Neural Network Basic

Onte nts

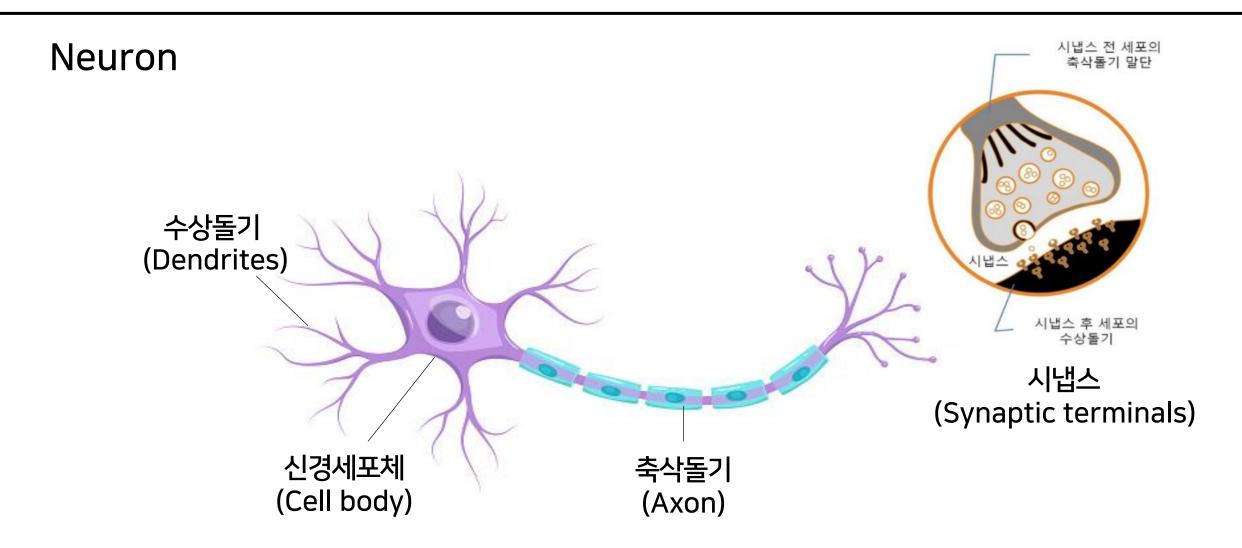
Unit 01 | Perceptron

Unit 02 | Backpropagation

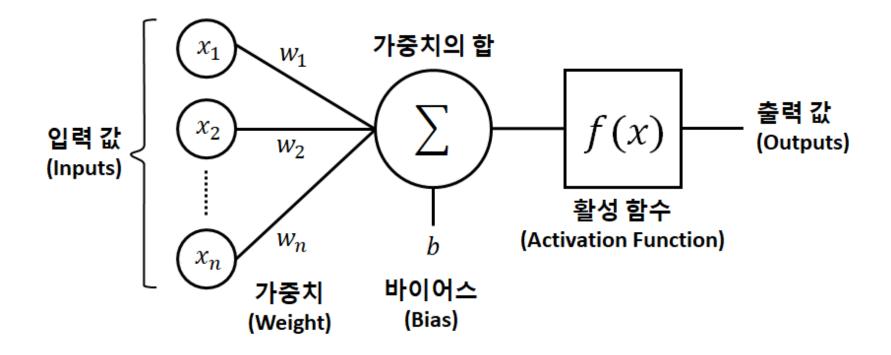
01 | Perceptron

인공신경망이란?

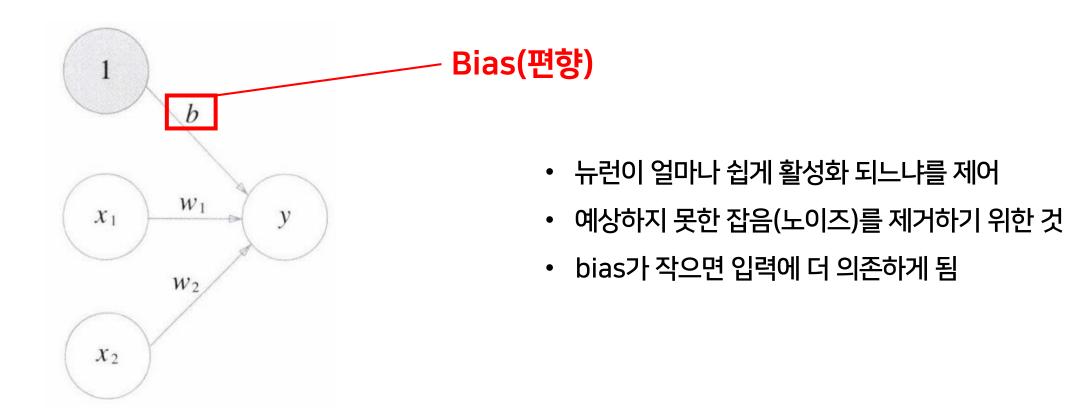
수학적 논리학이 아닌 <mark>인간의 두뇌를 모방</mark>하여 수많은 간단한 처리기들(뉴런)의 네트워크를 통해 문제를 해결하는 기계학습 모델 **29**



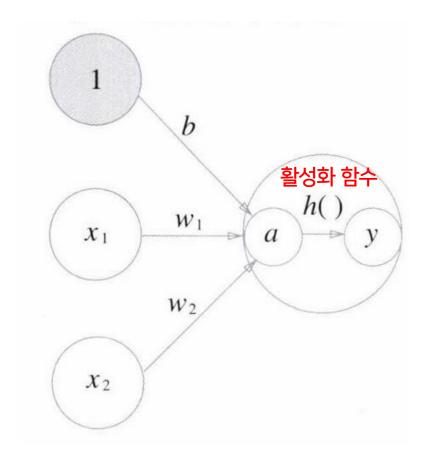
Perceptron



Perceptron



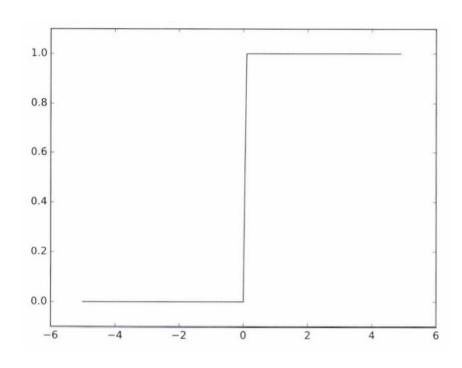
활성화 함수



$$a = b + w_1 x_1 + w_2 x_2$$
$$y = h(a)$$

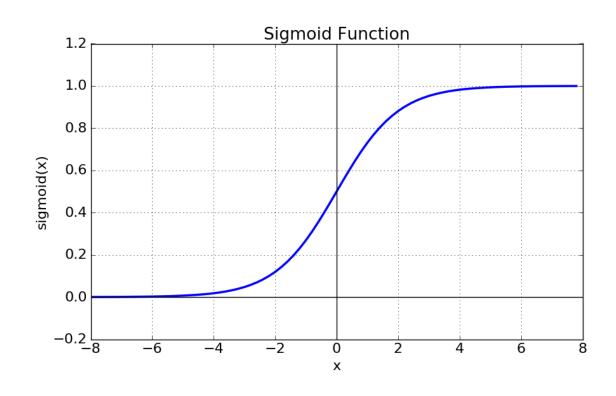
$$h(a) = \begin{cases} 0 \ (a \le 0) \\ 1 \ (a > 0) \end{cases}$$

활성화 함수 - Step function



- 출력이 0 또는 1
- 활성화할지 말지 여부만 반환

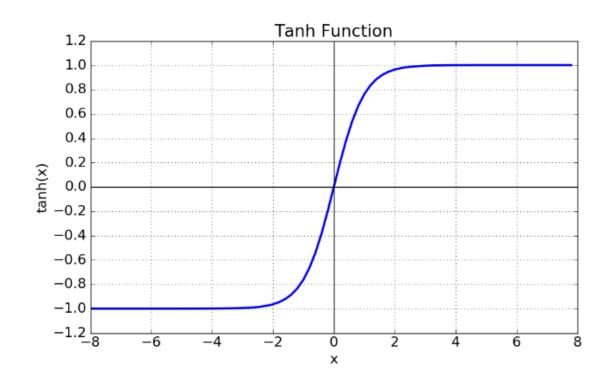
활성화 함수 - Sigmoid



$$h(x) = \frac{1}{1 + \exp(-x)}$$

- 0에서 1 사이의 값 출력
- 활성화 여부가 아닌, 활성화 정도를 반환
- 1에 가까울수록 많이 활성화됐다는 뜻

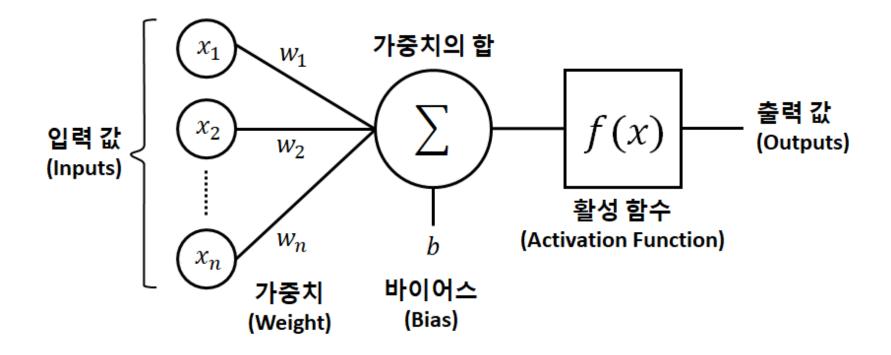
활성화 함수 - Tanh



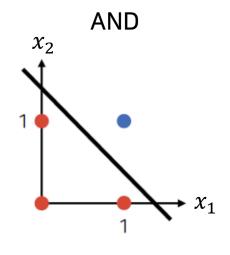
$$tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

- -1에서 1 사이의 값 출력
- 항상 양수만 나오는 sigmoid의 문제를 해결

Perceptron

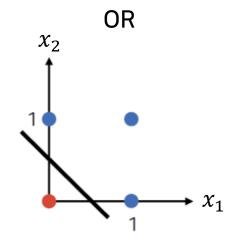


Perceptron 연산



$w_0 = 1.0, w_1 = 1.0, b = -1$

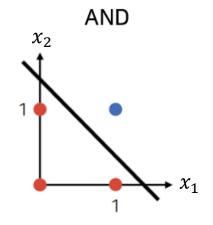
x_1	<i>x</i> ₂	S	У
0	0	-1.5	0
0	1	-0.5	0
1	0	-0.5	0
1	1	0.5	1



$$w_0 = -0.5, w_1 = 1.0, b = 1.0$$

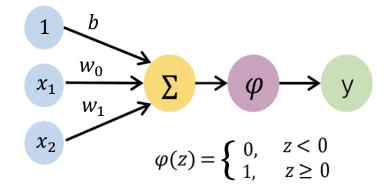
x_1	x_2	S	У
0	0	-0.5	0
0	1	0.5	1
1	0	0.5	1
1	1	1.5	1

Perceptron 연산



$$w_0 = 1.0, w_1 = 1.0, b = -1.5$$

<i>x</i> ₁	<i>x</i> ₂	S	У
0	0	-1.5	0
0	1	-0.5	0
1	0	-0.5	0
1	1	0.5	1



$$\varphi(w_1x_1 + w_2x_2 + w_3) = y$$

①
$$(1.0 \times 0) + (1.0 \times 0) + (-1.5) = -1.5$$

 $\varphi((1.0 \times 0) + (1.0 \times 0) + (-1.5)) = 0$

②
$$(1.0\times0) + (1.0\times1) + (-1.5) = -0.5$$

 $\varphi((1.0\times0) + (1.0\times1) + (-1.5)) = 0$

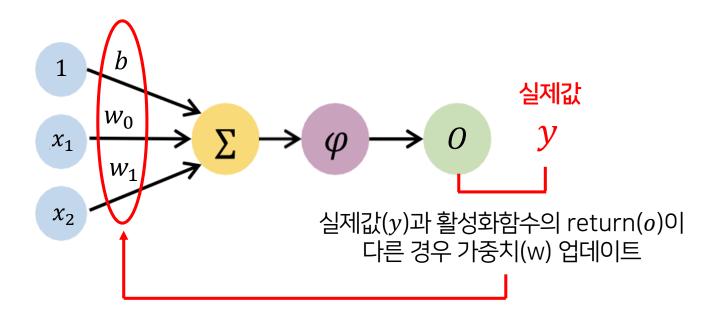
③
$$(1.0 \times 1) + (1.0 \times 0) + (-1.5) = -0.5$$

 $\varphi((1.0 \times 1) + (1.0 \times 0) + (-1.5)) = 0$

(4)
$$(1.0 \times 1) + (1.0 \times 1) + (-1.5) = 0.5$$

 $\varphi((1.0 \times 1) + (1.0 \times 1) + (-1.5)) = 1$

Perceptron 학습



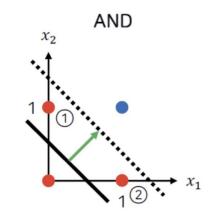
가중치 조정 식

$$w_i \leftarrow w_i + \frac{\eta}{\eta}(y - o)x_i$$

학습률(learning rate)

너무 작으면 학습 속도가 매우 느리고 너무 크면 오차의 최솟값을 지나칠 수 있음

Perceptron 학습



$$w_1 = 0.55, w_2 = 0.55, b = -0.65$$

x_1	x_2	0	у
0	0	0	0
0	1	1	0
1	0	1	0
1	1	1	1

$$w_i \leftarrow w_i + \eta(y - o)x_i \qquad \eta = 0.05$$

$$b \leftarrow b + 0.05(0 - 1) \times 1 \qquad b \leftarrow -0.65 + 0.05(0 - 1) \times 1 = -0.7$$

$$w_1 \leftarrow w_1 + 0.05(0 - 1) \times 0 \qquad w_1 \leftarrow 0.55 + 0.05(0 - 1) \times 0 = 0.55$$

$$w_2 \leftarrow w_2 + 0.05(0 - 1) \times 1 \qquad w_2 \leftarrow 0.55 + 0.05(0 - 1) \times 1 = 0.5$$

$$w_1 \leftarrow w_1 + 0.05(0 - 1) \times 0$$
 $w_1 \leftarrow 0.55 + 0.05(0 - 1) \times 0 = 0.55$ $w_2 \leftarrow w_2 + 0.05(0 - 1) \times 1$ $w_2 \leftarrow 0.55 + 0.05(0 - 1) \times 1 = 0.5$

$$b \leftarrow b + 0.05(0 - 1) \times 1$$

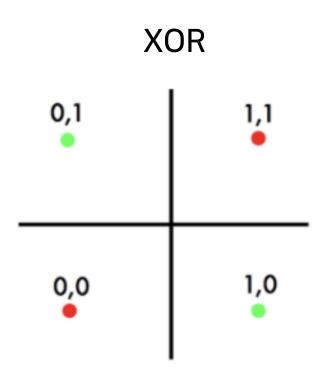
$$2 \quad w_1 \leftarrow w_1 + 0.05(0 - 1) \times 1$$

$$w_2 \leftarrow w_2 + 0.05(0 - 1) \times 0$$

$$b \leftarrow -0.65 + 0.05(0 - 1) \times 1 = -0.7$$

 $w_1 \leftarrow 0.55 + 0.05(0 - 1) \times 1 = 0.5$
 $w_2 \leftarrow 0.55 + 0.05(0 - 1) \times 0 = 0.55$

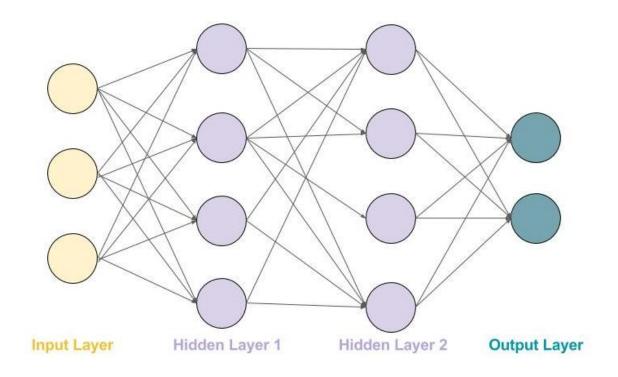
Perceptron의 한계



1969년 MIT AI랩 창립자 Minskey& Papert "현재의 퍼셉트론으로는 XOR 문제를 해결할 수 없다."

02 | Backpropagation

Multi Layer Perceptron



MLP를 학습시키는 방법

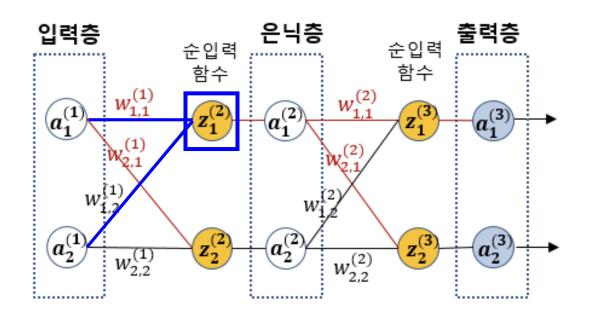
: 오류 역전파

역전파(Backpropagation)

순전파(Feedforward) 알고리즘에서 발생한 오차를 줄이기 위해 새로운 가중치를 업데이트하고, 새로운 가중치로 다시 학습하는 과정

역전파(Backpropagation)

순전파(Feedforward) 알고리즘에서 발생한 오차를 줄이기 위해 새로운 가중치를 업데이트하고, 새로운 가중치로 다시 학습하는 과정



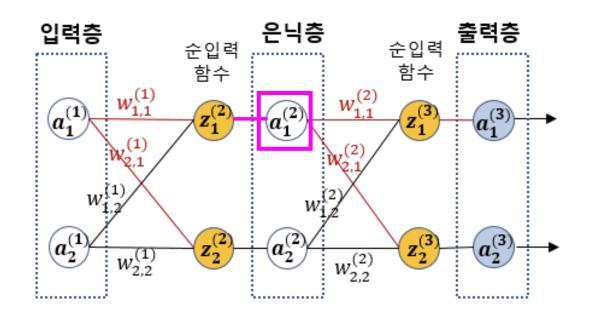
$$\phi(z) = \frac{1}{1 + e^{-z}}$$

$$z_1^{(2)} = w_{1,1}^{(1)} a_1^{(1)} + w_{1,2}^{(1)} a_2^{(1)}$$

$$a_1^{(2)} = \phi(z_1^{(2)})$$

$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$a_1^{(3)} = \phi(z_1^{(3)})$$



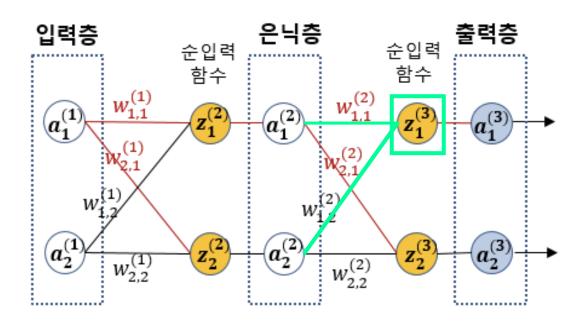
$$\phi(z) = \frac{1}{1 + e^{-z}}$$

$$z_1^{(2)} = w_{1,1}^{(1)} a_1^{(1)} + w_{1,2}^{(1)} a_2^{(1)}$$

$$a_1^{(2)}=\,\phi(z_1^{(2)})$$

$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$a_1^{(3)} = \phi(z_1^{(3)})$$



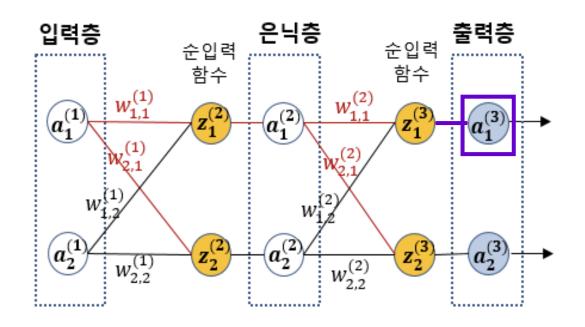
$$\phi(z) = \frac{1}{1 + e^{-z}}$$

$$z_1^{(2)} = w_{1,1}^{(1)} a_1^{(1)} + w_{1,2}^{(1)} a_2^{(1)}$$

$$a_1^{(2)} = \phi(z_1^{(2)})$$

$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$a_1^{(3)} = \phi(z_1^{(3)})$$



$$\phi(z) = \frac{1}{1 + e^{-z}}$$

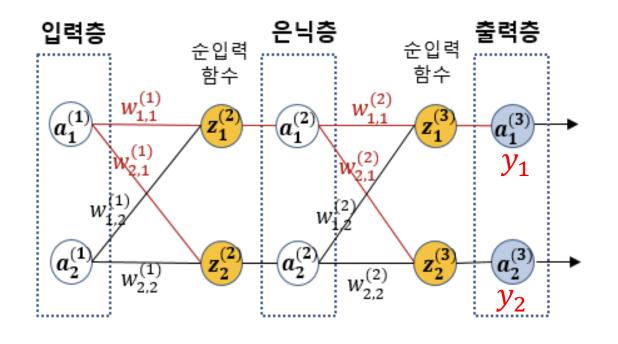
$$z_1^{(2)} = w_{1,1}^{(1)} a_1^{(1)} + w_{1,2}^{(1)} a_2^{(1)}$$

$$a_1^{(2)} = \phi(z_1^{(2)})$$

$$z_1^{(3)} = w_{1.1}^{(2)} a_1^{(2)} + w_{1.2}^{(2)} a_2^{(2)}$$

$$a_1^{(3)} = \phi(z_1^{(3)})$$

손실함수(Cost Function)

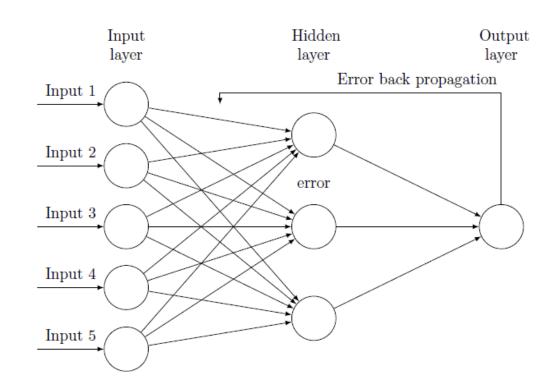


MSE =
$$\frac{1}{2N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

$$J_1 = \frac{1}{2}(a_1^{(3)} - y_1)^2$$

$$J_2 = \frac{1}{2}(a_2^{(3)} - y_2)^2$$

역전파(Backpropagation)



- Input과 output 값을 알고 있는 상태에서 신경망을 학습시키는 방법
- 출력부터 반대 방향으로 순차적으로 편미분을 수행해 가면서 weight와 bias 값을 갱신시킴

$$w_j = w_j - \eta \frac{\partial J_{total}}{\partial w_j}$$

가중치 업데이트 식

편미분

다변수함수의 특정 변수를 제외한 나머지 변수를 상수로 생각하여 미분

$$z = f(x, y) = x^2 + xy + y^2$$

$$\frac{\partial z}{\partial x} = 2x + y, \qquad \frac{\partial z}{\partial y} = 2y + x$$

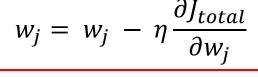
$$\Delta f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = (2x + y, 2y + x)$$

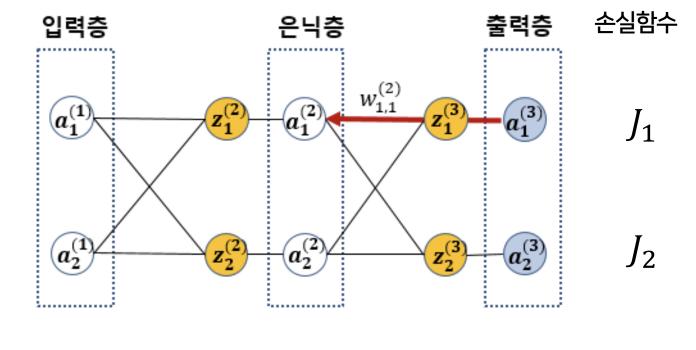
Chain Rule

연쇄 법칙, 합성 함수를 미분할 때의 계산 공식

$$f(g(x))' = f'(g(x))g'(x)$$
 $y = f(x), u = g(x)$ 일 때, $\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \cdot \frac{\partial u}{\partial x}$ 성립

역전파(Backpropagation)

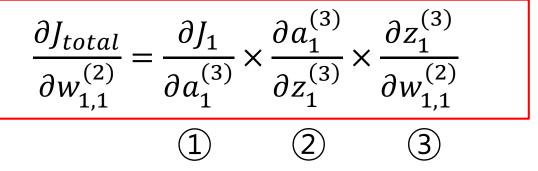


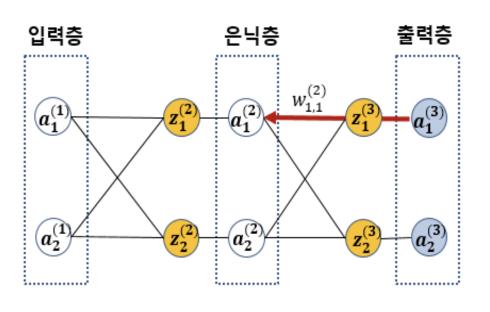


$$w_{1,1}^{(2)} = w_{1,1}^{(2)} - \frac{\partial J_{total}}{\partial w_{1,1}^{(2)}}$$

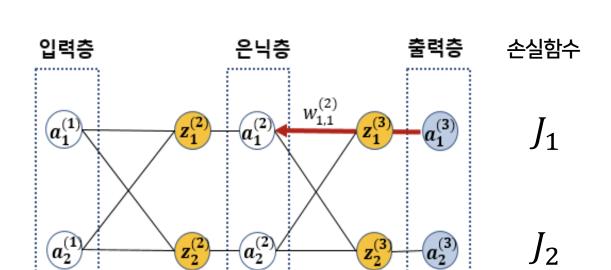
역전파의 출발노드인 $a_1^{(3)}$ 의 J_{total} 은 J_1

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \frac{\partial J_1}{\partial a_1^{(3)}} \times \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}}$$





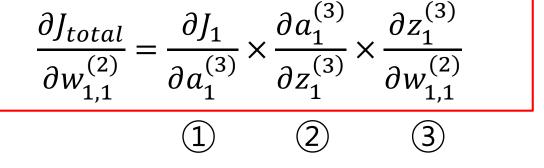
참고:
$$J_1 = \frac{1}{2} \left(a_1^{(3)} - y_1 \right)^2$$

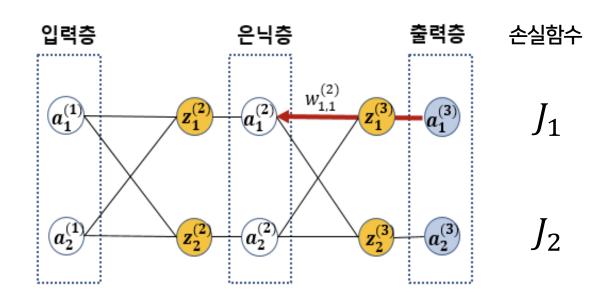


$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \frac{\partial J_1}{\partial a_1^{(3)}} \times \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}}$$

$$\boxed{1} \qquad \boxed{2} \qquad \boxed{3}$$

참고:
$$a_1^{(3)} = \phi(z_1^{(3)})$$
 $\sigma'(x) = \frac{\partial}{\partial x} \frac{1}{1+e^{-x}}$ $= \frac{e^{-x}}{(1+e^{-x})^2}$ $= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}}$ $= \sigma(x)(1-\sigma(x))$



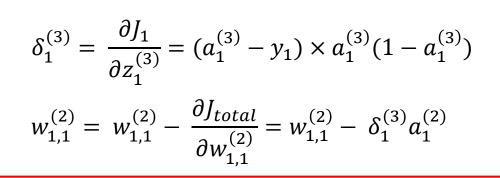


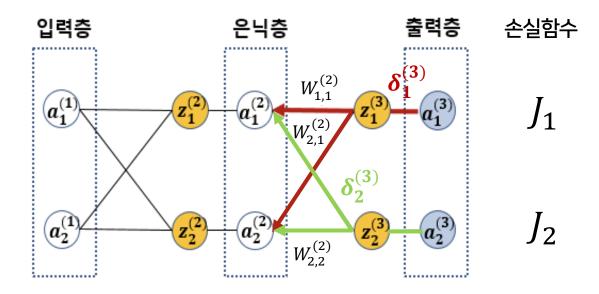
참고:
$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \frac{\partial J_1}{\partial a_1^{(3)}} \times \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}} = \underbrace{\left[(a_1^{(3)} - y_1) \times a_1^{(3)} (1 - a_1^{(3)}) \times a_1^{(2)} \right]}_{(2)} \times \underbrace{\left[a_1^{(3)} - y_1 \times a_1^{(3)} (1 - a_1^{(3)}) \times a_1^{(3)} (1 - a_1^{(3)}) \times a_1^{(3)} \right]}_{(2)} \times \underbrace{\left[a_1^{(3)} - y_1 \times a_1^{(3)} (1 - a_1^{(3)}) \times a_1^{(3)} (1 -$$

$$w_{1,1}^{(2)} = w_{1,1}^{(2)} - \frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = w_{1,1}^{(2)} - \delta_1^{(3)} a_1^{(2)}$$

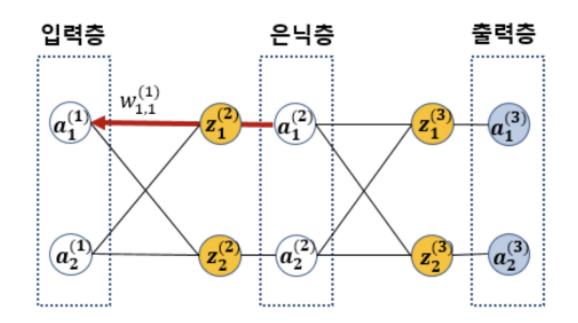
역전파(Backpropagation)





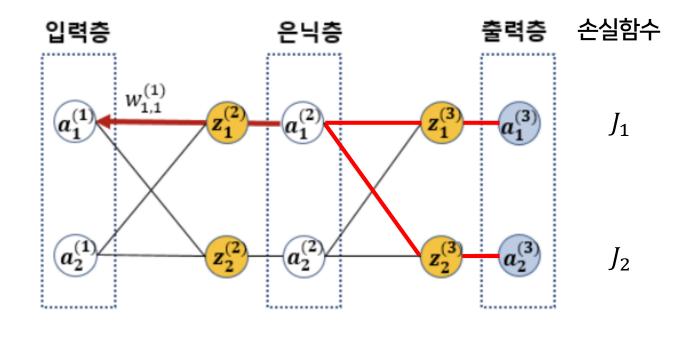
같은 방식으로

$$\begin{split} \delta_2^{(3)} &= \frac{\partial J_2}{\partial z_2^{(3)}} = \left(a_2^{(3)} - y_2\right) \times a_2^{(3)} \left(1 - a_2^{(3)}\right) \\ w_{2,1}^{(2)} &= w_{2,1}^{(2)} - \delta_2^{(3)} a_1^{(2)} \\ w_{2,2}^{(2)} &= w_{2,2}^{(2)} - \delta_2^{(3)} a_2^{(2)} \end{split}$$



$$w_j = w_j - \eta \frac{\partial J_{total}}{\partial w_j}$$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

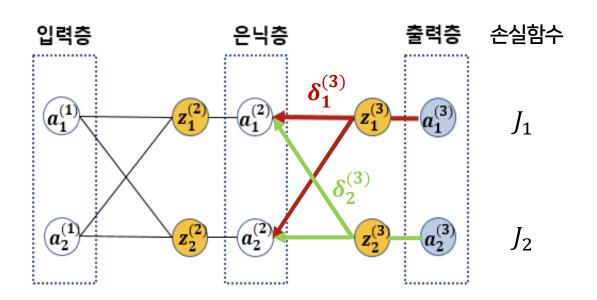


$$w_j = w_j - \eta \frac{\partial J_{total}}{\partial w_j}$$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

$$a_1^{(2)}$$
의 J_{total} 은 J_1+J_2

$$\frac{\partial J_{total}}{\partial a_1^{(2)}} = \frac{\partial J_1}{\partial a_1^{(2)}} + \frac{\partial J_2}{\partial a_1^{(2)}} = \frac{\partial J_1}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial a_1^{(2)}} + \frac{\partial J_2}{\partial z_2^{(3)}} \times \frac{\partial z_2^{(3)}}{\partial a_1^{(2)}}$$



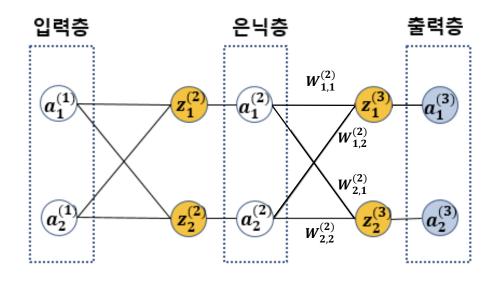
$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

$$\delta_{1}^{(3)} = \frac{\partial J_{1}}{\partial z_{1}^{(3)}} = \left(a_{1}^{(3)} - y_{1}\right) \times a_{1}^{(3)} \left(1 - a_{1}^{(3)}\right)$$

$$\delta_{2}^{(3)} = \frac{\partial J_{2}}{\partial z_{2}^{(3)}} = \left(a_{2}^{(3)} - y_{2}\right) \times a_{2}^{(3)} \left(1 - a_{2}^{(3)}\right)$$

$$\frac{\partial J_{total}}{\partial a_{1}^{(2)}} = \frac{\partial J_{1}}{\partial a_{1}^{(2)}} + \frac{\partial J_{2}}{\partial a_{1}^{(2)}} = \frac{\partial J_{1}}{\partial z_{1}^{(3)}} \times \frac{\partial z_{1}^{(3)}}{\partial a_{1}^{(2)}} + \frac{\partial J_{2}}{\partial z_{2}^{(3)}} \times \frac{\partial z_{2}^{(3)}}{\partial a_{1}^{(2)}}$$

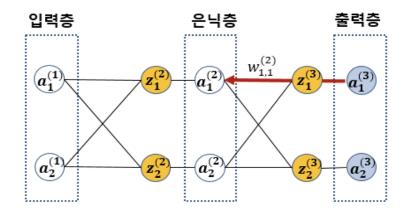
$$= \delta_{1}^{(3)} w_{1,1}^{(2)} + \delta_{2}^{(3)} w_{2,1}^{(2)}$$



$$z_1^{(3)} = w_{1,1}^{(2)} a_1^{(2)} + w_{1,2}^{(2)} a_2^{(2)}$$
$$z_2^{(3)} = w_{2,1}^{(2)} a_1^{(2)} + w_{2,2}^{(2)} a_2^{(2)}$$

$$\frac{\partial J_{total}}{\partial w_{1.1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1.1}^{(1)}}$$

$$\begin{split} \delta_{1}^{(3)} &= \frac{\partial J_{1}}{\partial z_{1}^{(3)}} = \left(a_{1}^{(3)} - y_{1}\right) \times a_{1}^{(3)} \left(1 - a_{1}^{(3)}\right) \\ \delta_{2}^{(3)} &= \frac{\partial J_{2}}{\partial z_{2}^{(3)}} = \left(a_{2}^{(3)} - y_{2}\right) \times a_{2}^{(3)} \left(1 - a_{2}^{(3)}\right) \\ \frac{\partial J_{total}}{\partial a_{1}^{(2)}} &= \frac{\partial J_{1}}{\partial a_{1}^{(2)}} + \frac{\partial J_{2}}{\partial a_{1}^{(2)}} = \frac{\partial J_{1}}{\partial z_{1}^{(3)}} \times \frac{\partial z_{1}^{(3)}}{\partial a_{1}^{(2)}} + \frac{\partial J_{2}}{\partial z_{2}^{(3)}} \times \frac{\partial z_{2}^{(3)}}{\partial a_{1}^{(2)}} \\ &= \delta_{1}^{(3)} w_{1,1}^{(2)} + \delta_{2}^{(3)} w_{2,1}^{(2)} \end{split}$$



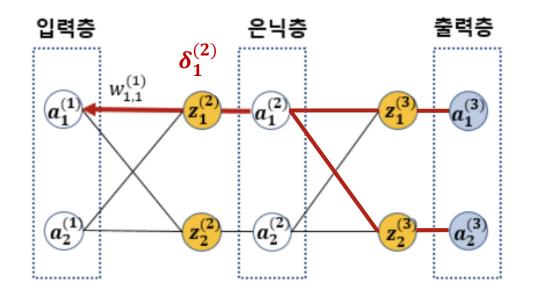
$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \frac{\partial J_1}{\partial a_1^{(3)}} \times \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \times \frac{\partial z_1^{(3)}}{\partial w_{1,1}^{(2)}}$$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(2)}} = \left(a_1^{(3)} - y_1\right) \times \underline{a_1^{(3)}} \left(1 - a_1^{(3)}\right) \times \underline{a_1^{(2)}}$$

입력층 은닉층 출력층
$$a_1^{(1)}$$
 $a_1^{(2)}$ $a_1^{(2)}$ $a_1^{(3)}$ $a_2^{(3)}$ $a_2^{(3)}$

$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \frac{\partial J_{total}}{\partial a_1^{(2)}} \times \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \times \frac{\partial z_1^{(2)}}{\partial w_{1,1}^{(1)}}$$

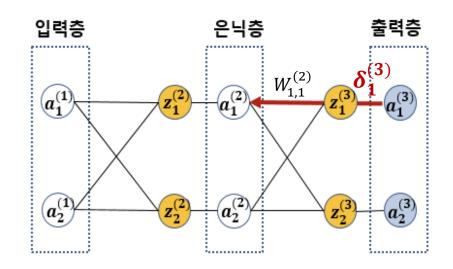
$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \left(\delta_1^{(3)} w_{1,1}^{(2)} + \delta_2^{(3)} w_{2,1}^{(2)} \right) \times \underline{a_1^{(2)} \left(1 - a_1^{(2)} \right) \times \underline{a_1^{(1)}}}$$



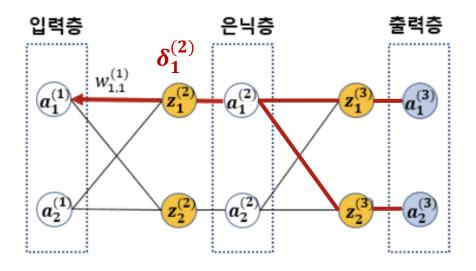
$$\frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = \left(\delta_1^{(3)} w_{1,1}^{(2)} + \delta_2^{(3)} w_{2,1}^{(2)}\right) \times a_1^{(2)} \left(1 - a_1^{(2)}\right) \times a_1^{(1)}$$

$$\delta_1^{(2)} = \left(\delta_1^{(3)} w_{1,1}^{(2)} - \delta_2^{(3)} w_{2,1}^{(2)}\right) \times a_1^{(2)} (1 - a_1^{(2)})$$

$$w_{1,1}^{(1)} = w_{1,1}^{(1)} - \frac{\partial J_{total}}{\partial w_{1,1}^{(1)}} = w_{1,1}^{(1)} - \delta_1^{(2)} a_1^{(1)}$$

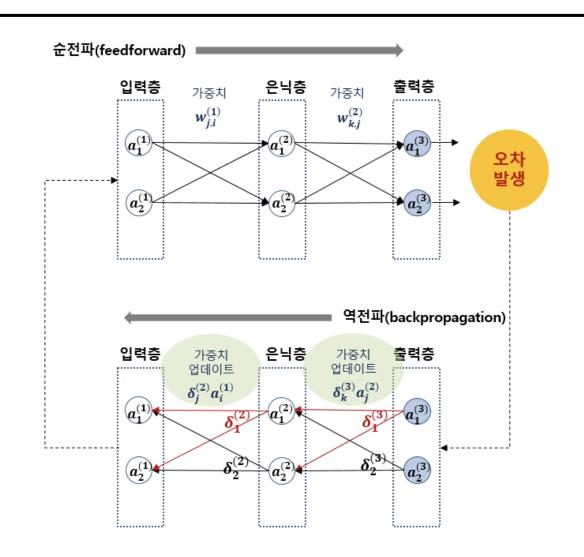


$$w_{1,1}^{(2)} = w_{1,1}^{(2)} - \delta_1^{(3)} a_1^{(2)}$$



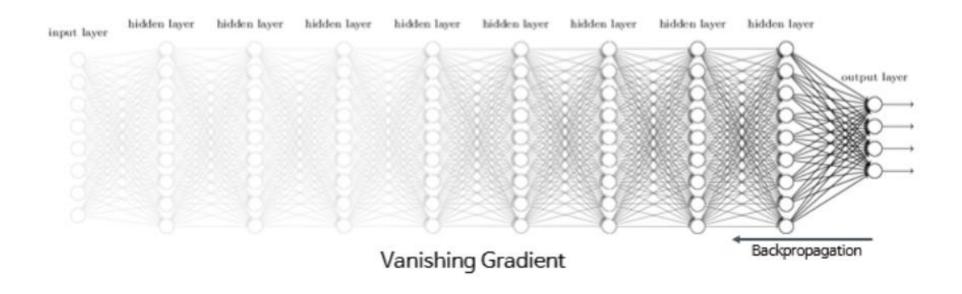
$$w_{1,1}^{(1)} = w_{1,1}^{(1)} - \delta_1^{(2)} a_1^{(1)}$$

$$\begin{split} w_{j,i}^{(l)} &= w_{j,i}^{(l)} - \delta_j^{(l+1)} a_i^{(l)} \\ \delta_j^{(3)} &= \left(a_j^{(3)} - y_j \right) \times a_j^{(3)} \left(1 - a_j^{(3)} \right) \\ \delta_j^{(2)} &= \left(\delta_1^{(3)} w_{1,j}^{(2)} + \delta_2^{(3)} w_{2,j}^{(2)} \right) \times a_j^{(2)} \left(1 - a_j^{(2)} \right) \end{split}$$

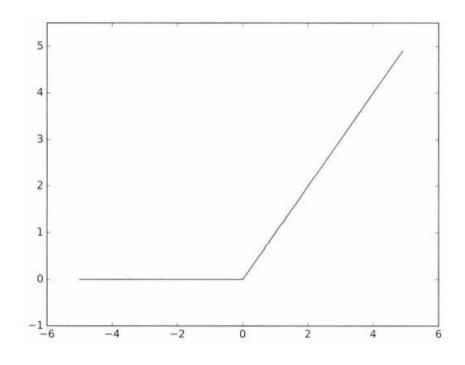


Vanishing Gradient Problem

깊이가 깊은 심층신경망에서는 역전파 알고리즘이 입력층으로 전달됨에 따라 그래디언트가 점점 작아져 결국 가중치 매개변수가 업데이트 되지 않는 경우가 발생



활성화함수 - ReLU 함수



$$h(x) = \begin{cases} x & (x > 0) \\ 0 & (x \le 0) \end{cases}$$

- Vanishing gradient problem 해결!
- 입력이 0 넘으면 값이 클수록 많이 활성화
- 입력이 0보다 작으면 무조건 비활성화

Unitㅣ과제

"week3_NeuralNetworkBasic_assignment.pdf" 파일의 문제들을 상세한 풀이과정과 함께 풀어주세요.

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Q & A

들어주셔서 감사합니다.