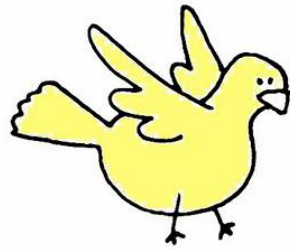
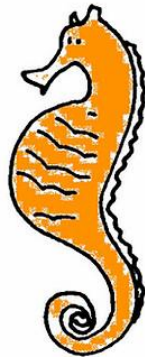


# Categorical data continued and introduction to quantitative data analysis

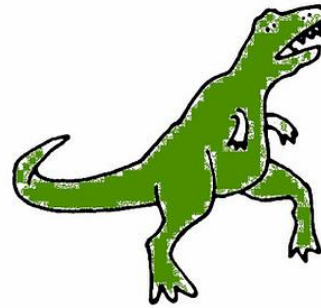
## CATEGORICAL DATA:



I am a bird.  
I am yellow.  
I am awesome.



I am a seahorse.  
I am orange.  
I am super awesome.

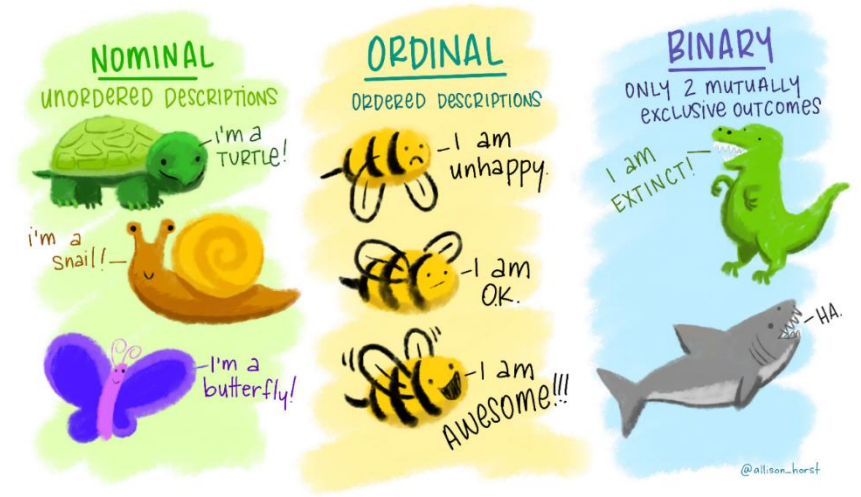


I am a T-rex.  
I am green.  
I am extinct.

# Overview

## Review of:

- Quarto documents and R
- Categorical data concepts and R



## Brief discussion of analyzing two categorical variables

If there is time: quantitative data

Graphing the shape: histograms and outliers

Measures of the central tendency: mean and median

# Announcement: homework 1

Homework 1 is due on Gradescope on  
Sunday, September 7<sup>th</sup> at 11pm

[library\(SDS1000\)](#)

[goto\\_homework\(1\)](#)

The TA office hours are on Canvas if  
you need help with the homework

Lynda's practice sessions

- Thursday: 3:00–5:00 PM
- Friday: 10:00 AM–12:00 PM



# Announcement: homework 1

Instructions for how to submit homework on Gradescope are on Canvas

- Please mark all pages that answers correspond to on Gradescope!

Be sure to also "show your work" by printing out any values you report

- Although don't print out hundreds of access pages of numbers

Ask/answer questions on Ed Discussions, but don't give away the solutions!



Review: Quarto

# Quarto

Quarto (.qmd files) allow you to embed written descriptions, R code and the output to create a reproducible research document!

Everything in R chunks is executed as code:

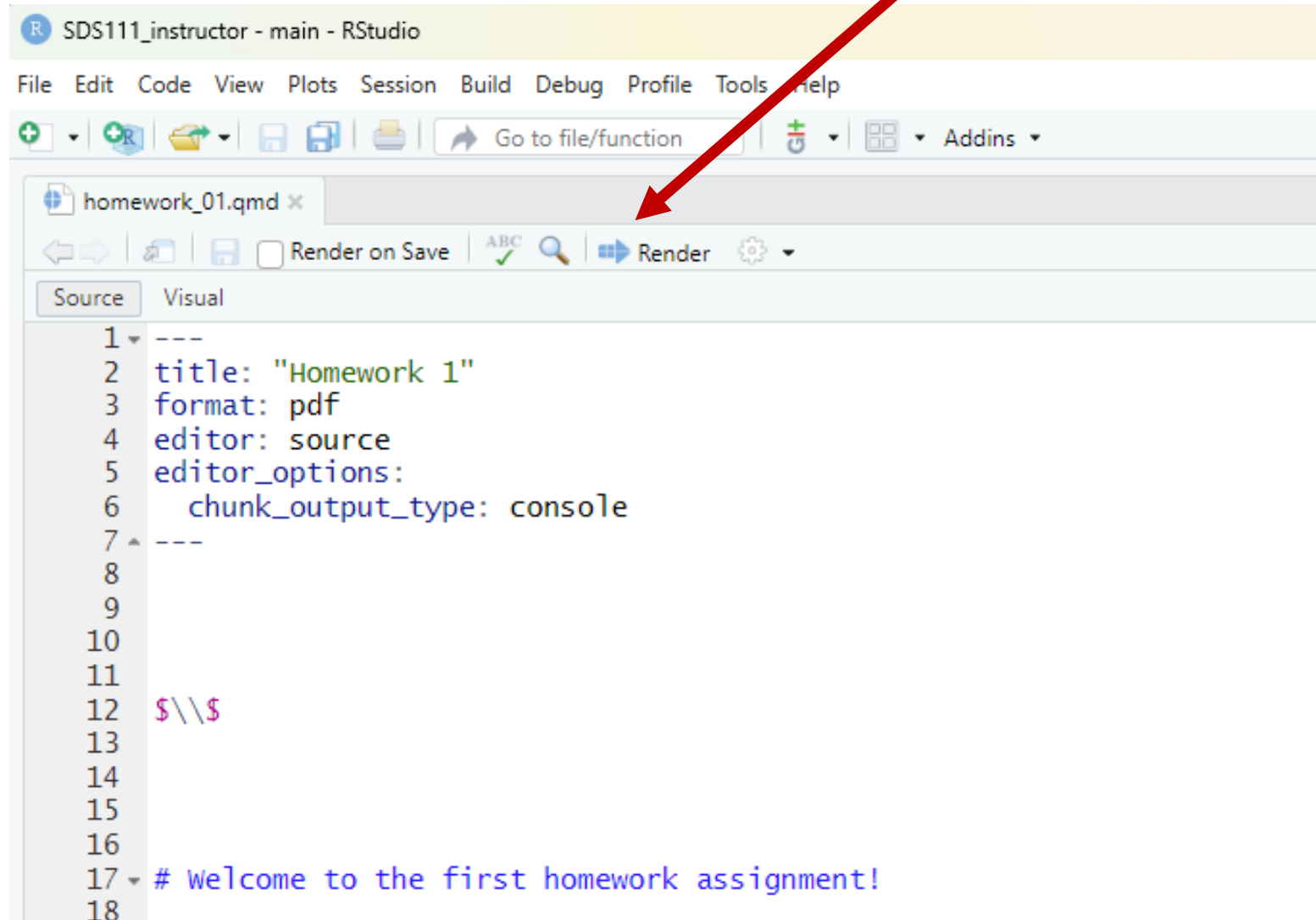
```
```${r}  
  # this is a comment  
  # the following code will be executed  
  2 + 3  
```
```

Everything outside R chunks appears as text



# Render to a pdf

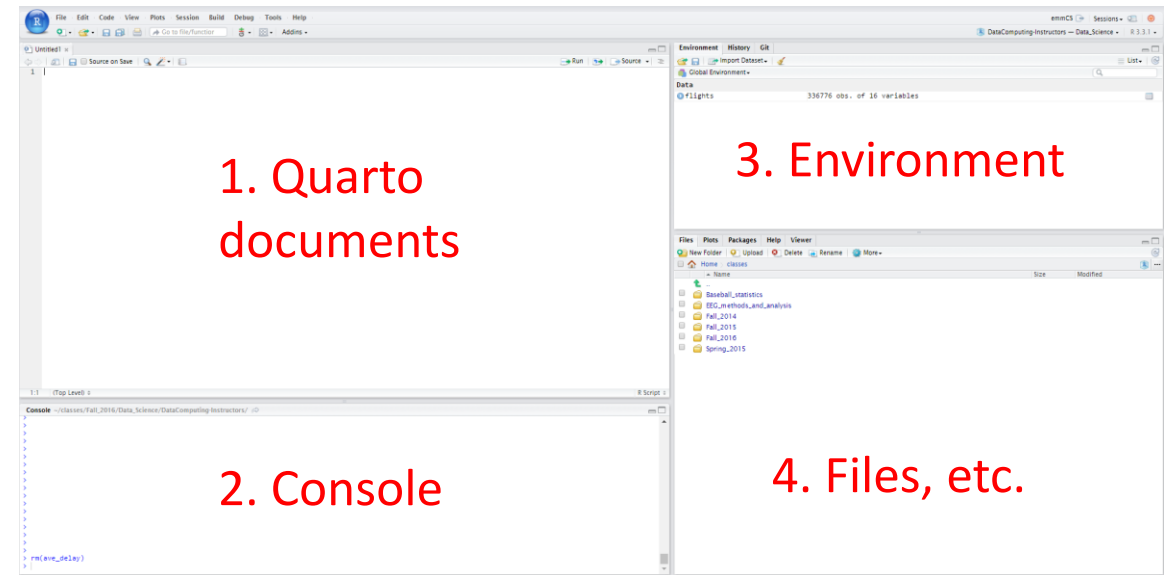
Renders to a pdf document  
(which you will submit to Gradescope)



# Quarto and the global environment

**Note:** When you **render** a Quarto document, your Quarto document **does not have access to objects in the global environment**

- i.e., it can't access any objects you created at the console



Why is this a good thing???

**Takeway:** All object you use in your Quarto document must be defined/created in the Quarto document



# Formatting in Quarto

We can add formatting to text outside the code chunks

Examples:

`## Level 2 header`

**Level 2 header**

---

`**bold**`

**bold**

LaTeX {  
`$\pi$`  
`$x_{outcome}$`

$\pi$

$x_{outcome}$

# To repeat: avoid hard to debug code!

Only change a few lines at a time and then render your document to make sure everything is working!

If your document isn't rendering:

- **For code chunks:** use the `# symbol` to comment out code until you can find the line of code that is giving the error message
- **Outside of code chunk:** cut out part of the document until it renders and then paste it back

Questions?



Quick review of R...

# Review: R Basics

Arithmetic:

```
> 2 + 2
```

```
> 7 * 5
```

Assignment of values to ***objects***:

```
> a <- 4
```

```
> b <- 7
```

```
> z <- a + b
```

```
> z
```

```
[1] 11
```

# Review: Character strings and Booleans

```
> a <- 7
```

```
> s <- "s is a terrible name for an object"
```

```
> b <- TRUE
```

```
> class(a)
```

```
[1] numeric
```

```
> class(s)
```

```
[1] character
```

# Review: Functions

Functions use parenthesis: `functionName(x)`

```
> sqrt(49)
```

```
> tolower("DATA is AWESOME!")
```

To get help

```
> ? sqrt
```

One can add comments to your code

```
> sqrt(49)  # this takes the square root of 49
```

# Review: Vectors

Vectors are ordered sequences of numbers or letters

The `c()` function is used to create vectors

```
> v <- c(5, 232, 5, 543)
```

```
> s <- c("statistics", "data", "science", "fun")
```

One can access elements of a vector using square brackets `[]`

```
> s[4]      # what will the answer be?
```

We can also apply functions to vectors

```
> length(v)  # this tells us how many elements there are in a vector
```



# Data frames

Data frames contain structured data

Below is a data frame (from the homework) where people were asked their opinions about the [Oxford comma](#)

|   | respondent_id | care_oxford_comma | gender | age   | household_income    |
|---|---------------|-------------------|--------|-------|---------------------|
| 1 | 3292953864    | Some              | Male   | 30-44 | \$50,000 - \$99,999 |
| 2 | 3292950324    | Not much          | Male   | 30-44 | \$50,000 - \$99,999 |
| 3 | 3292942669    | Some              | Male   | 30-44 | NA                  |
| 4 | 3292932796    | Some              | Male   | 18-29 | NA                  |
| 5 | 3292932522    | Not much          | NA     | NA    | NA                  |

# Data frames

Suppose our Oxford comma survey data was stored in an object called `comma_survey`

We can extract the columns of a data frame as vector objects using the `$` symbol

```
gender <- comma_survey$gender
```

|   | respondent_id | care_oxford_comma | gender | age   | household_income    |
|---|---------------|-------------------|--------|-------|---------------------|
| 1 | 3292953864    | Some              | Male   | 30-44 | \$50,000 - \$99,999 |
| 2 | 3292950324    | Not much          | Male   | 30-44 | \$50,000 - \$99,999 |
| 3 | 3292942669    | Some              | Male   | 30-44 | NA                  |
| 4 | 3292932796    | Some              | Male   | 18-29 | NA                  |
| 5 | 3292932522    | Not much          | NA     | NA    | NA                  |

# Questions?



# Categorical variables

# Motivation: The sprinkle business

**ACME**  
CORPORATION



PERFECT  
Corporation



ACME corporation believes that if they had the correct ratio (proportion) of red sprinkles that PERFECT corporation uses, their sales will increase

# Where do samples/data come from?

To assess the proportion of sprinkles that PERFECT corporation uses, AMCE sampled 100 of PERFECT corporation's sprinkles

- The **sample size** is 100 ( $n = 100$ )

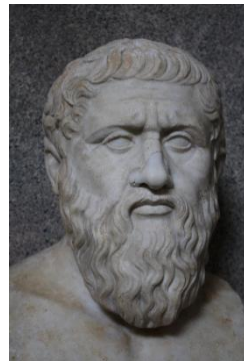


|   |        |
|---|--------|
| 1 | orange |
| 2 | red    |
| 3 | green  |
| 4 | white  |
| 5 | white  |
| 6 | white  |
| 7 | white  |
| 8 | white  |
| 9 | red    |

# Population parameters vs. sample statistics

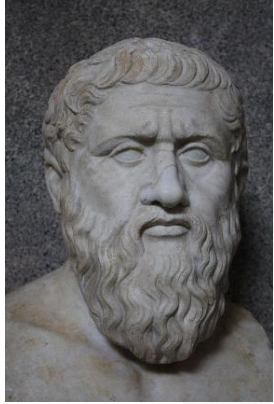
A **statistic** is a number that is computed from ***data in a sample***

A **parameter** is a number that describes some aspect of a ***population***

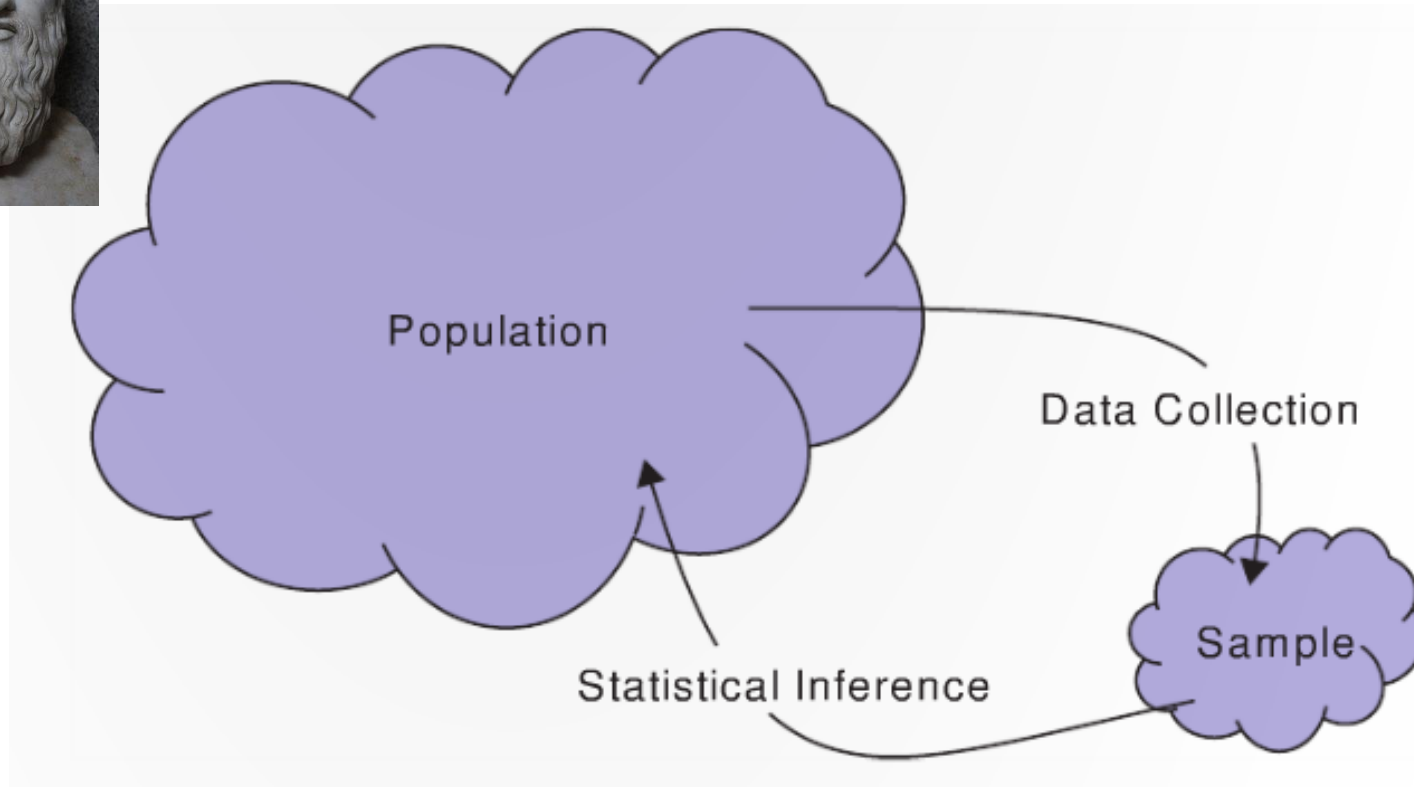


?

# Parameters and statistics



Parameters



statistics





# Proportions

For a *single **categorical variable***, the main ***statistic*** of interest is the *proportion* in each category

- E.g., the proportion of red sprinkles

$$\text{Proportion in a category} = \frac{\text{number in that category}}{\text{total number}}$$

# Example proportion of red sprinkles

The sample

- orange, red, green, white, white, white, ..., pink

The proportion for a **sample** is denoted  $\hat{p}$  (pronounced “p-hat”)

- $\hat{p}_{\text{red}} = 13/100 = 0.13$

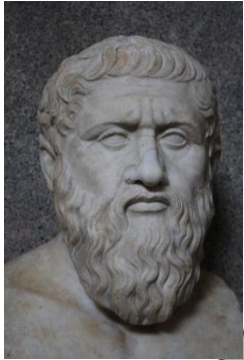
The proportion for a **population** is denoted  $\pi$  (the book uses  $p$ )

- $\pi_{\text{red}}$  proportion if we had measured all sprinkles in the population

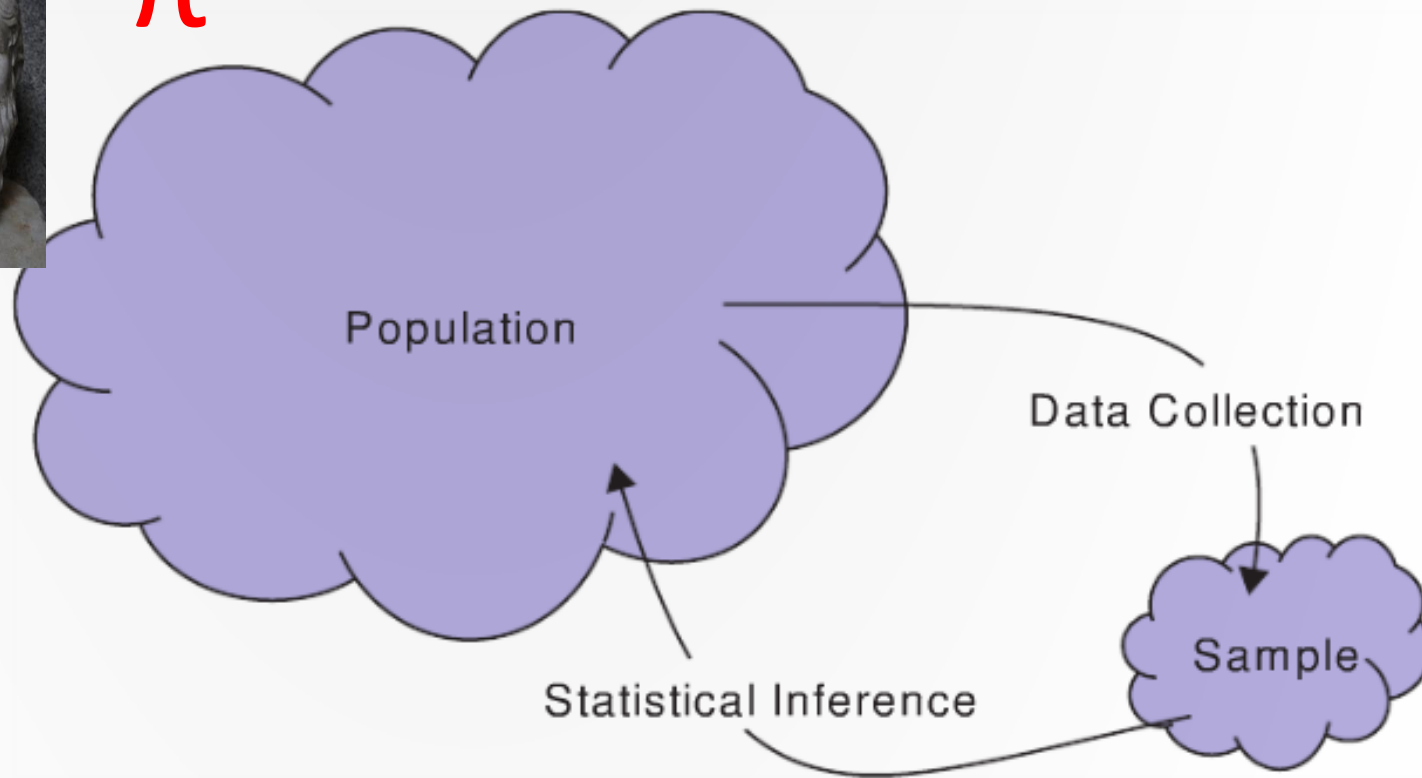
$\hat{p}$  is a **point estimate** of  $\pi$

- i.e.,  $\hat{p}$  our best guess of what  $\pi$  is

# Sample vs. Population proportion



$\pi$



Different samples yield different values for the statistic

$$\hat{p}_{s1\_red} = 0.13$$

$$\hat{p}_{s2\_red} = 0.11$$

$$\hat{p}_{s3\_red} = 0.15$$

$\hat{p}$



# Calculating counts on a categorical variable

The count of how many items are in each category can be summarized in a ***frequency table***

| Color | green | orange | pink | red | white | yellow |  | Total |
|-------|-------|--------|------|-----|-------|--------|--|-------|
| Count | 20    | 11     | 9    | 13  | 36    | 11     |  | 100   |

In R: `my_table <- table(my_vector)`

# Calculating proportions (relative frequencies)

We can convert a frequency table into a ***relative frequency table*** by dividing each cell by the total number of items

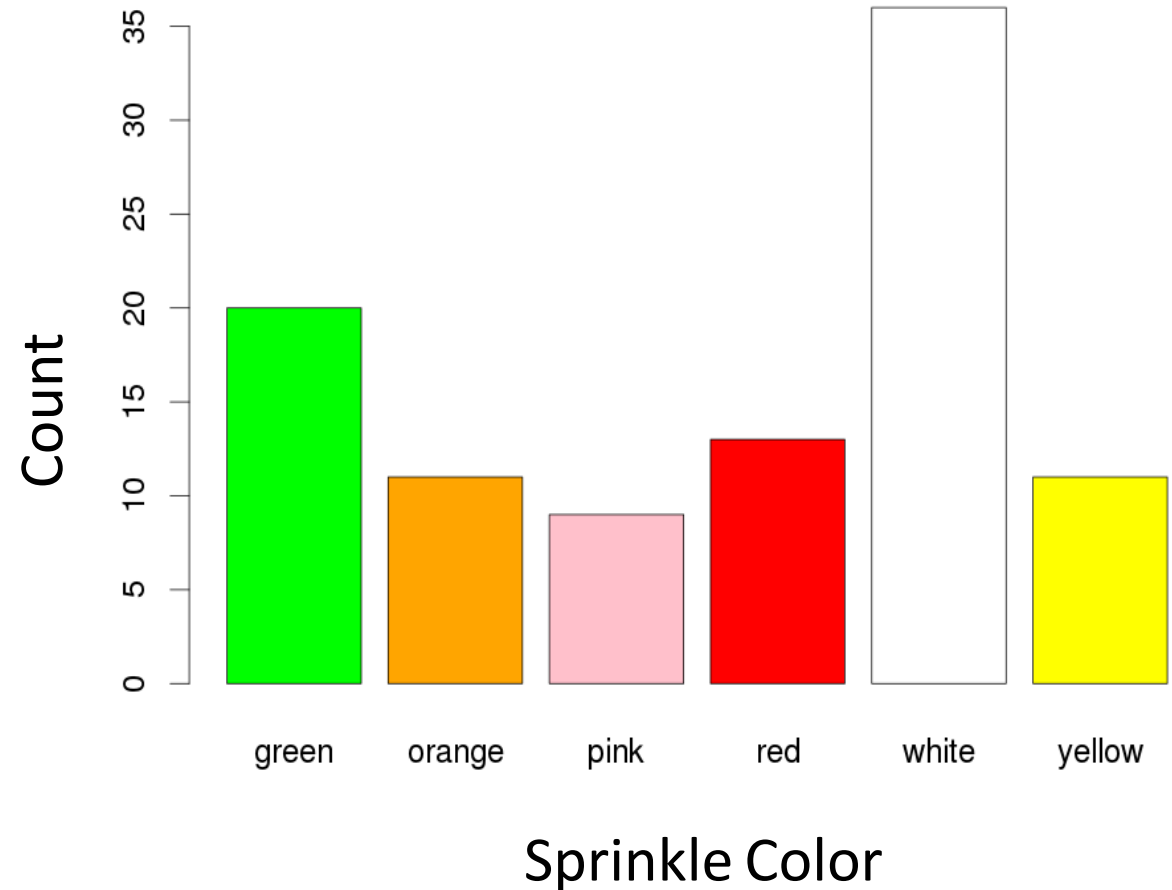
| Color | green | orange | pink | red | white | yellow |  | Total |
|-------|-------|--------|------|-----|-------|--------|--|-------|
| Count | .20   | .11    | .09  | .13 | .36   | .11    |  | 1     |

In R: `prop.table(my_table)`

# Visualizing categorical data: The bar plot

A bar plot shows the number of items in each category

The height of each bar corresponds to the number of items in a given category



In R: `barplot(my_table)`

# Visualizing categorical data: The pie chart

A pie chart plots the proportion of items in each category

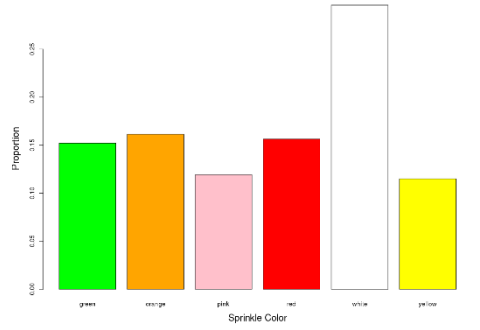
The area of each segment corresponds to the proportion of items in that segment

In R: `pie(my_table)`

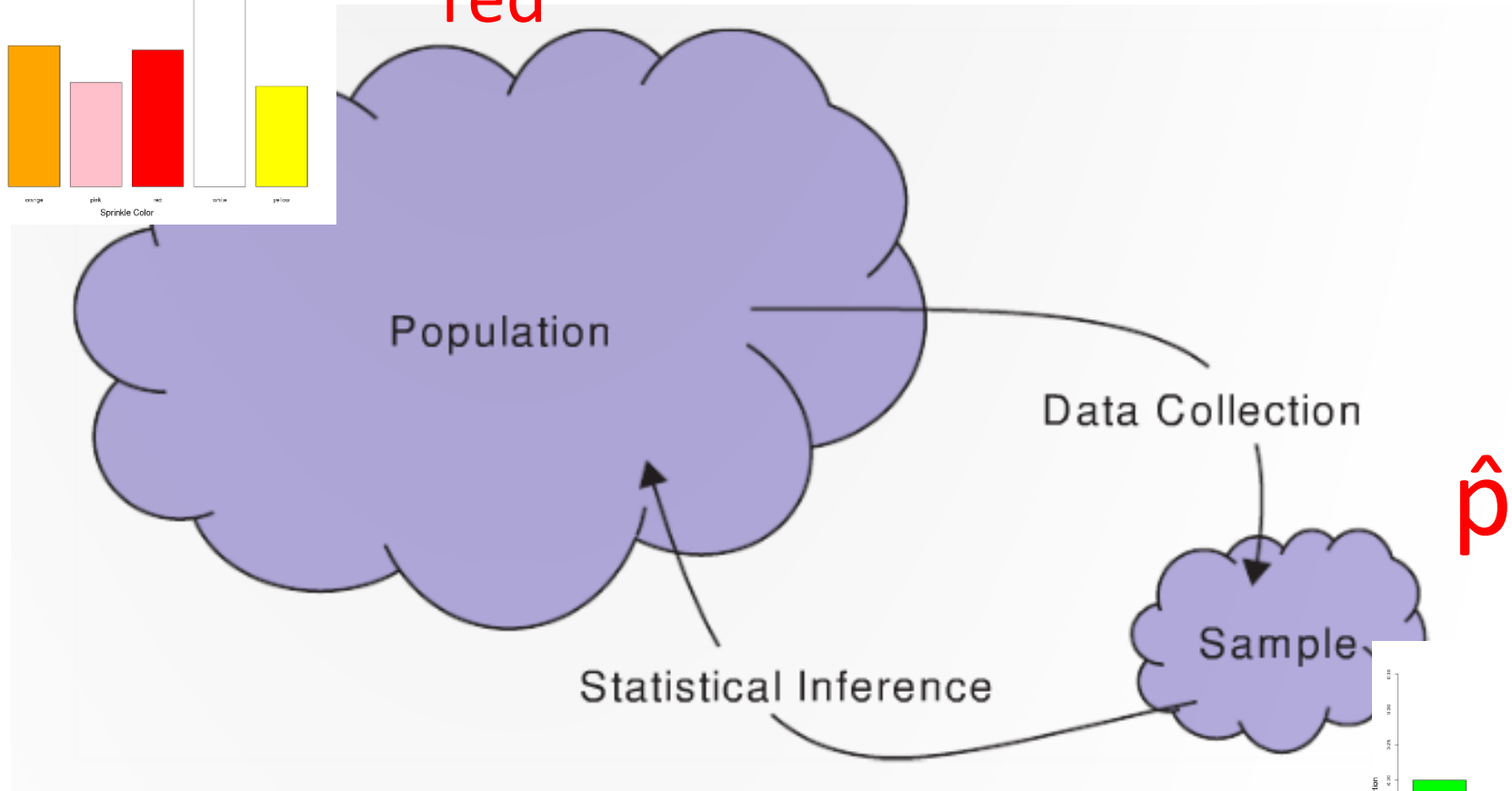


# Summary: Sample and Population proportion

Categorical  
distribution

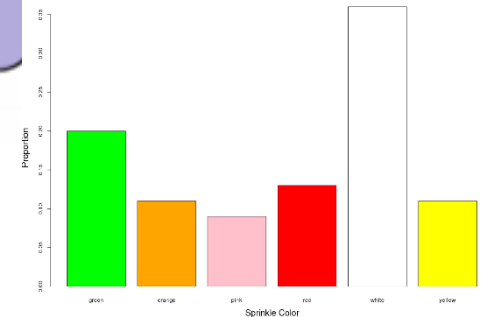


$\pi_{\text{red}}$



Bar chart

$\hat{p}_{\text{red}}$





Example of categorical data: Presidential approval ratings



Attend the practice sessions to try this example!

Let's sample virtual sprinkles in R...



# Sampling virtual sprinkles

```
library(SDS100)
```

```
sprinkle_sample <- get_sprinkle_sample(100)
```

```
sprinkle_count_table <- table(sprinkle_sample)
```

```
sprinkle_prop_table <- prop.table(sprinkle_count_table)
```

```
barplot(sprinkle_count_table)
```

```
pie(sprinkle_count_table)
```

Two categorical variables

# Two categorical variables

Sometimes we have measured two categorical variables for each case, and we want to investigate if there is a relationship between the levels of these categorical variables

- E.g., Suppose we have measure sprinkle **color** and **size**, and we want to investigate whether there is a relationship between these variables

A **two-way table** shows the relationship between two categorical variables

- The category levels for one of the variables (factors) are listed down the rows
- The category levels for the other variable (factor) are listed across the columns
- Each cell in the table counts the number of cases that are in both the row and column categories

|   | color  | size   |
|---|--------|--------|
| 1 | orange | large  |
| 2 | green  | large  |
| 3 | white  | medium |
| 4 | green  | small  |
| 5 | red    | large  |

|       |        | Size  |        |       |
|-------|--------|-------|--------|-------|
|       |        | Large | Medium | Small |
| Color | Green  | 5     | 7      | 8     |
|       | Orange | 3     | 2      | 8     |
|       | Pink   | 3     | 3      | 2     |
|       | Red    | 10    | 3      | 7     |
|       | White  | 10    | 14     | 7     |
|       | Yellow | 2     | 3      | 3     |
| Total |        | 33    | 32     | 35    |

In R: `table(vector1, vector2)`

# Two categorical variables

Sometimes we are interested in the proportion of one variable, given the other variable is a fixed value

- E.g., the proportion of large sprinkles that are red:  $\hat{p}_{\text{red}|\text{large}}$

We can calculate these values by looking at the proportion in the relevant column or row

- $\hat{p}_{\text{red}|\text{large}} = 10/33 = 0.303$

Note: In general:  $\hat{p}_{A|B} = \hat{p}_{B|A}$

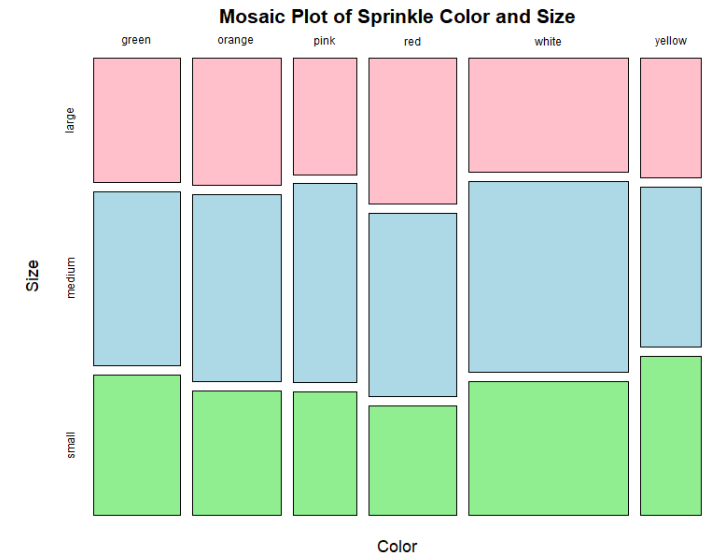
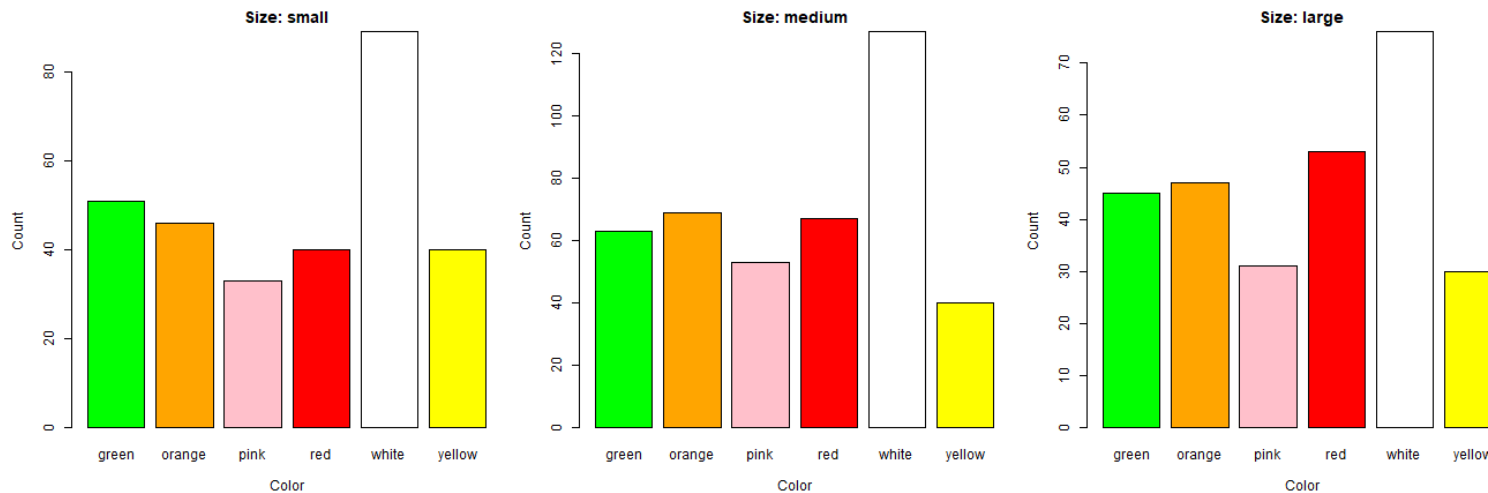
- $\hat{p}_{\text{large}|\text{red}} = 10/20 = 0.5$

|        | Size  |        |       | Total |
|--------|-------|--------|-------|-------|
|        | Large | Medium | Small |       |
| Green  | 5     | 7      | 8     | 20    |
| Orange | 3     | 2      | 8     | 13    |
| Pink   | 3     | 3      | 2     | 8     |
| Red    | 10    | 3      | 7     | 20    |
| White  | 10    | 14     | 7     | 31    |
| Yellow | 2     | 3      | 3     | 8     |
| Total  | 33    | 32     | 35    | 100   |

# Brief mention: Visualizing two categorical variables

**Faceted bar plot:** A series of bar plots split into panels (“facets”) by a categorical variable, making it easier to compare patterns across groups

**Mosaic plot:** A graphical display of contingency tables where the area of each tile is proportional to the cell frequency, showing relationships between categorical variables



# Let's try it in R!

We will use the data from the class survey with the variables:

- The month you were born in
- Whether you were older or younger than other students in your class



# Summary of concepts

1. A **statistic** is a number that is computed from ***data in a sample***
  - The number of items in a sample is called the ***sample size*** and is usually denoted with the symbol  $n$
2. A **parameter** is a number that describes some aspect of a ***population***
3. A **point estimate** is using a value of a statistic as a guess for the value of a parameter
4. **When calculating proportions:**
  - The proportion statistic is denoted  $\hat{p}$
  - The population proportion is denoted  $\pi$
  - Thus  $\hat{p}$  is a ***point estimate*** of  $\pi$
5. Proportions can be summarized in a **relative frequency table** and can be visualized using **bar plots** and **pie charts**
6. **Two-way tables** can be used to summarize data from two categorical variables

# Summary of R

# a vector of character strings (or factors)

```
my_sample <- c("orange", "red", "green", "white", " white", ... )
```

# creating a table using the table() function

```
my_table <- table(my_sample)
```

# creating a frequency table using the prop.table() function

```
prop.table(my_table)
```

# creating bar and pie charts

```
barplot(my_table)
```

```
pie(my_table)
```

# Quantitative variables

# Descriptive statistics for one quantitative variable

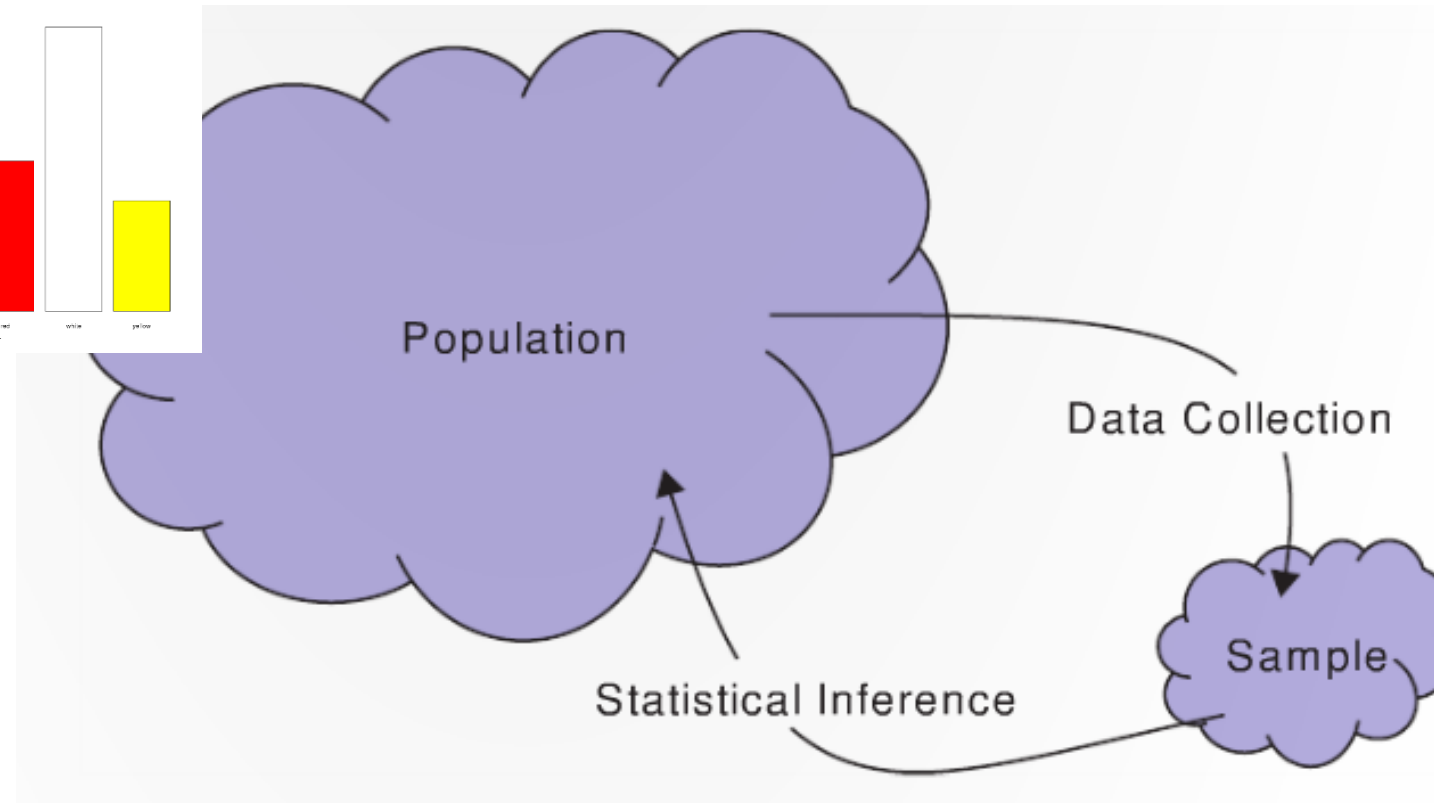
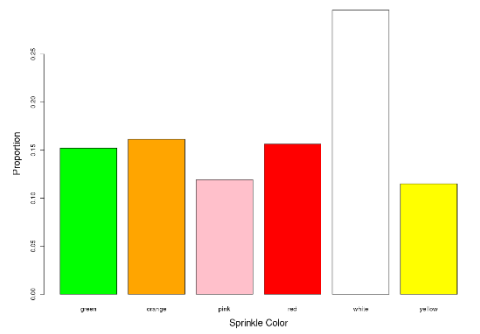
We will be looking at:

- What is the general 'shape' of the data
- Where are the values centered
- How do the data vary

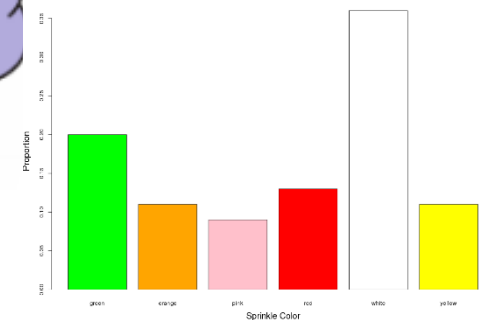
There are all properties of how the data is ***distributed***

# For categorical data we had...

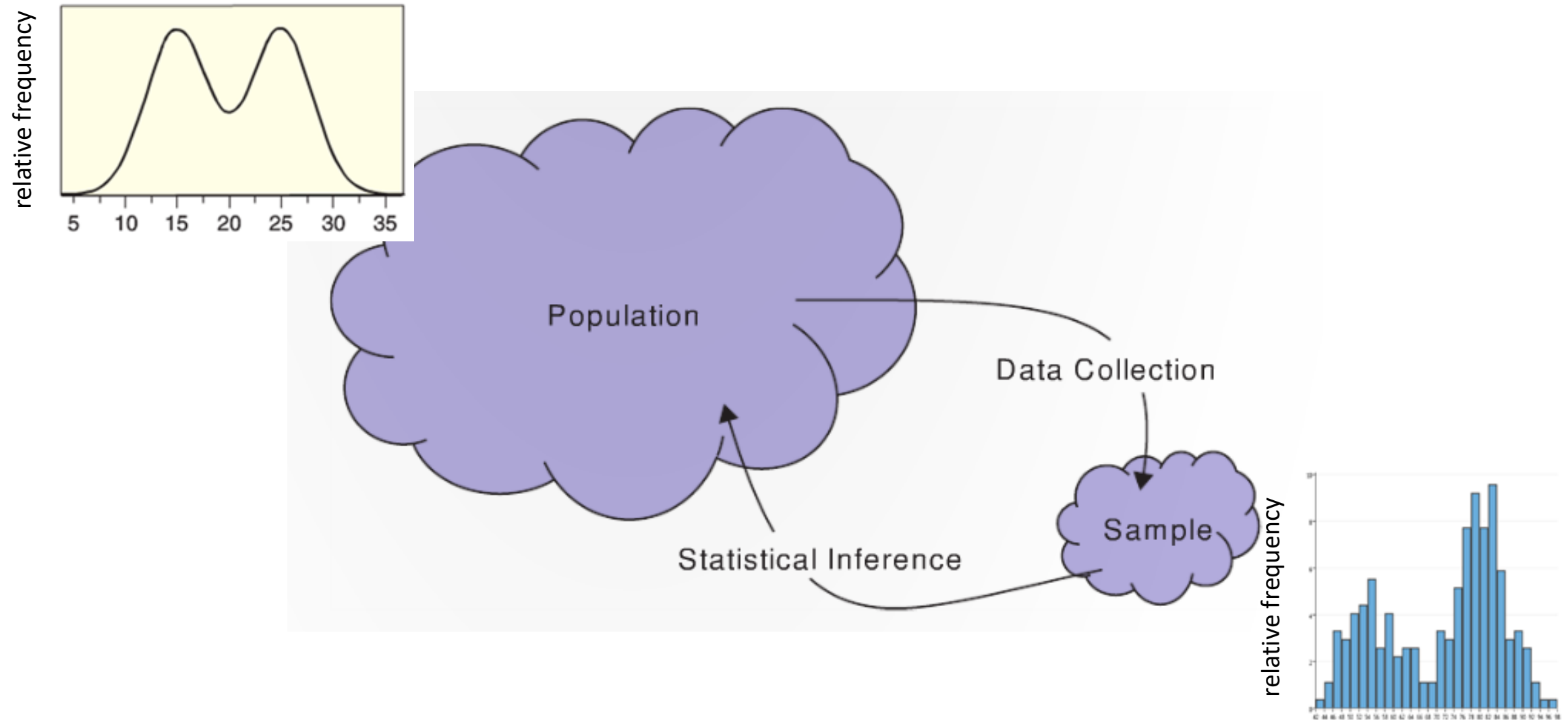
Categorical  
Distribution ( $\pi$ )



Bar chart ( $\hat{p}$ )



# Population distributions and sample histograms

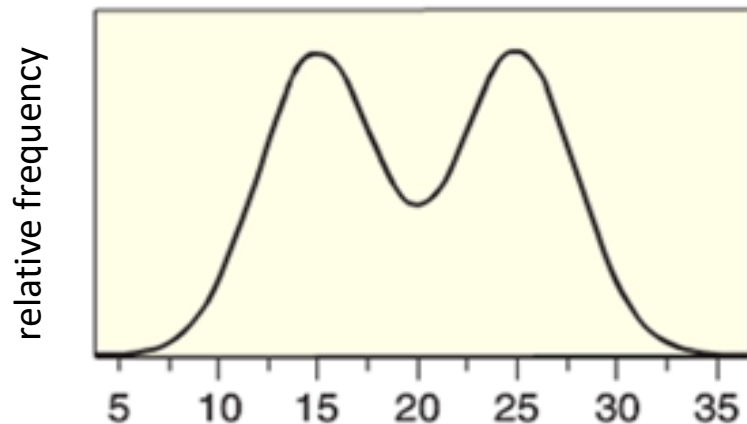


# Histograms

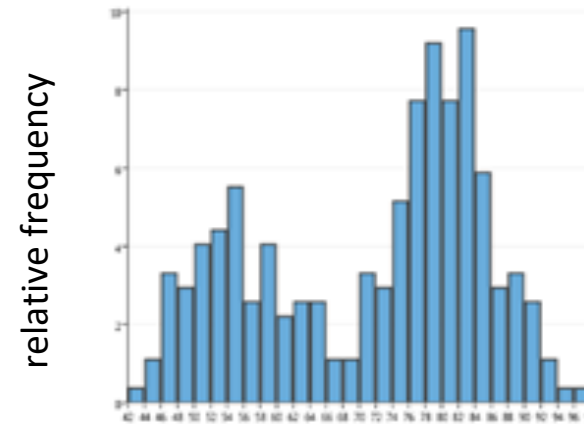
Histograms are a way of visualizing a sample of quantitative data

- They are similar to bar charts but for quantitative variables
- They aim to give a picture of how the data is distributed

Continuous distribution



Histogram



# Gapminder data and data frames

# get a data frame with information about the countries in the world

> load("gapminder\_2007.rda")

> View(gapminder\_2007)

|   | country     | continent | year | lifeExp | pop      | gdpPercap  |
|---|-------------|-----------|------|---------|----------|------------|
| 1 | Afghanistan | Asia      | 2007 | 43.828  | 31889923 | 974.5803   |
| 2 | Albania     | Europe    | 2007 | 76.423  | 3600523  | 5937.0295  |
| 3 | Algeria     | Africa    | 2007 | 72.301  | 33333216 | 6223.3675  |
| 4 | Angola      | Africa    | 2007 | 42.731  | 12420476 | 4797.2313  |
| 5 | Argentina   | Americas  | 2007 | 75.320  | 40301927 | 12779.3796 |

Hans Rosling's [gapminder](#)



# Gapminder data

## Questions:

1. What are the observational units (cases)?
2. What are the variables?
3. Are the variable categorical or quantitative?
4. What is the population?

|   | country     | continent | year | lifeExp | pop      | gdpPercap  |
|---|-------------|-----------|------|---------|----------|------------|
| 1 | Afghanistan | Asia      | 2007 | 43.828  | 31889923 | 974.5803   |
| 2 | Albania     | Europe    | 2007 | 76.423  | 3600523  | 5937.0295  |
| 3 | Algeria     | Africa    | 2007 | 72.301  | 33333216 | 6223.3675  |
| 4 | Angola      | Africa    | 2007 | 42.731  | 12420476 | 4797.2313  |
| 5 | Argentina   | Americas  | 2007 | 75.320  | 40301927 | 12779.3796 |

# Gapminder: life expectancy in different countries

Let's look at the life expectancy in different countries, which is a quantitative variable

# pull a vector of life expectancies from the data frame

```
life_expectancy <- gapminder_2007$lifeExp
```

# Histograms – countries life expectancy in 2007

Life expectancy for different countries for 142 countries in the world:

- 43.83, 72.30, 76.42, 42.73, ...

To create a histogram we create a set of intervals

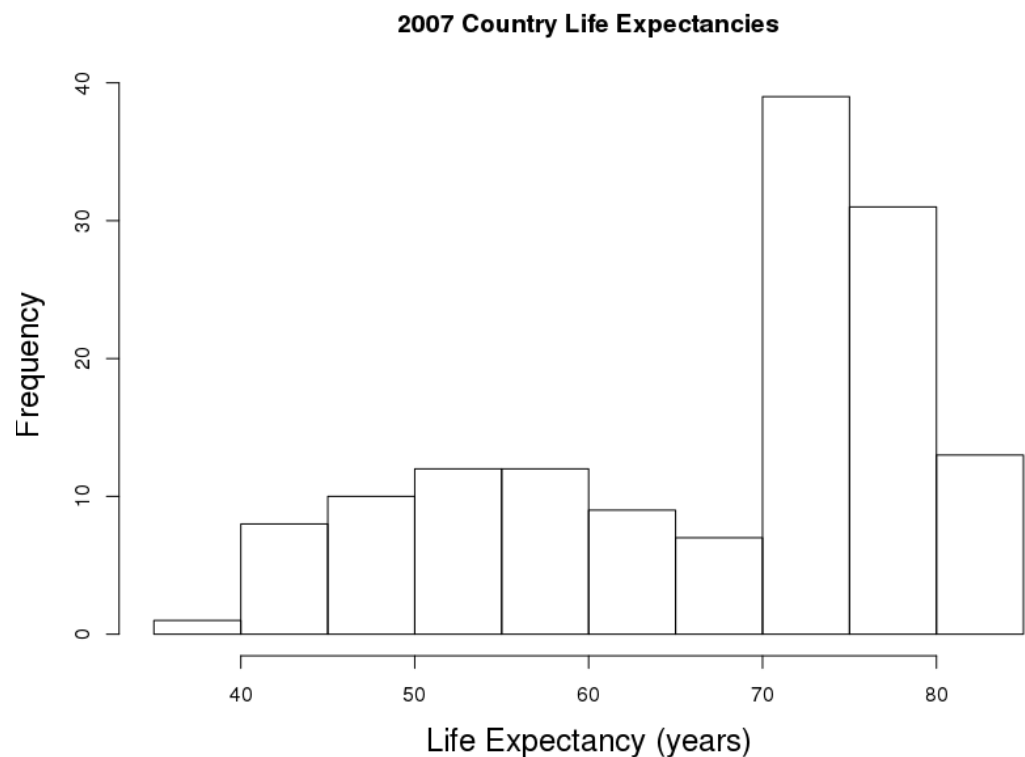
- 35-40, 40-45, 45-50, ... 75-80, 80-85

We count the number of points that fall in each interval

We create a bar chart with the counts in each bin

# Histograms – countries life expectancy in 2007

| Life Expectancy | Frequency Count |
|-----------------|-----------------|
| (35 – 40]       | 1               |
| (40 – 45]       | 8               |
| (45 – 50]       | 10              |
| (50 – 55]       | 12              |
| (55 – 60]       | 12              |
| (60 – 65]       | 9               |
| (65 – 70]       | 7               |
| (70 – 75]       | 39              |
| (75 – 80]       | 31              |
| (80 – 85]       | 13              |



R: `hist(v)`

# Gapminder: life expectancy in different countries

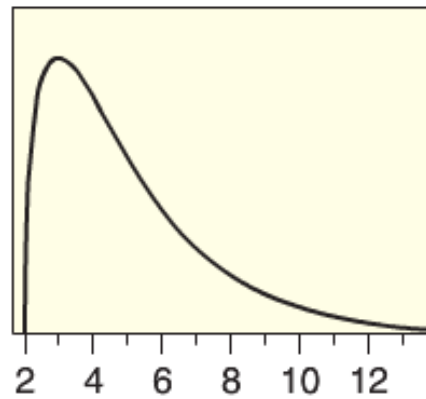
Try creating a histogram of the life expectancy in different countries using the `hist()` function

# pull a vector of life expectancies from the data frame

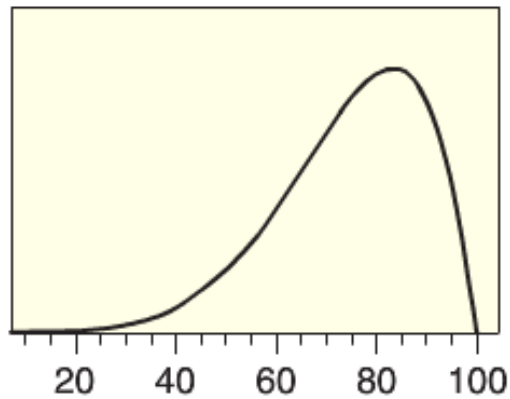
```
> life_expectancy <- gapminder_2007$lifeExp
```

```
> hist(life_expectancy)
```

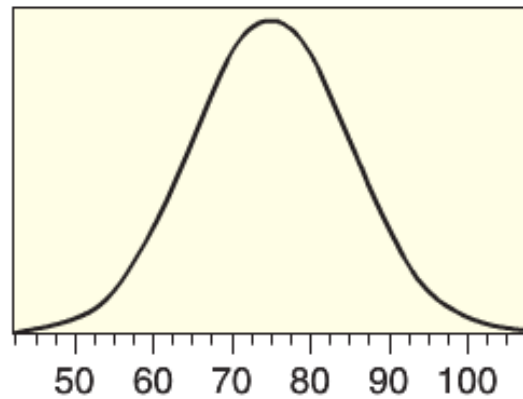
# Common shapes for distributions



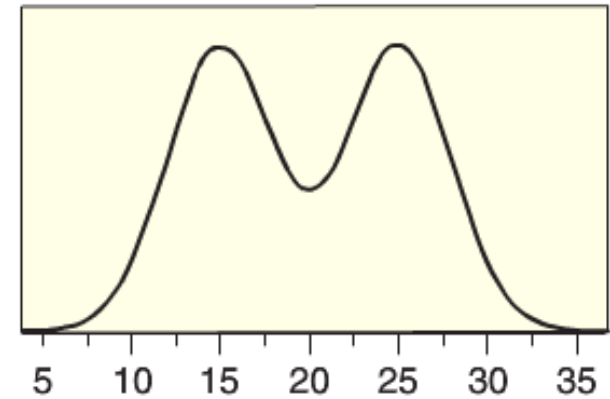
(a) Skewed to the right



(b) Skewed to the left

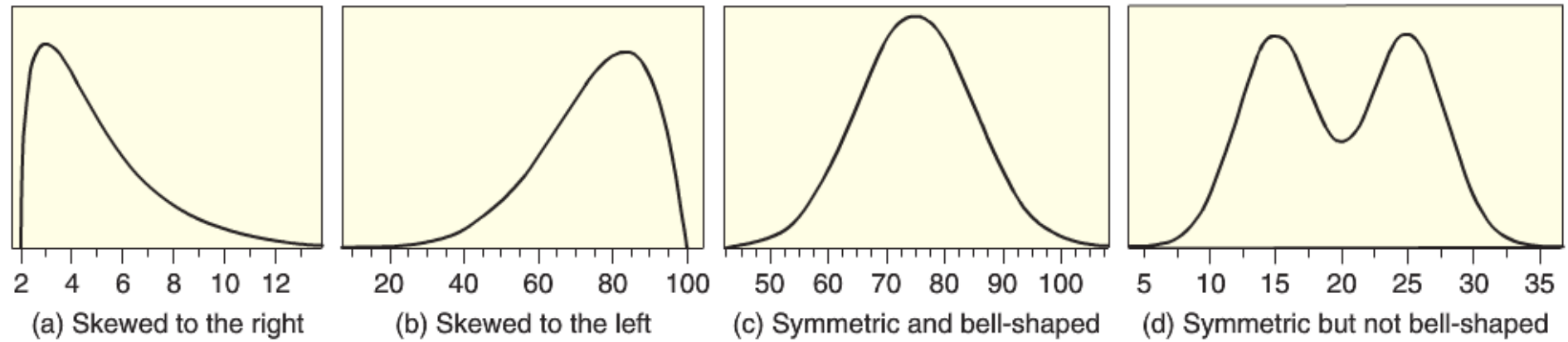
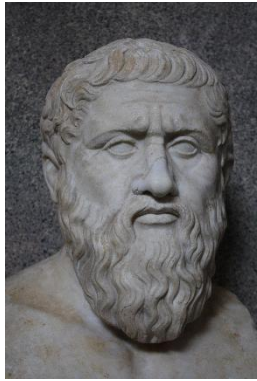


(c) Symmetric and bell-shaped

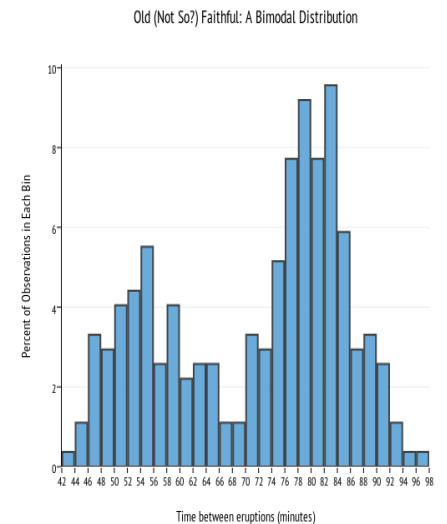
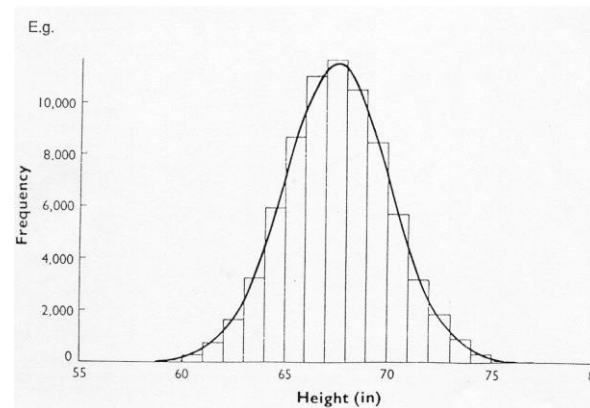
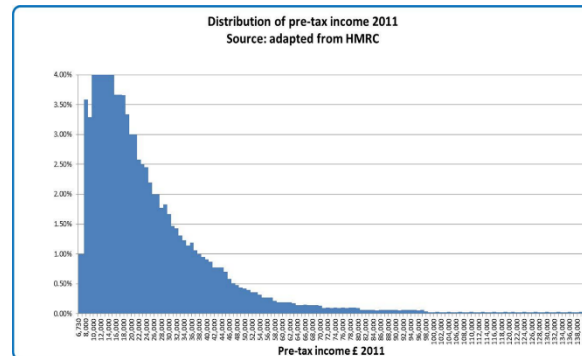


(d) Symmetric but not bell-shaped

# Plato and shadows: distributions and histograms

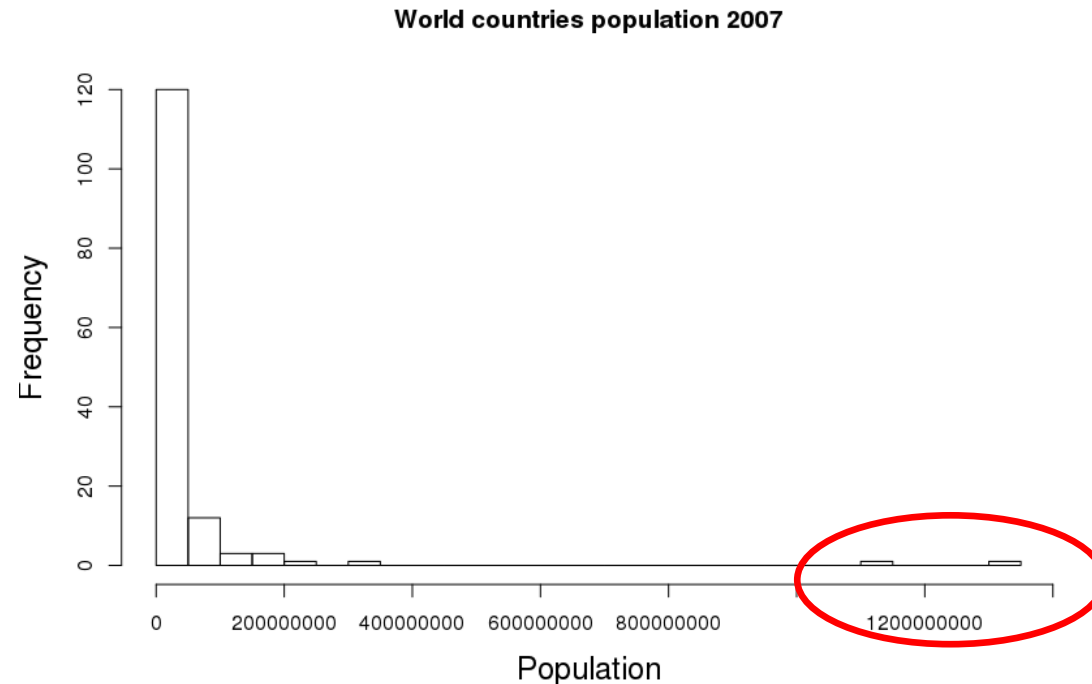


## Income distribution



# Outliers

An **outlier** is an observed value that is notably distinct from the other values in a dataset by being much smaller or larger than the rest of the data.



Outliers can potentially have a large influence on the statistics you calculate

- One should examine outliers in more detail to understand what is causing them



Descriptive statistics for the center of a distribution

# Descriptive statistics for the center of a distribution

Graphs are useful for visualizing data to get a sense of what the data look like

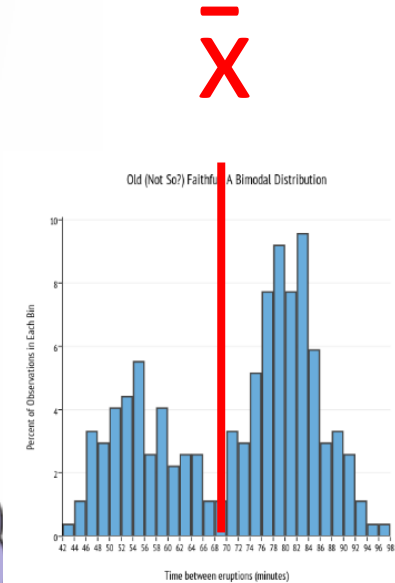
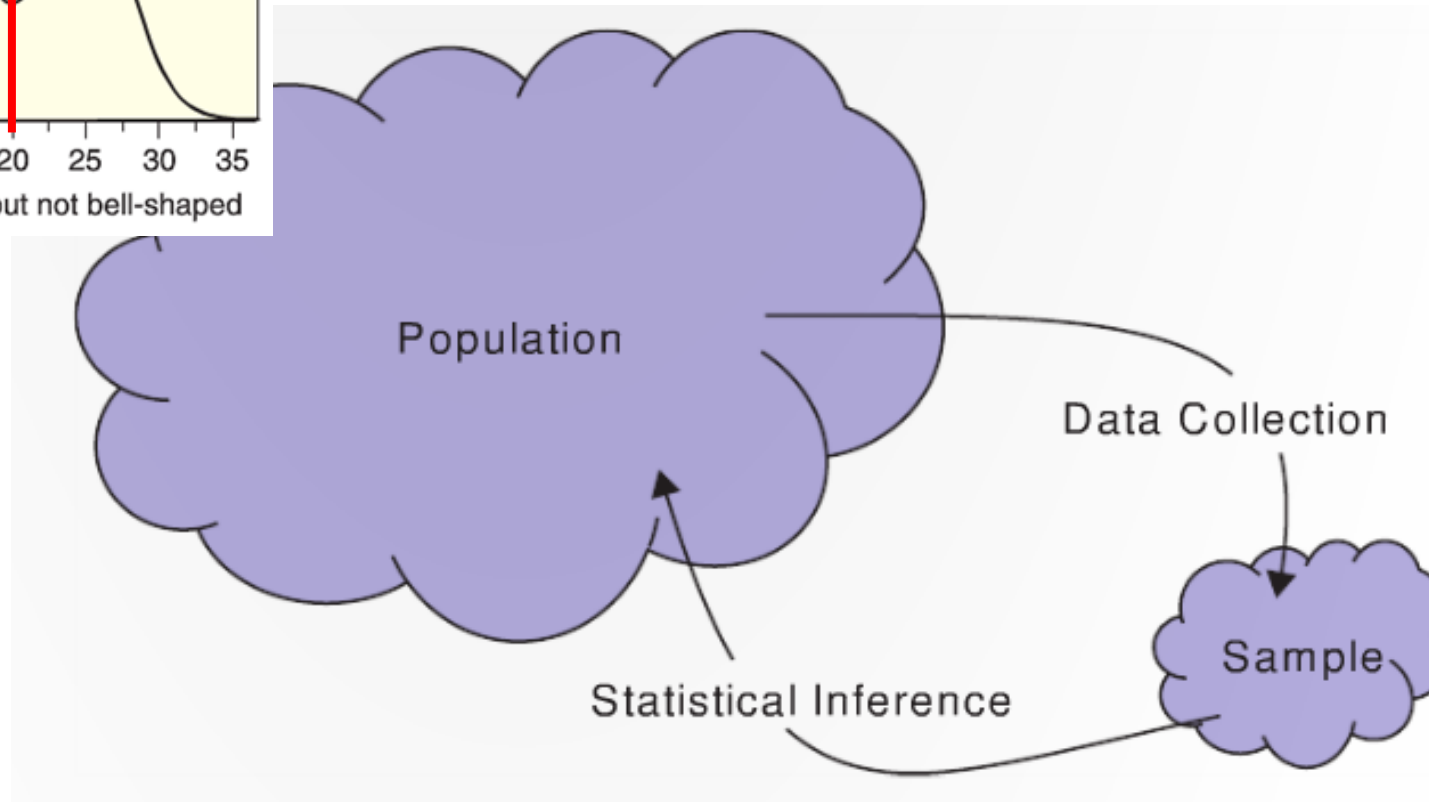
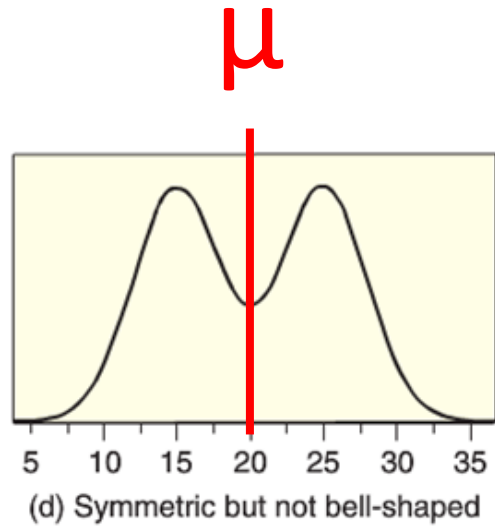
We can also summarize data numerically

**Question:** what is a numerical summary of a sample of data called?

**A: a statistic!**

Two important statistics that can be used to describe the center of the data are the **mean** and the **median**

# Sample and population mean



# The mean

$$\text{Mean} = \frac{\text{Sum of all data values}}{\text{Number of data values}}$$

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \sum_{i=1}^n \frac{x_i}{n} = \frac{1}{n} \sum_{i=1}^n x_i$$

R: `mean(x)`

R: `mean(x, na.rm = TRUE)`

Give the proper notation:  $\mu$  vs.  $\bar{x}$  ?

We measure the height of 50 randomly chosen Yale students

We measure the height of all Yale students

Can you calculate the mean of the countries life expectancy in R?

```
> life_expectancy <- gapminder_2007$lifeExp  
> mean(life_expectancy)
```

# The median

The **median** of a data set of size  $n$  is

- If  $n$  is odd: The middle value of the sorted data
- If  $n$  is even: The average of the middle two values of the sorted data

The median splits the data in half

R: `median(v)`  
`median(v, na.rm = TRUE)`

# Resistance

We say that a statistics is **resistant** if it is relatively unaffected by extreme values (outliers).

The median is resistant when the mean is not

Example:

Mean US salary = \$72,641

Median US salary = \$51,939

# Summary of concepts

1. A **probability distribution** shows the **relative likelihood** that we will get a data point in the population with a particular value

- (for a more precise definition take a class in probability)

2. Distributions can have different shapes

- E.g., left skewed, right skewed, bell shaped, etc.

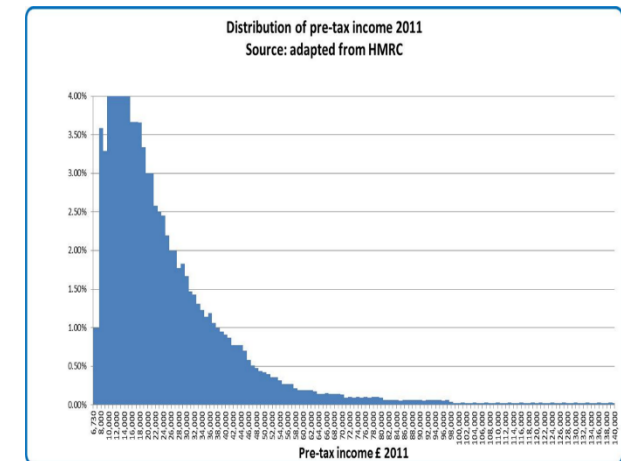
3. The **mean** is one measure of central tendency

- Sample mean is denoted  $\bar{x}$  (statistic)
- Population mean is denoted  $\mu$  (parameter)

4. The **median** is another measure of central tendency

- The median is resistant to outliers while the mean is not

## Income distribution





# Summary of R

## **Data frames** contain structured data

- We can view a data frame in R Studio (not in Markdown) using:  
    > `View(my_data_frame)`
- We can extract vectors from a data frame using:  
    > `my_vec <- my_data_frame$my_var`

We can get a sense of how quantitative data is distributed by creating a histogram

> `hist(my_vec)`

We can calculate measures of central tendency using:

> `mean(my_vec)`  
> `median(my_vec)`