

Practice Session 2 Answers

Part 1 : Measures of central tendency & Measures of spread for quantitative data

1.1 Calculating the Sample Standard Deviation by Hand

Here is the formula for the sample mean:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

And here is the formula for the sample standard deviation:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Using the above data, perform the following calculations. Complete the table for calculating the sample standard deviation.

Cost \$\$\$	b. Deviations ($x_i - \bar{x}$)	c. Deviations squared $(x_i - \bar{x})^2$
850		
900		
1400		
1200		
1050		
750		
1250		
1050		
565		
1000		
a. mean = _____		

d. Sum of squared deviations $\sum_{i=1}^n (x_i - \bar{x})^2 =$

e. Sum of squared deviations divided by n - 1: $\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} =$

$$\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

f. Take the square root to get s: $=$ $= s$

1.2 Five-Number Summary

Using the numbers from the previous exercise, do the following:

- Find the 5-number summary (minimum, Q1, median, Q3, maximum)
- Check your work using R functions

Answers

```
v<- c( 565, 750, 850, 900, 1000, 1050, 1050, 1200, 1250, 1400 )
v
```

```
[1] 565 750 850 900 1000 1050 1050 1200 1250 1400
```

```
## you can use function fivenum()

fivenum(v)
```

```
[1] 565 850 1025 1200 1400
```

```
## you can use function summary()

summary(v)
```

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
565.0	862.5	1025.0	1001.5	1162.5	1400.0

1.3 Boxplots and Histograms

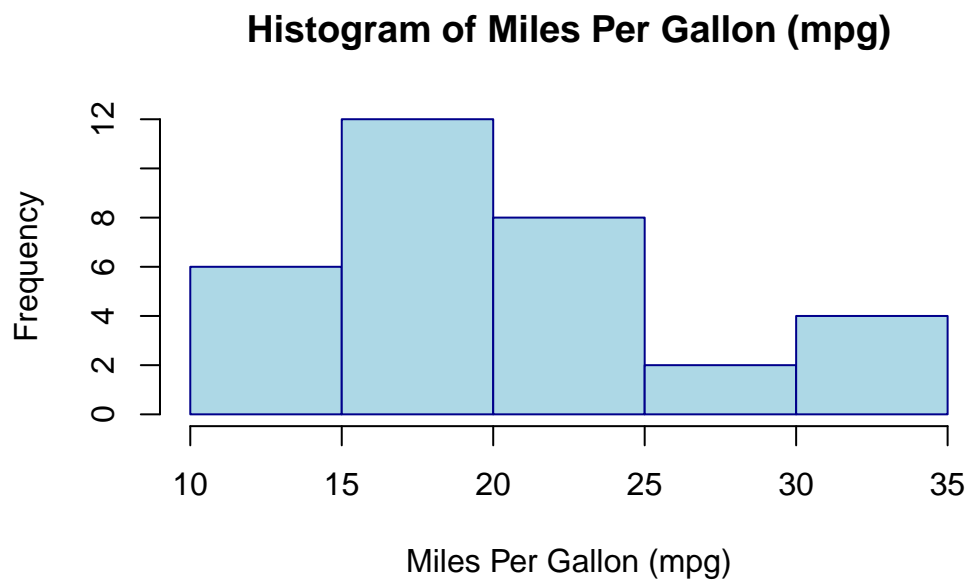
Consider the `mtcars` data set. This data is built into R, so you can access it directly; no downloads required! First, create a histogram of the variable `mpg`. Then create a boxplot of `mpg`.

1. How do these two plots compare?
2. Create a boxplots of `mpg` per number of cylinders `cyl`.

Answer

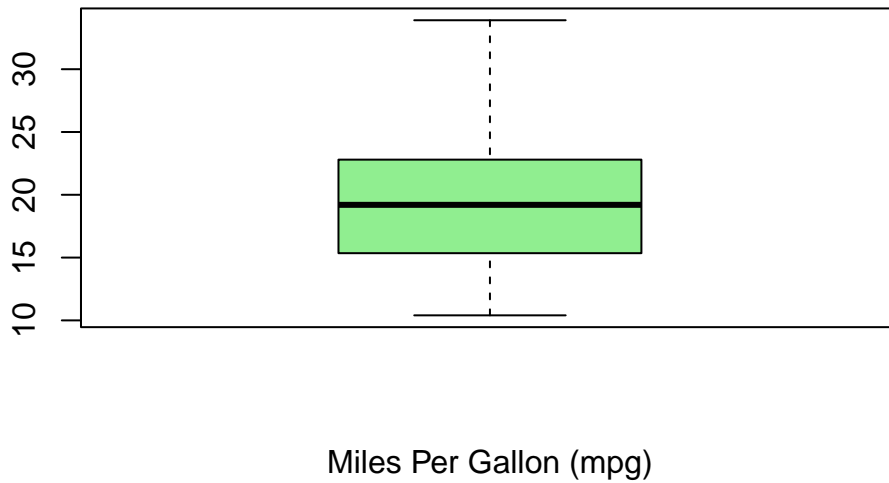
```
# Create a histogram of 'mpg'

hist(mtcars$mpg,
     main = "Histogram of Miles Per Gallon (mpg)",
     xlab = "Miles Per Gallon (mpg)",
     ylab = "Frequency",
     col = "lightblue",
     border = "darkblue")
```

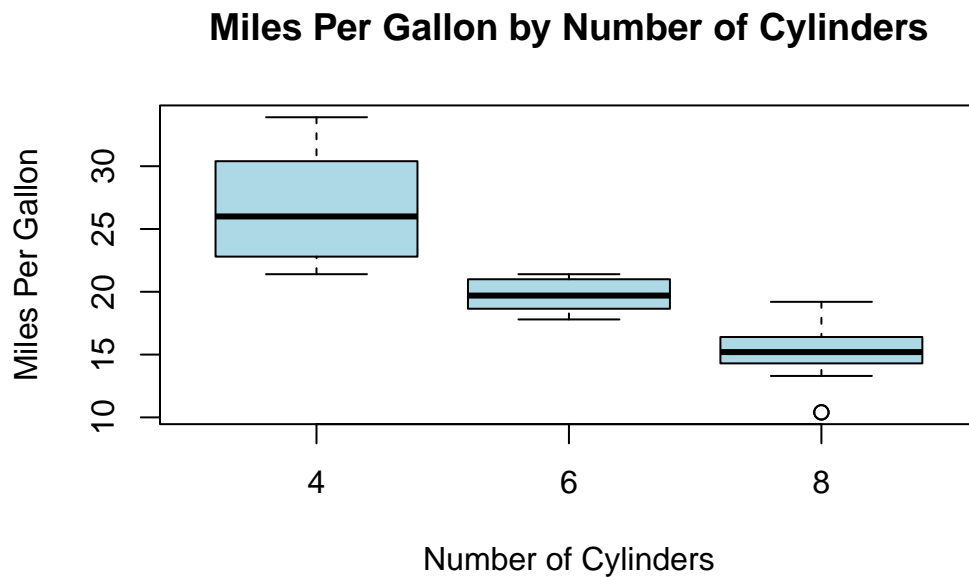


```
# Create a boxplot of 'mpg':  
  
boxplot(mtcars$mpg,  
        main = "Boxplot of Miles Per Gallon (mpg)",  
        xlab = "Miles Per Gallon (mpg)",  
        col = "lightgreen")
```

Boxplot of Miles Per Gallon (mpg)



```
# create boxplot of mpg per cylender:  
  
# Ensure 'cyl' is treated as a factor (categorical variable) for grouping  
mtcars$cyl <- as.factor(mtcars$cyl)  
  
# Create the boxplot  
boxplot(mpg ~ cyl, data = mtcars,  
        main = "Miles Per Gallon by Number of Cylinders",  
        xlab = "Number of Cylinders",  
        ylab = "Miles Per Gallon",  
        col = "lightblue")
```



1.4 Quantitative data : histograms and outliers

Generate histograms for each of the following data sets. Use the `$` command to access the individual data sets. For each histogram, add the mean to the plot using `abline()`. Do you see any potential outliers? Also calculate the five-number summary for each using R.

```
set.seed(999)
s2_data = data.frame(
  dat1 = -rchisq(1000, df = 1),
  dat2 = rchisq(1000, df = 1),
  dat3 = runif(1000),
  dat4 = rnorm(1000),
  dat5 = sample(c(rnorm(1000, mean = 2), rnorm(1000, mean = 10)), size = 1000)
)
```

Answers:

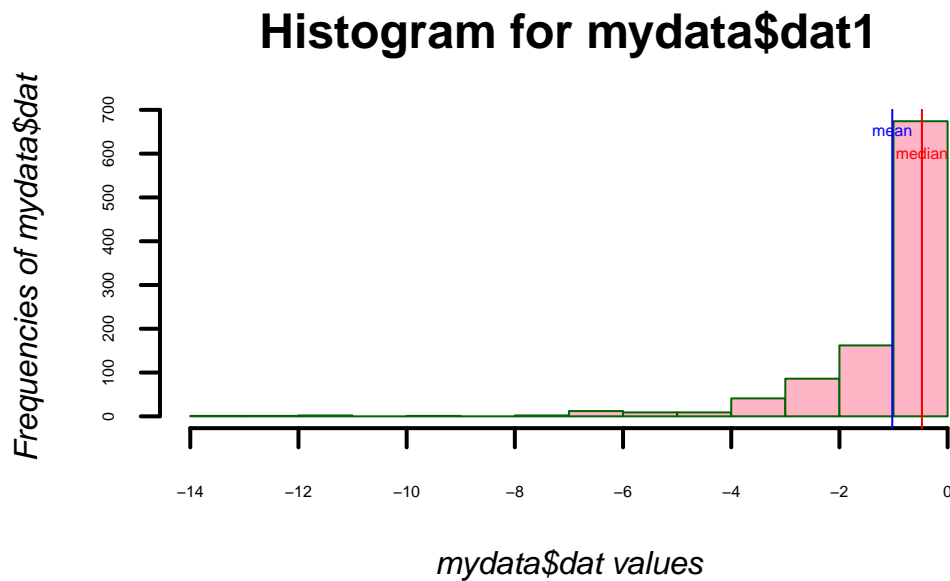
```
mean1 <- mean (s2_data$ dat1 )
mean1
```

```
[1] -1.022716
```

```
median1 <- median (s2_data$ dat1 )  
median1
```

```
[1] -0.4746566
```

```
hist1<- hist( s2_data$ dat1,  
  
             col = "pink1",                # bins color  
             main = "Histogram for mydata$dat1", # title  
             xlab = "mydata$dat values",      # x-axis label  
             ylab = "Frequencies of mydata$dat", # y-axis label  
             border = "darkgreen",          # bins border's color  
             lwd = 2,                      # border thickness  
             cex.main = 1.5,               # title size  
             cex.lab = 1,                  # axis labels size  
             cex.axis = 0.5,               # tick labels size  
             font.main = 2,                 # bold title  
             font.lab = 3)                 # italic axis labels  
  
abline(v= mean1, col="blue" )  
abline(v= median1, col="red2" )  
  
text(x = median1, y = 600, cex= 0.5, labels = "median",  
     adj = 0.5, col = "red") # Add label slightly above the line  
  
text(x = mean1, y = 650, cex= 0.5, labels = "mean",  
     adj = 0.5, col = "blue") # Add label slightly above the line
```



1.5 Percentiles

Compute the 25th, 50th, and 75th percentile for the 5 data sets in the `s2_data` data.frame. Which has the smallest median? Which as the largest?

1.6 Normal Distribution and ± 2 Standard Deviations

The normal distribution (also known as the “bell-curve”) occurs very frequently in mathematics, statistics, and the natural and social sciences. Which of the 5 data sets in the `s2_data` data.frame appears to be normally distributed?

Using this data set, find the mean and standard deviation, then calculate 1 standard deviations above, and 1 standard deviations below the mean. What percentiles do these values correspond to?

1.7 Z-Scores

Read the following description on Z-scores, then answer the question below.

5.1 Standardizing with z-Scores

Expressing a distance from the mean in standard deviations *standardizes* the performances. To **standardize** a value, we subtract the mean and then divide this difference by the standard deviation:

$$z = \frac{y - \bar{y}}{s}$$

■ NOTATION ALERT

We always use the letter z to denote values that have been standardized with the mean and standard deviation.

The values are called **standardized values**, and are commonly denoted with the letter z . Usually we just call them **z-scores**.

z -scores measure the distance of a value from the mean in standard deviations. A z -score of 2 says that a data value is two standard deviations above the mean. It doesn't matter whether the original variable was measured in fathoms, dollars, or carats; those units don't apply to z -scores. Data values below the mean have negative z -scores, so a z -score of -1.6 means that the data value was 1.6 standard deviations below the mean. Of course, regardless of the direction, the farther a data value is from the mean, the more unusual it is, so a z -score of -1.3 is more extraordinary than a z -score of 1.2.

- 15. Temperatures** A town's January high temperatures average 2°C with a standard deviation of 6° , while in July the mean high temperature is 24° and the standard deviation is 5° . In which month is it more unusual to have a day with a high temperature of 13° ? Explain.

Part 2 : The Relationship Between Two Quantitative Variables: Correlation and Regression

In this session you might use the formula of the correlation between two quantitative variables:

$$r_{xy} = \frac{1}{(n-1)s_x s_y} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

Remember that the fitted regression line is defined by the equation:

- $\hat{y} = a + bx$, or
- $\text{Response} = a + b \cdot (\text{Explanatory})$
- $\text{Residuals} = \text{observed} - \text{predicted} = y - \hat{y}$

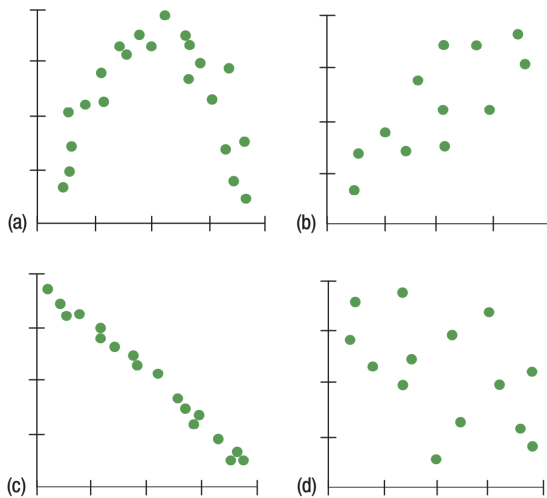
Where:

- Response: is the response variable or the dependent variable
- Explanatory: is the independent variable
- a: is the y-intercept
- b: is the slope of the regression line

You may use the following R functions: `plot()`, `lm()`, `cor()`, `abline()`. And you might need to download Lock5Data using `library(Lock5Data)`.

2.1 Describe scatterplots and correlation

Here are several scatterplots. The calculated correlations are 0.006, - 0.977, - 0.487, and 0.777. Match each scatter plot with the appropriate correlation coefficient.



Answers:

- 0.006
- 0.777

- c. • 0.977
- d. • 0.487

2.2 Create scatterplots in R

Load the data `FloridaLakes` from `library(Lock5Data)`.

1. Describe the type of each of the variables `pH`, `Calcium`, and `Alkalinity`.
2. Create a three scatter plots for each pair of the variables: `pH` vs `Calcium`, `pH` vs `Alkalinity`, and `Calcium` vs `Alkalinity`. Add the main title to each plot.
3. What is the correlation coefficient between `pH` and `Calcium`. Is it positive or negative?
4. What do these coefficients mean in the context of this data ?
5. Try to calculate the correlation coefficient between `pH` and `Calcium` without using R function.

Answers:

```
# download the data and load it into R
library(Lock5Data)
data(FloridaLakes)

# Describe the structure of the variables of FloridaLake
#str(FloridaLakes)
```

1. Describe the type of each of the variables `pH` and `Calcium`, `Alkalinity`.

The variables `pH` and `Calcium`, `Alkalinity` are quantitative variables (continuous)

2. Create a scatter plots for each of the pair of variables (add the main title of the plot).

```
# Create scatter pH vs Calcium
par(mfrow= c(1,3))

plot(FloridaLakes$pH,
     FloridaLakes$Calcium,
     main = "pH vs Calcium Scatter plot")

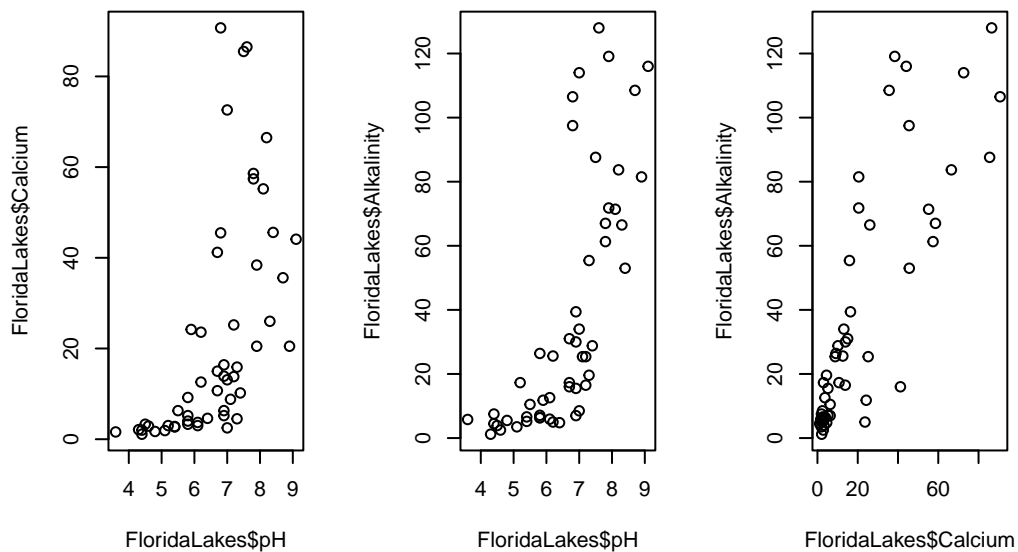
# Create scatter plot pH vs Alkalinity
```

```
plot(FloridaLakes$pH,
     FloridaLakes$Alkalinity ,
     main = "ph vs Alkalinity Scatter plot")

# Create scatter plot Calcium vs Alkalinity

plot(FloridaLakes$Calcium, FloridaLakes$Alkalinity,
     main = "Calcium vs Alkalinity Scatter plot")
```

ph vs Calcium Scatter plot ph vs Alkalinity Scatter plot Calcium vs Alkalinity Scatter plot



3. What is the correlation coefficient between pH and Calcium. Is it positive or negative.

```
# correlation

cor_pH_Cal <- cor(FloridaLakes$pH, FloridaLakes$Calcium)

cor_pH_Alkal <- cor(FloridaLakes$pH, FloridaLakes$Alkalinity)

cor_Cal_Alkal <- cor(FloridaLakes$Calcium, FloridaLakes$Alkalinity)

cor_pH_Cal
```

```
[1] 0.5771327
```

```
cor_pH_Alk
```

```
[1] 0.7191657
```

```
cor_Cal_Alk
```

```
[1] 0.8326042
```

4. What do these coefficients mean in the context of this data.

A correlation coefficient of 0.577 between pH and Calcium means that there is a linear positive association of moderate strength of 0.577.

5. calculate the correlation coefficient between pH and Calcium without using R function.

```
## Method 2 to calculate the correlation using the formula
```

```
n<- length(FloridaLakes$pH)
```

```
X<-FloridaLakes$pH
```

```
Y<- FloridaLakes$Calcium
```

```
cor_pH_Cal<- sum((X - mean(X))*(Y-mean(Y)))*(1/(n - 1))*1/sd(X)*1/sd(Y)
```

```
cor_pH_Cal
```

```
[1] 0.5771327
```