# Practice Session 5 Answers

In this practice section we will introduce the bootstrap distribution, bootstrap confidence interval, and hypothesis testing. You may use the functions: `sample()`, `do_it()` to generate the bootstrap distribution. `SDS1000::cnorm` and `qnorm` to find the critical value corresponding to a specific **confidence level**.

## Part 1: Confidence interval concept

**Practice 1:**

True or False/ Confidence interval interpretation

A catalog sales company promises to deliver orders placed on the Internet within 3 days. Follow-up calls to a few randomly selected customers show that a 95% confidence interval for the proportion of all orders that arrive on time is $85\% \pm 5\%$.

1.) Between 80% and 90% of all orders arrive on time.

2.) 95% of all random samples of customers will show that 85% of orders arrive on time.

3.) The interval between 80% and 90% gives a plausible range of values for where the true population parameter lies since 95% of intervals created will contain the population proportion.

4.) For a given sample size, higher confidence means a larger margin of error.

5.) For a specified confidence level, smaller samples provide smaller margins of error.

**Answers:**

1.) False. This statement implies certainty. There is no level of confidence in the statement.

2.) False. Different samples will give different results. Many fewer than 95% of samples are expected to have exactly 88% on-time orders.

3.) True.

4.) True. For a given sample size, higher confidence means a larger margin of error.

5.) False. Smaller samples lead to larger standard errors, which lead to larger margins of error.

## Part 2: Construct Bootstrap Distribution

Here's the clever idea: We don't have the population, but we have a sample. Probably the sample is similar to the population in many ways.So let's sample from our sample. We'll call it `bootstrapping`. We want samples **the same size** as our original sample, so we will need to **sample with replacement**.This means that we may pick some members of the population more than once and others not at all. We'll replicate this many times.

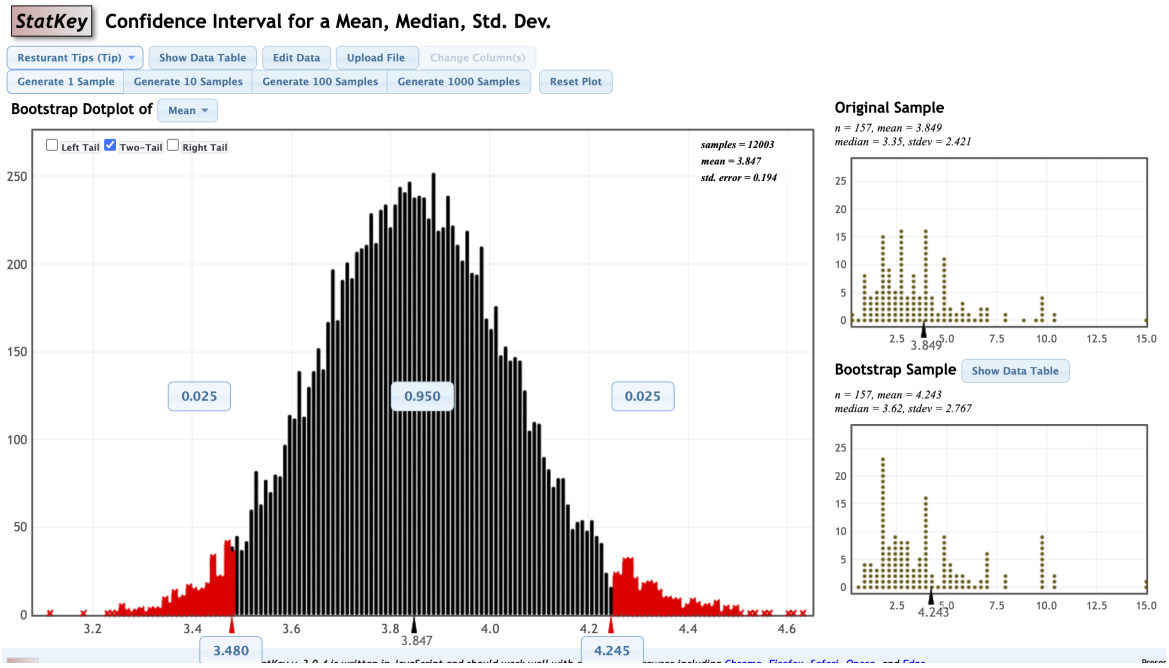
**Generating a Bootstrap Distribution:**

- Generate bootstrap samples by **sampling with replacement** from the original sample, using the **same sample size**.
- Compute the **statistic of interest** (called a bootstrap statistic), for each of the bootstrap samples.
- Collect the **samples statistics** for many bootstrap samples to create a **bootstrap distribution**.

**Example:**

Using `StatKey`website link. Try to play with the creation of the bootstrap distribution from different data. The following picture shows the website and an example of bootstrap CI for the variable `tip` from the dataset `Restaurant tips` .

`https://www.lock5stat.com/StatKey/bootstrap_1_quant/bootstrap_1_quant.html`




### Practice 2:

The data `ExerciseHours` provide an in-class survey of statistics students asking them about the amount of exercise per week.

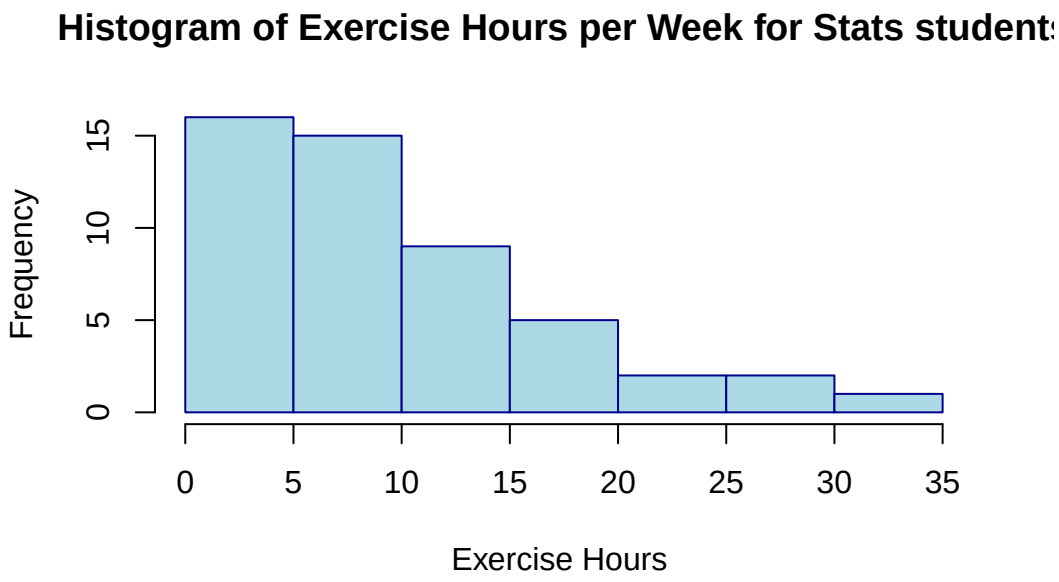
1.) **First**, create `histogram` of `Exercise`. What is the `sample size` of the data ?

```
library(Lock5Data)
library(SDS1000)
data(ExerciseHours)

hours<- ExerciseHours$Exercise

hist(hours,

     main = "Histogram of Exercise Hours per Week for Stats students",
     xlab = "Exercise Hours ",
     ylab = "Frequency",
     col = "lightblue",
     border = "darkblue" )
```




```
n <- length(hours)
```

2.) **Second**, create a one bootstrap sample from `Exercise` . You might use the functions `sample()`.

```
one_sample <- sample(hours,
                     size = n, replace = T)
```

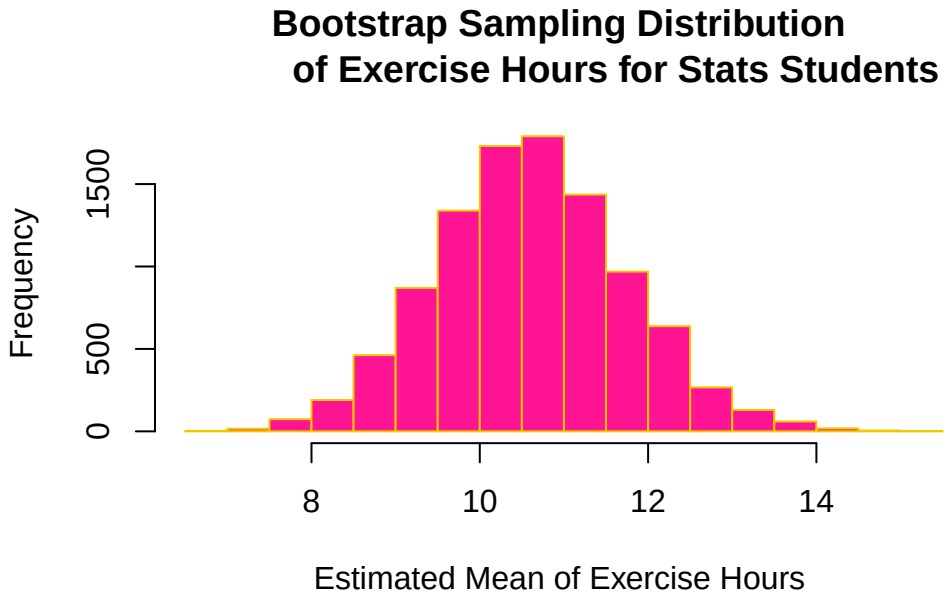
3.) **Third**, you might replicate this one sample `10000` times `with replacement` using the function `do_it()` to create a `bootstrap sampling ditribution`

```
boot_dist<- do_it(10000)*{
  one_sample = sample(hours, size = n, replace = T)
  mean( one_sample)
}
```

4.) **Fourth**, create a `histogram` for the bootstrap distribution of the variable `Exercise`

```
hist(boot_dist,
            main ="Bootstrap Sampling Distribution
            of Exercise Hours for Stats Students  ",
```

```
        xlab = "Estimated Mean of Exercise Hours ",
        ylab = "Frequency",
        col = "deeppink",
        border = "gold2" )
```




Congrats! you created bootstrap sampling distribution from one sample !

### Part 3: Construct Bootstrap Confidence Interval

*Reminder:*

The steps to Construct Bootstrap Confidence Interval are:

- 1. Compute the statistic from the original sample.
- 2. Create a bootstrap distribution by re-sampling from the sample.
  - Same size samples as the original sample.
  - With replacement.
  - Compute the statistic for each sample.

  - The distribution of these statistics is the bootstrap distribution.

- 3. Estimate the standard error SE by computing the standard deviation of the bootstrap distribution.
- 4. Create the 95% CI using the formula: $statistic \pm 2 * SE$.
- 5. Interpret the confidence interval within the context.

**Practice 3:**

From the `ExerciseHours` bootstrap sampling distribution you have created in the previous question, create a 95% CI for the the sample mean of `Exercise`.

1.) **First**, calculate the mean of `Exercise` from your original sample

```
x_bar <- mean(hours )
x_bar
```

```
[1] 10.6
```

2.) **Second**, create the bootstrap sampling distribution of the `Exercise`. Ww did it in the previous question (practice 2).

3). **Third**, calculate the standard error of our bootstrap sampling distribution of the `Exercise`.

```
boot_se <- sd(boot_dist )
boot_se
```

```
[1] 1.129671
```

4.) **Fourth**, calculate the 95% CI, which is based on the formula: $statistic \pm 2 * SE$

```
# calculate the $95\%$ CI
#CI_lower <- x_bar - 2*boot_se
#CI_upper <- x_bar + 2*boot_se

CI95_lower_upper <- x_bar + 2*boot_se*c(-1,1)
CI95_lower_upper
```

```
[1]  8.340658 12.859342
```

Congrats! you created a 95% CI with bootstrap distribution !

5.) **Fifth**. Interpret the confidence interval within the context.

We are 95% confident that the mean hours of students exercise is between 8.327 12.872

## Part 4: Extra Practice/ Create Bootstrap Confidence Interval with Different Confidence Levels

**Practice 4:**

*Note*: You might use the function `SDS1000::cnorm()` or ,`qnorm()` to help you find the critical values.

1.) Create a 90% CI of the mean `Exercise` in `R`.

```
# a $90\%$  CI of the mean `Exercise`

CI90_cv <- SDS1000::cnorm(0.9)
CI90_cv
```

```
[1] 1.644854
```

```
CI90_lower_upper <- x_bar + CI90_cv*boot_se*c(-1,1)
CI90_lower_upper
```

```
[1]  8.741857 12.458143
```

2.) Calculate a 99% confidence interval of the mean `exercise` using the function `qnorm()` .

```
# a  $99\%$ confidence interval of the mean `exercise`

CI99_cv <- qnorm(0.995)
CI99_cv
```

```
[1] 2.575829
```

```r
CI99_lower_upper <- x_bar+(CI99_cv)*boot_se*c(-1,1)
CI99_lower_upper
```

```
[1]  7.690161 13.509839
```

3) Compare the three confidence intervals you have obtained from the three different confidence levels: 95% , 90%, and, 99% .

## Part 5: Extra Practice/ Introduction to Hypothesis testing

**Statistical Tests:**

A statistical test is used to determine whether results from a sample are convincing enough to allow us to conclude something about the population.

We have two competing claims about the population, the **null hypothesis**, denoted by `H0`, and the **alternative hypothesis**, denoted by `Ha`.

**Practice 5:**

State the null and alternative hypotheses for the statistical test described:

1.) Testing to see if there is evidence that a mean is less than 50.

2.) Testing to see if there is evidence that a proportion is greater than 0.3.

3.) Testing to see if there is evidence that the mean of group A is not the same as the mean of group B.

4.) Testing to see if there is evidence that the correlation between two variables is positive.

**Answers:**

1. $H_0 : \mu = 50$ vs $H_a : \mu < 50$
2. $H_0 : \pi = 0.3$ vs $H_a : \pi > 0.3$
3. $H_0 : \mu_A = \mu_B$ vs $H_a : \mu_A \neq \mu_B$
4. $H_0 : \rho = 0$ vs $H_a : \rho > 0$