misused as a measure of surprise and what a proper conceptualization within the social sciences would need to consider.

That surprise is a reaction to the unexpected can within the social sciences be understood to reflect a *discrepancy between results and predictions made about those results based on theory*. Here, predictions are generally formalized as *hypotheses* about a parameter of interest (Shaughnessey et al., 2015; Toshkov, 2016), whose theoretical basis may range from weak (e.g., intuition-based) to strong (e.g., well-established theory). That such discrepancy is (informally) recognized as being synonymous with surprise is made evident by statistically discernible differences between expectations and results almost ubiquitously being referred to as surprising and worthy of discussion. For example, Cochrane et al. (1979) studied the political values of individuals in British neighborhoods with door-to-door interviews. Based on theory by Rokeach (1973), they expected individuals with different political ideologies (e.g., liberals, communists, fascists) to differ in their prioritization of freedom. Instead, they found that individuals across all ideologies prioritized freedom similarly, and this unexpected discrepancy between the predictions by Rokeach (1973) and the results by Cochrane et al. (1979) were discussed and later explicitly referred to as 'surprising' by other researchers (e.g., Sterling et al., 2019: 2). Surprise can thus informally be characterized as the discrepancy between results and theory, and a formal conceptualization would thus need to reflect this property.

Another related characteristic of surprise that may be recognized from its informal usage is that it is entirely *relative* to the predictions of the theory to which results are being compared. This is made evident by researchers

recognizing that results may be surprising when evaluated against one theory, but that they can also be entirely unsurprising when compared to another. For example, Osmundsen et al. (2021) investigate the behavior of American individuals on the social media *X* (then known as *Twitter*). In contrast to the expectations of *ignorance theory*, where fake news sharing would expectedly be attributable to individuals being unable to discern between true and fake information (Osmundsen et al., 2021: 1001; based on Altay et al., 2020; Pennycook and Rand, 2019; Pennycook et al., 2020), the authors made the remark that it was better explained as based in hostility toward political opponents, in line with the *negative partisanship* literature (Osmundsen et al., 2021: 1012; see Abramowitz and Webster, 2018). While this result could thus be considered surprising when compared to one theory, it was entirely unsurprising when compared to another. This means that an appropriate conceptualization of surprise needs to account for it being *relative* to the different theories that results are being compared against.

A final characteristic of surprise gained from analyzing its popular use is that it is *dynamic*, evolving as science progresses. This is made evident by researchers using surprising results to argue for revisions, if not discardment, of theory, with the aim being to bring expectations in greater alignment with results and thereby reduce future surprise. For example, in the discussion of the previously noted study by Osmundsen et al. (2021), they use their results to argue for revising future expectations about fake news sharing, stating that 'If people care primarily about a story's ability to hurt political enemies, we should *not* be *surprised* if fact checking fails to reduce sharing.' (Osmundsen et al., 2021: 1013, emphasis added) Such revisions fit the self-correcting ideal of science, whereby theories are revised or discarded in favor of theories with a higher explanatory power (i.e., *verisimilitude*, Miller, 1974; Popper, 1959; see also Dellsén, 2024; Lakatos, 1970). In the event that similar results were to appear in future studies, they should accordingly incur *less* surprise.

That example also corroborates the notion that, whether phrased in terms of 'weak' explanatory power, 'bad' fit, or a similar term, discrepancy between results and predictions are indicative of the *epistemic limitations* of the theory underlying those predictions<sup>2</sup> (French and Murphy, 2023: 1448-1449). As such, an appropriate conceptualization of surprise should thus enable researchers to motivate revisions or discardment of theory, and that the better the predictions of a theory, the less surprising the results should appear.

Based on these initial considerations, it is apparent that surprise can currently be considered vaguely defined within the social sciences. Nonetheless, based on an analysis of its common usage, it has become clear that it can be characterized as the discrepancy between results and predictions derived from theory, with surprise being relative to, and co-evolving with, theory. Higher degrees of surprise are indicative of the limitations of a theory, which may be used to motivate its revision or discardment. These insights will prove important for any formal conceptualization of surprise within the social sciences, since a proper measure of surprise will need to account for these characteristics to be suitable for that discipline. While some may argue that a vague conceptualization of surprise is sufficient for the social sciences, the pitfalls of this informality is demonstrated in the following section by considering the misuse of the *p*-value as an implicit measure of surprise.

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<sup>2</sup>Indeed, at least in a deterministic world, a researcher making predictions using a theory with perfect explanatory power will never be surprised by the results (Itti and Baldi, 2009). As a note of reference, it may thus be stated that Laplace's (1951) *demon* should be incapable of being surprised (for arguments against a deterministic world, see, e.g., Masi, 2023).

## 2.2 $p$ -value

In the classical statistical framework, also known as the *Frequentist* approach (for an introductory text, see, e.g., Agresti, 2018), the *probability value* (i.e.,  $p$ -value) denotes *the probability of at least as extreme an estimated parameter value from the sample given that the null-hypothesis about the population parameter is true* (Clayton, 2021: 291). This dense definition presupposes some conceptual understanding, which can be briefly explained.

The  $p$ -value is conceptually part of the *Null-Hypothesis Significance Testing* procedure (NHST; Cohen, 1994; Field, 2018: 72-82; Kruschke, 2014: 297-334), which involves the researcher formulating a *null-hypothesis* (i.e.,  $H_0$ ) about a parameter in a population of interest (i.e.,  $\theta$ ), which typically takes the form of a *nil hypothesis* by stating that the parameter is exactly zero (i.e.,  $H_0 : \theta = 0$ ; Cohen, 1994: 999-1000). This null-hypothesis contrasts an *alternative hypothesis* (i.e.,  $H_a$ ) about the population parameter that the researcher has derived from theory. By convention, however,  $H_a$  takes the form of a non-nil hypothesis (e.g.,  $H_a : \theta \neq 0$ ), which seldom reflects any substantive prediction of said theory (cf. Meehl, 1967).

In line with *hypothetico-deductive reasoning* (Popper, 1959), the purpose of NHST is then to build confidence for  $H_a$  through attempts at falsifying  $H_0$ . This is accomplished by collecting data from the population of interest, generally in the form of a sufficiently large and representative sample, before calculating the parameter from this data using a mathematical formula known as an *estimator* (Stock and Watson, 2019: 105; Wooldridge, 2019: 715-721), which provides an estimate of the parameter in the population based on the collected data. The estimated parameter from that sample (i.e.,  $\hat{\theta}_s$ ) is then compared to the *sampling distribution* of its estimator (i.e.,  $\hat{\theta}$ ; Field, 2018: 61-64), which is a distribution of hypothetical estimates derived from an infinite number of alternative, random samples taken from that population if  $H_0$  is true. Since Frequentists define probability as *the long-run relative frequency* (Agresti, 2018: 79), the  $p$ -value is calculated as the area under the curve in this sampling distribution of  $\hat{\theta}$  that is at least as extreme as  $\hat{\theta}_s$ . If the  $p$ -value is lower than some threshold (i.e.,  $\alpha$ ) specified before collecting the data (e.g.,  $\alpha = 0.05$ , with  $p \leq \alpha$ ) then  $H_0$  is rejected, which would be consistent with  $H_a$  being true, and the estimated parameter is referred to as 'statistically significant'. By contrast, if the  $p$ -value is higher than  $\alpha$ , then  $H_0$  fails to be rejected, which is consistent with  $H_0$  being true, and the estimated parameter is referred to as 'statistically insignificant' (Note that  $H_0$  can never be 'accepted' within the logic of NHST).

From this conceptualization of the  $p$ -value, it is evident that the intended purpose of this statistic is that of *statistical inference*, that is, inferring results from sample to population (Agresti, 2018: 16-17). Nowhere in its conceptualization is a consideration for expressing the degree of discrepancy between results and predictions based in theory (i.e., either a small or large  $p$ -value could be consistent with any degree of actual surprise). While the  $p$ -value is relative to a prediction based in the null-hypothesis, the null-hypothesis generally does *not* reflect serious predictions based in theory (i.e., what social theory would be considered substantive if its sole prediction involved a parameter-value being non-zero?), and the null-hypothesis generally fails to evolve with a theory by generally remaining a nil hypothesis no matter the context, making the  $p$ -value too static to reflect the dynamics of surprise.

These issues can be further recognized with a simple example. Suppose one conducts a *randomized controlled trial* (RCT; Deaton and Cartwright, 2018) on the *entire* population of interest, where half of individuals are exposed either to a placebo condition or some treatment, which given one theory is expected to change these individuals, while another theory expects it *not* to change anything. Even if the treatment is found to induce a change or not, the

$p$ -value is entirely irrelevant to the purposes of this experiment, because there is no need for statistical inference (because the 'sample' is the entire population).<sup>3</sup> Since an assessment of the surprise the results remains relevant in spite of the irrelevance of the  $p$ -value, this means that an assessment of surprise must be separate from it.

These conceptual limitations can be further realized by considering the mathematical formula underlying the  $p$ -value. While it can be calculated in numerous ways, the most widely used version of the  $p$ -value involves an assumption that the sampling distribution of the estimator used to calculate the parameter of interest ( $\hat{\theta}$ ) is *Gaussian* (i.e., Normal; Wooldridge, 2019: 723-724), which under a nil hypothesis, means that the distribution can be characterized with the following zero-mean *Normal probability distribution* ( $\mathcal{N}$ ; Agresti, 2018: 84-91):

$$\hat{\theta} \sim \mathcal{N}(0, \sigma_{\hat{\theta}}^2) = \frac{1}{\sqrt{2\pi\sigma_{\hat{\theta}}^2}} e^{-\frac{x^2}{2\sigma_{\hat{\theta}}^2}} \quad (1)$$

where  $\sigma_{\hat{\theta}}$  refers to the dispersion of this sampling distribution of  $\hat{\theta}$  (i.e., the *standard error*; Field, 2018: 61-64);  $\pi = 3.141 \dots$  is the ratio between the circumference and diameter of a circle; and  $e = \sum_{n=0}^{\infty} \frac{1}{n!} = 2.718 \dots$  is the base of the exponential function.

If  $H_0 : \theta = 0$  and  $H_a : \theta \neq 0$ , the direction by which  $\hat{\theta}_s$  deviates from 0 is irrelevant, and the  $p$ -value is accordingly referred to as *two-tailed* (Field, 2018: 79-81), which given the symmetry of the distribution in (1) is given by the definite integral (cf. Stock and Watson, 2019: 111):

$$p = 2 \int_{|\hat{\theta}_s|}^{\infty} \mathcal{N}(0, \sigma_{\hat{\theta}}) dx \quad (2)$$

where  $\int$  denotes the integration operator and  $|\hat{\theta}_s|$  is the absolute value of  $\hat{\theta}_s$ .

Since an application of formula (2) requires calculating  $\sigma_{\hat{\theta}}$  using infinitely many samples from a population in the possibly hypothetical scenario where  $H_0$  is true,  $\sigma_{\hat{\theta}}$  is generally estimated. For example, if the parameter of interest is the population mean (i.e.,  $\theta = \mu$ ) with  $\hat{\mu} = \frac{1}{n} \sum_{i=1}^n x_i$  being its estimator (Field, 2018: 27), where  $x_i$  denotes the  $i$ th value from an  $n$ -length vector constituting a random sample of independent and identically distributed (*iid*) values drawn from a normally distributed population of interest; and the population variance is estimated as  $\hat{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \hat{\mu})^2$  (Field, 2018: 30), then the estimator for the dispersion of the sampling distribution for the estimated mean is (Stock and Watson, 2019: 113, equation 3.9):  $\hat{\sigma}_{\hat{\mu}} = \frac{\hat{\sigma}}{\sqrt{n}}$ .

Now, since  $\hat{\sigma}^2$  is an *unbiased* and *consistent* estimator of  $\sigma^2$  (Stock and Watson, 2019: 112; Wooldridge, 2019: 716-723, 766),  $\hat{\sigma}^2$  and  $\sigma^2$  converge as the sample size ( $n$ ) increases towards infinity, while  $\sqrt{n}$  increases without bound. A mathematical analysis of  $\hat{\sigma}_{\hat{\mu}}$  thus makes it evident that the estimated dispersion of the sampling distribution approaches zero as  $n$  increases towards infinity:

$$\lim_{n \rightarrow \infty} \frac{\hat{\sigma}}{\sqrt{n}} = 0 \quad (3)$$

The implications of equation (3) pose a detrimental issue for the ability of the  $p$ -value to reflect surprise. This can be made more clear by first converting the estimated parameter into a standardized format called a *test statistic*

<sup>3</sup>While one may argue that calculating the  $p$ -value in this example could help express uncertainty, it should be emphasized that the only uncertainty that can be expressed by the  $p$ -value is that of *sampling uncertainty*, which is only relevant for statistical inference and therefore irrelevant in the example.

(Field, 2018: 78-79). In relation to an assumption of a normal sampling distribution, this test statistic is referred to as a  $z$ -value, which involves the rescaling formula (Field, 2018: 37):  $z = \frac{\hat{\mu} - \mu_{H_0}}{\hat{\sigma}_{\hat{\mu}}}$ , where  $\mu_{H_0}$  denotes the mean under the null-hypothesis. If the difference between the population parameter and the population parameter under the null-hypothesis is non-zero (i.e.,  $\mu - \mu_{H_0} \neq 0$ ), this  $z$ -value can be recognized to approach infinity as  $\hat{\sigma}_{\hat{\mu}}$  decreases towards zero, since  $\hat{\mu} - \mu_{H_0}$  converges to  $\mu - \mu_{H_0}$  as  $n$  increases towards infinity due to  $\hat{\mu}$  being unbiased and consistent, while  $\mu_{H_0}$  is a constant (Stock and Watson, 2019: 104-107; Wooldridge, 2019: 716-723). Similar to (3), this is expressed as:

$$\lim_{\hat{\sigma}_{\hat{\mu}} \rightarrow 0} \frac{\hat{\mu} - \mu_{H_0}}{\hat{\sigma}_{\hat{\mu}}} = \infty \quad (4)$$

which has been established to occur in (3) when  $n \rightarrow \infty$ . Due to the  $z$ -standardization, the Normal distribution in (2) has become a standard Normal, and substituting  $\hat{\theta}_s$  for  $z$ , it can finally be realized that the limit of the  $p$ -value as  $z$  increases towards infinity is zero:

$$\lim_{z \rightarrow \infty} 2 \int_{|z|}^{\infty} \mathcal{N}(0, 1) dx = 0 \quad (5)$$

as demonstrated to occur in (4) when  $\hat{\sigma}_{\hat{\mu}} \rightarrow 0$ , which happens when  $\mu - \mu_{H_0} \neq 0$  and  $n \rightarrow \infty$  as shown in (3).

The implication of these limits are critical. For any instance where the null-hypothesis is not *exactly* true — which likely applies to most of the phenomena studied in the social sciences<sup>4</sup> — anything becomes ‘statistically significant’ with a sufficiently large sample size. In the event that one equates a low  $p$ -value with surprise, this unfortunately means that *almost anything becomes surprising with a large enough sample size*, irrespective of how expected the results actually are under the given theory.<sup>5</sup> Using the  $p$ -value as a measure of surprise is thus conceptually flawed and runs counter to the characteristics of actual surprise recognized by social science researchers as previously identified in section (2.1).

While one may argue that the extent of this informal practice of using the  $p$ -value as a measure of surprise is too negligible for consideration, it may be counter-argued that such misuse is best addressed as early as possible, so as to prevent it from becoming a formal practice. Such a development may seem improbable, but this happened recently within the field of epidemiology, where Cole et al. (2021) introduced the ‘ $S$ -value’ as a formal measure of surprise. Readers of this paper, however, could quickly realize that this ‘ $S$ ’-value unfortunately had a one-to-one correspondence with the  $p$ -value through the formula (Cole et al., 2021: 192):  $S \equiv -\log_2(p)$ . Similar to criticisms made against other measures derived solely from the  $p$ -value<sup>6</sup>, this means that the  $S$ -value fails to incorporate, nor communicate, any new information besides that already expressed by the  $p$ -value. The  $S$ -value is

<sup>4</sup>That the null-hypothesis is generally false within the social sciences can be realized by considering that hypotheses generally concern point values (e.g.,  $H_0 : \theta = 0$ ) despite most studied phenomena arguably being continuous, and the assumptions for the data-generating processes of stochastic continuous phenomena involve distributions where a point hypothesis has an *a priori* zero probability of being true (cf. Cohen, 1994: 1000; Loftus, 1991, 2001; Steiger and Fouladi, 1997; Thompson, 1996; Tukey, 1991: 100; Vicente and Torenvliet, 2000: 260). In psychology, the notion that the nil hypothesis is always false is referred to as Meehl’s (1978) conjecture (Gigerenzer, 2004: 601).

<sup>5</sup>The issue cannot be addressed by specifying the null-hypothesis to fit the point prediction of a given theory, since the most negligible difference between the result and the prediction can produce a vanishingly small  $p$ -value given a sufficiently large sample.

<sup>6</sup>Somewhat unsurprisingly, this is *not* the first time that the  $p$ -value has been inappropriately repurposed to express phenomena outside its intended scope. As an infamous example, Killeen (2005a) similarly sought to address the fact that the  $p$ -value cannot express the probability that the results of a study will replicate in another study (cf. Gigerenzer, 2018). The mathematical formula underlying the proposed solution by Killeen (2005a) was widely criticized (e.g., Iverson et al., 2009; Macdonald, 2005; Maraun and Gabriel, 2010; Serlin, 2010; for the response, see, Killeen, 2005b), since it was evident that the formula had a one-to-one correspondence with the  $p$ -value, and it thus failed to incorporate any information about the probability of replication not already communicated by the  $p$ -value. Instead, as similarly argued here in relation to surprise, it was argued that a (re)conceptualization of replicability was better founded in Bayesian inference (e.g., Macdonald, 2005).

thus as susceptible to the exact same issues identified here for the  $p$ -value in its ability to express surprise<sup>7</sup>, making it just as *inappropriate*.

Now, while this paper serves to solve this misuse of the  $p$ -value, the aim is *not* to portray researchers negatively for partaking in this inappropriate practice. Indeed, the  $p$ -value is infamous for being particularly susceptible to misinterpretation (Clayton, 2021: 291; Gigerenzer, 2018: 206), with examples including it supposedly being an index of scientific significance, reliability, replicability, effectsize, and the probability of  $H_0$ ,  $H_a$ , or data having been generated by chance, resulting in various criticisms and cautions against its (mis)use (American Statistical Association, 2016; Clayton, 2021; Field, 2018: 97-110; Gelman and Stern, 2006; Kruschke, 2014: 297-329; McShane et al., 2019). Instead, as similarly achieved by other researchers identifying questionable practices within the scientific community (e.g., Bishop, 2006; Flake and Fried, 2020; Gelman, 2024; Ioannidis et al., 2017; Lenz and Sahn, 2021; Ritchie, 2020; Simmons et al., 2011), the aim of this paper is to improve the social scientific practice by informing researchers of this inappropriate practice and to propose a possible solution.

### 3 Solution

While the previous section sought to describe the current, informal conceptualization of surprise and explain why the  $p$ -value is an inappropriate measure of surprise, whose use constitutes an issue for the social sciences, this section serves to provide a solution to that problem with a formal (re)conceptualization of surprise. This solution is partly founded in similar (re)conceptualizations proposed for other disciplines (e.g., Bencomo and Belaggoun, 2014; Burkhart et al., 2025; Burnham and Anderson, 2001; Itti and Baldi, 2009; Mello et al., 2025; Seehars et al., 2014). To that end, central concepts of Bayesian inference (Section 3.1) and Information theory (Section 3.2) are presented and used to build that conceptualization and identify an appropriate measure for it.

#### 3.1 Bayesian inference

The *Bayesian statistical framework*, also known as Bayesian inference, is an approach to statistical inference that differs from its Frequentist counterpart by defining probability in relation to (inter-)subjective beliefs (for an introduction, see, e.g., McElreath, 2019). Understanding this framework presupposes some familiarity with *Bayes' theorem* (Bayes and Price, 1763; Laplace, 1951), which can be expressed in relation to the probability of two discrete events as (McElreath, 2019: 49):

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)} \quad (6)$$

where  $P(A)$  denotes the probability of the event  $A$ , which is referred to as the *prior* probability;  $P(B)$  denotes the probability of event  $B$ , sometimes called the *marginal likelihood*;  $P(B|A)$  is the probability of event  $B$  given that event  $A$  has occurred, denoted the *likelihood*; and  $P(A|B)$  is the probability of event  $A$  given that event  $B$  has occurred, which is referred to as the *posterior* probability.

The components of (6) are generally reconceptualized for the purposes of statistical inference (Gelman et al.,

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<sup>7</sup>This can be made evident by considering that  $\lim_{p \rightarrow 0} -\log_2(p) = \infty$ , which was established by equations (3) through (5) to occur as  $n \rightarrow \infty$ , which means that, based on the logic of the  $S$ -value, almost anything will inevitably become surprising with a sufficiently large sample size, no matter how expected the results were in advance. Despite these issues, other researchers have worryingly advocated for the adoption of the  $S$ -value (e.g., Greenland et al., 2022; Rafi and Greenland, 2020), further revealing the need for a proper formal (re)conceptualization of surprise.

2014; Kruschke, 2014; Levy and Mislevy, 2020) so that the parameter of interest ( $\theta$ ) substitutes  $A$ , while the collected data ( $\mathcal{D}$ ) substitutes  $B$ . This means that  $P(\theta)$  denotes the (prior) probability of  $\theta$ ,  $P(\mathcal{D}|\theta)$  the probability of the data given the parameter, and  $P(\theta|\mathcal{D})$  the (posterior) probability of  $\theta$  given  $\mathcal{D}$ , with this latter component being of primary interest in Bayesian inference.

For practical purposes  $P(\mathcal{D})$  can often be ignored due to relying on *Markov-Chain Monte Carlo methods* (MCMC; Brooks et al., 2011), which leaves  $P(\theta)$  and  $P(\mathcal{D}|\theta)$  relevant for calculating  $P(\theta|\mathcal{D})$ . While those familiar with *likelihood-based inference* (King, 1998) will know that  $P(\mathcal{D}|\theta)$  is often easy to derive from the statistical model used in the analysis,  $P(\theta)$  is by comparison rarely known. To solve that issue, this prior is specified as a probability distribution using *a priori* knowledge about  $\theta$ . The discretion involved in specifying this prior distribution is where Bayesian inference becomes (inter-)subjective, though results generally approximate Frequentist results with a sufficiently large sample size (Cover and Thomas, 2006: 388; Hastie et al., 2017: 272; King, 1998: 30; Kruschke, 2014: 113), which means that Bayesian inference could be considered a generalization of the arguably less subjective<sup>8</sup> likelihood-based approach (cf. King, 1998: 28). The difference between the two approaches is thus primarily in the definition, and thus interpretation, of probability.

Unlike other approaches, this conceptualization Bayesian inference provides a coherent statistical framework for characterizing uncertainty (cf. Cox, 1946; Gelman et al., 2014; Jaynes, 2003; Savage, 2017) and enables one to answer questions about the probability of parameters given the data,  $P(\theta|\mathcal{D})$ . This is impossible in the Frequentist framework, since that approach solely relies on the likelihood and fails to consider prior information in relation to  $\theta$  (Kruschke, 2014; Wagenmakers et al., 2010). It also means that results from a Bayesian analysis are in the form of probability distributions, which conceptually express how credible different estimated values of  $\theta$  are given the data. While these distributions could be summarized with point estimates (e.g., mean, dispersion) or intervals (e.g., 95% credibility interval) for practical purposes, as done in the Frequentist approach, this is theoretically unnecessary, since the posterior distribution comprehensively details one's knowledge about  $\theta$  based on the prior knowledge and data (McElreath, 2019: 58). While Bayesian inference can involve *p*-values and hypothesis tests (see, e.g., Makowski et al., 2019a; Wagenmakers et al., 2010), such procedures related to NHST are *not* native to this framework, which instead of the hypothetico-deductive approach, is more naturally based in *abductive reasoning* (see, e.g., Dellsén, 2024).

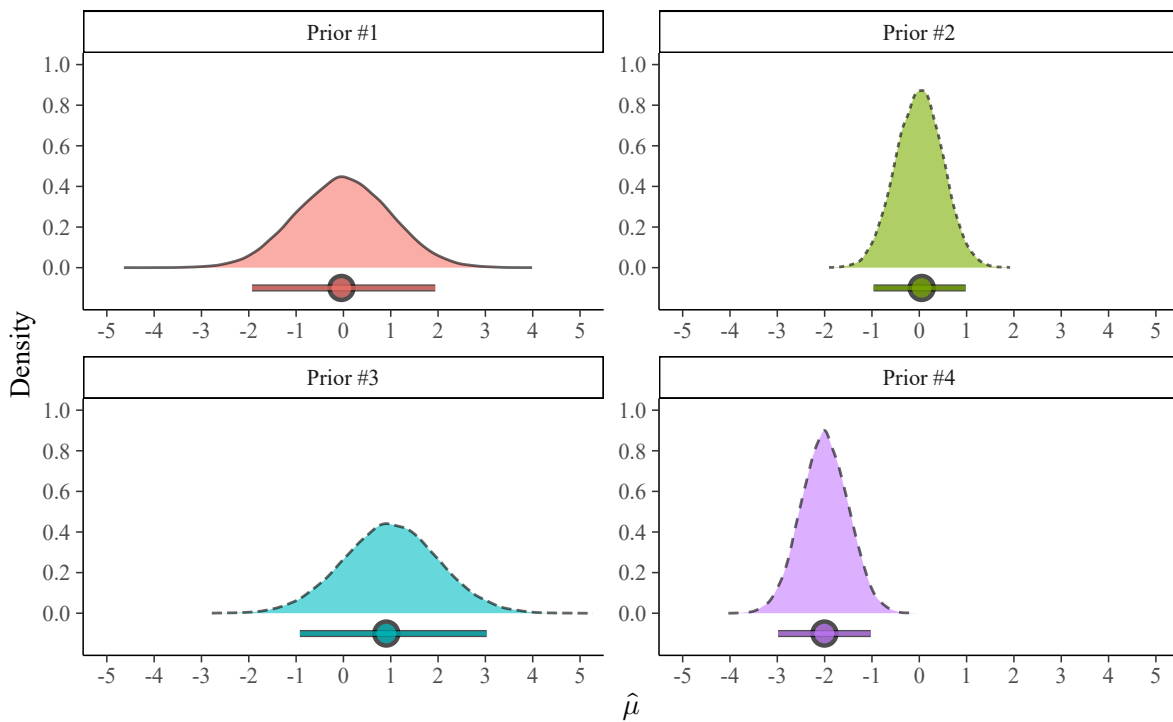
Now, consistent with work in other disciplines (e.g., Itti and Baldi, 2009), the conceptualization of Bayes theorem used for statistical inference can be recognized to *naturally* and *coherently* capture the informally recognized characteristics of surprise identified in section (2.1). With surprise being the discrepancy between results and expectations/predictions based in theory, it can be equivalently understood as a discrepancy between (inter-)subjective states of knowledge *prior* to seeing the results and the state of knowledge *posterior* to seeing the results. When theory is formalized, data-driven, and well-established among peers within the social sciences, the prior thus characterizes the state of knowledge as a relatively intersubjective evidence-based belief about a phenomenon, describing what the researcher expects the results to be. As established, since the prior distribution is conceptually a comprehensive statement about this former state of knowledge, it can be appropriately used to characterize the expectations based in theory.

<sup>8</sup>Since all human processing of information and knowledge is necessarily (inter-)subjectively dependent (cf. Husserl, 1983; see also Kant, 1781), the Frequentist statistical framework, including its likelihood-based approach, is *not* 'objective' (i.e., independent of (inter-)subjective experience), despite claims to the contrary (e.g., Mayo, 1996: 89).

### 3.1.1 The prior reflects current knowledge

Characterizing the predictions of a theory as state of knowledge expressible as a prior distribution can be difficult, especially if it only makes vague predictions about the parameters relevant to that theory. To therefore help build intuition around them, four examples of different specifications are provided here.

At no loss of generality, suppose  $X$  is a normally distributed variable in some population of interest:  $X_i \sim \mathcal{N}(\mu, \sigma)$  for all  $i$  in  $1, \dots, N$ , where one wishes to estimate the mean  $\mu$  using a representative and sufficiently large sample. To enable Bayesian inference, it is necessary to specify its prior,  $P(\mu)$ , which can be achieved in numerous ways depending on one's prior information about it. As shown visually in figure (1), the first prior (Prior #1) is specified as a normal distribution:  $\mathcal{N}(0, 1)$ , which in this context can be taken to reflect a diffuse state of knowledge, where there is complete theoretical ignorance about sign of the parameter (i.e., is the mean positive or negative), but its relatively substantial width (i.e., size of the dispersion) implies that mean could be relatively larger irrespective of its sign. This prior may be referred to as regularizing (McElreath, 2019: 214-216), or 'skeptical, yet persuadable', which is a type of weakly informative prior that is critical with respect to the sign of a parameter of interest, but which remain wide enough to be influenced by the results.



**Figure 1.** Examples of Prior Distributions

**NOTE:** Prior #1  $\sim \mathcal{N}(0, 1)$ ; Prior #2  $\sim \mathcal{N}(0, 0.5)$ ; Prior #3  $\sim \mathcal{N}(1, 1)$ ; Prior #4  $\sim \mathcal{N}(-2, 0.5)$ . The geometrics below the distributions indicate the mode (circle) and the 95% HD CI (bar). Monte Carlo samples = 40,005.

The second prior (Prior #2) is similar to the first, but by comparison, it is much narrower, which implies a stronger expectation that the mean will not be that large irrespective of the sign. This contrasts the third prior (Prior #3), which has the expectation that mean is positive, but its similar width to the first prior means it reflects a less certain expectation. The last prior (Prior #4) characterizes an opposite expectation to the third prior, since it expects the mean to be negative, and its greater narrowness reflects a greater degree of certainty about this theoretical belief.

### 3.1.2 The posterior as updated knowledge

While the state of knowledge prior to the results could be difficult to precisely express as a probability distribution, there is no need for the researcher to specify the state of knowledge posterior to the results, since this resulting probability distribution is a function of the chosen prior distribution and the data. Following Itti and Baldi (2009), the posterior can thus be considered an updated state of knowledge, which is useful, since it is informative about how the expectations of that theory should change based on the data to enable better predictions in the future.

Since the posterior is the product of the prior and the data, this conceptualization helps reflect the aforementioned characteristics of surprise as being relative and dynamic in relation to the theories to which results are being compared. Since different prior specifications can reflect the varying predictions made across theories, the posterior could theoretically be compared to each to enable an assessment of how 'surprising' the results are given each theory. For example, let the posterior distribution resulting from an analysis be identical to the fourth example of the priors (Prior #4) previously shown in figure (1), meaning it is characterized as a normal probability distribution with a mean parameter of  $-2$  and a dispersion of  $0.5$ :  $\mathcal{N}(-2, 0.5)$ . Assume that these aforementioned priors represent a set of predictions made from different theories, and that the data is large enough to make the posterior independent of these priors.

Comparing the posterior to each prior then makes it clear that the discrepancies between them are *not* equivalent. When evaluated against the the fourth prior (Prior #4), the results fit perfectly with the predictions made *a priori* about the parameter, which would be fully consistent with a complete lack of surprise, which is fully consistent with how social science researchers use the term. While the differences between the posterior mean (i.e.,  $-2$ ) and the mean of priors one and two (i.e.,  $0$ ) are the same, the results, however, would intuitively appear more 'surprising' when compared against the second prior, since that distribution is more narrow and thus express greater certainty about the parameter values as implied by its lower dispersion (i.e.,  $0.5$  vs.  $1$ ). As such, while the third prior has the highest discrepancy with the posterior in terms of the mean, the dispersion of the distribution should also be considered when evaluating the degree of surprise under this Bayesian conceptualization, which will prove important when moving from a conceptualization of surprise towards an appropriate measurement of it.

Using these concepts from Bayesian inference, surprise has thus far been conceptualized as a discrepancy between states of knowledge, with these states specifically being expressible as a prior and posterior probability distribution. For both the posterior and prior distributions to be valid, they should ideally reflect an intersubjective state of knowledge about a phenomenon of interest as based in well-established theory and quality data. The advantage of this approach has been demonstrated to arise from the conceptual properties of Bayesian inference that enable comprehensive statements to be made with relative ease about such states of knowledge. A shortcoming of this approach, however, is that Bayesian inference can be considered conceptually weak in terms of meaningfully quantifying the degree of discrepancy between states of knowledge, which is why the following section turns to the key concepts of Information theory that will build upon this Bayesian-based conceptualization to enable an appropriate measurement of surprise.

## 3.2 Information theory

Information theory is a relatively recent mathematical field that is primarily concerned with the efficient communication and compression of information (e.g., Hartley, 1928; Nyquist, 1924, 1928; Shannon, 1948, 1949; Wiener,

1961), but its concepts has proven useful across numerous disciplines, including statistical modeling (e.g., Akaike, 1973; Hastie et al., 2017; Mielniczuk, 2022) and Bayesian inference (e.g., Chaloner and Verdinelli, 1995; Jaynes, 1968; Vehtari et al., 2017; Watanabe, 2010). Using terminology closely related to statistical mechanics (Jaynes, 1957a; 1957b), information is generally conceptualized within this discipline as the 'entropy' of a random variable, which is the key concept in which an appropriate measure of surprise is founded. Since this and related concepts have seen relatively sparse use within the social sciences, they are briefly presented here.

### 3.2.1 Entropy

In Information theory, *entropy* is 'a measure of the average uncertainty in the random variable' (Cover and Thomas, 2006: 5; Shannon, 1948), making it synonymous with randomness, unpredictability, and disorganization (Berrett et al., 2019: 288; Cover and Thomas, 2006: 135; Jaynes, 1957a: 622; Rényi, 1961: 547). It can be thought of as the information needed, on average, to describe a random variable (Berrett et al., 2019: 288; Cover and Thomas, 2006: 19), which means that the more random the variable, the higher its entropy.

For a finite discrete random variable  $X$ , the *Shannon-Boltzmann entropy*,  $H(X)$ , is defined by the mathematical formula (Back et al., 2018: 1; Cover and Thomas, 2006: 5, equation 1.1; Rényi, 1961: 547):

$$H(X) \equiv \sum_{x \in X} p(x) \log \frac{1}{p(x)} \quad (7)$$

$$= - \sum_{x \in X} p(x) \log p(x) \quad (8)$$

where  $\sum$  denotes the summation operator applied over the unique elements of  $X$ ;  $p(x) = P(X = x) \geq 0$  is the probability mass function for  $X$ ; and  $\log$  is the logarithmic function, where it is taken by convention that  $0 \log 0 \equiv 0$  (Cover and Thomas, 2006: 14). Since the base of the logarithm is generally taken to be 2, this means that the scale of entropy is in *binary units*<sup>9</sup> (i.e., *bits*, which are *not* to be confused with the related concept of binary digits; Cover and Thomas, 2006; Delgado-Bonal and Marshak, 2019; Shannon, 1948). Being measured in bits means that the entropy can be meaningfully interpreted as the expected number of binary questions (i.e., Yes/No questions) needed to know a value of  $X$  with certainty (Cover and Thomas, 2006: 16).

With this formula, the entropy can generally be stated to be a non-negative real number, whose upper bound is the logarithm of the number of unique elements in  $X$  (i.e.,  $H \in \mathbb{R}^{[0; \log(|X|)]}$ ; Cover and Thomas, 2006: 29). For example, a uniform *Bernoulli* ( $\mathcal{B}$ ) variable  $X$  (i.e.,  $X \sim \mathcal{B}(p = 0.5)$ ) has an entropy of 1 bit. This is an intuitive upper bound for Bernoulli variables, since it reflects the *equiprobability* of the elements, making any element of  $X$  maximally unpredictable. This also means that if  $p$  either increases or decreases, the entropy decreases, becoming 0 if  $p \in \{0, 1\}$ . This is similarly an intuitive lower bound, since either value makes  $X$  constant, and an entropy of 0 thus reflects that we *a priori* know the values of  $X$  and can predict them with absolute certainty (Cover and Thomas, 2006: 14-15). As such, the more uniform a distribution, the higher its entropy (Delgado-Bonal and Marshak, 2019).

While this conceptualization of entropy possess numerous desirable mathematical properties for measuring information, for example, being insensitive to the actual values of  $X$  by depending solely upon the their probabil-

<sup>9</sup>Popular alternatives to the base 2 for the logarithmic function are the base of the exponential  $e$  and 10, which means that the natural logarithm (i.e.,  $\ln$ ) would be used for the former. Instead of bits, this produces entropies on the scale of *natural units* (i.e., *nats*) and *decimal units* (i.e., *diits*), respectively (Back et al., 2018; Cover and Thomas, 2006: 14; Delgado-Bonal and Marshak, 2019).

ities (Cover and Thomas, 2006: 14), it is, however, limited to *discrete* random variables, which is unfortunate since most phenomena of interest within the social sciences are arguably *continuous*.<sup>10</sup>

In the event that  $X$  instead denotes a continuous random variable, its uncertainty is instead captured by the concept of *differential entropy*, which is 'the entropy of a continuous random variable' and is denoted  $h(x)$  (Cover and Thomas, 2006: 243). If the distribution of  $X$  can be characterized by the probability density function  $f(x)$ , its differential entropy is calculated with the formula (Ahmad and Lin, 1975: 373; Beirlant et al., 2001: 1; Berrett et al., 2019: 288; Cover and Thomas, 2006: 243, equation 8.1; Hastie et al., 2017: 561, equation 14.82; Shannon, 1948):

$$h(X) \equiv - \int_S f(x) \log f(x) \, dx \quad (9)$$

where  $\int_S$  denote the integration operator applied over the *support set* of  $X$  with respect to  $x$  (i.e., where  $f(x) > 0$ ).<sup>11</sup> Unlike entropy, the differential entropy can be negative (Cover and Thomas, 2006: 244), generally making it a real-valued scalar (i.e.,  $h(X) \in \mathbb{R}$ ), which can be considered harder to interpret. Similarly, by involving an integral, it is harder to calculate, though closed-form expressions do exist for some known probability distributions, such as the Normal, whose differential entropy can easily be calculated by merely knowing its dispersion (Cover and Thomas, 2006: 244):

$$h(X) = \frac{1}{2} \log 2\pi e \sigma^2 \quad (10)$$

For example, if  $X$  is a standard normal variable (i.e.,  $X \sim \mathcal{N}(0, 1)$ ), this means that its differential entropy is  $\frac{1}{2} \log 2\pi e \approx 2.047$  bits.

Following Delgado-Bonal and Marshak (2019), the concept of entropy can be found useful in relation to the states of knowledge previously conceptualized using Bayesian inference in section (3.1). Since that tentative conceptualization used random variables and their probability distributions to characterize prior and posterior knowledge, that approach can easily be combined with these insights from Information theory, leading to the notion that the different states of knowledge implied by the prior and posterior distributions simply express different states of information. Specifically, the prior distribution has an entropy, which express the degree of uncertainty in one's information about the parameter of interest  $\theta$ ; the wider that distribution, the greater the uncertainty about  $\theta$ . The posterior distribution, by comparison, differs from the prior to the extent that the analyzed data has increased or reduced the uncertainty around  $\theta$ . For example, the normal prior distributions with a dispersion ( $\sigma$ ) of 1 previously specified (see Section 3.1.1) have a differential entropy of approximately 2.047 bits, while those with a dispersion of 0.5 have a differential entropy of approximately 1.047 bits.

The example demonstrates that the concept of (differential) entropy can be imported to Bayesian inference and used to characterize the information contained within a state of knowledge. However, the aforementioned example also makes it clear that a mere comparison of (differential) entropy between the prior and posterior could fail to consider changes the plethora of ways by which the prior and posterior distributions could differ, since the absolute discrepancy between the differential entropy of the posterior previously mentioned in section (3.1.2)

<sup>10</sup>In statistical analysis, rather than studying a phenomenon of interest by itself, researchers generally study the parameters of a statistical model theorized to generate said phenomenon. This often involves *generalized linear models* (GLMs; Field, 2018; Kruschke, 2014) that assume *linearity* (and thus continuity) in its parameters (Wooldridge, 2019: 763).

<sup>11</sup>Since (9) involves an integral, the calculation of differential entropy presupposes that the density function and integral for  $X$  exists (Cover and Thomas, 2006: 243-244).

and the first and third prior (i.e.,  $\Delta h = 1$  bits), and the second and fourth prior (i.e.,  $\Delta h = 0$  bits), are the same, despite having different means. This contradicts the established intuition suggesting that the degrees of discrepancy between them, and thus the induced surprise, should differ.

While the concept of (differential) entropy has thus shown to be a useful foundation to link concepts from Bayesian inference and Information theory, it is insufficient for appropriately measuring surprise. This can be mitigated by introducing the final concept of relative entropy.

### 3.2.2 Relative entropy

The *relative entropy*, also known as the Kullback-Leibler (1951) divergence (KL divergence; e.g., Pérez-Cruz, 2019: 1) quantifies information divergence by being 'a measure of the distance between two distributions.' (Cover and Thomas, 2006: 19). Unlike entropy, which quantifies the uncertainty of a distribution, the relative entropy quantifies the divergence between distributions, with 'divergence' simply referring to the 'statistical distance' between them (which should *not* be mistaken for a *geometric* distance; Nielsen, 2019: 4). It has proven 'central to information theory and statistics' (Pérez-Cruz, 2019: 1084; see also Cover and Thomas, 2006: 347-408) and seen application across numerous disciplines (e.g., Burnham and Anderson, 2001; Chaloner and Verdinelli, 1995).

For two discrete random variables  $p$  and  $q$ , the relative entropy is denoted  $D(p||q)$  and mathematically defined by formula (11) (Cover and Thomas, 2006: 19; Delgado-Bonal and Marshak, 2019: equation 5); and if  $p$  and  $q$  are continuous random variables with probability density functions  $f$  and  $g$ , it is given by formula (12) (Cover and Thomas, 2006: 251; Pérez-Cruz, 2019: 1).

$$D(p||q) \equiv \sum_{x \in X} p(x) \log \frac{p(x)}{q(x)} \quad (11)$$

$$\equiv \int f \log \frac{f}{g} \quad (12)$$

While the properties of relative entropy for the discrete and continuous case are mostly the same (Cover and Thomas, 2006: 251), for example, they satisfy the property of *information monotonicity* by being invariant to reparameterizations (cf. Nielsen, 2019: 3), the relative entropy is zero *if and only if* (iff)  $p = q$  for the discrete case, and  $f = g$  (almost) everywhere for the continuous case (Cover and Thomas, 2006: 252-253; see also Pérez-Cruz, 2019: 1). Since it is taken by convention that  $0 \frac{0}{0} \equiv 0$ ,  $0 \frac{0}{q} \equiv 0$ , and  $p \frac{p}{0} \equiv \infty$  (Cover and Thomas, 2006: 19), the relative entropy can for both (11) and (12) be infinite, with finite values only occurring for the continuous case when the support set of  $f$  is contained in the support set of  $g$  (i.e., if  $f$  is *absolutely continuous* with respect to  $g$ ). Thus, the relative entropy is generally a non-negative scalar on the *extended real number line* (i.e.,  $D(p||q) \in \bar{R}^{[0;\infty]}$ ), and if a base of 2 is used for the logarithm, the relative entropy expresses the information divergence in bits, with a unit of surprise being denoted one 'wow' (cf. Itti and Baldi, 2009).

While the relative entropy was developed to quantify how easy it is to discriminate between two distributions (Kullback and Leibler, 1951: 79), it is generally interpreted as a quantification of how inefficient it is to assume that a random variable is distributed as  $q$  when it is actually distributed as  $p$  (Cover and Thomas, 2006: 19). This has gained widespread appeal, since researchers may interpret the relative entropy as the mean of the *log-likelihood ratio* (Pérez-Cruz, 2019: 1). This makes it useful for statistical hypothesis testing and model selection (Cover and Thomas, 2006: 375-380; see, e.g., King, 1998: 84-86; McElreath, 2019: 202-214), since it can be used to justify

preferring a statistical model with the lowest relative entropy on the basis of *parsimony* (i.e., Occam's razor; Cover and Thomas, 2006: 1).

That interpretation can be recognized as similarly useful for the current purposes of conceptualizing surprise, since it may be meaningfully rephrased so that the relative entropy 'reflects the gain of information resulting from adding the knowledge in  $p$  relative to  $q$ , or the gain of information when we learn that the real probability function was  $p$  instead of  $q$  as we thought initially.' (Delgado-Bonal and Marshak, 2019: 7) Such an interpretation fits the ongoing Bayesian (re)conceptualization of surprise by enabling the relative entropy between the prior and posterior distribution to be meaningfully interpreted as the gain in information resulting from adding the knowledge of the data to the prior. This means that divergence, by merit of being conceptually based in Bayesian inference and Information theory, coherently reflects how unexpected the results are compared to one's expectations (cf. Itti and Baldi, 2009), with zero divergence/surprise occurring iff the results exactly match one's expectations.

This leads to the completed, formal conceptualization of surprise as *the relative entropy between prior and posterior knowledge*. Within this conceptualization, knowledge and information are equated, while the prior and posterior are taken to reflect two states of knowledge, with the discrepancy between them being the degree of surprise. Higher degrees of discrepancy thus reflect greater surprise, measured in units of the *base* used for the logarithmic function, which here is specified as 2 so that the units of surprise are in bits.

This conceptualization fits perfectly with the informal notion of surprise identified among social science researchers (Section 2.1). This is evident, because the informal conceptualization of surprise involved a discrepancy between results and expectations/predictions about those results based in theory, which is directly reflected in this formal conceptualization, since the relative entropy measures the divergence (i.e., discrepancy) between the posterior knowledge (i.e., results) and the prior knowledge (i.e., predictions based in theory). It also reflects the *relative* and *dynamic* characteristics of surprise, since the degree of surprise within this formal conceptualization is always relative to the prior, with different degrees of surprise being possible if the posterior is compared to different prior specifications representing different theoretical expectations, and as the prior is continuously updated based on new data to better reflect reality, the relative entropy, and thus the surprise, will lessen. This formal (re)conceptualization of surprise thus offers a theoretically coherent understanding of surprise that fits with social science researchers' intuitive use of the word, making it particularly suited for that field.

Now, perhaps attributable to its coherent conceptualization and intuitive interpretation, this is not the first time that relative entropy has been proposed as part of a formal conceptualization of surprise across different scientific fields. For example, it has previously been proposed within the disciplines of clinical care (e.g., Burkhart et al., 2025), ecology (e.g., Burnham and Anderson, 2001), software engineering (e.g., Bencomo and Belaggoun, 2014), and computer science (e.g., Itti and Baldi, 2009). However, while these proposals are undoubtedly helpful in reducing the mispractice of using the  $p$ -value as an implicit measure of surprise, their choice to not just conceptualize surprise as the relative entropy, but also as its measure, can be considered suboptimal when considering some of its limitations.

A limitation of relative entropy is that it is *not* a 'proper' metric in the geometric sense of the word. This is because it is *asymmetric*, which generally means that  $D(p||q) \neq D(q||p)$ , and it fails to satisfy the *triangle inequality* (Cover and Thomas, 2006: 19; Delgado-Bonal and Marshak, 2019: 7), whereby it lacks a series of intuitive properties that one may generally attribute to a distance. For example, suppose  $A$ ,  $B$ , and  $C$  represent

points in Euclidean space. Let the measure of the distance travelled be asymmetric and in violation of the triangle inequality. Then, the measured distance directly from  $A$  to  $C$  may *not* be the same as the distance travelled directly from  $C$  to  $A$ , and either of these may be longer than the distance travelled from  $A$  to  $B$  to  $C$ . This is counter-intuitive (cf. Cover and Thomas, 2006: 367; for an alternative view, see, e.g., McElreath, 2019: 208-209), since from real-world experience, one would expect distances to be symmetric, with stops and detours either adding or not influencing the distance travelled. These limitations thus make the relative entropy harder to interpret and invites researcher discretion by being sensitive to the arguably arbitrary choice of whether to calculate it between  $p$  and  $q$  or between  $q$  and  $p$ .

The relative entropy/KL divergence is by no means the only measure of divergence, with numerous symmetric alternatives (e.g., Jeffreys's (1946) divergence; Cha, 2007), some of which can be considered proper geometric measures of distance. For example, the Bhattacharyya (1943; 1946) distances, the Earth mover's distance (Kantorovich, 1939; Monge, 1781; Vaseršteĭn, 1969), and the Mahalanobis' (1936) generalized distance (Kullback and Leibler, 1951: 79). While each of these measures of distance possess advantages and disadvantages, the one proposed here to measure surprise within the social sciences is the Jensen-Shannon divergence, that, while based on the relative entropy, possesses none of the limitations mentioned here.

### 3.2.3 Jensen-Shannon divergence

Formally introduced by Lin (1991; see also Wong and You, 1985), the *Jensen-Shannon divergence* (JS divergence) is a popular alternative measure of divergence (Cha, 2007; Nielsen, 2019). Based on the KL divergence, it is mathematically defined with the formula (Lin, 1991: 146; Nielsen, 2019: 3):

$$JS(p||q) \equiv \frac{D(p||M) + D(q||M)}{2} \quad (13)$$

where  $M = \frac{p+q}{2}$  is the *mixture distribution* of  $p$  and  $q$ . From this formula, it can be recognized that the JS divergence is related to the KL divergence by being the mean of the relative entropies between  $p$  and  $q$  and their mixture distribution (Lin, 1991: 146). JS divergence shares numerous properties of the KL divergence useful to conceptualizing and measuring information divergence (Lin, 1991; Nielsen, 2019: 3), for example, being zero iff  $p = q$  (Lin, 1991: 146-147).

While the JS divergence is closely related to the KL divergence, it possesses numerous advantages. For example, the JS divergence is always finite due to involving the mixture distribution, since this means that  $p$  and  $q$  need not have overlapping support sets (i.e., they need *not* be absolutely continuous with respect to each other; Lin, 1991: 146-147).

Another advantage is that the JS divergence generally is a non-negative, real-valued scalar with an upper-bound of  $\log 2$  (i.e.,  $JS \in \mathbb{R}^{[0;\log 2]}$ ; Lin, 1991: 147; Nielsen, 2019: 3), which means that the upper bound is 1 if using base 2 for the logarithm. These bounds can be considered common for measures within the social sciences, for example, explained variance ( $R^2$ , Gelman et al., 2019), congeneric reliability ( $\rho_c$ ; Jöreskog, 1971), and Sklar's  $\omega$  (Hughes, 2023), making this measure more intuitively interpretable within that discipline.

Perhaps more importantly, unlike relative entropy, the JS divergence is symmetric, and by taking its square-root (i.e.,  $\sqrt{JS}$ ), one obtains the *Jensen-Shannon distance* (JS distance), which also satisfies the triangle inequality (Nielsen, 2019: 3), making it a proper geometric measure of distance. Hence, these Jensen-Shannon-based mea-

asures may be preferred to the KL divergence as the actual measure of the 'relative entropy' between prior and posterior knowledge, and with its approximately linear properties as opposed to the more quadratic behavior of the JS divergence, the JS distance may ultimately be preferred in terms of quantifying surprise. That the JS distance is conceptually appropriate is further corroborated by the notion that it was originally conceptualized in relation to Bayesian decision-making (Lin, 1991: 147-148), making it a natural choice for use in relation to this Bayesian Information-theoretic conceptualization of surprise.

These considerations thus serve this formal conceptualization of surprise by the JS divergence enabling a symmetric measurement of the discrepancy between the prior and posterior distributions, with the JS distance providing an intuitively interpretable quantification of the distance between prior and posterior knowledge, with its  $0 - 1$  bounds (when using base 2) further permitting an interpretation of this distance as the (percentage) degree of surprise. This means that, unlike Itti and Baldi's (2009) conceptualization of 'wow' units, the JS distance thus provides a measure of the *degree* to which one is 'wow'ed'.

If the prior and posterior distribution are normal, then the JS distance is relatively easily computed using numerical integration. For example, if one again considers the normal prior distributions in figure (1) in section (3.1.1) with the posterior being equivalent to Prior #4, then the JS divergence between the posterior and each prior can be computed as approximately .684, .913, .910, and .000 bits, respectively. These values respectively correspond to JS distances of .827, .955, .954, and .000. Within this conceptualization, the distance between the posterior and the second prior can, for example, be meaningfully interpreted as a degree of surprise of 95.5%.

These values fit the conceptual expectations about surprise, since the degree of surprise implied by the discrepancy between the posterior and Prior #4 is effectively zero (i.e., 0%), consistent with the complete absence of surprise when results match the theoretical expectations; the degree of surprise is higher when the theoretical expectation is more certain, as revealed by the higher distance between the posterior and the more narrow Prior #2 (i.e., 91.3%) than when compared to the wider Prior #1 (i.e., 68.4%); and the degree of surprise is also higher if the theoretical expectation incorrectly predicts the magnitude and direction of the results, with the posterior-prior distance being higher for Prior #3 (i.e., 91%) than Prior #1 (i.e., 68.4%), since the former expects the parameter of interest to be positive when it instead is negative.

While the JS divergence/distance is thus useful, these examples have the fundamental limitation of presupposing knowledge of the probability density functions of the prior and posterior. In practice, the prior and posterior need not necessarily be univariate Normal as these examples are, but can take on multiple forms, some of which have no closed-form expression. However, this is *not* a critical issue due to having built this conceptualization on Bayesian inference, whose reliance on MCMC procedures theoretically can be used to estimate almost any arbitrary distribution. While this solves the problem of identifying the distributions of the prior and posterior, this does not solve the problem of calculating the JS distance. This is a common problem in Information theory (Beirlant et al., 2001: 11), where, for example, (relative) entropy must be estimated from the data, which can require large and representative samples (cf. Back et al., 2018: 1; Schürmann and Grassberger, 1996; though see, e.g., Bonachela et al., 2008; Grassberger, 1988; Lesne et al., 2009). For the continuous case, this is further troubled by the need to estimate the density of the particular distribution, which is needed in relation to differential entropy, KL divergence, and the JS divergence. Fortunately, parametric assumptions can sometimes be justified to ease estimation (e.g., Jaynes, 1968: 236-237), and a multitude of density estimators otherwise exist (e.g., Ahmad and

Lin, 1975; Berrett et al., 2019; Delgado-Bonal and Marshak, 2019; Hall and Morton, 1993; Pérez-Cruz, 2019), which may be utilized to compute the JS distance.

## 4 Demonstration

Having thus far provided a formal (re)conceptualization of surprise suited for the social sciences, this section provides a practical demonstration in relation to empirical data to show how one can use this to measure surprise. In line with the principles of *open science* (Christensen et al., 2019; Xie, 2014), all data processing and analysis was done with the R programming language (R Core Team, 2025) in the RStudio IDE (Posit Team, 2025).<sup>12</sup>

This demonstration is based on the research article by Todorov et al. (2005) published in the prestigious journal *Science*. Their article details a series of studies conducted on (under)graduate students at Princeton University in the United States (US; see Todorov et al., 2005, and their supplementary materials), who in one study were individually exposed for 1-second to pairs of standardized black-and-white head-shot photographs of real-world political candidates from the Democratic party or the Republican party, who were either the winner or the runner-up to elections for the US Senate or House of Representatives. The sequence of pairs and the left-right placement of photos were randomized, with participants being instructed to choose the more competent-looking candidate. Based on that data, the authors found that these 'inferences of competence based solely on facial appearance predicted the outcomes of U.S. congressional elections better than chance' (Todorov et al., 2005: 1623). This result was arguably rather surprising, since the dominant theory at the time assumed voting choices to be 'based primarily on rational and deliberative considerations' (Todorov et al., 2005: 1623). Despite the recognized novelty of these results, the degree of surprise was never quantified, making this study suitable for a demonstration of this paper's conceptualization of surprise.

For this demonstration, a publicly available subset of the data by Todorov et al. (2005), provided in Imai (2017), is used. This data constitutes a 119 by 10 matrix, where each row is a pair of political candidates, while the columns are variables indicating the election year, US state, the name of the winner and runner-up, the political party affiliation of each candidate, their absolute vote share, and average competence rating by the participants. With no missing values, the data provides a statistical power of 80% to detect a difference as small as 0.473 at the 5% statistical discernibility level.

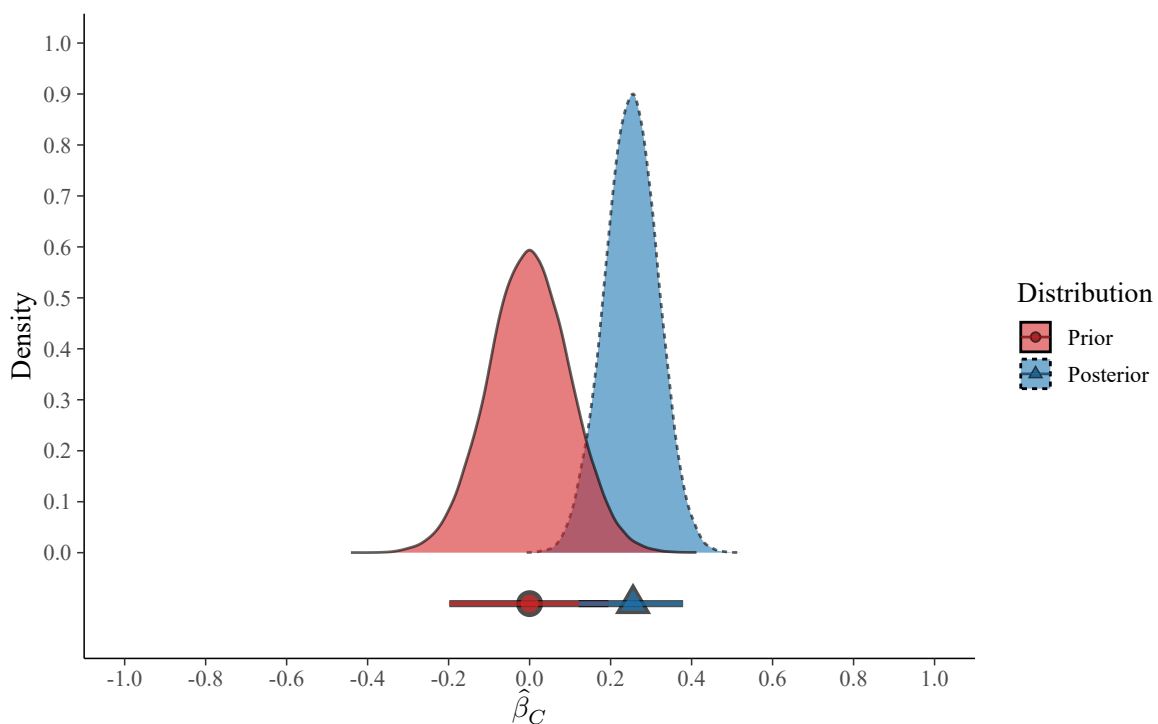
$$\begin{aligned}
 V_i | \mu_i, \sigma &\sim \mathcal{N}(\mu_i, \sigma^2) \text{ for } i \text{ in } 1, \dots, n \\
 \mu_i &= \beta_C C_i \text{ for } i \text{ in } 1, \dots, n \\
 \beta_C &= \mathcal{N}^{[-1;1]}(0, 0.1) \\
 \sigma &= \mathcal{N}^{[0;1]}(1, 0.1)
 \end{aligned} \tag{14}$$

In accordance with Todorov et al. (2005; see also Imai, 2017: 139-148), data is processed so that the variables are reduced to a mean competence score (i.e.,  $C$ ) and a difference in relative vote share (i.e.,  $V$ ) for each candidate.  $V$  is then modeled as a conditionally linear function of  $C$ , with the coefficient of  $C$  (i.e.,  $\beta_C$ ) thus being the parameter of interest, while an intercept and residual dispersion ( $\sigma$ ) serve as auxiliary parameters.

<sup>12</sup>For software specifications and replication materials, see section 8.1 of the Appendix.

For ease of specifying a prior for  $\beta_C$  and  $\sigma$ , both of these variables are rescaled using *Fisher z-scoring* (henceforth denoted *standardization*; Field, 2018: 37), meaning that values greater than zero indicate a competence score in favor of the democratic candidate for  $C$ , while values greater than zero indicate a vote share in favor of the democratic candidate for  $V$ . By standardizing the variables, this eliminates the need to estimate the intercept and introduces the following bounds for  $\beta_C$  and  $\sigma$ :  $0 \leq \sigma \leq 1$  and  $-1 \leq \beta_C \leq 1$  (cf. Agresti, 2018: 273; Wooldridge, 2019: 184:185), which need to be accounted for in their respective prior specifications.

Using this paper's formal conceptualization of surprise, the priors for  $\beta_C$  and  $\sigma$  should ideally be specified so that they accurately reflect the expectations/predictions of theory. Otherwise, the discrepancy between the posterior and prior will otherwise *not* reflect the genuine surprise of the results. While Todorov et al. (2005: 1623) make numerous arguments that can be used to justify a theoretical expectation of facial competence as having a zero influence on the vote share, the certainty around this expectation is unclear. This is partly because the authors refer to numerous existing studies on facial evaluations, competence, and political candidates (e.g., Hassin and Trope, 2000; Haxby et al., 2000; Kinder et al., 1980; Todorov and Uleman, 2003; Winston et al., 2002), and while this helps the authors' argument, it also has the effect of reducing the certainty of opposing theories, thus making eventual positive results comparatively less surprising.



**Figure 2.** Estimated Effect of Facial Competence on Relative Vote Share

**NOTE:** Prior and posterior distributions of the estimated effect of facial competence on relative vote share. Prior/posterior draws = 40,005. The geometrics below the distributions indicate the mode (circle) and the 95% HDI (bar). Jensen-Shannon divergence = .526 bits. Results based on data ( $n = 119$ ) by Todorov et al. (2005; provided in Imai, 2017).

Nevertheless, for the purposes of this demonstration, the prior for  $\beta_C$  is specified as a Normal distribution, with a mean of 0 and a standard deviation of 0.1, which in the context of Todorov et al.'s (2005) study can be taken to reflect a state of knowledge, where one would be comfortably skeptical about the influence of facial competence on vote outcomes. Stated differently, the expectation implied by this prior is that one would believe

with 95% confidence that the absolute effect would maximally correspond to a ‘small’ or ‘medium’ effect (i.e.,  $|r| \approx 0.2$ ; cf. Cohen, 1988; Funder and Ozer, 2019; Gignac and Szodorai, 2016; Lovakov and Agadullina, 2021). Using a similar logic for the prior specification of  $\sigma$ , the statistical model used to estimate  $\beta_C$  is formalized using common notation (e.g., Gelman et al., 2014; Levy and Mislevy, 2020) and shown in model (14).

This model is then fit to the data using the *brms* R package (Bürkner, 2017, 2018), which exploits the *No-U-Turn sampler* (NUTS; Hoffman and Gelman, 2014) of the *STAN probabilistic programming language* (Stan Development Team, 2025b) to enable an efficient Hamiltonian MCMC sampling (Kruschke, 2014: 400-416). To assess convergence of this sampler, 15 independent sampling chains are used, each with 2,000 warm-up samples to help them identify the posterior in advance of actual sampling, with 40,005 samples then being drawn in total. This applies to the sampling of both the prior and the posterior distributions, since both are needed for computing the JS distance. Visual inspections of *trace plots* (Kruschke, 2014: 179-180) and metric assessments of the *Gelman-Rubin convergence metric* ( $\hat{R}$ ) and the *Monte-Carlo standard error* (MCSE) found results consistent with convergence (i.e.,  $\hat{R} \leq 0.01$ , Vehtari et al., 2021;  $\text{MCSE} \leq 0.01$ , Kruschke, 2014: 186-187). Accounting for autocorrelation in the MCMC samples reveals *effective sample sizes* (ESS) that could be considered sufficiently high ( $\text{ESS} \geq 10,000$ ; Kruschke, 2014: 182-184). The explanatory power of this model (i.e., its  $R^2$ ; Gelman et al., 2019) is .073 (SD = .035; 95% HDI[.010; .140]), which can be interpreted as ‘weak’ (cf. Cohen, 1988) with 89.3% credibility. As revealed by a *posterior predictive p-value* ( $PP_P$ ; Gelman et al., 1996) of .970, this shortcoming is likely attributable to the model’s tendency to make over-exaggerated predictions.

**Table 1.** Relationship between Facial Competence & Relative Vote Share

$\hat{\theta}$	Parameter	Posterior	$\hat{R}$	MCSE	ESS	Prior	Surprise
$\hat{\beta}_C$	Facial Competence	.251 (.066) [.123; .380]	1.001	.001	14,865	-.001 (.100) [-.199; .194]	.726
$\hat{\sigma}$	RMSE	.932 (.041) [.857; 1.000]	1.002	.000	11,603	.921 (.060) [.805; 1.000]	.161

**NOTE:** Mean estimated standardized parameters, with posterior SDs in parentheses and 95% HDIs in brackets.  $n = 119$ . MCMC samples = 40,005.  $\hat{R}$  = Gelman-Rubin convergence metric (Vehtari et al., 2021). MCSE = Monte-Carlo standard error (Kruschke, 2014: 186-187). ESS = Effective sample size (Kruschke, 2014: 182-184). Surprise = Jensen-Shannon distance. Bayesian  $R^2$  (Gelman et al., 2019) = .073 (SD = .035; 95% HDI[.010; .140]). Posterior predictive  $p$ -value ( $PP_P$ ; Gelman et al., 1996) = 0.97.

The sampled prior and posterior are displayed in figure (2), with a metric summary being provided in table (1). Inspecting the prior distribution reveals an expected estimated effect of facial competence on vote share before seeing the data of -.001 (SD = .100; 95% HDI[-.199; .194]). By comparison, the posterior distribution reveals an expected estimated effect of facial competence on vote share after seeing the data of .251 (SD = .066; 95% HDI[.123; .380]), which can be interpreted as a ‘medium’-sized effect (cf. Funder and Ozer, 2019) with 55.4% credibility.

The regularizing effect of the theoretically-motivated ‘skeptical, yet persuadable’ prior on the results can be made evident by comparing the posterior to the results from a Frequentist version of model (14), estimated with *ordinary least squares* (OLS; Wooldridge, 2019: 70-73) and the *non-parametric bootstrap* (Efron, 1979, 2003; Efron and Tibshirani, 1994), with 40,005 bootstrapped samples being used to compute the standard error (SE) and a 95% *bias corrected and accelerated interval* (BCaI; Efron, 1987; DiCiccio and Efron, 1996; Makowski et al., 2019b). This yields an expected estimated effect of facial competence on vote share of .432 (SE = .074; 95% BCaI[.275; .565]). The relatively substantial and credible difference between the Bayesian and Frequentist

approach (i.e.,  $\Delta\hat{\beta}_C = -.181$ ;  $SD = .098$ ; 95% HDI $[-.026; -.318]$ ) is likely attributable to the relatively small sample used in this demonstration (i.e.,  $n = 119$ ), and it indicates that the prior will have greater influence on an assessment of surprise in relation to this data, since the posterior is relatively sensitive to its specification.

Based on this paper's conceptualization, the degree of surprise resulting from this study can then be measured as the Jensen-Shannon distance. While the posterior and prior could be considered approximately normal, enabling a parametric estimation of their discrepancy (e.g., Jaynes, 1968: 236-237), this will *not* always be the case, which is why the computation of the JS distance involved in this demonstration instead considers a *non-parametric kernel density plug-in estimator* (KDE), and while such an estimator can be considered imperfect (cf. Berrett et al., 2019; Pérez-Cruz, 2019), it is popular and generally adequate for estimating the densities of the univariate distributions (e.g., Ahmad and Lin, 1975; Hall and Morton, 1993; Parzen, 1962; Rosenblatt, 1956), especially considering the relatively large number of samples (i.e., 40, 000+). Provided in the Appendix (see Section 8.2), this function used for this example also mimics similar estimators (e.g., Drost, 2018), for example, by applying common adjustments to avoid a division by zero, and while functional, these adjustment can be considered suboptimal. While the function uses an efficient KDE (Wand, 2025), similarly applied for density estimation in relation to Bayes factors (Makowski et al., 2019b; Wagenmakers et al., 2010), to make it scalable for the large number of MCMC samples commonly drawn in relation to Bayesian inference, it is by no means perfect, and it merely serves as a functional demonstration for the purposes of this paper.

Computing the JS divergence in this way yields an informational divergence between the posterior and prior of .526 bits, which can be translated into the JS distance to arrive at an estimated degree of surprise of 72.6%. In line with an intuitive/qualitative assessment of these results, this degree of surprise can be considered relatively large.

## 5 Discussion

[Work-in-progress]

## 6 Conclusion

[Work-in-progress]

## 7 Acknowledgements

[Work-in-progress]

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## 8 Appendix

This appendix contains the specifications for the software used in this paper (Section 8.1) and an R function implementing the KDE plug-in estimator that was used to compute the Jensen-Shannon distance from MCMC samples (Section 8.2) as demonstrated in section (4). Besides this appendix, supplementary materials, including replication files, are made freely available online on the OSF project page dedicated to this paper (DOI: 10.17605/OSF.IO/GQ6C8).

### 8.1 Software Specifications

This document was written with *markdown* (Gruber, 2014) and  $\text{\LaTeX}$  (Lamport, 1986; The LaTeX Project, 2025), using  $\text{\pdfTeX}$ (v3.141592653-2.6-1.40.2;  $\text{\MiKTeX}$ [v25.4]; Thành and Knuth, 2025). Compiled on a Windows 11 x64 (build 26200) operating system (OS; Microsoft, 2024) on the 2026-02-01 13:58 CET (Timezone: Europe/Copenhagen), the machine has 16 cores available, and the range of doubles is  $2.23 \times 10^{-308}$  -  $1.8 \times 10^{+308}$ .

All data processing and analyses were conducted on the aforementioned software using the R programming language (v4.4.3 [2025-02-28 ucrt]; R Core Team, 2025) in the RStudio IDE (2025.5.1.513; Posit Team, 2025), which besides the base R packages by R Core Team (2025) used the following packages (including their dependencies, which were *not* necessarily used): *abind* (v1.4.8; Plate and Heiberger, 2024), *arrayhelpers* (v1.1.0; Beleites, 2020), *backports* (v1.5.0; Lang et al., 2024), *base64enc* (v0.1.3; Urbanek, 2015), *bayestestR* (v0.16.1; Makowski et al., 2019b), *bayesplot* (v1.13.0; Gabry et al., 2019), *bookdown* (v0.43; Xie, 2016, 2025a), *boot* (v1.3.31; Canty and Ripley, 2024; Davison and Hinkley, 1997), *bridgesampling* (v1.1.2; Gronau et al., 2020), *brms* (v2.22.0; Bürkner, 2017, 2018), *Brodbingnag* (v1.2.9; Hankin, 2007), *broom* (v1.0.8; Robinson et al., 2025), *cachem* (v1.1.0; Chang, 2024a), *checkmate* (v2.3.3; Lang, 2017), *cli* (v3.6.5; Csárdi, 2025), *cluster* (v2.1.8; Maechler et al., 2024), *coda* (v0.19.4.1; Plummer et al., 2006), *codetools* (v0.2.20; Tierney, 2024), *colorspace* (v2.1.1; Zeileis et al., 2009, 2020), *curl* (v6.4.0; Ooms, 2025a), *data.table* (v1.17.8; Barrett et al., 2025), *datawizard* (v1.1.0; Patil et al., 2022), *devtools* (v2.4.5; Wickham et al., 2022), *digest* (v0.6.37; Eddelbuettel, 2024), *distributional* (v0.5.0; O’Hara-Wild et al., 2024), *dplyr* (v1.1.4; Wickham et al., 2023a), *effectsize* (v1.0.1; Ben-Shachar et al., 2020), *ellipsis* (v0.3.2; Wickham, 2021), *evaluate* (v1.0.4; Wickham and Xie, 2025), *farver* (v2.1.2; Pedersen et al., 2024), *fastmap* (v1.2.0; Chang, 2024b), *filehash* (v2.4.6; Peng, 2006), *forcats* (v1.0.0; Wickham, 2023a), *foreach* (v1.5.2; Microsoft and Weston, 2022), *foreign* (v1.5.2; Microsoft and Weston, 2022), *Formula* (v1.2.5; Zeileis and Croissant, 2010), *fs* (v1.6.6; Hester et al., 2025), *gdata* (v3.0.1; Warnes et al., 2024), *generics* (v0.1.4; Wickham et al., 2025c), *ggdist* (v3.3.3; Kay, 2024a, 2025), *ggplot2* (v3.5.2; Wickham, 2016), *glmnet* (v4.1.9; Friedman et al., 2010), *glue* (v1.8.0; Hester and Bryan, 2024), *gridExtra* (v2.3; Auguie, 2017), *gtable* (v0.3.6; Wickham and Pedersen, 2024), *gtools* (v3.9.5; Warnes et al., 2023), *Hmisc* (v5.2.3; Harrell Jr, 2025), *hms* (v1.1.3; Müller, 2023), *htmlTable* (v2.4.3; Gordon et al., 2024), *htmltools* (v0.5.8.1; Cheng et al., 2024), *htmlwidgets* (v1.6.4; Vaidyanathan et al., 2023), *httpuv* (v1.6.16; Cheng et al., 2025), *HuraultMisc* (v1.1.1; Hurault, 2021), *inline* (v0.3.21; Sklyar et al., 2025), *insight* (v1.3.1; Lüdtke et al., 2019), *iterators* (v1.0.14; Revolution Analytics and Weston, 2022), *jomo* (v2.7.6; Quartagno and Carpenter, 2023), *jsonlite* (v2.0.0; Ooms, 2014), *kableExtra* (v1.4.0; Zhu, 2024), *KernSmooth* (v2.23.26; Wand, 2025), *knitr* (v1.50; Xie, 2014, 2015, 2025b), *later* (v1.4.2; Chang et al., 2025b), *lattice* (v0.22.6; Sarkar, 2008), *lifecycle* (v1.0.4; Henry and Wickham, 2023), *lme4* (v1.1.37; Bates et al.,

2015), loo (v2.8.0; Vehtari et al., 2024), lubridate (v1.9.4; Grolemund and Wickham, 2011), magrittr (v2.0.3; Bache and Wickham, 2022), MASS (v7.3.64; Venables and Ripley, 2002), Matrix (v1.7.2; Bates et al., 2025), matrixStats (v1.5.0; Bengtsson, 2025), memoise (v2.0.1; Wickham et al., 2021), mice (v3.18.0; van Buuren and Groothuis-Oudshoorn, 2011), mime (v0.13; Xie, 2025c), miniUI (v0.1.2; Cheng, 2025a), minqa (v1.2.8; Bates et al., 2024), mitml (v0.4.5; Grund et al., 2023), mvtnorm (v1.3.3; Genz and Bretz, 2009), nlme (v3.1.167; Pinheiro et al., 2025), nloptr (v2.2.1; Johnson, 2008), nnet (v7.3.20; Venables and Ripley, 2002), pacman (v0.5.1; Rinker and Kurkiewicz, 2018), pan (v1.9; Zhao and Schafer, 2023), parameters (v0.27.0; Lüdecke et al., 2020), pillar (v1.11.0; Müller and Wickham, 2025a), pkgbuild (v1.4.8; Wickham et al., 2025b), pkgconfig (v2.0.3; Csárdi, 2019), pkgload (v1.4.0; Wickham et al., 2024b), Polychrome (v1.5.4; Coombes et al., 2019), posterior (v1.6.1; Bürkner et al., 2025), profvis (v0.4.0; Wickham et al., 2024c), promises (v1.3.3; Cheng, 2025b), purrr (v1.1.0; Wickham and Henry, 2025), pwr (v1.3.0; Champely, 2020), QuickJSR (v1.8.0; Johnson, 2025), R6 (v2.6.1; Chang, 2025), rbibutils (v2.3; Boshnakov and Putman, 2024), RColorBrewer (v1.1.3; Neuwirth, 2022), Rcpp (v1.1.0; Eddelbuettel, 2013; Eddelbuettel and Balamuta, 2018; Eddelbuettel and François, 2011; Eddelbuettel et al., 2025), RcppParallel (v5.1.10; Allaire et al., 2025), Rdpack (v2.6.4; Boshnakov, 2025), readr (v2.1.5; Wickham et al., 2024d), reformulas (v0.4.1; Bolker, 2025), remotes (v2.5.0; Csárdi et al., 2024), rlang (v1.1.6; Henry and Wickham, 2025), rmarkdown (v2.29; Allaire et al., 2024), rpart (v4.1.24; Therneau and Atkinson, 2025), rstan (v2.32.7; Stan Development Team, 2025a), rstantools (v2.4.0; Gabry et al., 2024), rstudioapi (v0.17.1; Ushey et al., 2024), scales (v1.4.0; Wickham et al., 2025d), scatterplot3d (v0.3.44; Ligges and Mächler, 2003), scrutiny (v0.5.0; Jung, 2024), sessioninfo (v1.2.3; Wickham et al., 2025a), shape (v1.4.6.1; Soetaert, 2024), shiny (v1.11.1; Chang et al., 2025a), StanHeaders (v2.32.10; Stan Development Team, 2020), stringi (v1.8.7; Gagolewski, 2022), stringr (v1.5.1; Wickham, 2023b), survival (v3.8.3; Therneau, 2024), svUnit (v1.0.6; Grosjean, 2025), tensorA (v0.36.2.1; van den Boogaart, 2023), tibble (v3.3.0; Müller and Wickham, 2025b), tidybayes (v3.0.7; Kay, 2024b), tidyr (v1.3.1; Wickham et al., 2024e), tidyselect (v1.2.1; Henry and Wickham, 2024), tidyverse (v2.0.0; Wickham et al., 2019), tikzDevice (v0.12.6; Sharpsteen and Bracken, 2023), timechange (v0.3.0; Spinu, 2024), tzdb (v0.5.0; Vaughan, 2025), urlchecker (v1.0.1; R Core team et al., 2021), usethis (v3.1.0; Wickham et al., 2024a), V8 (v7.0.0; Ooms, 2025b), vctrs (v0.6.5; Wickham et al., 2023b), weights (v1.1.2; Pasek, 2025), withr (v3.0.2; Hester et al., 2024), xfun (v0.52; Xie, 2025d), and xtable (v1.8.4; Dahl et al., 2019). For the random number generator used by MCMC, the seed (i.e., 896663432) was exogenously predetermined by the random R package (v0.2.6; Eddelbuettel, 2017).

## 8.2 Jensen-Shannon distance function

An implementation of the Jensen-Shannon distance function using R (v4.4.3 [2025-02-28 ucrt]; R Core Team, 2025), and an example of its application in relation to the posterior and prior from Bayesian inference in relation to empirical data using the brms R package (v2.22.0; Bürkner, 2017, 2018), is provided in the code below:

```

1 # Function for computing the Jensen-Shannon (JS) distance/divergence based on MCMC
  ↳ samples from the posterior and prior distributions using default parameter
  ↳ values
2 #
3 # @param p An n-dimensional vector of distributional samples

```

```

4 # @param q An n-dimensional vector of distributional samples
5 # @param n An integer indicating the length of the grid used for estimating the
  ↪ densities
6 # @param epsilon A small, positive real-valued scalar used to avoid instances of
  ↪ division by zero
7 # @param base A small, positive real-valued scalar used as the base of the
  ↪ logarithmic function
8 # @param type A character that determines whether the output is the JS distance or
  ↪ divergence
9 # @return A non-negative real-valued scalar reflecting the estimated discrepancy
  ↪ between the distributions
10
11 # install.packages("KernSmooth")
12 JSD <- function(p = NULL, q = NULL, n = 10000, epsilon = 1e-05, base = 2, type = c(
  ↪ "Distance", "Divergence")){
13   # Parameter checks
14   stopifnot(length(p)==length(q))
15   stopifnot(is.numeric(p))
16   stopifnot(is.numeric(q))
17   stopifnot(is.numeric(n))
18   stopifnot(n > 1)
19   stopifnot(is.numeric(epsilon))
20   stopifnot(epsilon > 0)
21   stopifnot(is.numeric(base))
22   stopifnot(base > 0)
23   type <- match.arg(type)
24
25   # Define a common grid
26   grid <- seq(min(c(p, q)), max(c(p, q)), length.out = n)
27
28   # Use Kernsmooth::bkde to estimate densities on the same grid if available,
  ↪ otherwise use stats::density
29   if(requireNamespace("KernSmooth", quietly = TRUE)){
30     require(KernSmooth)
31     p_density <- KernSmooth::bkde(p, gridsize = n, range.x = c(min(c(p, q)), max(c(
  ↪ p, q))))$y
32     q_density <- KernSmooth::bkde(q, gridsize = n, range.x = c(min(c(p, q)), max(c(
  ↪ p, q))))$y
33   }
34   else{
35     p_density <- stats::density(p, bw = "SJ", kernel = "gaussian", from = min(grid)
  ↪ , to = max(grid), n = length(grid))$y

```

```
36   q_density <- stats::density(q, bw = "SJ", kernel = "gaussian", from = min(grid)
37     ↪ , to = max(grid), n = length(grid))$y
38 }
39
40 # Avoid any zeros
41 p_density <- pmax(p_density, epsilon)
42 q_density <- pmax(q_density, epsilon)
43
44 # Ensure normalized densities
45 dx <- diff(grid)[1]
46 p_density <- p_density / sum(p_density * dx)
47 q_density <- q_density / sum(q_density * dx)
48
49 # Compute mixture
50 m <- 0.5 * (p_density + q_density)
51
52 # Compute JSD
53 jensen_shannon_divergence <- 0.5 * sum(p_density * log(p_density / m) * dx) +
54   0.5 * sum(q_density * log(q_density / m) * dx)
55
56 # Change to user-specified base
57 jensen_shannon_divergence/log(base)
58
59 if(type == "Distance"){
60   jensen_shannon_distance <- sqrt(jensen_shannon_divergence)
61   return(jensen_shannon_distance)
62 }
63 else{
64   return(jensen_shannon_divergence)
65 }
```

```
1 # Example
2 # install.packages("tidyverse")
3 library("tidyverse")
4
5 # Load data
6 # install.packages("devtools")
7 # library("devtools")
8 # devtools::install_github("kosukeimai/qss-package", build_vignettes = TRUE)
9 data(face, package = "qss")
10 df <- face; rm(face)
11
```

```
12 # Recode data
13 df <- df %>%
14   mutate(
15     d.share = d.votes / (d.votes + r.votes),
16     r.share = r.votes / (d.votes + r.votes),
17     d.diff.share = d.share - r.share,
18     d.diff.share = standardize(d.diff.share),
19     d.comp = standardize(d.comp)
20 ) %>%
21   select(d.comp, d.diff.share) %>%
22   rename(
23     democratic_competence = d.comp,
24     democratic_relative_support = d.diff.share
25   )
26
27 # Standardize data matrix
28 # install.packages("datawizard")
29 library("datawizard")
30 df <- apply(df, 2, datawizard::standardize) %>% as.data.frame()
31
32 # Specify MCMC
33 # install.packages("parallel")
34 library("parallel")
35 posterior_samples <- 4000
36 chains <- cores <- parallel::detectCores()-1
37 warmup <- 1000
38 iter <- ceiling((posterior_samples / chains) + warmup)
39 posterior_samples <- (iter - warmup)*chains
40
41 # Fit BGLM
42 # install.packages("brms")
43 library("brms")
44 democratic_relative_support_competence_fit <- brms::brm(
45   democratic_relative_support ~ 0 + democratic_competence,
46   data = df,
47   prior = c(
48     brms::set_prior("normal(0, 0.1)", class = "b", lb = -1, ub = 1),
49     brms::set_prior("normal(1, 0.1)", class = "sigma", lb = 0, ub = 1)
50   ),
51   family = gaussian(
52     link = "identity"
53   ),
```

```
54 sample_prior = "yes",
55 cores = cores,
56 chains = chains,
57 seed = 896663432,
58 iter = iter,
59 warmup = warmup
60 )
61 # summary(democratic_relative_support_competence_fit)
62
63 # Extract draws
64 democratic_relative_support_competence_draws <- brms::as_draws_df(
65   ↪ democratic_relative_support_competence_fit)
66 b_democratic_competence_posterior <-
67   ↪ democratic_relative_support_competence_draws$b_democratic_competence
68 b_democratic_competence_prior <-
69   ↪ democratic_relative_support_competence_draws$prior_b
70
71 # Compute JSKL divergence
72 JSD(b_democratic_competence_prior, b_democratic_competence_posterior)
```