

Practice Session 2 Answers

Part 1 : Measures of central tendency & Measures of spread for quantitative data

1.1 Calculating the Sample Standard Deviation by Hand

Here is the formula for the sample mean:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

And here is the formula for the sample standard deviation:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Using the above data, perform the following calculations. Complete the table

for calculating the sample standard deviation.

Cost \$\$\$	b. Deviations ($x_i - \bar{x}$)	c. Deviations squared $(x_i - \bar{x})^2$
850		
900		
1400		
1200		
1050		
750		
1250		
1050		
565		
1000		
a. mean = _____		

d. Sum of squared deviations $\sum_{i=1}^n (x_i - \bar{x})^2 =$ _____

e. Sum of squared deviations divided by $n - 1$: $\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} =$ _____

$$\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

f. Take the square root to get s: $=$ _____ $= s$

1.2 Five-Number Summary

Using the numbers from the previous exercise, do the following:

- Find the 5-number summary (minimum, Q1, median, Q3, maximum)
- Check your work using R functions

Answers

```
v<- c( 565, 750, 850, 900, 1000, 1050, 1050, 1200, 1250, 1400 )  
v
```

```
[1] 565 750 850 900 1000 1050 1050 1200 1250 1400
```

```
## you can use function fivenum()  
  
fivenum(v)
```

```
[1] 565 850 1025 1200 1400
```

```
## you can use function summary()  
  
summary(v)
```

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
565.0	862.5	1025.0	1001.5	1162.5	1400.0

1.2 Boxplots

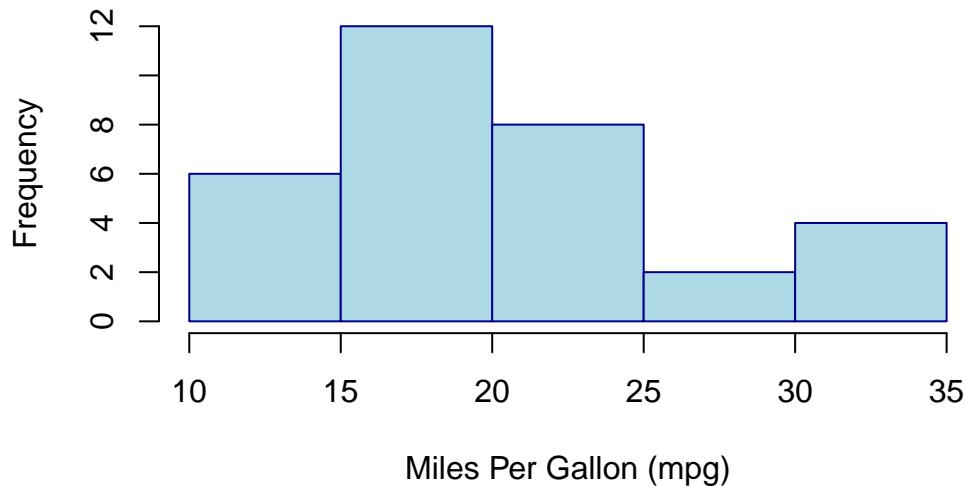
Consider the `mtcars` data set. This data is built into R, so you can access it directly; no downloads required! First, create a histogram of the variable `mpg`. Then create a boxplot of `mpg`.

1. How do these two plots compare?
2. Create a boxplots of `mpg` per number of cylinders `cyl`.

Answer

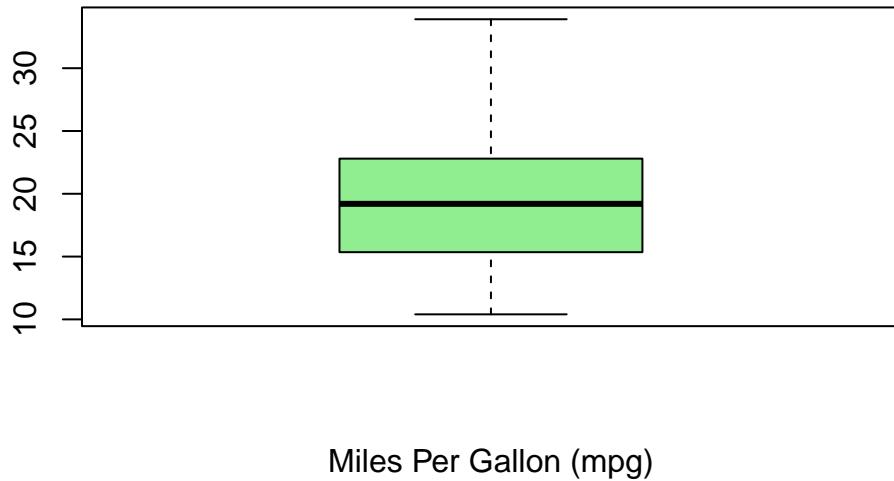
```
# Create a histogram of 'mpg'  
  
hist(mtcars$mpg,  
      main = "Histogram of Miles Per Gallon (mpg)",  
      xlab = "Miles Per Gallon (mpg)",  
      ylab = "Frequency",  
      col = "lightblue",  
      border = "darkblue")
```

Histogram of Miles Per Gallon (mpg)



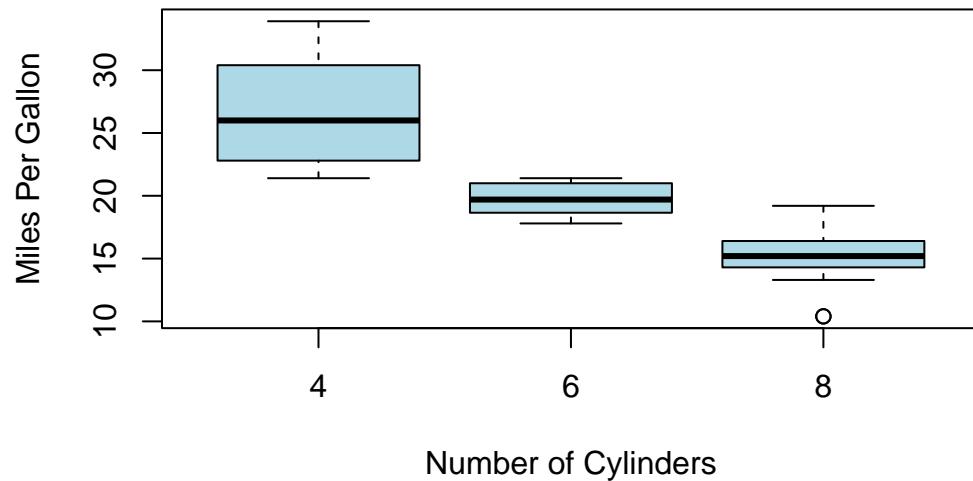
```
# Create a boxplot of 'mpg':  
  
boxplot(mtcars$mpg,  
        main = "Boxplot of Miles Per Gallon (mpg)",  
        xlab = "Miles Per Gallon (mpg)",  
        col = "lightgreen")
```

Boxplot of Miles Per Gallon (mpg)



```
# create boxplot of mpg per cylinder:  
  
# Ensure 'cyl' is treated as a factor (categorical variable) for grouping  
mtcars$cyl <- as.factor(mtcars$cyl)  
  
# Create the boxplot  
boxplot(mpg ~ cyl, data = mtcars,  
        main = "Miles Per Gallon by Number of Cylinders",  
        xlab = "Number of Cylinders",  
        ylab = "Miles Per Gallon",  
        col = "lightblue")
```

Miles Per Gallon by Number of Cylinders



1.3 Quantitative data : histograms and outliers

Generate histograms for each of the following data sets. Use the `$` command to access the individual data sets. For each histogram, add the mean to the plot using `abline()`. Do you see any potential outliers? Also calculate the five-number summary for each using R.

```
set.seed(999)
s2_data = data.frame(
  dat1 = -rchisq(1000, df = 1),
  dat2 = rchisq(1000, df = 1),
  dat3 = runif(1000),
  dat4 = rnorm(1000),
  dat5 = sample(c(rnorm(1000, mean = 2), rnorm(1000, mean = 10)), size = 1000)
)
```

Answers:

```
mean1 <- mean (s2_data$ dat1 )
mean1
```

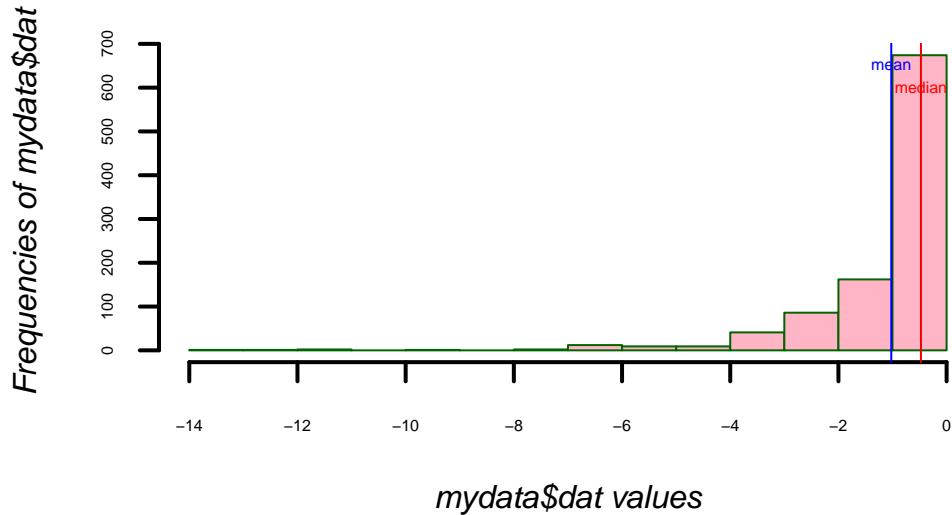
```
[1] -1.022716
```

```
median1 <- median (s2_data$ dat1 )  
median1
```

```
[1] -0.4746566
```

```
hist1<- hist( s2_data$ dat1,  
  
              col = "pink1",                                # bins color  
              main = "Histogram for mydata$dat1",          # title  
              xlab = "mydata$dat values",                   # x-axis label  
              ylab = "Frequencies of mydata$dat",           # y-axis label  
              border = "darkgreen",                          # bins border's color  
              lwd = 2,                                     # border thickness  
              cex.main = 1.5,                               # title size  
              cex.lab = 1,                                  # axis labels size  
              cex.axis = 0.5,                               # tick labels size  
              font.main = 2,                               # bold title  
              font.lab = 3)                                # italic axis labels  
  
abline(v= mean1, col="blue" )  
abline(v= median1, col="red2" )  
  
text(x = median1, y = 600,  cex= 0.5, labels = "median",  
      adj = 0.5, col = "red") # Add label slightly above the line  
  
text(x = mean1, y = 650,  cex= 0.5, labels = "mean",  
      adj = 0.5, col = "blue") # Add label slightly above the line
```

Histogram for mydata\$dat1



1.4 Percentiles

Compute the 25th, 50th, and 75th percentile for the 5 data sets in the `s2_data` data.frame. Which has the smallest median? Which has the largest?

1.5 Normal Distribution and +/- 2 Standard Deviations

The normal distribution (also known as the “bell-curve”) occurs very frequently in mathematics, statistics, and the natural and social sciences. Which of the 5 data sets in the `s2_data` data.frame appears to be normally distributed?

Using this data set, find the mean and standard deviation, then calculate 1 standard deviations above, and 1 standard deviations below the mean. What percentiles do these values correspond to?

Note: approximately, there is 68% of data between 1 standard deviations above, and 1 standard deviations below the mean, in the normal distribution.

```
quantile(s2_data$ dat4, c(0.16, 0.84) )
```

```
16%          84%
-0.9581738  0.9486344
```

1.6 Z-Scores

Read the following description on Z-scores, then answer the question below.

5.1 Standardizing with z-Scores

Expressing a distance from the mean in standard deviations *standardizes* the performances. To **standardize** a value, we subtract the mean and then divide this difference by the standard deviation:

$$z = \frac{y - \bar{y}}{s}$$

NOTATION ALERT

We always use the letter z to denote values that have been standardized with the mean and standard deviation.

The values are called **standardized values**, and are commonly denoted with the letter z . Usually we just call them **z -scores**.

z -scores measure the distance of a value from the mean in standard deviations. A z -score of 2 says that a data value is two standard deviations above the mean. It doesn't matter whether the original variable was measured in fathoms, dollars, or carats; those units don't apply to z -scores. Data values below the mean have negative z -scores, so a z -score of -1.6 means that the data value was 1.6 standard deviations below the mean. Of course, regardless of the direction, the farther a data value is from the mean, the more unusual it is, so a z -score of -1.3 is more extraordinary than a z -score of 1.2.

- 15. Temperatures** A town's January high temperatures average 2°C with a standard deviation of 6° , while in July the mean high temperature is 24° and the standard deviation is 5° . In which month is it more unusual to have a day with a high temperature of 13° ? Explain.

Part 2 : The Relationship Between Two Quantitative Variables: Correlation and Regression

In this session you might use the formula of the correlation between two quantitative variables:

$$r_{xy} = \frac{1}{(n-1)s_x s_y} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$$

Remember that the fitted regression line is defined by the equation:

- $\hat{y} = a + bx$, or
- Response = $a + b \cdot$ (Explanatory)
- Residuals = observed - predicted = $y - \hat{y}$

Where:

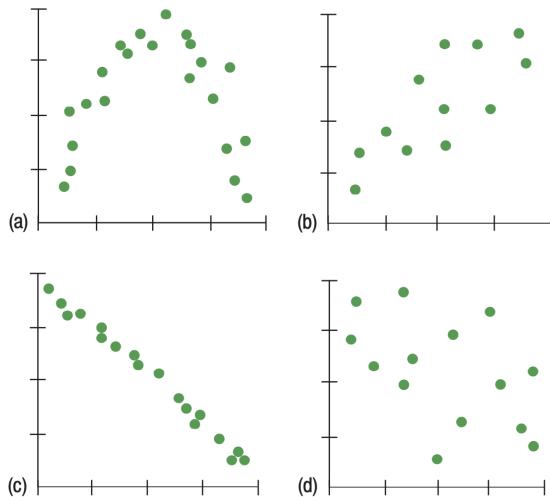
- Response: is the response variable or the dependent variable
- Explanatory: is the independent variable
- a: is the y-intercept
- b: is the slope of the regression line

You may use the following R functions: `plot()`, `lm()`, `cor()`, `abline()`. And you might need to download Lock5Data using `library(Lock5Data)`.

Part 2: Two quantitative variables/ Scatterplot and Correlation

2.1 Describe scatterplots

Here are several scatterplots. The calculated correlations are 0.006, -0.977, -0.487, and 0.777. Match each scatter plot with the appropriate correlation coefficient.



Answers:

- a. 0.006
- b. 0.777
- c. • 0.977
- d. • 0.487

2.2 Create scatterplots in R

Load the data `FloridaLakes` from library(`Lock5Data`).

1. Describe the type of each of the variables `pH`, `Calcium`, and `Alkalinity`.
2. Create a three scatter plots for each pair of the variables: `pH` vs `Calcium`, `pH` vs `Alkalinity`, and `Calcium` vs `Alkalinity`. Add the main title to each plot.
3. What is the correlation coefficient between `pH` and `Calcium`. Is it positive or negative?
4. What do these coefficients mean in the context of this data ?
5. Try to calculate the correlation coefficient between `pH` and `Calcium` without using R function.

Answers:

```
# download the data and load it into R
library(Lock5Data)
data(FloridaLakes)

# Describe the structure of the variables of FloridaLake
#str(FloridaLakes)
```

1. Describe the type of each of the variables pH and Calcium, Alkalinity.

The variables pH and Calcium, Alkalinity are quantitative variables (continuous)

2. Create a scatter plots for each of the pair of variables (add the main title of the plot).

```
# Create scatter pH vs Calcium
par(mfrow= c(1,3))

plot(FloridaLakes$pH,
      FloridaLakes$Calcium,
      main = "pH vs Calcium Scatter plot")

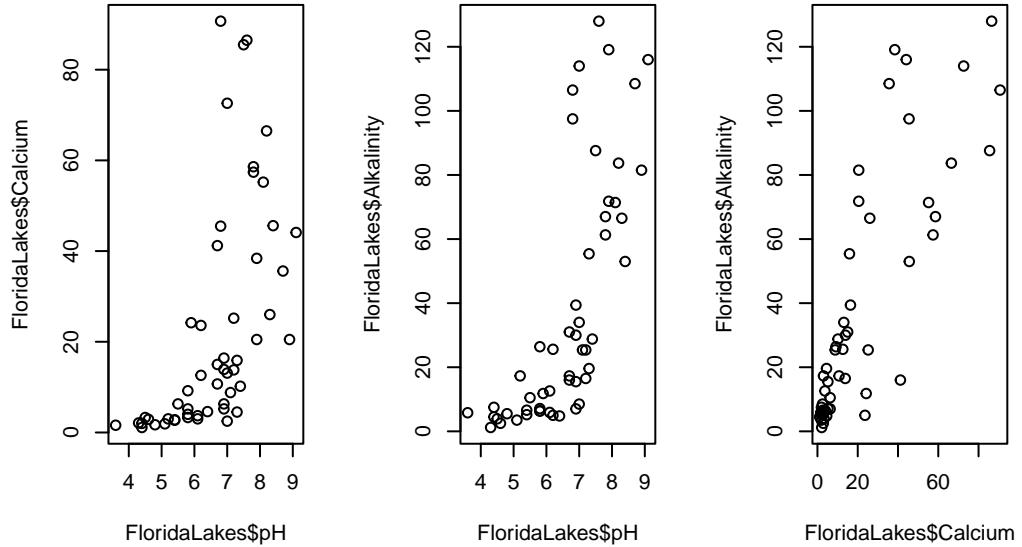
# Create scatter plot pH vs Alkalinity

plot(FloridaLakes$pH,
      FloridaLakes$Alkalinity ,
      main = "pH vs Alkalinity Scatter plot")

# Create scatter plot Calcium vs Alkalinity

plot(FloridaLakes$Calcium, FloridaLakes$Alkalinity,
      main = "Calcium vs Alkalinity Scatter plot")
```

ph vs Calcium Scatter plot ph vs Alkalinity Scatter plot Calcium vs Alkalinity Scatter



3. What is the correlation coefficient between pH and Calcium. Is it positive or negative.

```
# correlation

cor_pH_Cal <- cor(FloridaLakes$pH, FloridaLakes$Calcium)

cor_pH_Alk <- cor(FloridaLakes$pH, FloridaLakes$Alkalinity)

cor_Cal_Alk <- cor(FloridaLakes$Calcium, FloridaLakes$Alkalinity)

cor_pH_Cal
```

```
[1] 0.5771327
```

```
cor_pH_Alk
```

```
[1] 0.7191657
```

```
cor_Cal_Alk
```

```
[1] 0.8326042
```

4. What do these coefficients mean in the context of this data.

A correlation coefficient of 0.577 between pH and Calcium means that there is a linear positive association of moderate strength of 0.577.

5.calculate the correlation coefficient between pH and Calcium without using R function.

```
## Method 2 to calculate the correlation using the formula

n<- length(FloridaLakes$pH)

X<-FloridaLakes$pH

Y<- FloridaLakes$Calcium

cor_pH_Cal<- sum((X - mean(X))*(Y-mean(Y)))*(1/(n - 1))*1/sd(X)*1/sd(Y)

cor_pH_Cal
```

```
[1] 0.5771327
```