CHAPTER 1 FUNCTIONS

1.1 FUNCTIONS AND THEIR GRAPHS

1. domain $= (-\infty, \infty)$; range $= [1, \infty)$

2. domain $= [0, \infty)$; range $= (-\infty, 1]$

3. domain $= [-2, \infty)$; y in range and $y = \sqrt{5x+10} \geq 0 \Rightarrow$ y can be any positive real number $\Rightarrow$ range $= [0, \infty)$.

4. domain $= (-\infty, 0] \cup [3, \infty)$; y in range and $y = \sqrt{x^2 - 3x} \geq 0 \Rightarrow$ y can be any positive real number $\Rightarrow$ range $= [0, \infty)$.

5. domain $= (-\infty, 3) \cup (3, \infty)$; y in range and $y = \frac{4}{3-t}$, now if $t < 3 \Rightarrow 3 - t > 0 \Rightarrow \frac{4}{3-t} > 0$, or if $t > 3$ $\Rightarrow 3 - t < 0 \Rightarrow \frac{4}{3-t} < 0 \Rightarrow$ y can be any nonzero real number $\Rightarrow$ range $= (-\infty, 0) \cup (0, \infty)$.

6. domain $= (-\infty, -4) \cup (-4, 4) \cup (4, \infty)$; y in range and $y = \frac{2}{t^2-16}$, now if $t < -4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0$, or if $-4 < t < 4 \Rightarrow -16 \leq t^2 - 16 < 0 \Rightarrow -\frac{2}{16} \leq \frac{2}{t^2-16} < 0$, or if $t > 4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0 \Rightarrow$ y can be any nonzero real number $\Rightarrow$ range $= (-\infty, -\frac{1}{8}] \cup (0, \infty)$.

7. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Is the graph of a function of x since any vertical line intersects the graph at most once.

8. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Not the graph of a function of x since it fails the vertical line test.

9. base $= x$; $(\text{height})^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow$ height $= \frac{\sqrt{3}}{2}x$; area is $a(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$; perimeter is $p(x) = x + x + x = 3x$.

10. $s =$ side length $\Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$; and area is $a = s^2 \Rightarrow a = \frac{1}{2}d^2$

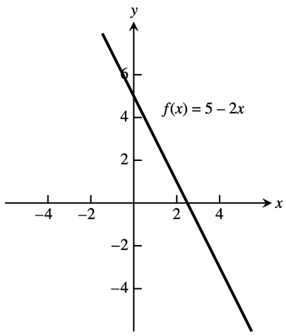
11. Let $D =$ diagonal length of a face of the cube and $\ell =$ the length of an edge. Then $\ell^2 + D^2 = d^2$ and $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$. The surface area is $6\ell^2 = \frac{6d^2}{3} = 2d^2$ and the volume is $\ell^3 = \left(\frac{d^2}{3}\right)^{3/2} = \frac{d^3}{3\sqrt{3}}$.

12. The coordinates of P are $\left(x, \sqrt{x}\right)$ so the slope of the line joining P to the origin is $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$ $(x > 0)$. Thus, $\left(x, \sqrt{x}\right) = \left(\frac{1}{m^2}, \frac{1}{m}\right)$.

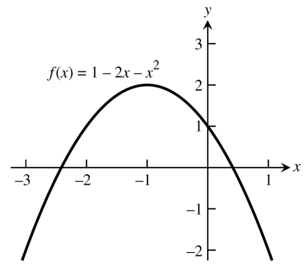
13. $2x + 4y = 5 \Rightarrow y = -\frac{1}{2}x + \frac{5}{4}$; $L = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + (-\frac{1}{2}x + \frac{5}{4})^2} = \sqrt{x^2 + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{25}{16}}$ $= \sqrt{\frac{5}{4}x^2 - \frac{5}{4}x + \frac{25}{16}} = \sqrt{\frac{20x^2 - 20x + 25}{16}} = \frac{\sqrt{20x^2 - 20x + 25}}{4}$

14. $y = \sqrt{x-3} \Rightarrow y^2 + 3 = x$; $L = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(y^2 + 3 - 4)^2 + y^2} = \sqrt{(y^2-1)^2 + y^2}$ $= \sqrt{y^4 - 2y^2 + 1 + y^2} = \sqrt{y^4 - y^2 + 1}$

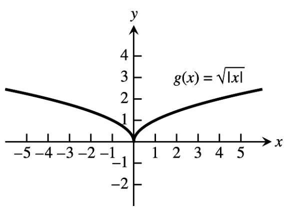
15. The domain is $(-\infty, \infty)$.



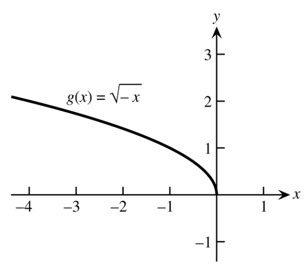
16. The domain is $(-\infty, \infty)$.



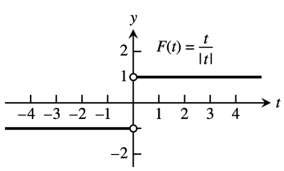
17. The domain is $(-\infty, \infty)$.



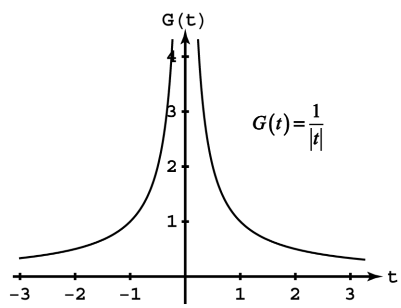
18. The domain is $(-\infty, 0]$.



19. The domain is $(-\infty, 0) \cup (0, \infty)$.



20. The domain is $(-\infty, 0) \cup (0, \infty)$.

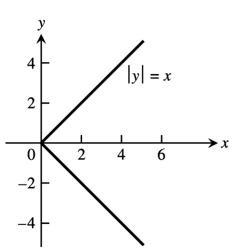


21. The domain is $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$

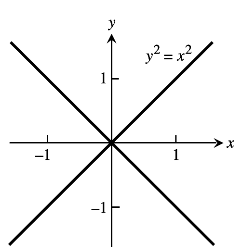
22. The range is $[2, 3)$.

23. Neither graph passes the vertical line test

(a)

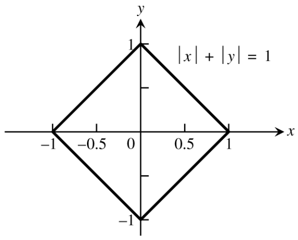


(b)

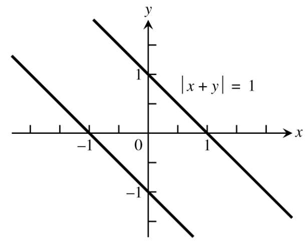


24. Neither graph passes the vertical line test

(a)



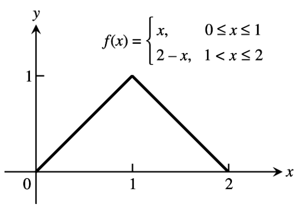
(b)



$$|x+y| = 1 \Leftrightarrow \left\{ \begin{array}{c} x+y=1 \\ \text{or} \\ x+y=-1 \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{c} y=1-x \\ \text{or} \\ y=-1-x \end{array} \right\}$$

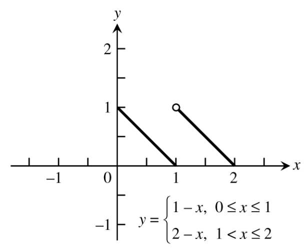
25.

x	0	1	2
y	0	1	0

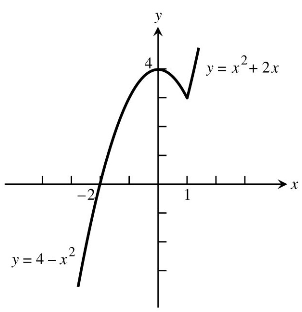


26.

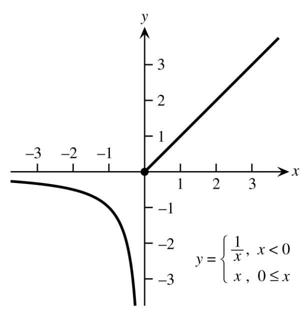
x	0	1	2
y	1	0	0



27. $F(x) = \begin{cases} 4 - x^2, & x \le 1 \\ x^2 + 2x, & x > 1 \end{cases}$



28. $G(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & 0 \le x \end{cases}$



29. (a) Line through $(0, 0)$ and $(1, 1)$: $y = x$; Line through $(1, 1)$ and $(2, 0)$: $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x + 2, & 1 < x \le 2 \end{cases}$$

(b) $$f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$$

30. (a) Line through $(0, 2)$ and $(2, 0)$: $y = -x + 2$

Line through $(2, 1)$ and $(5, 0)$: $m = \frac{0-1}{5-2} = \frac{-1}{3} = -\frac{1}{3}$, so $y = -\frac{1}{3}(x - 2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x + 2, & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \le 5 \end{cases}$$

(b) Line through $(-1, 0)$ and $(0, -3)$: $m = \frac{-3-0}{0-(-1)} = -3$, so $y = -3x - 3$

Line through $(0, 3)$ and $(2, -1)$: $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x - 3, & -1 < x \le 0 \\ -2x + 3, & 0 < x \le 2 \end{cases}$$

31. (a) Line through $(-1, 1)$ and $(0, 0)$: $y = -x$

Line through $(0, 1)$ and $(1, 1)$: $y = 1$

Line through $(1, 1)$ and $(3, 0)$: $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$, so $y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x & -1 \le x < 0 \\ 1 & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2} & 1 < x < 3 \end{cases}$$

(b) Line through $(-2, -1)$ and $(0, 0)$: $y = \frac{1}{2}x$

Line through $(0, 2)$ and $(1, 0)$: $y = -2x + 2$

Line through $(1, -1)$ and $(3, -1)$: $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x & -2 \le x \le 0 \\ -2x + 2 & 0 < x \le 1 \\ -1 & 1 < x \le 3 \end{cases}$$

32. (a) Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$: $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$, so $y = \frac{2}{T}\left(x - \frac{T}{2}\right) + 0 = \frac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$$

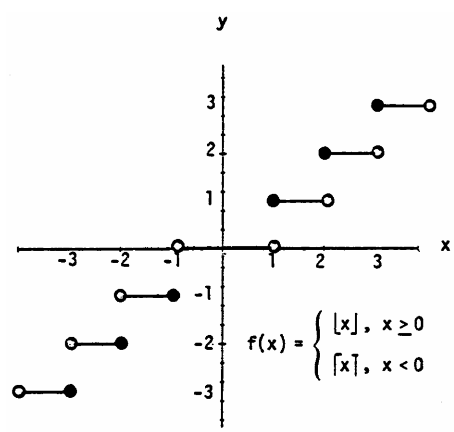
(b) $$f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$$

33. (a) $\lfloor x \rfloor = 0$ for $x \in [0, 1)$ (b) $\lceil x \rceil = 0$ for $x \in (-1, 0]$

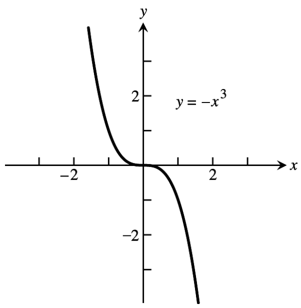
34. $\lfloor x \rfloor = \lceil x \rceil$ only when x is an integer.

35. For any real number x, $n \le x \le n+1$, where n is an integer. Now: $n \le x \le n+1 \Rightarrow -(n+1) \le -x \le -n$. By definition: $\lceil -x \rceil = -n$ and $\lfloor x \rfloor = n \Rightarrow -\lfloor x \rfloor = -n$. So $\lceil -x \rceil = -\lfloor x \rfloor$ for all $x \in \Re$.

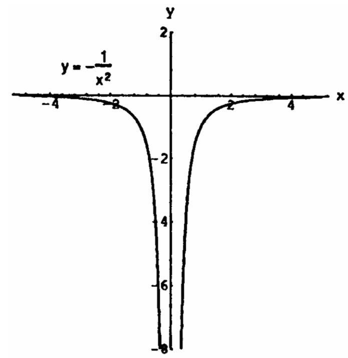
36. To find f(x) you delete the decimal or fractional portion of x, leaving only the integer part.



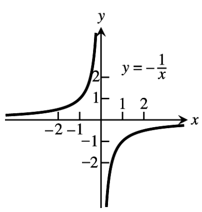
37. Symmetric about the origin
    Dec: $-\infty < x < \infty$
    Inc: nowhere



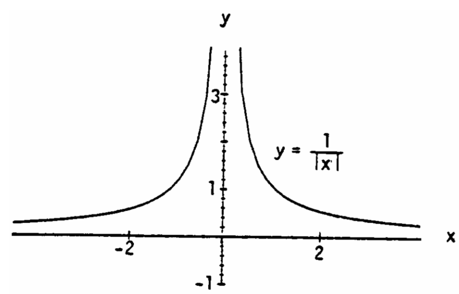
38. Symmetric about the y-axis
    Dec: $-\infty < x < 0$
    Inc: $0 < x < \infty$



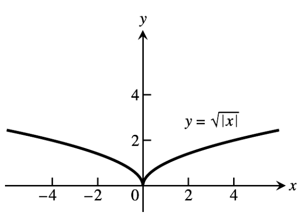
39. Symmetric about the origin
    Dec: nowhere
    Inc: $-\infty < x < 0$
    $0 < x < \infty$



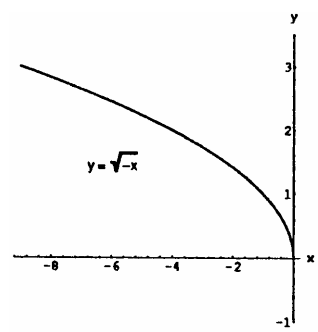
40. Symmetric about the y-axis
    Dec: $0 < x < \infty$
    Inc: $-\infty < x < 0$



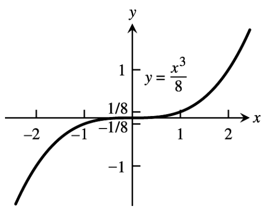
41. Symmetric about the y-axis
    Dec: $-\infty < x \le 0$
    Inc: $0 < x < \infty$



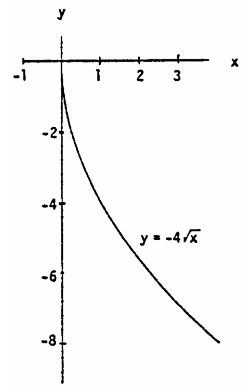
42. No symmetry
    Dec: $-\infty < x \le 0$
    Inc: nowhere



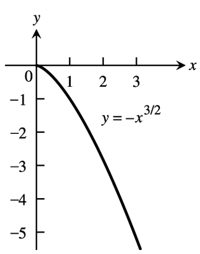
43. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < \infty$



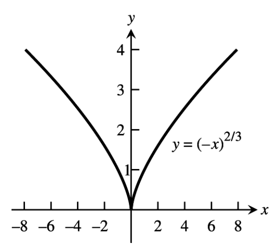
44. No symmetry
Dec: $0 \le x < \infty$
Inc: nowhere



45. No symmetry
Dec: $0 \le x < \infty$
Inc: nowhere



46. Symmetric about the y-axis
Dec: $-\infty < x \le 0$
Inc: $0 < x < \infty$



47. Since a horizontal line not through the origin is symmetric with respect to the y-axis, but not with respect to the origin, the function is even.

48. $f(x) = x^{-5} = \frac{1}{x^5}$ and $f(-x) = (-x)^{-5} = \frac{1}{(-x)^5} = -\left(\frac{1}{x^5}\right) = -f(x)$. Thus the function is odd.

49. Since $f(x) = x^2 + 1 = (-x)^2 + 1 = -f(x)$. The function is even.

50. Since $[f(x) = x^2 + x] \neq [f(-x) = (-x)^2 - x]$ and $[f(x) = x^2 + x] \neq [-f(x) = -(x)^2 - x]$ the function is neither even nor odd.

51. Since $g(x) = x^3 + x$, $g(-x) = -x^3 - x = -(x^3 + x) = -g(x)$. So the function is odd.

52. $g(x) = x^4 + 3x^2 - 1 = (-x)^4 + 3(-x)^2 - 1 = g(-x)$, thus the function is even.

53. $g(x) = \frac{1}{x^2 - 1} = \frac{1}{(-x)^2 - 1} = g(-x)$. Thus the function is even.

54. $g(x) = \frac{x}{x^2 - 1}$; $g(-x) = -\frac{x}{x^2 - 1} = -g(x)$. So the function is odd.

55. $h(t) = \frac{1}{t - 1}$; $h(-t) = \frac{1}{-t - 1}$; $-h(t) = \frac{1}{1 - t}$. Since $h(t) \neq -h(t)$ and $h(t) \neq h(-t)$, the function is neither even nor odd.

56. Since $|t^3| = |(-t)^3|$, $h(t) = h(-t)$ and the function is even.

57. $h(t) = 2t + 1$, $h(-t) = -2t + 1$. So $h(t) \neq h(-t)$. $-h(t) = -2t - 1$, so $h(t) \neq -h(t)$. The function is neither even nor odd.

58. $h(t) = 2|\,t\,| + 1$ and $h(-t) = 2|-t\,| + 1 = 2|\,t\,| + 1$. So $h(t) = h(-t)$ and the function is even.

59. $s = kt \Rightarrow 25 = k(75) \Rightarrow k = \frac{1}{3} \Rightarrow s = \frac{1}{3}t$; $60 = \frac{1}{3}t \Rightarrow t = 180$

60. $K = c\,v^2 \Rightarrow 12960 = c(18)^2 \Rightarrow c = 40 \Rightarrow K = 40v^2$; $K = 40(10)^2 = 4000$ joules

61. $r = \frac{k}{s} \Rightarrow 6 = \frac{k}{4} \Rightarrow k = 24 \Rightarrow r = \frac{24}{s}$; $10 = \frac{24}{s} \Rightarrow s = \frac{12}{5}$

62. $P = \frac{k}{v} \Rightarrow 14.7 = \frac{k}{1000} \Rightarrow k = 14700 \Rightarrow P = \frac{14700}{v}$; $23.4 = \frac{14700}{v} \Rightarrow v = \frac{24500}{39} \approx 628.2\ \text{in}^3$

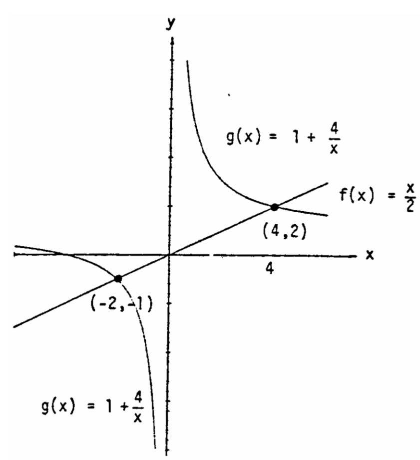
63. $v = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x$; $0 < x < 7$.

64. (a) Let h = height of the triangle. Since the triangle is isosceles, $\overline{AB}^{\,2} + \overline{AB}^{\,2} = 2^2 \Rightarrow \overline{AB} = \sqrt{2}$. So,
$h^2 + 1^2 = \left(\sqrt{2}\right)^2 \Rightarrow h = 1 \Rightarrow$ B is at $(0, 1) \Rightarrow$ slope of AB $= -1 \Rightarrow$ The equation of AB is
$y = f(x) = -x + 1$; $x \in [0, 1]$.
(b) $A(x) = 2x\,y = 2x(-x + 1) = -2x^2 + 2x$; $x \in [0, 1]$.

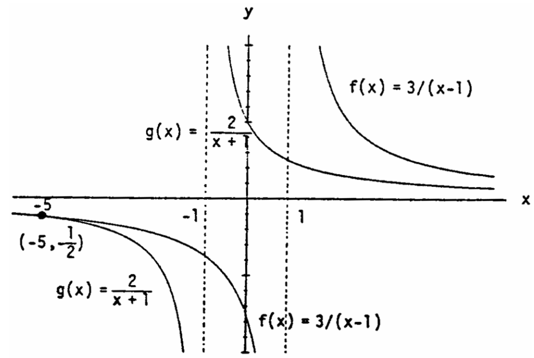
65. (a) Graph h because it is an even function and rises less rapidly than does Graph g.
(b) Graph f because it is an odd function.
(c) Graph g because it is an even function and rises more rapidly than does Graph h.

66. (a) Graph f because it is linear.
(b) Graph g because it contains $(0, 1)$.
(c) Graph h because it is a nonlinear odd function.

67. (a) From the graph, $\frac{x}{2} > 1 + \frac{4}{x} \;\Rightarrow\; x \in (-2, 0) \cup (4, \infty)$
(b) $\frac{x}{2} > 1 + \frac{4}{x} \;\Rightarrow\; \frac{x}{2} - 1 - \frac{4}{x} > 0$
$x > 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \;\Rightarrow\; \frac{x^2 - 2x - 8}{2x} > 0 \;\Rightarrow\; \frac{(x-4)(x+2)}{2x} > 0$
$\Rightarrow\; x > 4$ since x is positive;
$x < 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \;\Rightarrow\; \frac{x^2 - 2x - 8}{2x} < 0 \;\Rightarrow\; \frac{(x-4)(x+2)}{2x} < 0$
$\Rightarrow\; x < -2$ since x is negative;
sign of $(x - 4)(x + 2)$
$+ \quad - \quad +$
$-2 \quad 4$
Solution interval: $(-2, 0) \cup (4, \infty)$



68. (a) From the graph, $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow x \in (-\infty, -5) \cup (-1, 1)$



(b) Case $x < -1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} > 2$
$\Rightarrow 3x+3 < 2x-2 \Rightarrow x < -5.$
Thus, $x \in (-\infty, -5)$ solves the inequality.

Case $-1 < x < 1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} < 2$
$\Rightarrow 3x+3 > 2x-2 \Rightarrow x > -5$ which is true if $x > -1$. Thus, $x \in (-1, 1)$ solves the inequality.

Case $1 < x$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow 3x+3 < 2x-2 \Rightarrow x < -5$
which is never true if $1 < x$, so no solution here.

In conclusion, $x \in (-\infty, -5) \cup (-1, 1)$.

69. A curve symmetric about the x-axis will not pass the vertical line test because the points (x, y) and $(x, -y)$ lie on the same vertical line. The graph of the function $y = f(x) = 0$ is the x-axis, a horizontal line for which there is a single y-value, 0, for any x.

70. price $= 40 + 5x$, quantity $= 300 - 25x \Rightarrow R(x) = (40 + 5x)(300 - 25x)$

71. $x^2 + x^2 = h^2 \Rightarrow x = \frac{h}{\sqrt{2}} = \frac{\sqrt{2}h}{2}$; cost $= 5(2x) + 10h \Rightarrow C(h) = 10\left(\frac{\sqrt{2}h}{2}\right) + 10h = 5h\left(\sqrt{2} + 2\right)$

72. (a) Note that 2 mi = 10,560 ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and $(10{,}560 - x)$ feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10{,}560 - x)$.

(b) $C(0) = \$1,200,000$
$C(500) \approx \$1,175,812$
$C(1000) \approx \$1,186,512$
$C(1500) \approx \$1,212,000$
$C(2000) \approx \$1,243,732$
$C(2500) \approx \$1,278,479$
$C(3000) \approx \$1,314,870$
Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from the point P.

1.2 COMBINING FUNCTIONS; SHIFTING AND SCALING GRAPHS

1. $D_f: -\infty < x < \infty, D_g: x \ge 1 \Rightarrow D_{f+g} = D_{fg}: x \ge 1$. $R_f: -\infty < y < \infty, R_g: y \ge 0, R_{f+g}: y \ge 1, R_{fg}: y \ge 0$

2. $D_f: x + 1 \ge 0 \Rightarrow x \ge -1, D_g: x - 1 \ge 0 \Rightarrow x \ge 1$. Therefore $D_{f+g} = D_{fg}: x \ge 1$.
$R_f = R_g: y \ge 0, R_{f+g}: y \ge \sqrt{2}, R_{fg}: y \ge 0$

3. $D_f: -\infty < x < \infty, D_g: -\infty < x < \infty, D_{f/g}: -\infty < x < \infty, D_{g/f}: -\infty < x < \infty, R_f: y = 2, R_g: y \ge 1,$
$R_{f/g}: 0 < y \le 2, R_{g/f}: \frac{1}{2} \le y < \infty$

4. $D_f: -\infty < x < \infty, D_g: x \ge 0, D_{f/g}: x \ge 0, D_{g/f}: x \ge 0; R_f: y = 1, R_g: y \ge 1, R_{f/g}: 0 < y \le 1, R_{g/f}: 1 \le y < \infty$

5. (a) 2 (b) 22 (c) $x^2 + 2$
(d) $(x + 5)^2 - 3 = x^2 + 10x + 22$ (e) 5 (f) -2
(g) $x + 10$ (h) $(x^2 - 3)^2 - 3 = x^4 - 6x^2 + 6$

6. (a) $-\frac{1}{3}$ (b) 2 (c) $\frac{1}{x+1} - 1 = \frac{-x}{x+1}$
(d) $\frac{1}{x}$ (e) 0 (f) $\frac{3}{4}$
(g) $x - 2$ (h) $\frac{1}{\frac{1}{x+1}+1} = \frac{1}{\frac{x+2}{x+1}} = \frac{x+1}{x+2}$

7. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(4-x)) = f(3(4-x)) = f(12-3x) = (12-3x)+1 = 13-3x$

8. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x^2)) = f(2(x^2)-1) = f(2x^2-1) = 3(2x^2-1)+4 = 6x^2+1$

9. $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\frac{1}{x}\right)\right) = f\left(\frac{1}{\frac{1}{x}+4}\right) = f\left(\frac{x}{1+4x}\right) = \sqrt{\frac{x}{1+4x}+1} = \sqrt{\frac{5x+1}{1+4x}}$

10. $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\sqrt{2-x}\right)\right) = f\left(\frac{\left(\sqrt{2-x}\right)^2}{\left(\sqrt{2-x}\right)^2+1}\right) = f\left(\frac{2-x}{3-x}\right) = \frac{\frac{2-x}{3-x}+2}{3-\frac{2-x}{3-x}} = \frac{8-3x}{7-2x}$

11. (a) $(f \circ g)(x)$ (b) $(j \circ g)(x)$ (c) $(g \circ g)(x)$
(d) $(j \circ j)(x)$ (e) $(g \circ h \circ f)(x)$ (f) $(h \circ j \circ f)(x)$

12. (a) $(f \circ j)(x)$ (b) $(g \circ h)(x)$ (c) $(h \circ h)(x)$
(d) $(f \circ f)(x)$ (e) $(j \circ g \circ f)(x)$ (f) $(g \circ f \circ h)(x)$

13.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
(a)	$x-7$	$\sqrt{x}$	$\sqrt{x-7}$
(b)	$x+2$	$3x$	$3(x+2) = 3x+6$
(c)	x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
(d)	$\frac{x}{x-1}$	$\frac{x}{x-1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{x}{x-(x-1)} = x$
(e)	$\frac{1}{x-1}$	$1+\frac{1}{x}$	x
(f)	$\frac{1}{x}$	$\frac{1}{x}$	x

14. (a) $(f \circ g)(x) = |g(x)| = \frac{1}{|x-1|}$.
(b) $(f \circ g)(x) = \frac{g(x)-1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{1}{g(x)} = \frac{x}{x+1} \Rightarrow 1 - \frac{x}{x+1} = \frac{1}{g(x)} \Rightarrow \frac{1}{x+1} = \frac{1}{g(x)}$, so $g(x) = x+1$.
(c) Since $(f \circ g)(x) = \sqrt{g(x)} = |x|$, $g(x) = x^2$.
(d) Since $(f \circ g)(x) = f\left(\sqrt{x}\right) = |x|$, $f(x) = x^2$. (Note that the domain of the composite is $[0, \infty)$.)
The completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f \circ g)(x)$
$\frac{1}{x-1}$	$\lvert x \rvert$	$\frac{1}{\lvert x-1 \rvert}$
$x+1$	$\frac{x-1}{x}$	$\frac{x}{x+1}$
x^2	$\sqrt{x}$	$\lvert x \rvert$
$\sqrt{x}$	x^2	$\lvert x \rvert$

15. (a) $f(g(-1)) = f(1) = 1$ (b) $g(f(0)) = g(-2) = 2$ (c) $f(f(-1)) = f(0) = -2$
(d) $g(g(2)) = g(0) = 0$ (e) $g(f(-2)) = g(1) = -1$ (f) $f(g(1)) = f(-1) = 0$

16. (a) $f(g(0)) = f(-1) = 2-(-1) = 3$, where $g(0) = 0-1 = -1$
(b) $g(f(3)) = g(-1) = -(-1) = 1$, where $f(3) = 2-3 = -1$
(c) $g(g(-1)) = g(1) = 1-1 = 0$, where $g(-1) = -(-1) = 1$

(d) $f(f(2)) = f(0) = 2 - 0 = 2$, where $f(2) = 2 - 2 = 0$

(e) $g(f(0)) = g(2) = 2 - 1 = 1$, where $f(0) = 2 - 0 = 2$

(f) $f\left(g\left(\frac{1}{2}\right)\right) = f\left(-\frac{1}{2}\right) = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2}$, where $g\left(\frac{1}{2}\right) = \frac{1}{2} - 1 = -\frac{1}{2}$

17. (a) $(f \circ g)(x) = f(g(x)) = \sqrt{\frac{1}{x} + 1} = \sqrt{\frac{1+x}{x}}$

$(g \circ f)(x) = g(f(x)) = \frac{1}{\sqrt{x+1}}$

(b) Domain $(f \circ g)$: $(-\infty, -1] \cup (0, \infty)$, domain $(g \circ f)$: $(-1, \infty)$

(c) Range $(f \circ g)$: $(1, \infty)$, range $(g \circ f)$: $(0, \infty)$

18. (a) $(f \circ g)(x) = f(g(x)) = 1 - 2\sqrt{x} + x$

$(g \circ f)(x) = g(f(x)) = 1 - |x|$

(b) Domain $(f \circ g)$: $[0, \infty)$, domain $(g \circ f)$: $(-\infty, \infty)$

(c) Range $(f \circ g)$: $(0, \infty)$, range $(g \circ f)$: $(-\infty, 1]$

19. $(f \circ g)(x) = x \Rightarrow f(g(x)) = x \Rightarrow \frac{g(x)}{g(x) - 2} = x \Rightarrow g(x) = (g(x) - 2)x = x \cdot g(x) - 2x$

$\Rightarrow g(x) - x \cdot g(x) = -2x \Rightarrow g(x) = -\frac{2x}{1-x} = \frac{2x}{x-1}$

20. $(f \circ g)(x) = x + 2 \Rightarrow f(g(x)) = x + 2 \Rightarrow 2(g(x))^3 - 4 = x + 2 \Rightarrow (g(x))^3 = \frac{x+6}{2} \Rightarrow g(x) = \sqrt[3]{\frac{x+6}{2}}$

21. (a) $y = -(x+7)^2$ (b) $y = -(x-4)^2$

22. (a) $y = x^2 + 3$ (b) $y = x^2 - 5$

23. (a) Position 4 (b) Position 1 (c) Position 2 (d) Position 3

24. (a) $y = -(x-1)^2 + 4$ (b) $y = -(x+2)^2 + 3$ (c) $y = -(x+4)^2 - 1$ (d) $y = -(x-2)^2$

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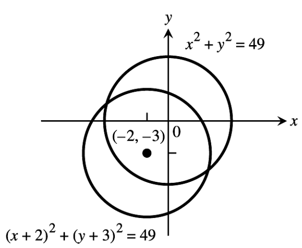

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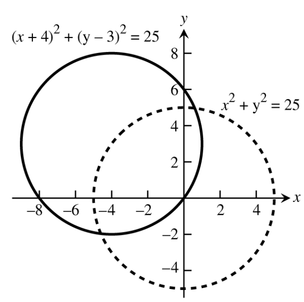

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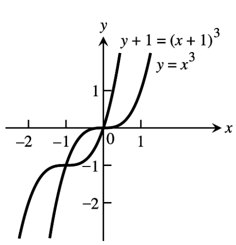

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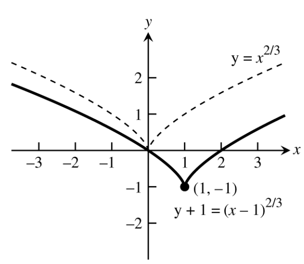

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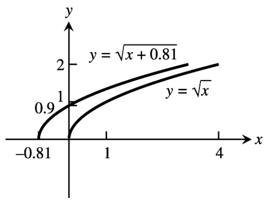

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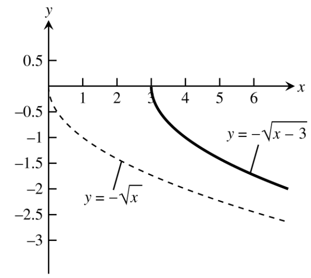

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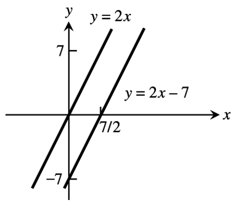

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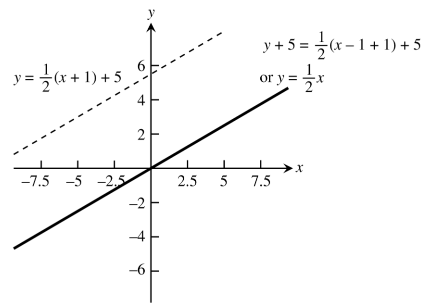

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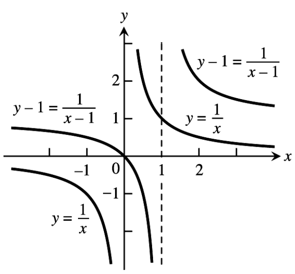

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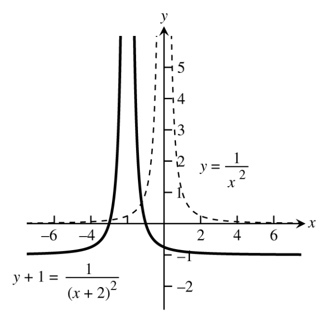

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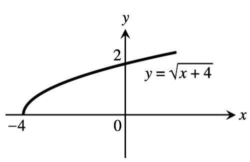

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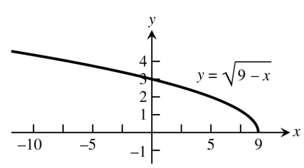

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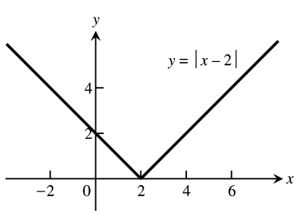

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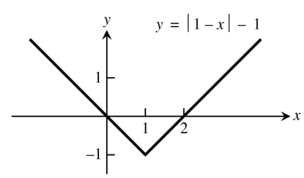

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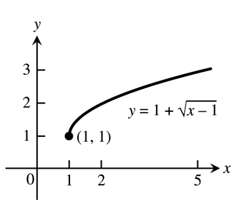

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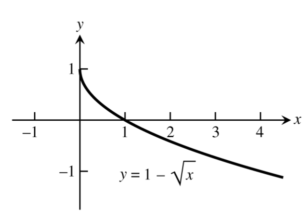

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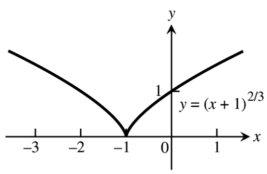

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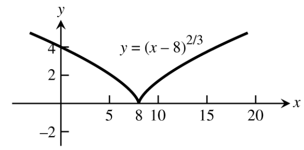

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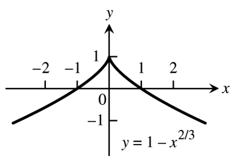

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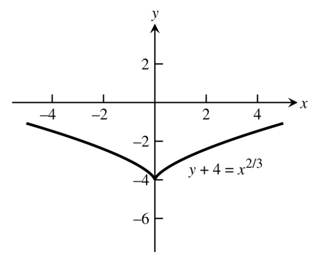

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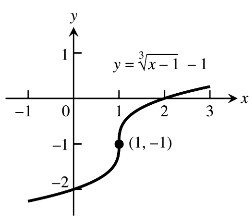

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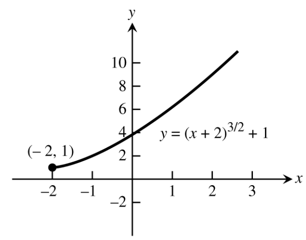

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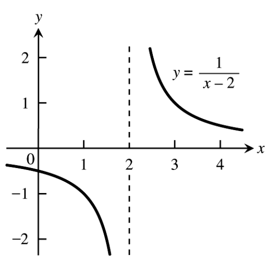

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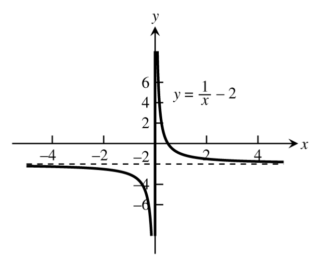

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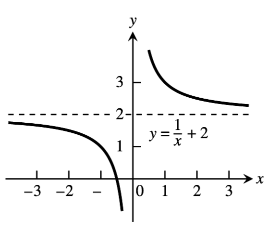

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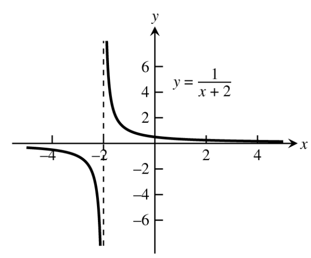

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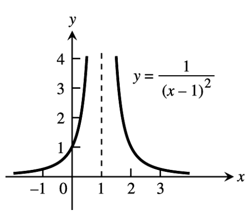

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