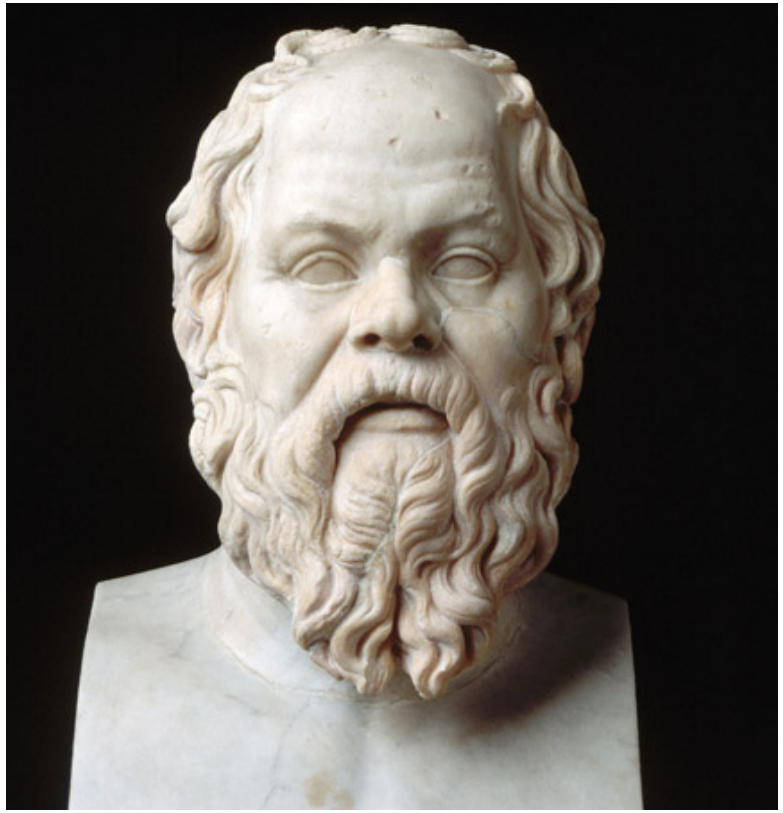
Review!



Review of descriptive statistics

1. Intro to data

What is Statistics?

What are…

- Observational units?
- Variables?
- Categorical variables?
- Quantitative variables?

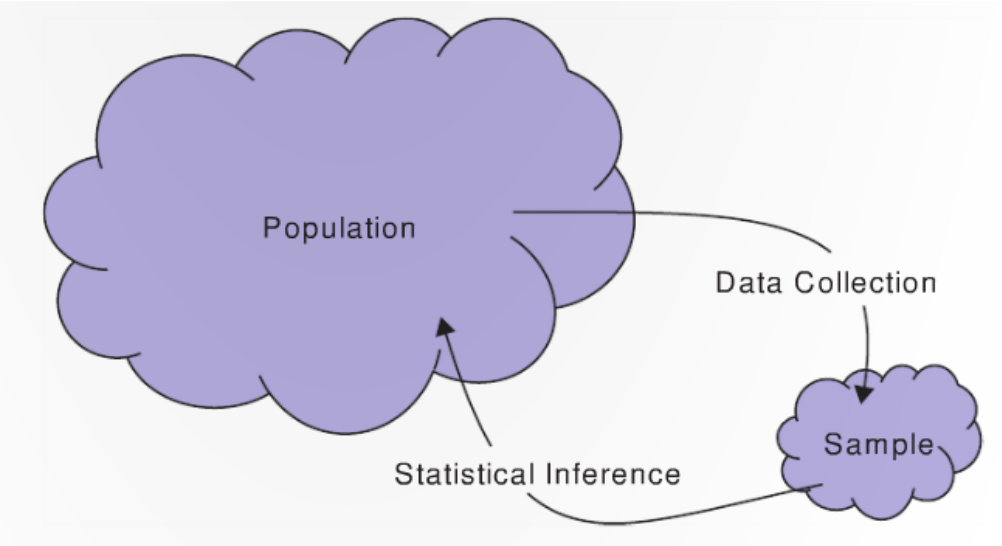
	flight	date	carrier	origin	dest	air_time	arr_delay
1	1545	1-1-2013	UA	EWR	IAH	227	11
2	1714	1-1-2013	UA	LGA	IAH	227	20
3	1141	1-1-2013	AA	JFK	MIA	160	33
4	725	1-1-2013	B6	JFK	BQN	183	-18
5	461	1-1-2013	DL	LGA	ATL	116	-25
6	1696	1-1-2013	UA	EWR	ORD	150	12
7	507	1-1-2013	B6	EWR	FLL	158	19

2. Sampling

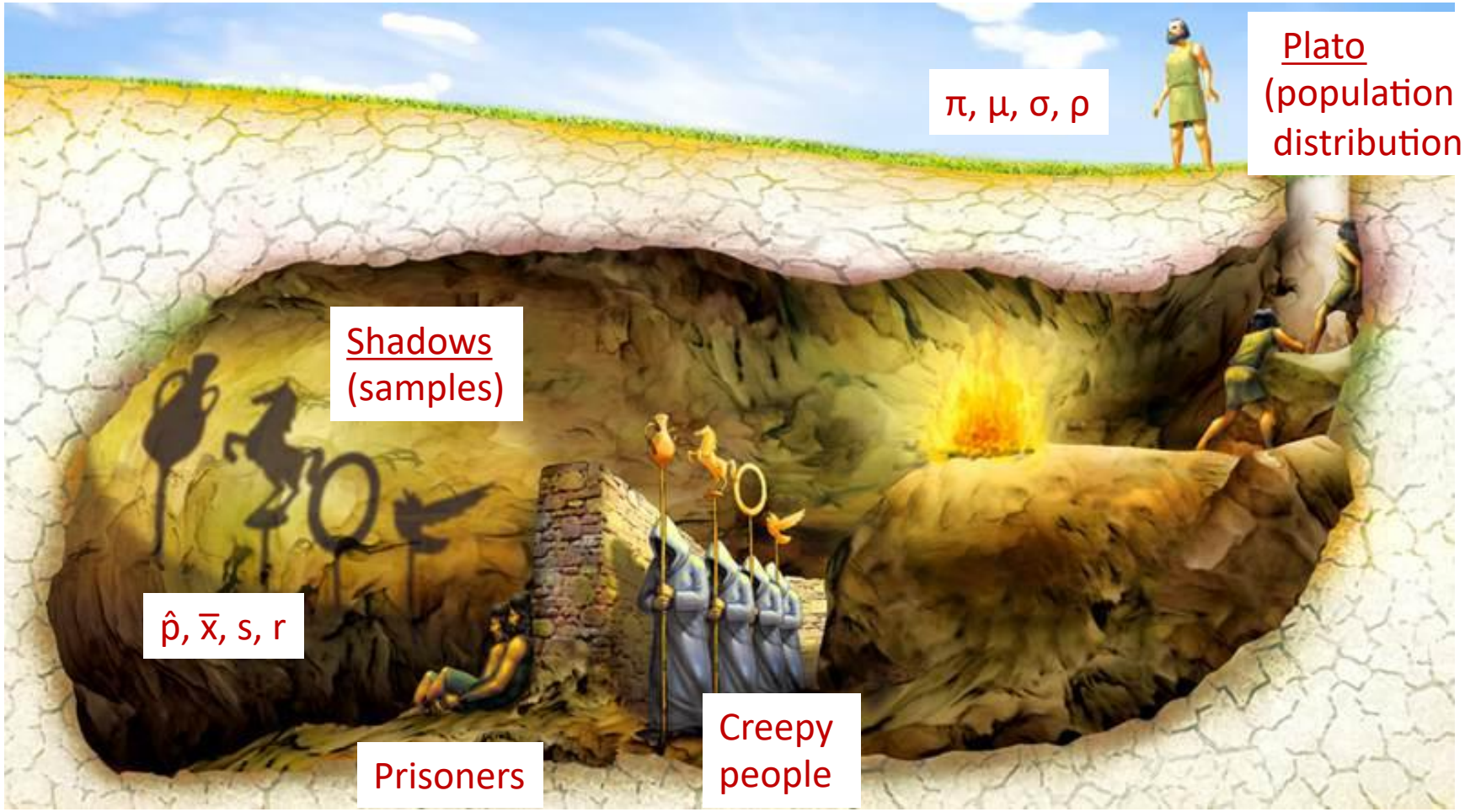
What is a …?

- sample
- population
- statistic
- parameter

What is statistical inference?

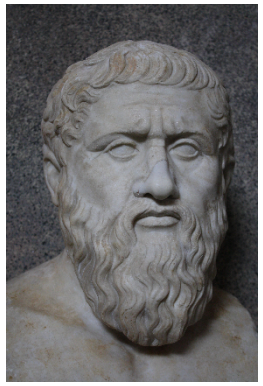


Plato's cave

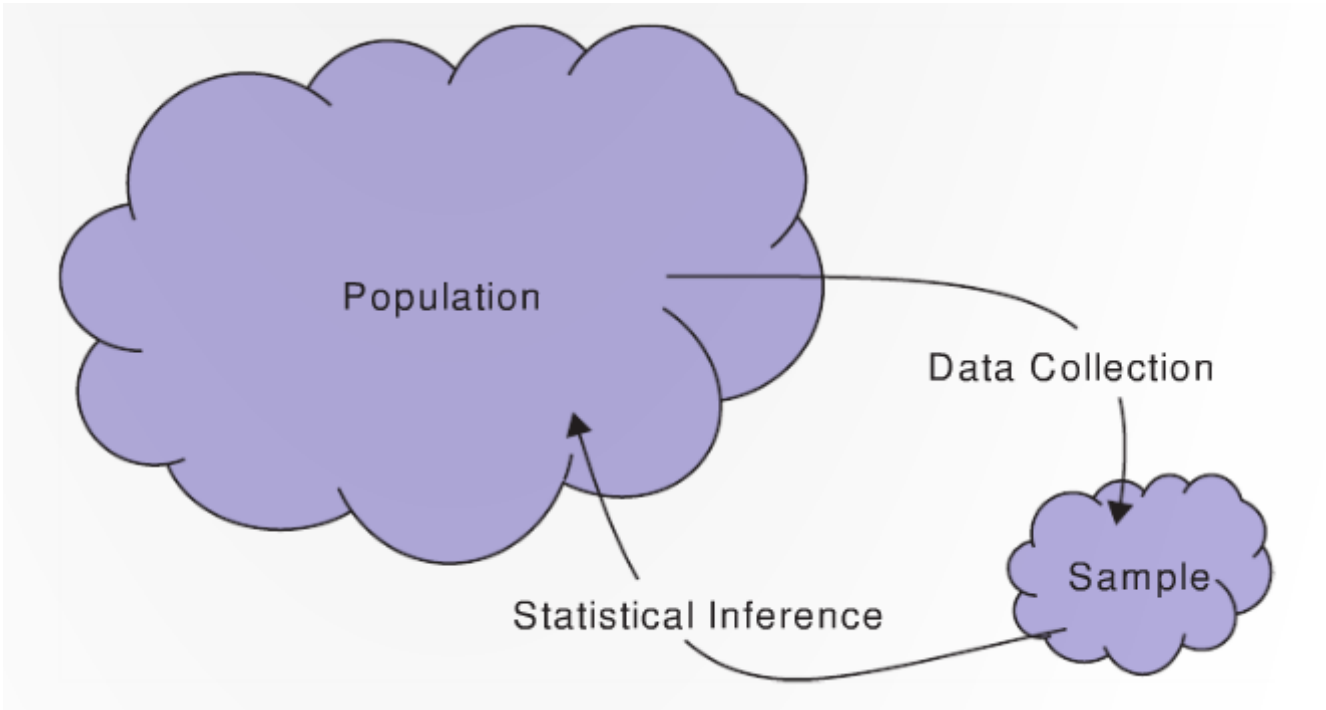


From The Republic (~ 380 BCE)

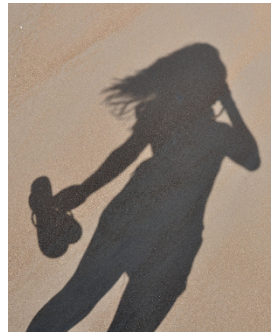
Population parameters vs. sample statistics



$\pi, \mu, \sigma, \rho, \beta$



$\hat{p}, \bar{x}, s, r, b$

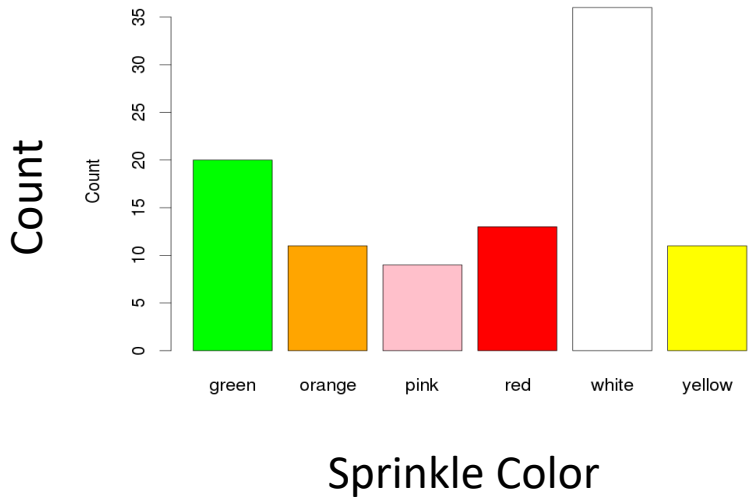


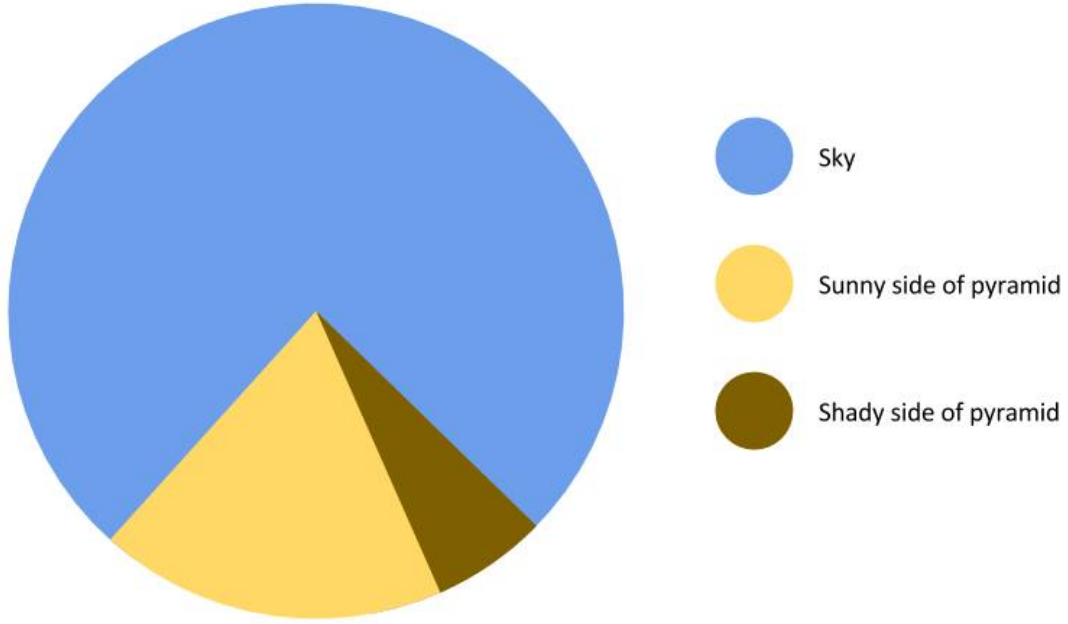
Categorical data

What is the main statistic we discussed for categorical data?

- π or $\hat{p}$
- proportion = number in category/total

How can we plot categorical data?

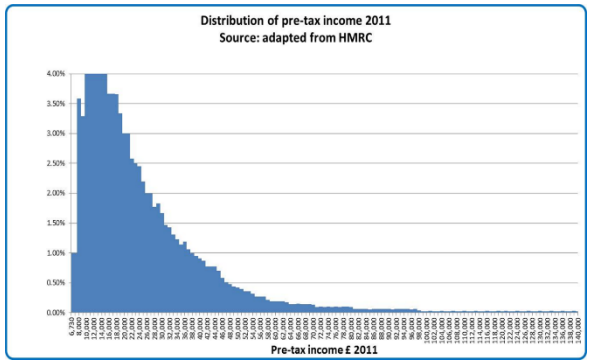


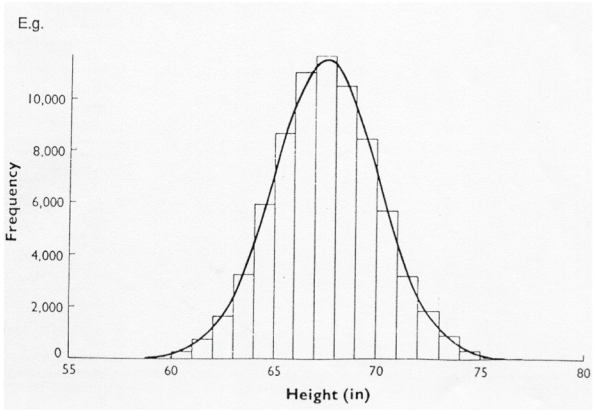


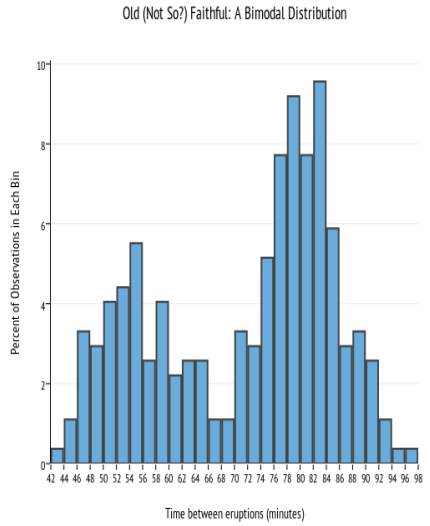
Quantitative data?

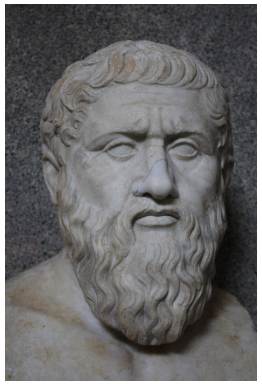
What is a good way to visualize the shape of quantitative data?

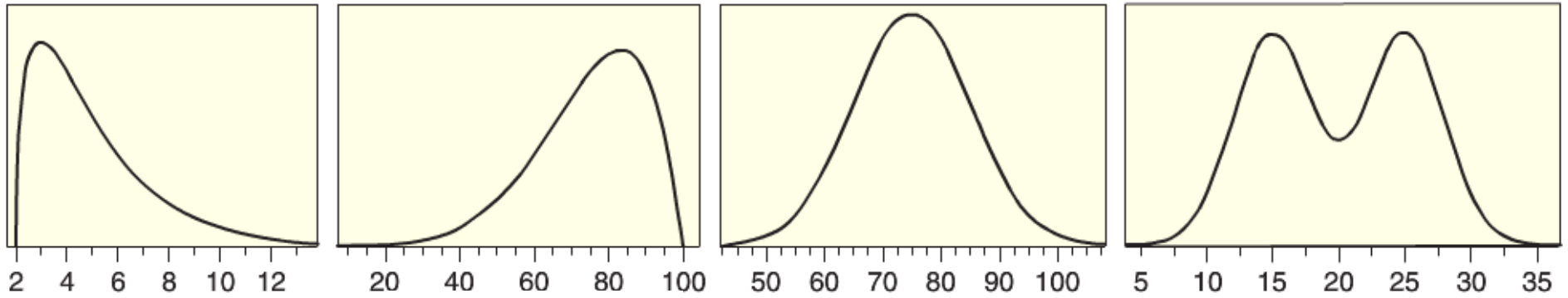
Income distribution



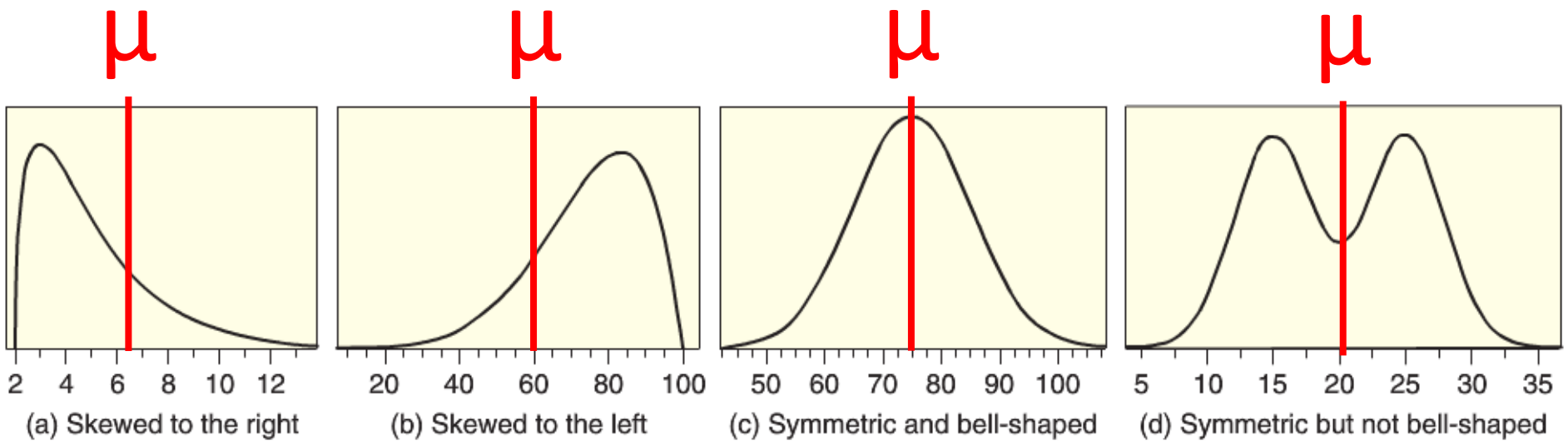






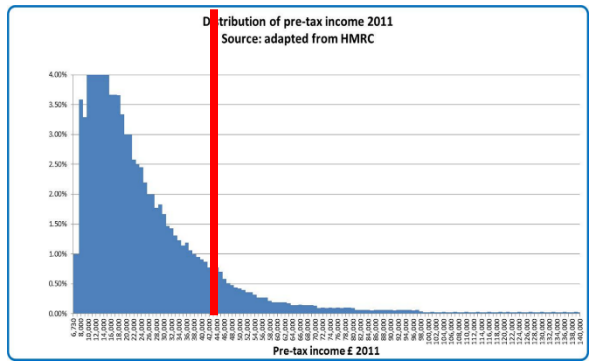


Measure of central tendency: the mean

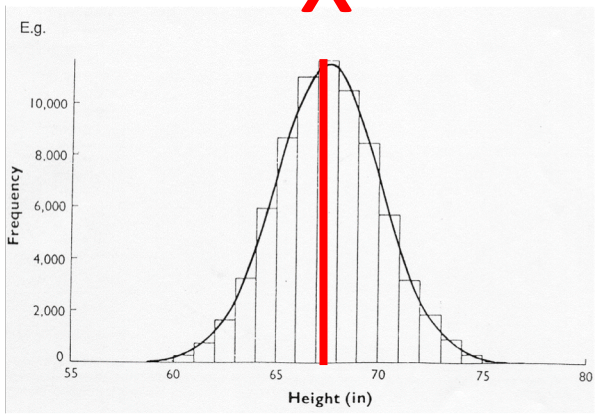


$$\frac{\sum_i^n x_i}{n}$$

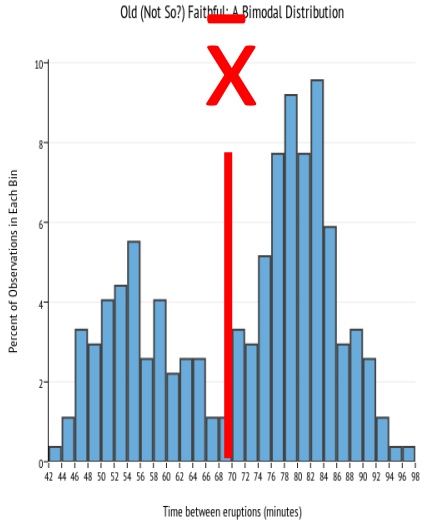
$\bar{x}$



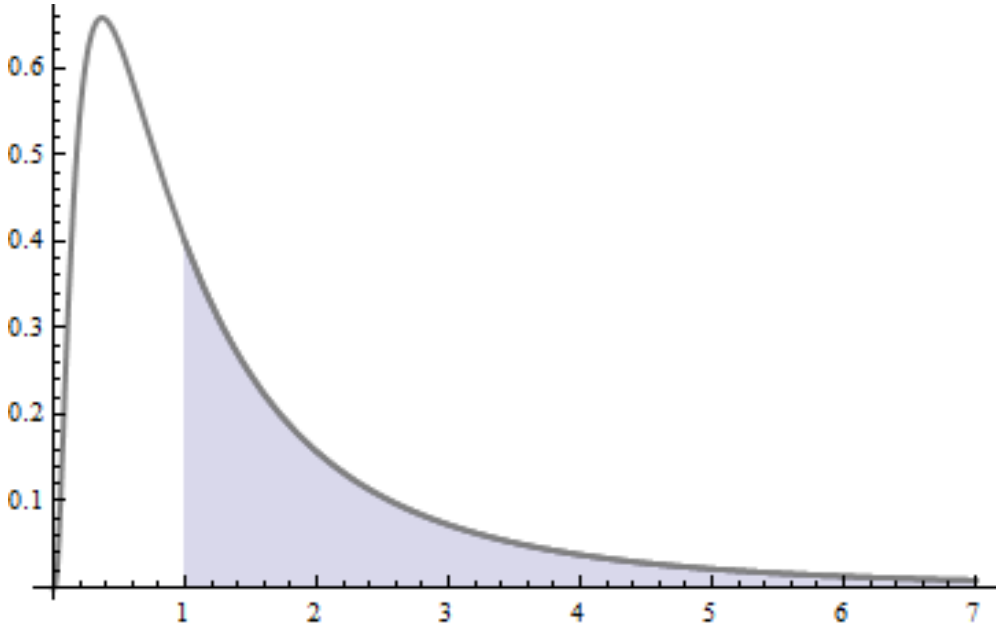
$\bar{x}$

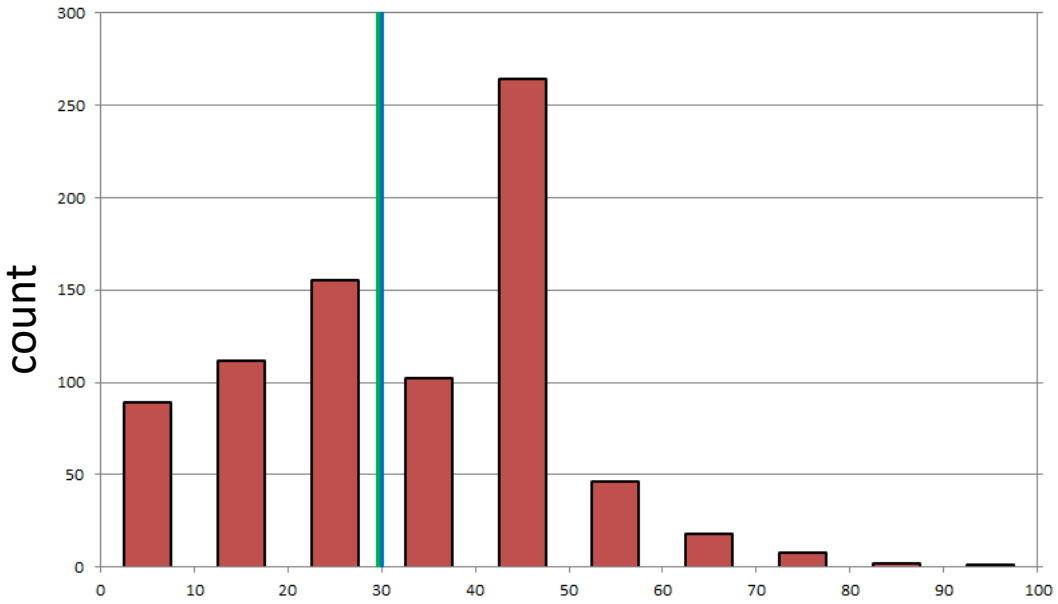


$\bar{x}$



Measure of central tendency: the median



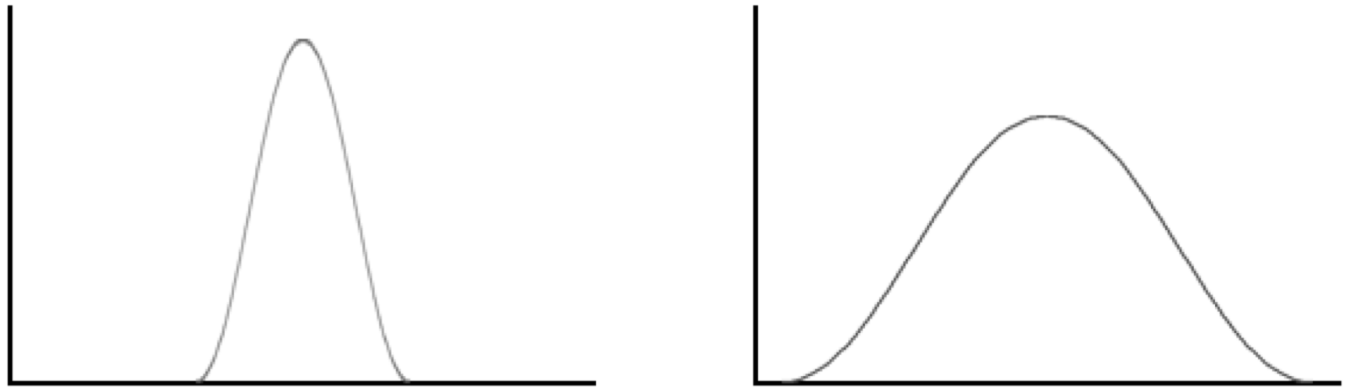


Which is resistant, the mean or the median?

The standard deviation

Which distribution has a larger standard deviation?

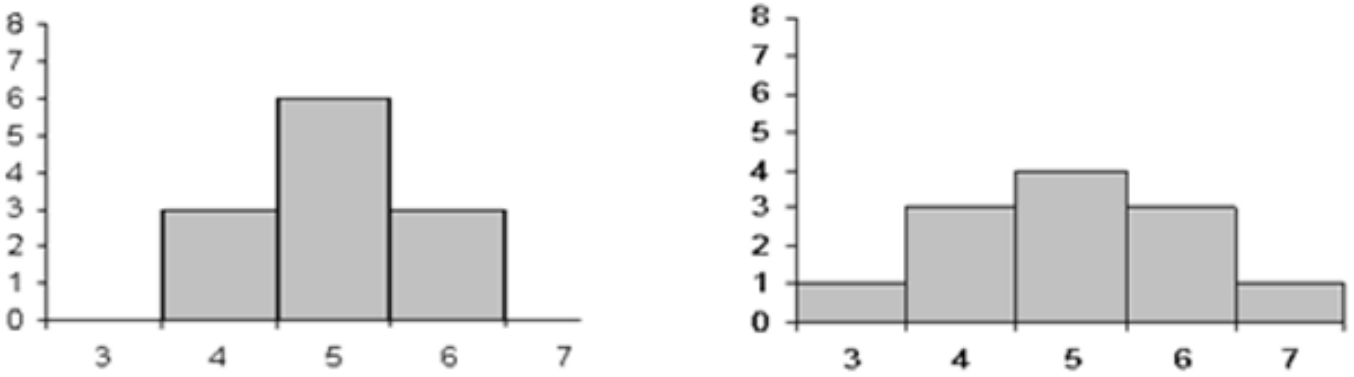
parameter σ



The standard deviation

Which distribution has a larger standard deviation?

statistic: s



What is the formula for the standard deviation?

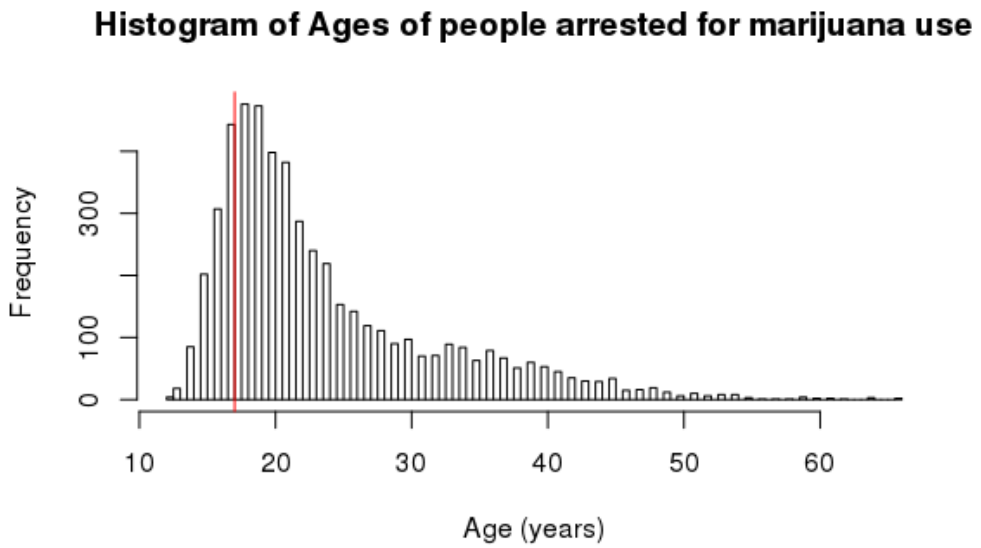
$$s = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

z-scores and percentiles

What is a z-score and why is it useful?

$$\text{z-score}(x_i) = \frac{x_i - \bar{x}}{s}$$

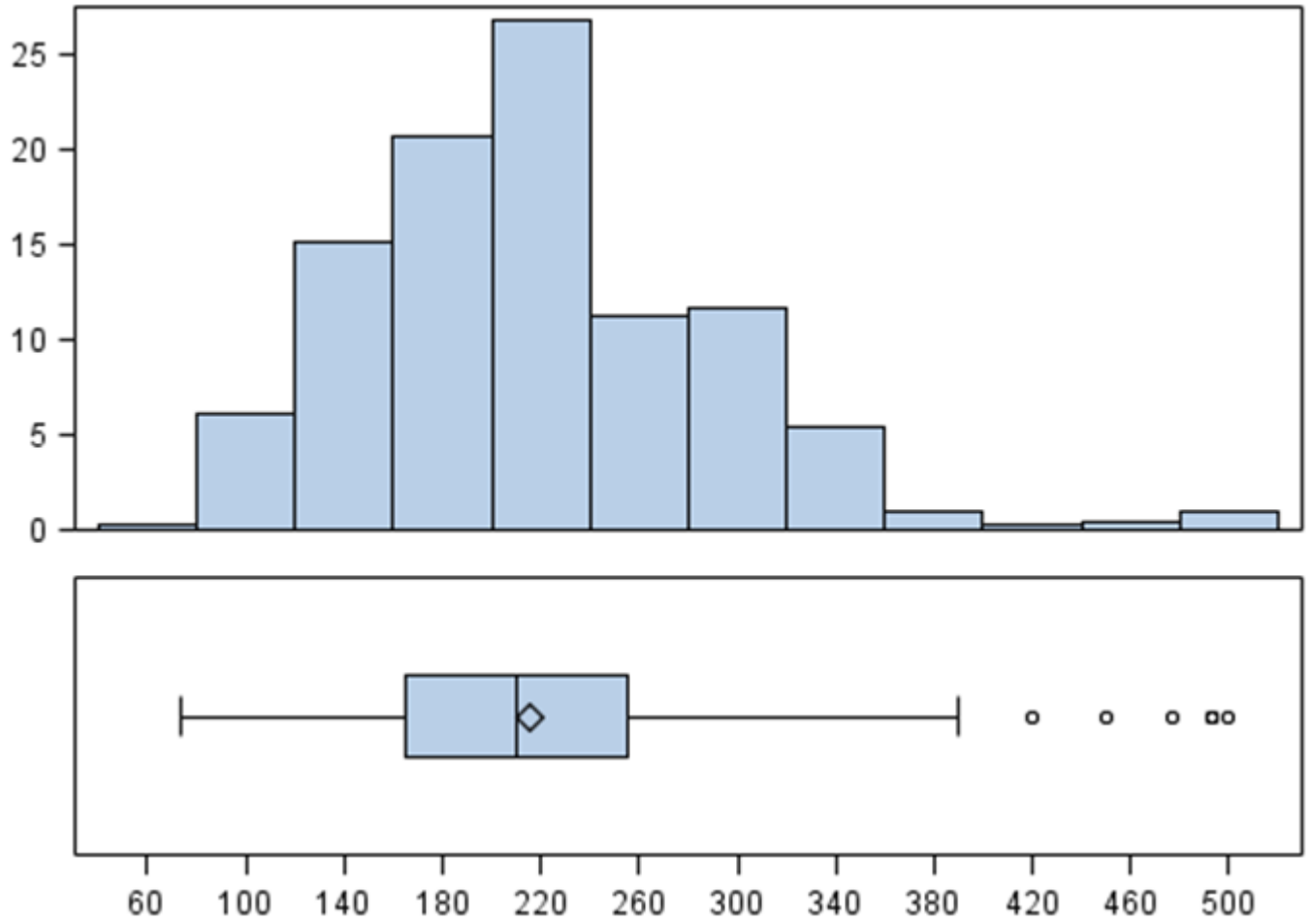
What is the p^{th} percentile?



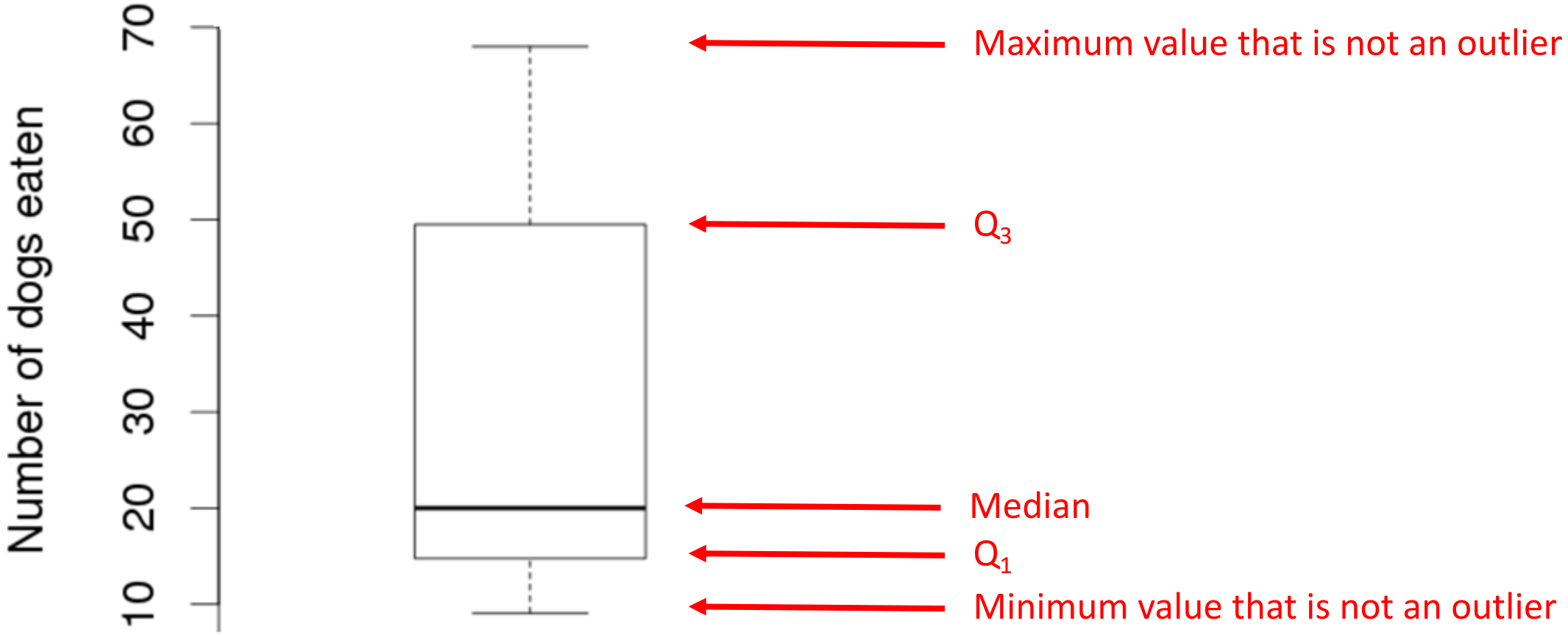
Normal pillow

What percent of the pillow's mass is ± 2 standard deviations from the mean?

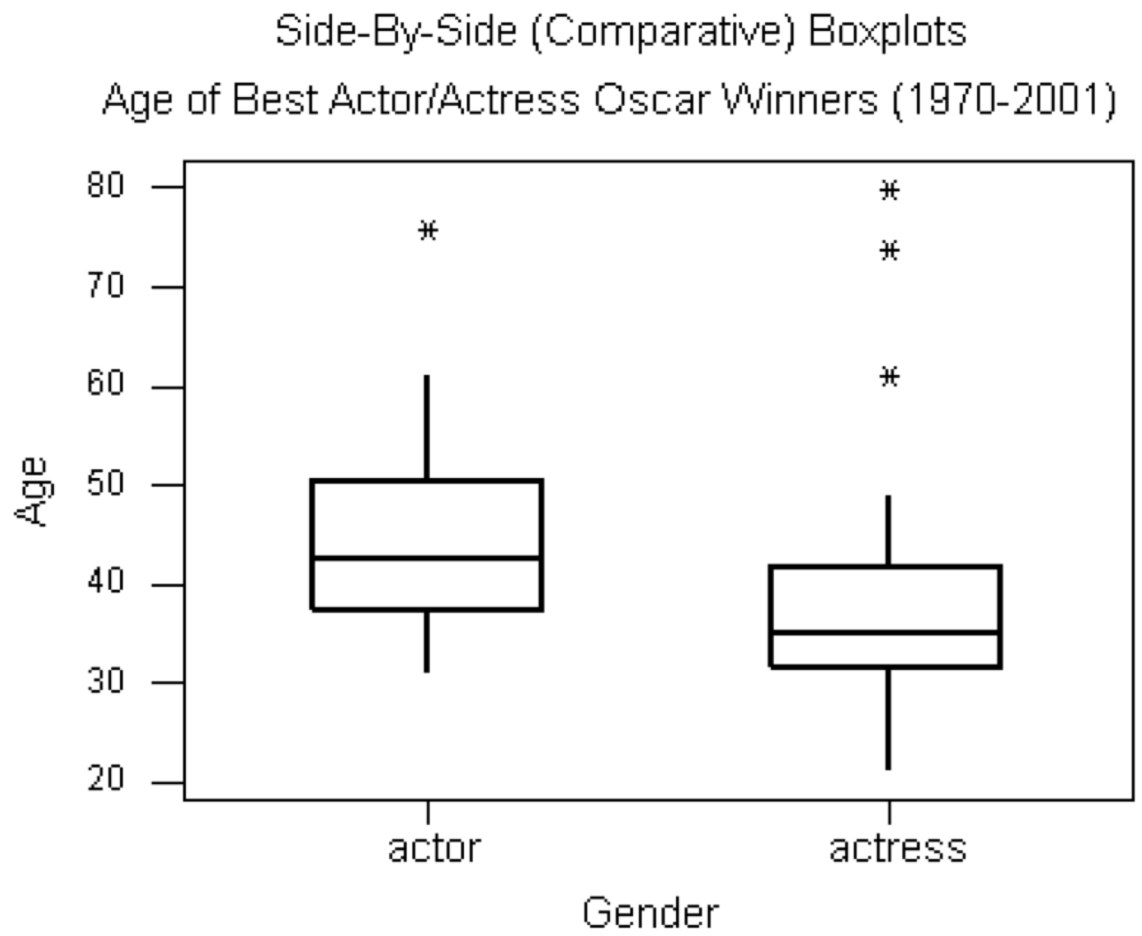
What is a 5 number summary and a boxplot?



What is a 5 number summary and a boxplot?

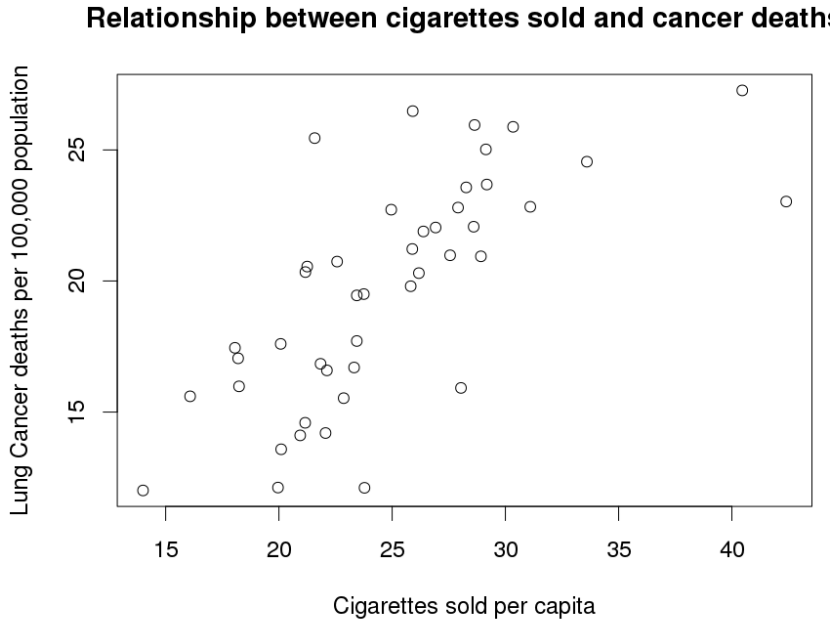


Side-by-side boxplots



Relationships between measures

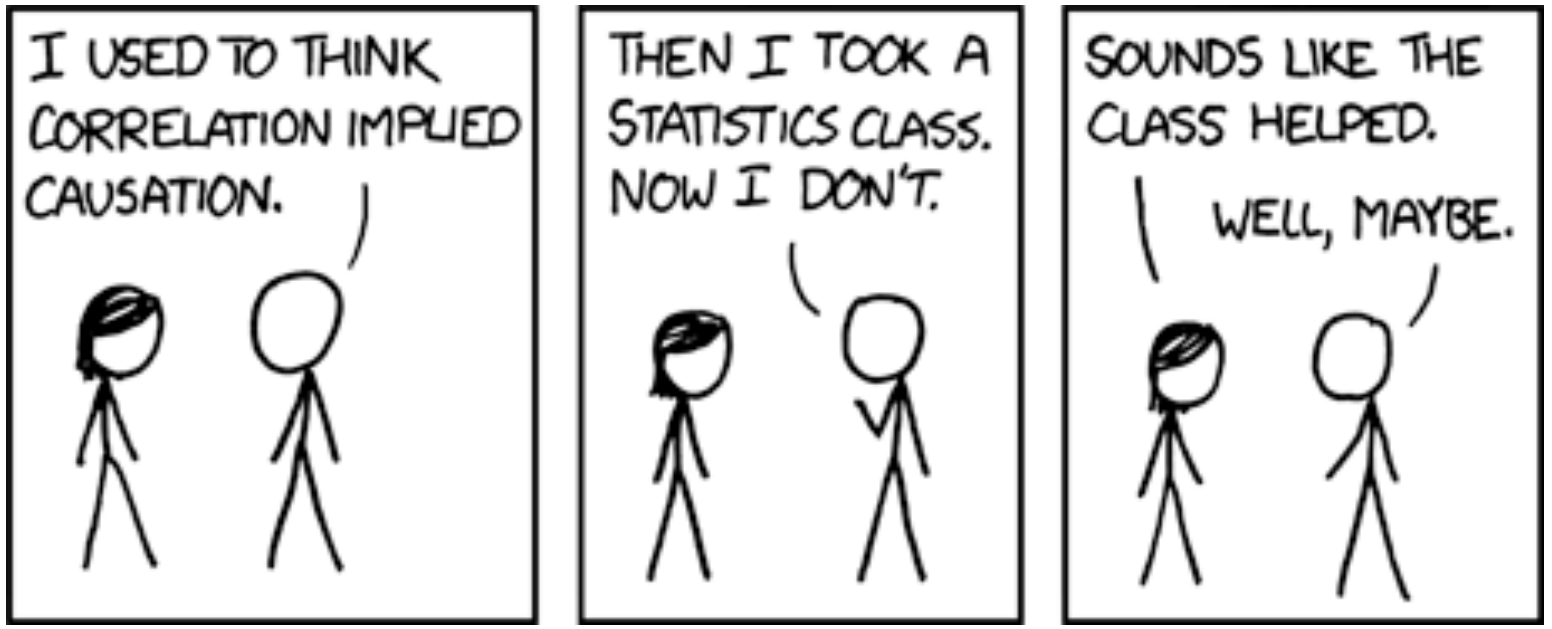
Q: What is this type of plot called?



Q: What statistic have we used to describe the linear relationship between quantitative variables?

$$r = \frac{1}{(n-1)} \sum_{i=1}^{n} \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

Does correlation imply causation?

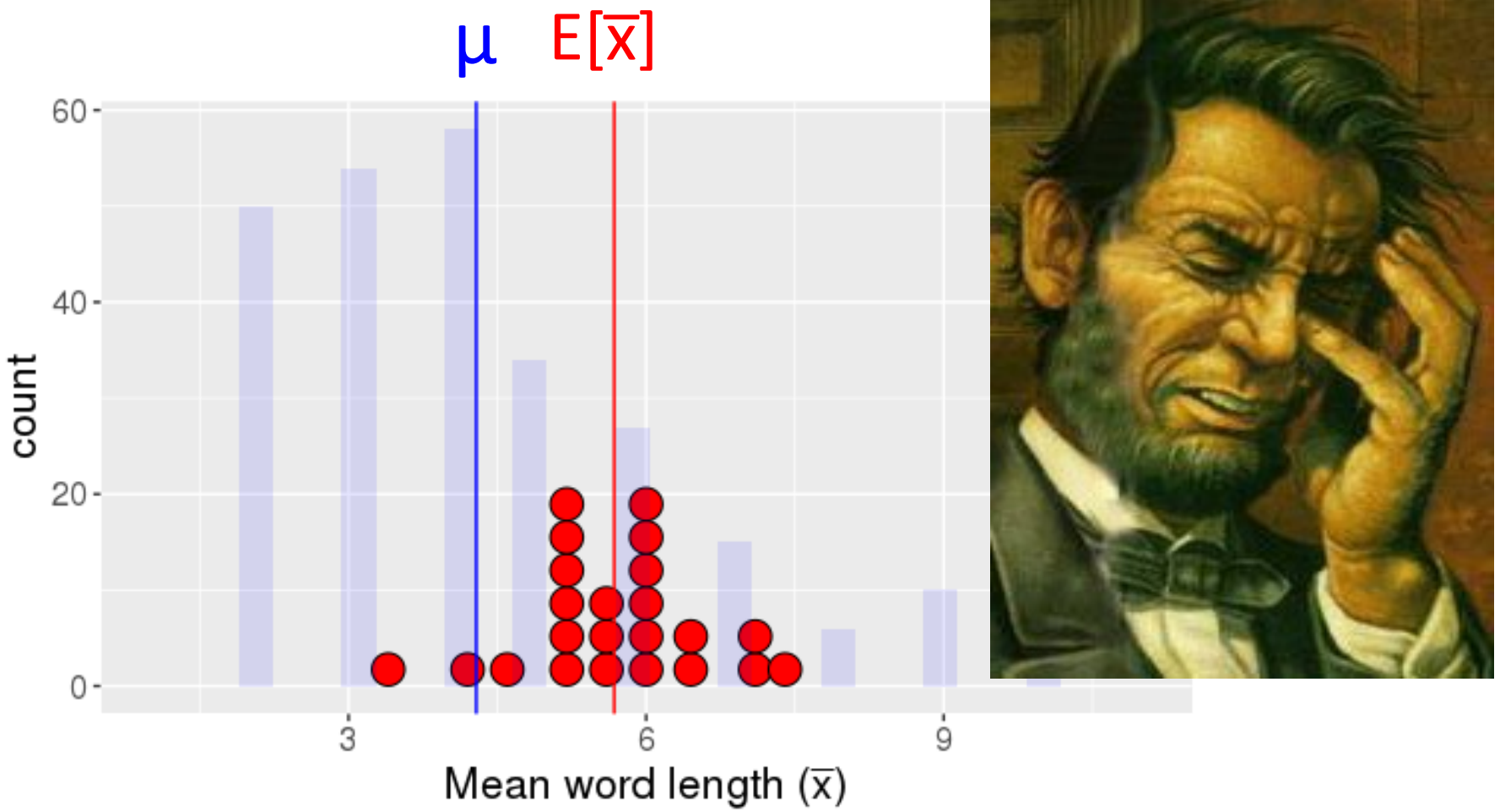


Bias and the Gettysburg address word length distribution

Bias is when our average statistic does not equal the population parameter

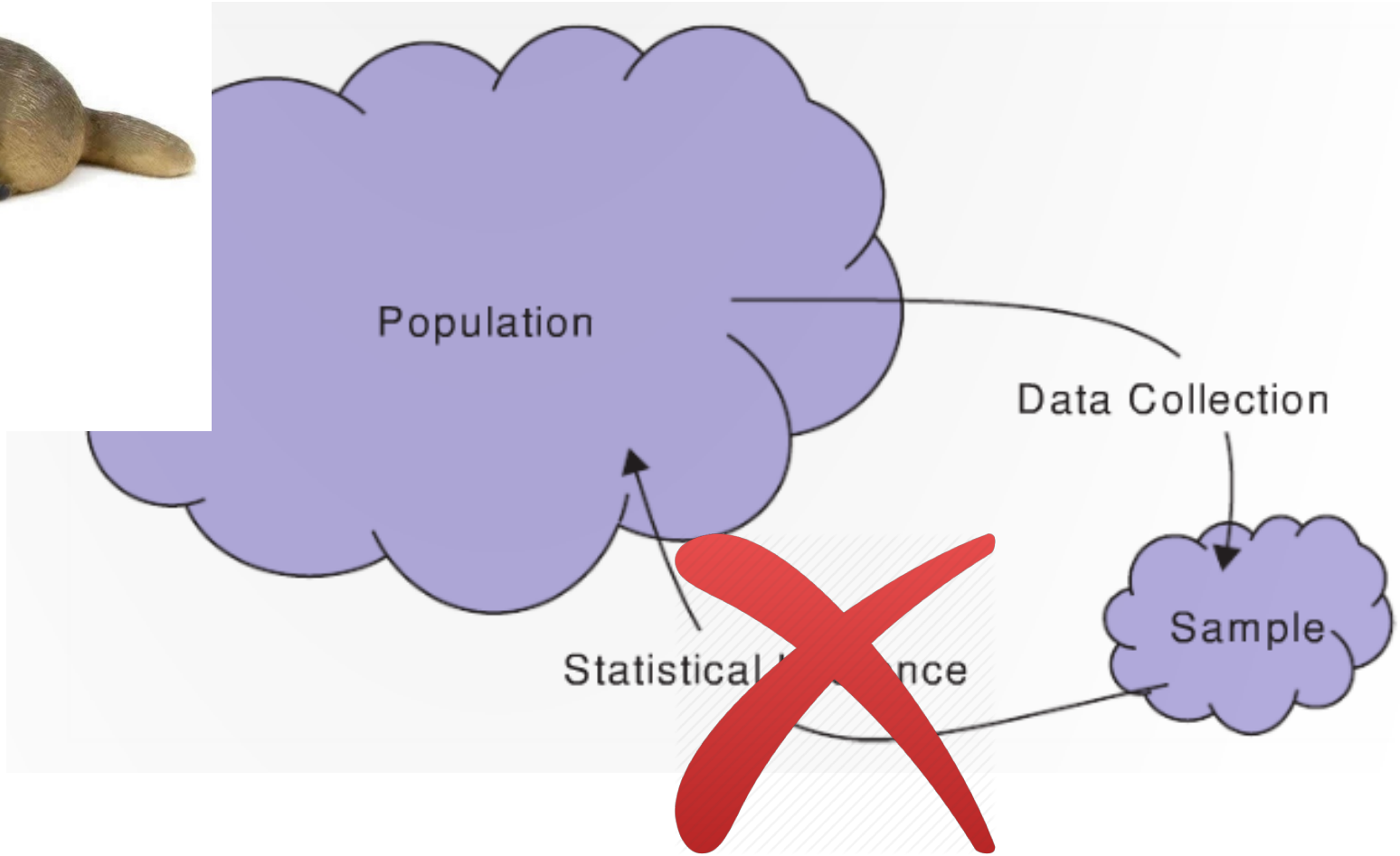
Here:

$E[\bar{x}] \neq \mu$



Statistical bias

μ



$\bar{x}$

To prevent bias: use simple random sample!

Simple random sample: each member in the population is equally likely to be in the sample.

Allows for generalizations to the population!

Soup analogy!

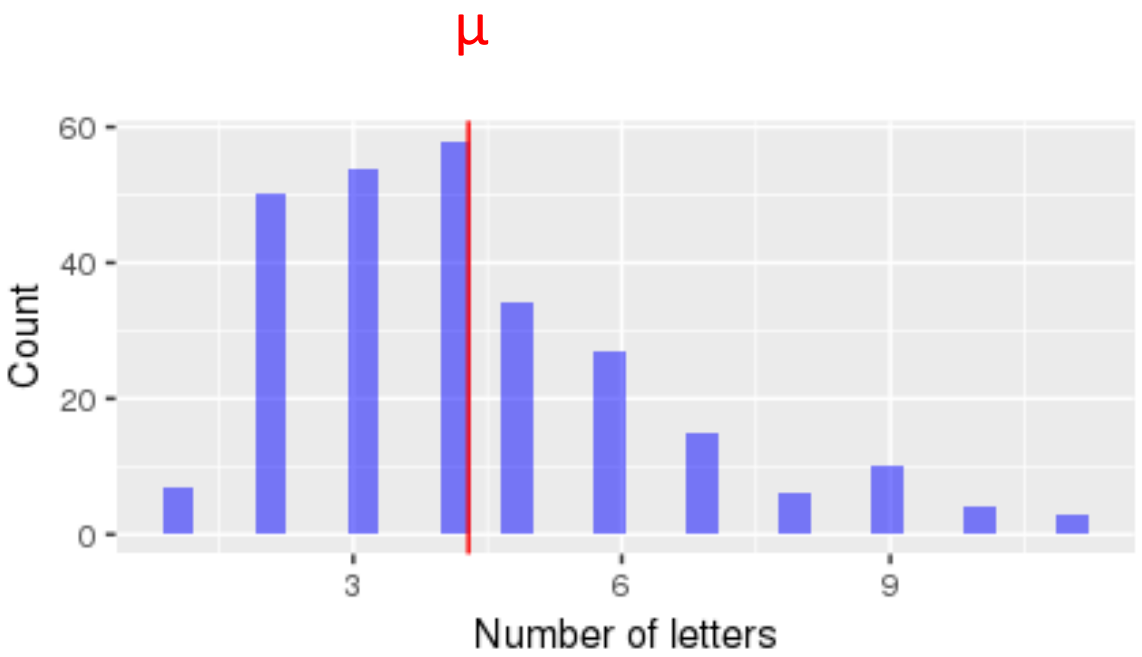
What is our primary focus in Statistics?

Sampling distribution

A **sampling distribution** is the distribution of sample statistics computed for different samples of the same size (n) from the same population

A sampling distribution shows us how the sample statistic varies from sample to sample

Gettysburg address word length sampling distribution



10, 3, 3, 3, 4,
3, 2, 6, 10, 5

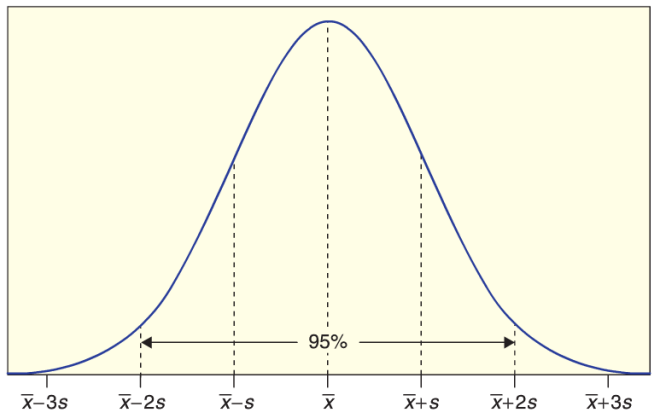
$\bar{x} = 5$

2, 6, 2, 6, 6,
2, 5, 3, 2, 9

$\bar{x} = 4.3$

3, 9, 3, 4, 4,
3, 6, 6, 2, 2

$\bar{x} = 4.2$

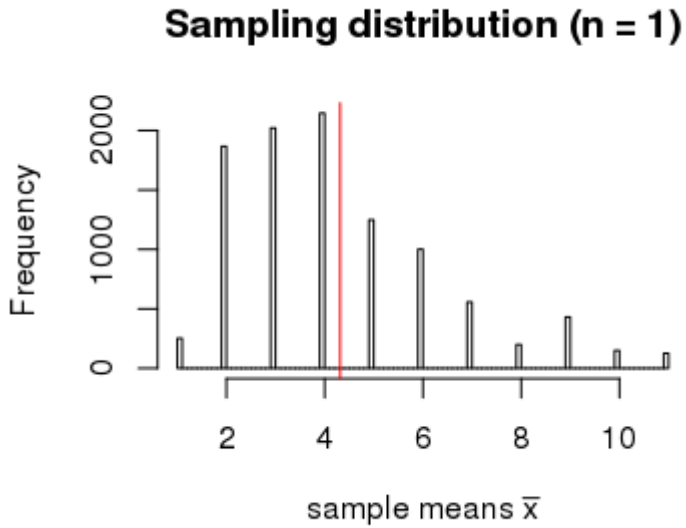


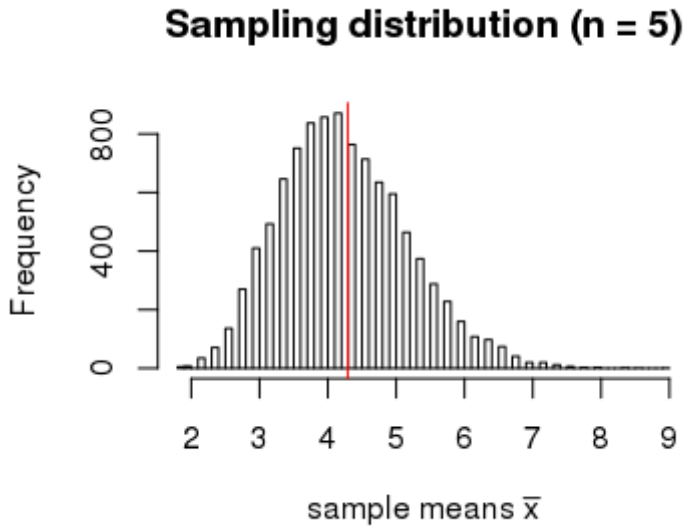
Sampling distribution!

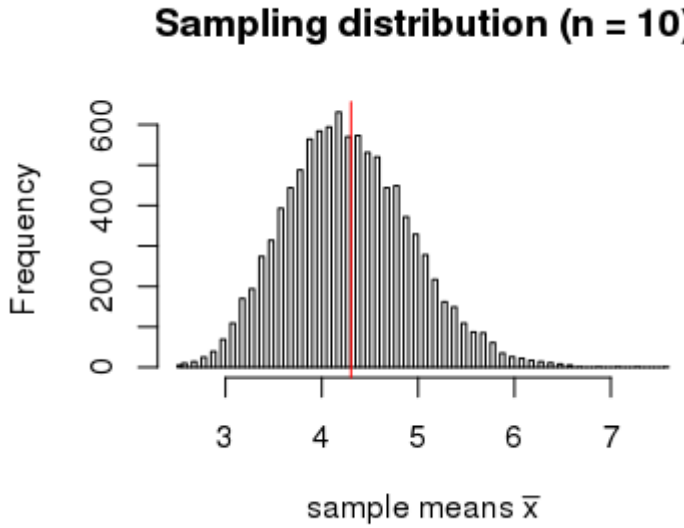
[Gettysburg sampling distribution app]

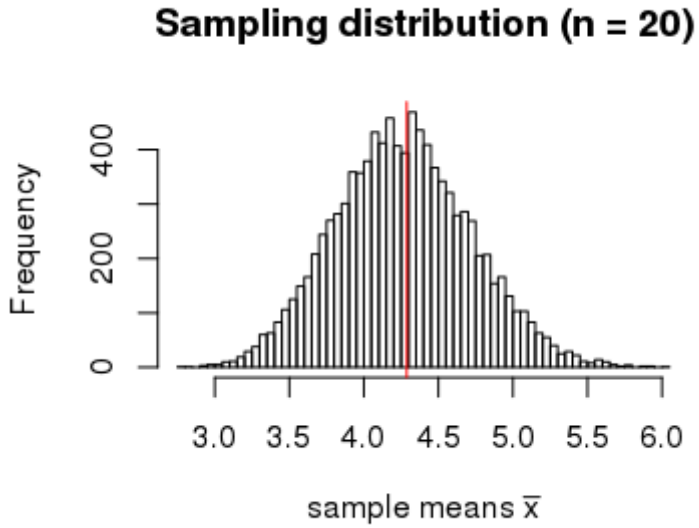
Creating a sampling distribution in R

```
sampling_dist <- do_it(10000)  *  {

        curr_sample <- sample(word_lengths, 10)
        mean(curr_sample)

}

hist(sampling_dist)
```









x-axis range 9 vs. 6

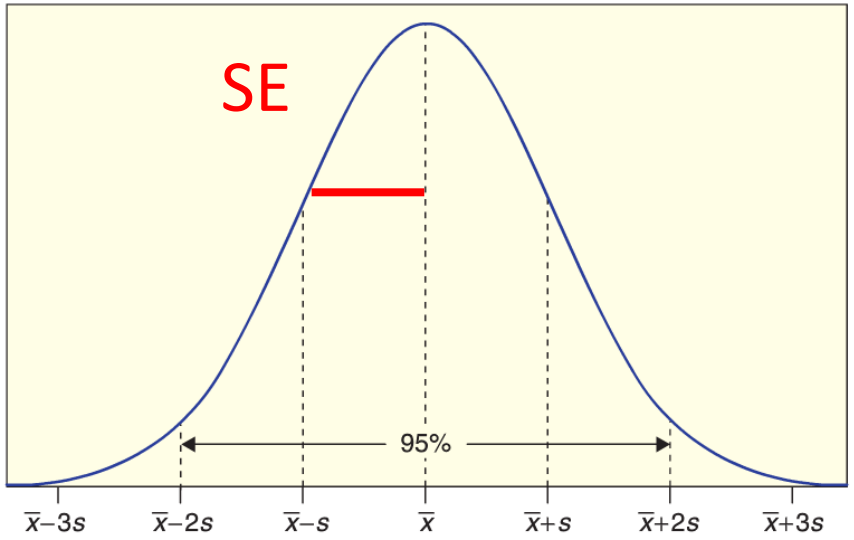
As the sample size n increases

1. The sampling distribution becomes more like a normal distribution
2. The sampling distribution points ($\bar{x}$'s) become more concentrated around the mean $E[\bar{x}] = \mu$

The standard error

The **standard error** of a statistic, denoted SE, is the standard deviation of the sample statistic

- i.e., SE is the standard deviation of the *sampling distribution*

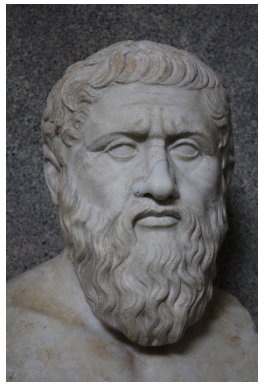


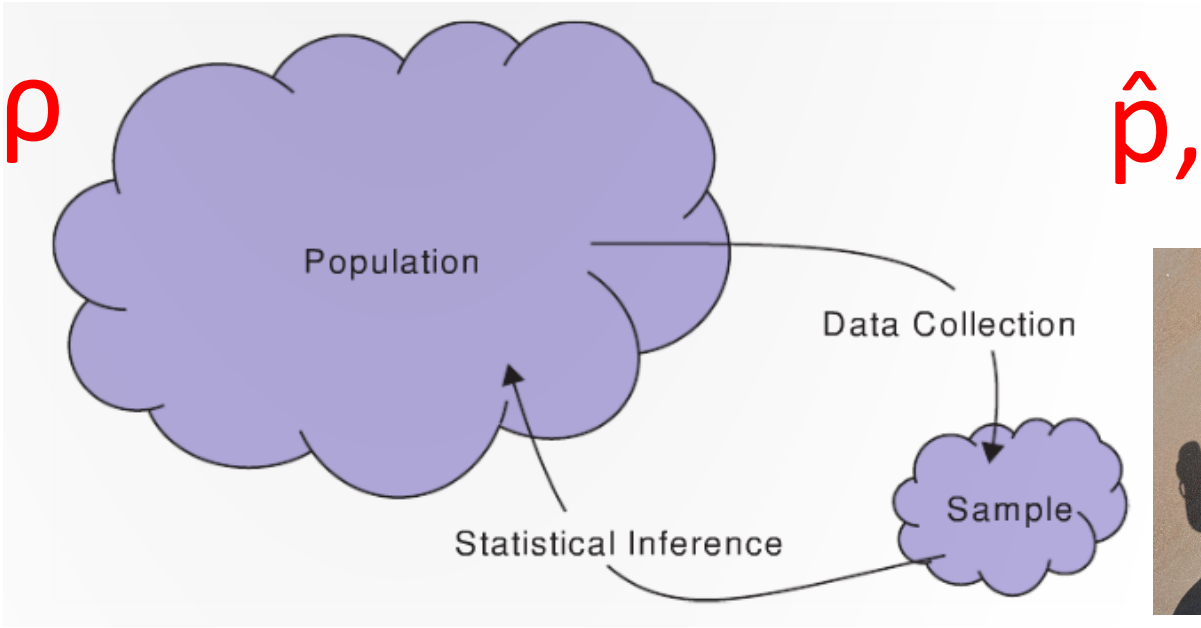
Back to the big picture: Inference

Statistical inference is…?

the process of drawing conclusions about the entire population based on information in a sample

π, μ, σ, ρ





$\hat{p}, \bar{x}, s, r$

Interval estimate based on a margin of error

We use the statistics from a sample as a **point estimate** for a population parameter

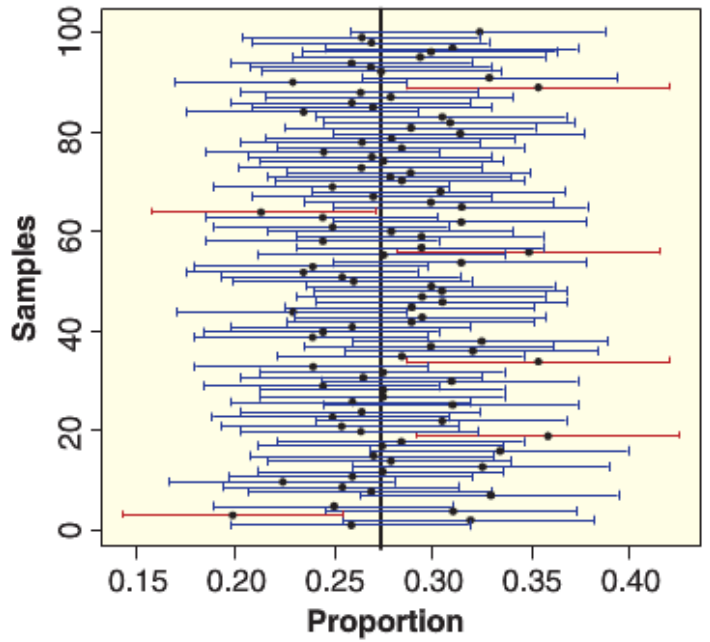
An **interval estimate** give a range of plausible values for a population parameter.

One common form of an interval estimate is:

Point estimate ± margin of error

Where the **margin of error** is a number that reflects the precision of the sample statistic as a point estimate for this parameter

Confidence Intervals

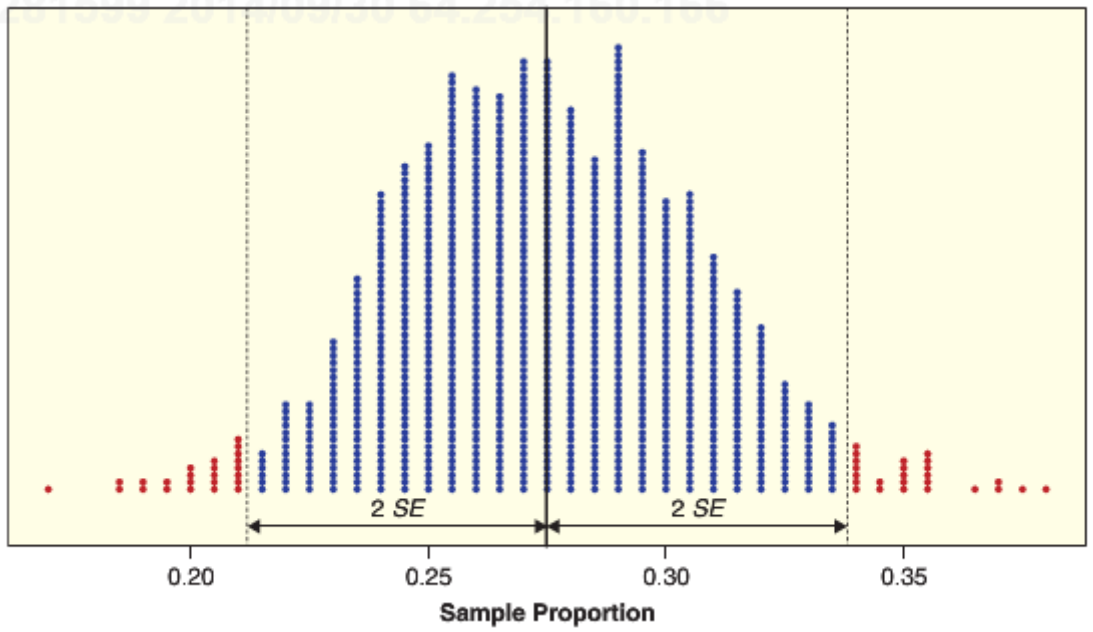


A **confidence interval** is an interval computed by a method that will contain the *parameter* a specified percent of times

The **confidence level** is the percent of all intervals that contain the parameter

Sampling distributions

For a sampling distribution that is a normal distribution, 95% of ***statistics*** lie within 2 standard deviations (SE) for the population mean



Thus if we had:

- A statistics value
- The SE

We could compute a 95% confidence interval!

$$CI_{95} = \bar{x} \pm 2 \cdot SE$$

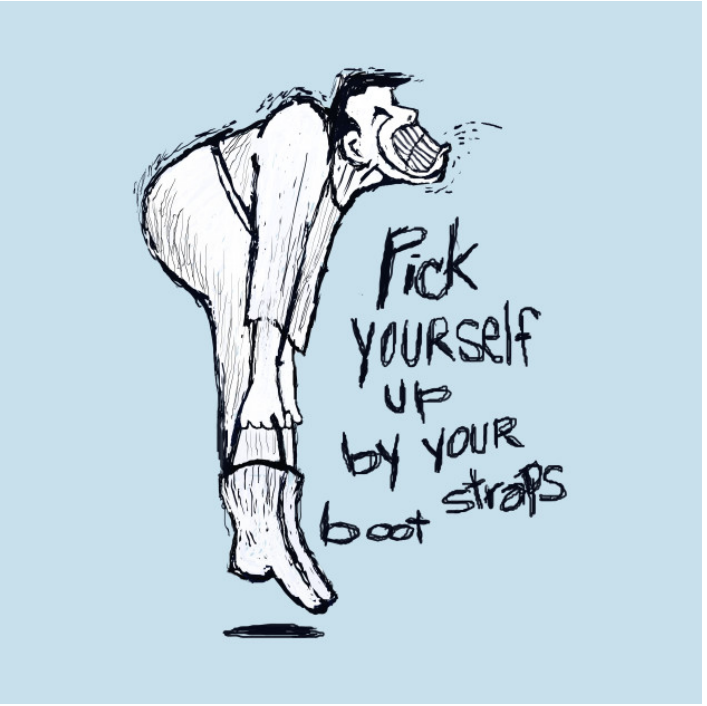
Sampling distributions

Unfortunately we can't calculate the sampling distribution ☹

- Therefore we can't get the SE from the sampling distribution ☹

We have to pick ourselves up by the bootstraps!

1. Estimate SE with $\hat{SE}$
2. Then use $\bar{x} \pm 2 \cdot \hat{SE}$ to get the 95% CI



Plug-in principle

Suppose we get a sample from a population of size *n*

We pretend that *the sample is the population* (plug-in principle)

1. We then sample *n* points *with replacement* from our sample, and compute our statistic of interest

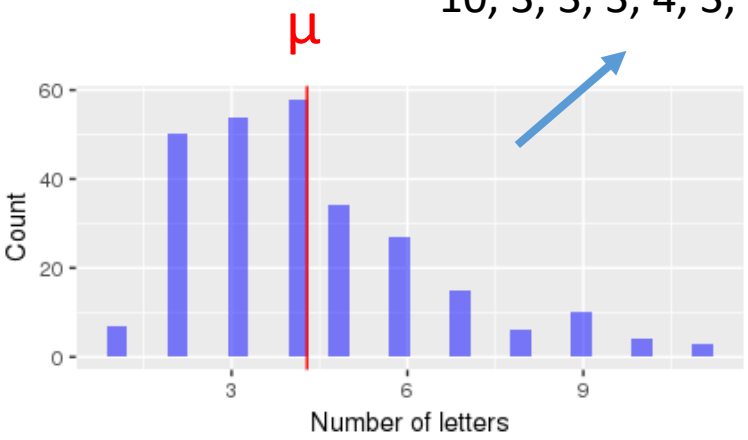
2. We repeat this process 1000's of times and get a ***bootstrap sample distribution***

3. The standard deviation of this bootstrap distribution (SE* bootstrap) is a good approximate for standard error SE from the real sampling distribution

Bootstrap distribution illustration

The sample (n = 10)

10, 3, 3, 3, 4, 3, 2, 6, 4, 5



3, 3, 3, 5, 3,
4, 5, 2, 2, 10

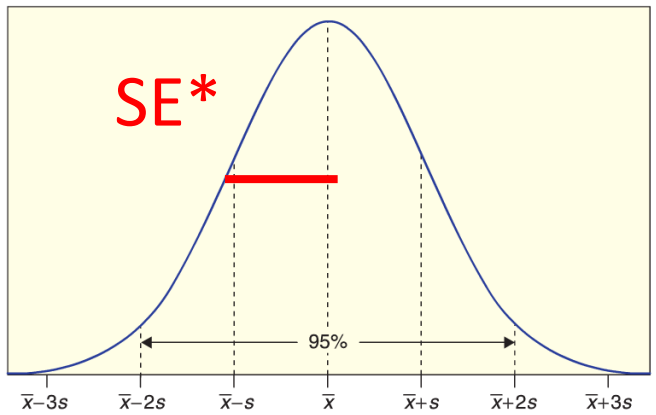
$\bar{x}^* = 4$

3, 3, 2, 3, 6,
4, 6, 5, 3, 6

$\bar{x}^* = 4.1$

5, 3, 2, 3, 3,
3, 10, 3, 4, 3

$\bar{x}^* = 3.9$



Bootstrap distribution!

Notice there is no 9's in the bootstrap samples

95% Confidence Intervals

When a bootstrap distribution for a sample statistic is approximately normal, we can estimate a 95% confidence interval using:

$$Statistic \pm 2 \cdot SE^*$$

Where SE* is the standard error estimated using the bootstrap

Bootstrap distribution in R

```
my_sample <- c(21, 29, 25, 19, 24, 22, 25, 26, 25, 29)

bootstrap_dist <-  do_it(10000) * {
        curr_boot <- sample(my_sample , 10, replace = TRUE)
        mean(curr_boot)
}

SE_boot <- sd(bootstrap_dist)
```

Bootstrap confidence interval in R

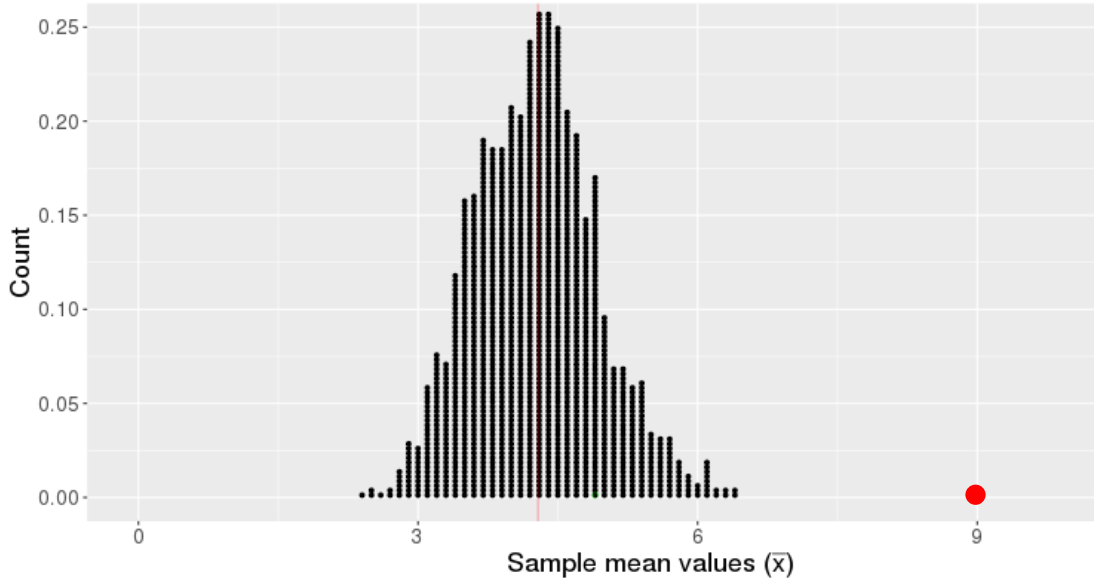
```
obs_mean <- mean(my_sample)

CI_lower <-  obs_mean  - 2 * SE_boot

CI_upper <-  obs_mean  + 2 * SE_boot
```

Basic hypothesis test logic

We start with a claim about a population parameter

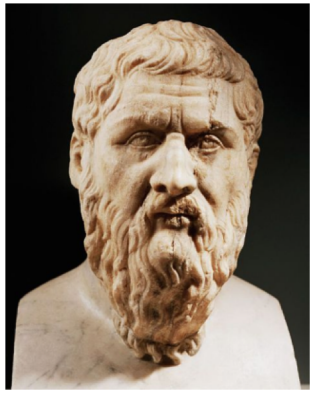
- E.g., μ = 4

This claim implies we should get a certain distribution of statistics



If our observed statistic is highly unlikely, we reject the claim

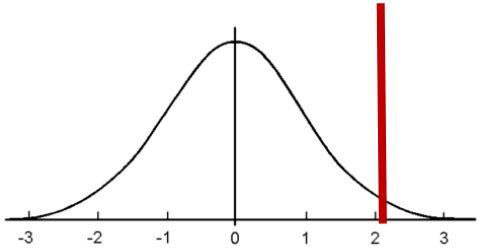
Five steps of hypothesis testing



1. State H_0 and H_A
   - Assume Gorgias (H_0) was right

2. Calculate the actual observed statistic

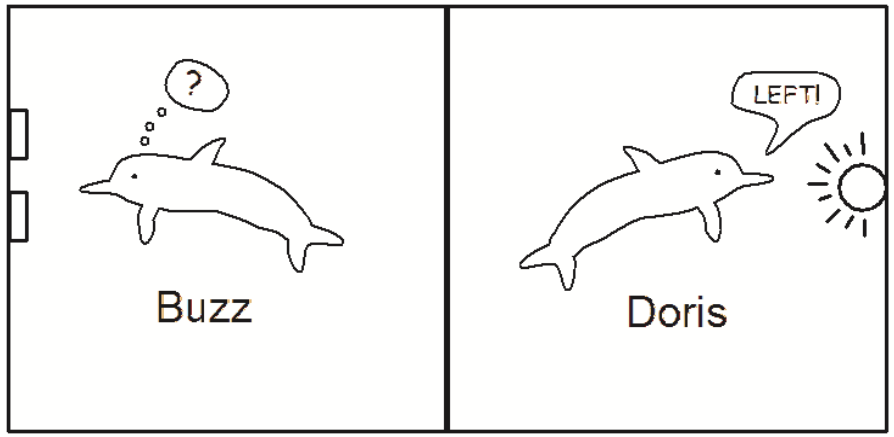
3. Create a distribution of what statistics would look like if Gorgias is right
   - Create the **null distribution** (that is consistent with H_0)

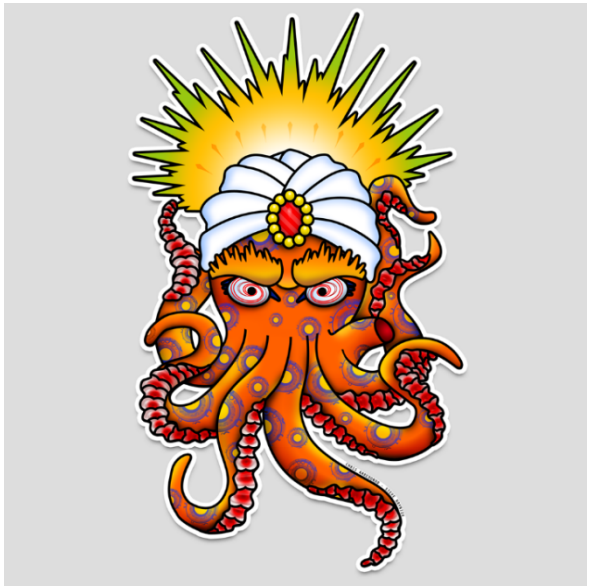


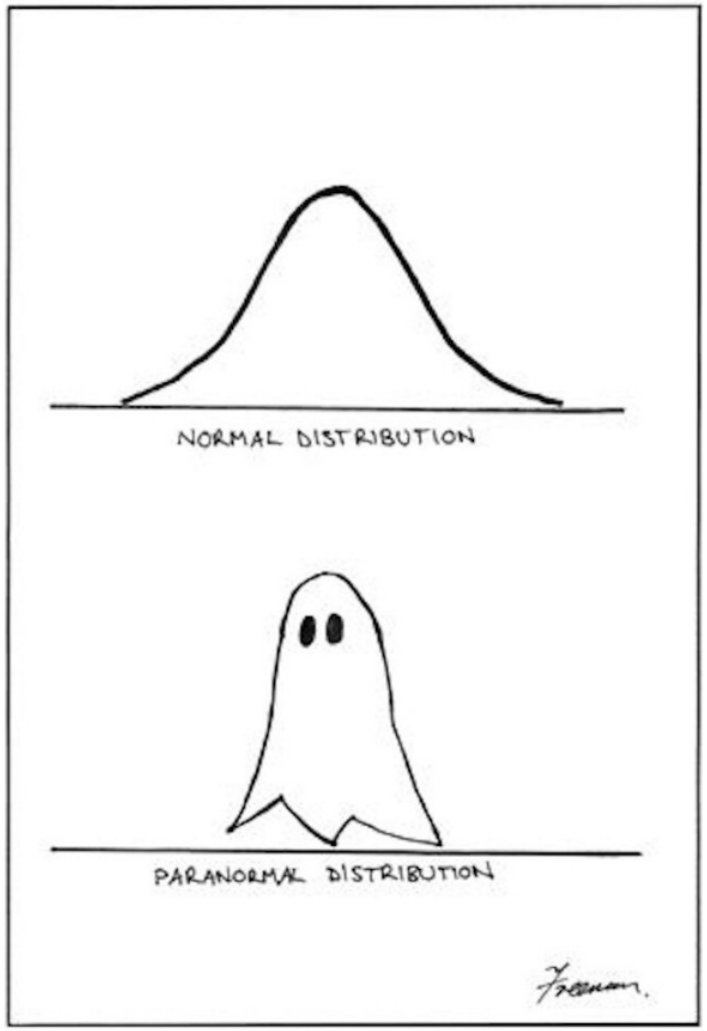
4. Get the probability we would get a statistic more than the observed statistic from the null distribution
   - p-value

5. Make a judgement
   - Assess whether the results are statistically significant

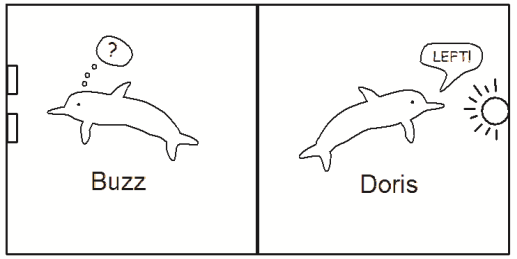
Hypothesis tests for a single proportion







Hypothesis tests for a single proportion



1. State the null hypothesis… and the alternative hypothesis
   - Buzz is just guessing so the results are due to chance: H_0: $\pi = 0.5$
   - Buzz is getting more correct results than expected by chance: H_A: $\pi > 0.5$

2. Calculate the observed statistic
   - Buzz got 15 out of 16 guesses correct, or $\hat{p} = .973$

3. Create a null distribution that is consistent with the null hypothesis
   - i.e., what statistics would we expect if Buzz was just guessing

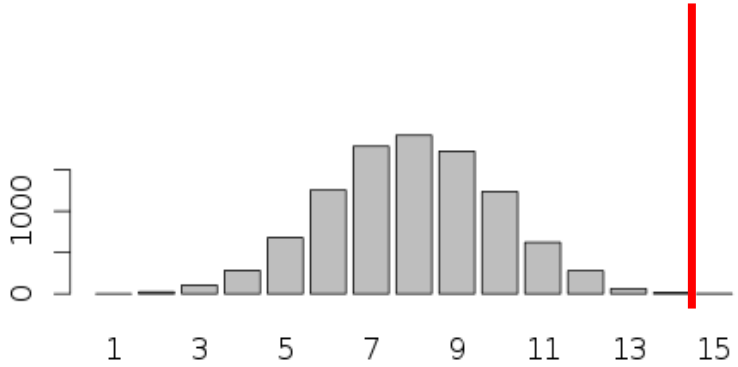
4. Examine how likely the observed statistic is to come from the null distribution
   - What is the probability that the dolphins would guess 15 or more correct?
   - i.e., what is the p-value

5. Make a judgement
   - If we have a small p-value, this means that $\pi = .5$ is unlikely and so $\pi > .5$
   - i.e., we say our results are 'statistically significant'

Getting p-values using ClassTools functions

Flipping coints many times:

```
flip_simulations  <-  do_it(10000) * {
      rflip_count(16,  prob = .5)
}
```



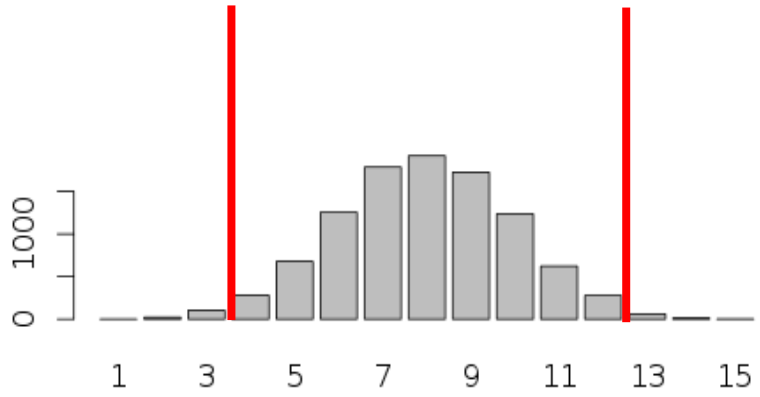
We can get the number of values as or more extreme than an observed statistic (obs_stat) using the pnull() function:

```
obs_stat <-  ?
p_value  <- pnull(obs_stat,  flip_simulations,  lower.tail = FALSE)
```

Estimating a p-value from a randomized distribution

For a one tailed alternative: Find the proportion of randomized samples that equal or exceed the original statistic in the direction (tail) indicated by the alternative hypothesis

For a two-tailed alternative: Find the proportion of randomization samples in the tails beyond the observed statistic and 1 - the observed statistic



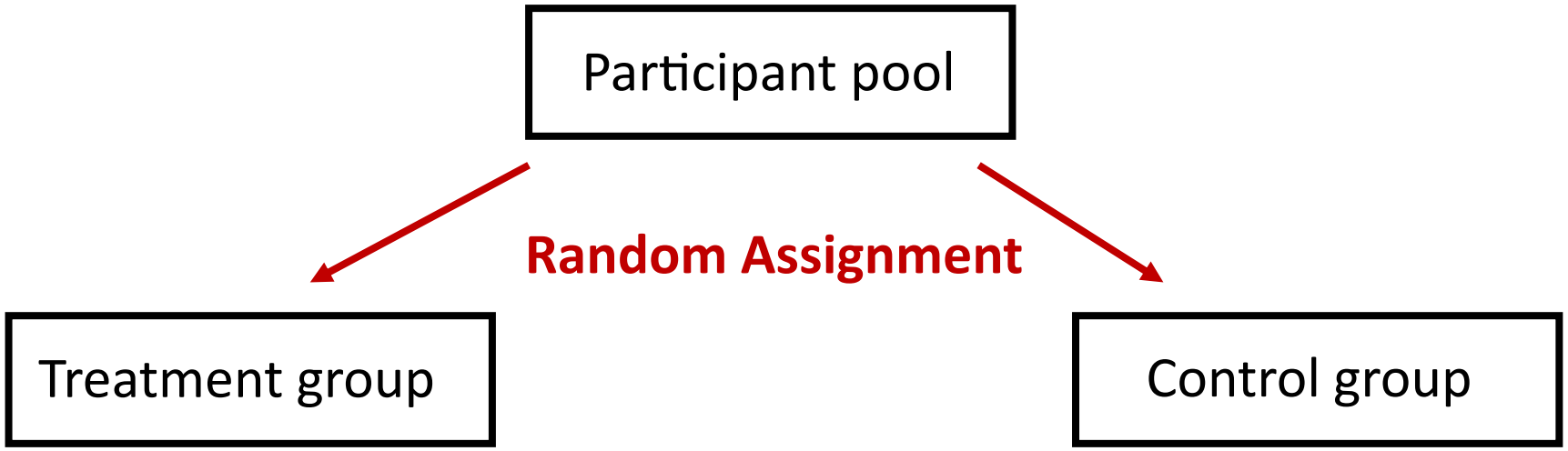
Hypothesis tests for comparing two means

Question: Can we find out the ***Truth*** of whether the pill effective?

Experimental design

Take a group of participant and ***randomly assign***:

- Half to a *treatment group* where they get the pill
- Half in a *control group* where they get a fake pill (placebo)
- See if there is more improvement in the treatment group compared to the control group

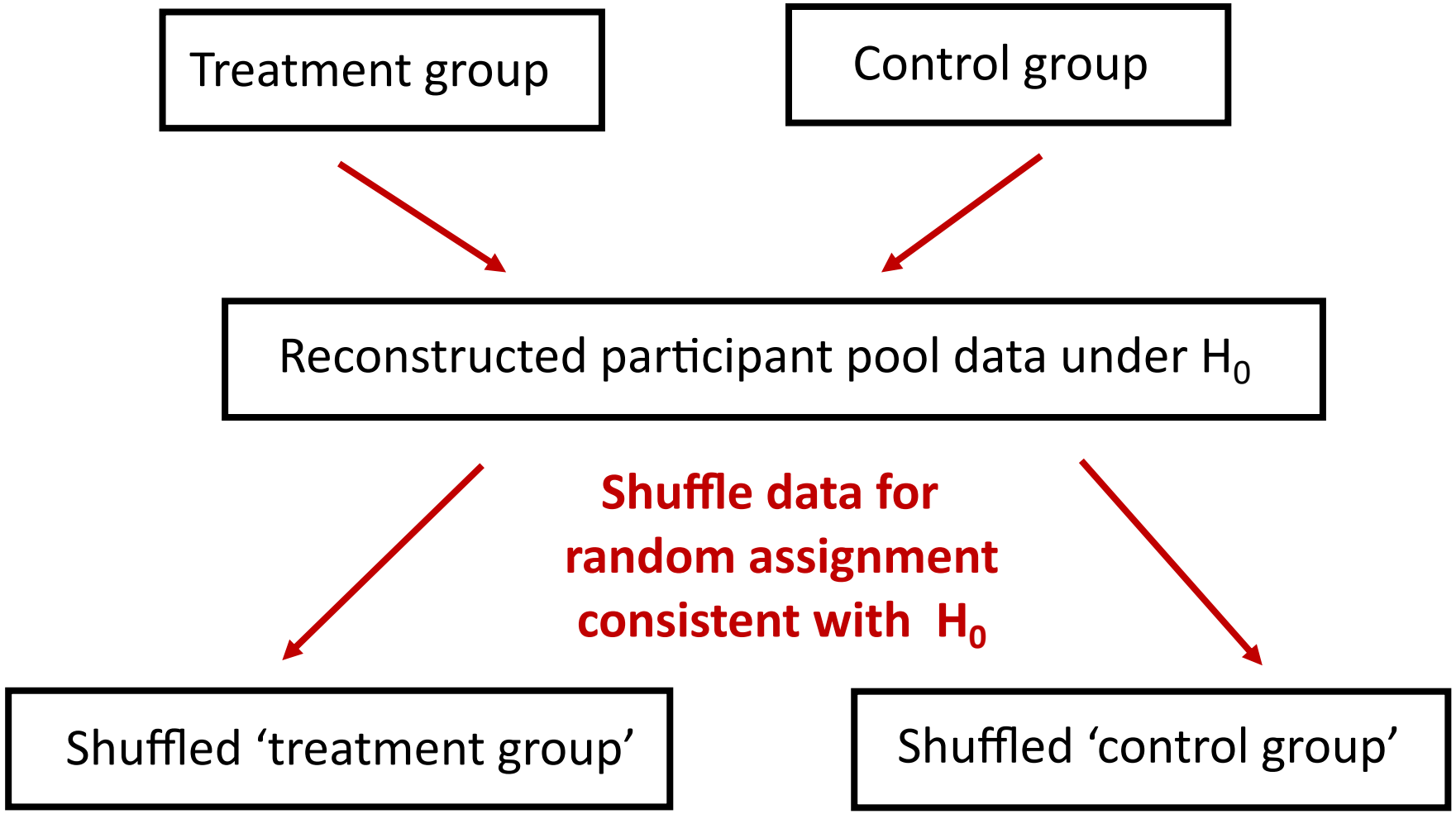


Hypothesis tests for differences in two group means

1) State the null and alternative hypothesis
   - H_0: $\mu_{Treatment} = \mu_{Control}$ or $\mu_{Treatment} - \mu_{Control} = 0$
   - H_A: $\mu_{Treatment} > \mu_{Control}$ or $\mu_{Treatment} - \mu_{Control} > 0$

2) Calculate statistic of interest
   - $\bar{x}_{Effect} = \bar{x}_{Treatment} - \bar{x}_{Control}$

3. Create the null distribution!



One null distribution statistic: $\bar{x}_{Shuff\_Treatment} - \bar{x}_{Shuff\_control}$

3. Creating a null distribution in R

```
# the data from the calcium study
treat <- c(7, -4, 18, 17, -3, -5,  1, 10, 11, -2)
control <- c(-1,  12,  -1,  -3,   3,  -5,   5,   2, -11,  -1,  -3)

# observed statistic
obs_stat <- mean(treat) - mean(control)

#  Combine data from both groups
combined_data <- c(treat, control)
```

3. Creating a null distribution in R

```
null_distribution <-  do_it(10000) * {

    shuff_data <- shuffle(combined_data)      # shuffle data

    # create fake treatment and control groups
    shuff_treat   <-  shuff_data[1:10]
    shuff_control  <-  shuff_data[11:21]

    # save the statistic of interest
    mean(shuff_treat) - mean(shuff_control)
}

p_value <- pnull(obs_stat, null_distribution, lower.tail = FALSE)
```

Testing more than two means

1. State the null and alternative hypotheses!

$\mathbf{H_0}$: $\mu_1 = \mu_2 = \ldots = \mu_k$
$\mathbf{H_A}$: $\mu_i \neq \mu_j$ for one pair of fields of study

2. Calculate an observed statistic
3. Create a null distribution
4. Calculate a p-value
5. Make a judgement

Questions?