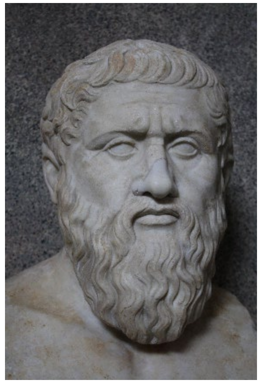
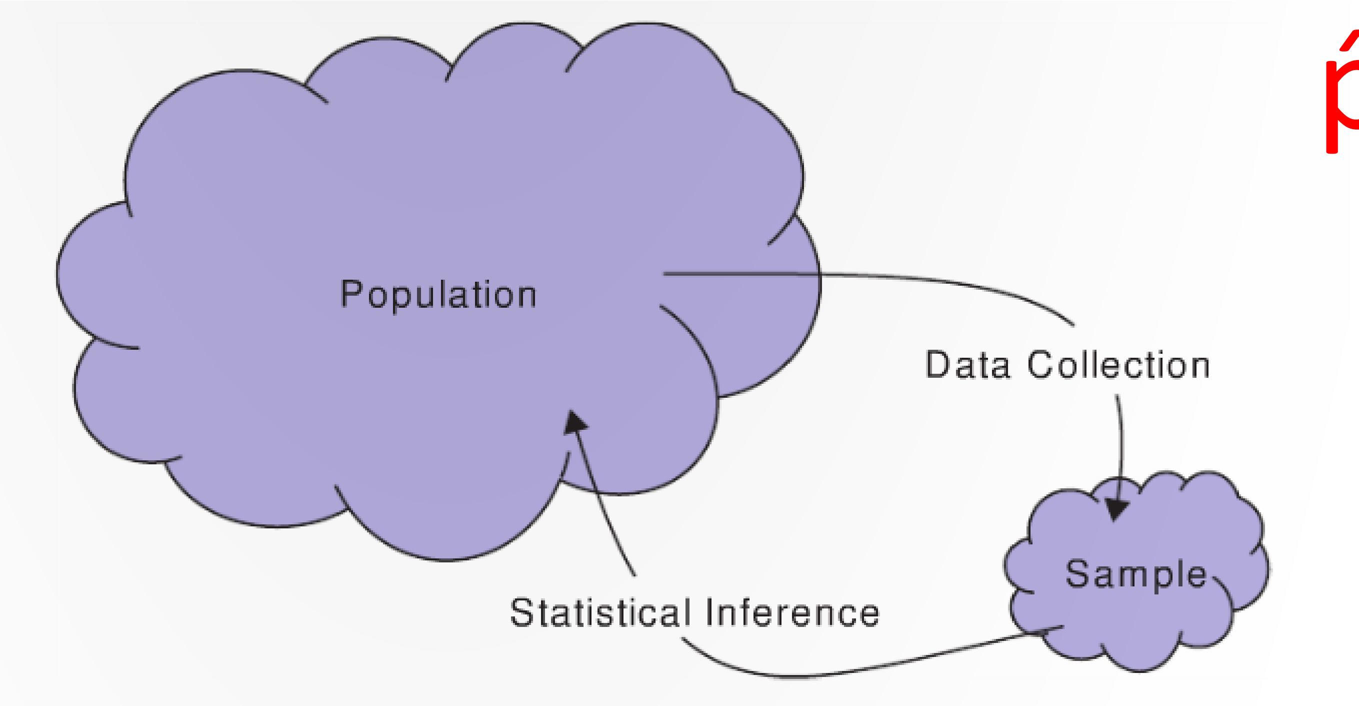
Quick review of the class

Central concepts in Statistics

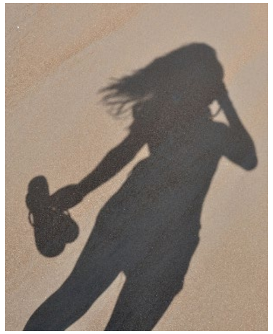
Population parameters vs. sample statistics



π, μ, σ, ρ, β



p̂, x̄, s, r, b



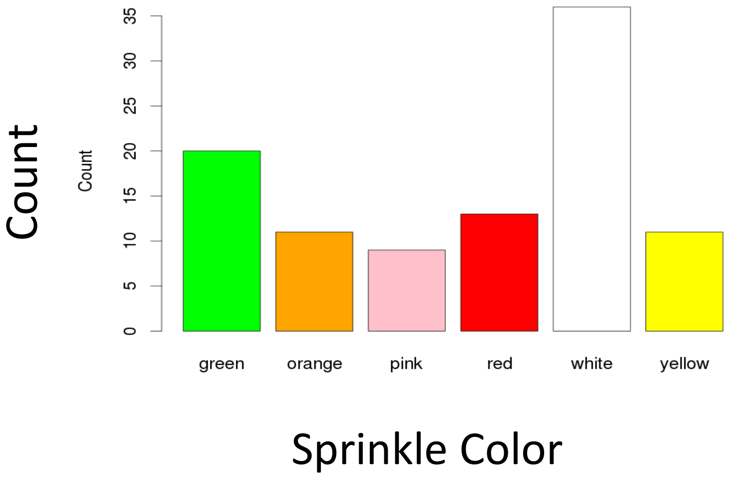
Descriptive statistics: exploring the shadows

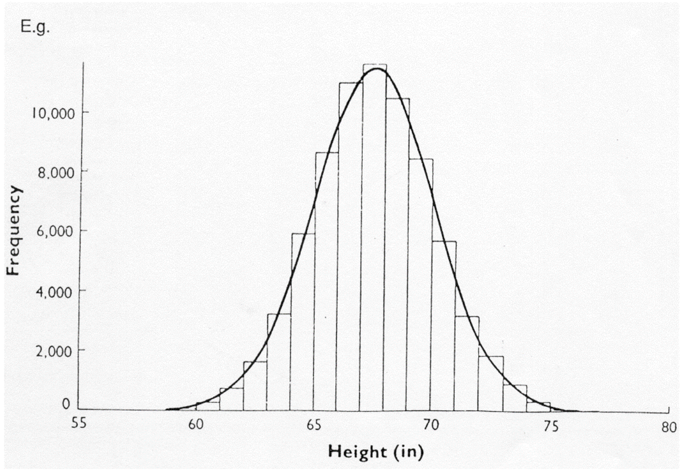
Describing structured data

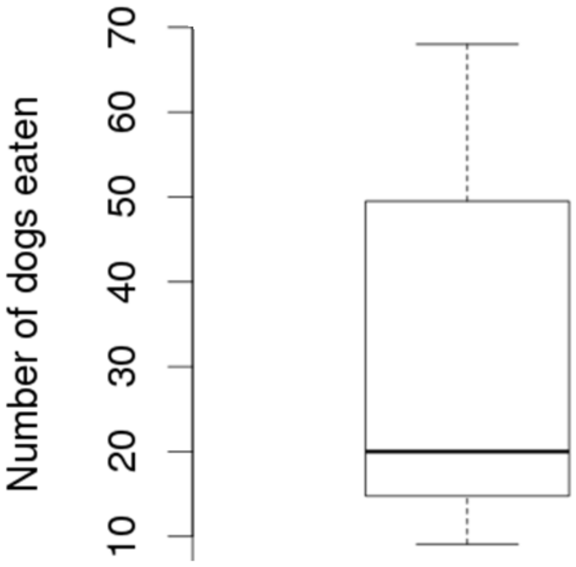
	transactionid	date_sold	make_bought	price_bought	zip_bought	mileage_bought
1	16966151	2014-09-27	Acura	30892.00	21043	40
2	16914863	2014-09-27	Toyota	25566.00	15108	297
3	15977620	2014-07-31	Nissan	34300.00	8753	0
4	18666685	2015-01-27	Subaru	30059.00	7446	10
5	14383133	2014-04-27	Honda	32508.00	97027	21
6	18196788	2014-12-18	Toyota	10819.66	95117	55246
7	15722278	2014-07-24	Audi	59630.00	90401	143

Categorical data

Quantitative data





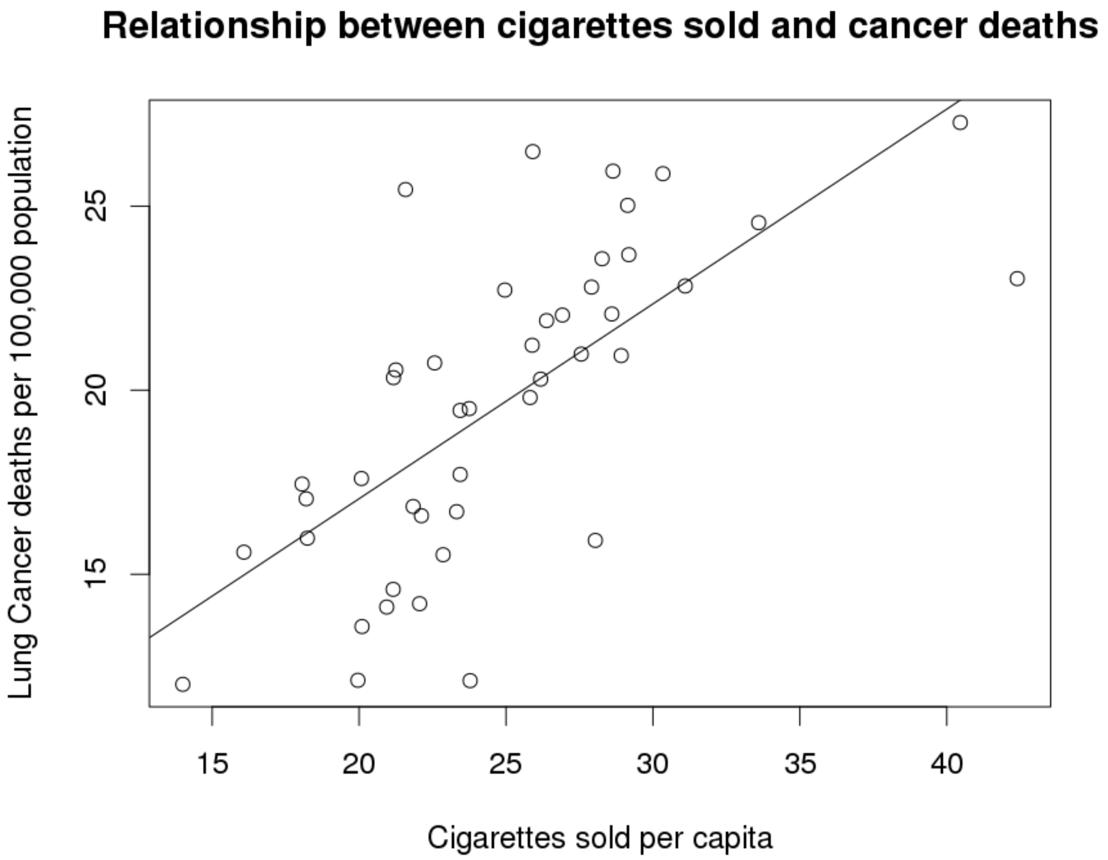


Proportion $\hat{p}$

Center: Mean $\bar{x}$, median

Spread: Standard deviation (s), IQR

Relationships between 2 quantitative variables



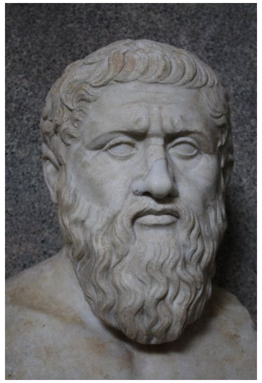
Correlation:

$$r = \frac{1}{(n-1)} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

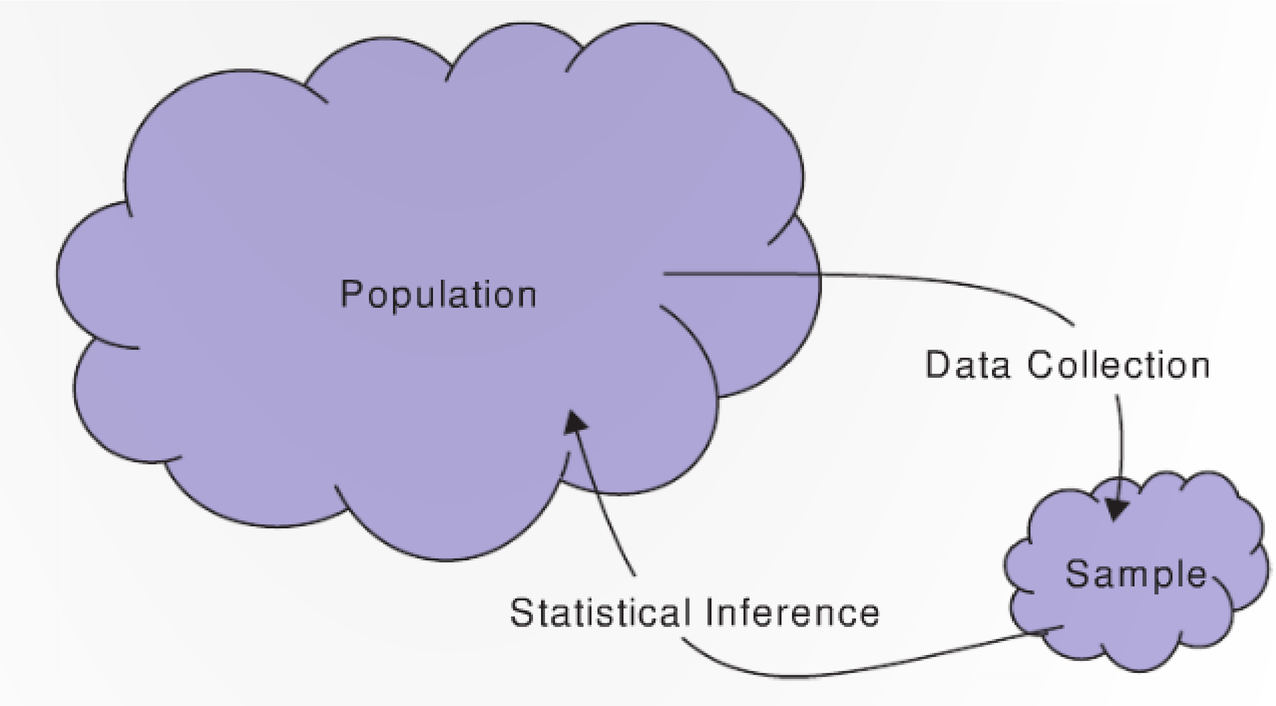
Regression:

$$\hat{y} = a + b \cdot x$$

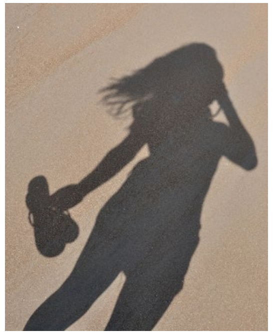
Statistical inference:
Confidence Intervals and Hypothesis Tests



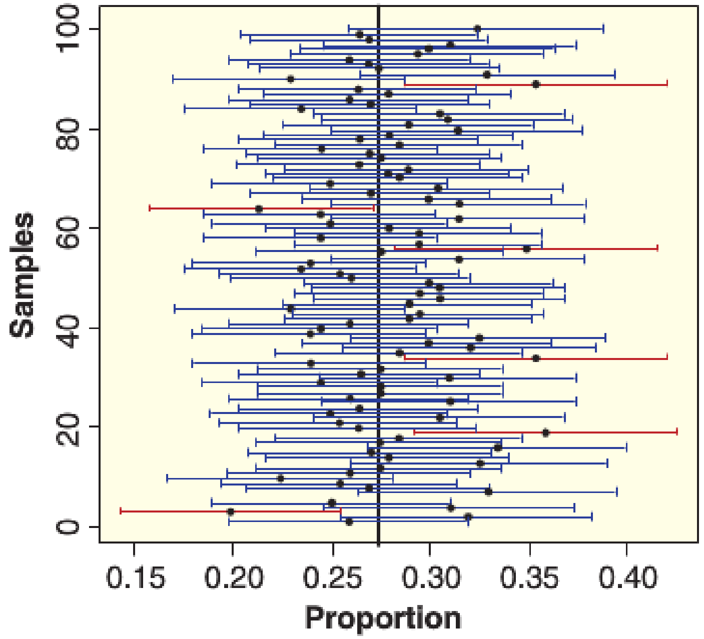
$\pi, \mu, \sigma, \rho, \beta$



$\hat{p}, \bar{x}, s, r, b$



Confidence Intervals



A **confidence interval** is an interval computed by a method that will contain the *parameter* a specified percent of times

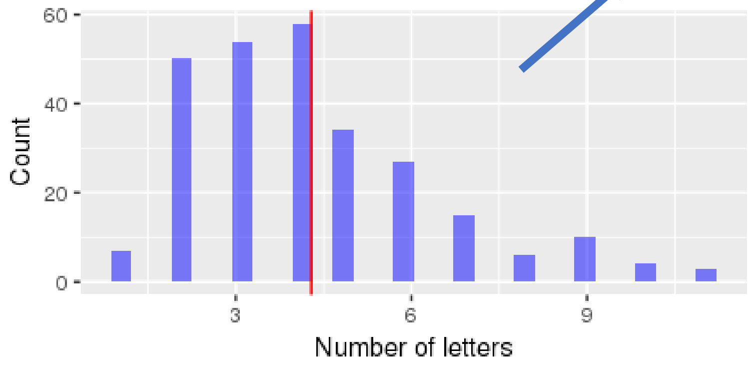
The **confidence level** is the percent of all intervals that contain the parameter

Computational methods for CIs: The Bootstrap

The sample (n = 10)

10, 3, 3, 3, 4, 3, 2, 6, 4, 5

μ



3, 3, 3, 5, 3,
4, 5, 2, 2, 10

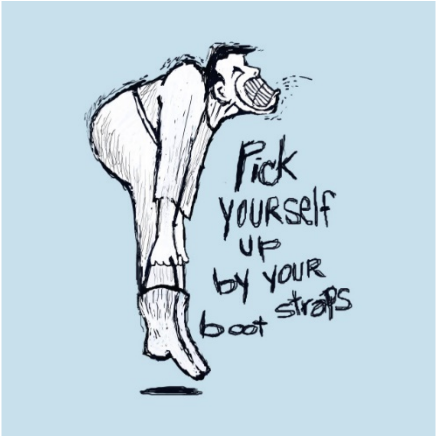
$\bar{x}^* = 4$

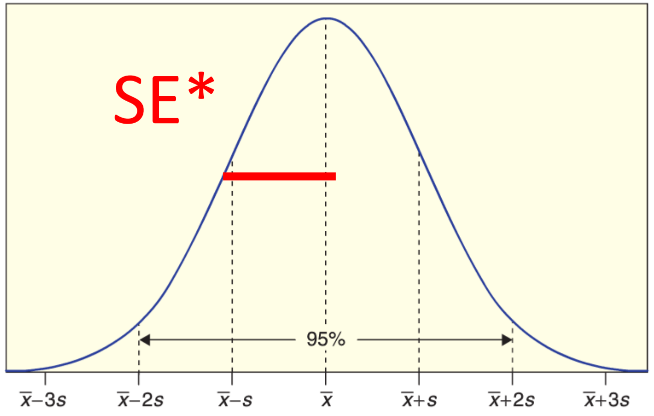
3, 3, 2, 3, 6,
4, 6, 5, 3, 6

$\bar{x}^* = 4.1$

5, 3, 2, 3, 3,
3, 10, 3, 4, 3

$\bar{x}^* = 3.9$





Bootstrap distribution!

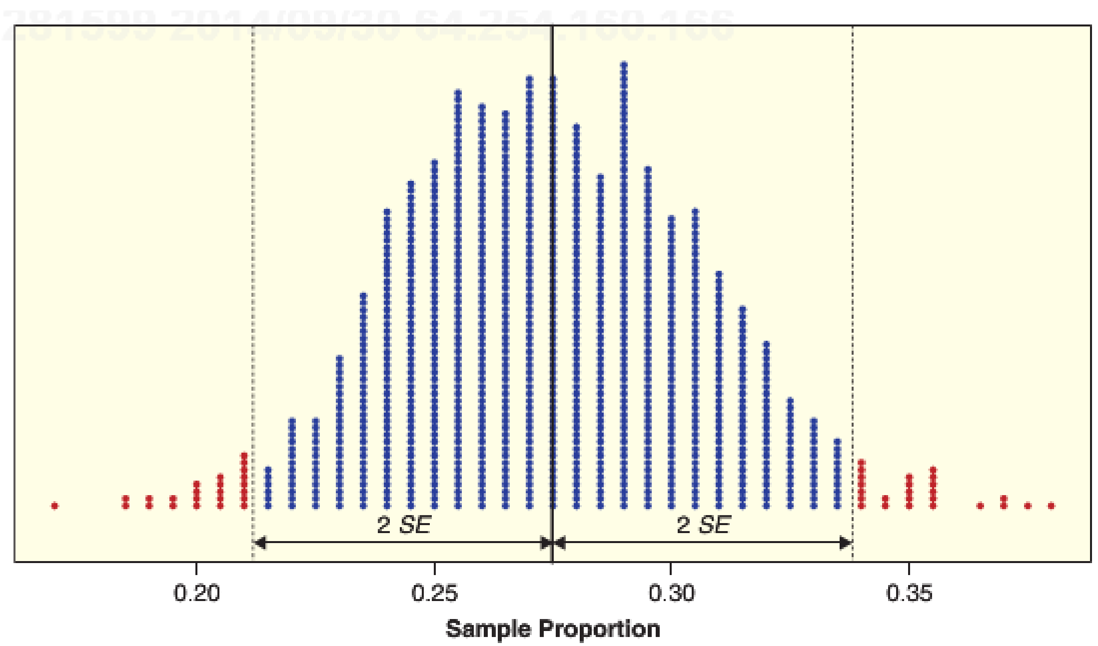
do_it(10000) * {...}

95% Confidence Intervals

When a bootstrap distribution for a sample statistic is approximately normal, we can estimate a 95% confidence interval using:

$$\textit{Statistic} \pm 2 \cdot SE^*$$

Where SE* is the standard error estimated using the bootstrap

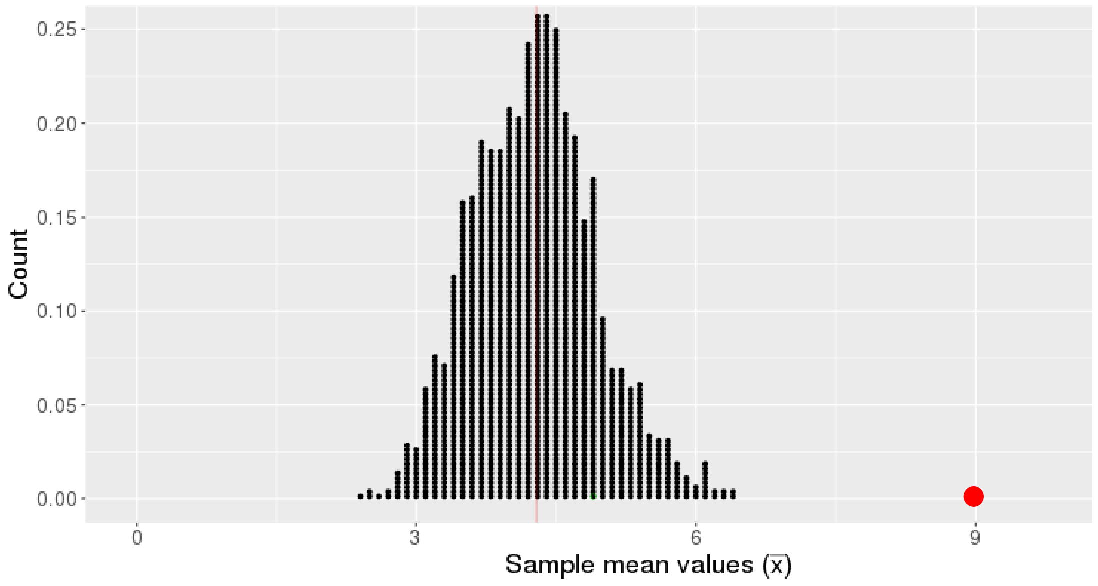


Hypothesis test logic

We start with a claim about a population parameter

- E.g., $\mu = 4$

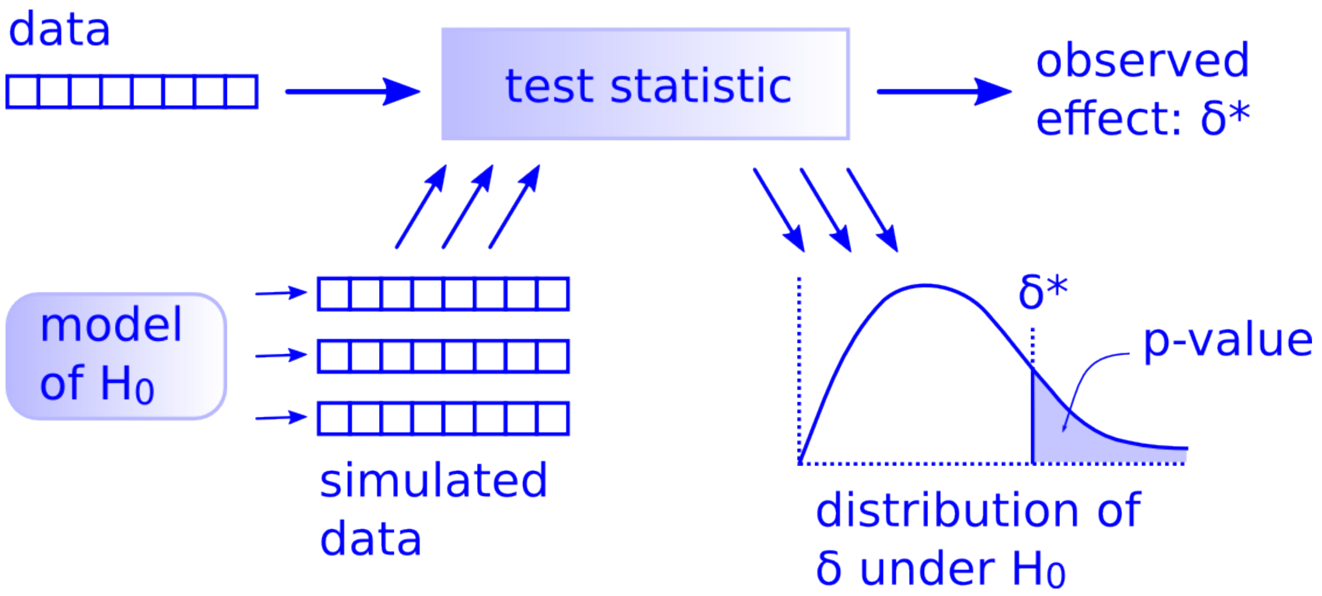
This claim implies we should get a certain distribution of statistics

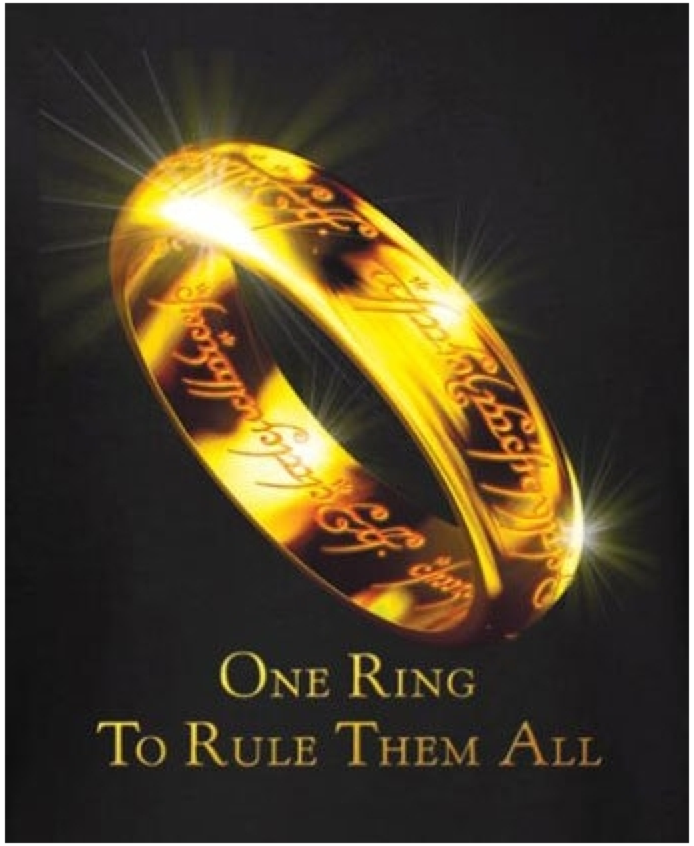


If our observed statistic is highly unlikely, we reject the claim

One test to rule them all

There is only one hypothesis test!





Just follow the 5 hypothesis tests steps!

Computational methods for hypothesis tests

Type of parameter	Simulation method
A single proportion π	
Comparing 2 or more means μ_1, μ_2, μ_k	
Correlation and regression ρ, β	

do_it(10000) * {…}

Computational methods for hypothesis tests

Type of parameter	Simulation method
A single proportion π	Coin flipping
Comparing 2 or more means μ_1, μ_2, μ_k	Combine and reassign
Correlation and regression ρ, β	Shuffle one of the columns

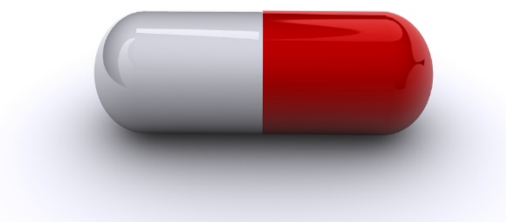
do_it(10000) * {…}

Computational methods for hypothesis tests

Type of parameter	Simulation method
A single proportion π	Coin flipping
Comparing 2 or more means μ_1, μ_2, μ_k	Combine and reassign
Correlation and regression ρ, β	Shuffle one of the columns

do_it(10000) * {…}

	5	3	2		7			8
6		1	5					2
2			9	1	3		5	
7	1	4	6	9	2			
	2						6	
			4	5	1	2	9	7
	6		3	2	5			9
1					6	3		4
8			1		9	6	7	



Treatment group

Control group

Reconstructed participant pool data under H_0

Shuffle and randomly assignment

Shuffled 'treatment group'

Shuffled 'control group'

Computational methods for hypothesis tests

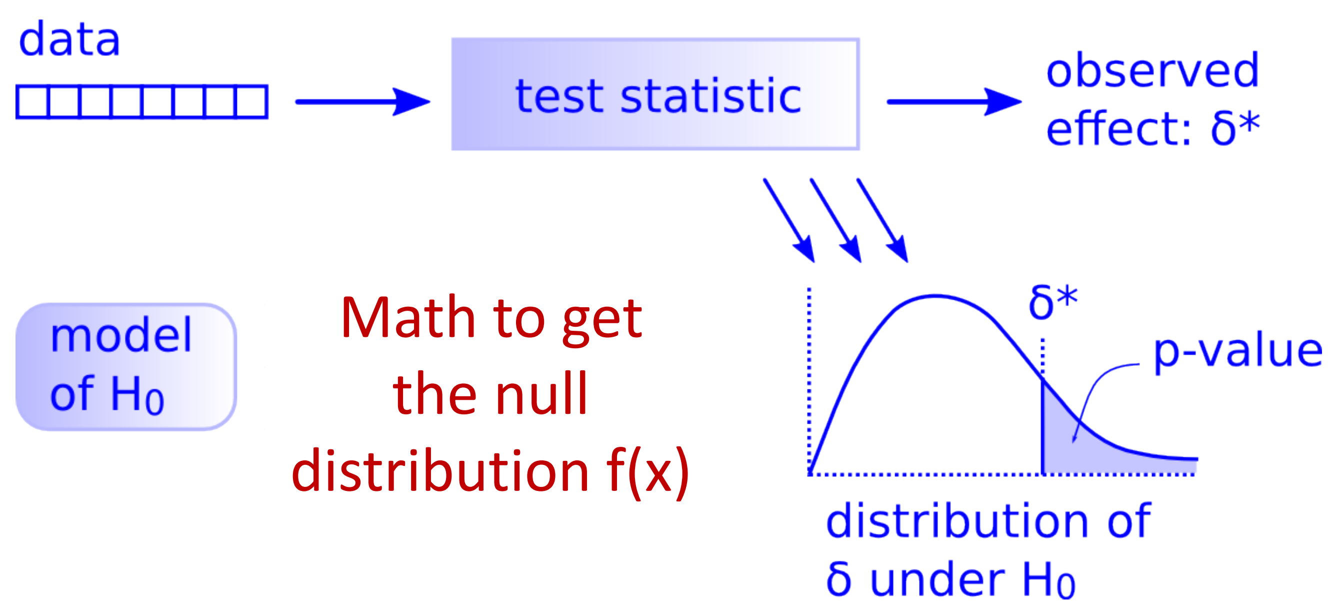
Type of parameter	Simulation method
A single proportion π	Coin flipping
Comparing 2 or more means μ_1, μ_2, μ_k	Combine and reassign
Correlation and regression ρ, β	Shuffle one of the columns

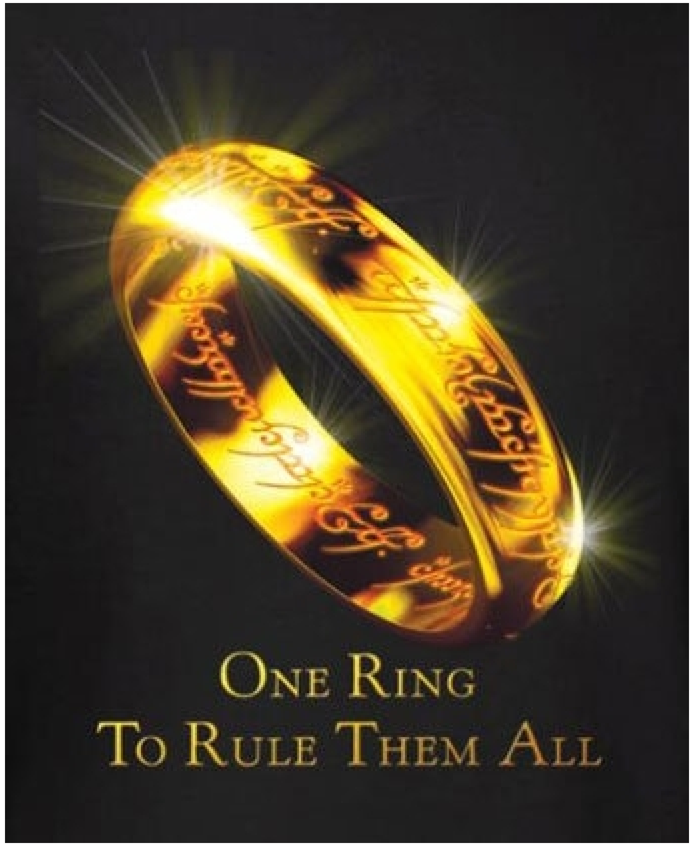
do_it(10000) * {…}

	Calories	Carbohydrates
AppleJacks	117	27
Boo Berry	118	27
Cap'n Crunch	144	31
Cinnamon Toast Crunch	169	32

Parametric methods for hypothesis tests

There is only one hypothesis test!





Just follow the ~5 hypothesis tests steps!

Parametric hypothesis tests and CIs

Type of parameter	Null distribution
One or two proportions π_1, π_2	
One or two means, regression μ_1, μ_2, β	
More than 2 proportions π_1, π_2, π_3	
More than 2 means μ_1, μ_2, μ_k	

Parametric hypothesis tests and CIs

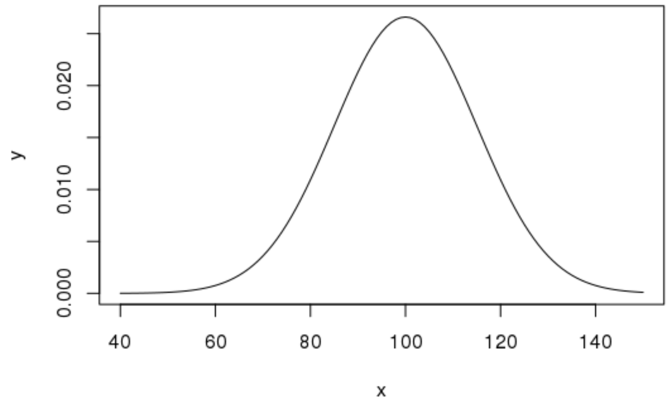
Type of parameter

- One or two proportions π_1, π_2
- One or two means, regression μ_1, μ_2, β
- More than 2 proportions π_1, π_2, π_3
- More than 2 means μ_1, μ_2, μ_k

Null distribution

- Normal distribution (z-test)
- t-distribution (t-test)
- χ^2-distribution (χ^2-test)
- F-distribution (ANOVA)

N(μ, σ)



$$\hat{p} \pm z^* \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$z = \frac{\hat{p}-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{n}}}$$

Parametric hypothesis tests and CIs

<u>Type of parameter</u>

One or two proportions π_1, π_2

One or two means, regression μ_1, μ_2, β

More than 2 proportions π_1, π_2, π_3

More than 2 means μ_1, μ_2, μ_k

<u>Null distribution</u>

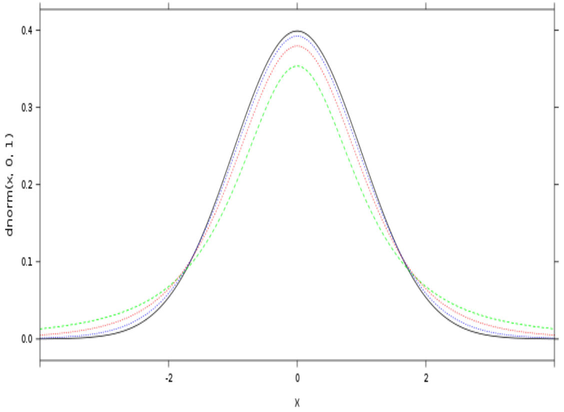
Normal distribution (z-test)

t-distribution (t-test)

χ^2-distribution (χ^2-test)

F-distribution (ANOVA)

t(df)

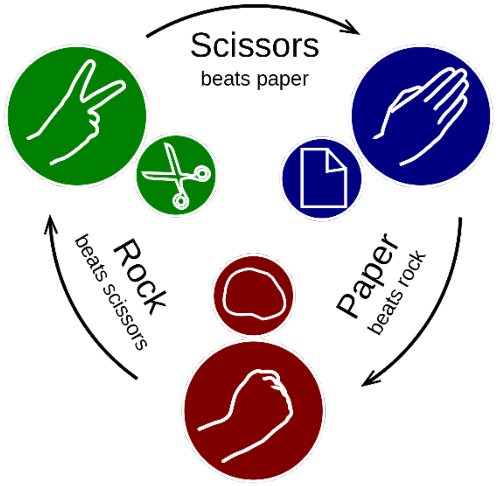


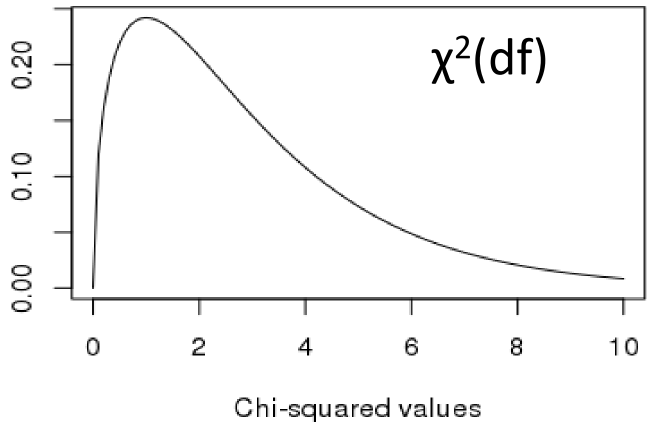
$$(\overline{x_1} - \overline{x_2}) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Parametric hypothesis tests and CIs

Type of parameter	Null distribution
One or two proportions π_1, π_2	Normal distribution (z-test)
One or two means, regression μ_1, μ_2, β	t-distribution (t-test)
More than 2 proportions π_1, π_2, π_3	χ^2-distribution (χ^2-test)
More than 2 means μ_1, μ_2, μ_k	F-distribution (ANOVA)





$$\chi^2 = \sum_{i=1}^{n} \frac{(Observed_i - Expected_i)^2}{Expected_i}$$

Parametric hypothesis tests and CIs

Type of parameter	Null distribution
One or two proportions π_1, π_2	Normal distribution (z-test)
One or two means, regression μ_1, μ_2, β	t-distribution (t-test)
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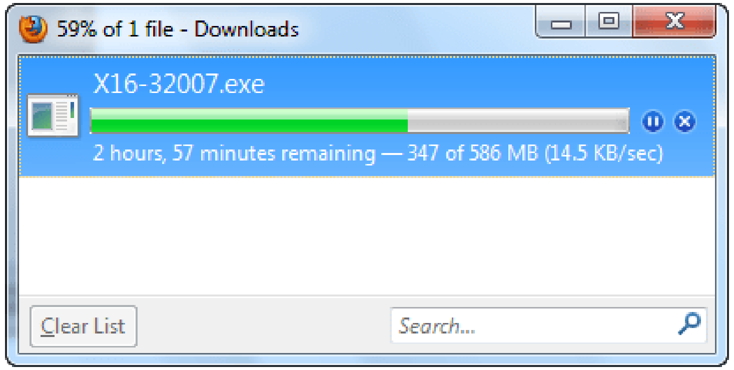
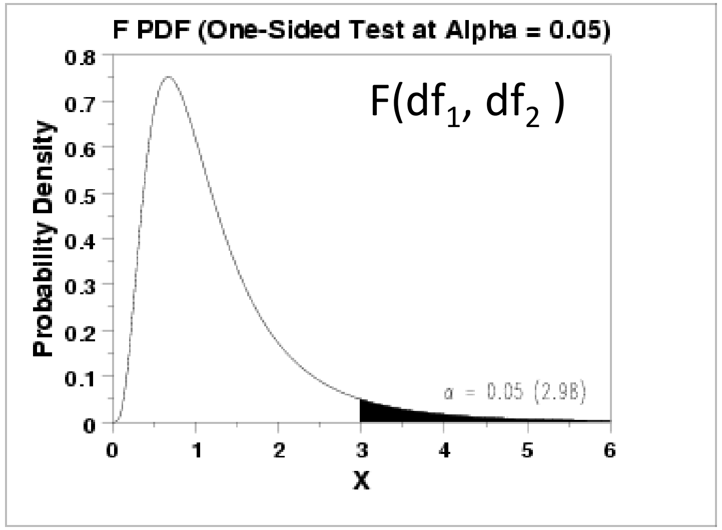
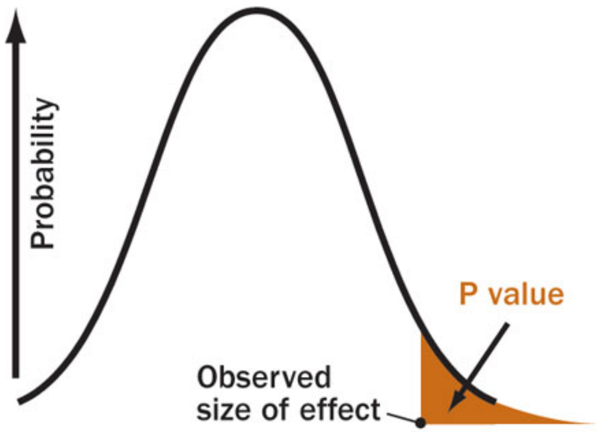




$$F = \frac{\text{between-group variability}}{\text{within-group variability}}$$

$$= \frac{\frac{1}{K-1}\sum_{i=1}^{K} n_i(\bar{x}_i - \bar{x}_{tot})^2}{\frac{1}{N-K}\sum_{i=1}^{K}\sum_{j=1}^{n_i}(x_{ij} - \bar{x}_i)^2}$$

Two theories of hypothesis testing



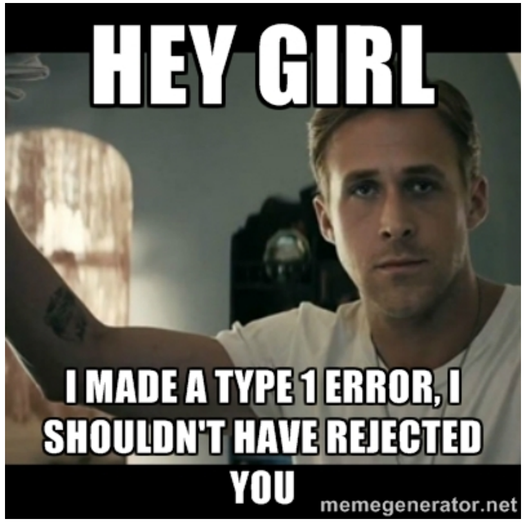
Significance testing

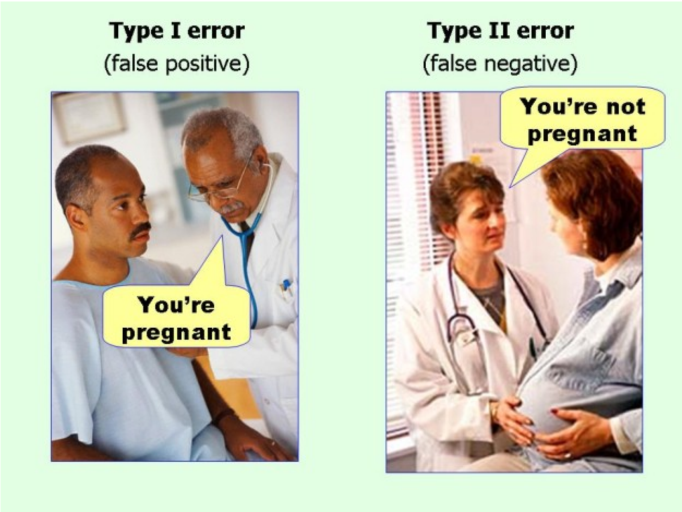
- Fisher
- P-value as strength of evidence

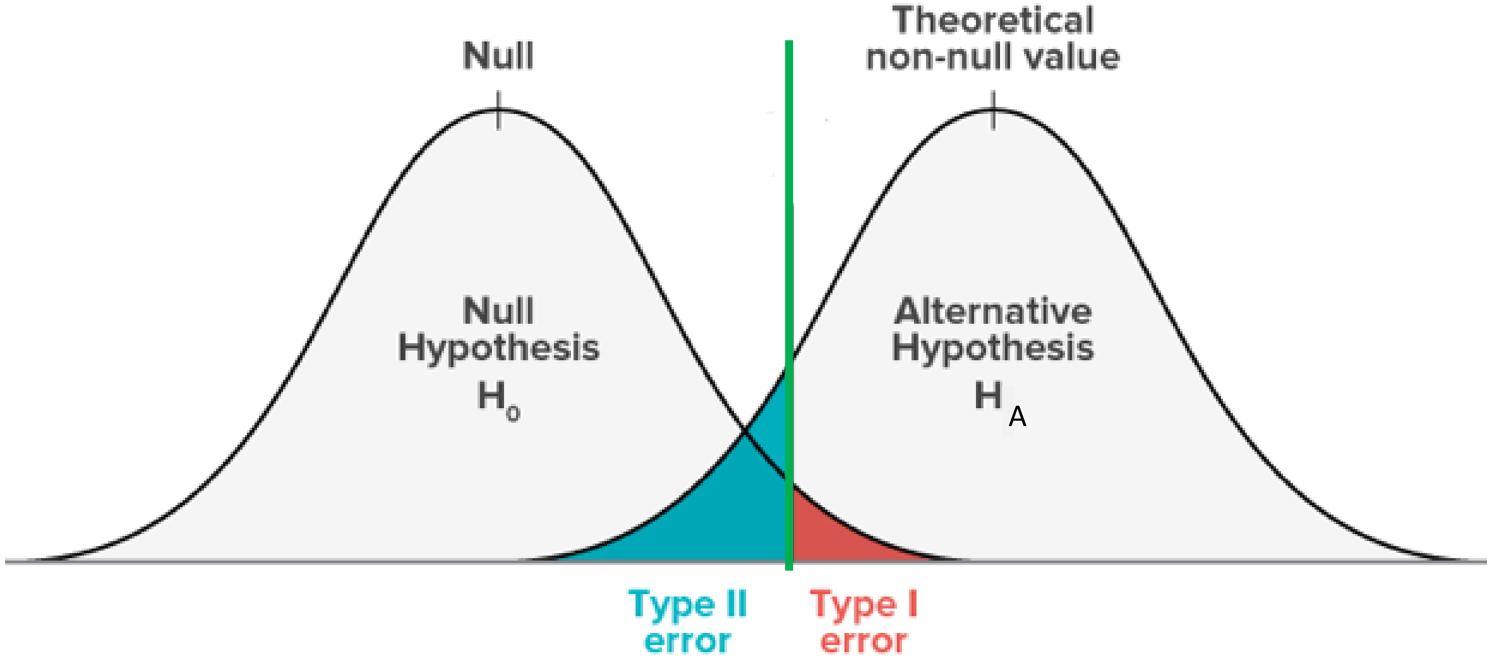
Hypothesis testing

- Neyman and Pearson
- Make a formal decision to reject or not reject (p-value < α)

Neyman-Pearson Type I and Type II Errors





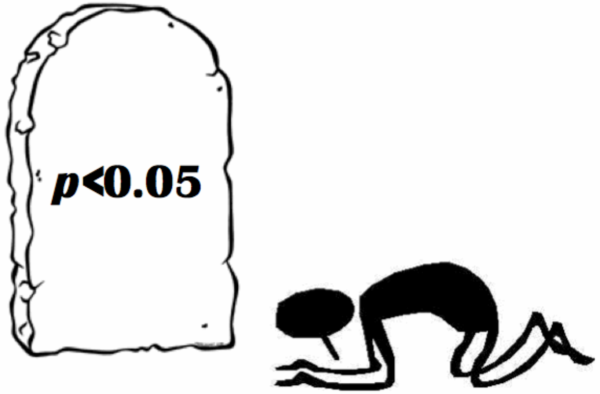


	Reject H_0	Do not reject H_0
H_0 is true	Type I error (α) (false positive)	No error
H_A is true (H_0 is false)	No error	Type II error (β) (false negative)

Problems with the NP hypothesis tests

<u>Problem 1</u>: we are interested in the results of a specific experiment, not whether we are right most of the time

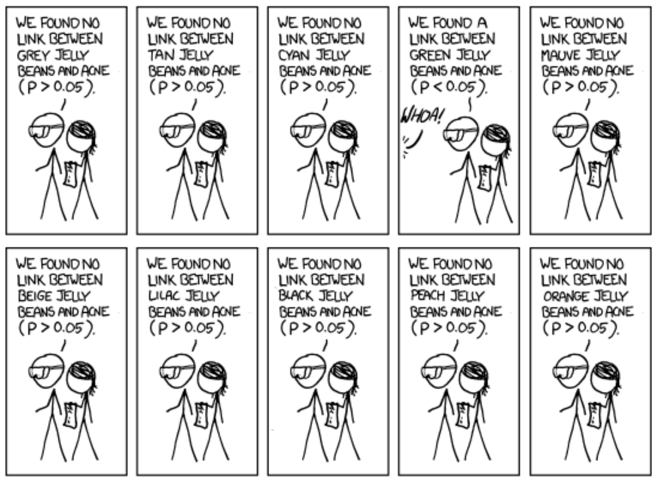
- E.g., 95% of these statements are true:
  - Calcium is good for your heart, Paul is psychic, Buzz and Doris can communicate, …



<u>Problem 2</u>: Arbitrary thresholds for alpha levels

- P-value = 0.051, we don't reject H_0?

<u>Problem 3</u>: running many tests can give rise to a high number of type 1 errors



Next steps in Statistics

Probability and Statistical theory (S&DS 238, 240, 241, 242)

- Parametric probability models and theory

Data Science (S&DS 123, 230, 262)

- Learn more advanced ways to visualize and manipulate data in R and Python

Linear models class   (S&DS 312; Stats2 at other schools)

- Multiple regression
- Learn more advanced forms of ANOVA (multi-way/repeated measures)

Machine Learning (S&DS 355, 363, 365)

- Algorithms for making predictions

Many more advanced classes!

One last question…

What was the worst joke of the semester?

That's all folks!

apart from one last thing...