Relationships between two quantitative variables

Overview

Quick review

- z-scores, percentiles, boxplots

Relationships between two quantitative variables

- Scatter plots
- Correlation

If there is time

- Simple linear regression

Announcement: homework 2

Homework 2 is due on Gradescope on Sunday, February 1st at 11pm

```
library(SDS1000)
goto_homework(2)
```

Note: you will also need to fill out a worksheet survey on sampling data from the Gettysburg address as part of the reflection for the homework by **Monday February 2nd**

Keep attending the practice sessions!

Announcement: mini-exam

The mini-exam will **in class** **on Thursday February 5th**

- Last 30 minutes of class

You should know/understand:

1. All the symbols we have used in class represent
   - E.g., A professor believes the average height of Yale students is 69 inches. How can you write this using the symbols we discussed in class?

	Statistic	Parameter
Mean	$\bar{x}$	μ
Standard deviation	s	σ
Proportion	$\hat{p}$	π
Correlation	r	ρ
regression slope	b	β

2. Should be able to answer simple questions about the R code we have used
   - E.g., What does this R code do? `boxplot(my_vec)`

Reducing the weight of the exam to be 4% of your grade (from 6%)

- Weight of final exam going up from 30% to 32%

Review: z-scores

The z-scores tells how many standard deviations a value is from the mean

$$\text{z-score}(x_i) = \frac{x_i - \bar{x}}{s}$$

<u>Which statistic is most impressive?</u>

Z-score FGPct = 0.868

Z- score Points = 2.698

Z-score Assists = 1.965

Z-score Steals = 1.771

This is LeBron's best statistic (relative to his peers)

But should we be impressed with this?

- i.e., maybe z-scores of 2.7 are common?

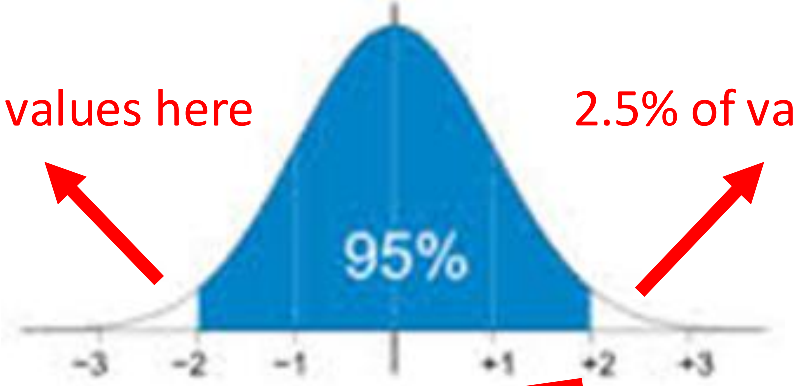
The normal pillow

Question: What percent of the pillow's fluff is within ± 2 standard deviations from the mean?

Question: If the values are normally distributed, how frequently should we expect a randomly selected z-score to be more than 2?

2.5% of values here

2.5% of values here



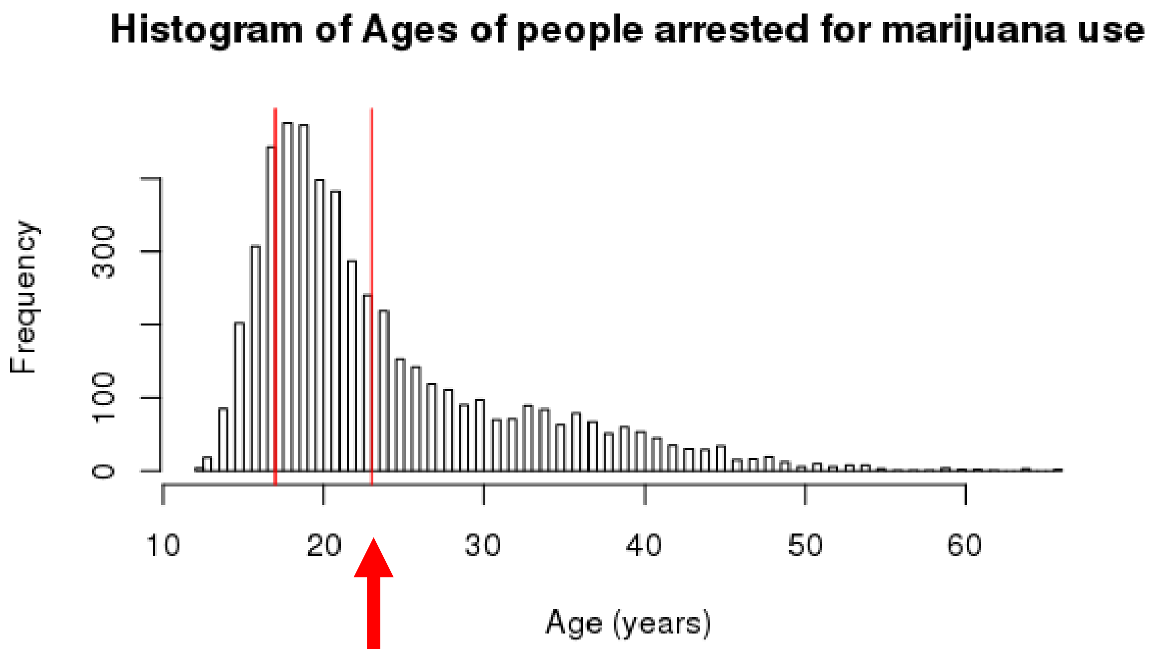
97.5% of values are less than 2

Review: quantiles (percentiles)

The **p^th percentile** is a value ***x*** which is greater than p percent of the data

- i.e., p% of the data is less than x

Let's look at the age people were arrested for using marijuana in Toronto



60th percentile value is 23

i.e., 60% of the arrested were of ages 23 or less

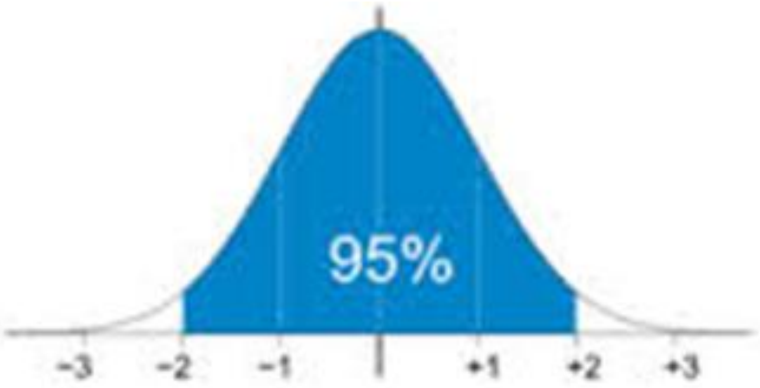
In R: quantile(Arrests$age, .6)

Preview: Quantiles of the normal distribution

We can also get quantiles of a normal distribution

Question: Suppose our data is has a (standard) normal distribution. What is the 97.5% percentile of data?

- i.e., What z-score value is greater than 97.5% of the data in a standard normal distribution?



In R: `qnorm(.975)`

The quantile universe

Five-Number Summary = (minimum, Q_1, median, Q_3, maximum)

Q_1 = 25th percentile, Q_3 = 75th percentile

Range = maximum – minimum

Interquartile range (IQR) = $Q_3 - Q_1$

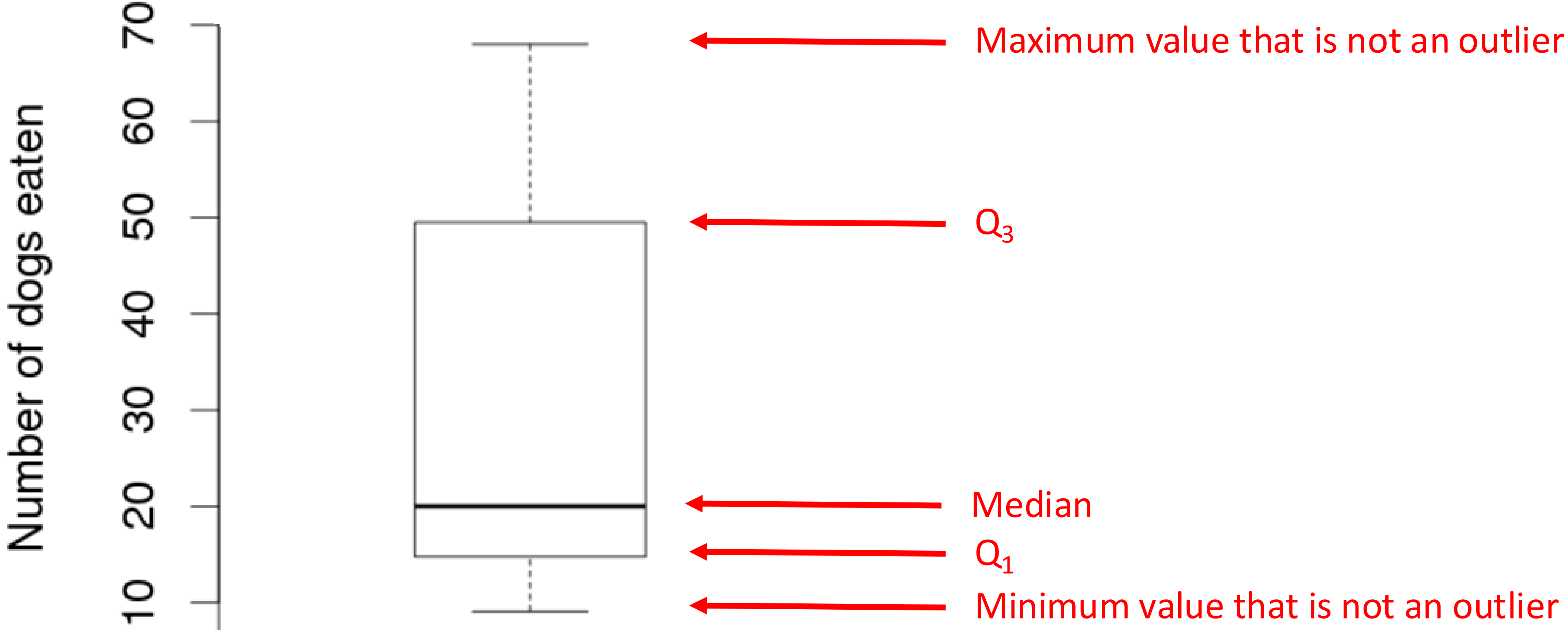
As a rule of thumb, we call a data value an **outlier** if it is:

Smaller than: $Q_1 - 1.5 * IQR$

Larger than: $Q_3 + 1.5 * IQR$

In R: `fivenum(v)`

Box plot of the number of hot dogs eaten by the men's contest winners 1980 to 2010

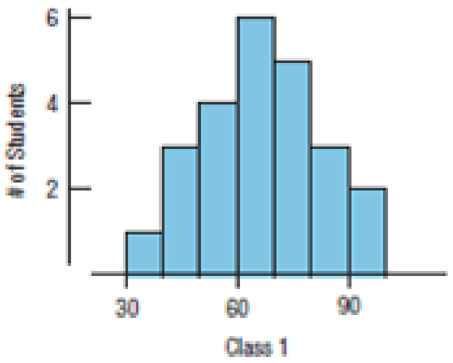


R: `boxplot(v)`

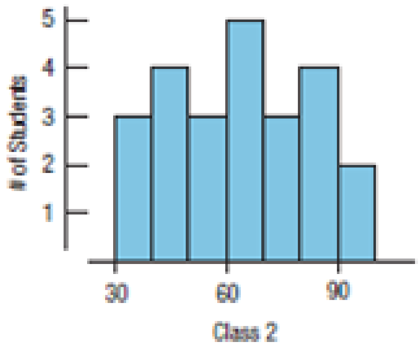
Box plots extract key statistics from histograms

Question: which box plot goes with which histogram?

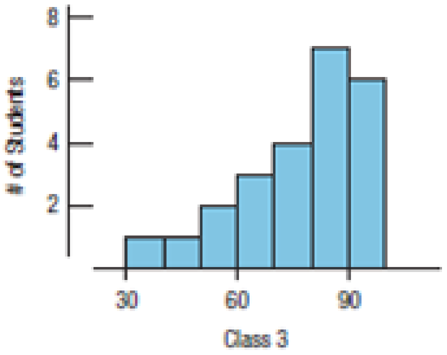
Histogram 1

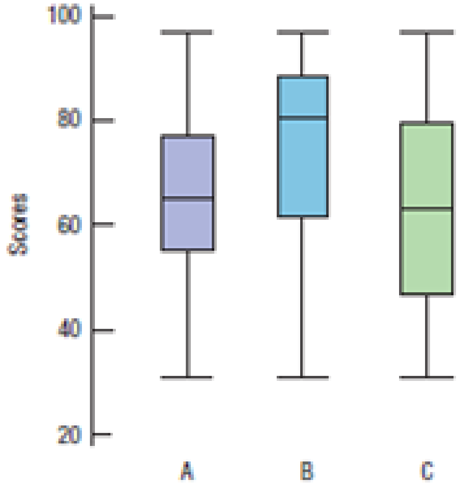


Histogram 2



Histogram 3

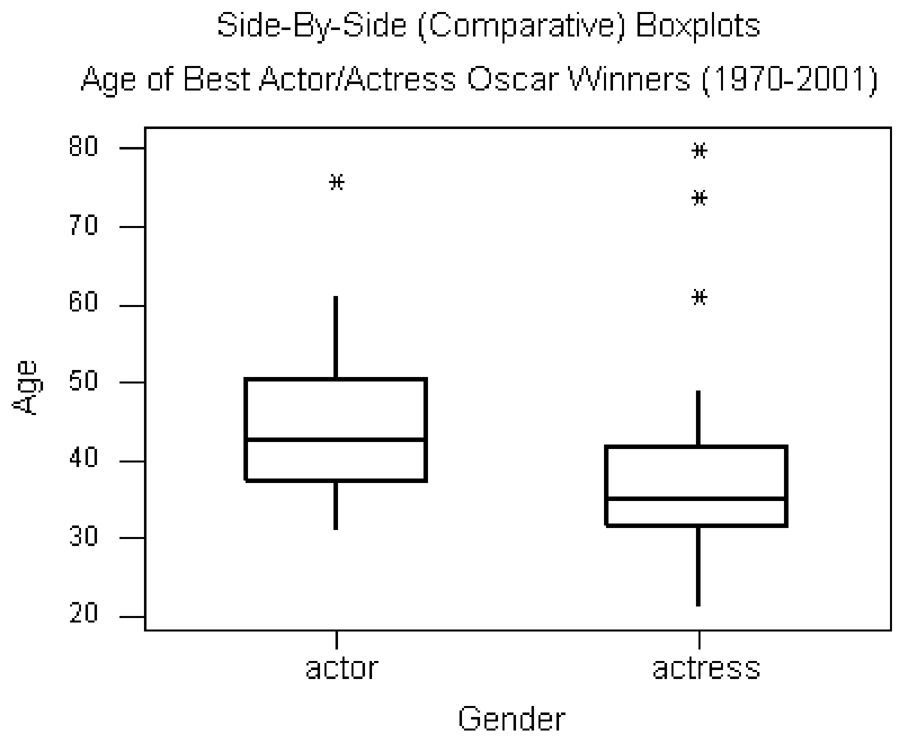




Comparing quantitative variables across categories

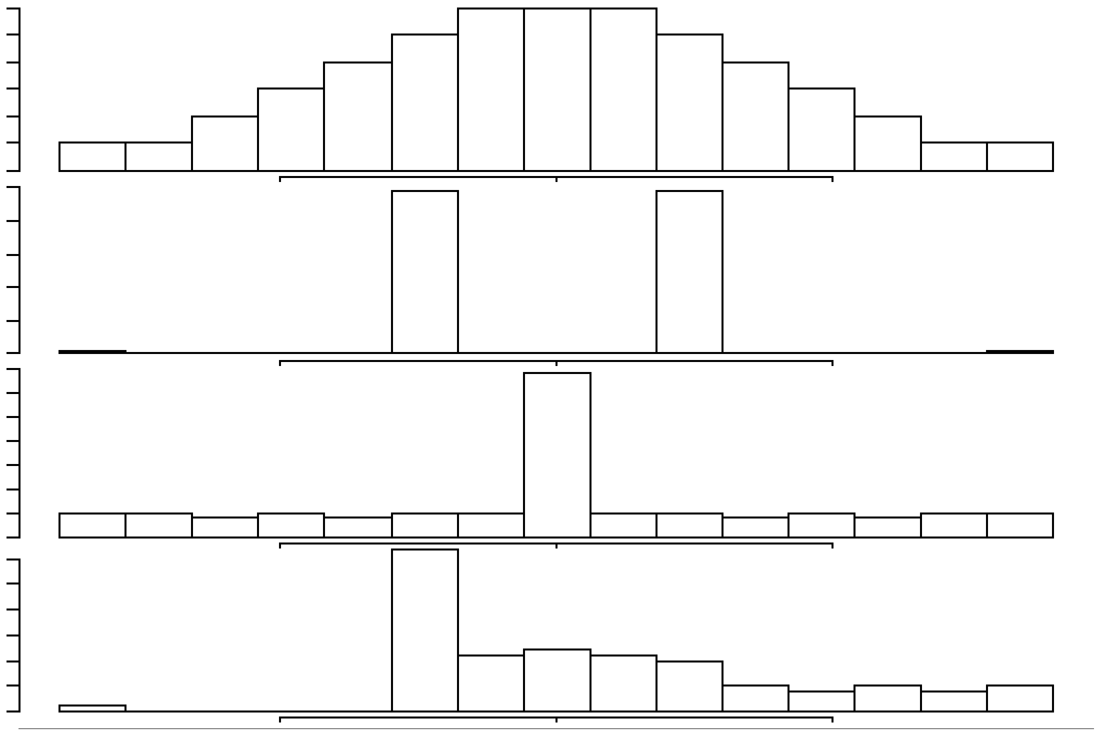
Often one wants to compare quantitative variables across categories

Side-by-Side graphs are a way to visually compare quantitative variables across different categories



boxplot(v1, v2, names = c("name 1", "name 2"), ylab = "y-axis name")

Box plots don’t capture everything



Do you think the box plots for these distributions look similar?

Questions?

Relationships between two quantitative variables

Two quantitative variables

In 1968, Joseph Fraumeni published a paper in the Journal of the National Cancer Institute that examined the relationship between smoking and different types of cancer

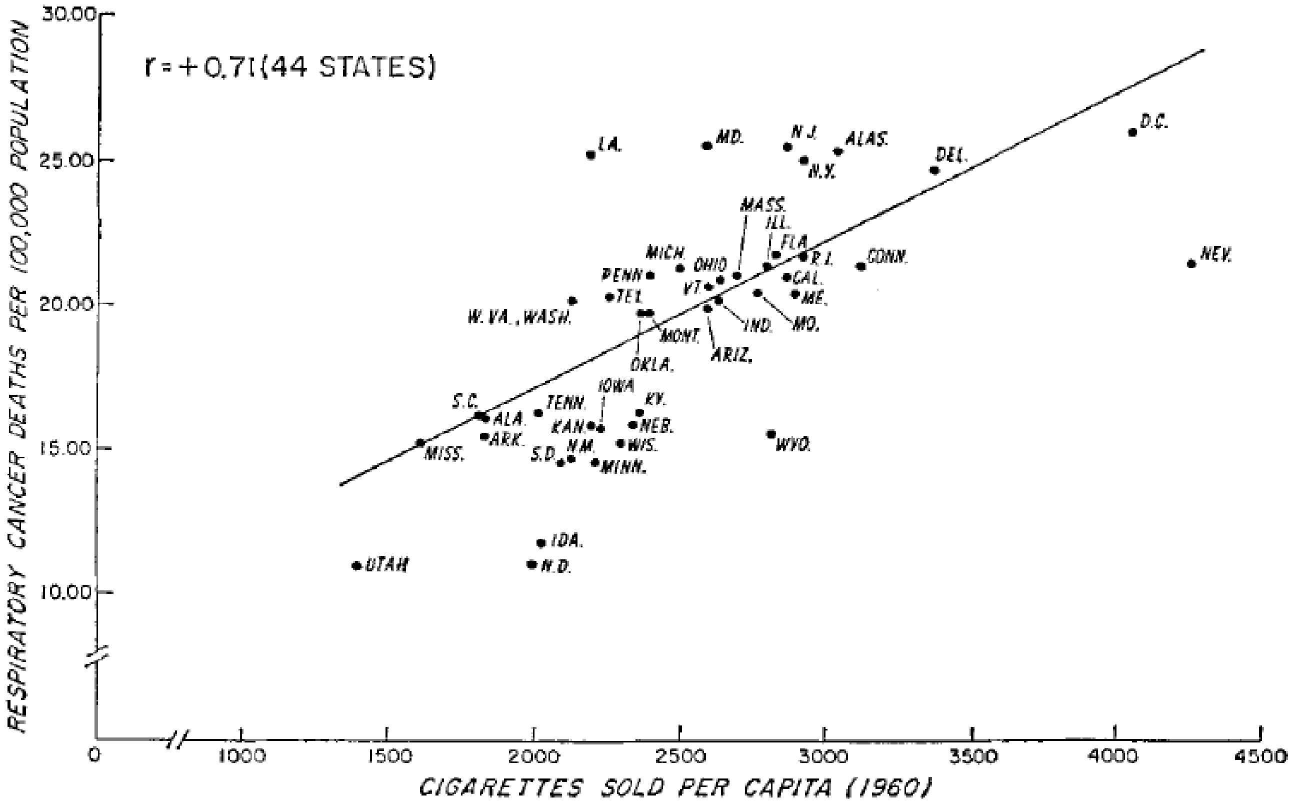
Explanatory variable

Response variable

State	Cig per capita	Bladder	Lung	Kidney	Leukemia
AL	1,820	2.9	17.05	1.59	6.15
AZ	2,582	3.52	19.8	2.75	6.61
AR	1,824	2.99	15.98	2.02	6.94
CA	2,860	4.46	22.07	2.66	7.06
CT	3,110	5.11	22.83	3.35	7.2
DE	3,360	4.78	24.55	3.36	6.45
DC	40,460	5.6	27.27	3.13	7.08

Relationship between smoking and lung cancer

TEXT-FIGURE 2.—Correlation between average annual age-adjusted death rates for respiratory tract cancer (1956–61) and *per capita* cigarette sales (1960) in 44 States.



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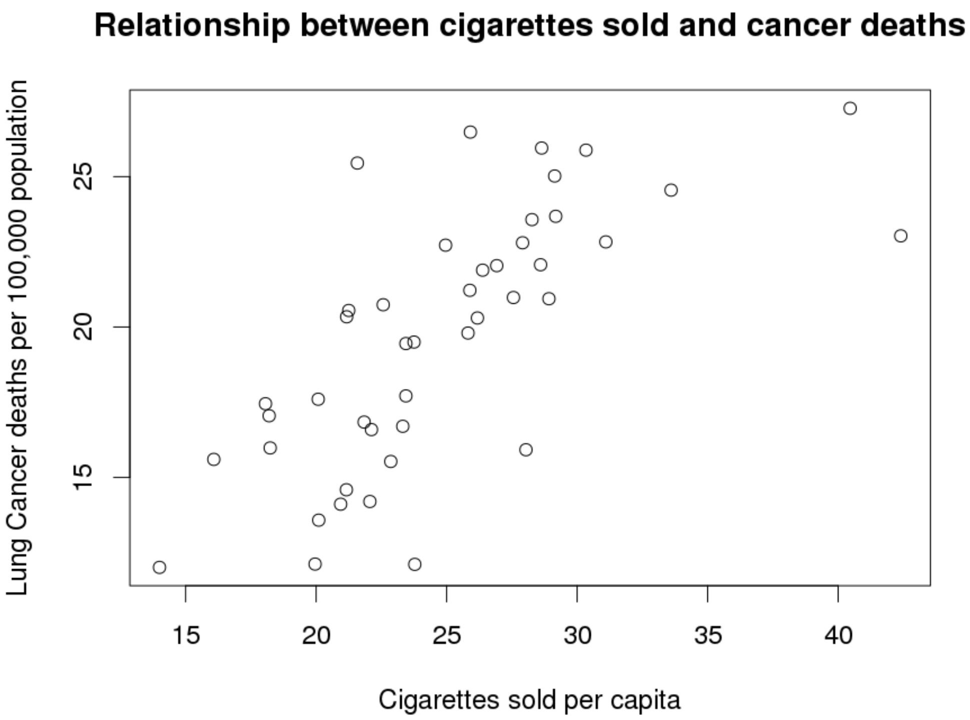
Scatterplot

A **scatterplot** graphs the relationship between two variables

- Each axis represents the value of one variables
- Each point shows the value for the two variables for a single data case



If there is an explanatory and response variable, then the explanatory variable is put on the x-axis and the response variable is put on the y-axis

R: `plot(x, y)`

Questions when looking at scatterplots

Do the points show a clear trend?

- Does it go upward or downward?
- How much scatter around the trend?

Does the trend seem be linear (follow a line) or is it curved?

Are there any outlier points?

Questions when looking at scatterplots

Do the points show a clear trend?

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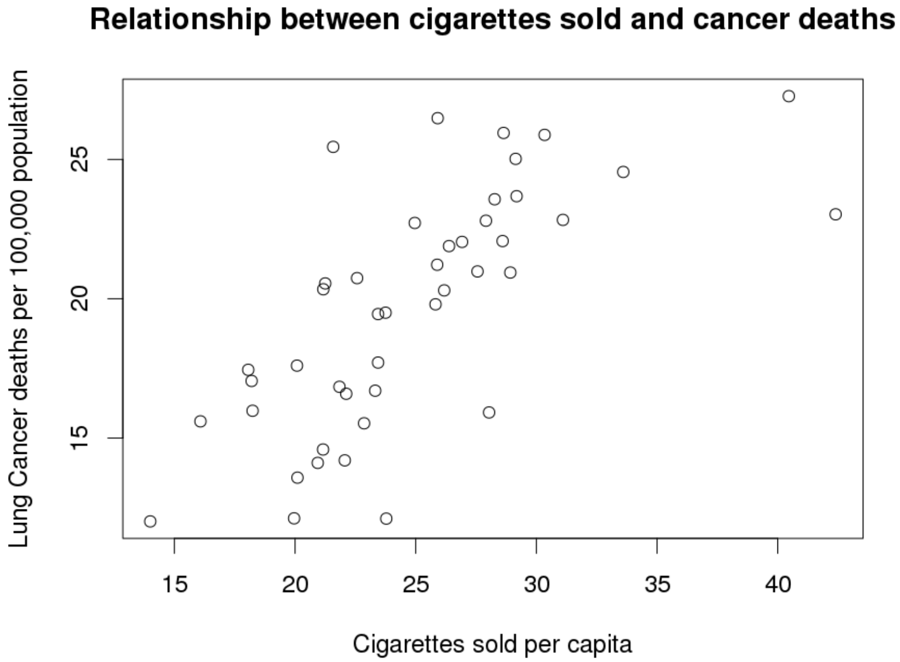
Smoking and cancer



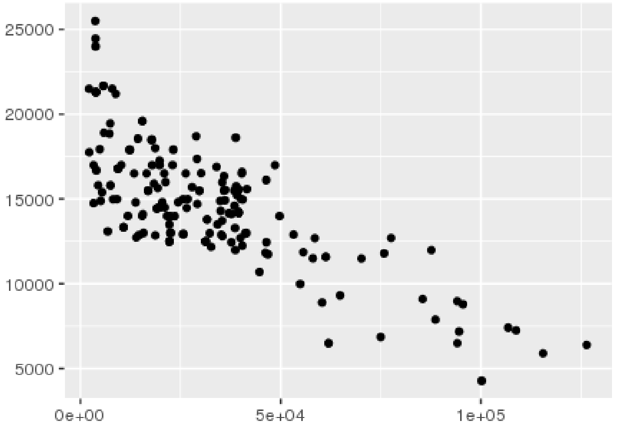
The correlation coefficient

The **correlation** is measure of the strength and direction of a linear association between two variables

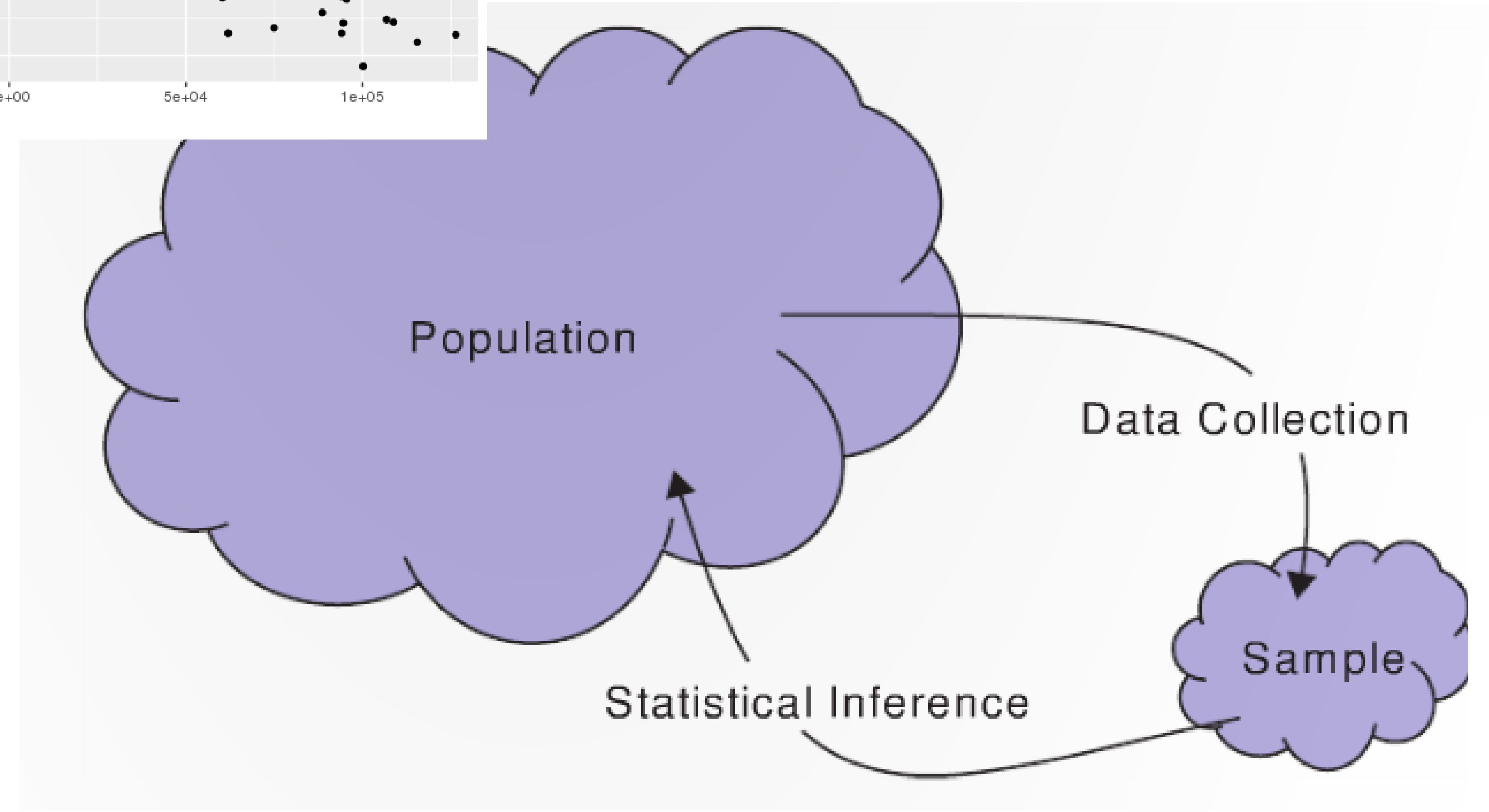
$$r = \frac{1}{(n-1)} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

- The correlation for a sample is denoted with **r**
- The correlation in the population is denoted with **ρ**
  (the Greek letter rho)

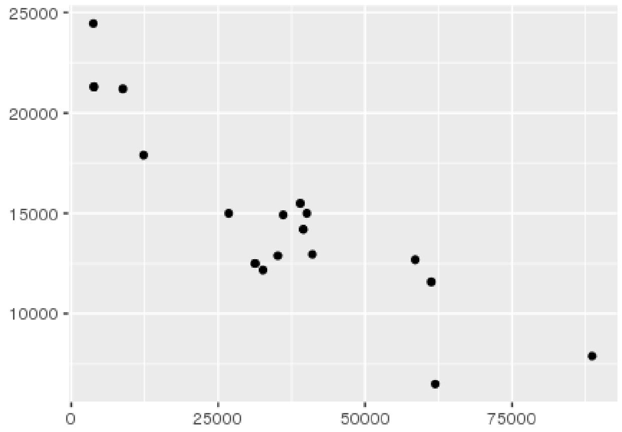
R: `cor(x, y)`



ρ parameter



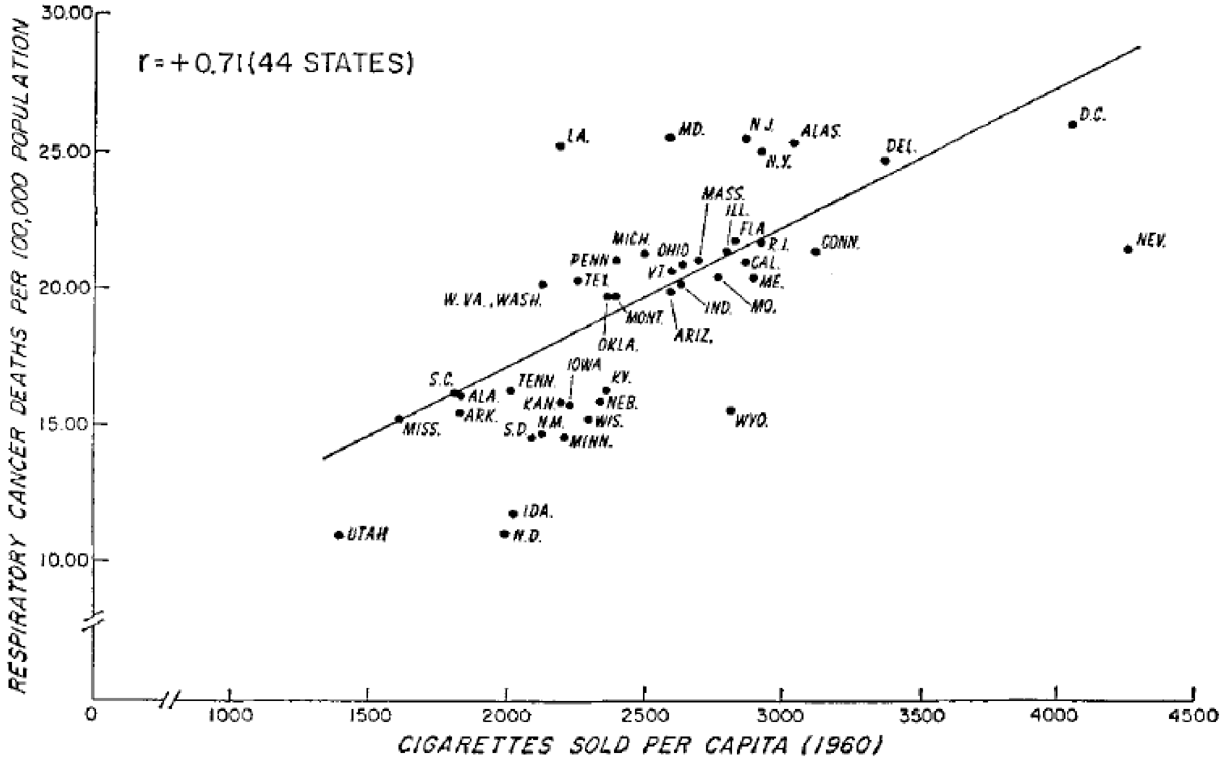
r statistic



Smoking and lung cancer correlation?

The **correlation** is measure of the strength and direction of a <u>linear association</u> between two variables

TEXT-FIGURE 2.—Correlation between average annual age-adjusted death rates for respiratory tract cancer (1956–61) and *per capita* cigarette sales (1960) in 44 States.



r = 0.71

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Properties of the correlation

Correlation as always between -1 and 1: $-1 \leq r \leq 1$

The sign of r indicates the direction of the association

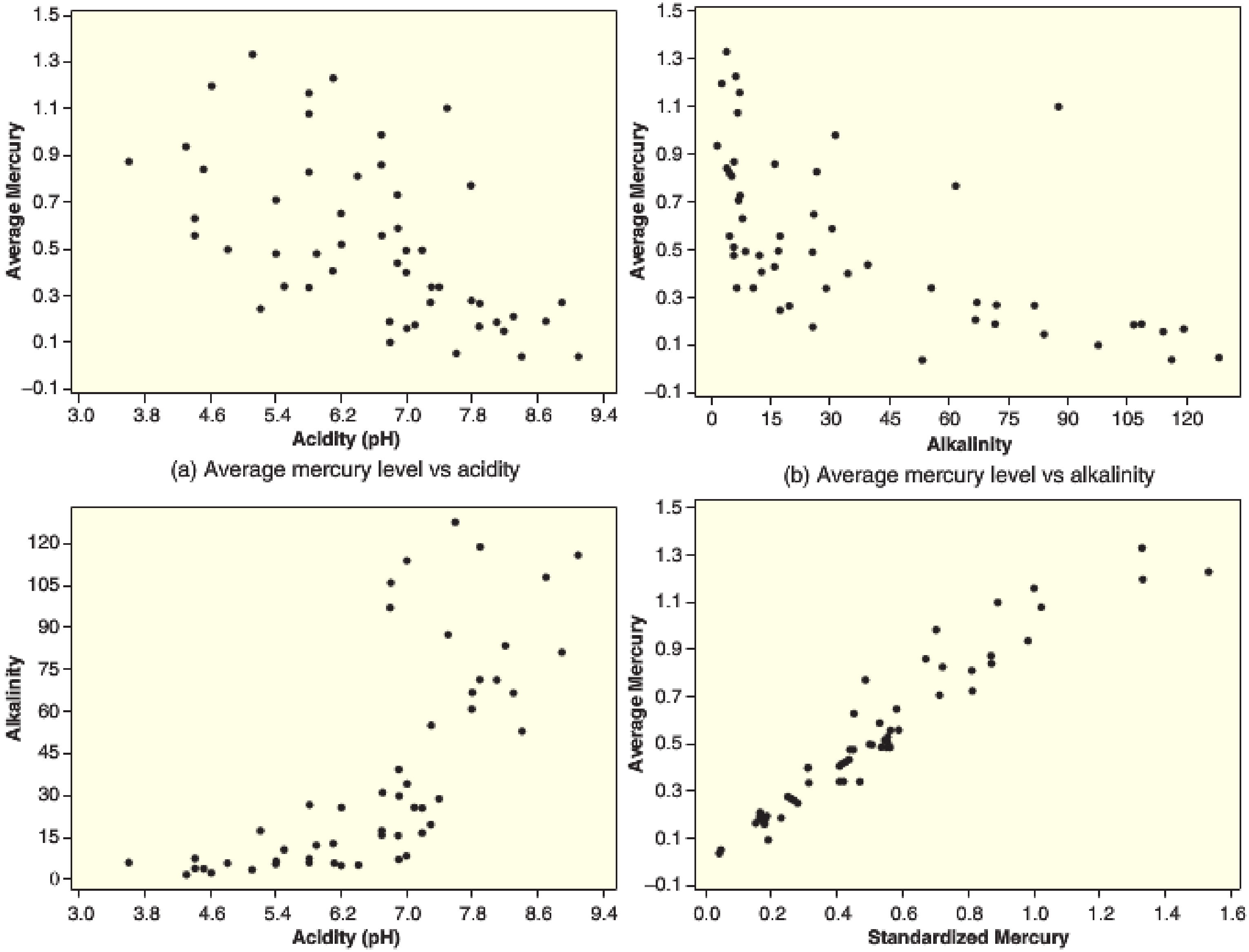
Values close to ± 1 show strong linear relationships, values close to 0 show no linear relationship

Correlation is symmetric: $r = cor(x, y) = cor(y, x)$

$$r = \frac{1}{(n-1)} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

Florida lakes

Correlation game



(a) Average mercury level vs acidity

(b) Average mercury level vs alkalinity

(c) Alkalinity vs acidity

(d) Average vs standardized mercury levels

Let’s calculate some correlations in R

```
# load the data
load("smoking_cancer.Rda")

# create a scatter plot and calculate the correlation
  plot(smoking$CIG, smoking$LUNG)
  cor(smoking$CIG, smoking$LUNG)
```

	STATE	CIG	BLAD	LUNG	KID	LEUK
1	AL	1820	2.90	17.05	1.59	6.15
2	AZ	2582	3.52	19.80	2.75	6.61
3	AR	1824	2.99	15.98	2.02	6.94
4	CA	2860	4.46	22.07	2.66	7.06
5	CT	3110	5.11	22.83	3.35	7.20

Number of cigarette’s sold per capita

Cancer rates per 10k for bladder, lung, kidney and leukemia

Try it in R

Correlation cautions

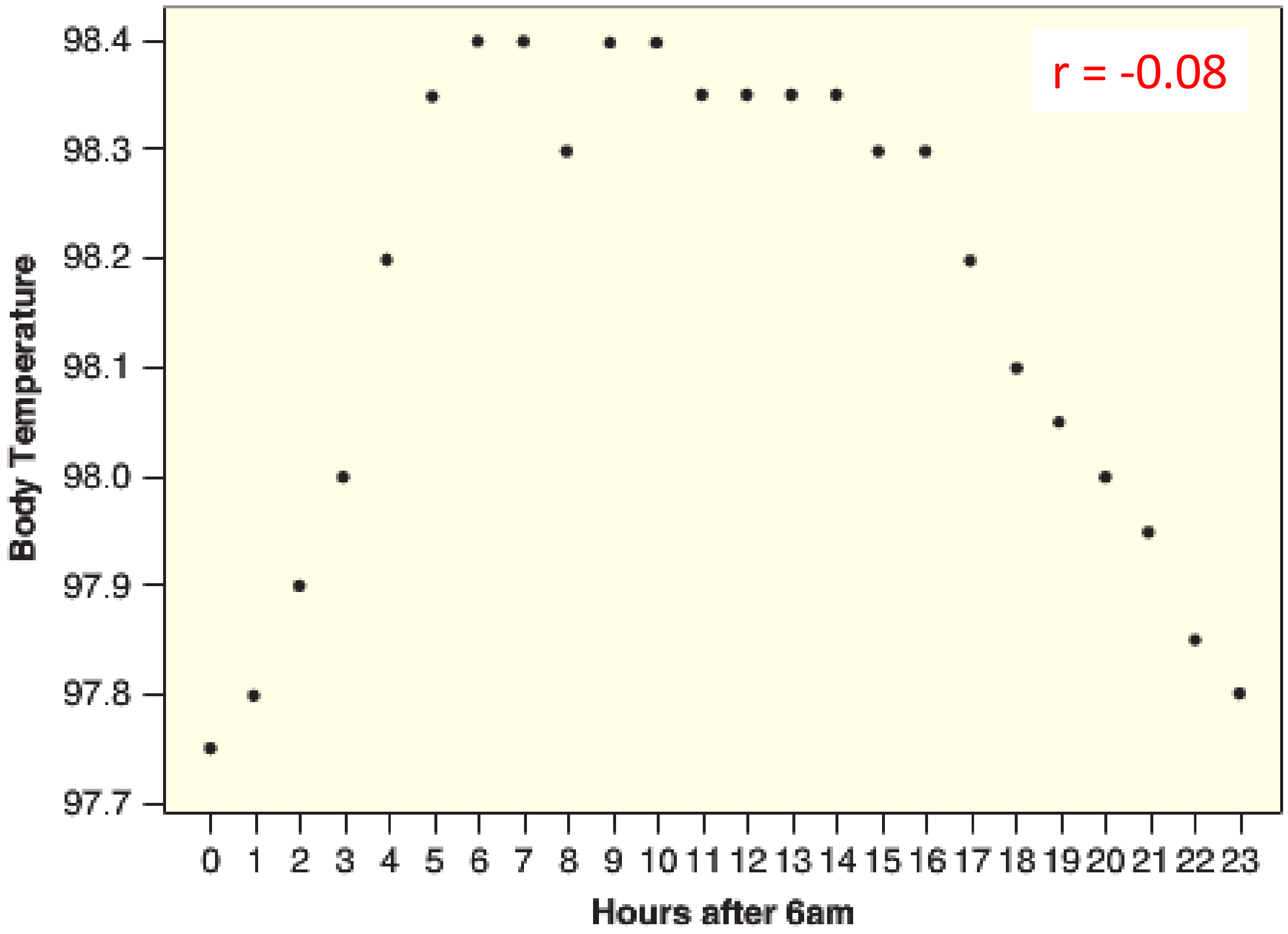
Correlation caution #1

A strong positive or negative correlation does not (necessarily) imply a cause and effect relationship between two variables

Correlation caution #2

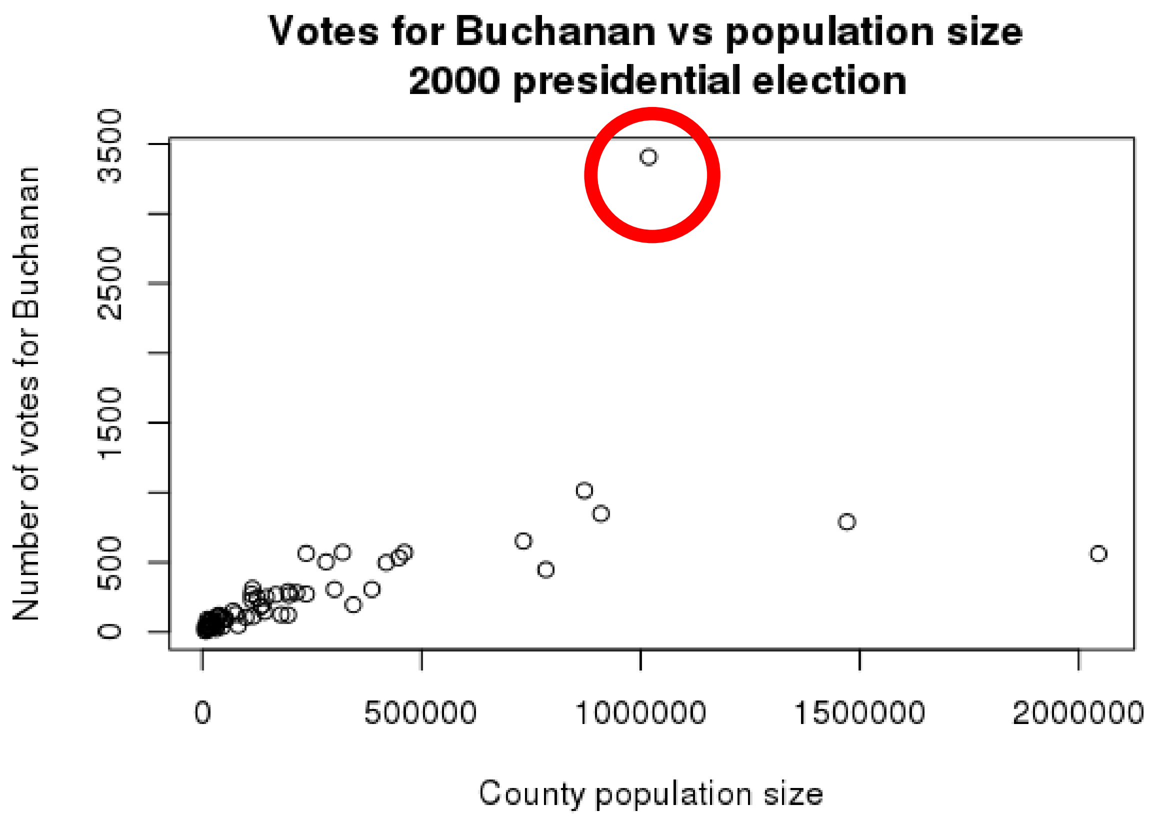
A correlation near zero does not (necessarily) mean that two variables are not associated. Correlation only measures the strength of a linear relationship

Body temperature as a function of time of the day



Correlation caution #3

Correlation can be heavily influenced by outliers. **Always plot your data!**



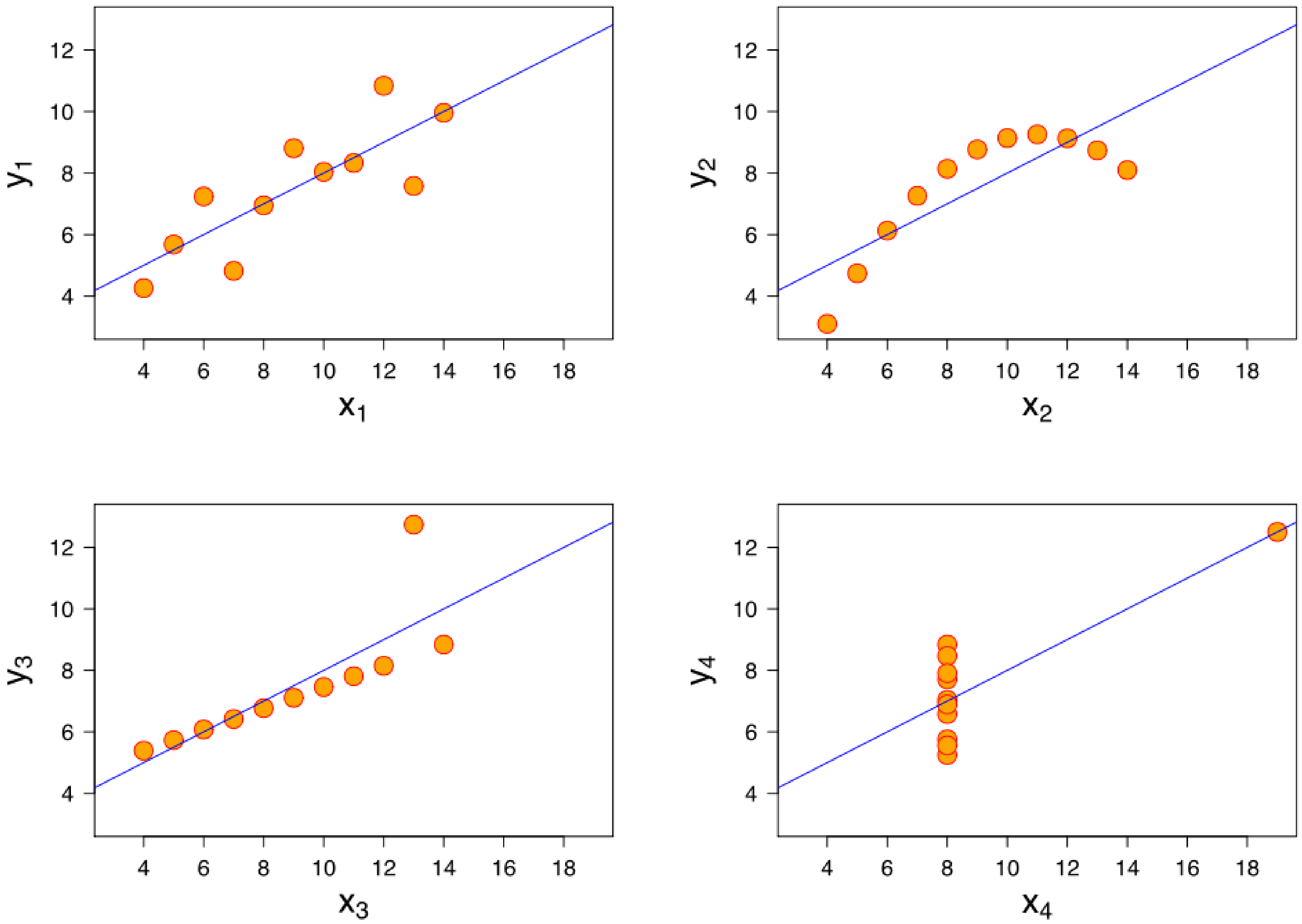
With Palm Beach

r = 0.61

Without Palm Beach

r = .78

Anscombe's quartet (r = 0.81)



Linear regression

Regression

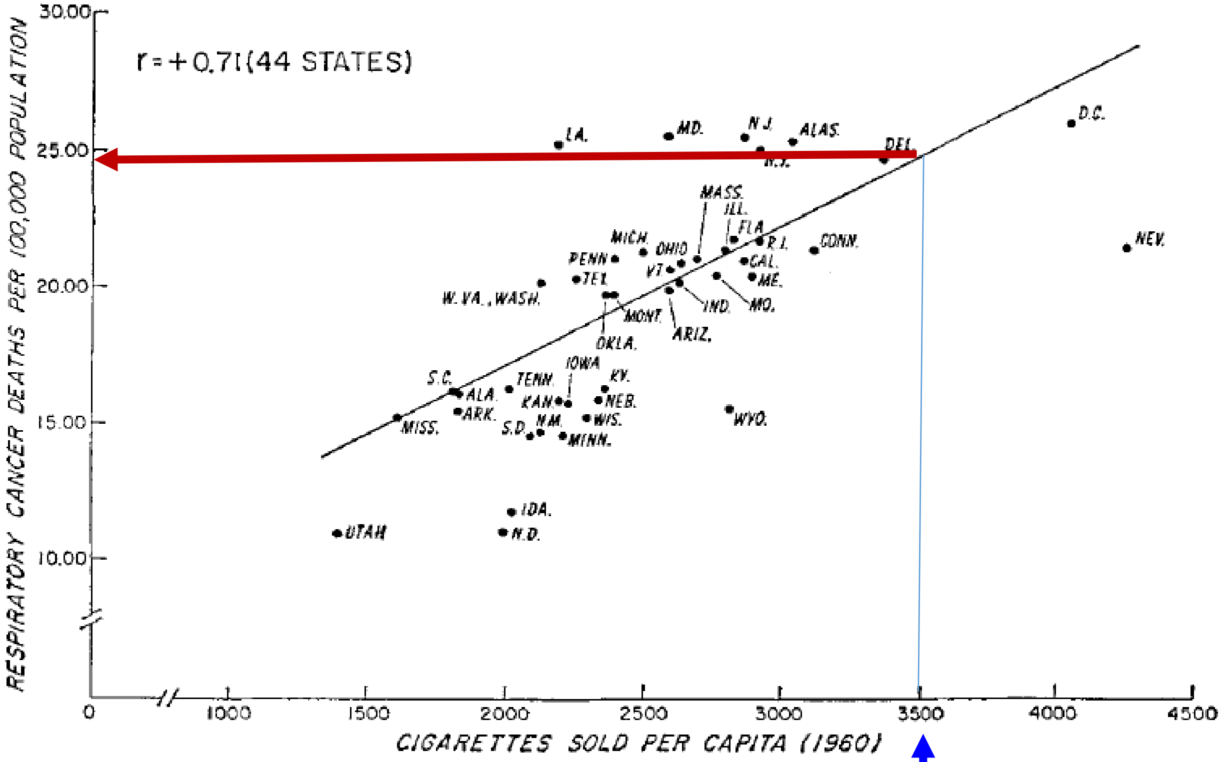
Regression is method of using one variable ***x*** *to predict* the value of a second variable ***y***

- i.e., $\hat{y} = f(x)$

In **linear regression** we fit a line to the data, called the **regression line**

Cigarette cancer regression line

TEXT-FIGURE 2.—Correlation between average annual age-adjusted death rates for respiratory tract cancer (1956–61) and *per capita* cigarette sales (1960) in 44 States.



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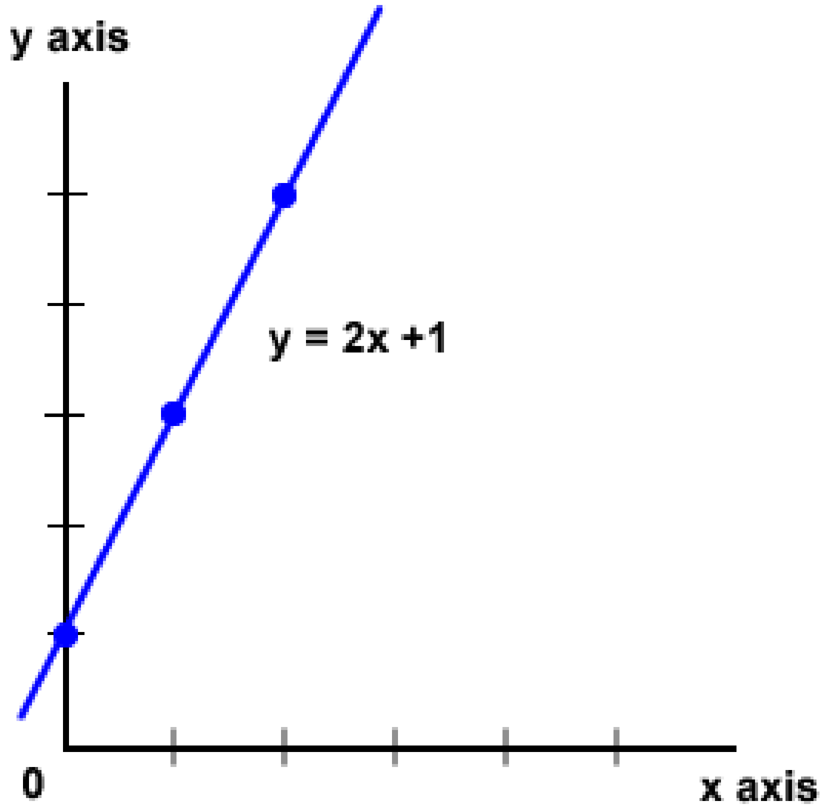
$x_i = 3500$

Equation for a line

What is the equation for a line?

$$\hat{y} = a + b \cdot x$$



Regression lines

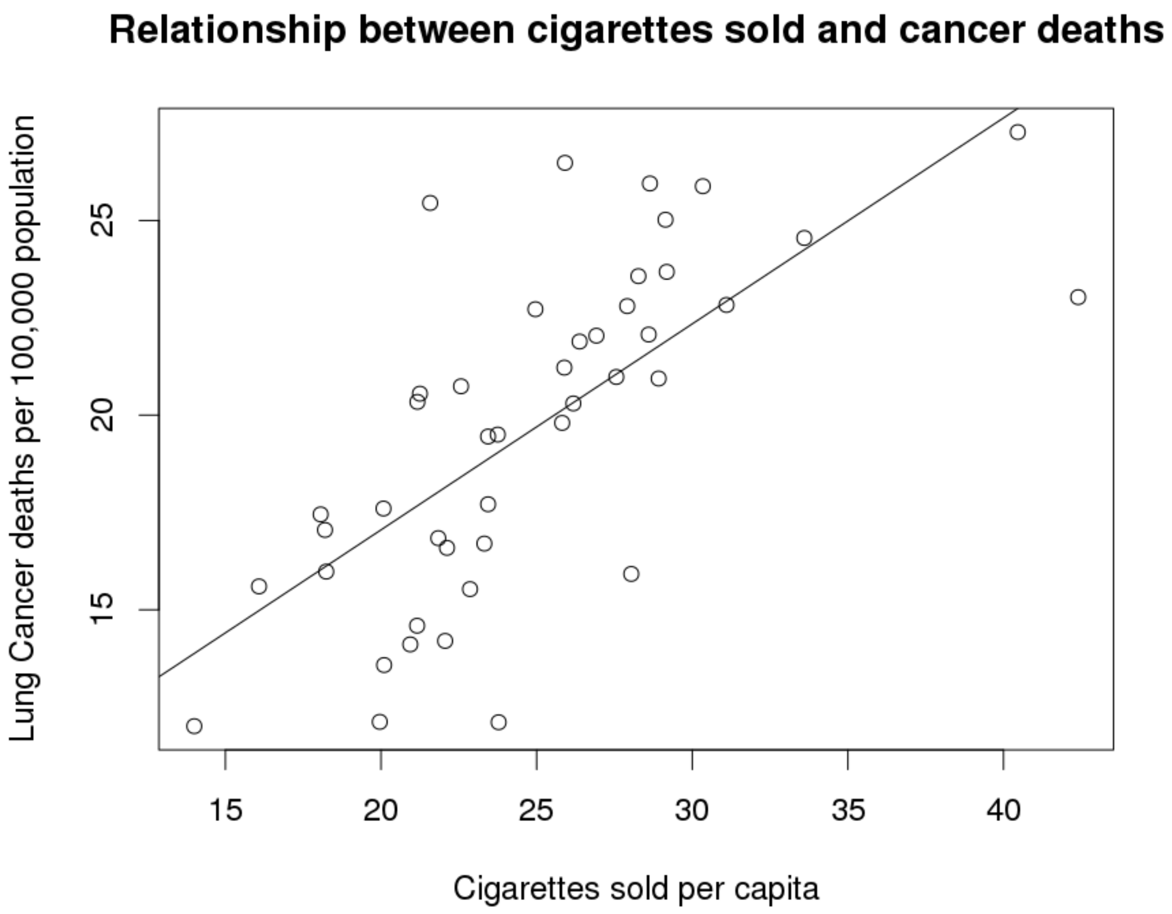
$\hat{y} \;=\; a \;+\; b \cdot x$

$Response \;= a \;+\; b \cdot Explanatory$

The slope ***b*** represents the predicted change in the response variable *y* given a one unit change in the explanatory variable *x*

The intercept ***a*** is the predicted value of the response variable y if the explanatory variable x were 0

Cancer smoking regression line



$\hat{y} = a + b \cdot x$

$a = 6.47$

$b = 0.0053$

R: `lm(y ~ x)`

$\hat{y} = 6.47 + 0.0053 \cdot x$

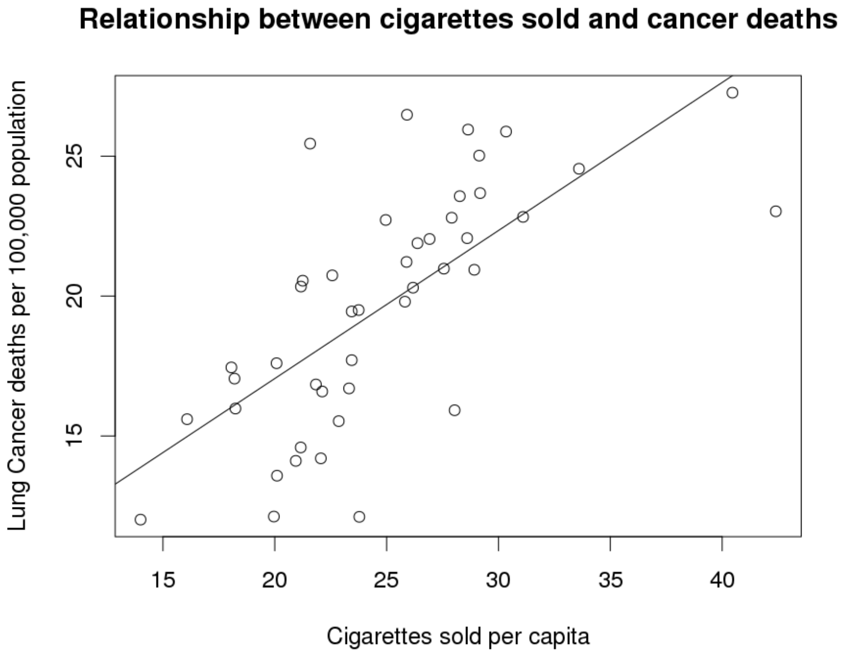
Using the regression line to make predictions

If a state sold 2,500 cigarettes per person

How many cancer deaths (per 100,000 people) would you expect?

$a = 6.47, \quad b = .0053$

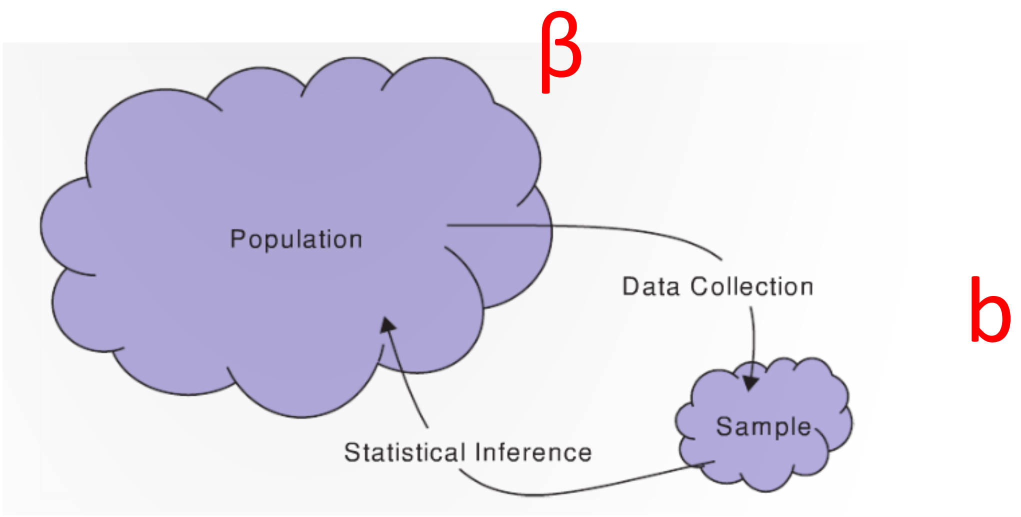
$\hat{y} = 6.47 + .0053 \cdot x$



Notation

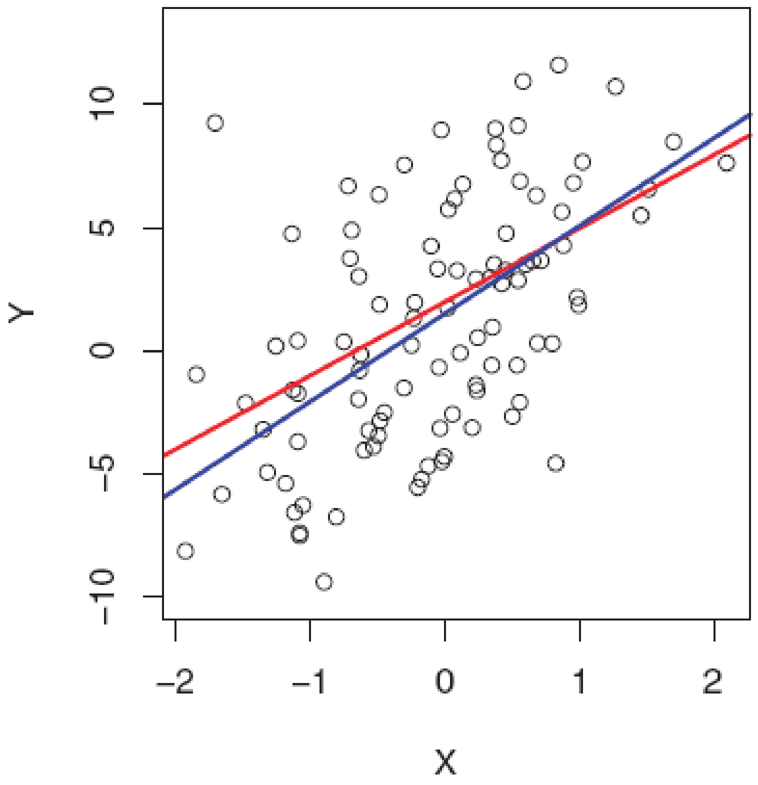
The letter **b** is typically used to denote the slope of the sample

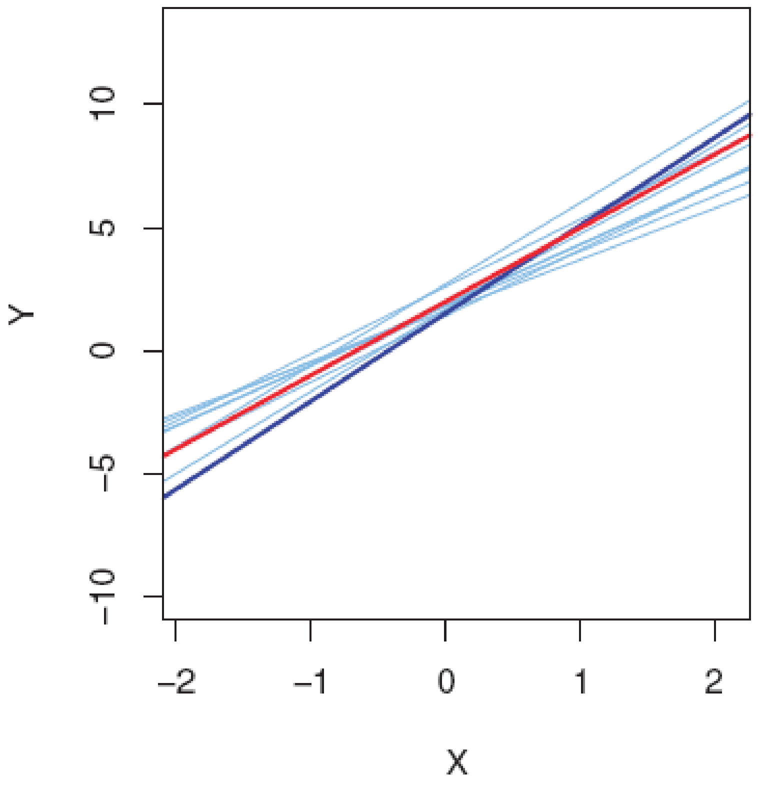
The Greek letter **β** is used to denote the slope of the population



Population: β

Sample estimates: b



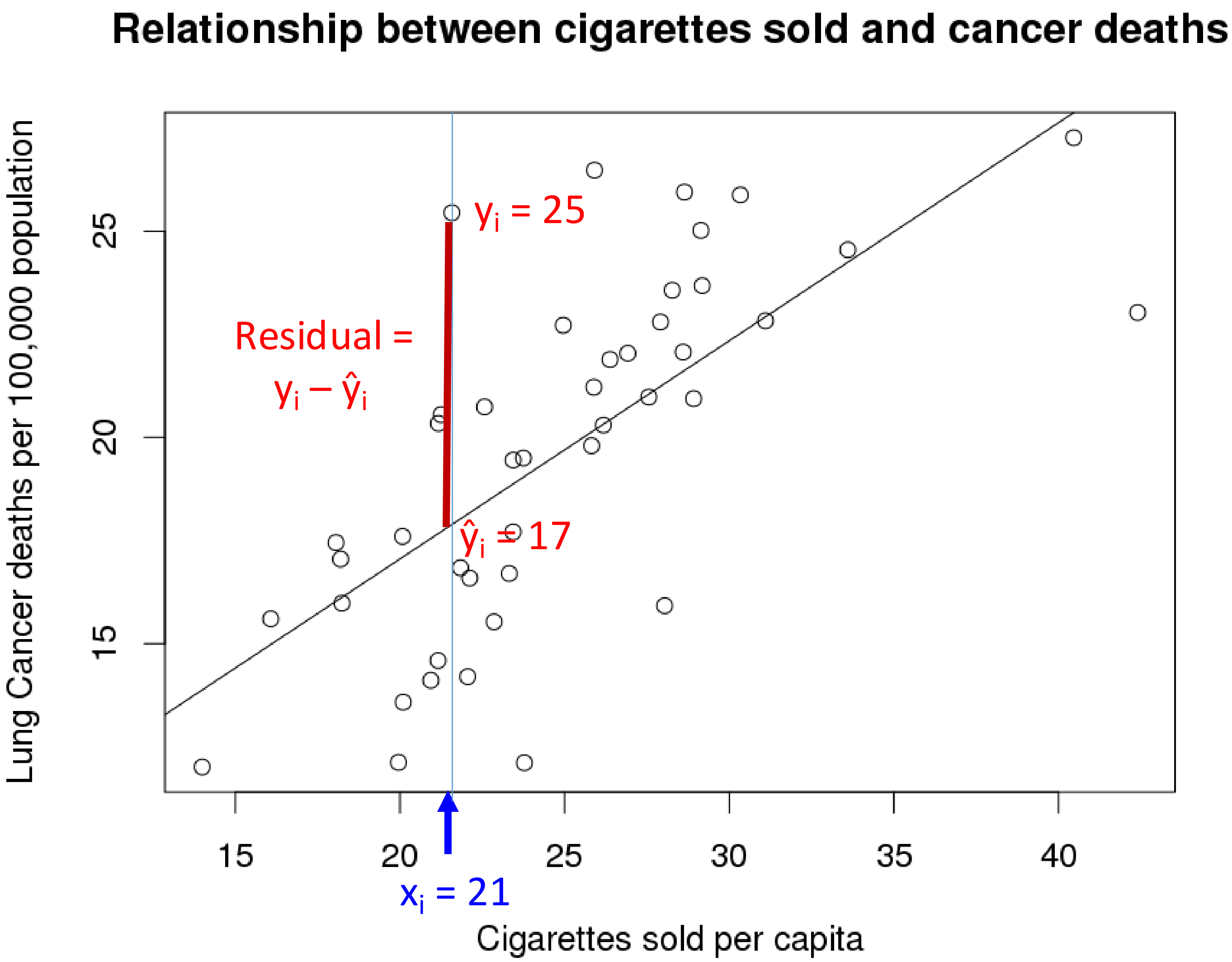


Residuals

The **residual** is the difference between an observed (y_i) and a predicted value ($\hat{y}_i$) of the response variable

$$Residual_i = Observed_i - Predicted_i = y_i - \hat{y}_i$$

Cancer smoking residuals



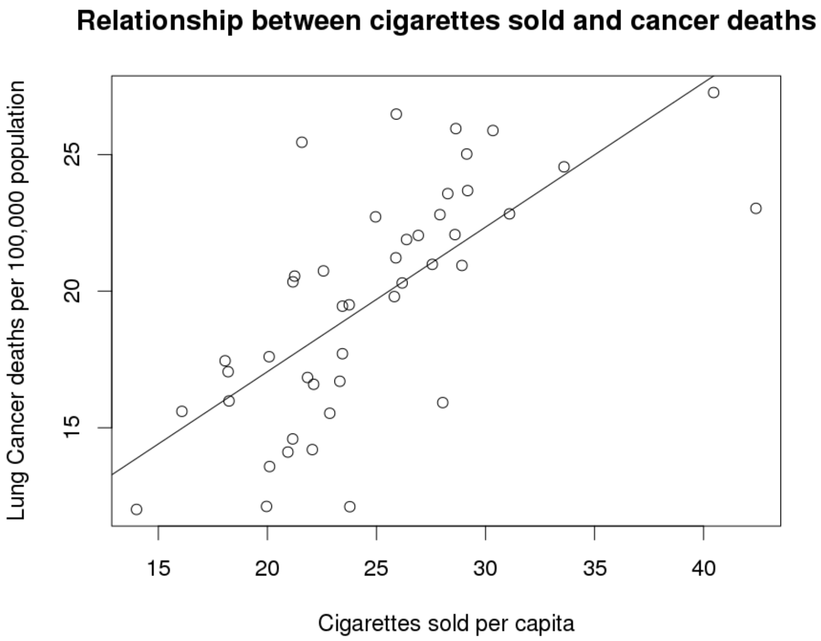
Cancer smoking residuals

$$\hat{y} = 6.47 + 0.0053 \cdot x$$

Cig per Capita (x)	Cancer obs (y)	Cancer pred ($\hat{y}$)	Residuals ($y - \hat{y}$)
1,820	17.05	16.10	0.95
2,582	19.80	20.13	-0.33
1,824	15.98	16.12	-0.14
2,860	22.07	21.60	0.47
3,110	22.83	22.93	-0.10
3,360	24.55	24.25	0.30
40,460	27.27	27.88	-0.61

Least squares line

The **least squares line**, also called **‘the line of best fit’**, is the line which <u>minimizes the sum of squared residuals</u>



[Try to find the line of best fit]

Cancer smoking residuals

Cancer obs (y)	Cancer pred ($\hat{y}$)	Residuals ($y - \hat{y}$)	Residuals^2 ($(y - \hat{y})^2$)
17.05	16.10	0.95	0.90
19.80	20.13	-0.33	0.11
15.98	16.12	-0.14	0.02
22.07	21.60	0.47	0.22
22.83	22.93	-0.10	0.01
24.55	24.25	0.30	0.09
27.27	27.88	-0.61	0.37
23.57	21.24	2.14	4.59

$$\hat{y} = a + b \cdot x$$

Find the a and b

That minimizes the sum of the squared residuals

Regression caution # 1

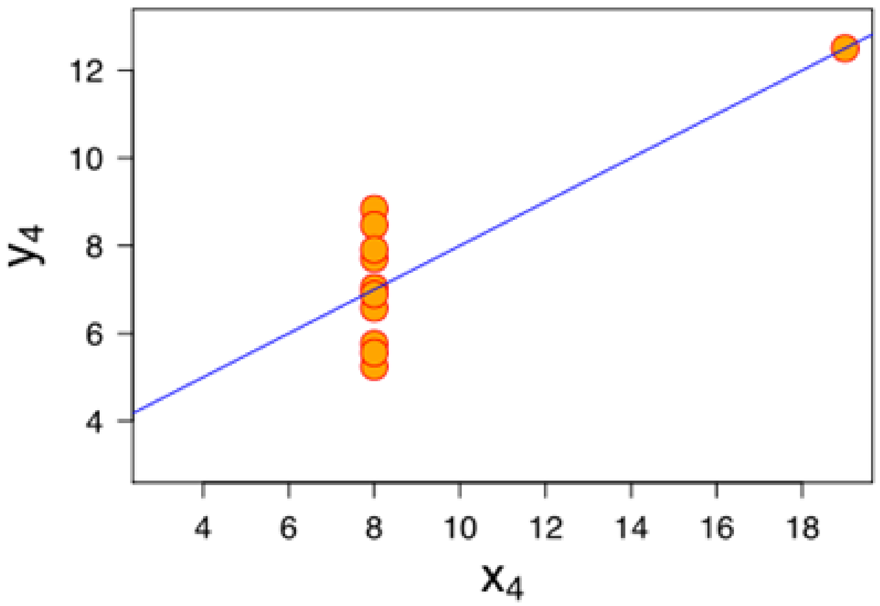
Avoid trying to apply the regression line to predict values far from those that were used to create the line. i.e., do not extrapolate too far

Regression caution # 2

Plot the data! Regression lines are only appropriate when there is a linear trend in the data.

Regression caution #3

Be aware of outliers – they can have an huge effect on the regression line



Linear regression in R

Regression lines in R

```
# load the data
  load("states_smoking.rda")

# create a scatter plot and calculate the correlation
  plot(smoking$CIG, smoking$LUNG)

# fit a regression model
  lm_fit <- lm(smoking$LUNG ~ smoking$CIG)

# get the a and b coefficients
  coef(lm_fit)

# add a regression line to the plot
  abline(lm_fit)
```

Try it in R!

Concepts for the relationship between two quantitative variables

A **scatterplot** graphs the relationship between two variables

The **correlation** is measure of the strength and direction of a linear association between two variables

- Value between -1 and 1

In **linear regression** we fit a line to the data, called the **regression line**

- We get coefficients for the slope (b) and the y-intercept (a)

The **residual** is the difference between an observed (y_i) and a predicted value ($\hat{y}_i$) of the response variable

- The regression line minimizes the sum of squared residuals