Practice Session 9_answers
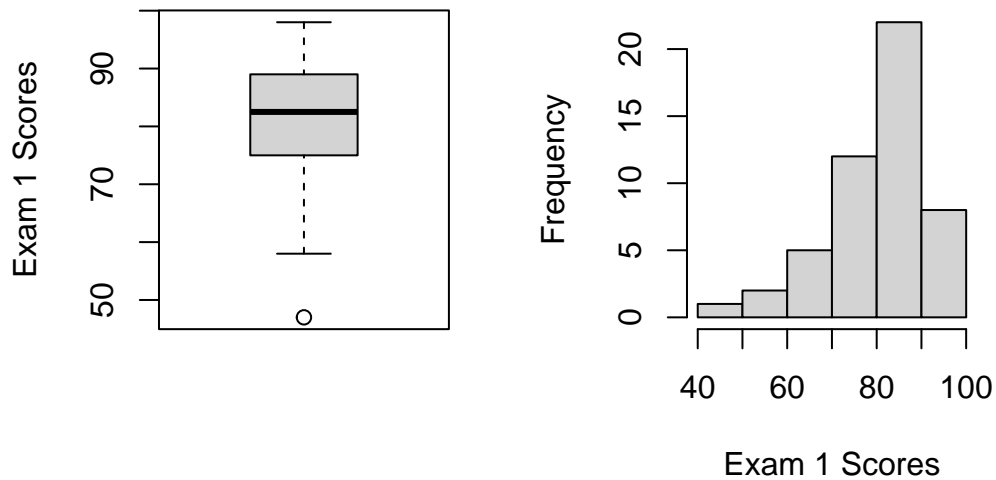
Question 1: Type I, Type II, or Neither?

a) Type I Error

b) No error

c)

- No error
- Type I Error
- Type II Error
- No error

If the disease is dangerous, then a Type II error (saying a patient doesn't have the disease when they really do) can be a big issue. We would want to make the Type II error lower. However, this may make the Type I error larger as a result.

Question 2: Single Sample T Test

a)

```
library(Lock5Data)
par(mfrow = c(1, 2))
boxplot(StatGrades$Exam1, main = "", ylab = "Exam 1 Scores")
hist(StatGrades$Exam1, main = "", xlab = "Exam 1 Scores")
```

There appears to be one outlier. We might want to investigate this score further.

b)

- $H_0 : \mu = 80$
- $H_A : \mu \neq 80$

c)

```
n1 = length(StatGrades$Exam1) # get sample size (n = 50)

mean_exam1 = mean(StatGrades$Exam1) # compute sample mean
sd_exam1 = sd(StatGrades$Exam1)
se_exam1 = sd_exam1 / sqrt(n1) # compute standard error

T_stat_exam1 = (mean_exam1 - 80) / se_exam1
T_stat_exam1
```
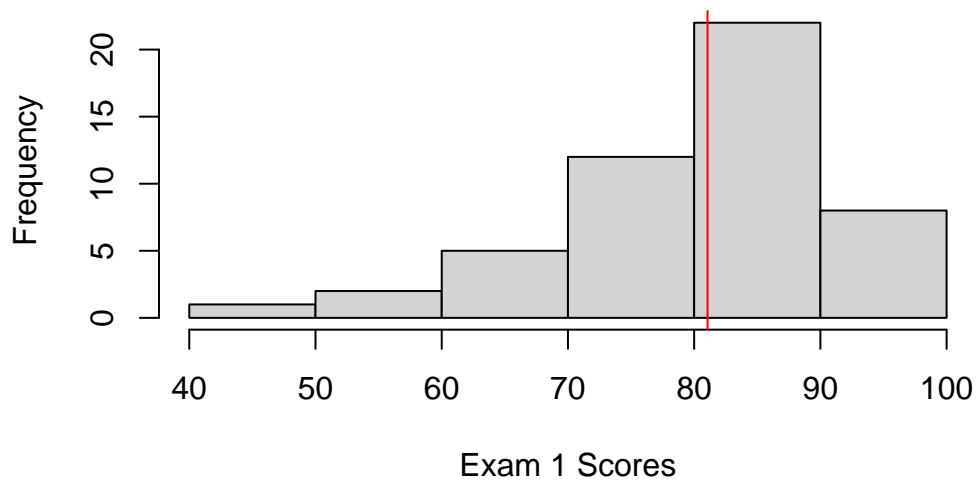
```
[1] 0.7145975
```

d)

```
# df = 50 - 1 = 49
pt(T_stat_exam1, df = n1 - 1, lower.tail = F)
```

```
[1] 0.2391243
```

Since our p-value is greater than 0.05, we will fail to reject the null hypothesis. We therefore do not have evidence that this year's exam scores are significantly higher than 80.

e)

```
par(mfrow = c(1, 1))
hist(StatGrades$Exam1, main = "", xlab = "Exam 1 Scores")
abline(v = mean_exam1, col = "red")
```
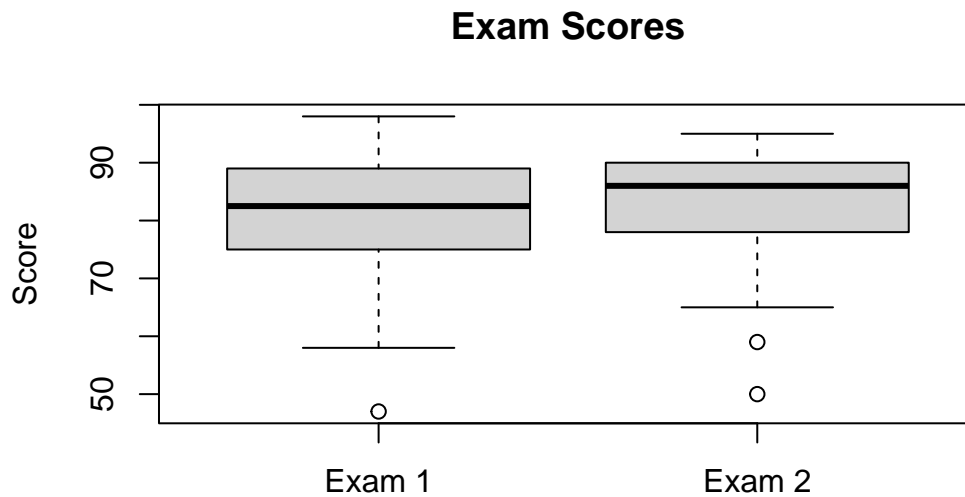

It seems that the sample mean wasn't very far from the null value of 80, which supports the lack of significance in our test.

Question 3: Two Sample T Test

a)

```r
boxplot(StatGrades$Exam1, StatGrades$Exam2,
        names = c("Exam 1", "Exam 2"),
        main = "Exam Scores",
        ylab = "Score")
```

Exam Scores

The median scores appear to be similar for both exams. Exam 1 scores appear to have more variability than exam 2 scores. There are two outliers in the exam 2 scores.

b)

- $H_0 : \mu_1 = \mu_2$
- $H_A : \mu_1 \neq \mu_2$

c)

```
n2 = length(StatGrades$Exam2) # get sample size (n = 50)

mean_exam2 = mean(StatGrades$Exam2) # compute sample mean
sd_exam2 = sd(StatGrades$Exam2)

# Compute standard error for two means
se_two_means = sqrt(sd_exam1^2/n1 + sd_exam2^2/n2)
T_stat_two_means = (mean_exam2 - mean_exam1) / se_two_means
T_stat_two_means
```

[1] 1.026024

d)

Since we are doing a two-sided test, we need to multiply the p-value by 2.

```
2 * pt(T_stat_two_means, df = 50 - 1, lower.tail = F)
```

[1] 0.3099189

Since our p-value is greater than 0.05, we will fail to reject the null hypothesis. We therefore do not have evidence that this year's students performed differently on the two exams.

e)

- $H_0 : \mu_1 = \mu_{final}$
- $H_A : \mu_1 < \mu_{final}$

Note that we are doing a one-sided test, with the alternative being improvement on the final compared to the first exam.

```
n_final = length(StatGrades$Final) # get sample size (n = 50)

mean_final = mean(StatGrades$Final) # compute sample mean
sd_final = sd(StatGrades$Final)

# Compute standard error for two means
se_one_final = sqrt(sd_exam1^2/n1 + sd_final^2/n_final)
T_stat_one_final = (mean_final - mean_exam1) / se_one_final
T_stat_one_final
```

```
[1] 2.264897
```

```
pt(T_stat_one_final, df = 50 - 1, lower.tail = F)
```

```
[1] 0.01398826
```

Since our p-value is less than **0.05**, we will reject the null hypothesis. We therefore have evidence that student scores improved on the final compared to the first exam.

Question 4: Paired T Test

a)

Let D be the true difference between the exam **1** and exam **2** scores. We will perform the following two-sided test:

- $H_0 : D = 0$
- $H_A : D \neq 0$

b)

```
diff_exams = StatGrades$Exam2 - StatGrades$Exam1
```

c)

```
n_diff = length(diff_exams)

mean_diff = mean(diff_exams)
se_diff = sd(diff_exams) / sqrt(n_diff)
```

d)

```
T_stat_diff = mean_diff / se_diff
```

e)

```
2 * pt(T_stat_diff, df = n_diff - 1, lower.tail = F)
```

```
[1] 0.01411695
```

Since our p-value is less than **0.05**, we will reject the null hypothesis. We therefore have evidence that this year's students performed differently on the two exams.

Part 2: Permutations tests for one mean and diffrence between two means:

1.) Test for the diffrence between two means with permuations:

The same local statistics instructor believes that students perform the same on the first exam (`Exam1`) and the second exam (`Exam2`). Again, the data is available in the `StatsGrades` data set from the `Lock5Data` library.
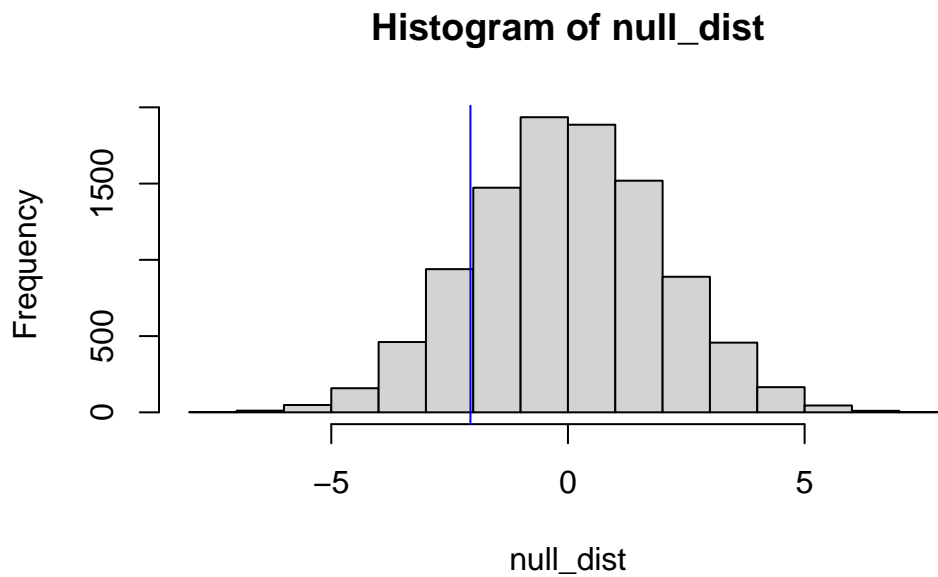
1.) Create a null distribution and create its histogram.

```
n=50
library(SDS1000)
combined_data <- c(StatGrades$Exam1, StatGrades$Exam2)
null_dist <- do_it (10000) * {
shuff_data <- shuffle(combined_data)
shuff_grp1 <- shuff_data[1:50]
shuff_grp2 <- shuff_data[51:100]
mean(shuff_grp2) - mean(shuff_grp1)
}
```

```
hist(null_dist)

obs_stat <- mean_exam1 -mean_exam2

abline(v = obs_stat, col = "blue")
```

Histogram of null_dist

2.) Calculate p-value

```
p_value <- pnull(obs_stat, null_dist, lower.tail = T)
p_value
```

```
[1] 0.1576
```

```
2*p_value
```

```
[1] 0.3152
```

```
# 0.3072
```

2.) Test for one mean using permutations: :

(Let go back to he the question of testing one mean).

According to a local statistics instructor, the average score on his first exam of the semester (Exam1) is historically around 80%. The data is available in the **StatsGrades** data set from the **Lock5Data** library. let Create a bootstrap distribution with a mean of the hypothesised value

1.) Calculate the mean of the data.

```
mean_exam1 <-  mean(StatGrades$Exam1) # compute sample mean
```

2.) Subtract this mean from the data.

```
subtract_mean <- StatGrades$Exam1- mean_exam1
```

3.) Add the hypothesized value to the data.

```
modified_data <- subtract_mean + 80

mean(modified_data )
```

```
[1] 80
```

4.) Now create a bootstrap distribution with the last modified data you have created in the previous step.
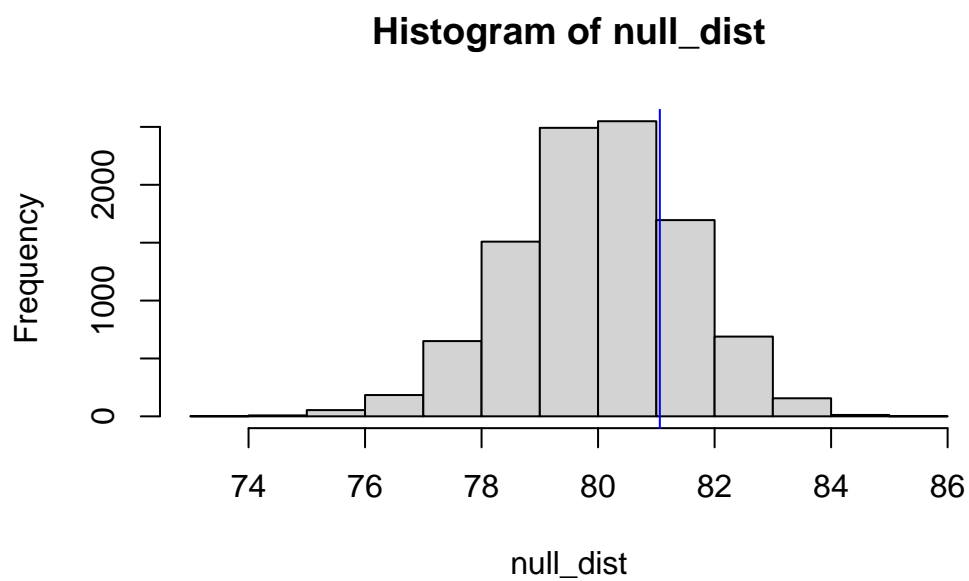
```
n=50
my_sample <- modified_data

boot_sample <- sample(my_sample, n, replace = TRUE) #sample with replacement
null_dist   <- do_it(10000) * {
curr_boot <- sample(my_sample, n, replace = TRUE)
mean(curr_boot)
}


hist( null_dist )

obs_stat <- mean_exam1

abline(v = obs_stat, col = "blue")
```

Histogram of null_dist

5) calculate the p-value:

```r
p_value <- pnull(obs_stat, null_dist, lower.tail = F )
p_value
```

```
[1] 0.246
```