

Appendix

Code used to produce plots, tables, and draw conclusions. All written by Edward Huber for purposes of this analysis.

In [1]:

```
library(tidyr)
library(ggplot2)
library(MASS)
quality = read.csv('airquality.csv')
```

Registered S3 methods overwritten by 'ggplot2':

method	from
[.quosures	rlang
c.quosures	rlang
print.quosures	rlang

In [2]:

```
head(quality)
```

No	year	month	day	hour	pm2.5	DEWP	TEMP	PRES	cbwd	lws	ls	lr
1	2010	1	1	0	NA	-21	-11	1021	NW	1.79	0	0
2	2010	1	1	1	NA	-21	-12	1020	NW	4.92	0	0
3	2010	1	1	2	NA	-21	-11	1019	NW	6.71	0	0
4	2010	1	1	3	NA	-21	-14	1019	NW	9.84	0	0
5	2010	1	1	4	NA	-20	-12	1018	NW	12.97	0	0
6	2010	1	1	5	NA	-19	-10	1017	NW	16.10	0	0

In [3]:

```
nrow(quality)
```

43824

In [4]:

```
quality2 = drop_na(quality)
row.names(quality2) = 1:nrow(quality2)
names(quality2) = tolower(names(quality2))
nrow(quality2)
```

41757

In [5]:

```
head(quality2)
```

no	year	month	day	hour	pm2.5	dewp	temp	pres	cbwd	iws	is	ir
25	2010	1	2	0	129	-16	-4	1020	SE	1.79	0	0
26	2010	1	2	1	148	-15	-4	1020	SE	2.68	0	0
27	2010	1	2	2	159	-11	-5	1021	SE	3.57	0	0
28	2010	1	2	3	181	-7	-5	1022	SE	5.36	1	0
29	2010	1	2	4	138	-7	-5	1022	SE	6.25	2	0
30	2010	1	2	5	109	-7	-6	1022	SE	7.14	3	0

In [6]:

```
library(tidyverse)
library(lubridate)
```

Registered S3 method overwritten by 'rvest':

```
method          from
read_xml.response xml2
— Attaching packages — tidyverse
rse 1.2.1 —
✓ tibble 2.1.1      ✓ dplyr 0.8.0.1
✓ readr 1.3.1      ✓ stringr 1.4.0
✓ purrr 0.3.3      ✓ forcats 0.4.0
— Conflicts — tidyverse_conflicts() —
✗ dplyr::filter() masks stats::filter()
✗ dplyr::lag()     masks stats::lag()
✗ dplyr::select() masks MASS::select()
```

Attaching package: 'lubridate'

The following object is masked from 'package:base':

date

In [7]:

```
#editing the formate of the data for plotting
quality2 = quality2 %>%
  mutate(date = make_date(year, month, day))

quality2$datetime = as.POSIXct(paste(quality2$date,
  quality2$hour), format="%Y-%m-%d %H")

quality3 = quality2[-c(1,2,3,4,5,10, 14)]

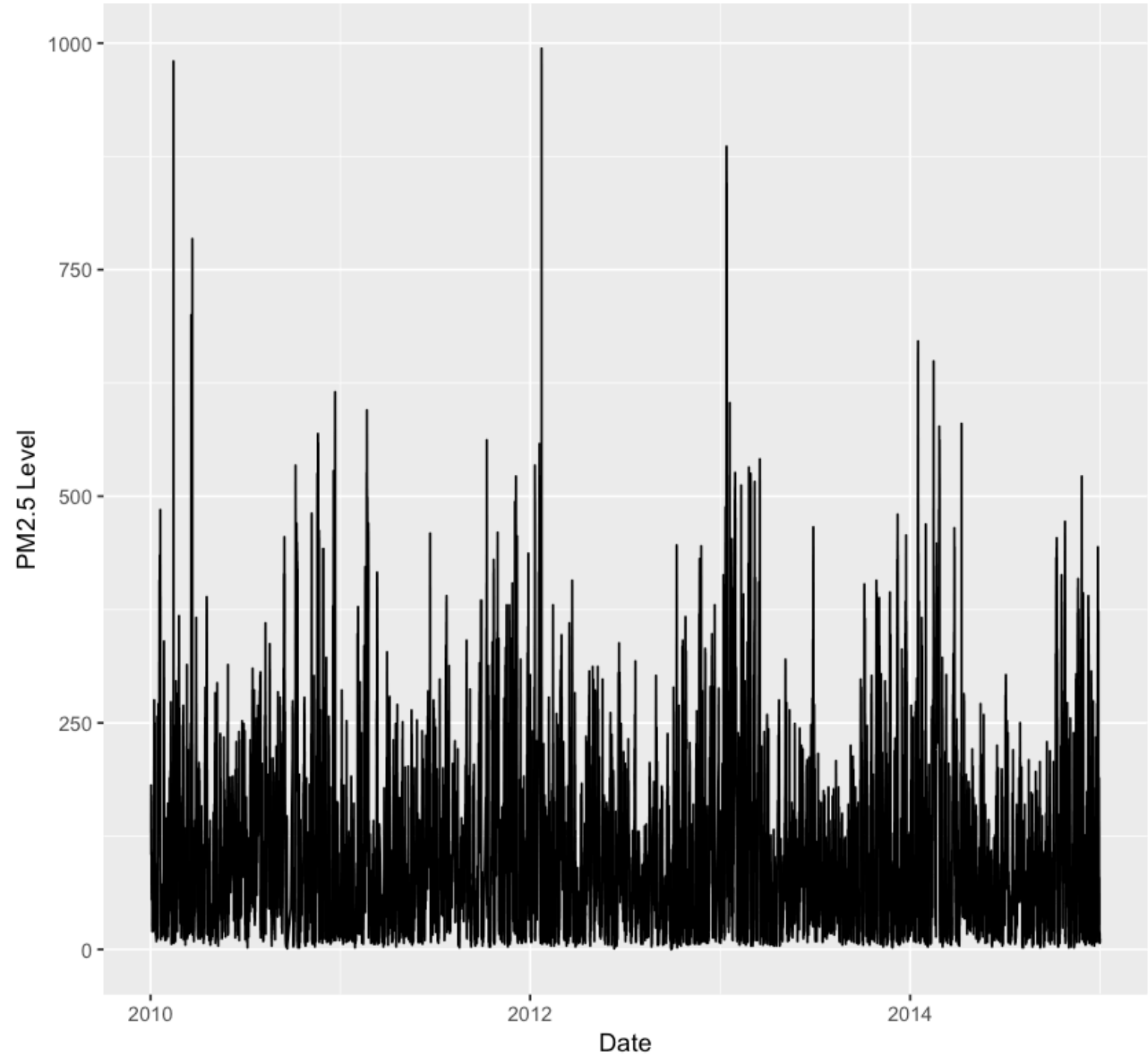
head(quality3)
```

pm2.5	dewp	temp	pres	iws	is	ir	datetime
129	-16	-4	1020	1.79	0	0	2010-01-02 00:00:00
148	-15	-4	1020	2.68	0	0	2010-01-02 01:00:00
159	-11	-5	1021	3.57	0	0	2010-01-02 02:00:00
181	-7	-5	1022	5.36	1	0	2010-01-02 03:00:00
138	-7	-5	1022	6.25	2	0	2010-01-02 04:00:00
109	-7	-6	1022	7.14	3	0	2010-01-02 05:00:00

In [8]:

```
ggplot(data = quality3, aes(x = datetime, y = pm2.5)) +
  geom_line() +
  labs(x = "Date", y = "PM2.5 Level",
    title = "PM2.5 Concentration Varies Greatly Over Time",
    subtitle = "Beijing, China: 2010-2014")
```

PM2.5 Concentration Varies Greatly Over Time
Beijing, China: 2010-2014



In [9]:

```
#creating new dataframe for use in summarising statistics
quality4 = quality3[-c(8)]
head(quality4)
```

pm2.5	dewp	temp	pres	iws	is	ir
129	-16	-4	1020	1.79	0	0
148	-15	-4	1020	2.68	0	0
159	-11	-5	1021	3.57	0	0
181	-7	-5	1022	5.36	1	0
138	-7	-5	1022	6.25	2	0
109	-7	-6	1022	7.14	3	0

In [10]:

```
summary(quality4)
```

pm2.5		dewp		temp		pres	
Min.	: 0.00	Min.	:-40.00	Min.	:-19.0	Min.	: 991
1st Qu.:	29.00	1st Qu.:	-10.00	1st Qu.:	2.0	1st Qu.:	1008
Median :	72.00	Median :	2.00	Median :	14.0	Median :	1016
Mean :	98.61	Mean :	1.75	Mean :	12.4	Mean :	1016
3rd Qu.:	137.00	3rd Qu.:	15.00	3rd Qu.:	23.0	3rd Qu.:	1025
Max.	:994.00	Max.	: 28.00	Max.	: 42.0	Max.	:1046

iws		is		ir	
Min.	: 0.45	Min.	: 0.00000	Min.	: 0.0000
1st Qu.:	1.79	1st Qu.:	0.00000	1st Qu.:	0.0000
Median :	5.37	Median :	0.00000	Median :	0.0000
Mean :	23.87	Mean :	0.05534	Mean :	0.1949
3rd Qu.:	21.91	3rd Qu.:	0.00000	3rd Qu.:	0.0000
Max.	:565.49	Max.	:27.00000	Max.	:36.0000

In [11]:

```
#plotting each variable vs. pm2.5 levels
par(mfrow = c(3,2), col.axis = "white", col.lab = "white", tck = 0)

ggplot(data = quality4, aes(x = dewp, y = pm2.5)) + geom_point()
ggplot(data = quality4, aes(x = temp, y = pm2.5)) + geom_point()
ggplot(data = quality4, aes(x = pres, y = pm2.5)) + geom_point()
ggplot(data = quality4, aes(x = iws, y = pm2.5)) + geom_point()
ggplot(data = quality4, aes(x = is, y = pm2.5)) + geom_point()
ggplot(data = quality4, aes(x = ir, y = pm2.5)) + geom_point()

#ggarrange(dewp, temp, pres, iws, is, ir, ncol = 3, nrow = 2)
```

pm2.5

1000
750
500
250
0

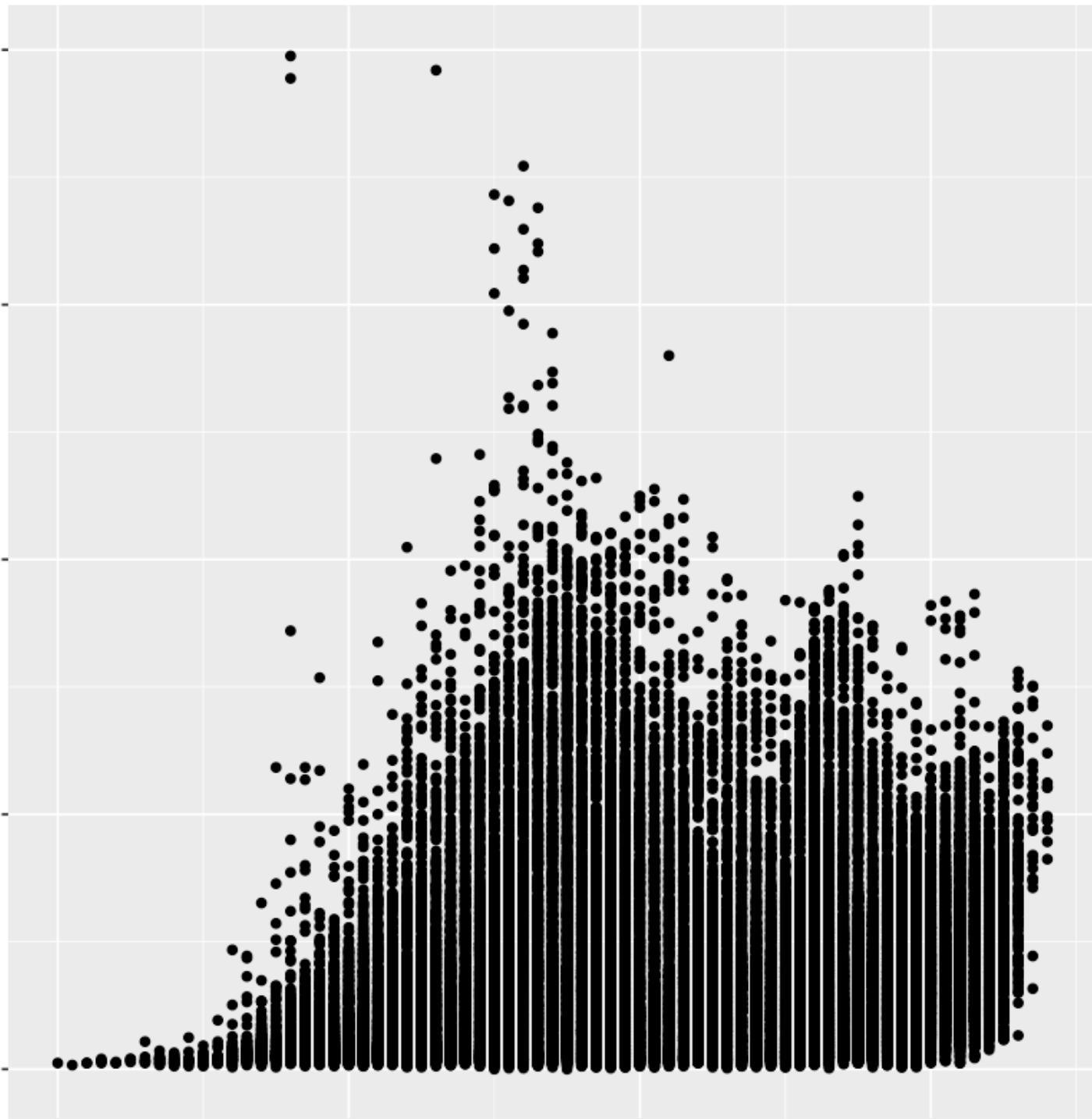
-40

-20

0

20

dewp



pm2.5

1000
750
500
250
0

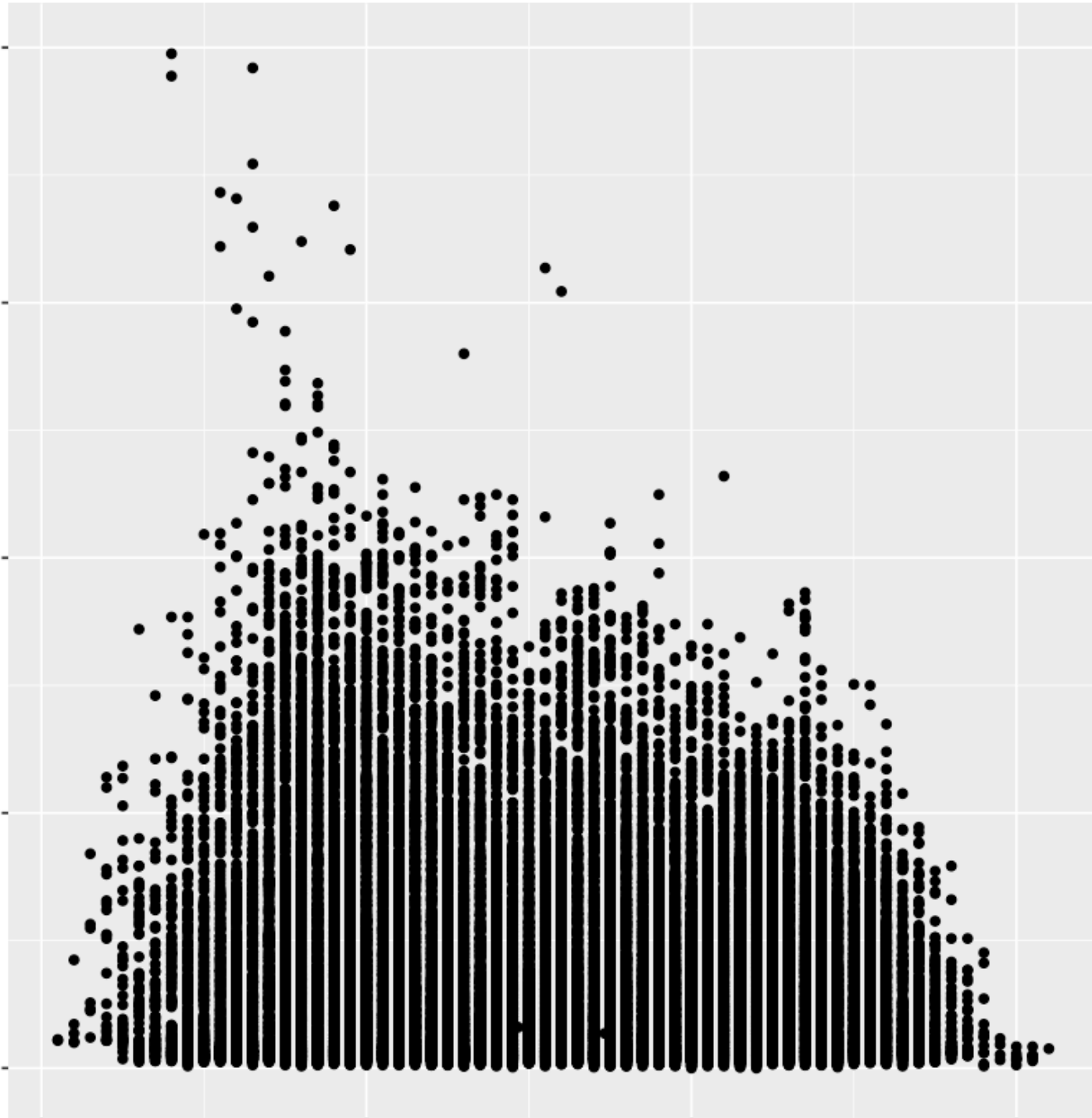
-20

0

20

40

temp

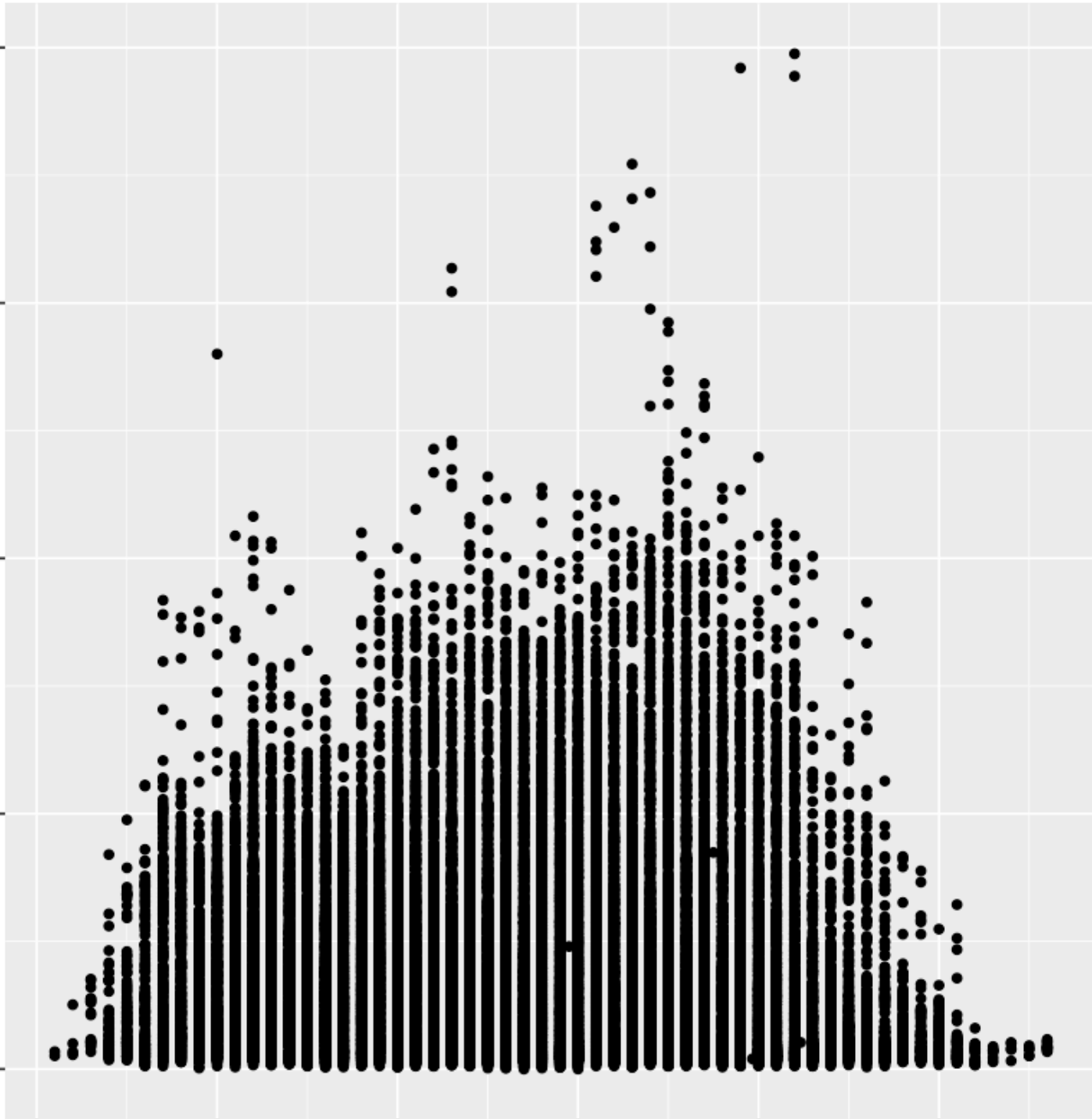


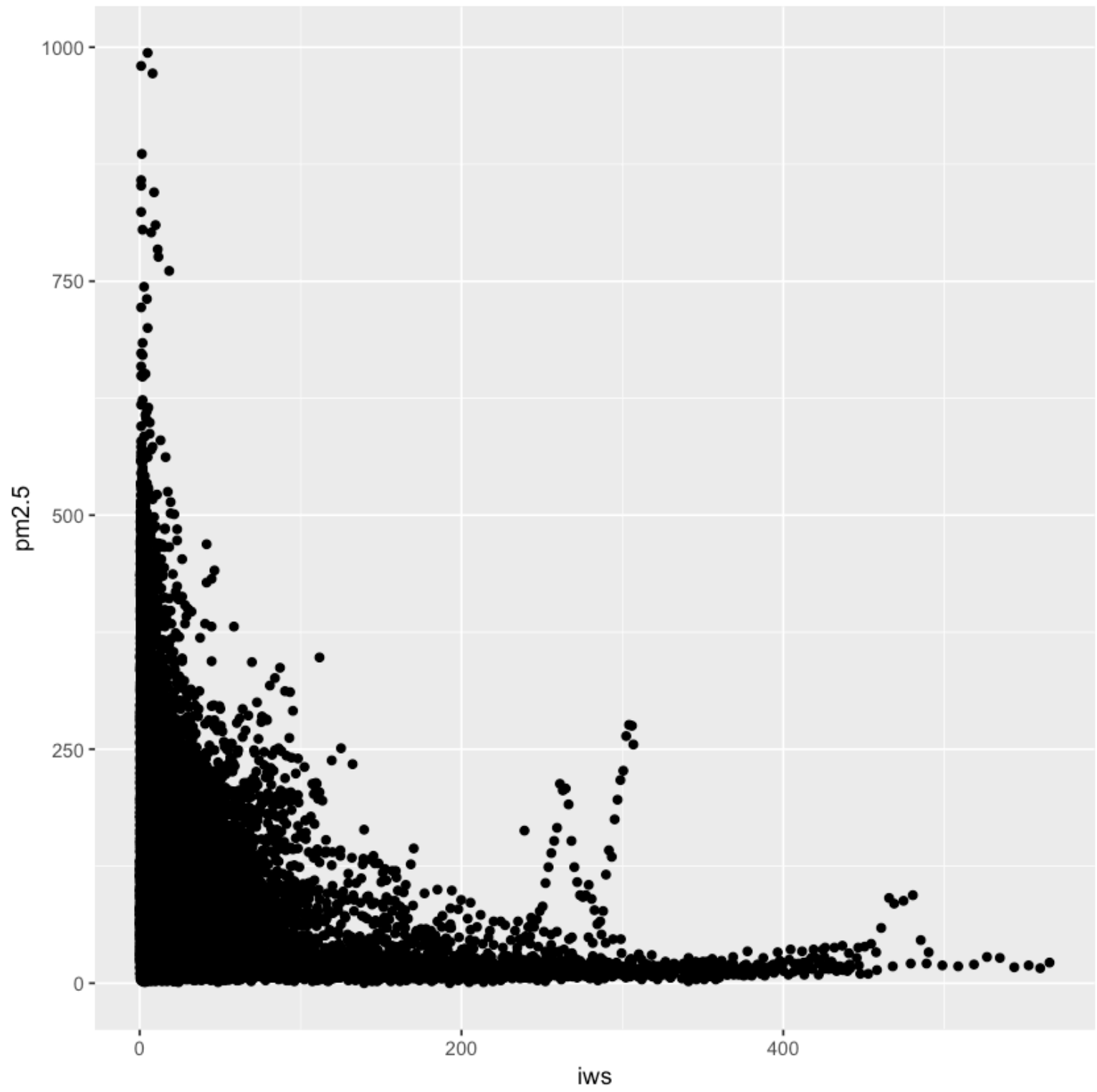
pm2.5

1000
750
500
250
0

990 1000 1010 1020 1030 1040

pres





pm2.5

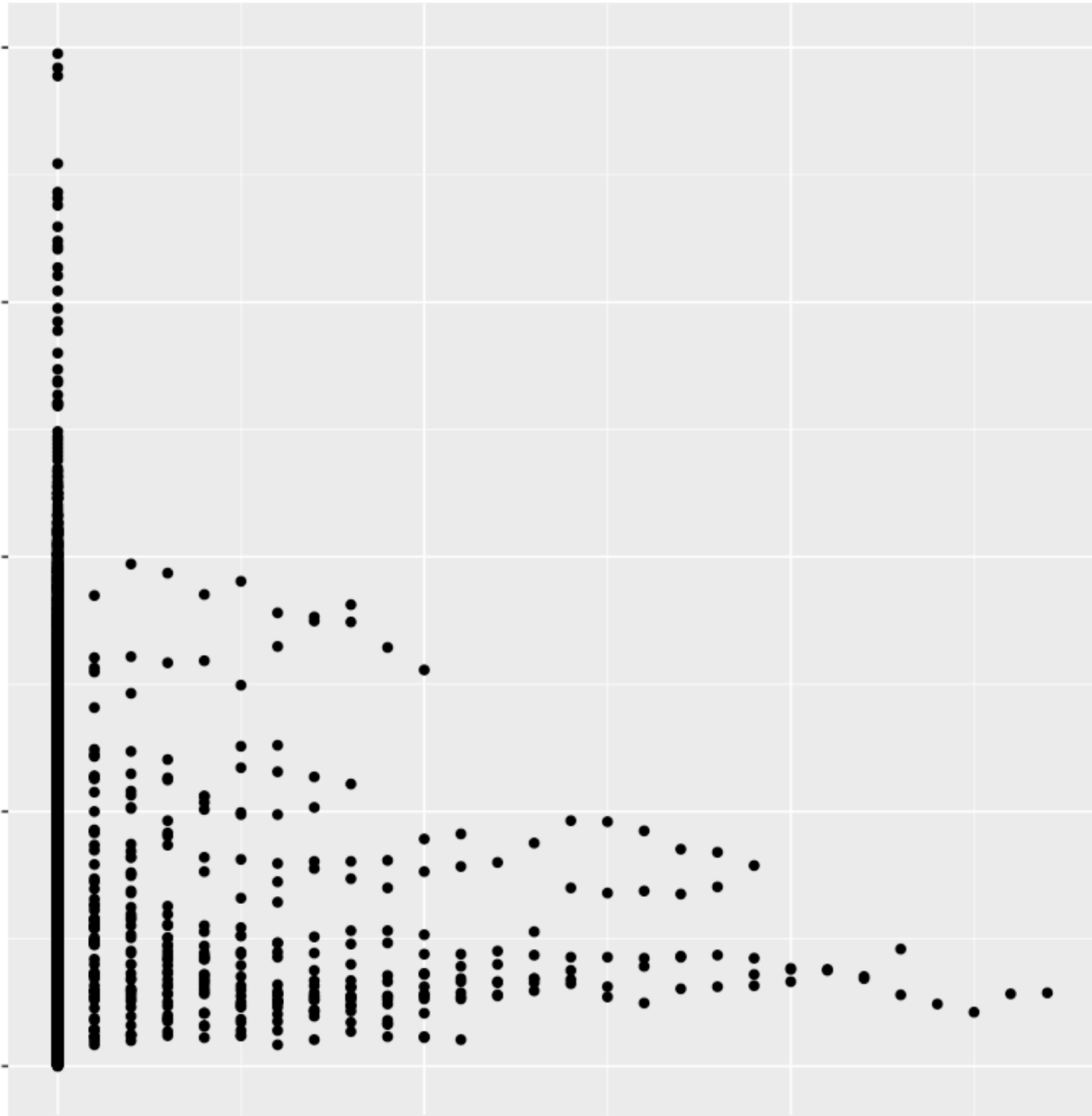
1000
750
500
250
0

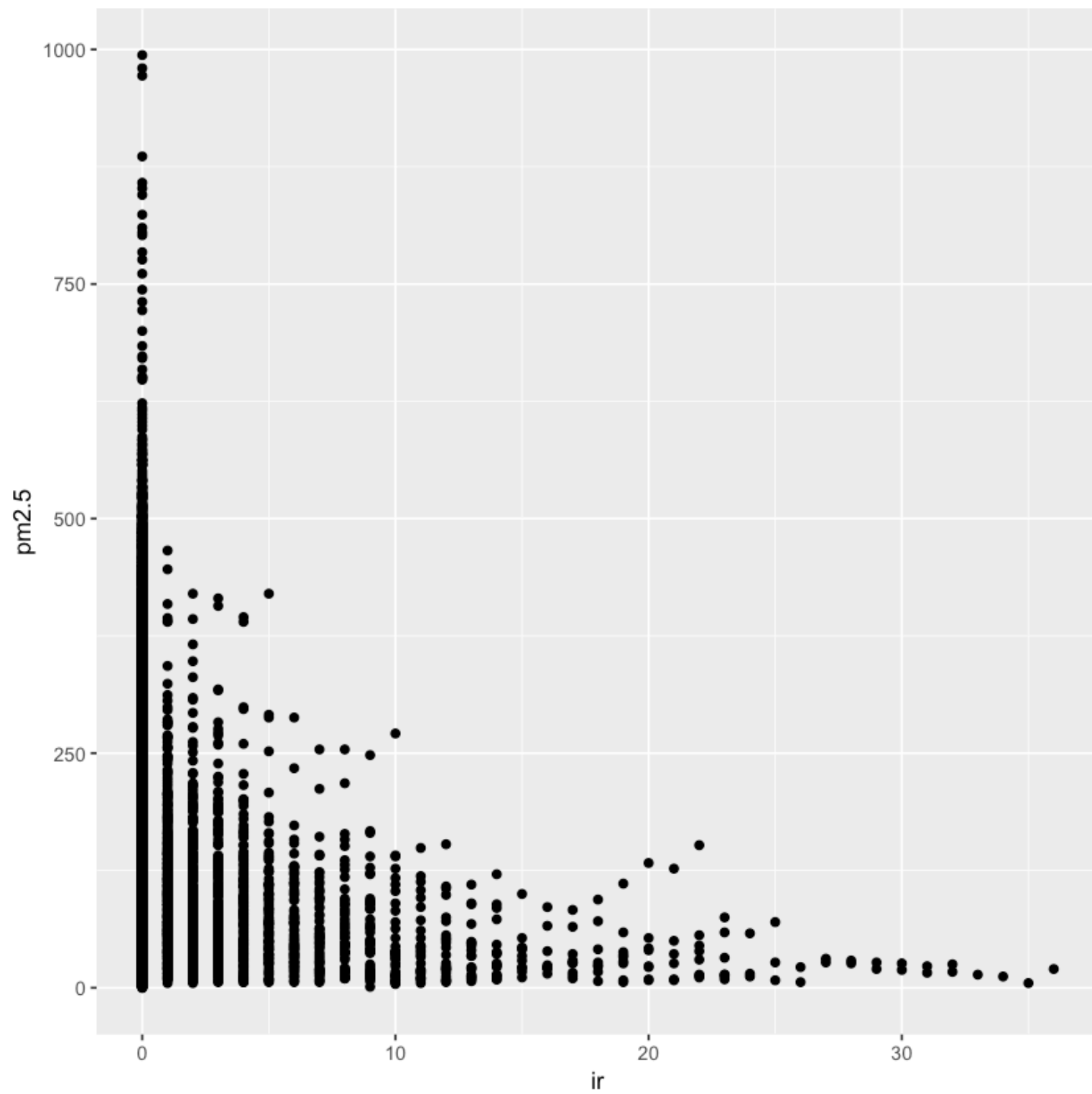
0

10

is

20





In [12]:

```
#creating initial model
quality.lm = lm(pm2.5 ~ dewp + temp +
                pres + iws + is + ir, data = quality4)
summary(quality.lm)
anova(quality.lm)
```

```
Call:
lm(formula = pm2.5 ~ dewp + temp + pres + iws + is + ir, data = quality4)
```

Residuals:

Min	1Q	Median	3Q	Max
-163.00	-52.31	-15.68	33.09	874.89

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1.728e+03	7.299e+01	23.680	< 2e-16 ***
dewp	4.282e+00	5.346e-02	80.109	< 2e-16 ***
temp	-6.068e+00	6.836e-02	-88.764	< 2e-16 ***
pres	-1.529e+00	7.135e-02	-21.431	< 2e-16 ***
iws	-2.616e-01	8.436e-03	-31.015	< 2e-16 ***
is	-2.267e+00	5.097e-01	-4.448	8.7e-06 ***
ir	-7.206e+00	2.816e-01	-25.593	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 80.46 on 41750 degrees of freedom
Multiple R-squared: 0.2361, Adjusted R-squared: 0.236
F-statistic: 2151 on 6 and 41750 DF, p-value: < 2.2e-16

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
dewp	1	10397039.4	10397039.366	1606.12865	0.000000e+00
temp	1	59142103.7	59142103.736	9136.23809	0.000000e+00
pres	1	2868883.7	2868883.672	443.18350	7.097183e-98
iws	1	6796714.5	6796714.523	1049.95254	1.666189e-227
is	1	102830.7	102830.737	15.88523	6.741610e-05
ir	1	4239916.8	4239916.839	654.97991	2.363577e-143
Residuals	41750	270262531.2	6473.354	NA	NA

In [13]:

```
#calculating F-statistic for initial model
qf(.95, df1 = 6, df2 = 41750)
```

2.09881381597824

In [14]:

```
#creating a matrix in order to calculate t-values for each variable and critical t for the model
n = 41757
p = 7
quality4['B0'] = rep(1, 41757)
y = matrix(quality4$pm2.5, ncol = 1)
X = matrix(c(quality4$B0, quality4$dewp, quality4$temp, quality4$pres, quality4$irs,
             quality4$is, quality4$ir), ncol = 7, byrow = FALSE)
beta.hat = solve(t(X)%*%X)%*%t(X)%*%y
SSres = as.vector(t(y)%*%y - t(beta.hat)%*%t(X)%*%y)
sig.hat = SSres/(n-p)
SSreg = as.vector(t(beta.hat)%*%t(X)%*%y - n*mean(y)^2)
C = solve(t(X)%*%X)
beta.se = sqrt(sig.hat*diag(C))
t = beta.hat/beta.se
critical.t = qt(1-0.5/2, n-p)
abs(t)
critical.t
```

23.680393

80.108929

88.764250

21.431247

31.014553

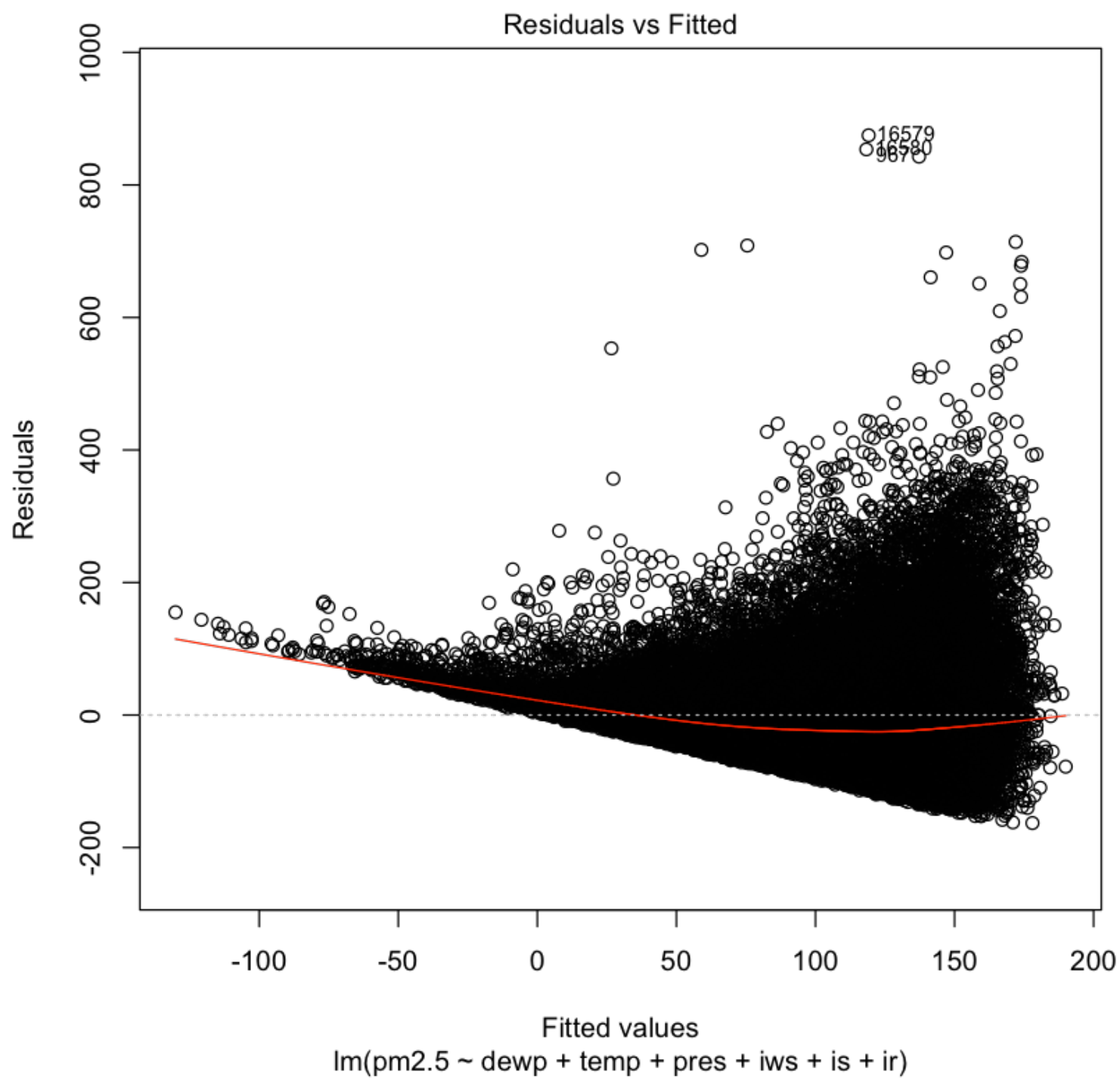
4.447625

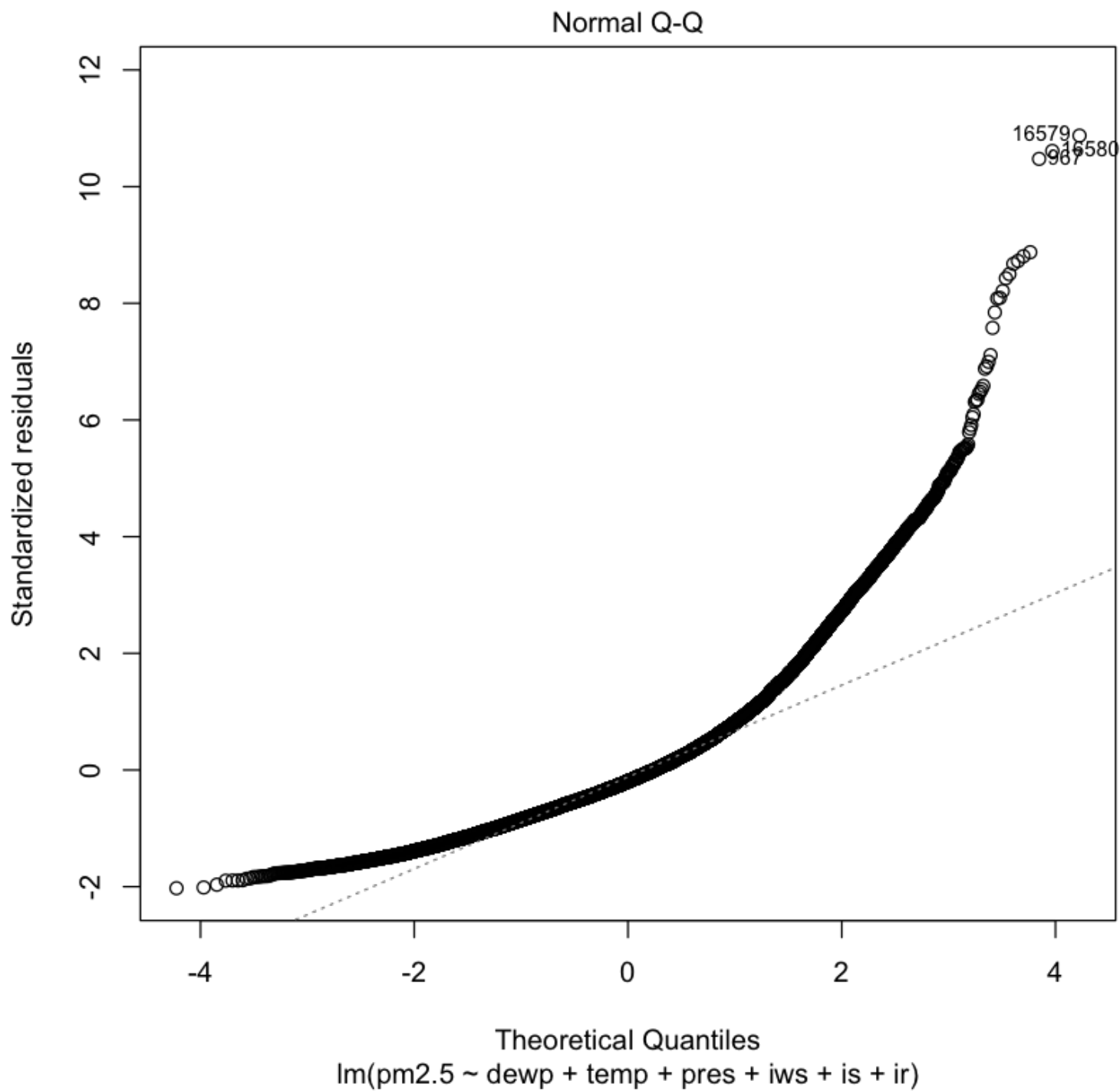
25.592575

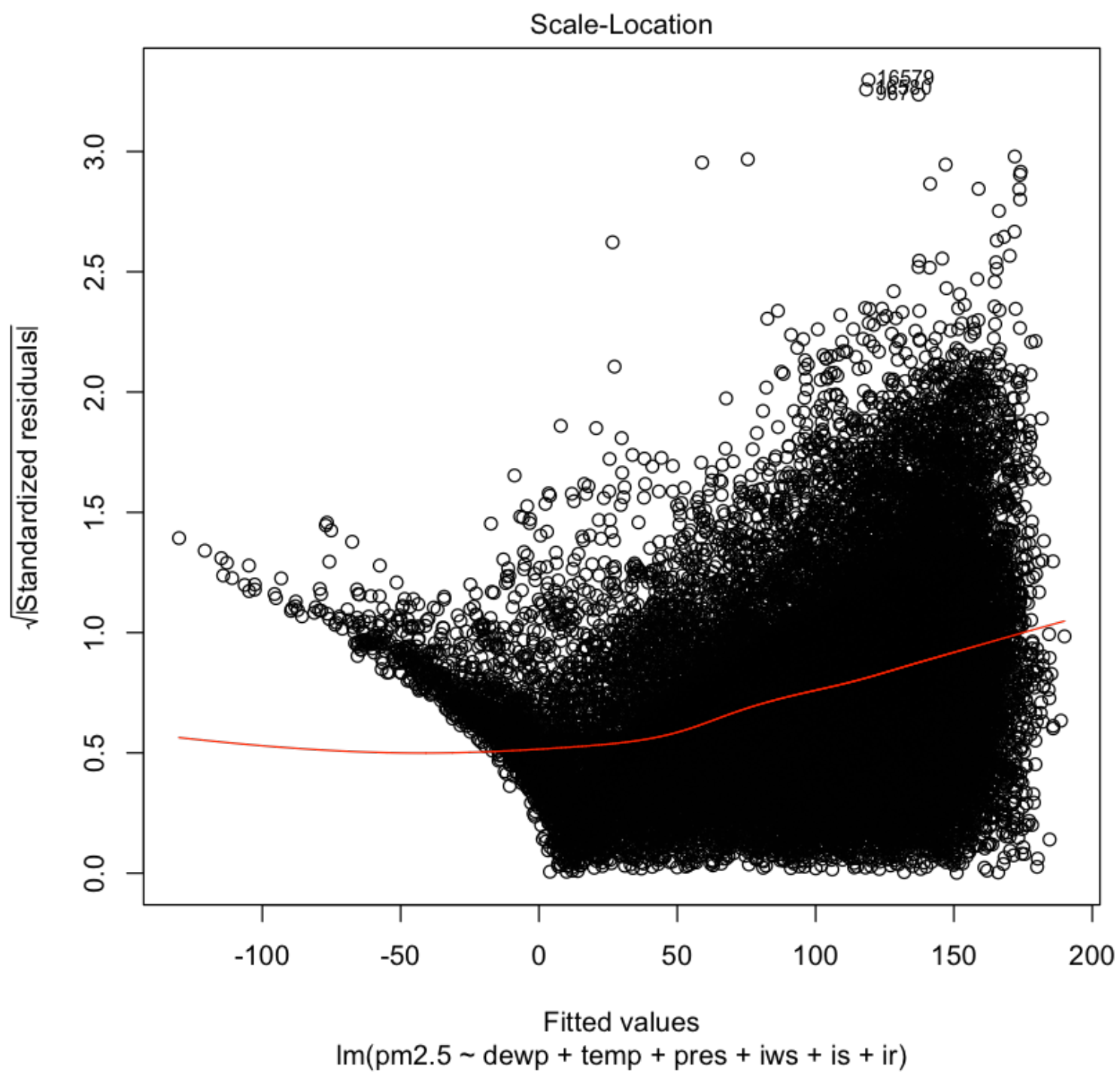
0.674495626527342

In [45]:

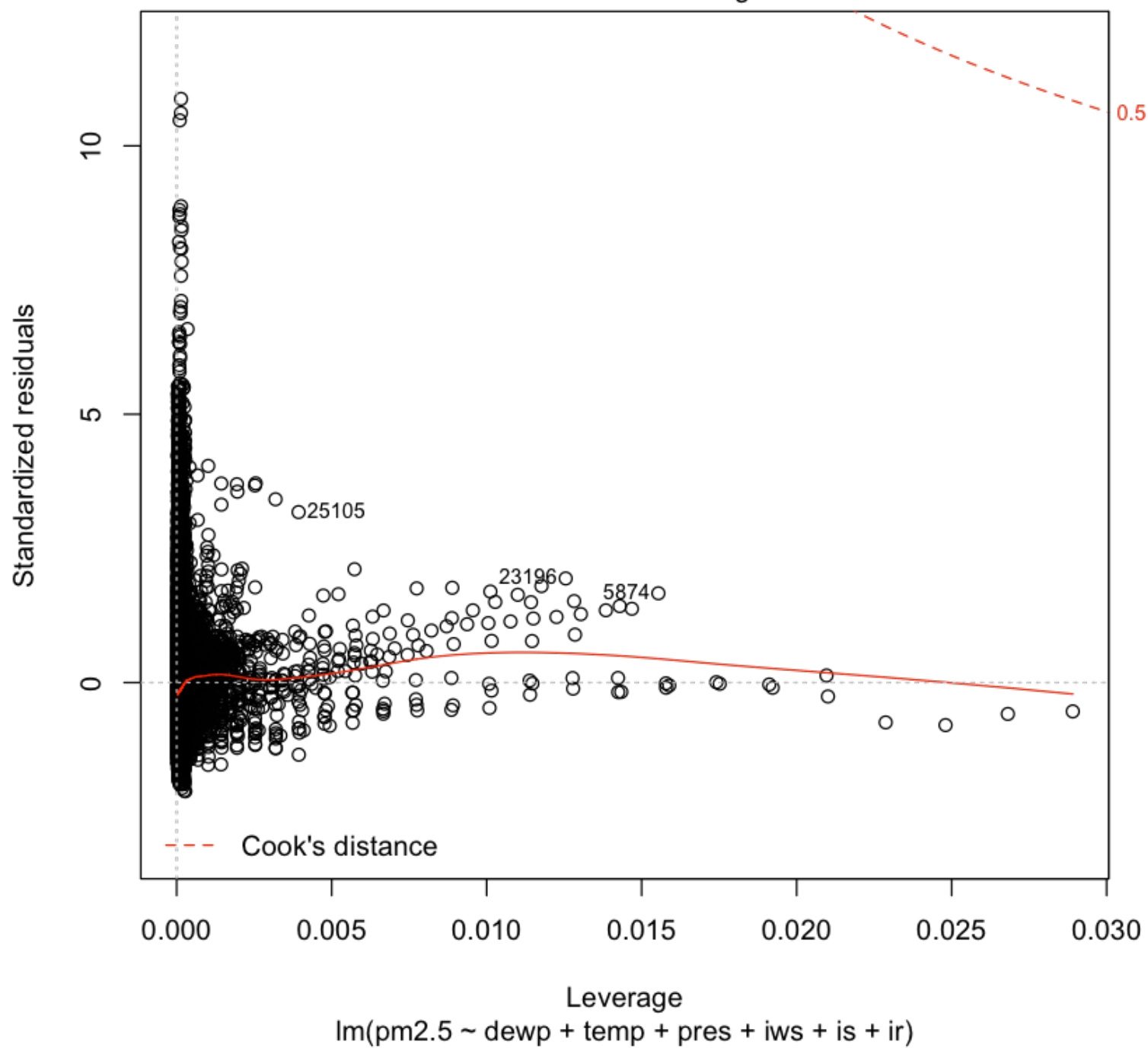
```
#plotting normal and residual vs fitted plots for the full model
plot(quality.lm)
```







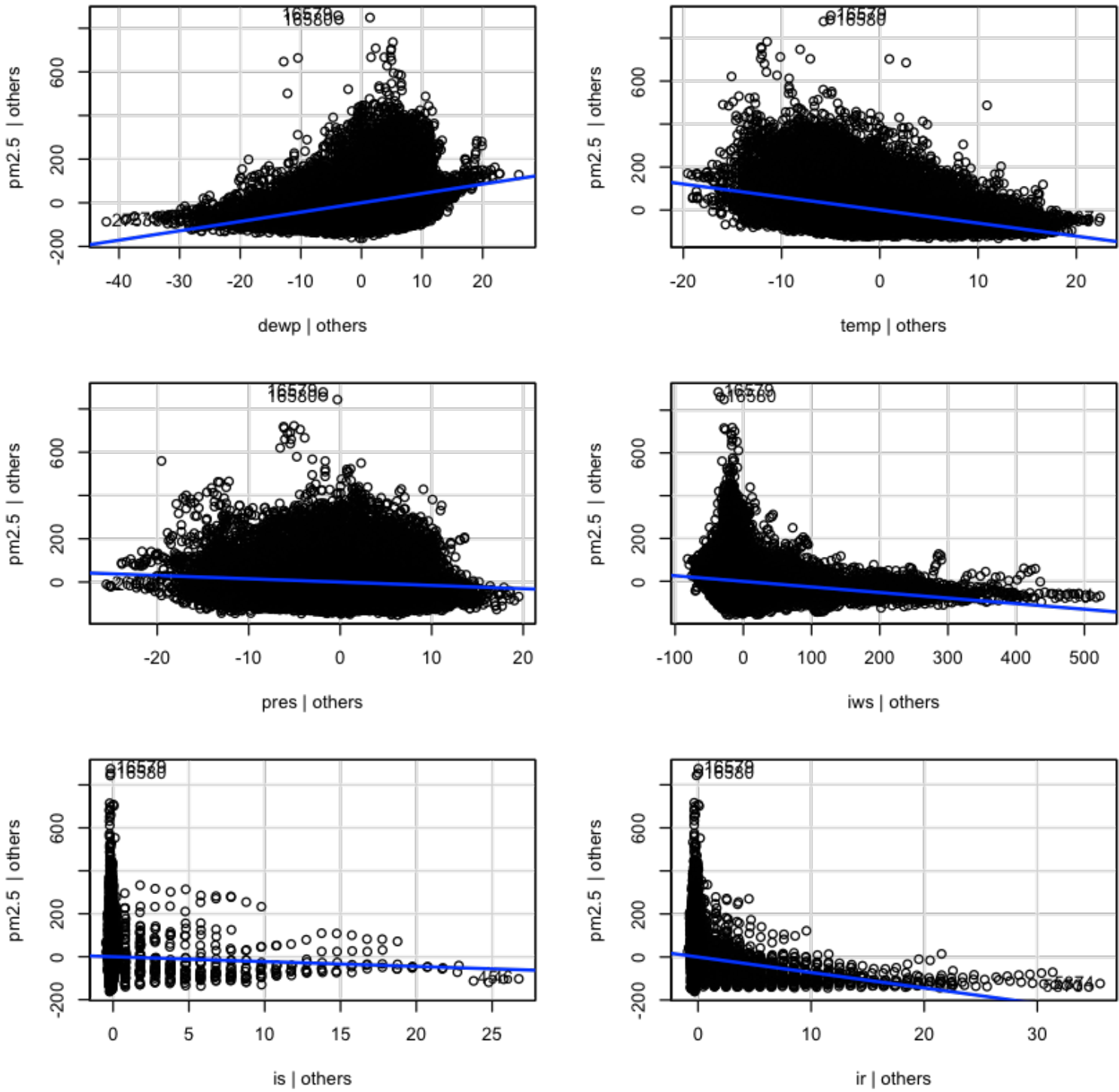
Residuals vs Leverage



In [130]:

```
avPlots(quality.lm)
```

Added-Variable Plots



In [46]:

```
#looking at residuals relationships
studentized.residuals = rstandard(quality.lm)
r.student.residuals = rstudent(quality.lm)
residuals = data.frame(studentized.residuals, r.student.residuals)
residuals$ID = seq.int(nrow(quality2))
head(residuals)
```

studentized.residuals	r.student.residuals	ID
0.06219617	0.06219543	1
0.24802944	0.24802665	2
0.11832494	0.11832354	3
0.23187989	0.23187727	4
-0.27154686	-0.27154385	5
-0.67645439	-0.67645000	6

In [48]:

```
#calculating min in order to transform the model appropriately based on log transformation
min(quality2$temp)
min(quality2$dewp)
min(quality2$pm2.5)
min(quality2$ir)
min(quality2$is)
min(quality2$iws)
min(quality2$pres)
```

-19
-40
0
0
0
0.45
991

In [52]:

```
# creating logarithmic transformed model
quality.lm2 = lm(log(pm2.5 + 1) ~ log(temp + 20) + log(dewp + 41) + log(ir + 1)
+ log(iws) + log(is + 1) + log(pres), data = quality2)#log(temp + 20) + log(dewp
+ 41), data = quality2)
summary(quality.lm2)
```

```
Call:
lm(formula = log(pm2.5 + 1) ~ log(temp + 20) + log(dewp + 41) +
    log(ir + 1) + log(iws) + log(is + 1) + log(pres), data = quality
2)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-3.9139	-0.5299	0.0624	0.5947	4.6837

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	66.246346	4.878310	13.580	<2e-16	***
log(temp + 20)	-1.364319	0.017341	-78.677	<2e-16	***
log(dewp + 41)	1.863805	0.019710	94.561	<2e-16	***
log(ir + 1)	-0.317148	0.012597	-25.176	<2e-16	***
log(iws)	-0.137644	0.002805	-49.069	<2e-16	***
log(is + 1)	0.024739	0.023314	1.061	0.289	
log(pres)	-9.250335	0.697051	-13.271	<2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8196 on 41750 degrees of freedom
Multiple R-squared: 0.3358, Adjusted R-squared: 0.3357
F-statistic: 3518 on 6 and 41750 DF, p-value: < 2.2e-16

In [53]:

```
# creating log transformed and partial model
quality.lm3 = lm(log(pm2.5 + 1) ~ log(temp + 20) + log(dewp + 41) + log(ir + 1)
+
    log(iws) + log(pres),
    data = quality2)#log(temp + 20) + log(dewp + 41), data = quality2)
summary(quality.lm3)
anova(quality.lm3)
```

```
Call:
lm(formula = log(pm2.5 + 1) ~ log(temp + 20) + log(dewp + 41) +
    log(ir + 1) + log(iws) + log(pres), data = quality2)
```

```
Residuals:
      Min       1Q   Median       3Q      Max
-3.9147 -0.5297  0.0626  0.5943  4.6923
```

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  66.046737   4.874689   13.55  <2e-16 ***
log(temp + 20) -1.366840   0.017177  -79.57  <2e-16 ***
log(dewp + 41)  1.866705   0.019520   95.63  <2e-16 ***
log(ir + 1)    -0.317493   0.012593  -25.21  <2e-16 ***
log(iws)       -0.137392   0.002795  -49.16  <2e-16 ***
log(pres)      -9.221829   0.696534  -13.24  <2e-16 ***
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

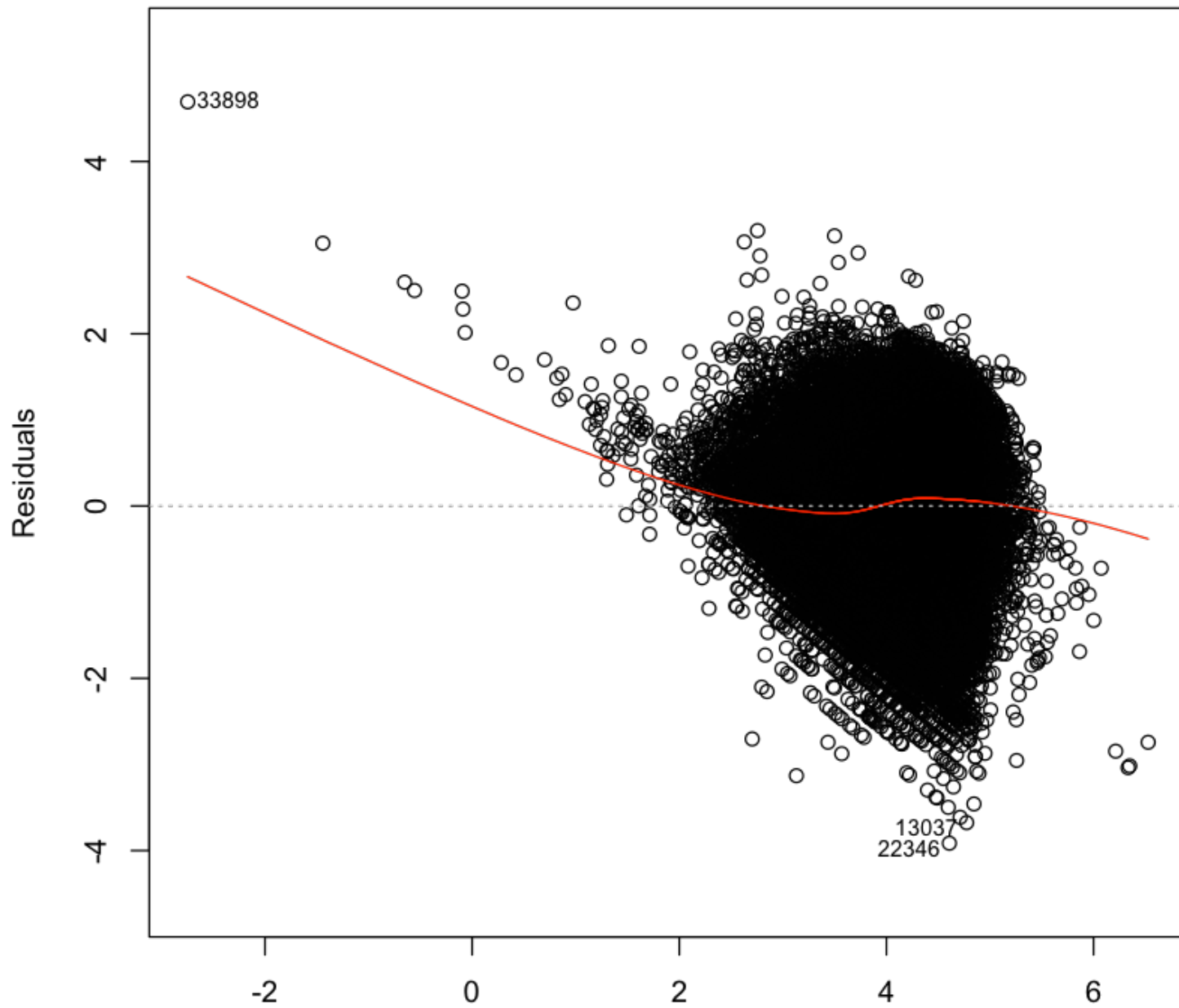
```
Residual standard error: 0.8196 on 41751 degrees of freedom
Multiple R-squared:  0.3358,    Adjusted R-squared:  0.3357
F-statistic: 4221 on 5 and 41751 DF,  p-value: < 2.2e-16
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
log(temp + 20)	1	21.48726	2.148726e+01	31.9896	1.560116e-08
log(dewp + 41)	1	11928.36177	1.192836e+04	17758.5943	0.000000e+00
log(ir + 1)	1	512.46088	5.124609e+02	762.9367	1.949623e-166
log(iws)	1	1597.19997	1.597200e+03	2377.8644	0.000000e+00
log(pres)	1	117.73934	1.177393e+02	175.2869	6.239461e-40
Residuals	41751	28043.94445	6.716952e-01	NA	NA

```
In [57]:
```

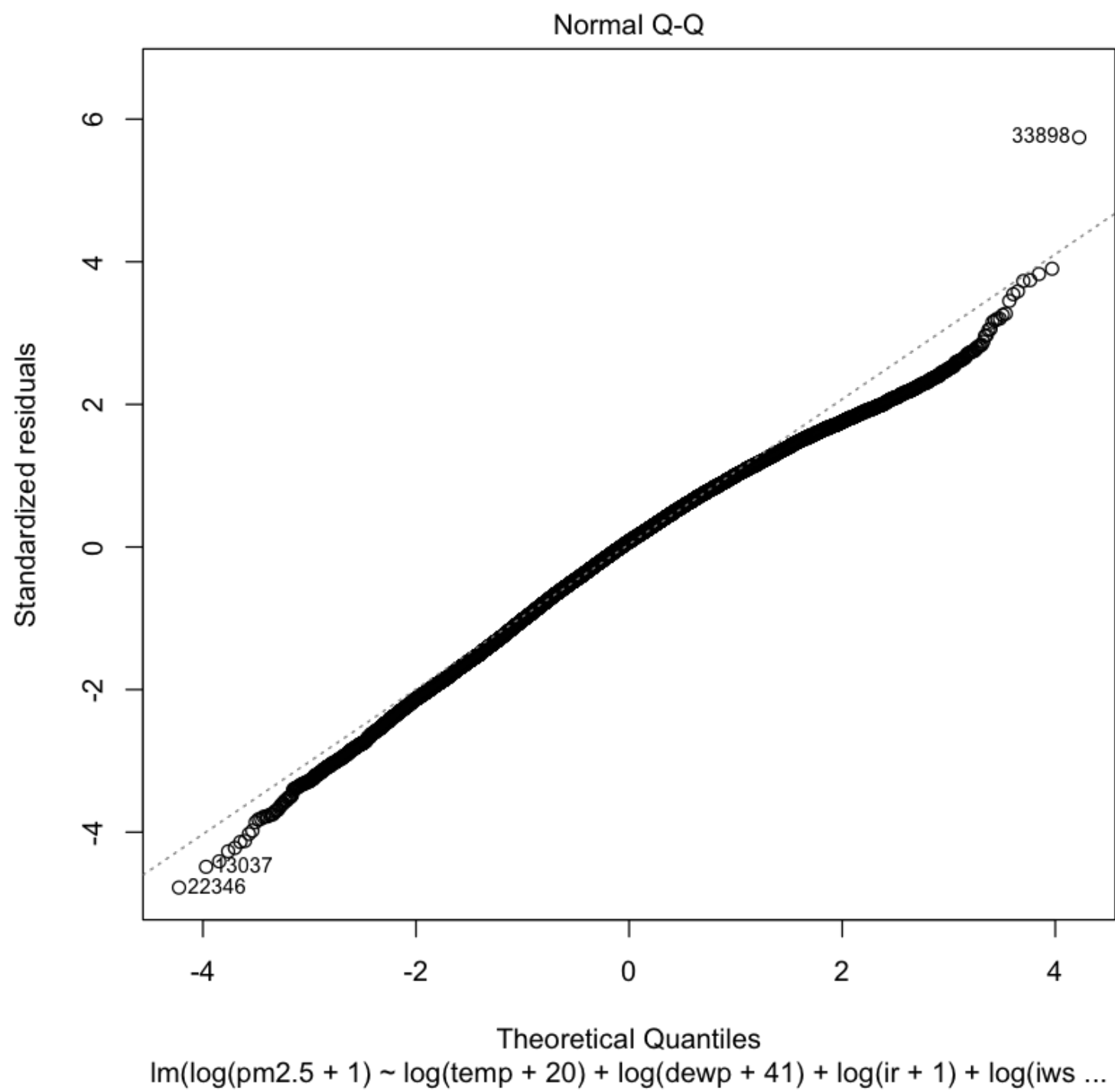
```
#plotting residual vs fitted and normal plot for the partial, transformed model
plot(quality.lm3)
```

Residuals vs Fitted

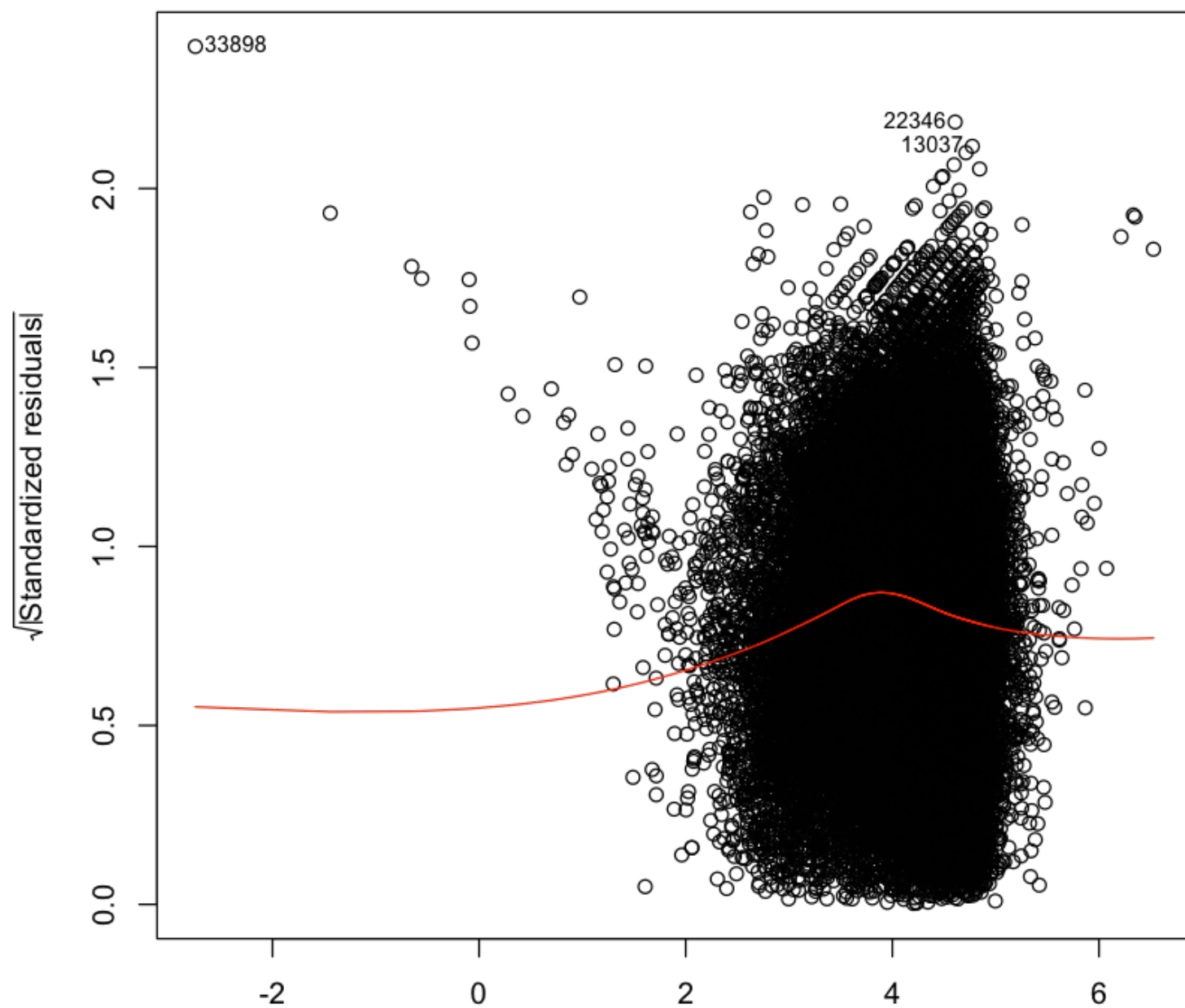


Fitted values

$\ln(\log(\text{pm2.5} + 1) \sim \log(\text{temp} + 20) + \log(\text{dewp} + 41) + \log(\text{ir} + 1) + \log(\text{iws} \dots$



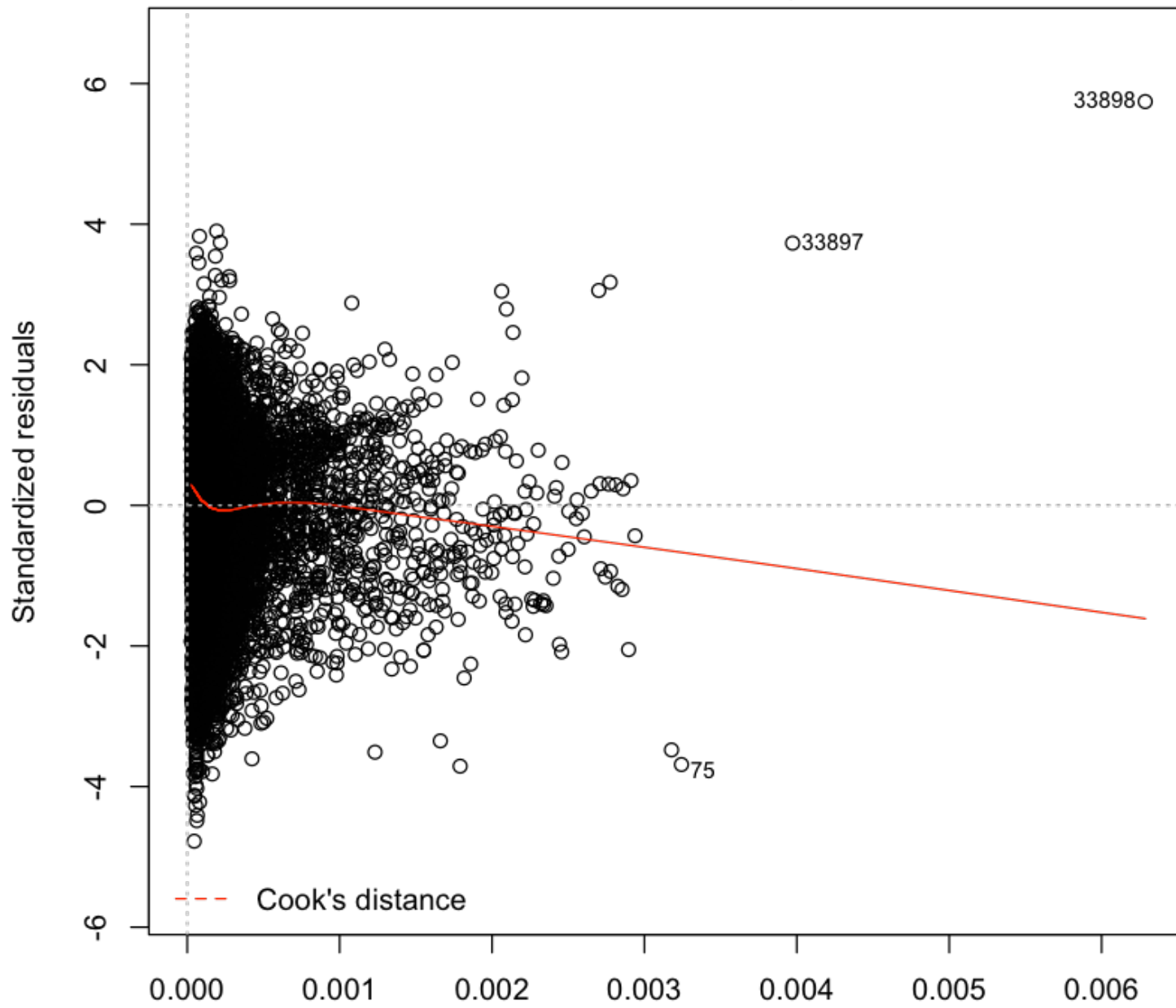
Scale-Location



Fitted values

$\ln(\log(\text{pm2.5} + 1) \sim \log(\text{temp} + 20) + \log(\text{dewp} + 41) + \log(\text{ir} + 1) + \log(\text{iws} \dots$

Residuals vs Leverage

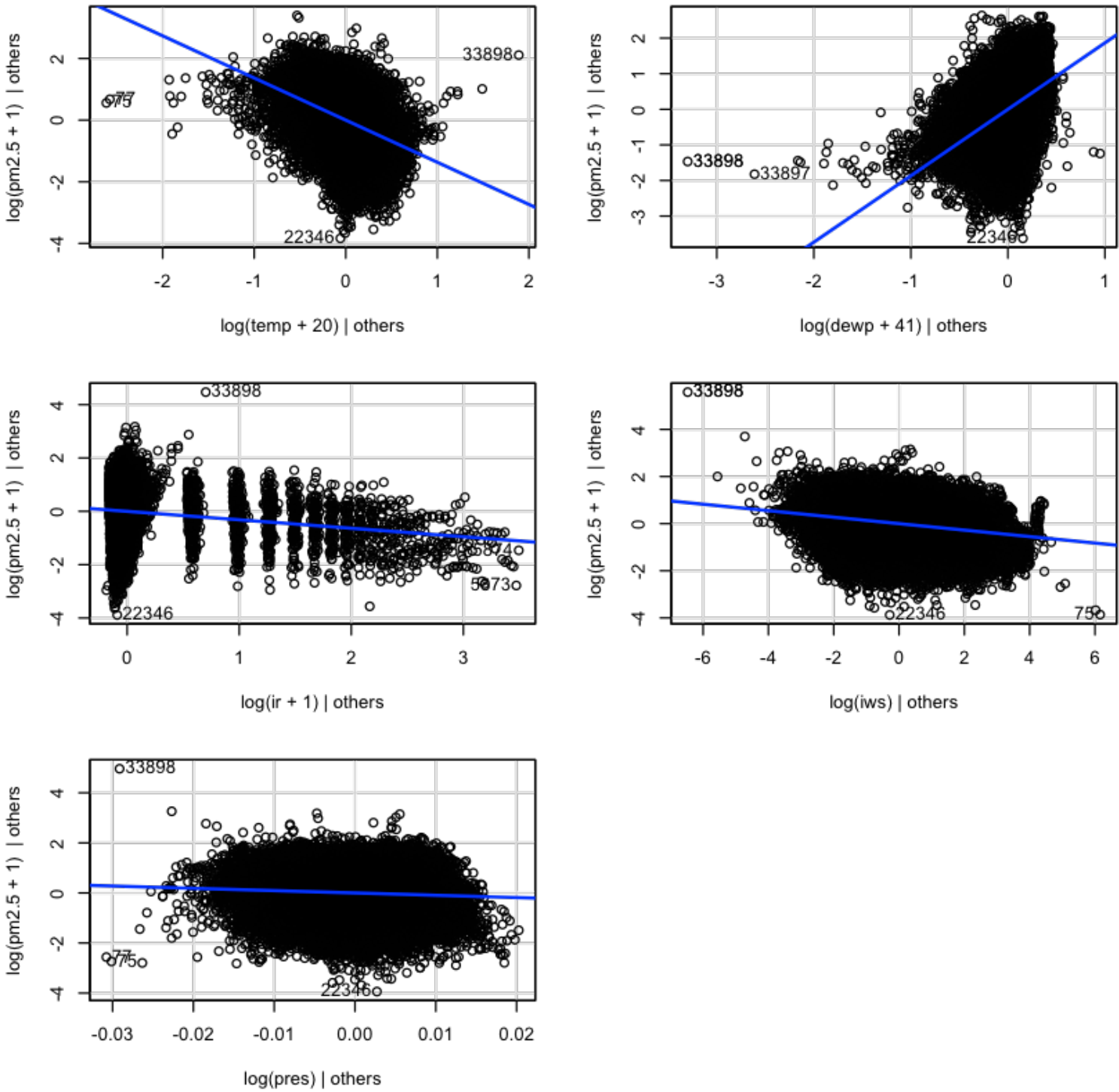


Leverage
 $\text{lm}(\log(\text{pm2.5} + 1) \sim \log(\text{temp} + 20) + \log(\text{dewp} + 41) + \log(\text{ir} + 1) + \log(\text{iws} \dots$

In [60]:

```
avPlots(quality.lm3)
```

Added-Variable Plots



In [68]:

```
#calculating PRESS statistics for each model as an indicator of predictive accuracy
full = sum((resid(quality.lm) / (1-lm.influence(quality.lm)$hat))^2)
full
trans1 = sum((resid(quality.lm2) / (1-lm.influence(quality.lm2)$hat))^2)
trans1
trans2 = sum((resid(quality.lm3) / (1-lm.influence(quality.lm3)$hat))^2)
trans2
```

270338852.262718

28052.5727840162

28052.5011265684

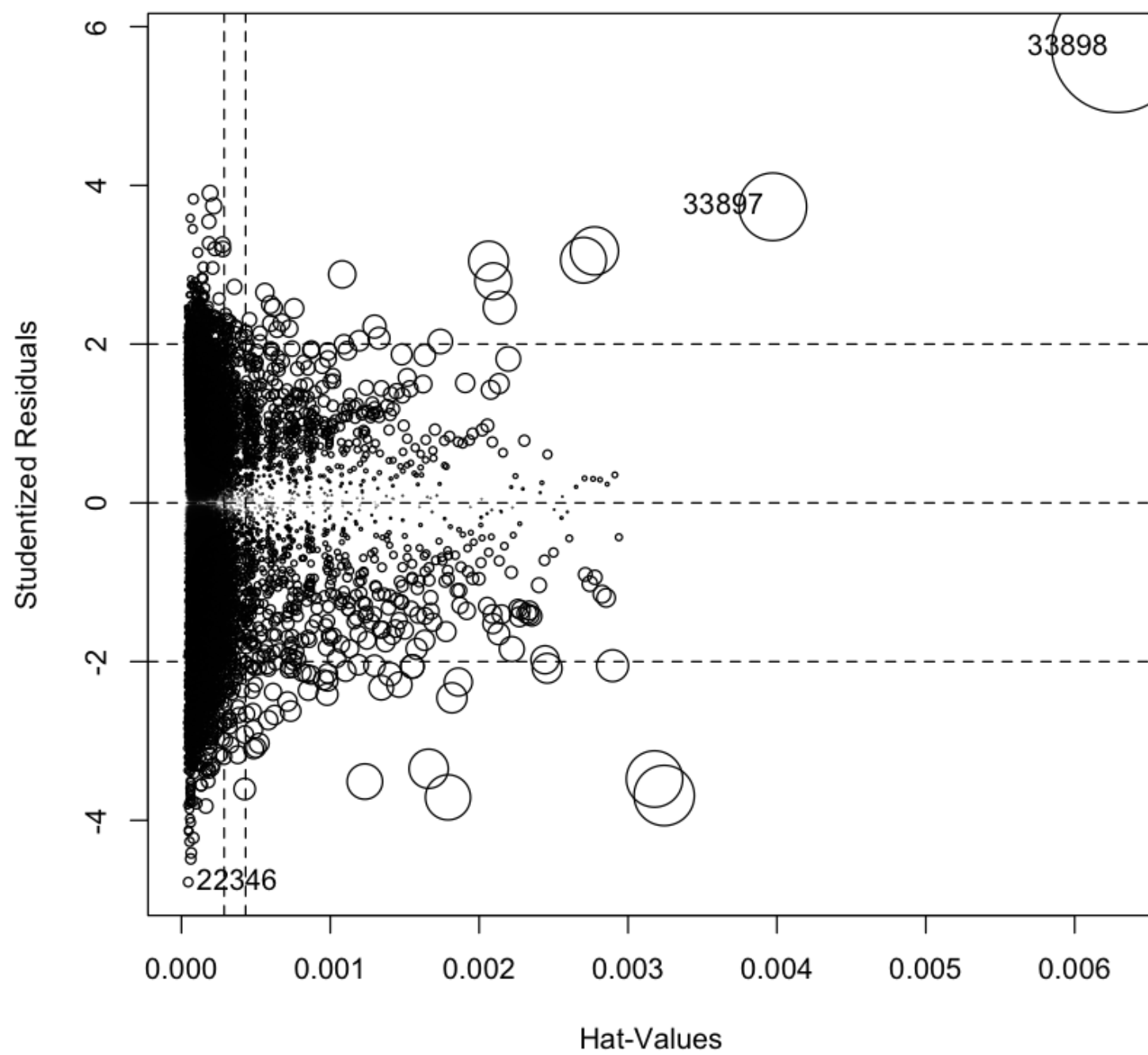
In [69]:

```
library(car)
```

In [70]:

```
# looking at influential points in the more accurate partial, transformed model
influencePlot(quality.lm3)
```

	StudRes	Hat	CookD
22346	-4.777892	4.571148e-05	0.0001738359
33897	3.730552	3.972530e-03	0.0092481860
33898	5.745626	6.286264e-03	0.0347793890



In [91]:

```
#creating a dataframe to simplify elimination of influential points
```

```
quality2$cooks = cooks.distance(quality.lm3)
quality2$outlier = ifelse(quality2$cooks < 4/nrow(quality2),
                          'keep', 'delete')
```

```
nrow(quality2)
```

```
quality3 = quality2[!(quality2$outlier == 'delete'),]
```

```
nrow(quality3)
```

```
nrow(quality2) - nrow(quality3)
```

41757

40063

1694

In [78]:

```
#creating new model, partial and transformed, using dataframe that has eliminate  
d influential points
```

```
quality.lm4 = lm(log(pm2.5 + 1) ~ log(temp + 20) + log(dewp + 41) + log(ir + 1)  
+
```

```
    log(iws) + log(pres),
```

```
    data = quality3)#log(temp + 20) + log(dewp + 41), data = quality2)
```

```
summary(quality.lm4)
```

```
anova(quality.lm4)
```

```
Call:
lm(formula = log(pm2.5 + 1) ~ log(temp + 20) + log(dewp + 41) +
    log(ir + 1) + log(iws) + log(pres), data = quality3)
```

```
Residuals:
      Min       1Q   Median       3Q      Max
-2.77128 -0.50820  0.05001  0.55719  2.29667
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  69.915960    4.717281   14.82  <2e-16 ***
log(temp + 20) -1.567131    0.017143  -91.42  <2e-16 ***
log(dewp + 41)  2.061474    0.019344  106.57  <2e-16 ***
log(ir + 1)    -0.283592    0.015382  -18.44  <2e-16 ***
log(iws)       -0.133139    0.002675  -49.77  <2e-16 ***
log(pres)      -9.783992    0.673881  -14.52  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

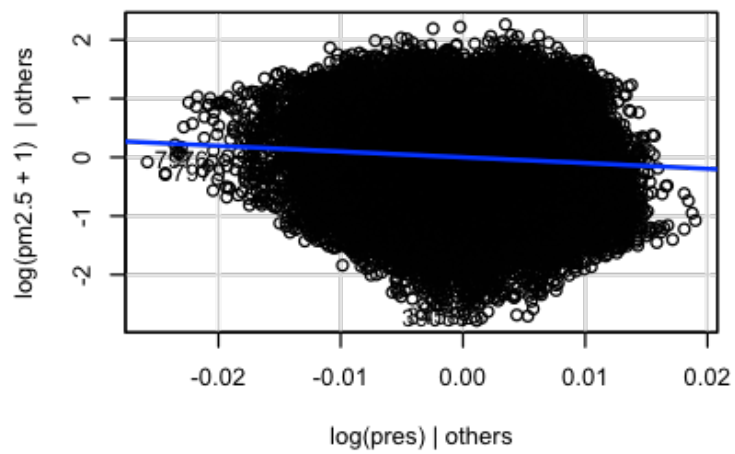
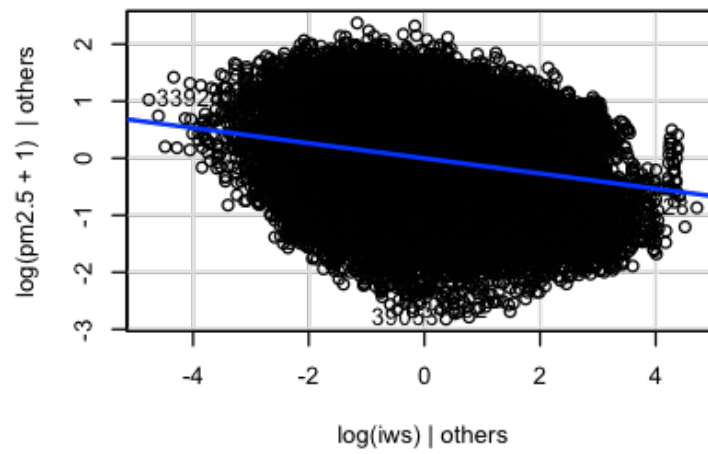
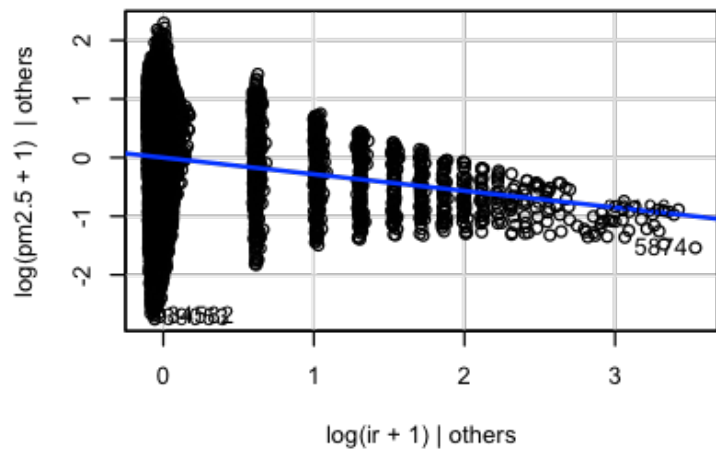
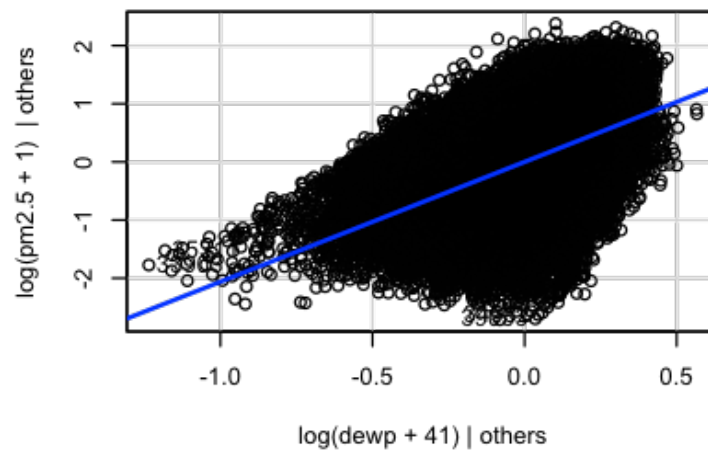
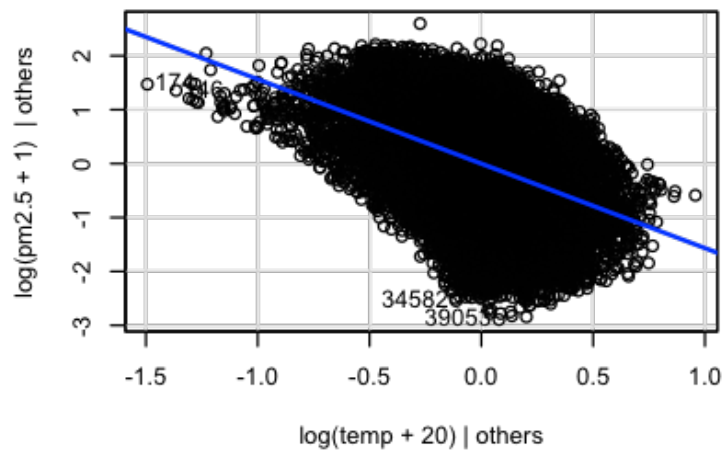
```
Residual standard error: 0.7584 on 40057 degrees of freedom
Multiple R-squared:  0.3887,    Adjusted R-squared:  0.3886
F-statistic: 5094 on 5 and 40057 DF,  p-value: < 2.2e-16
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
log(temp + 20)	1	4.304299	4.304299e+00	7.483632	6.228975e-03
log(dewp + 41)	1	12897.795002	1.289780e+04	22424.641208	0.000000e+00
log(ir + 1)	1	228.072824	2.280728e+02	396.536869	8.324667e-88
log(iws)	1	1399.218430	1.399218e+03	2432.739182	0.000000e+00
log(pres)	1	121.242749	1.212427e+02	210.797672	1.216175e-47
Residuals	40057	23039.252650	5.751617e-01	NA	NA

```
In [80]:
```

```
#viewing added variable plots for the final, partial model
avPlots(quality.lm4)
```

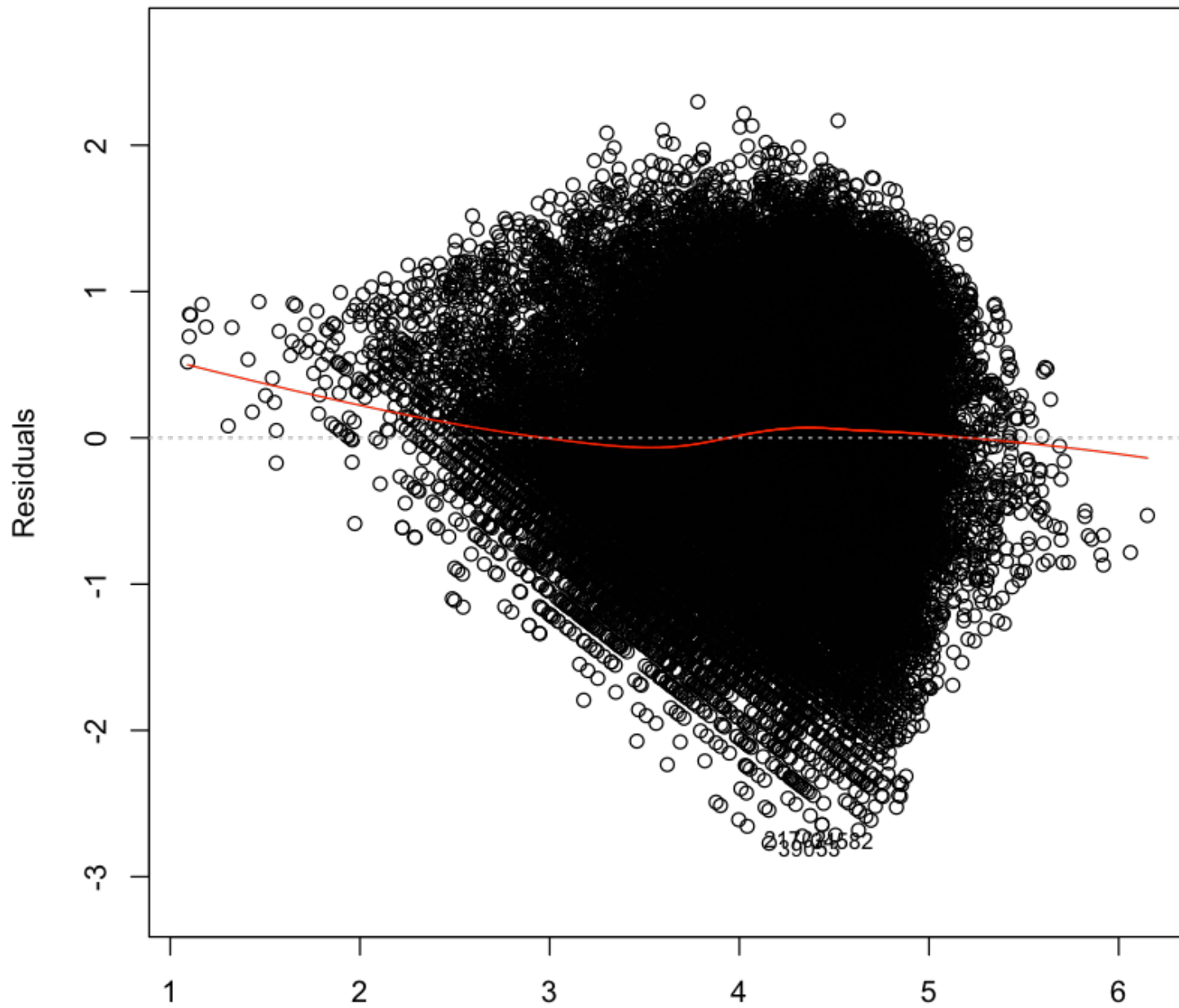
Added-Variable Plots



In [81]:

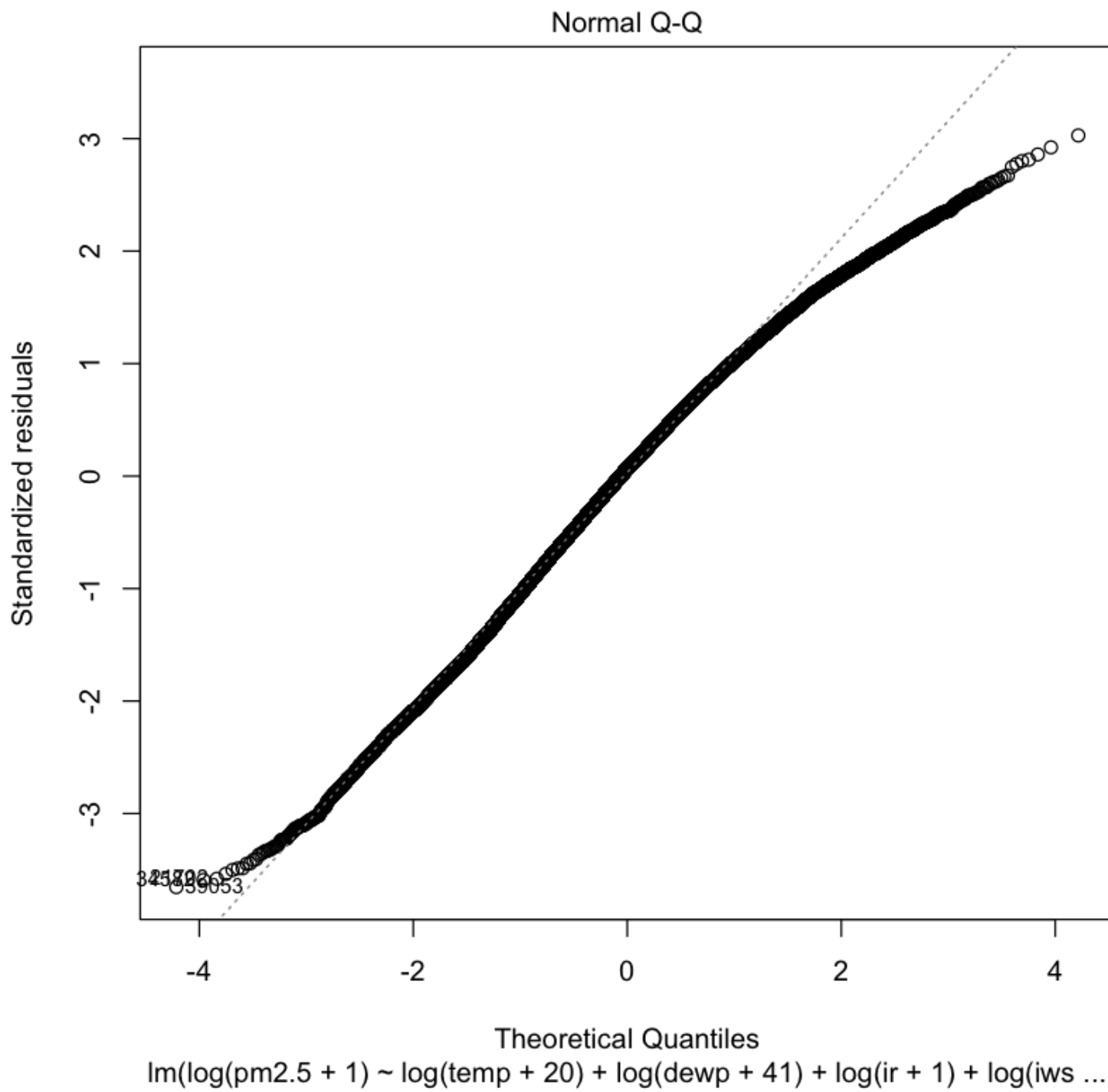
```
#plotting residual vs fitted and normal plot for the final model
plot(quality.lm4)
```

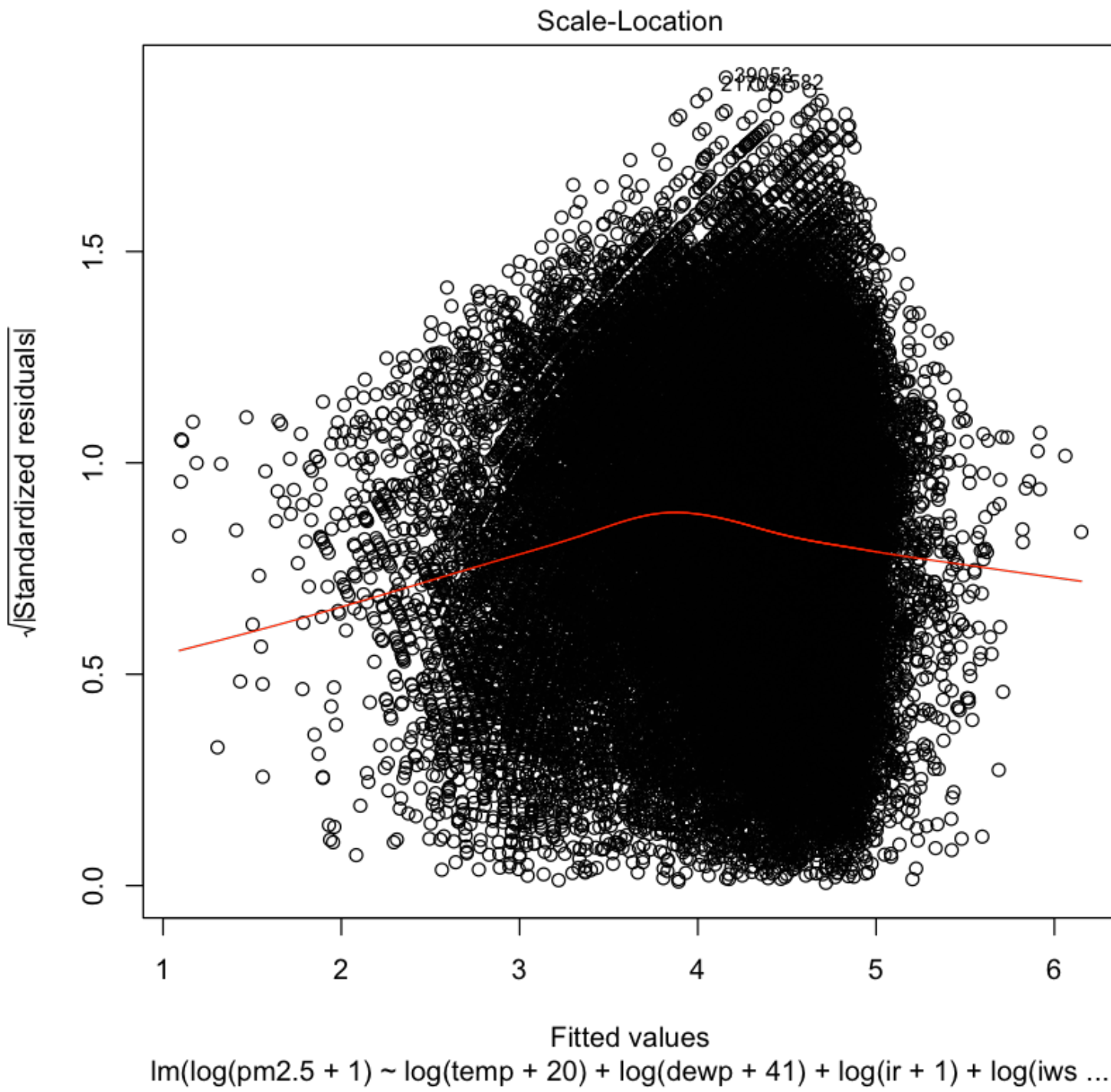
Residuals vs Fitted

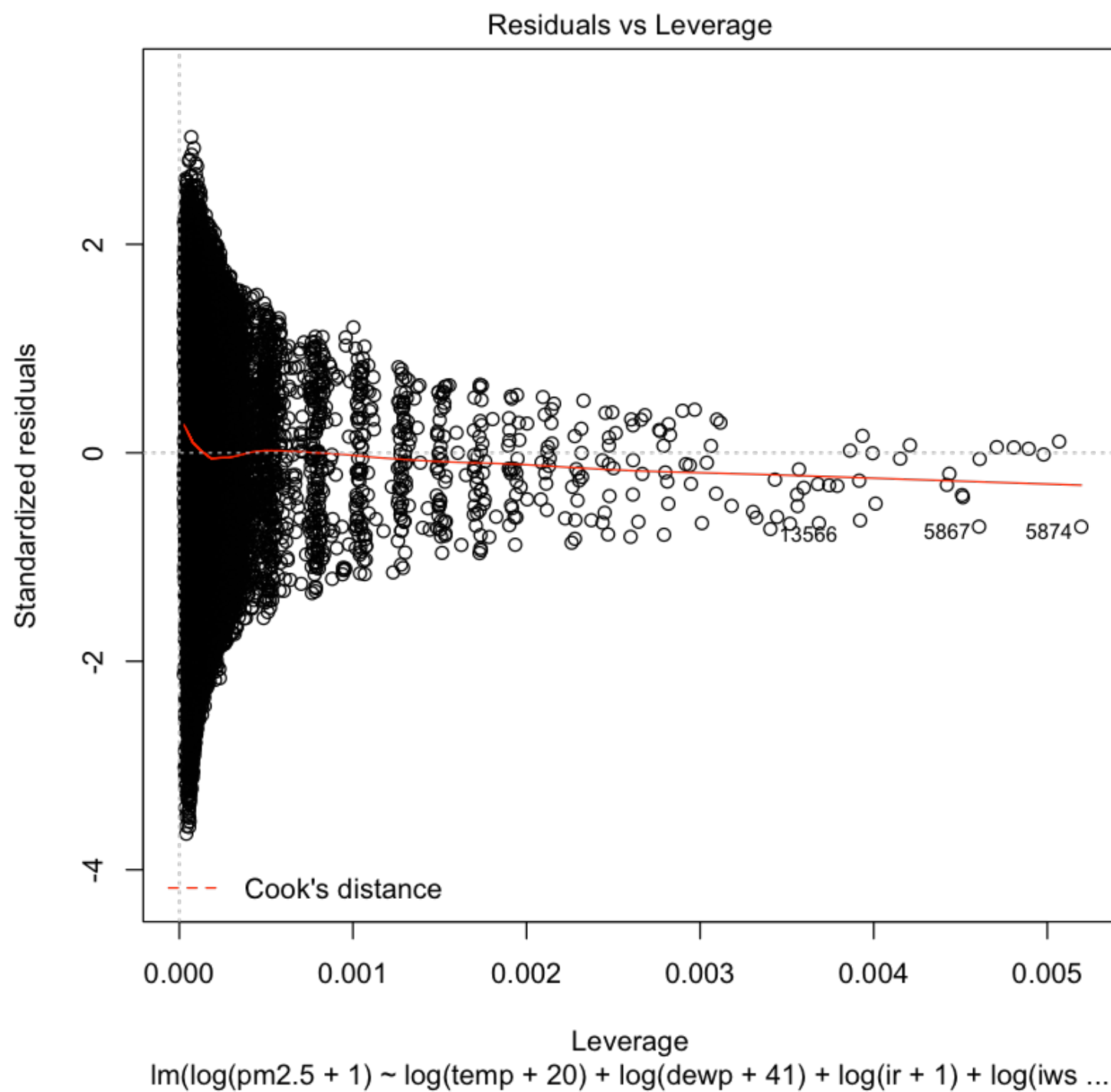


Fitted values

$\ln(\log(\text{pm2.5} + 1) \sim \log(\text{temp} + 20) + \log(\text{dewp} + 41) + \log(\text{ir} + 1) + \log(\text{iws} \dots$



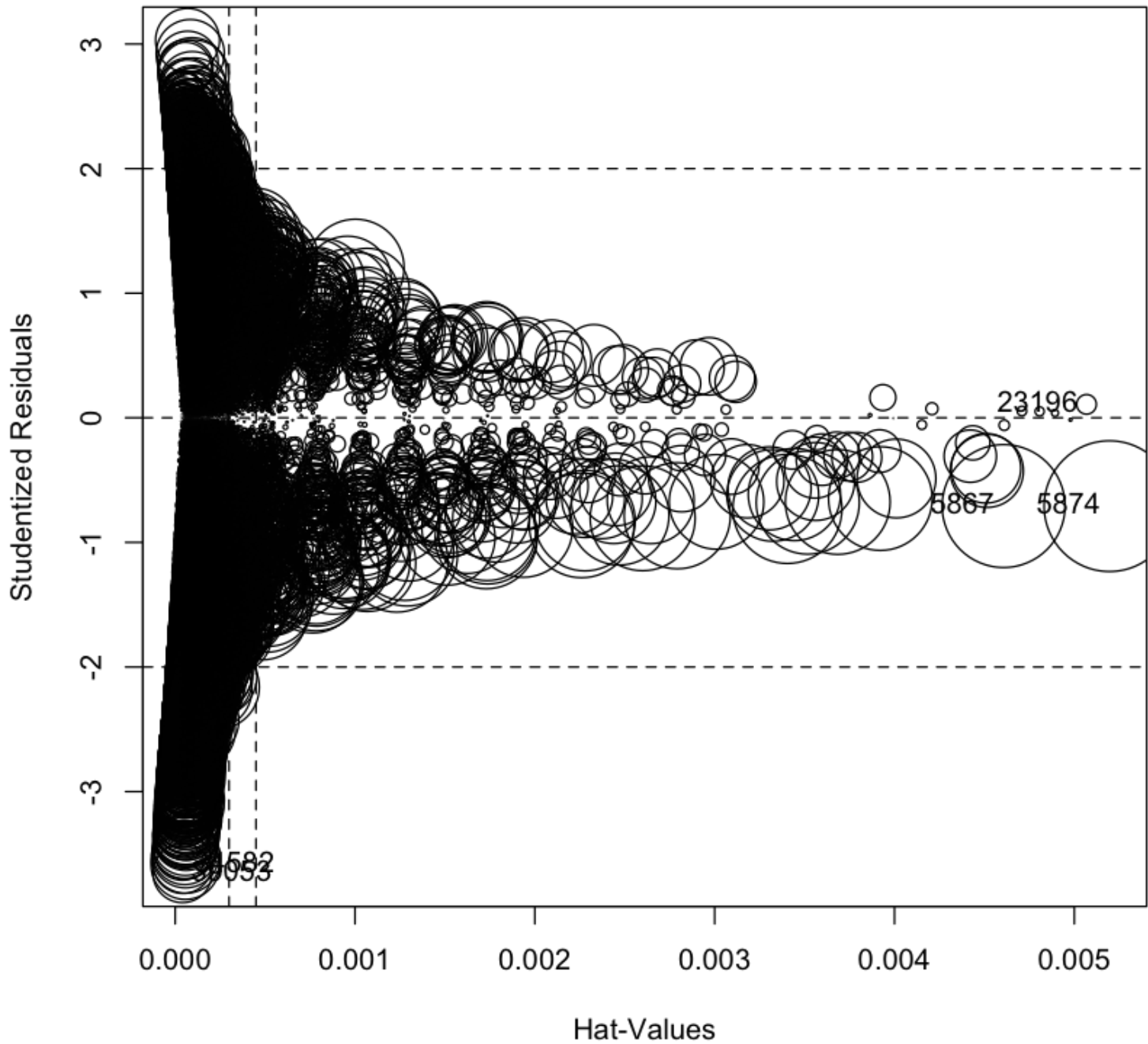




In [82]:

```
#calculating influential points and plot, to take a look at what has changed  
influencePlot(quality.lm4)
```

	StudRes	Hat	CookD
5867	-0.7083800	4.606775e-03	3.870696e-04
5874	-0.7093382	5.195548e-03	4.379802e-04
23196	0.1079260	5.068072e-03	9.889201e-06
34582	-3.5905763	5.603128e-05	1.203658e-04
39053	-3.6547811	3.993937e-05	8.889066e-05



```
In [84]:
#calculating PRESS statistic for the final model
trans.outliers = sum((resid(quality.lm4) / (1-lm.influence(quality.lm4)$hat))^2)
trans.outliers

23044.9043608328
```

In [86]:

```
#calculating F - statistic for the final model
qf(.95, df1 = 5, df2 = 40057)
```

2.21432259342585

In [93]:

```
quality3 = quality3[-c(1,2,3,4,5,10,12,14,15,16,17)]
head(quality3)
```

pm2.5	dewp	temp	pres	iws	ir
129	-16	-4	1020	1.79	0
148	-15	-4	1020	2.68	0
159	-11	-5	1021	3.57	0
181	-7	-5	1022	5.36	0
138	-7	-5	1022	6.25	0
109	-7	-6	1022	7.14	0

In [95]:

```
#creating matrix from data to calculate t-values and critical t to determine sig
nificance.
n = 40063
p = 6
quality3['B0'] = rep(1, n)
y = matrix(quality3$pm2.5, ncol = 1)
X = matrix(c(quality3$B0,quality3$dewp, quality3$temp, quality3$pres, quality3$i
ws,
            quality3$ir), ncol = p, byrow = FALSE)
beta.hat = solve(t(X)%*%X)%*%t(X)%*%y
SSres = as.vector(t(y)%*%y - t(beta.hat)%*%t(X)%*%y)
sig.hat = SSres/(n-p)
SSreg = as.vector(t(beta.hat)%*%t(X)%*%y - n*mean(y)^2)
C = solve(t(X)%*%X)
beta.se = sqrt(sig.hat*diag(C))
t = beta.hat/beta.se
critical.t = qt(1-0.5/2, n-p)
abs(t)
critical.t
```

22.46548
83.59505
91.54862
20.17455
32.10869
21.34565

0.674495874891155

In [100]:

```
#calculating confidence interval on Betas in the final model
(confint(quality.lm4))
```

	2.5 %	97.5 %
(Intercept)	60.6699795	79.1619409
log(temp + 20)	-1.6007315	-1.5335308
log(dewp + 41)	2.0235587	2.0993897
log(ir + 1)	-0.3137412	-0.2534431
log(iws)	-0.1383821	-0.1278958
log(pres)	-11.1048140	-8.4631694