Inference on a single proportion using normal distributions

Overview

Questions about anything?

Hypothesis tests for a single proportion using normal distributions

Confidence for a single proportion using normal distributions

Confidence for a single proportion using the bootstrap

Hypothesis tests for a single proportion using normal distributions

A <u>BBC news article</u> from April 8th discussed the AstraZeneca vaccine.

The article stated...

The Medicines and Health Regulatory Agency (MHRA) looked into UK cases of rare blood clots in people who had recently received the Oxford-AstraZeneca vaccine...

Covid: How does the Oxford-AstraZeneca vaccine work?

(1) 4 days ago

<

Coronavirus pandemic



People under the age of 30 are to be offered an alternative to the Oxford-AstraZeneca vaccine, after a review into a possible link with extremely rare blood clots in adults.

Let's we run a hypothesis test to assess if people who received the vaccine had a higher rate of blood clots than expected

It found that 79 people experienced clots after receiving a first vaccine dose.

More than 20 million AstraZeneca vaccines doses had been administered across the UK by the end of March.

• The first doses were administered in December.

About four people in a million would normally be expected to develop this particular kind of blood clot - though the fact they are so rare makes the usual rate hard to estimate.

• Note: this 4 in a million is the rate that occurs over a year.

Step 1: State the null and alternative hypotheses

The MHRA found that 79 people experienced clots after receiving a first vaccine dose.

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Step 1: State the null and alternative hypotheses

About four people in a million would normally be expected to develop this particular kind of blood clot - though the fact they are so rare makes the usual rate hard to estimate.

- Note: This is 4 in a million over the course of a year
- The first doses were administered in December (4 months ago)

$$H_0$$
: $\pi = 1/1,000,000$

$$H_{\Delta}$$
: $\pi > 1/1,000,000$

Step 2: Calculate the observed statistic

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Step 2: Calculate the observed statistic

$$\hat{p} = 79/20,000,000 = 3.96 \times 10^{-6}$$

Can you calculate the corresponding z-statistic? $z=\frac{p-\pi_0}{\sqrt{\frac{\pi_0(1-\pi_0)}{\sqrt{\frac{\pi_0(1-\pi_0)}{2}}}}}$

$$z = \frac{p - \pi_0}{\sqrt{\frac{\pi_0(1 - \pi_0)}{n}}}$$

$$n = 20 \times 10^6$$

$$\pi_0 = 10^{-6}$$

n <- 20 * 10^6
$$pi_0 <- 1/10^6 \\ p_hat <- 79/(20 * 10^6) \\ SE <- sqrt((pi_0 * (1 - pi_0))/n) \\ z_stat <- (p_hat - pi_0)/SE \\ 13.19$$

Is a normal distribution valid to use as a null distribution here?

The normal approximation is reasonable good when we see 10 "positive" outcomes and 10 "negative" outcomes

$$n\pi \ge 10$$
 and $n(1-\pi) \ge 10$

Are these conditions met?

Step 3: Create the null distribution

What type of distribution is the appropriate distribution?

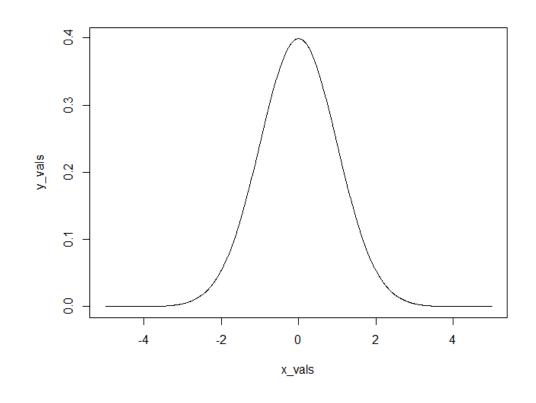
Can you plot this distribution in R?

Step 3: Create the null distribution

 $x_{vals} <- seq(-5, 5, length.out = 1000)$

y_vals <- dnorm(x_vals, 0, 1)</pre>

plot(x_vals, y_vals, type = "l")



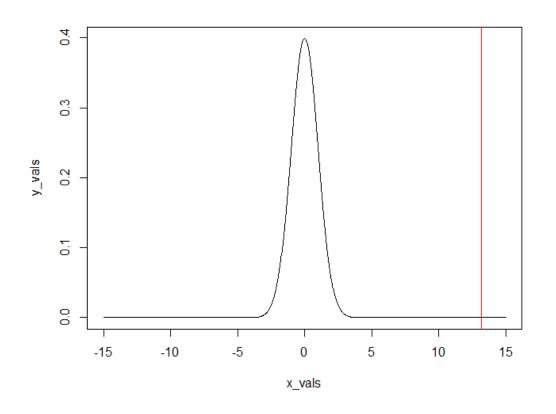
Step 4: Calculate the p-value

 $x_{vals} <- seq(-15, 15, length.out = 1000)$

y_vals <- dnorm(x_vals, 0, 1)</pre>

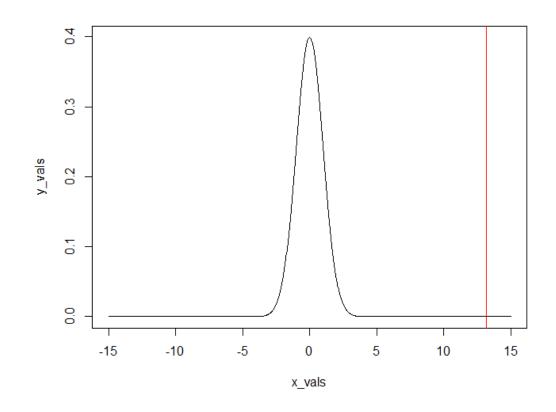
plot(x_vals, y_vals, type = "l")

abline(v = z_stat, col = "red")



Step 4: Calculate the p-value

pnorm(z_stat, 0, 1, lower.tail = FALSE)



Step 5: Make a decision

Does the AstroZeneca vaccine cause blood clots?

Should you take the AstroZeneca vaccine?



Confidence intervals for a single proportion using normal distributions

Let's create a 95% confidence interval for the likelihood of having a blood clot if you take the AstraZeneca vaccine

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Note: this 4 in a million is the rate that occurs over a year.

$$\hat{p} \pm z^* \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\hat{p} = 79/20,000,000 = 3.96 \times 10^{-6}$$

$$n = 20 \times 10^6$$

$$\hat{p} \pm z^* \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

z_star <- qnorm(.975)</pre>

$$[3.51 \times 10^{-6} \quad 4.39 \times 10^{-6}]$$
 in 4 months

[
$$1.05 \times 10^{-5}$$
 1.31×10^{-5}] in a year

Make a decision!

Should you take the AstroZeneca vaccine?



Fear of blood clots must not undermine the case for vaccinations

Editorial: The possibility of getting fatal blood clots from Covid are vastly higher than from the Oxford-AstraZeneca jab. We must avoid a catastrophic third wave, economic recession, and help build herd immunity

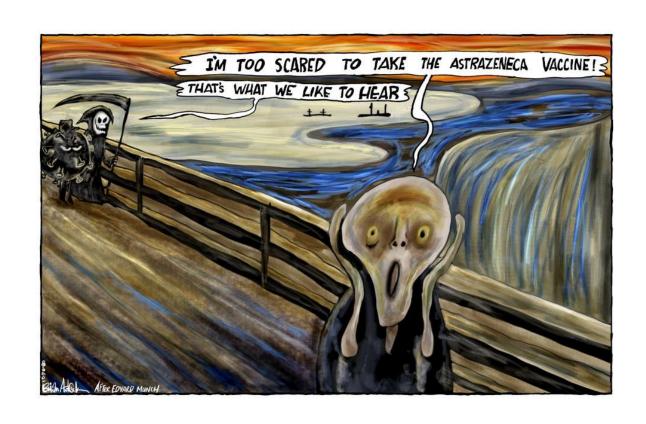
4 days ago | 43 comments











Confidence intervals for a single proportion using the bootstrap

Let's create a 95% confidence interval for the probability of having a blood clot if you take the AstraZeneca vaccine using the bootstrap

The MHRA found that 79 people experienced clots after receiving a first vaccine dose.

More than 20 million AstraZeneca vaccines doses had been administered across the UK by the end of March.

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About four people in a million would normally be expected to develop this particular kind of blood clot - though the fact they are so rare makes the usual rate hard to estimate.

Note: this 4 in a million is the rate that occurs over a year.

How should we start this analysis?

library(SDS100)

```
has_clot <- rep(TRUE, 79)
no_clot <- rep(FALSE, (20 * 10^6) - 79)
```

data_vec <- c(has_clot, no_clot)</pre>

Let's create **one** bootstrap statistic:

```
boot_data <- sample(data_vec, replace = TRUE)</pre>
```

boot proportion <- mean(boot data)</pre>

Let's do it 10 times!

```
boot_dist <- do_it(10) * {
  boot_data <- sample(data_vec, replace = TRUE)
  boot_proportion <- mean(boot_data)
}</pre>
```

Would doing this 10,000 times be reasonable?

```
install.packages(tictoc)
library(tictoc)
tic()
boot_dist <- do_it(10) * {
  boot_data <- sample(data_vec, replace = TRUE)</pre>
  boot_proportion <- mean(boot_data)</pre>
toc()
```

It took me around 19 seconds for 10 runs

So it would take over 5 hours for the typical 10,000 runs we use



Any other questions about anything?

