

# Hypothesis tests for more than two means

	5	3	2		7			8
6		1	5					2
2			9	1	3		5	
7	1	4	6	9	2			
	2						6	
			4	5	1	2	9	7
	6		3	2	5			9
1					6	3		4
8			1		9	6	7	

# Overview

Review/continuation of hypothesis testing for two means

Hypothesis tests for more than two means

If time: theories of hypothesis testing

# Hypothesis tests for comparing two means

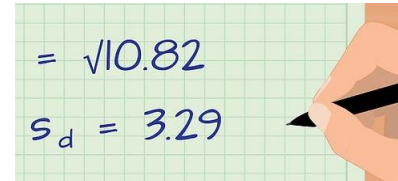


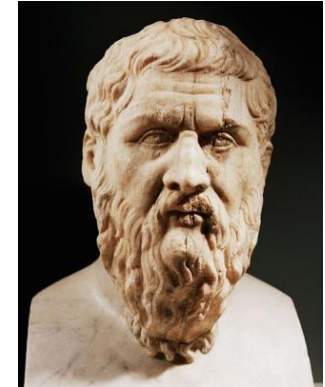
**Question:** Can we find out the ***Truth*** of whether the pill effective?

# Five steps of hypothesis testing

## 1. State $H_0$ and $H_A$

- Assume Gorgias ( $H_0$ ) was right


$$= \sqrt{10.82}$$
$$s_d = 3.29$$



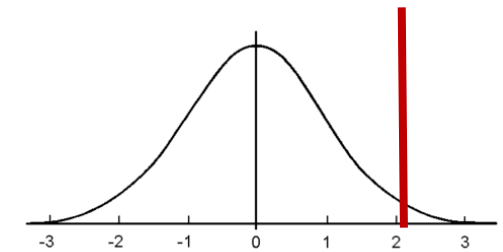
## 2. Calculate the actual observed statistic

## 3. Create a distribution of what statistics would look like if Gorgias is right

- Create the **null distribution** (that is consistent with  $H_0$ )

## 4. Get the probability we would get a statistic more than the observed statistic from the null distribution

- p-value



## 5. Make a judgement

- Assess whether the results are statistically significant



# Do mice who eat late at night get fat?

A study by Fonken et al, 2010, wanted to examine whether more weight was gained by mice who could eat late at night

Mice were randomly divided into 2 groups:

- Dark condition: 8 mice were given 8 hours of darkness at night (when they couldn't eat)
- Light condition: 9 were constantly exposed to light for 24 hours (so they could always eat)

1. State the null and alternative hypothesis

$$H_0: \mu_{\text{Light}} = \mu_{\text{Dark}} \quad \text{or} \quad \mu_{\text{Light}} - \mu_{\text{Dark}} = 0$$

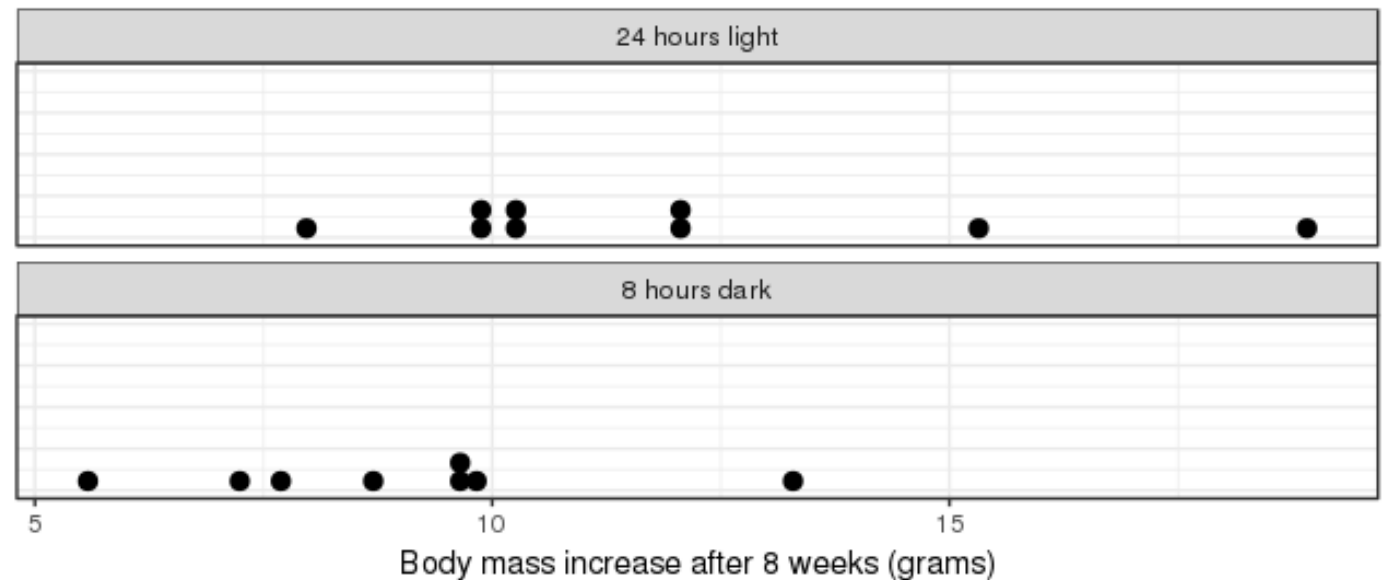
$$H_A: \mu_{\text{Light}} > \mu_{\text{Dark}} \quad \text{or} \quad \mu_{\text{Light}} - \mu_{\text{Dark}} > 0$$

# Hypothesis tests for differences in two group means

What is step 2?

2. Calculate statistic of interest

- $\bar{x}_{\text{effect}} = \bar{x}_{\text{Light}} - \bar{x}_{\text{Dark}}$



# Do mice who eat late at night get fat?

You can get the data from:

```
download_class_data("mice.Rda")  
load("mice.Rda")
```

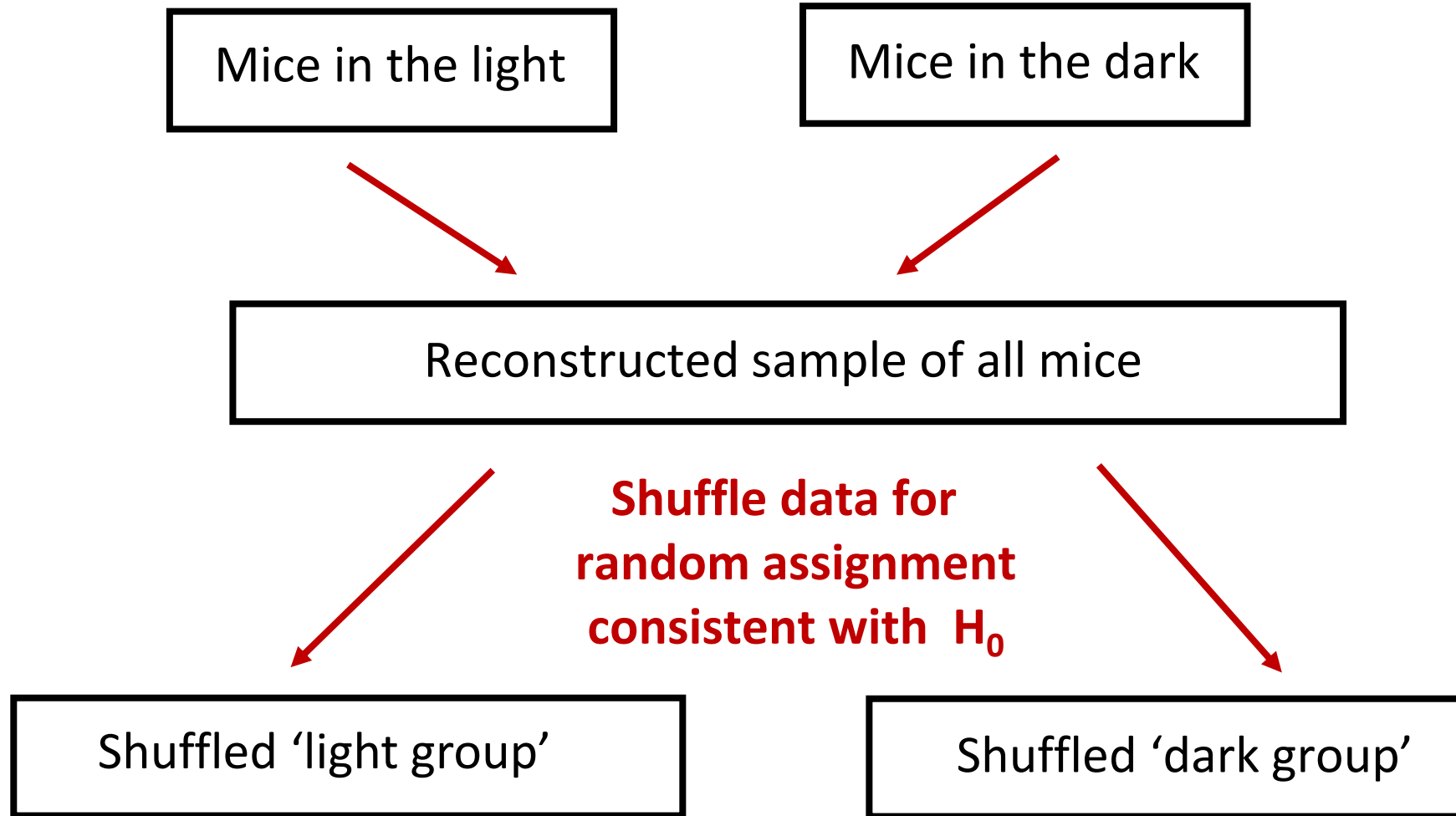
```
dark_BM_increase    # length(dark_BM_increase)  
light_BM_increase   # length(light_BM_increase)
```

Can you calculate the observed statistic (step 2)?

```
obs_stat <- mean(light_BM_increase) - mean(dark_BM_increase)
```

What's next?

### 3. Create the null distribution!



One null distribution statistic:  $\bar{X}_{\text{Shuff\_Light}} - \bar{X}_{\text{Shuff\_Dark}}$



# Do mice who eat late at night get fat?

What is the first thing we need to do for creating the null distribution?

```
combo_data <- c(light_BM_increase, dark_BM_increase)
```

How do we create one point in our null distribution?

```
# shuffle the data
```

```
shuff_data <- shuffle(combo_data)
```

```
# create fake light and dark data
```

```
shuff_light <- shuff_data[1:9]
```

```
shuff_dark <- shuff_data[10:17]
```

```
# compute fake statistic
```

```
mean(shuff_light) - mean(shuff_dark)
```

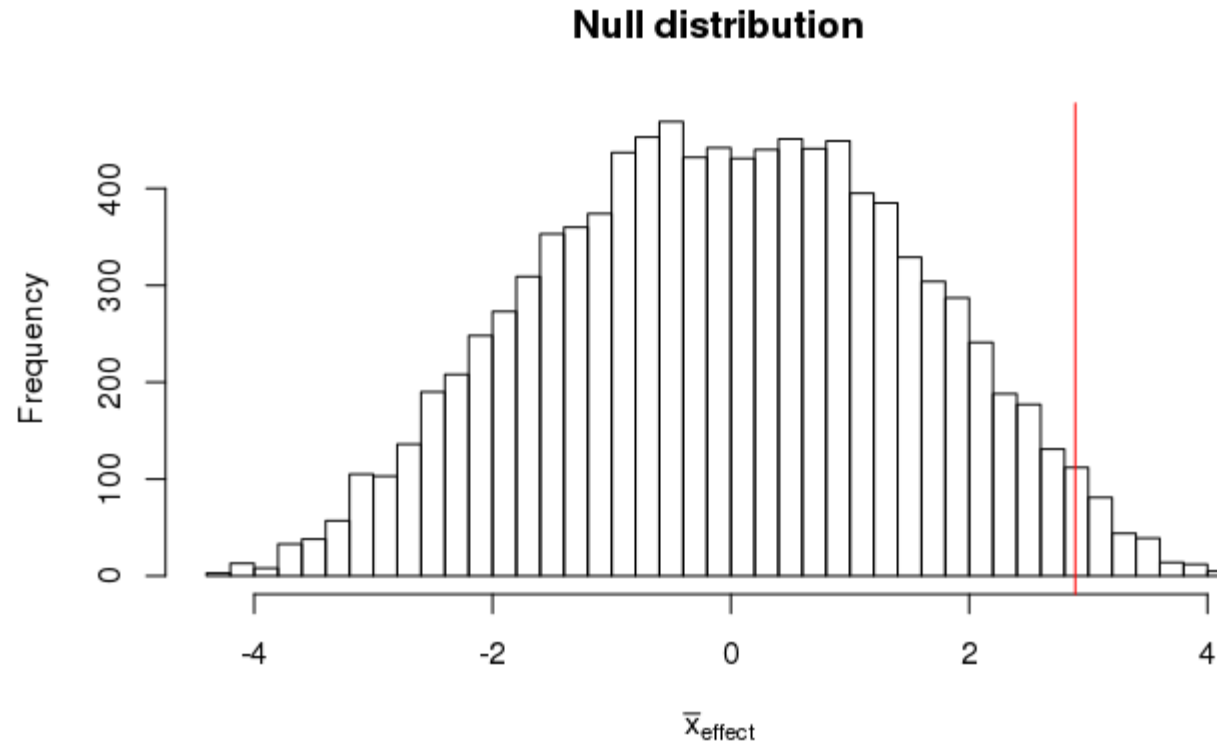
# Do mice who eat late at night get fat?

How do we create a full null distribution?

```
null_dist <- do_it(10000) * {  
  
  shuff_data <- shuffle(combo_data)  
  shuff_light <- shuff_data[1:9]  
  shuff_dark <- shuff_data[10:17]  
  mean(shuff_light) - mean(shuff_dark)  
  
}
```

# Do mice who eat late at night get fat?

Plot the null distribution: `hist(null_dist, breaks = 50)`



What do we do next?

# Do mice who eat late at night get fat?

Get the p-value

```
p_val <- pnull(obs_stat, null_dist, lower.tail = FALSE)
```

p-value = 0.02



# Comparing more than two means

A group of Hope College students wanted to see if there was an association between a student's major and the time it takes to complete a small Sudoku-like puzzle

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# Comparing more than two means

A group of Hope College students wanted to see if there was an association between a student's major and the time it takes to complete a small Sudoku-like puzzle

They grouped majors into four categories

- Applied science (as)
- Natural science (ns)
- Social science (ss)
- Arts/humanities (ah)

What is the first step of hypothesis testing?

# Sudoku by field

1. State the null and alternative hypotheses!

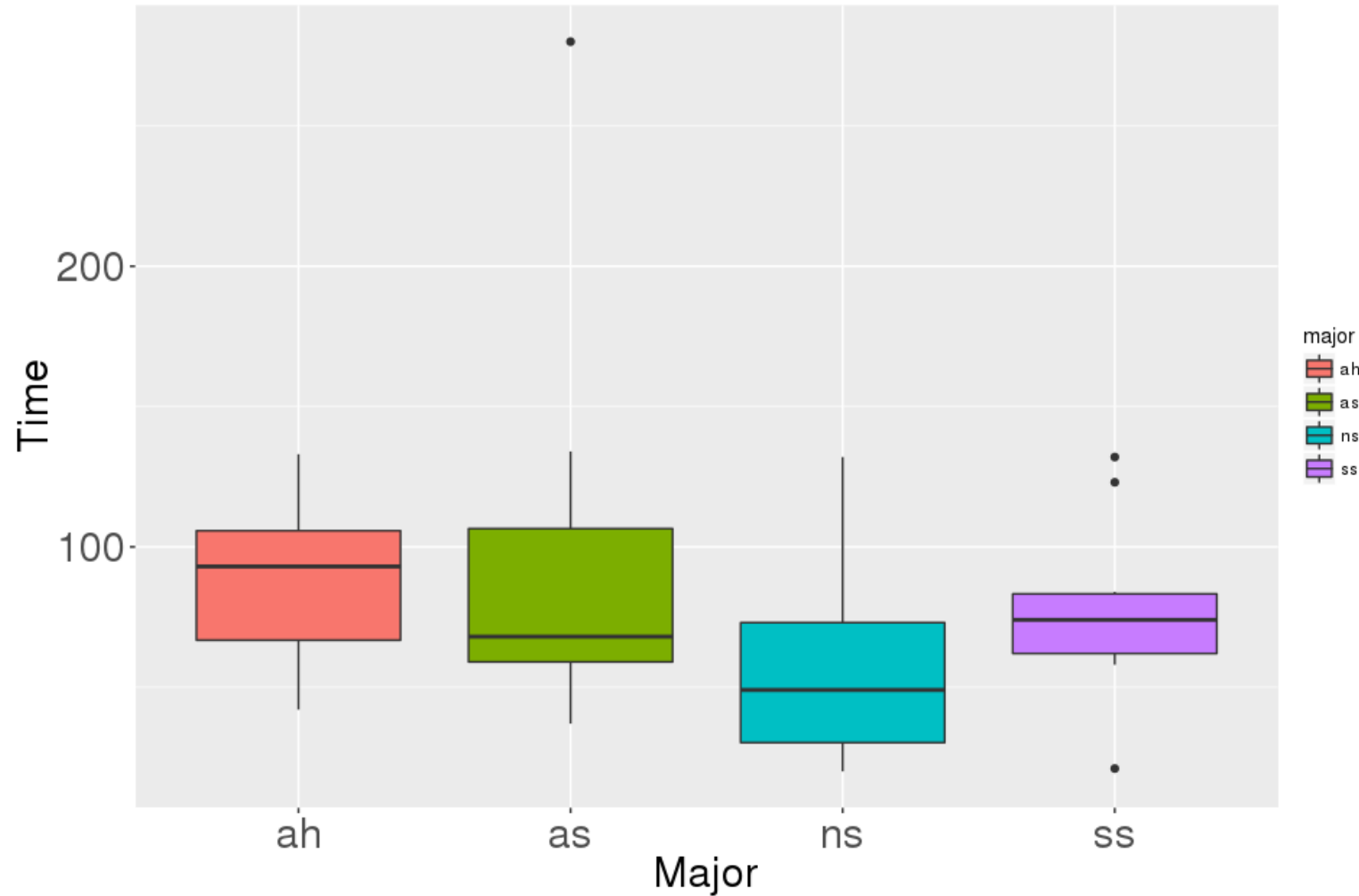
$$H_0: \mu_{as} = \mu_{ns} = \mu_{ss} = \mu_{ah}$$

$$H_A: \mu_i \neq \mu_j \text{ for one pair of fields of study}$$

What should we do next?

Let's plot the data first...

# Step 0: Plot of completion time by major



What should we do next?



# Sudoku by field

1. State the null and alternative hypotheses!

$$H_0: \mu_{as} = \mu_{ns} = \mu_{ss} = \mu_{ah}$$

$$H_A: \mu_i \neq \mu_j \text{ for one pair of fields of study}$$

Thoughts on the statistic of interest?

# Comparing multiple means

There are many possible statistics we could use. A few choices are:

1. Group range statistic:

$$\max \bar{x} - \min \bar{x}$$

2. Mean absolute difference (MAD):

$$(|\bar{x}_{as} - \bar{x}_{ns}| + |\bar{x}_{as} - \bar{x}_{ss}| + |\bar{x}_{as} - \bar{x}_{ah}| + |\bar{x}_{ns} - \bar{x}_{ss}| + |\bar{x}_{ns} - \bar{x}_{ah}| + |\bar{x}_{ss} - \bar{x}_{ah}|)/6$$

3. F statistic:

$$F = \frac{\text{between-group variability}}{\text{within-group variability}}$$

# Using the MAD statistic

Mean absolute difference (MAD):

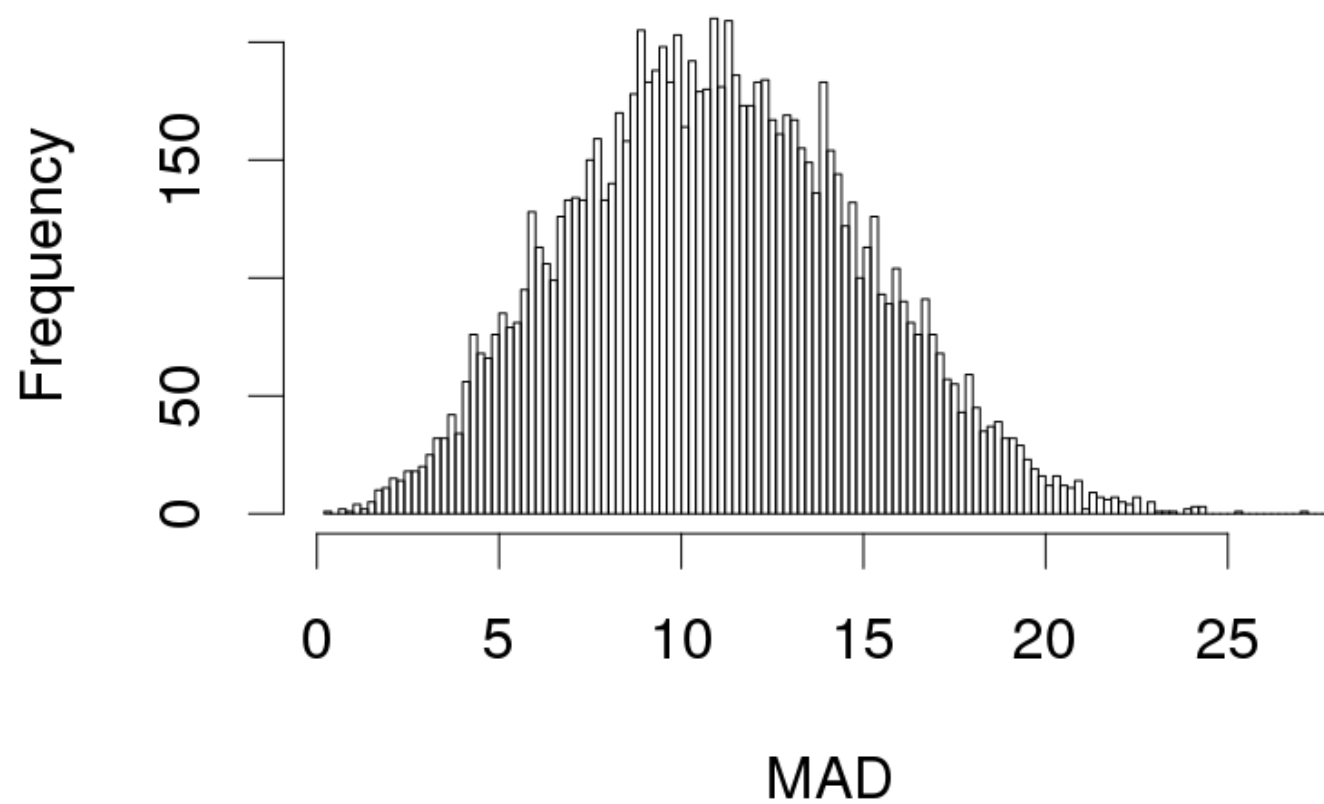
$$(|\bar{x}_{as} - \bar{x}_{ns}| + |\bar{x}_{as} - \bar{x}_{ss}| + |\bar{x}_{as} - \bar{x}_{ah}| + |\bar{x}_{ns} - \bar{x}_{ss}| + |\bar{x}_{ns} - \bar{x}_{ah}| + |\bar{x}_{ss} - \bar{x}_{ah}|)/6$$

Observed statistic value = 13.92

How can we create the null distribution?

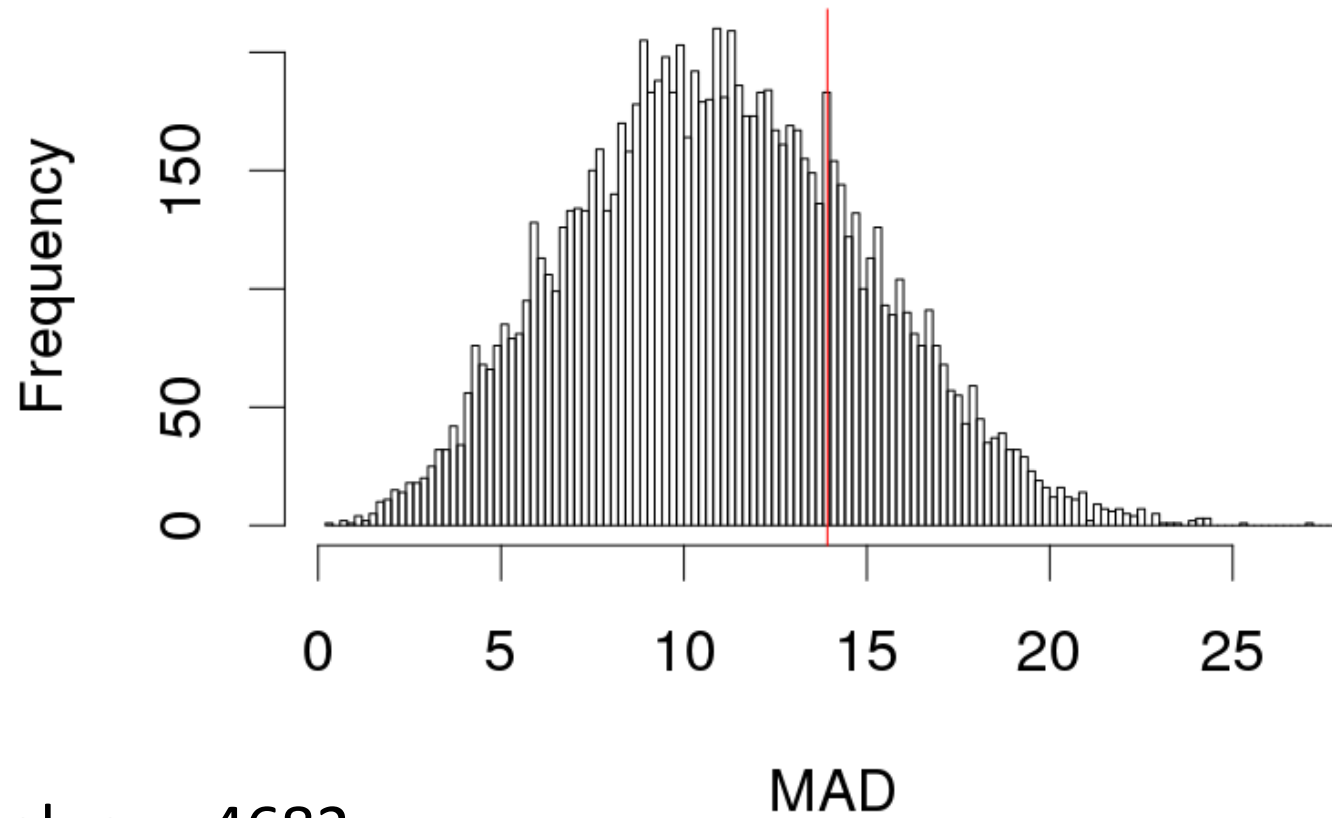
# Null distribution

## Null Distribution



# P-value

## Null Distribution



p-value = .4682

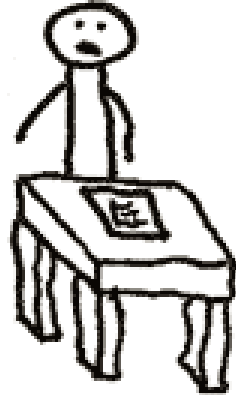
# Conclusions?



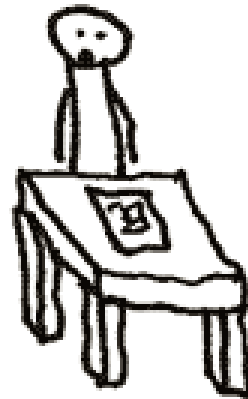
this sudoku is  
really hard!

that's a  
crossword  
puzzle

hmmm...



ohhh!



# Let's try this analysis in R...

Either run:

```
reinstall_class_package()
```

Or use workspace 3

- Link is on Canvas

```
# get the data
```

```
library(ClassTools)
```

```
download_class_data("MajorPuzzle.txt")
```

```
sudoku_data <- read.table("MajorPuzzle.txt", header = TRUE)
```



# Let's try this analysis in R...

```
# get the data
```

```
library(ClassTools)
```

```
download_class_data("MajorPuzzle.txt")
```

```
Sudoku_data <- read.table("MajorPuzzle.txt", header = TRUE)
```

```
# Extract vectors from the data frame (how do we do this?)
```

```
completion_time <- sudoku_data$time
```

```
major <- sudoku_data$major
```

# Let's try this analysis in R...

We can get the MAD statistic using the `get_MAD_stat()` function

`get_MAD_stat(data_vector, grouping_vector)`

- `data_vector`: a vector of quantitative data
- `grouping_vector`: a vector indicating which group the quantitative data is in

Can you get the MAD statistic for the sudoku data?

```
obs_stat <- get_MAD_stat(completion_time, major)
```

Can you visualize the data?

```
boxplot(completion_time ~ shuffled_majors)
```

# Let's try this analysis in R...

Q: How could we create one point in a null distribution?

- A: Shuffle the grouping\_vector (major vector) and calculate the MAD statistic

Q: How can we do this in R?

```
shuffled_majors <- shuffle(major)
```

```
get_MAD_stat(completion_time, shuffled_majors)
```

# Let's try this analysis in R...

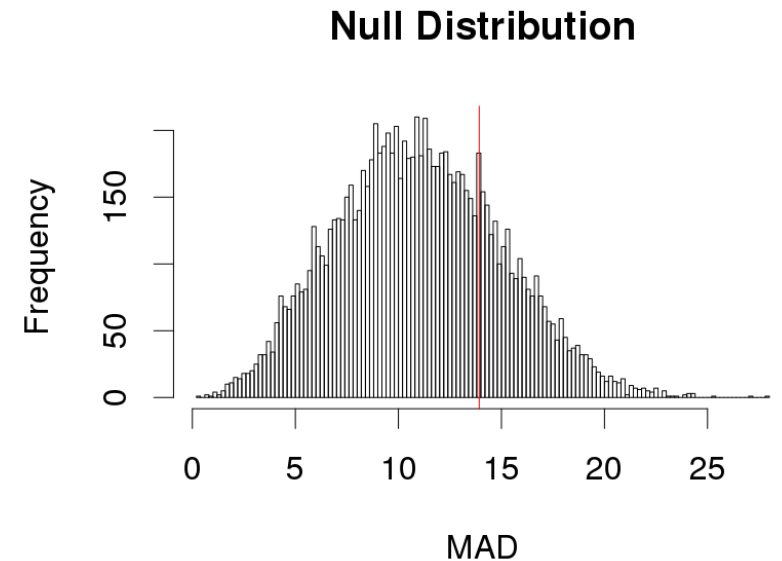
Q: How can we create a full null distribution?

```
null_dist <- do_it(10000) * {  
  shuffled_majors <- shuffle(major)  
  get_MAD_stat(completion_time, shuffled_majors)  
}
```

# visualize the null distribution

```
hist(null_dist, breaks = 100)
```

```
abline(v = obs_stat, col = "red")
```



# Let's try this analysis in R...

Q: What do we do next and how do we do it?

- A: We get the p-value

```
pnull(obs_stat, null_dist, lower.tail = FALSE)
```

