Measures of spread

Overview

Review of shapes distributions and central tendency

The standard deviation

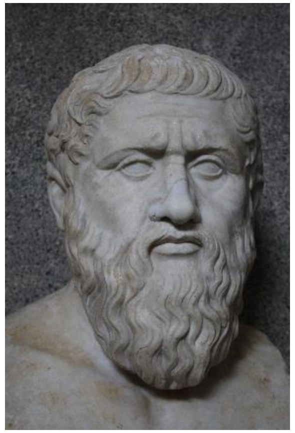
Z-scores

Percentiles

Review and continuation of…

Quantitative variables

Underlying concepts: the P’s and the S’s



P-Truth

- population or process
- parameter
- Plato (Greek symbols)

S-shadows

- sample
- statistic
- shadow (Latin symbols)

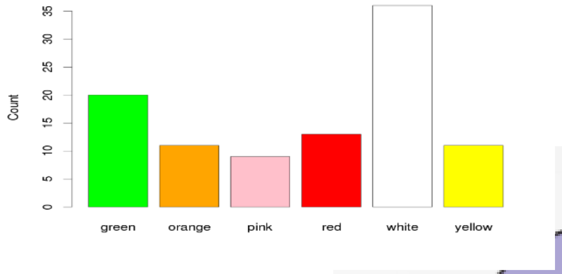
Review: Categorical data and proportions

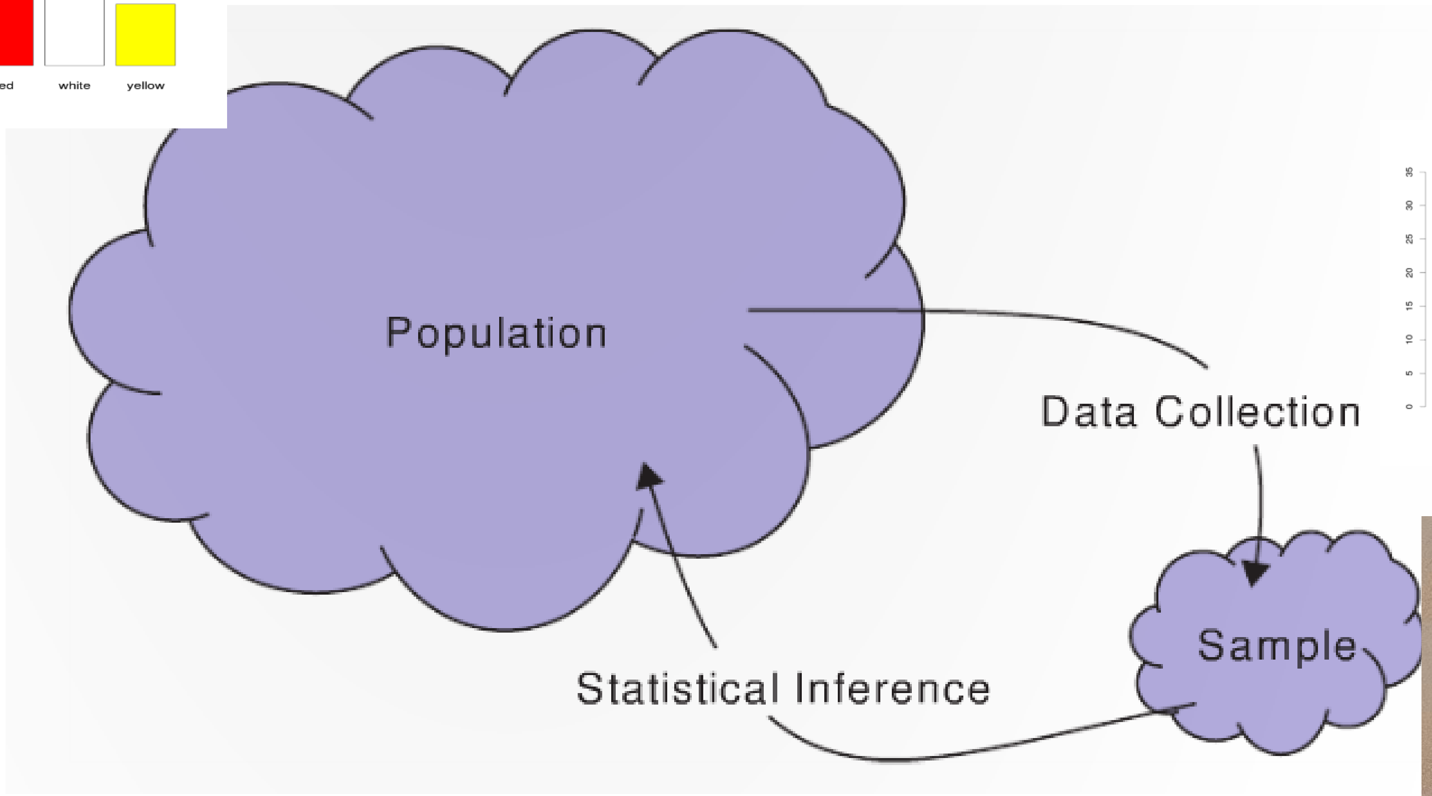
parameter

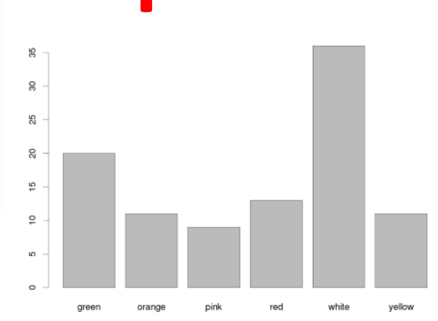
π

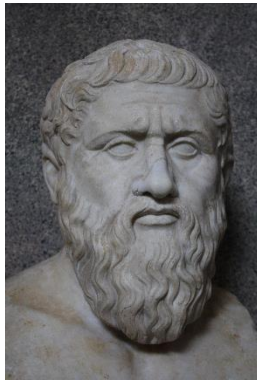
statistic

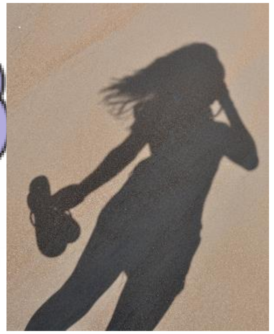
$\hat{p}$





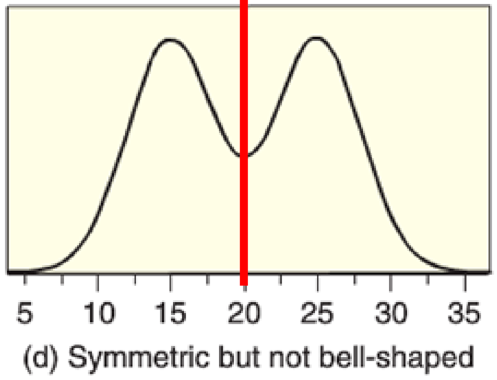






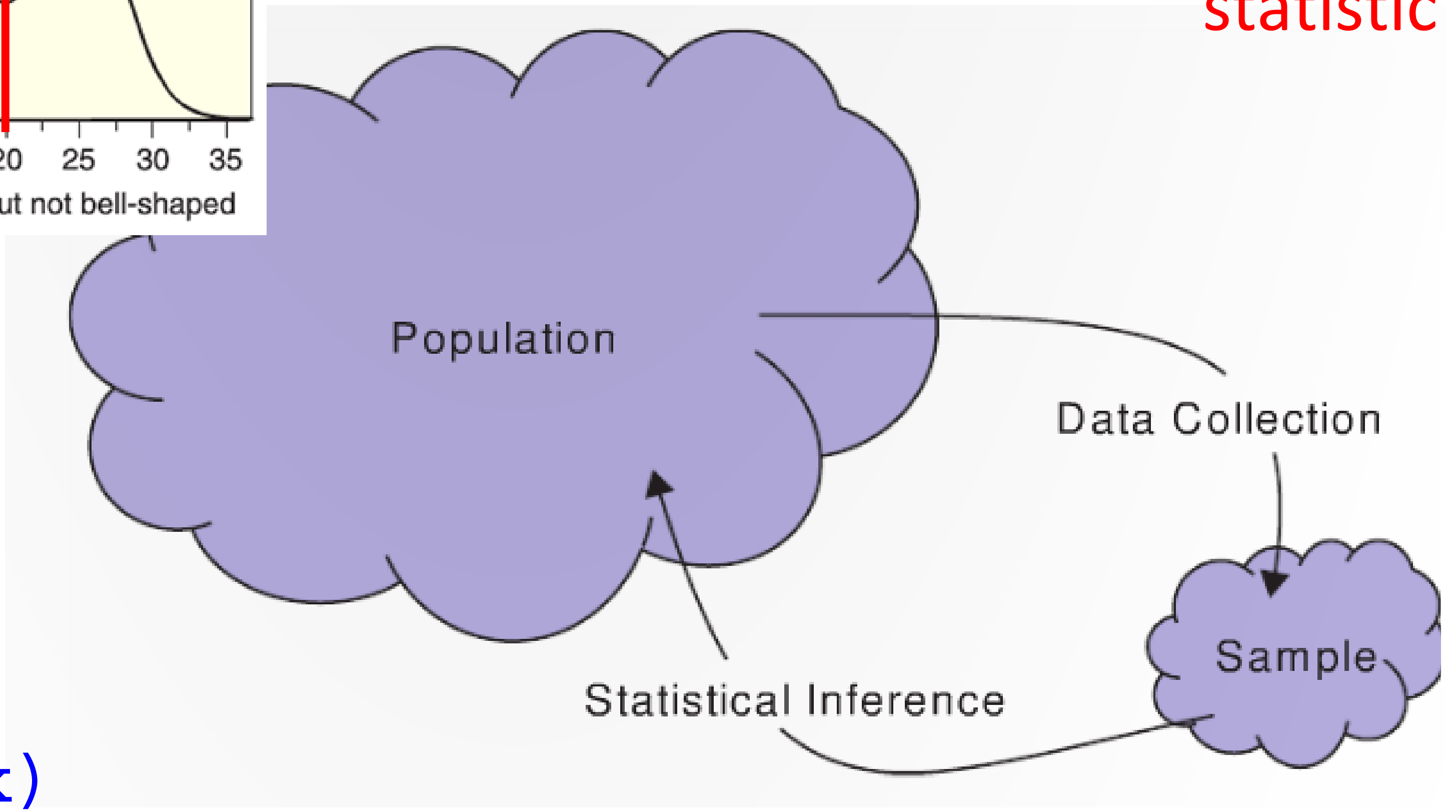
Review: Quantitative data and the mean

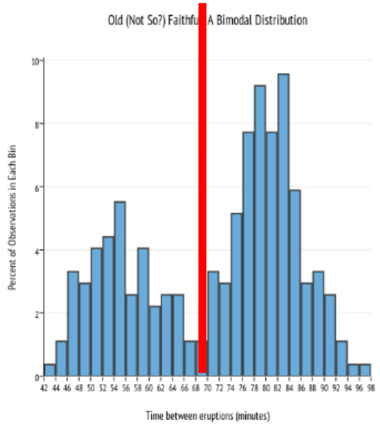
μ ← parameter



$$\bar{x} = \frac{\Sigma_i^n x_i}{n}$$

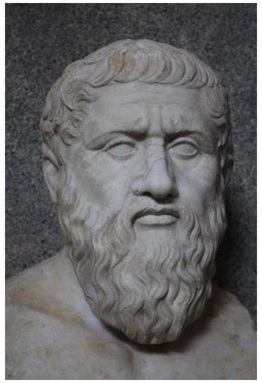
statistic → x̄

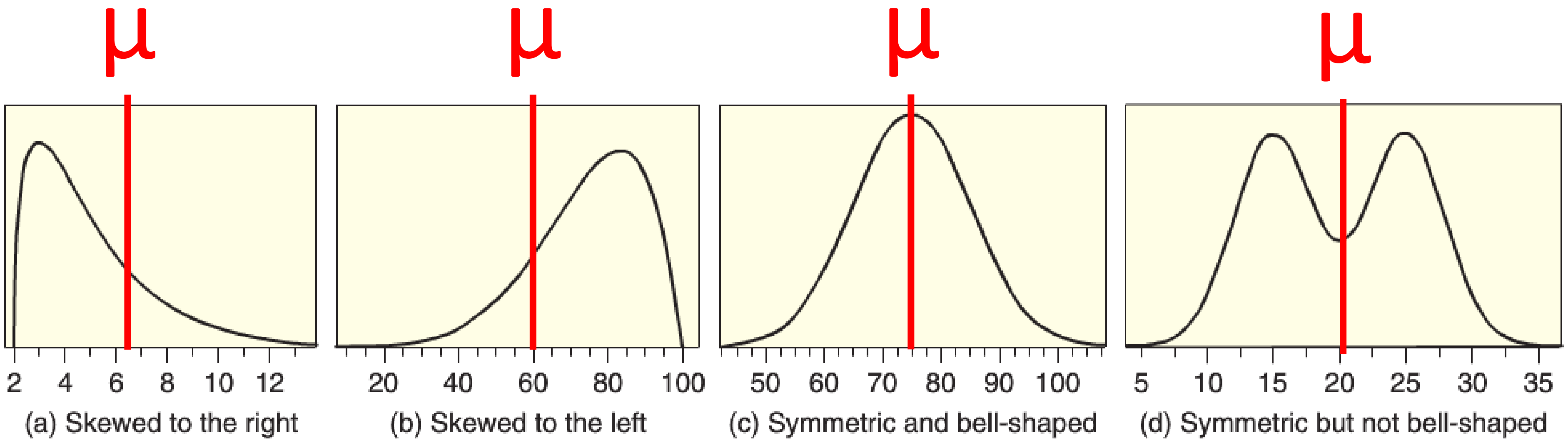


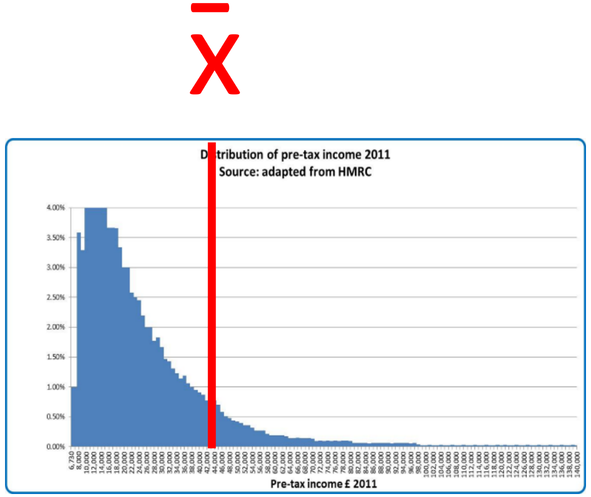


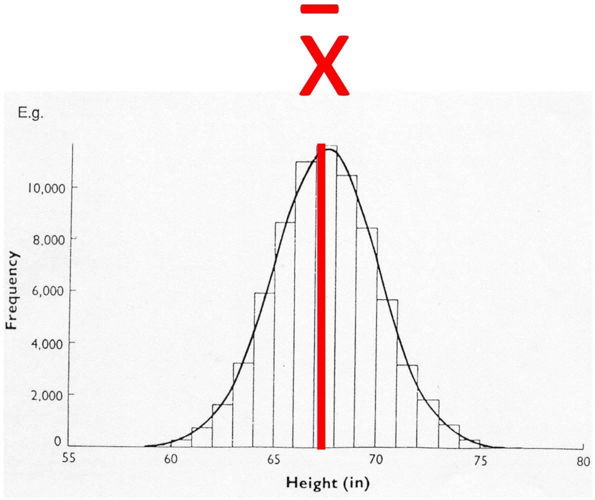
R: `mean(x)`

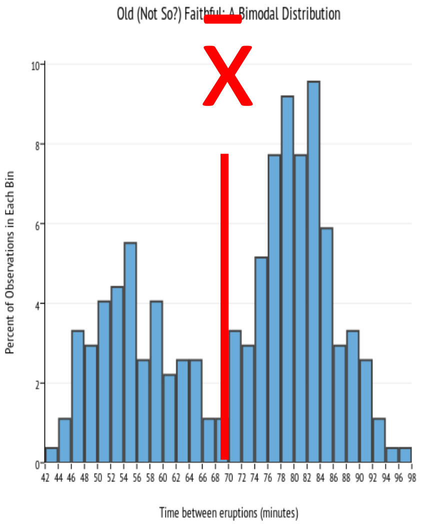
Means for differently shaped distributions





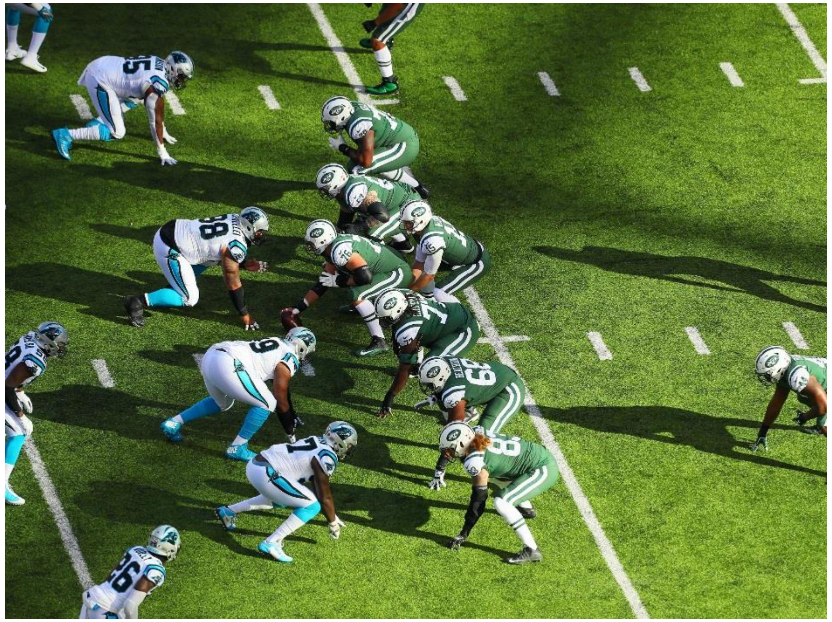


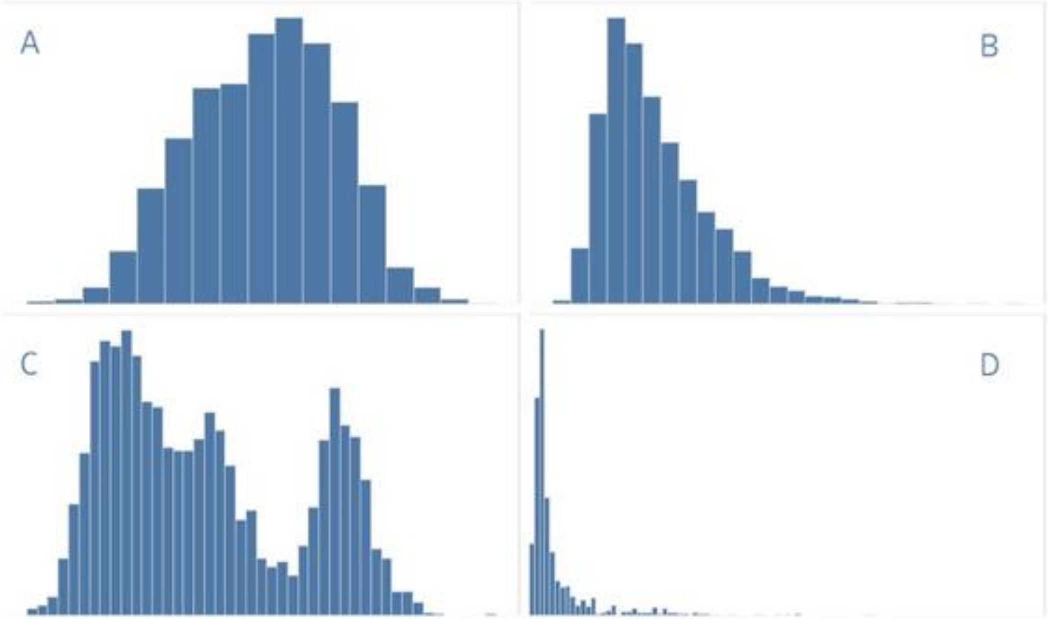




Neat facts – the average NFL player is:

- 1. **Age**: Is about 25 years old
- 2. **Height**: Is just over 6'2" in height
- 3. **Weight**: Weighs a little more than 244lbs
- 4. **Salary**: Makes slightly less than $1.5M in salary per year





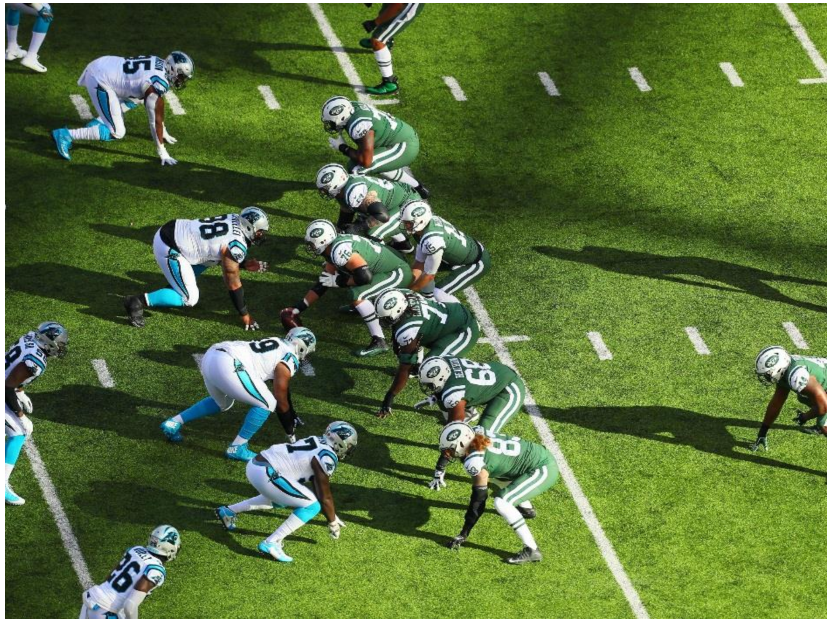
Question: Can you tell which histogram goes with which trait?

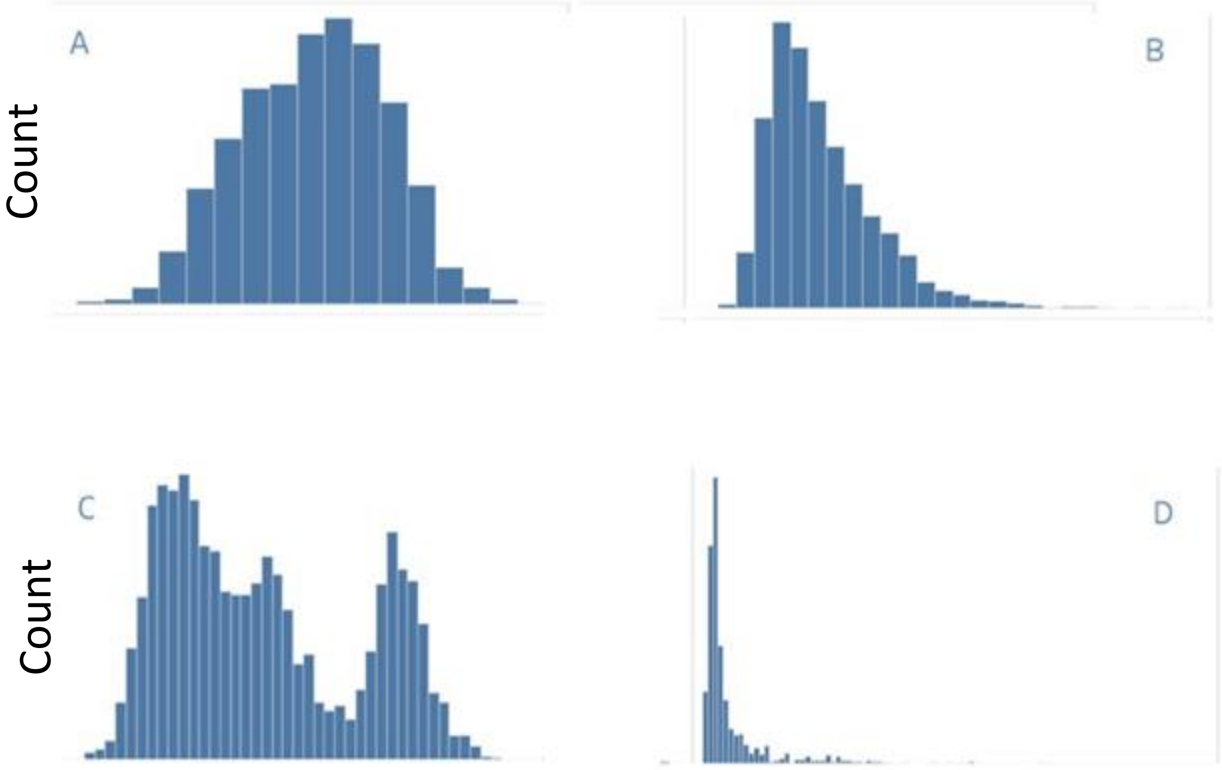
Task is to add the labels: **Age, Height, Weight, and Salary**

- Hint: There are a wide range of positions in football that have very different roles
  - E.g., placekickers only play for small factions of the game, while quarterbacks are essentially to a team's success

First: what is the label for the y-axis?

- A: Frequency or count



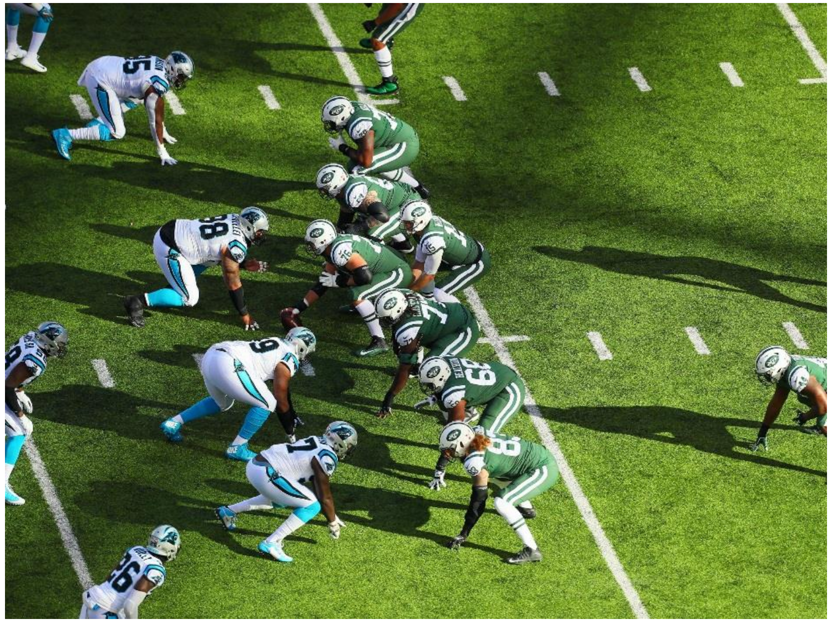


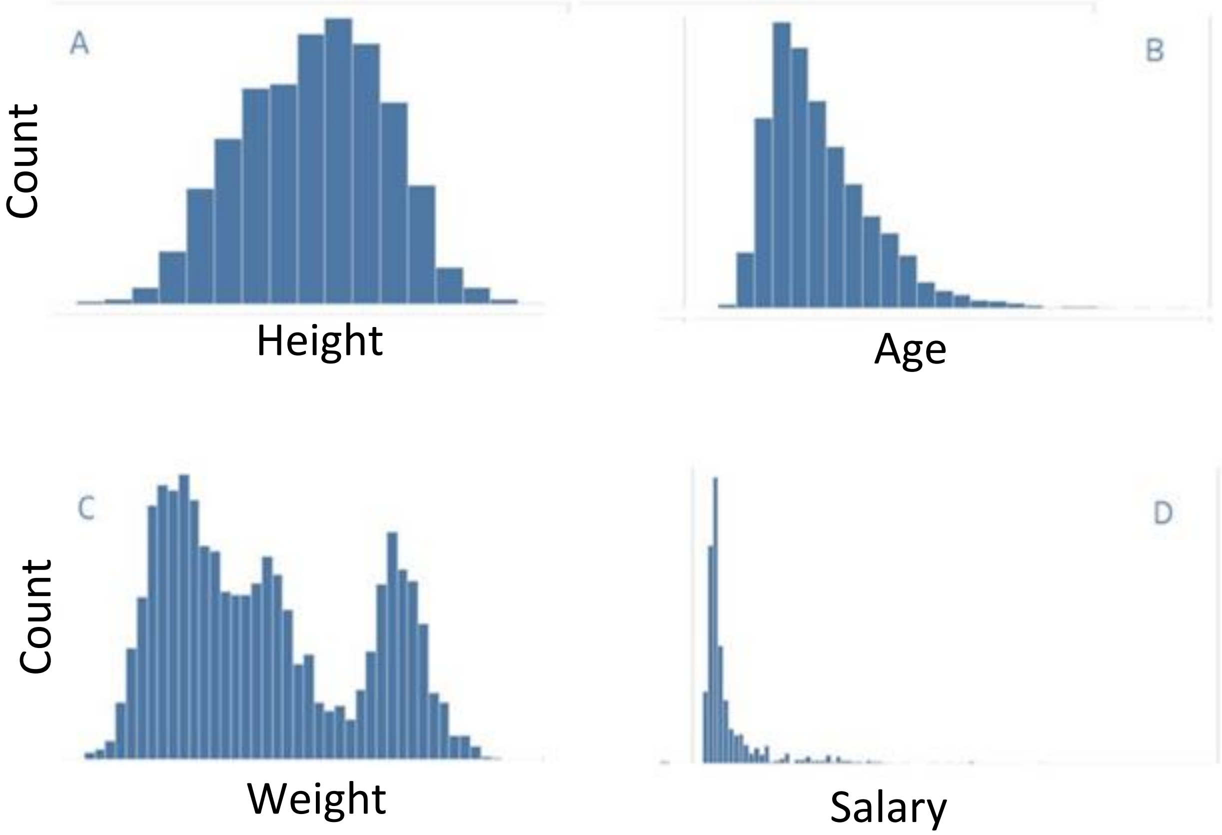
Task is to add the labels: **Age, Height, Weight, and Salary**

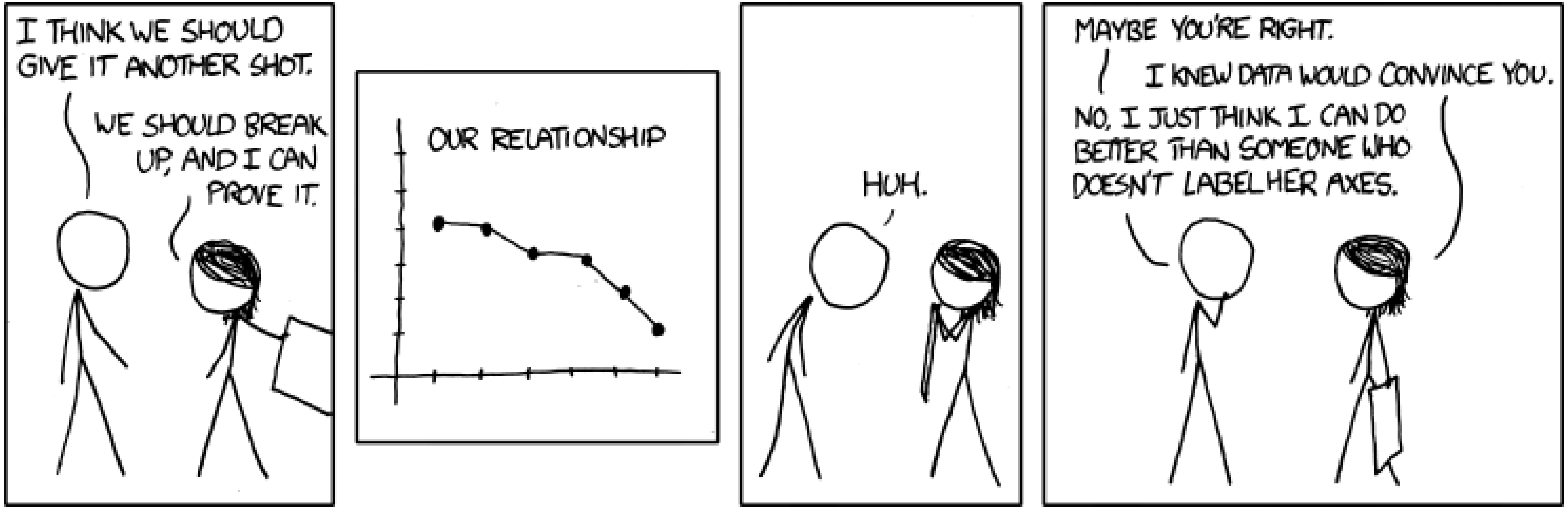
- Hint: There are a wide range of positions in football that have very different roles
  - E.g., placekickers only play for small factions of the game, while quarterbacks are essentially to a team's success

First: what is the label for the y-axis?

- A: Frequency or count







If you don’t want exes, label you axes!

Back to the Gapminder data...

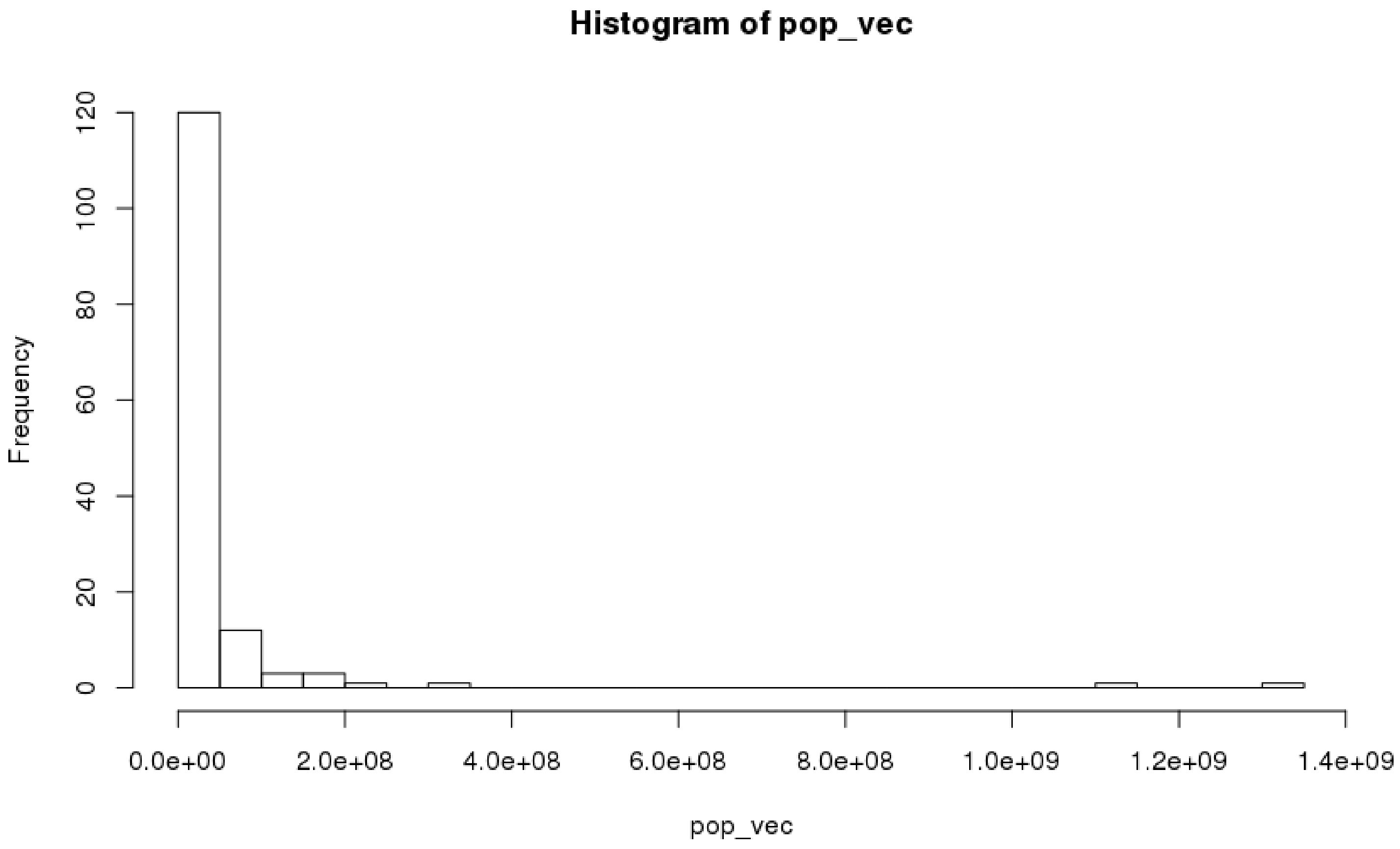
```
# get a data frame with information about the countries in the world
> download_data("gapminder_2007.Rda")   # SDS100 function - only need to run this once
> load("gapminder_2007.Rda")
```

	country	continent	year	lifeExp	pop	gdpPercap
1	Afghanistan	Asia	2007	43.828	31889923	974.5803
2	Albania	Europe	2007	76.423	3600523	5937.0295
3	Algeria	Africa	2007	72.301	33333216	6223.3675
4	Angola	Africa	2007	42.731	12420476	4797.2313
5	Argentina	Americas	2007	75.320	40301927	12779.3796

Can you plot a histogram of the population of each country with 20 bins?

```
> pop_vec <- gapminder_2007$pop   # first create a vector with the population of each country
> hist(pop_vec, breaks = 20)      # then create the histogram
```

What is missing from this histogram?



Axes labels could be more informative!

Labeling axes

Question: Can you figure out how to label the axes?

- > ? hist
- Answer: xlab and ylab!

```
> hist(pop_vec, breaks = 20,
       ylab = "Frequency",
       xlab = "Population",
       main = "World countries population in 2007")
```

The median

The **median** is a value that splits the data in half

- i.e., half the values in the data are smaller than the median and half are larger

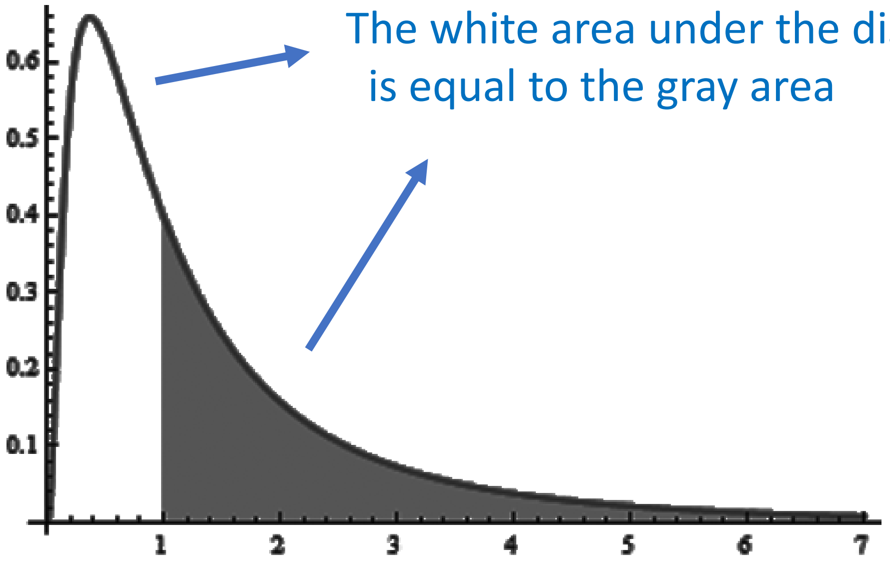
To calculate the median for a data sample of size n, sort the data and then:

- If n is odd: The middle value of the sorted data

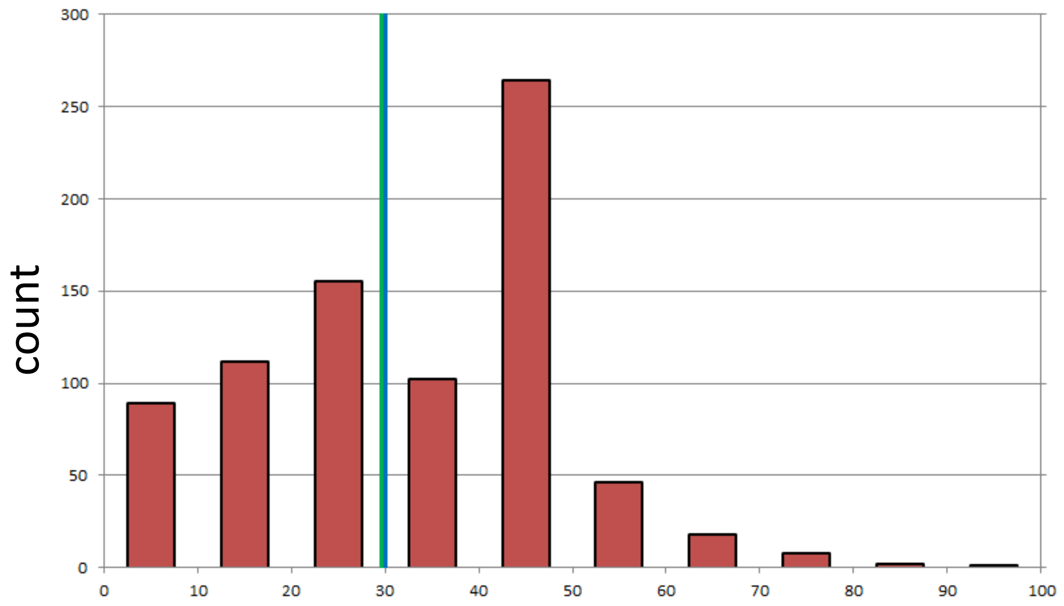
- If n is even: The average of the middle two values of the sorted data

The median

The sum of the heights of the bars on the left is equal to the sum of the heights of the bars on the right



The white area under the distribution is equal to the gray area



R: `median(v)`

`median(v, na.rm = TRUE)`

Example of calculating the mean and median

When an individual visits a webpage a ‘ping’ is generated

Below is a random sample of ping counts from 7 people who pinged a website at least once:

12, 45, 6, 4, 158, 10, 59

Question: What is the mean and median ping count in this sample?

A: mean = 42

median = 12

Mean = $\frac{\Sigma_i^n x_i}{n}$

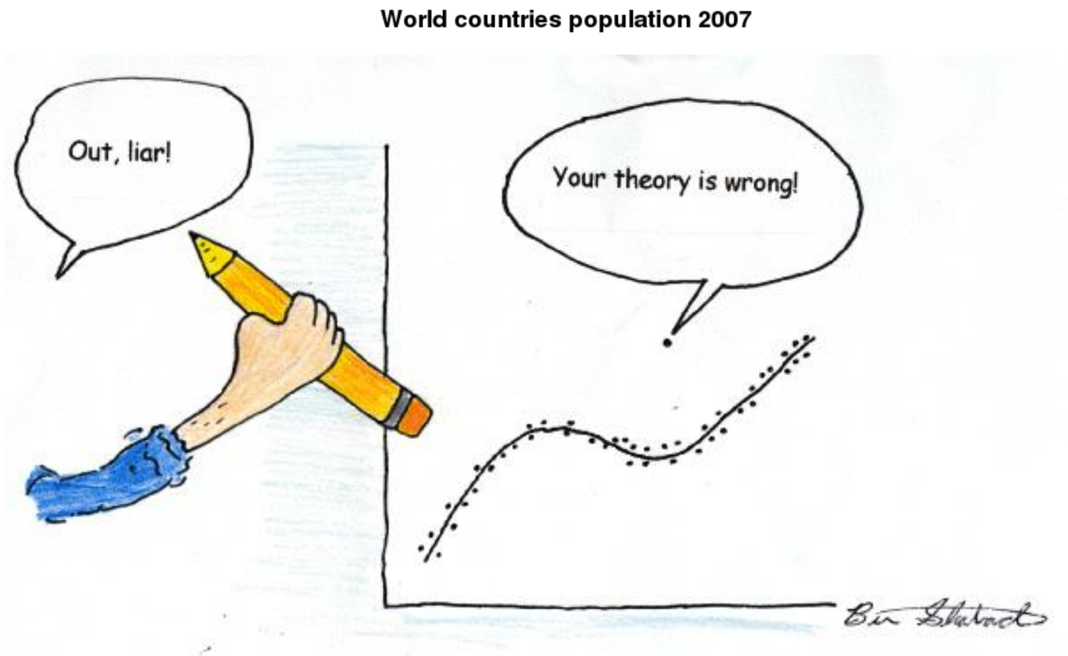
Review: outliers

Q: What is an **outlier?**

Q: Why are they problematic?

Q: What should you do if you have an outlier in your data?

Q: Is the mean and/or median resistant?



Review: outliers

Q: What is an **outlier?**

- A: An observed value that is notably distinct from the other values in a dataset

Q: Why are they problematic?

- A: can potentially have a large influence on the statistics you calculate

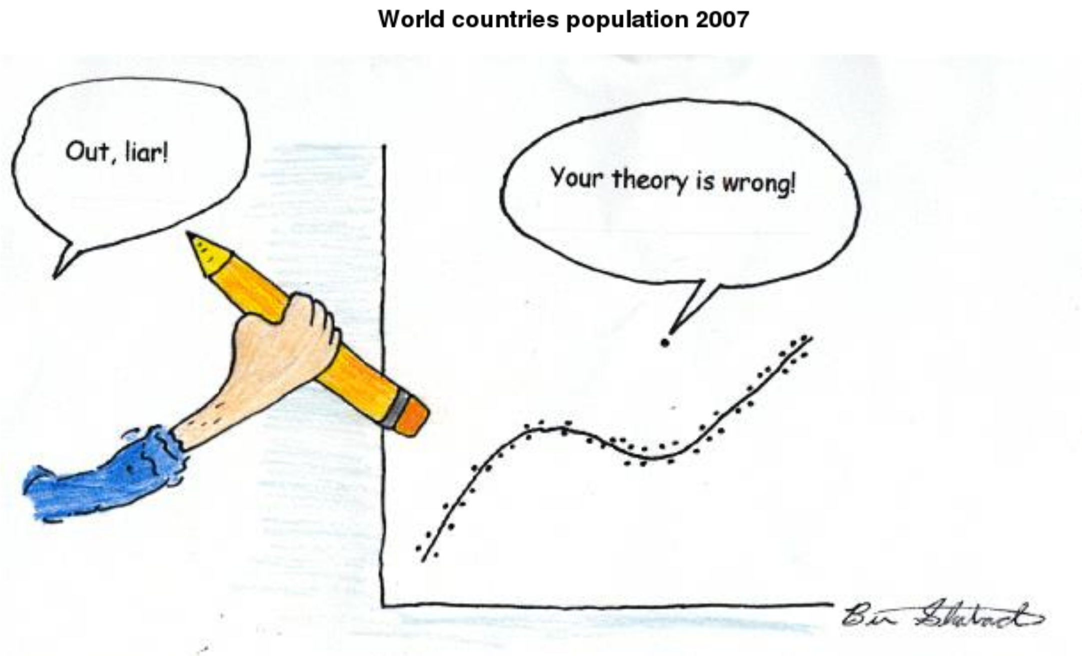
Q: What should you do if you have an outlier in your data?

A: See if you can understand what is causing it!

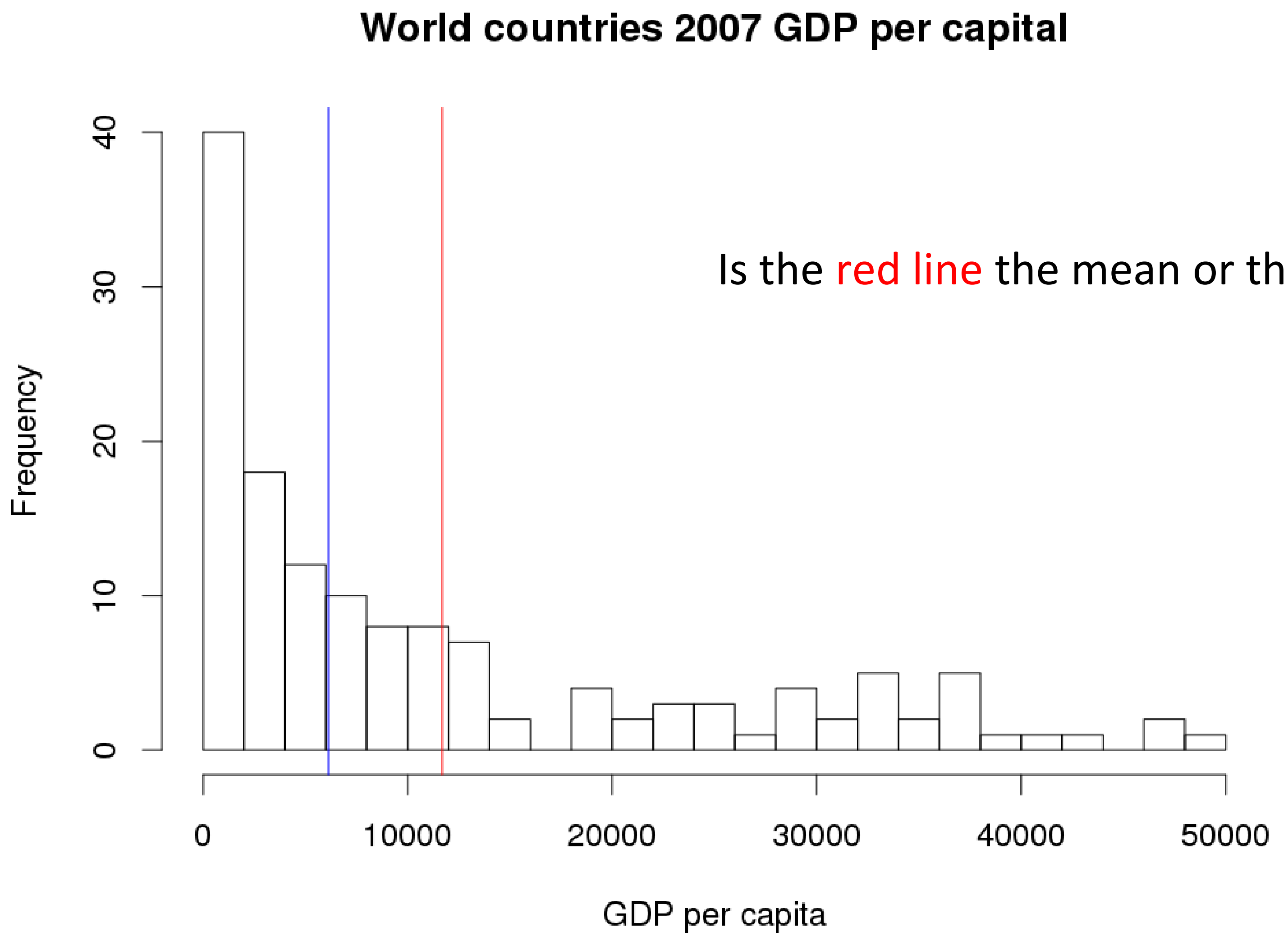
- If it’s an error, delete the point
- If it’s a real value, make sure it is not having a big effect on your conclusions, and/or use resistant statistics

Q: Is the mean and/or median resistant?

- A: The median is resistant while the mean is not



Measure of central tendency: mean and median

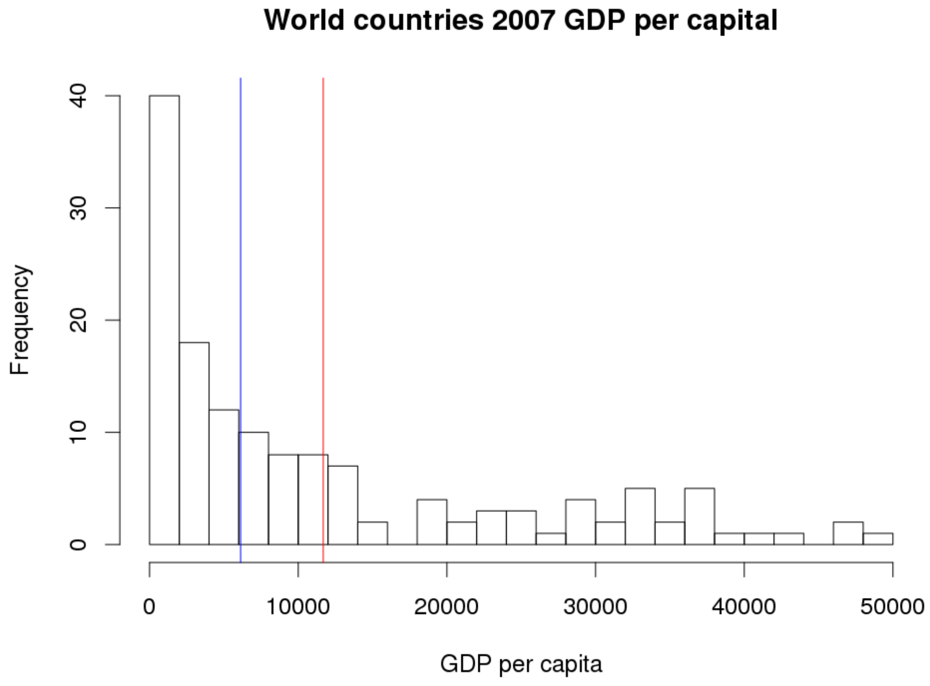


Is the red line the mean or the median?

Measures of spread

Characterizing the spread

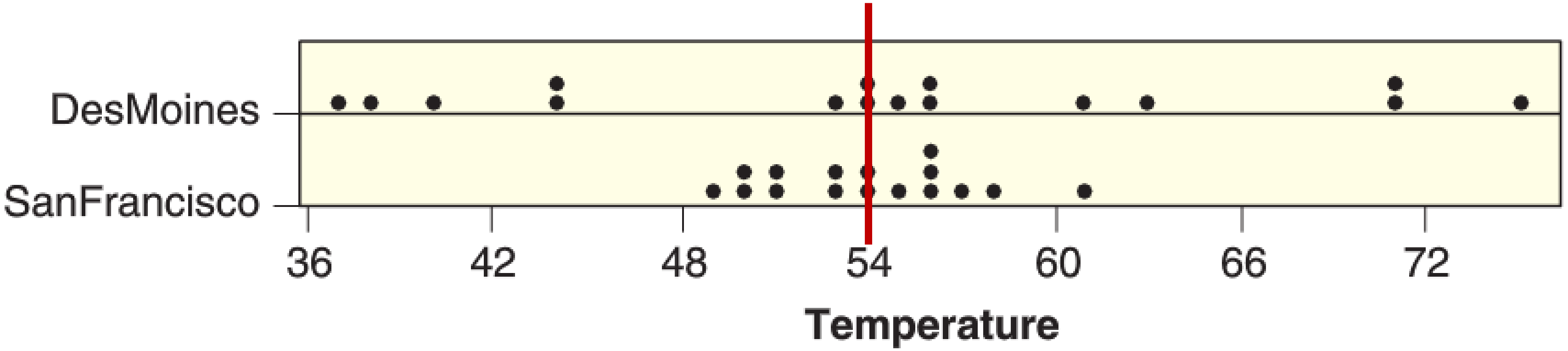
The mean and median are numbers that tell us about the center of a distribution



We can also use numbers to characterize how data is spread

Average monthly temperature:
Des Moines vs. San Francisco

Data measured on April 14th from 1997 to 2010:

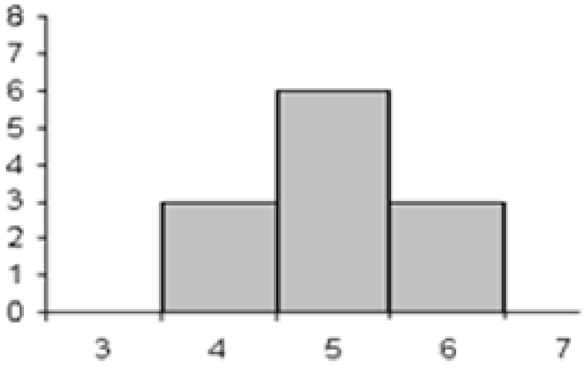


Mean temperature (°F): Des Moines = 54.49 San Fran = 54.01

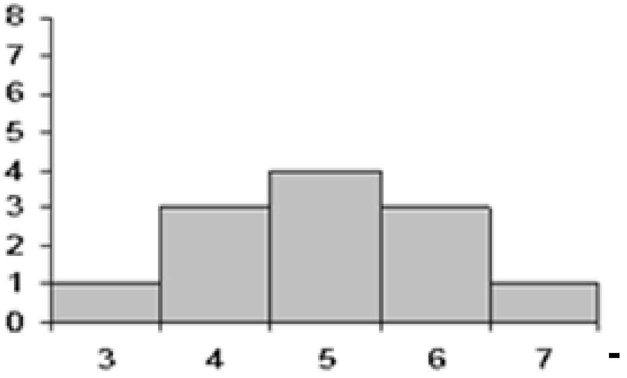
The standard deviation

The **standard deviation** (for a quantitative variable) is a measure of the spread of the data

Smaller standard deviation



Larger standard deviation



It gives a rough estimate for a typical distance a point is from the center

Notation

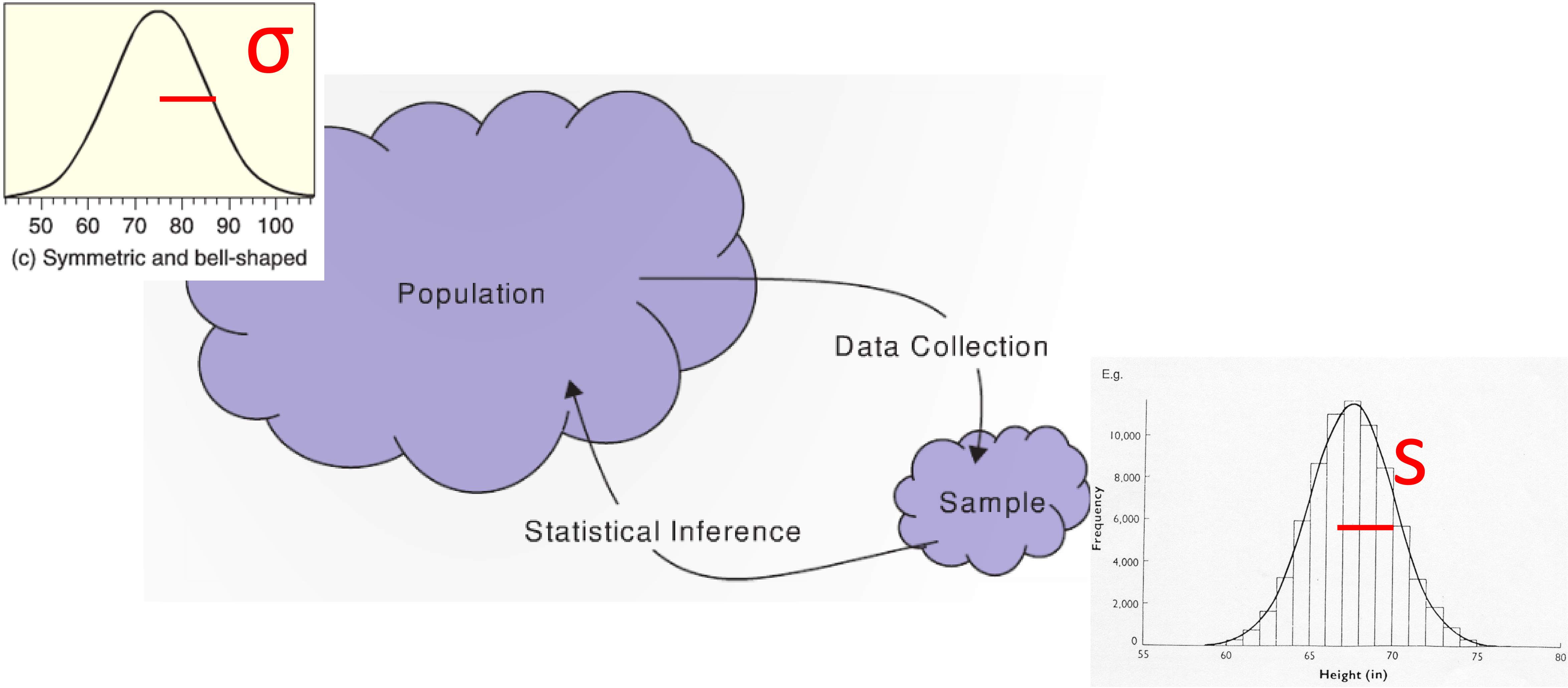
The standard deviation of the ***population*** is denoted **σ**

- It measure the spread of the data from the population mean μ

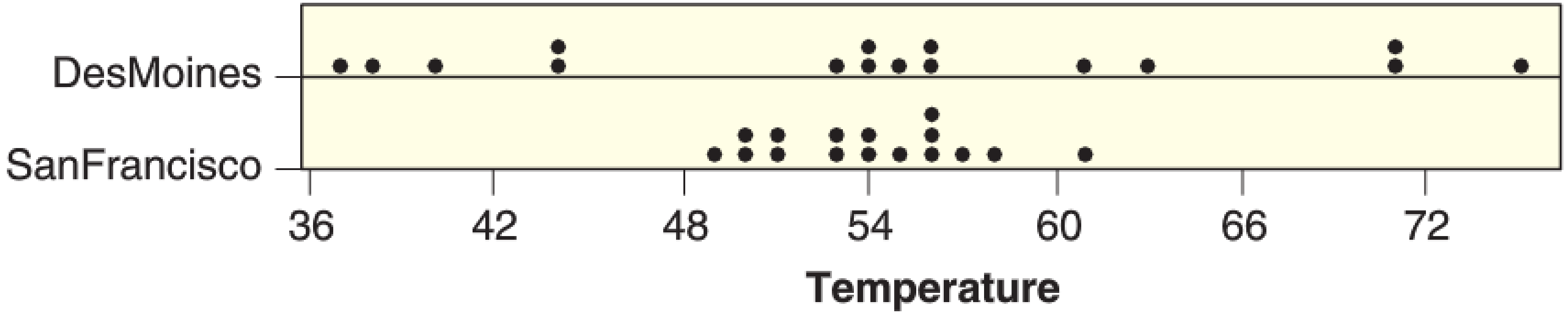
The standard deviation of a ***sample*** is denoted **s**

- It measure the spread of the data from the sample mean $\bar{x}$

Population and sample standard deviation



Which has the larger standard deviation?



s_{DM} = 11.73 ºF s_{SF} = 3.38 ºF

The standard deviation

The standard deviation can be computed using the following formula:

$$s = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

Example: computing the standard deviation

Suppose we had a sample with n = 4 points:

$x_1 = 8$, $x_2 = 2$, $x_3 = 6$, $x_4 = 4$,

We can compute the mean using the formula:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad = \tfrac{1}{4} \cdot (x_1 + x_2 + x_3 + x_4) \quad = \tfrac{1}{4} \cdot (8 + 2 + 6 + 4)$$

The standard deviation can be computed using the formula:

$$s = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n} (x_i - \overline{x})^2}$$

(remember order of operations!)

Hot dogs!

Every 4th of July, Nathan's Famous in NYC holds a hot dog eating contest where contestants try to eat as many hot dogs as they can in 10 minutes

$$s = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

Worksheet: Calculate the mean and standard deviation for the number of hot dogs eaten by the winners. Upload the filled out worksheet to Canvas.

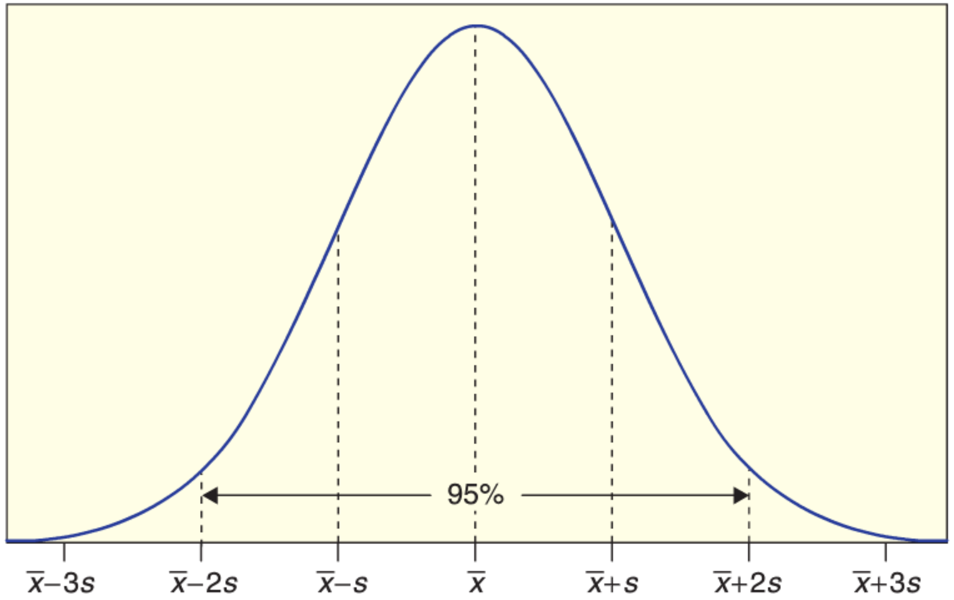
Normal distributions and z-scores

The 95% rule for *normal distributions*

A **normal distribution** is a common distribution that is symmetric and bell shaped

If a distribution of data is approximately normally distributed, about 95% of the data should fall within two standard deviations of the mean

i.e., 95% of the data is in the interval: $\bar{x}$ -2s to $\bar{x}$ +2s



The 95% rule for *normal distributions*

A **normal distribution** is a common distribution that is symmetric and bell shaped

If a distribution of data is approximately normally distributed, about 95% of the data should fall within two standard deviations of the mean

i.e., 95% of the data is in the interval: $\bar{x}$ -2s to $\bar{x}$ +2s

Example: IQ scores are normally distributed with a mean of 100 and a standard deviation of 15.

Question: what is the range of values that the middle 95% of IQ scores fall in?

The 95% rule for *normal distributions*

A **normal distribution** is a common distribution that is symmetric and bell shaped

If a distribution of data is approximately normally distributed, about 95% of the data should fall within two standard deviations of the mean

i.e., 95% of the data is in the interval: $\bar{x} - 2s$ to $\bar{x} + 2s$

Example: IQ scores are normally distributed with a mean of 100 and a standard deviation of 15.

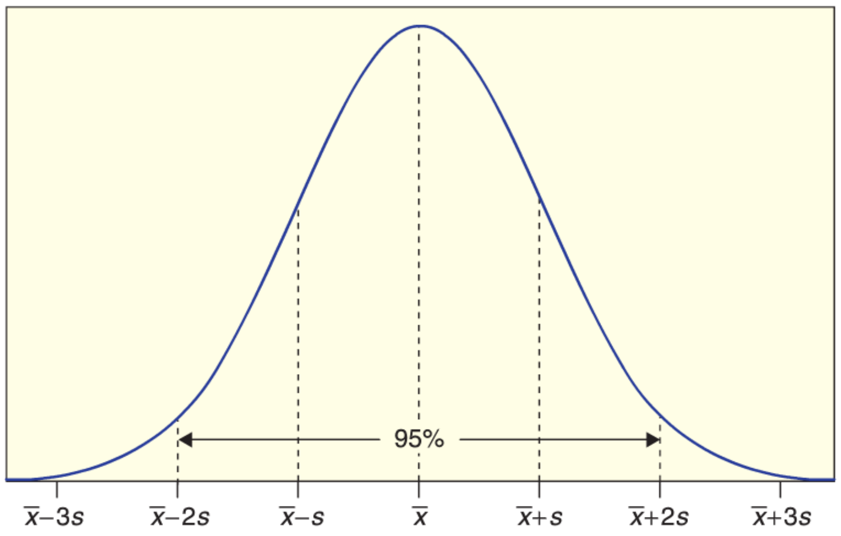
Question: what is the range of values that the middle 95% of IQ scores fall in?

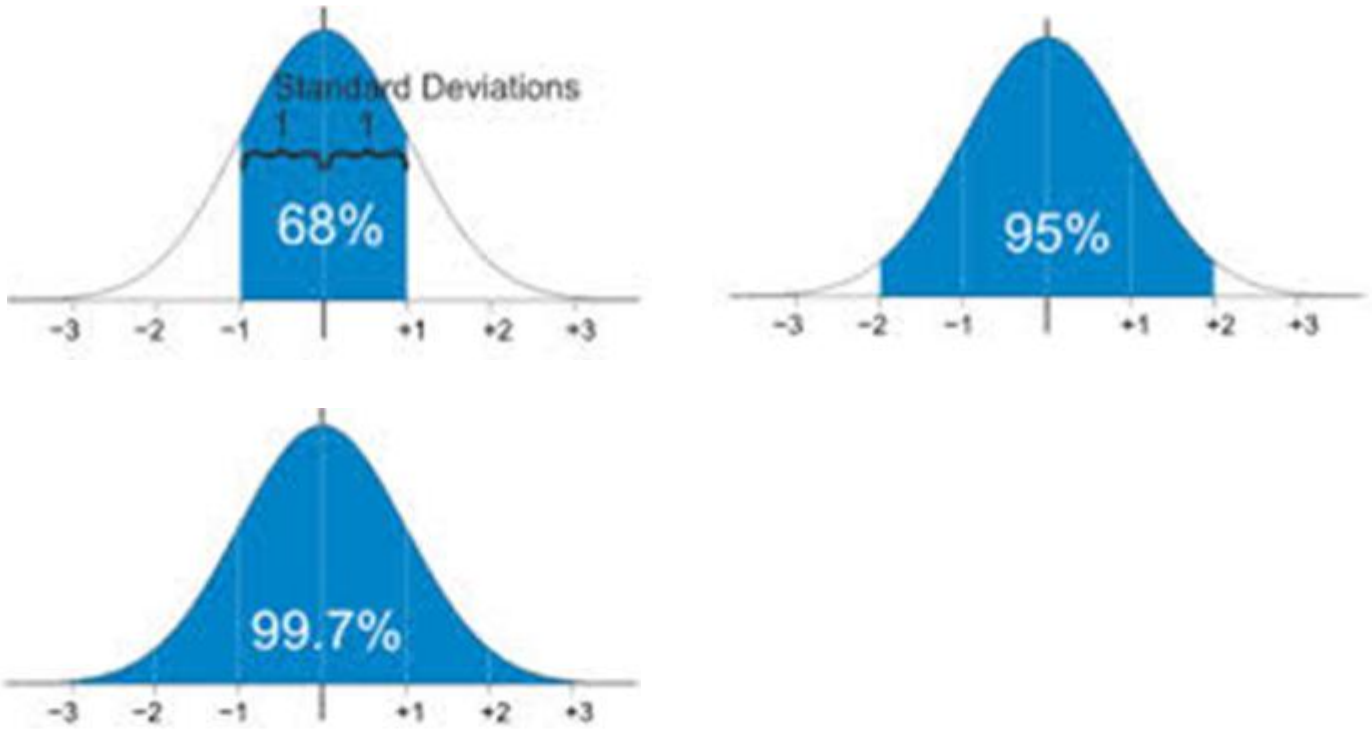
Answer: (100 – 30) to (100 + 30), 95% of IQ scores are in the range 70 to 130

The 68%, 95% and 99.7% rules for *normal distributions*

Other properties of normal distributions are:

- 68% of the data falls within **one** standard deviations of the mean
- 95% of the data falls within **two** standard deviations of the mean
- 99.7% of the data falls within **three** standard deviations of the mean





z-scores

The z-scores tells how many standard deviations a value is from the mean

- i.e., how far away a point x_i is from $\bar{x}$ in a way that is independent of the units of measurement

$$\text{z-score}(x_i) = \frac{x_i - \bar{x}}{s}$$

Which Accomplishment is most impressive?

LeBron James is a basketball player who had the following statistics in 2011:

- Field goal percentage (FGPct) = 0.510
- Points scored = 2111
- Assists = 554
- Steals = 124

The summary statistics of the NBA in 2011 are given below

$$\text{z-score}(x_i) = \frac{x_i - \bar{x}}{s}$$

	Mean	Standard Deviation
FGPct	0.464	0.053
Points	994	414
Assists	220	170
Steals	68.2	31.5

Question: Relative to his peers, which statistic is most and least impressive?

Which Accomplishment is most impressive?

LeBron James is a basketball player who had the following statistics in 2011:

- Field goal percentage (FGPct) = 0.510
- Points scored = 2111
- Assists = 554
- Steals = 124

The summary statistics of the NBA in 2011 are given below

z	=	$(x - \bar{x}) / s$		
Z-score FGPct	=	(0.510 - 0.464)/0.053	=	0.868
Z- score Points	=	(2111 - 994)/414	=	2.698
Z-score Assists	=	(554 - 220)/170	=	1.965
Z-score Steals	=	(124 - 68.2)/31.5	=	1.771

Percentiles

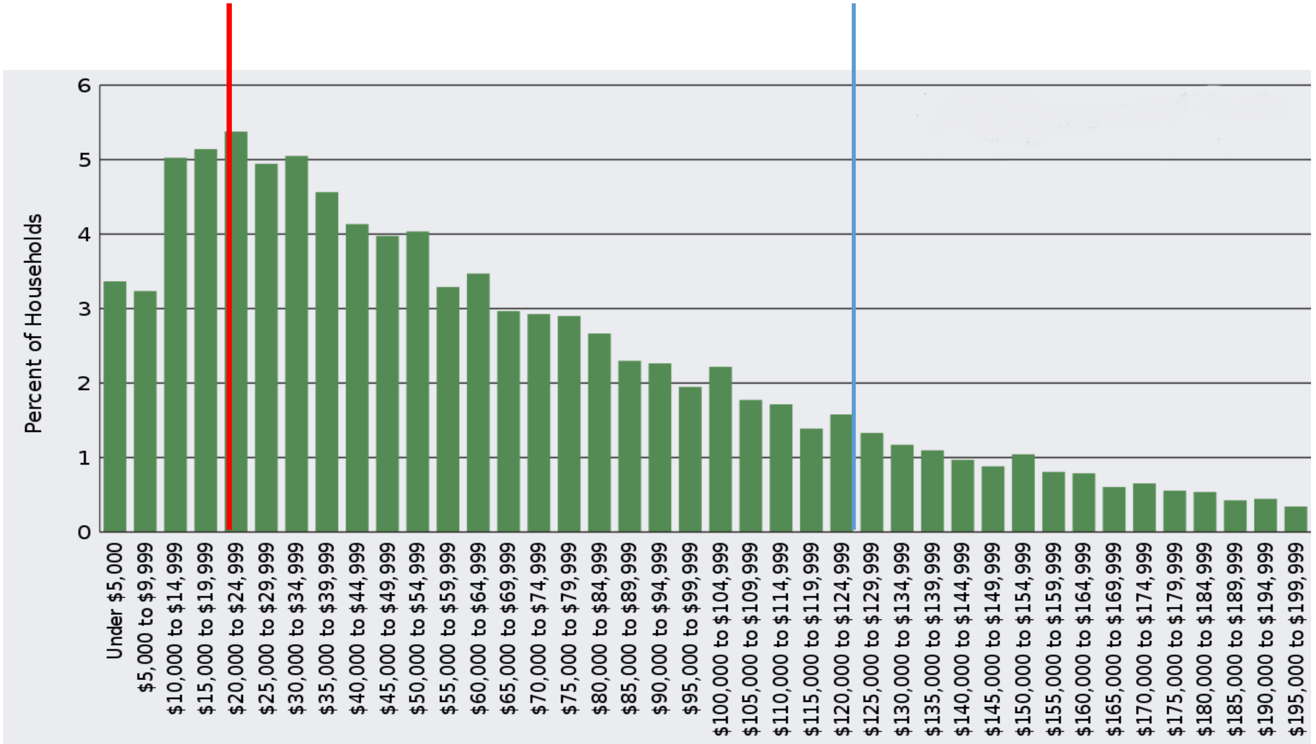
Percentiles

The **Pth percentile** is the value of a quantitative variable which is greater than P percent of the data

For the US income distribution what are the 20th and 80th percentiles?

20th percentile = $21,430

80th percentile = $112, 254




```
R: quantile(v, .95)
```

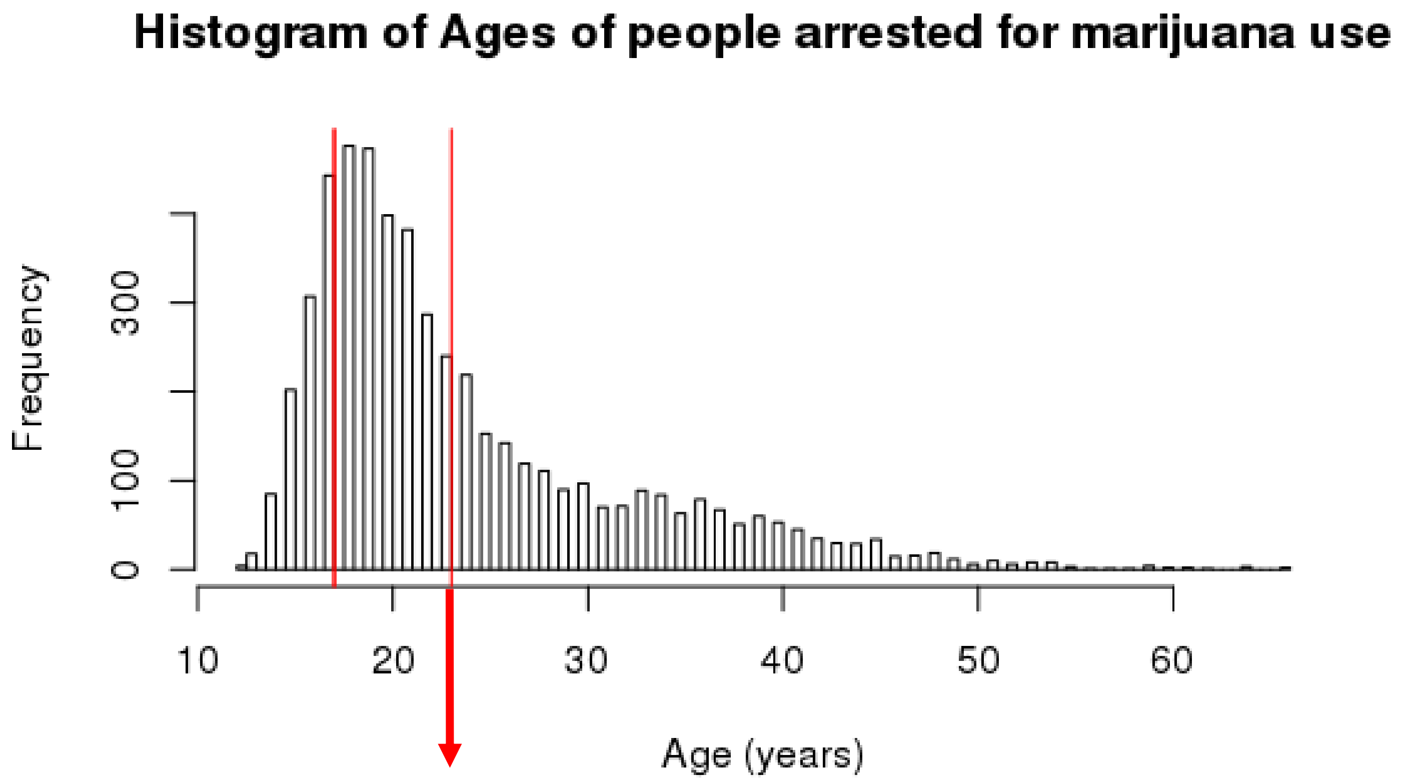
Age of marijuana arrests in Toronto

```
> install.packages('carData')
> library(carData)  # load the data
> quantile(Arrests$age,  .2)     # get the 20th percentile value from a vector of ages of arrests
```

20th percentile value is 17

i.e., 20% of the arrests were of ages 17 or less

Age of marijuana arrests in Toronto



60th percentile value is 23

i.e., 60% of the arrests were of ages 23 or less

```
> quantile(Arrests$age, c(.2, .6))  # get the 20th and 60th percentile values from a vector of ages of arrests
```

Five Number Summary

Five Number Summary = (minimum, Q_1, median, Q_3, maximum)

Q_1 = 25^{th} percentile (also called 1^{st} quartile)
Q_3 = 75^{th} percentile (also called 3^{rd} quartile)

Roughly divides the data into fourths

Range and Interquartile Range

Range = maximum – minimum

Interquartile range (IQR) = $Q_3 - Q_1$

Hot dog example – try this at home!

Try this at home: for the hot dog data calculate “by hand”

- The 5 number summary
- The range
- Interquartile range

<u>Cheat sheet:</u>

Five Number Summary = (minimum, Q_1, median, Q_3, maximum)

Range = maximum – minimum

Interquartile range (IQR) = $Q_3 - Q_1$

Q_1 = 25th percentile, Q_3 = 75th percentile

Answer in R: `fivenum(v)`

Year	Hot Dogs
2013	69
2012	68
2011	62
2010	54
2009	68
2008	59
2007	66
2006	54
2005	49
2004	54

Detecting of outliers

As a rule of thumb, we call a data value an **outlier** if it is:

Smaller than: $Q_1 - 1.5 * IQR$

Larger than: $Q_3 + 1.5 * IQR$

What is the range that a value would be called an outlier in the hot dog data?

Are there any outliers in the hot dog data?