Measures of spread continued

Overview

Homework 1 questions?

Quick review:

- Statistics for central tendency: mean and median
- Outliers
- Standard deviations

Z-scores

Percentiles

Boxplots

Correlation

Any questions about homework 1?

Homework 2 has been posted

- Use the link on Canvas to access homework 2 on R Studio Cloud
- Note: this homework is significantly longer than the first homework!

Remember to highlight each question when submitting your answers on Gradescope

Review: Descriptive statistics for one quantitative variable

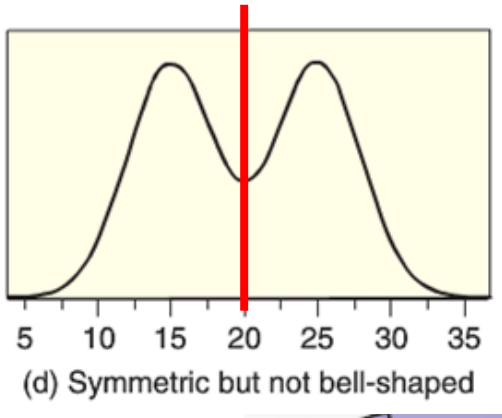
For quantitative data we look at:

- What is the general ‘shape’ of the data
- Where are the values centered (central tendency)
- How do the data vary (spread)

There are all properties of how the data is ***distributed***

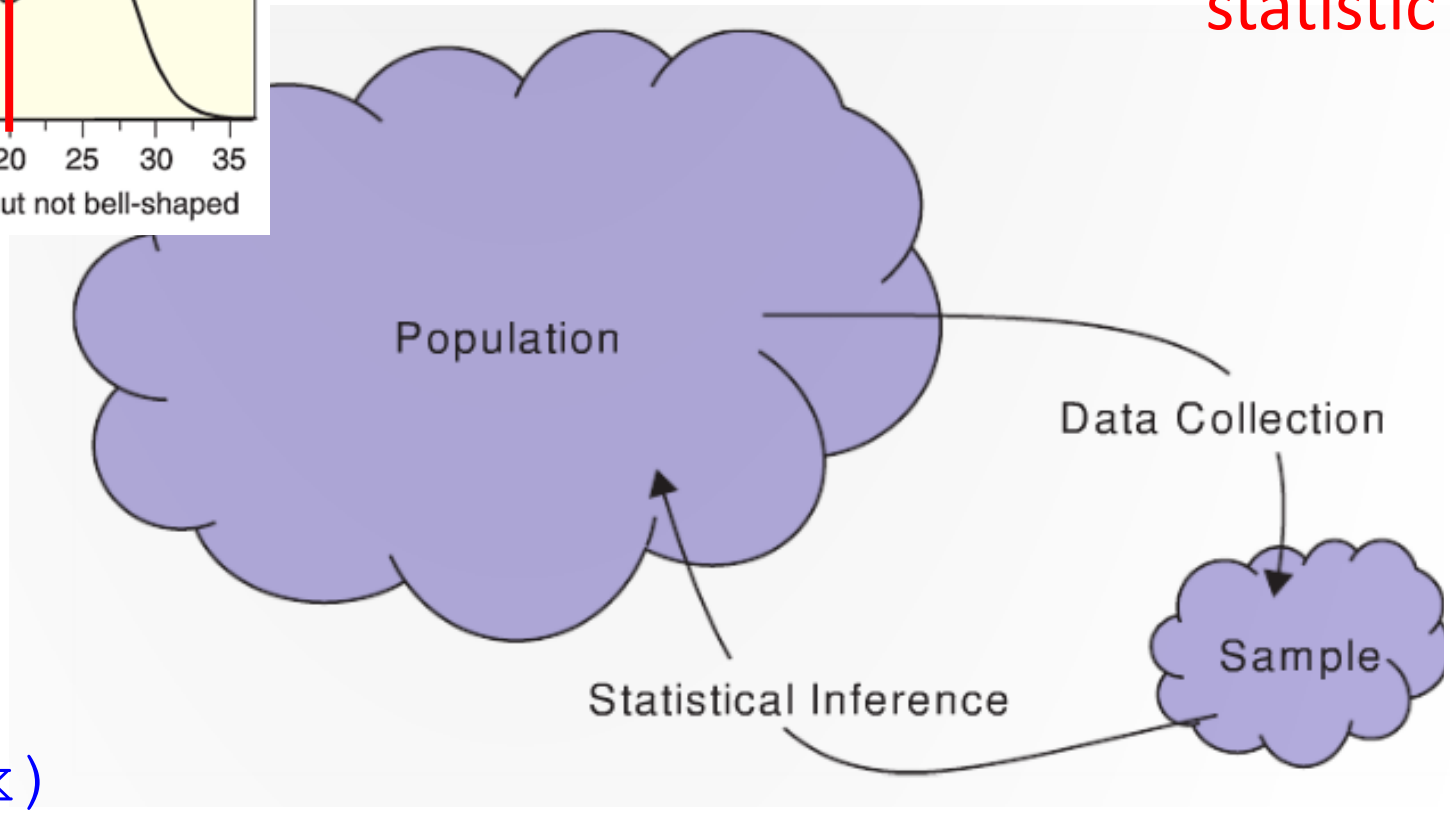
Review: Central Tendency and the mean

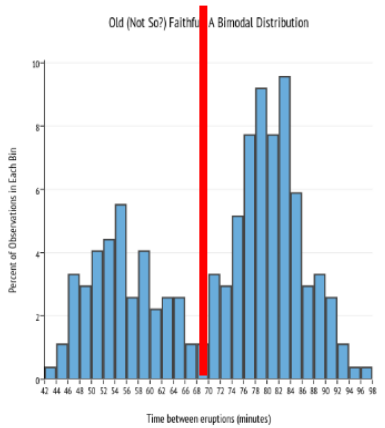
μ ← parameter



$$\bar{x} = \frac{\sum_i^n x_i}{n}$$

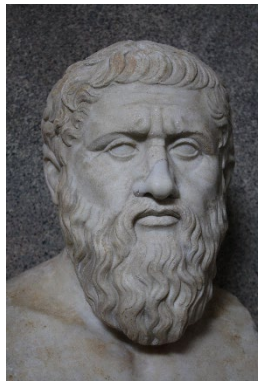
statistic → $\bar{x}$

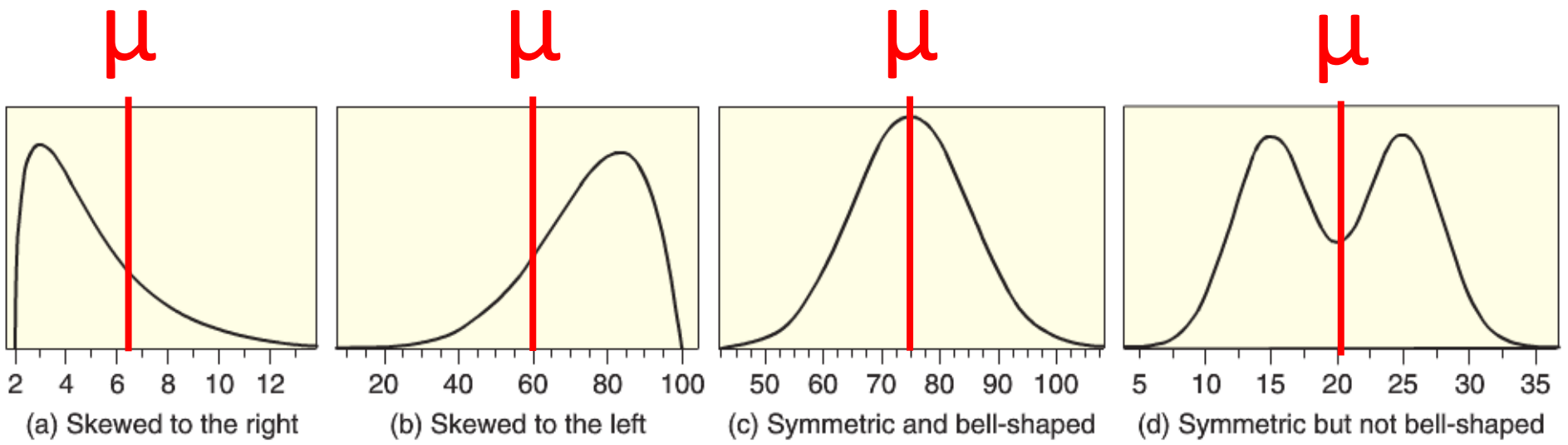




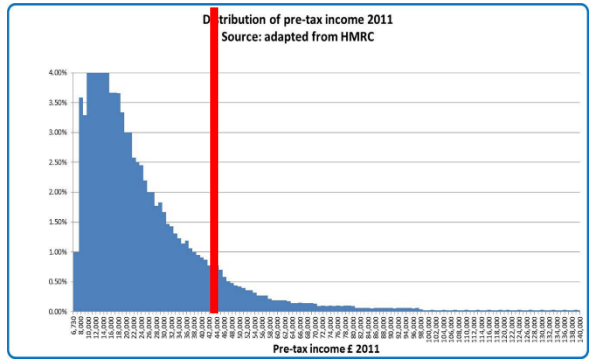
R: `mean(x)`

Review: Central Tendency and the mean

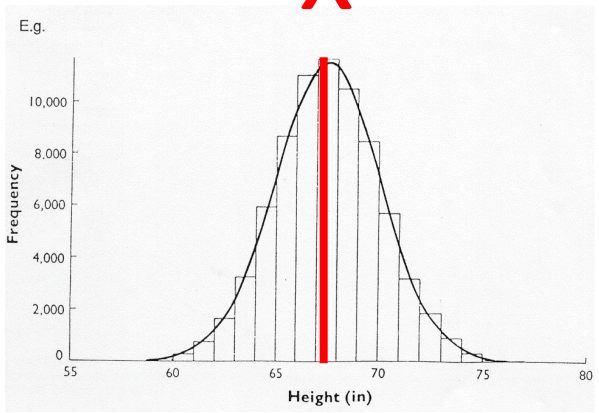




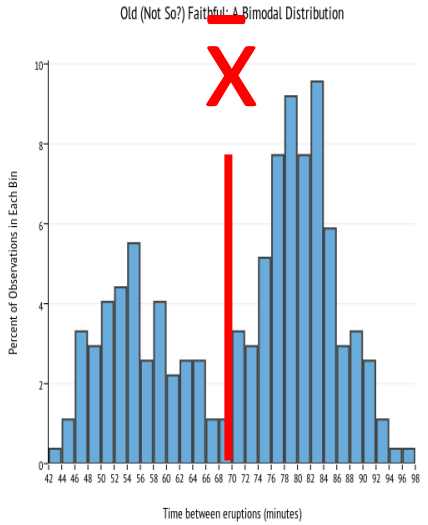
$\bar{x}$



$\bar{x}$



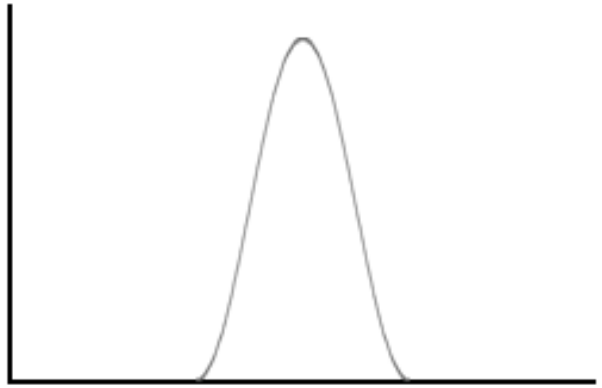
$\bar{x}$



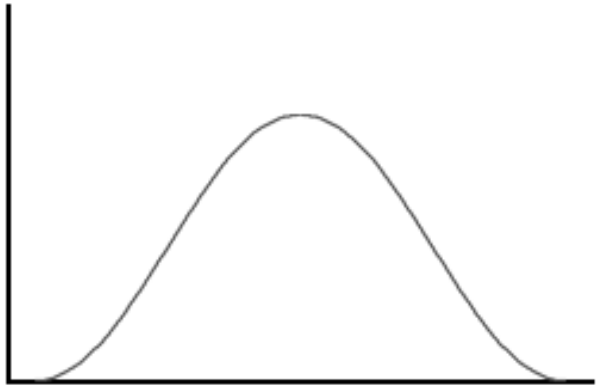
Review: the standard deviation is a measure of spread

The **standard deviation** (for a quantitative variable) is a measure of the spread spread of the data

Smaller standard deviation



Larger standard deviation



It gives a rough estimate for a typical distance a point is from the center

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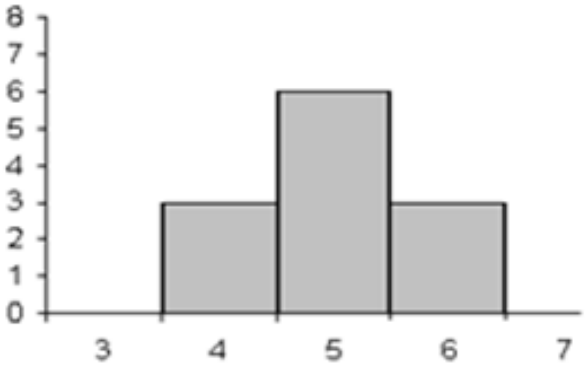
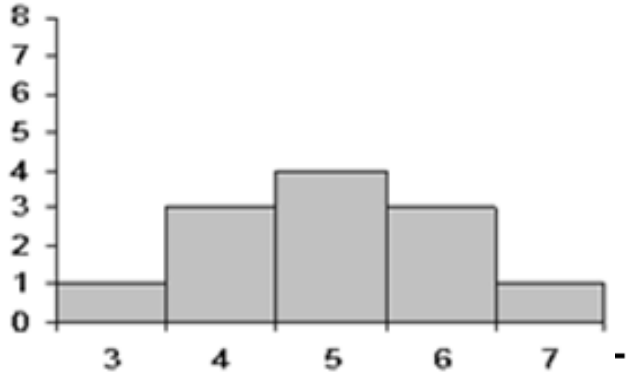


Larger standard deviation



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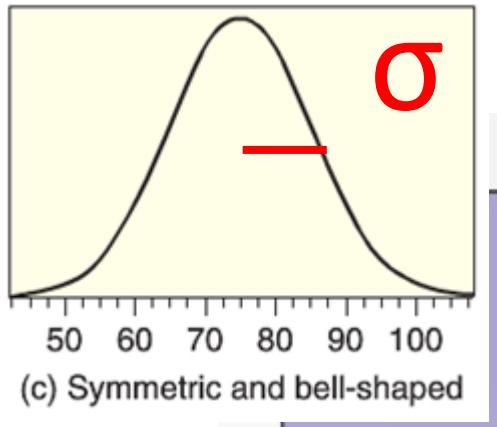
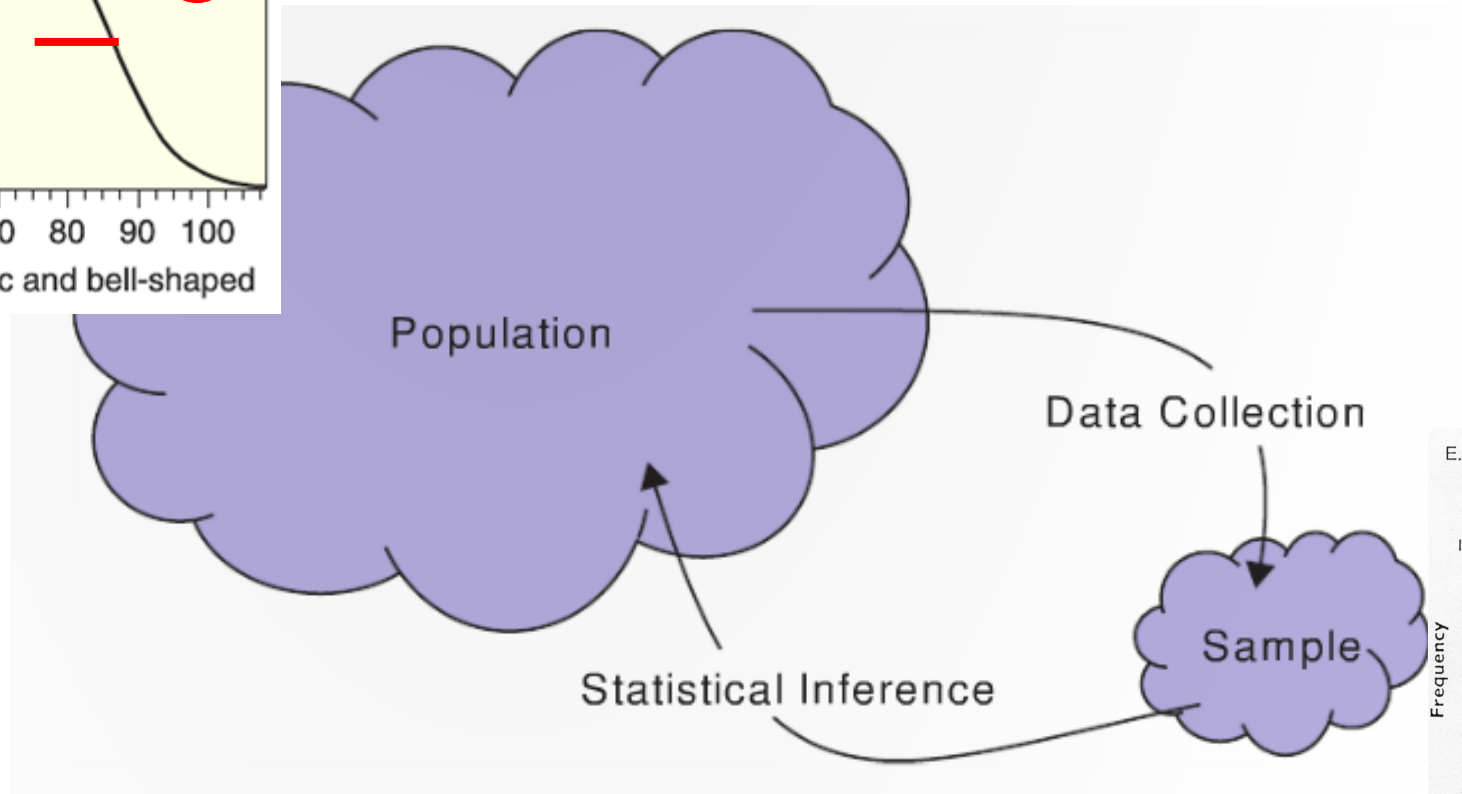
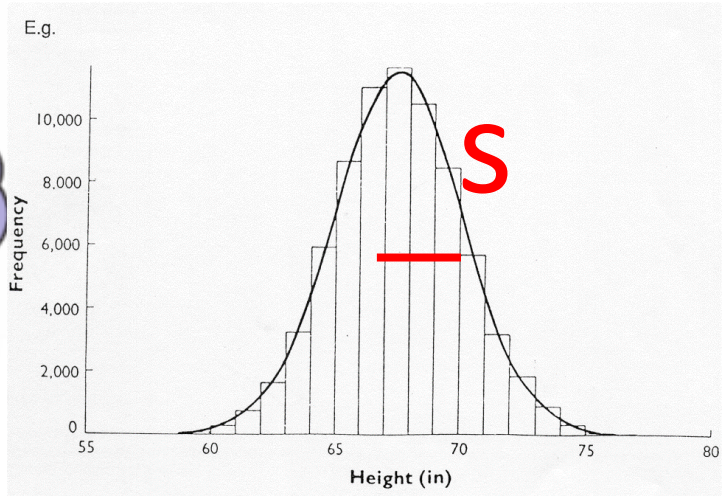
Population and sample standard deviation







Review: The standard deviation

The standard deviation can be computed using the following formula:

$$s = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

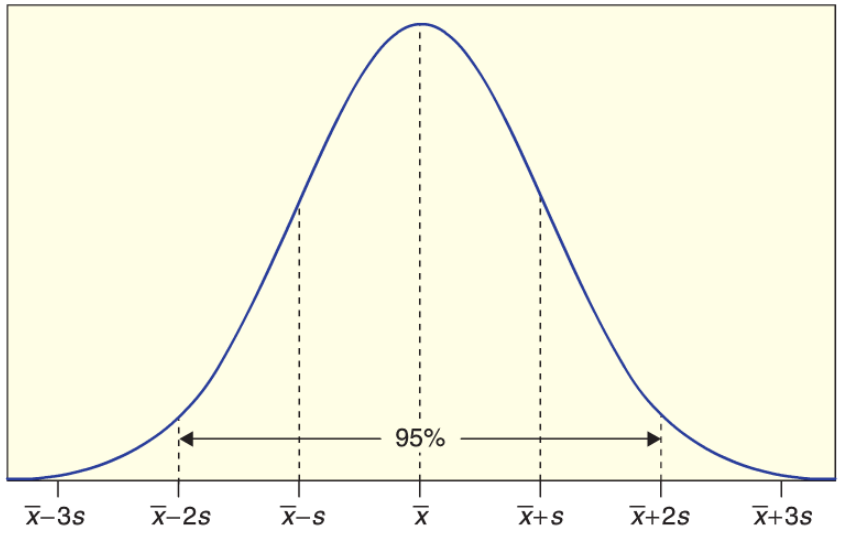
Homework 1: Were you able to calculate it “by hand”?

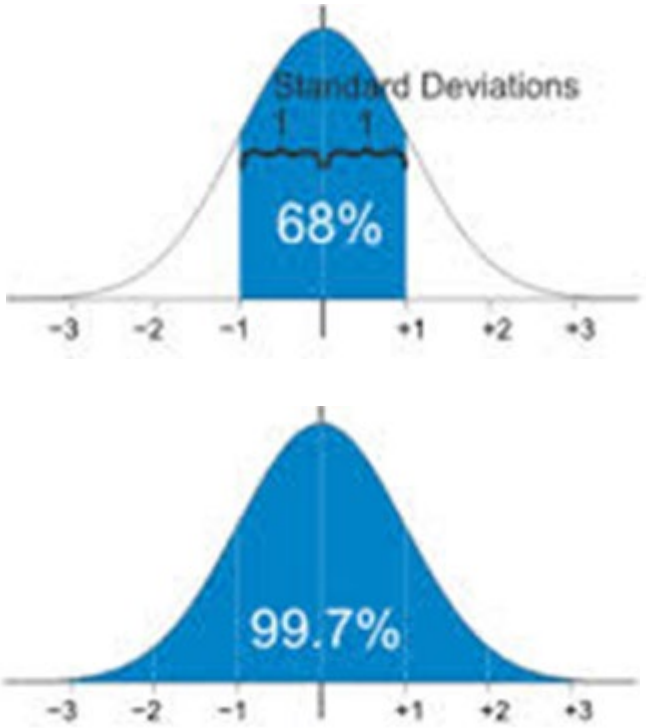
Review: The 95% rule for *normal distributions*

A **normal distribution** is a common distribution that is symmetric and bell shaped

For normal distributions

- 68% of the data falls within **one** standard deviations of the mean
- 95% of the data falls within **two** standard deviations of the mean
- 99.7% of the data falls within **three** standard deviations of the mean





z-Scores

The z-scores tells how many standard deviations a value is from the mean

- i.e., how far away a point x_i is from $\bar{x}$ in a way that is independent of the units of measurement

$$\text{z-score}(x_i) = \frac{x_i - \bar{x}}{s}$$

Which Accomplishment is most impressive?

LeBron James is a basketball player who had the following statistics in 2011:

- Field goal percentage (FGPct) = 0.510
- Points scored = 2111
- Assists = 554
- Steals = 124

The summary statistics of the NBA in 2011 are given below

$$\text{z-score}(x_i) = \frac{x_i - \bar{x}}{s}$$

	Mean	Standard Deviation
FGPct	0.464	0.053
Points	994	414
Assists	220	170
Steals	68.2	31.5

Relative to his peers, which statistic is most and least impressive?

Which Accomplishment is most impressive?

LeBron James is a basketball player who had the following statistics in 2011:

- Field goal percentage (FGPct) = 0.510
- Points scored = 2111
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The summary statistics of the NBA in 2011 are given below

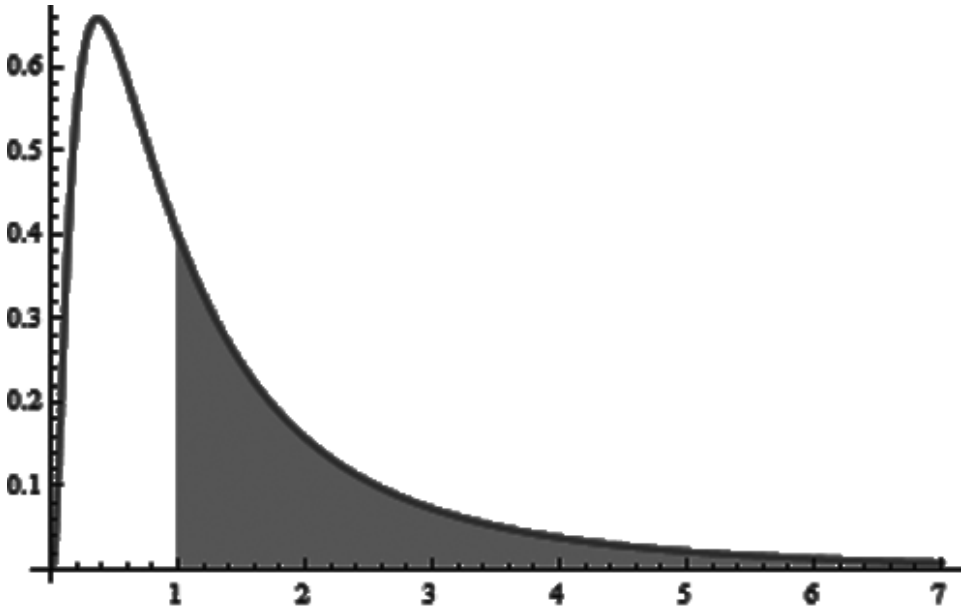
z	$=$	$(x - \bar{x}) / s$		
Z-score FGPct	=	(0.510 - 0.464)/0.053	=	0.868
Z- score Points	=	(2111 - 994)/414	=	2.698
Z-score Assists	=	(554 - 220)/170	=	1.965
Z-score Steals	=	(124 - 68.2)/31.5	=	1.771

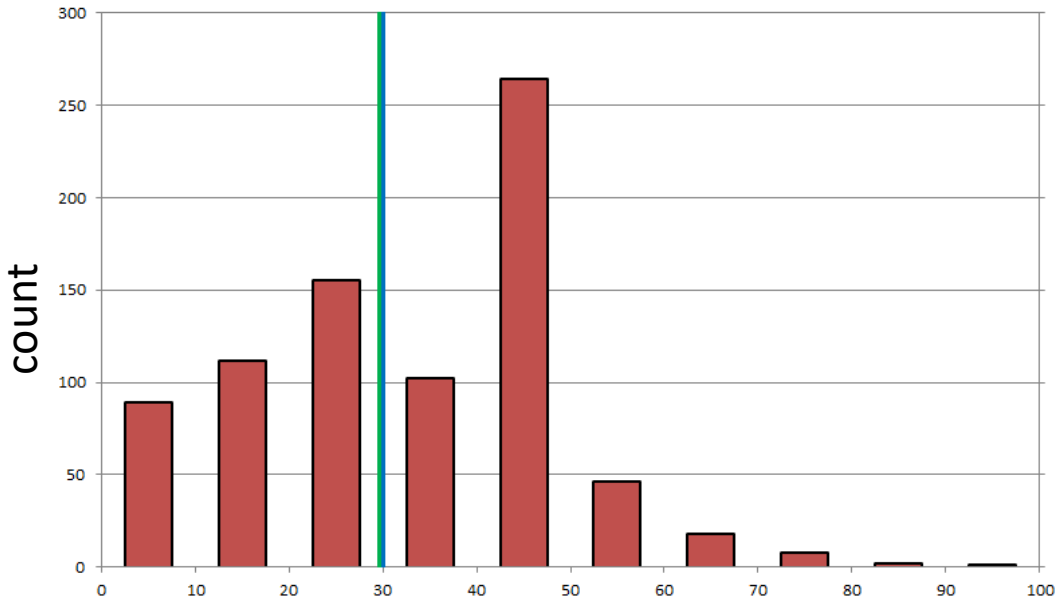
If you were given a vector of points for all players instead of $\bar{x}$ and s, would you be able to calculate z for LeBron?

Review: Central Tendency and the median

The **median** is a value that splits the data in half

- Sort the data and take the middle value (or the mean of the middle 2 values)





R: `median(v)`

Review: outliers

Q: What is an **outlier?**

- A: An observed value that is notably distinct from the other values in a dataset

Q: Why are they problematic?

- A: can potentially have a large influence on the statistics you calculate

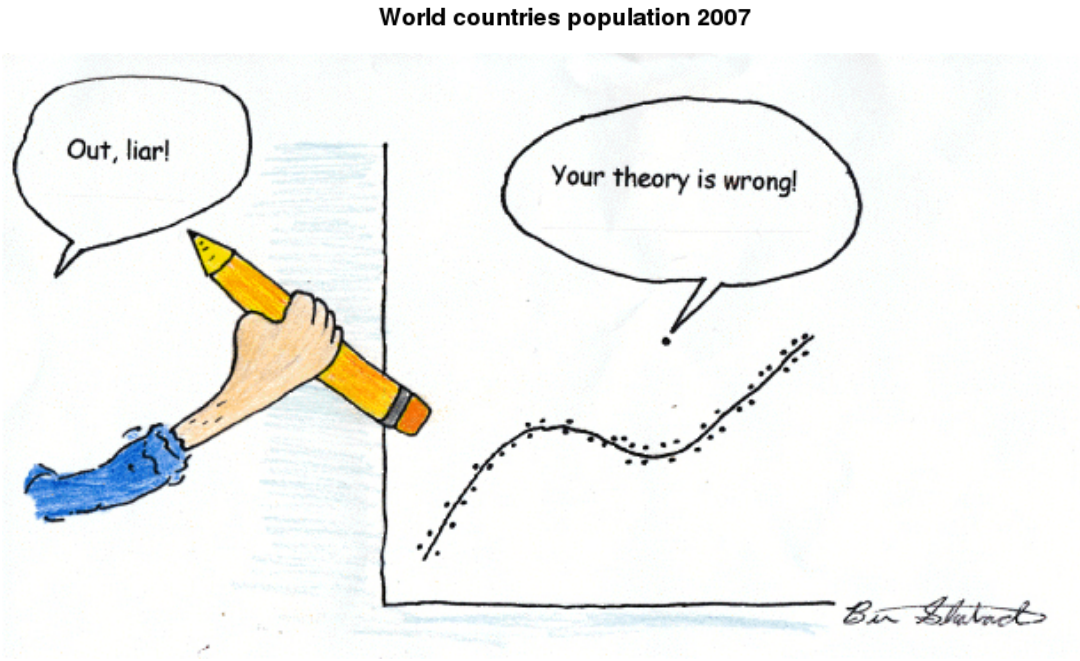
Q: What should you do if you have an outlier in your data?

A: See if you can understand what is causing it!

- If it's an error, delete the point
- If it's a real value, make sure it is not having a big effect on your conclusions, and/or use resistant statistics

Q: Is the mean and/or median resistant?

- A: The median is resistant while the mean is not



Percentiles (quantiles)

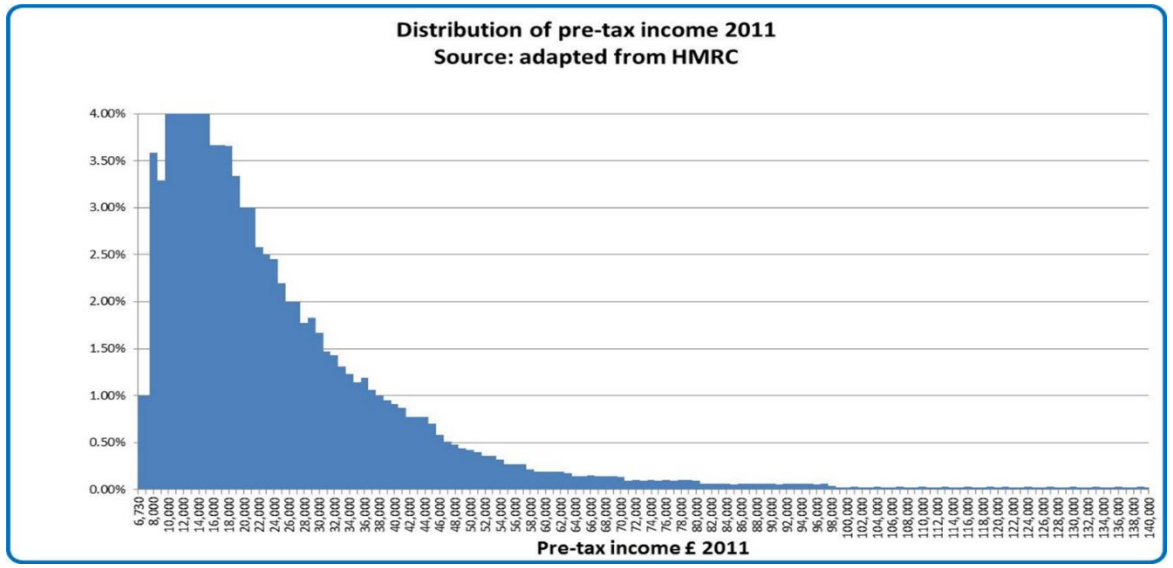
The **p^{th} percentile** is a quantitative value ***x***, such that ***p*** percent of the data is less than ***x***

The income distribution is shown below. What are the 25^{th} and 90^{th} percentiles?

25^{th} percentile = $27,794

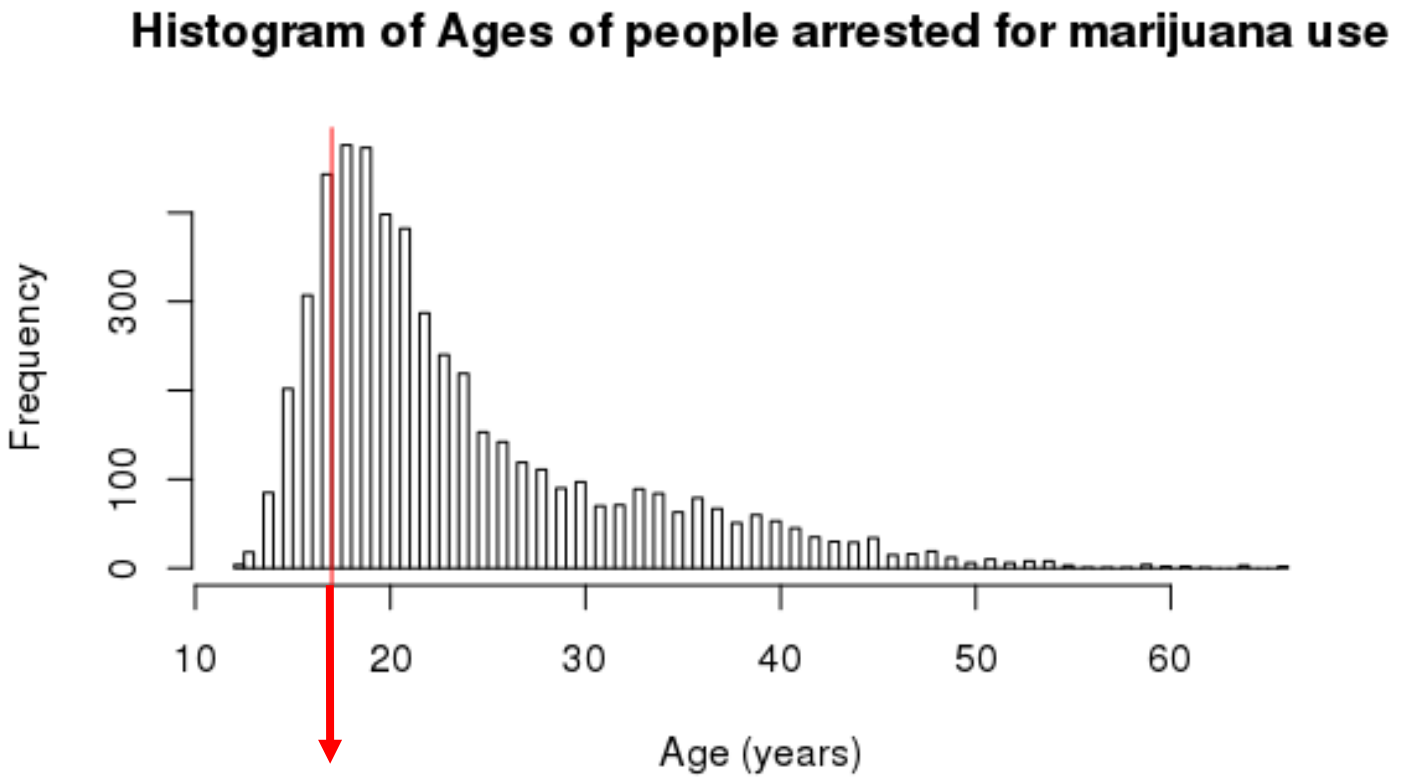
95^{th} percentile = $113, 820

Income distribution



R: `quantile(v, .95)`

Quantiles: age of marijuana arrests in Toronto

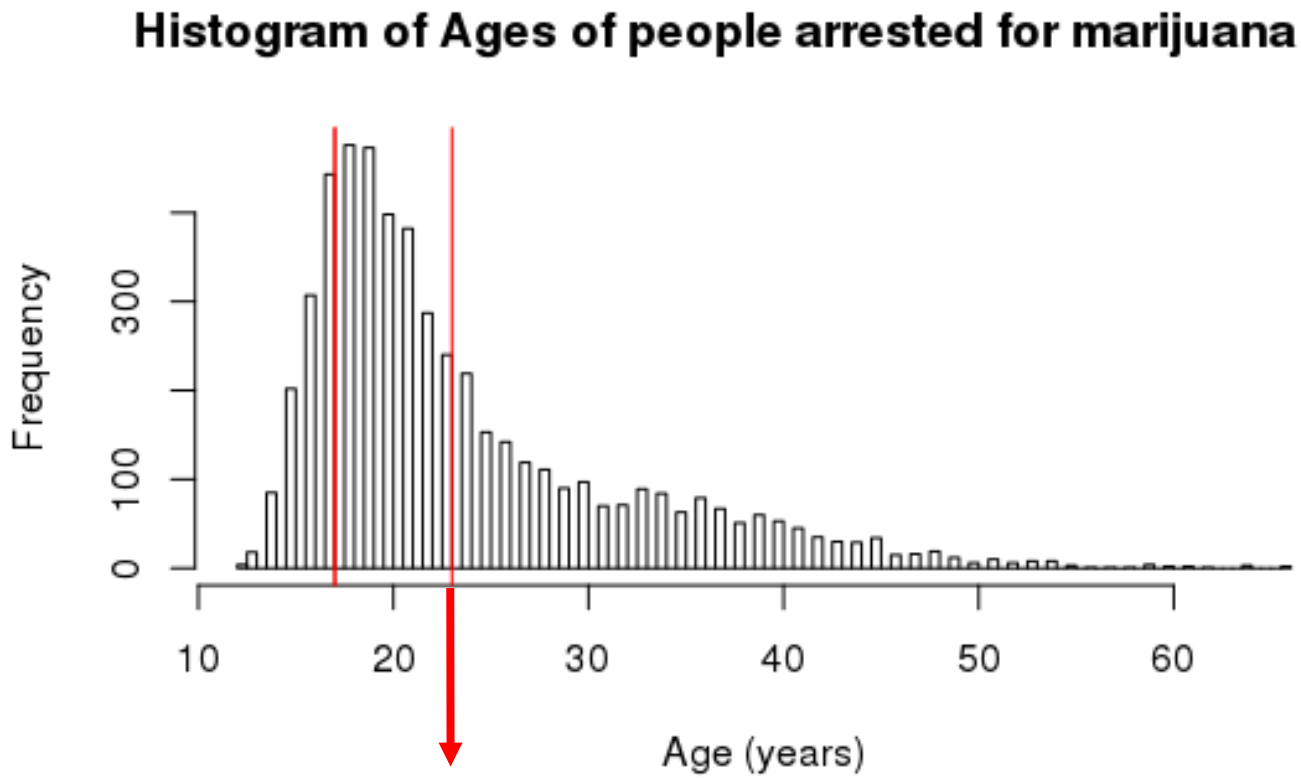


20th percentile value is 17

i.e., 20% of the arrests were of ages 17 or less

```
> install.packages('carData')
> library(carData)  # load the data
> quantile(Arrests$age,  .2)    # get the 20th percentile value from a vector of ages of arrests
```

Quantiles: age of marijuana arrests in Toronto



60th percentile value is 23

i.e., 60% of the arrests were of ages 23 or less

> quantile(Arrests$age, c(.2, .6)) # get the 20th and 60th percentile values from a vector of ages of arrests

Five Number Summary

Five Number Summary = (minimum, Q_1, median, Q_3, maximum)

Q_1 = 25^{th} percentile (also called 1^{st} quartile)

Q_3 = 75^{th} percentile (also called 3^{rd} quartile)

Roughly divides the data into fourths

Range and Interquartile Range

Range = maximum – minimum

Interquartile range (IQR) = $Q_3 - Q_1$

Hot dog example – try this at home!

Try this at home: for the hot dog data calculate "by hand"

- The 5 number summary
- The range
- Interquartile range

Cheat sheet:

Five Number Summary = (minimum, Q_1, median, Q_3, maximum)

Range = maximum – minimum

Interquartile range (IQR) = $Q_3 - Q_1$

Q_1 = 25th percentile, Q_3 = 75th percentile

Answer in R: `fivenum(v)`

Year	Hot Dogs
2013	69
2012	68
2011	62
2010	54
2009	68
2008	59
2007	66
2006	54
2005	49
2004	54

Detecting of outliers

As a rule of thumb, we call a data value an **outlier** if it is:

Smaller than: Q_1 - 1.5 * IQR

Larger than: Q_3 + 1.5 * IQR

What is the range that a value would be called an outlier in the hot dog data?

Are there any outliers in the hot dog data?

Boxplots

A **boxplot** is a graphical display of the 5 number summary and consists of:

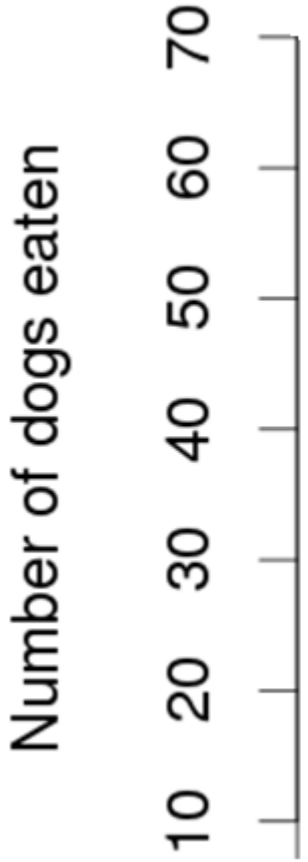
1. Drawing a box from Q_1 to Q_3

2. Dividing the box with a line (or dot) drawn at the median

3. Draw a line from each quartile to the most extreme data value that is not and outlier

4. Draw a dot/asterisk for each outlier data point.

Box plot of the number of hot dogs eaten by the men's contest winners 1980 to 2010



Maximum value that is not an outlier

Q_3

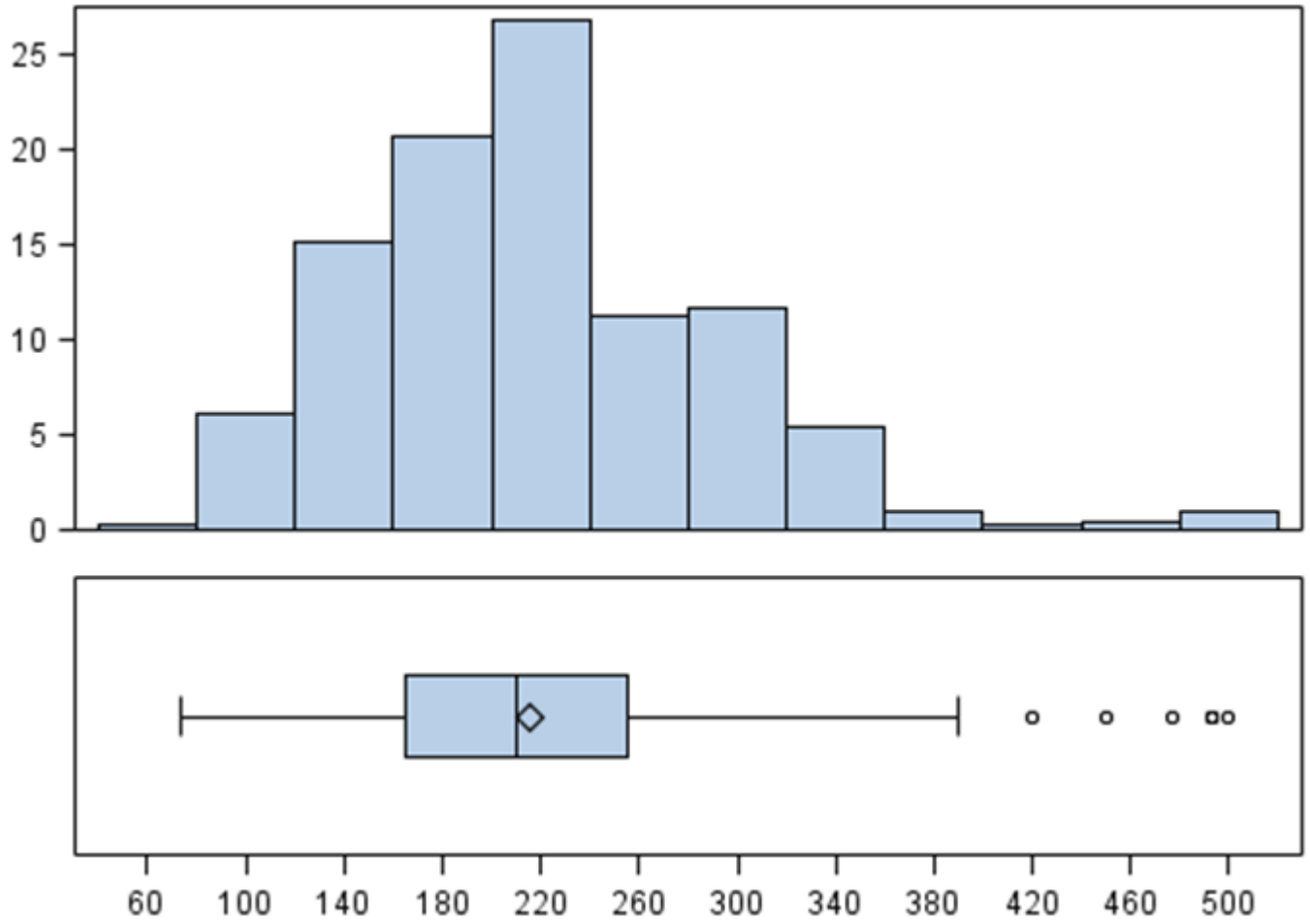
Median

Q_1

Minimum value that is not an outlier

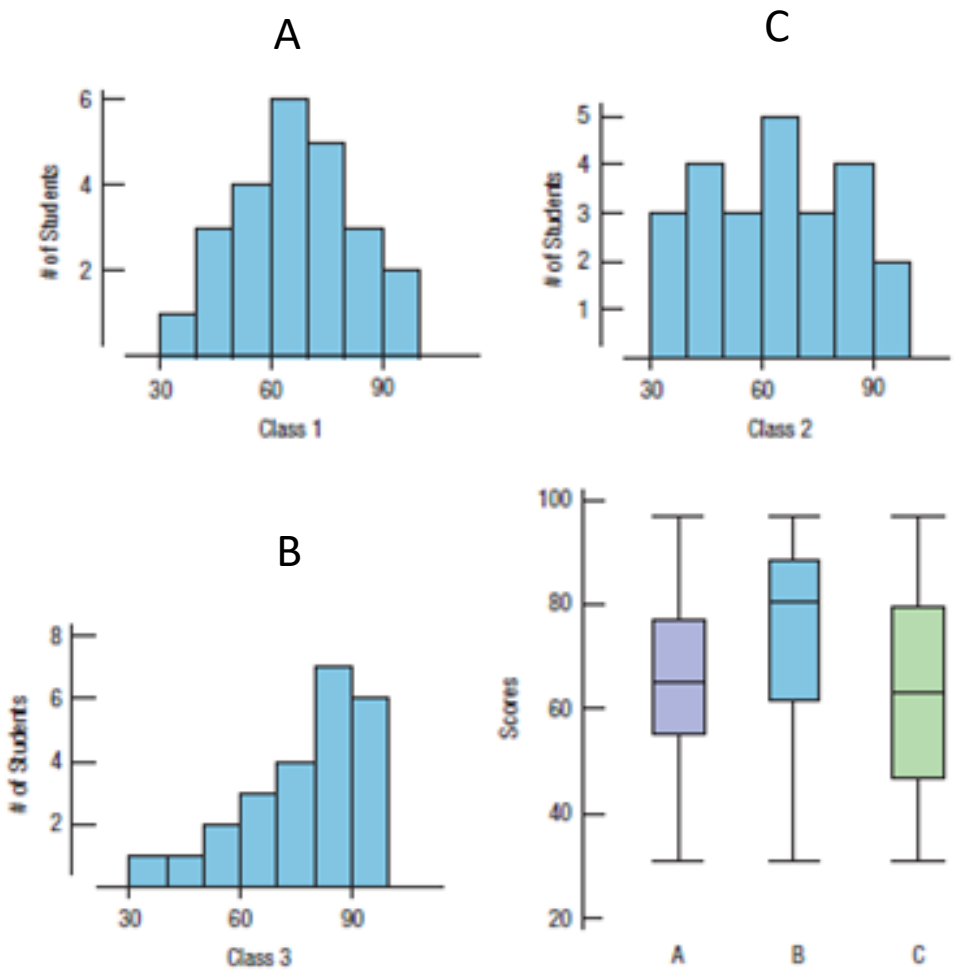
R: `boxplot(v)`

Box plots extract key statistics from histograms



Box plots extract key statistics from histograms

Question: which Boxplot goes with which histogram?

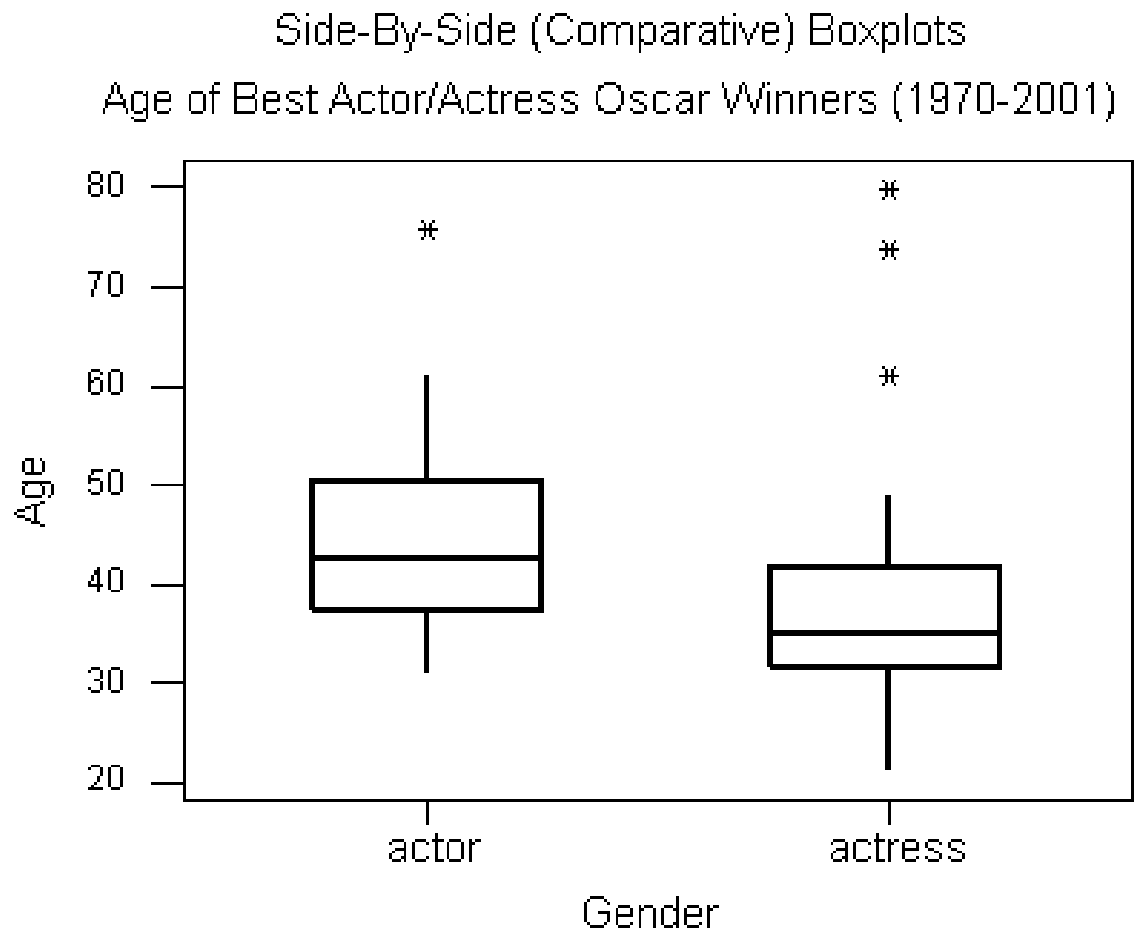


Comparing quantitative variables across categories

Often one wants to compare quantitative variables across categories

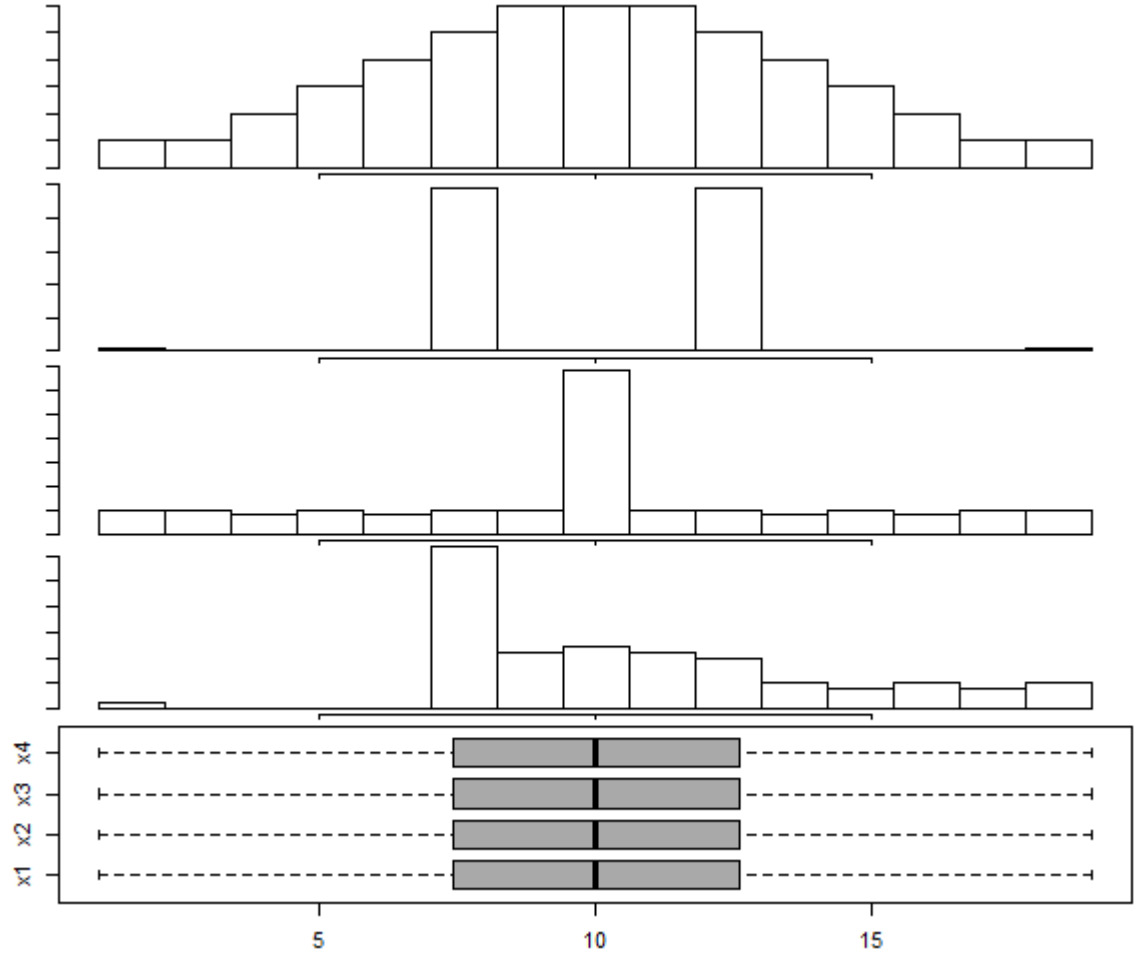
Side-by-Side graphs are a way to visually compare quantitative variables across different categories

Side-by-side boxplots



Boxplots don’t capture everything

Do you think the box plots for these distributions look similar?



Side-by-size boxplots in R

```
> boxplot(v1, v2,                    # compare two vectors v1 and v2
      names = c("name 1", "name 2"),      # labels below each boxplot
       ylab =  "y-axis name"          # y-axis label name
    )
```

Try it yourself, create histograms and boxplots for this data:

```
> download_class_data("distribution_vs_boxplot.Rda")
> load("distribution_vs_boxplot.Rda")
> boxplot(x1, x2, x3, x4)
```

Concepts for summarizing quantitative data

z-scores show how many standard deviations a point is from the mean

$$\text{z-score}(x_i) = \frac{x_i - \bar{x}}{s}$$

Quantiles show the value ***x,*** such that a fixed proportion of the data is less than x

- e.g., what is the value *x,* such that 20% of the data is less than *x*

Five Number Summary give key summary statistics of a data sample

- minimum, Q_1, median, Q_3, maximum

A **boxplot** is a visualization of the five number summary

- Side-by-side boxplots allow you to compare key summary statistics

Summary of R

We can compute a z-score for a value x, and a vector of values v using:

```
> the_mean <- mean(v)
> the_sd <- sd(v)
> the_zscore <-   (x - the_mean)/the_sd
```

We can compute quantiles using the quantile() function:

- `> quantile(v, .2)` or `quantile(v,  c(.25, .4))`

We can compute a five number summary using the fivenum() function:

```
> fivenum(v)
```

We can compute boxplots using the boxplot() function

```
> boxplot(v)
```