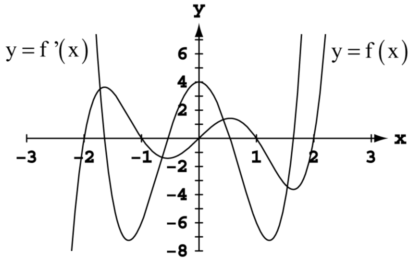
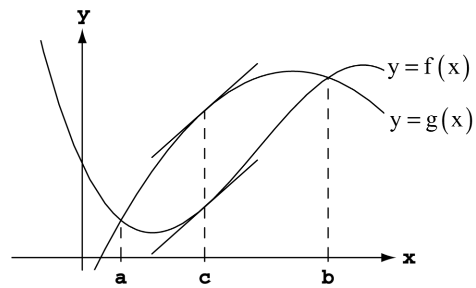
(b) Graphing $f(x) = x^5 - 5x^3 + 4x$ and $f'(x) = 5x^4 - 15x^2 + 4$ on $[-3, 3]$ by $[-7, 7]$ we see that each x-intercept of $f'(x)$ lies between a pair of x-intercepts of $f(x)$, as expected by Rolle's Theorem.



(c) Yes, since sin is continuous and differentiable on $(-\infty, \infty)$.

59. $f(x)$ must be zero at least once between a and b by the Intermediate Value Theorem. Now suppose that $f(x)$ is zero twice between a and b. Then by the Mean Value Theorem, $f'(x)$ would have to be zero at least once between the two zeros of $f(x)$, but this can't be true since we are given that $f'(x) \neq 0$ on this interval. Therefore, $f(x)$ is zero once and only once between a and b.

60. Consider the function $k(x) = f(x) - g(x)$. $k(x)$ is continuous and differentiable on $[a, b]$, and since $k(a) = f(a) - g(a)$ and $k(b) = f(b) - g(b)$, by the Mean Value Theorem, there must be a point c in (a, b) where $k'(c) = 0$. But since $k'(c) = f'(c) - g'(c)$, this means that $f'(c) = g'(c)$, and c is a point where the graphs of f and g have tangent lines with the same slope, so these lines are either parallel or are the same line.



61. $f'(x) \le 1$ for $1 \le x \le 4 \Rightarrow f(x)$ is differentiable on $1 \le x \le 4 \Rightarrow f$ is continuous on $1 \le x \le 4 \Rightarrow f$ satisfies the conditions of the Mean Value Theorem $\Rightarrow \frac{f(4) - f(1)}{4 - 1} = f'(c)$ for some c in $1 < x < 4 \Rightarrow f'(c) \le 1 \Rightarrow \frac{f(4) - f(1)}{3} \le 1$ $\Rightarrow f(4) - f(1) \le 3$

62. $0 < f'(x) < \frac{1}{2}$ for all $x \Rightarrow f'(x)$ exists for all x, thus f is differentiable on $(-1, 1) \Rightarrow f$ is continuous on $[-1, 1]$ $\Rightarrow$ f satisfies the conditions of the Mean Value Theorem $\Rightarrow \frac{f(1) - f(-1)}{1 - (-1)} = f'(c)$ for some c in $[-1, 1] \Rightarrow 0 < \frac{f(1) - f(-1)}{2} < \frac{1}{2}$ $\Rightarrow 0 < f(1) - f(-1) < 1$. Since $f(1) - f(-1) < 1 \Rightarrow f(1) < 1 + f(-1) < 2 + f(-1)$, and since $0 < f(1) - f(-1)$ we have $f(-1) < f(1)$. Together we have $f(-1) < f(1) < 2 + f(-1)$.

63. Let $f(t) = \cos t$ and consider the interval $[0, x]$ where x is a real number. f is continuous on $[0, x]$ and f is differentiable on $(0, x)$ since $f'(t) = -\sin t \Rightarrow f$ satisfies the conditions of the Mean Value Theorem $\Rightarrow \frac{f(x) - f(0)}{x - (0)} = f'(c)$ for some c in $[0, x] \Rightarrow \frac{\cos x - 1}{x} = -\sin c$. Since $-1 \le \sin c \le 1 \Rightarrow -1 \le -\sin c \le 1 \Rightarrow -1 \le \frac{\cos x - 1}{x} \le 1$. If $x > 0$, $-1 \le \frac{\cos x - 1}{x} \le 1$ $\Rightarrow -x \le \cos x - 1 \le x \Rightarrow |\cos x - 1| \le x = |x|$. If $x < 0$, $-1 \le \frac{\cos x - 1}{x} \le 1 \Rightarrow -x \ge \cos x - 1 \ge x$ $\Rightarrow x \le \cos x - 1 \le -x \Rightarrow -(-x) \le \cos x - 1 \le -x \Rightarrow |\cos x - 1| \le -x = |x|$. Thus, in both cases, we have $|\cos x - 1| \le |x|$. If $x = 0$, then $|\cos 0 - 1| = |1 - 1| = |0| \le |0|$, thus $|\cos x - 1| \le |x|$ is true for all x.

64. Let $f(x) = \sin x$ for $a \le x \le b$. From the Mean Value Theorem there exists a c between a and b such that $\frac{\sin b - \sin a}{b - a} = \cos c \Rightarrow -1 \le \frac{\sin b - \sin a}{b - a} \le 1 \Rightarrow \left|\frac{\sin b - \sin a}{b - a}\right| \le 1 \Rightarrow |\sin b - \sin a| \le |b - a|$.

65. Yes. By Corollary 2 we have $f(x) = g(x) + c$ since $f'(x) = g'(x)$. If the graphs start at the same point $x = a$, then $f(a) = g(a) \Rightarrow c = 0 \Rightarrow f(x) = g(x)$.

66. Assume f is differentiable and $|f(w) - f(x)| \le |w - x|$ for all values of w and x. Since f is differentiable, $f'(x)$ exists and $f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$ using the alternative formula for the derivative. Let $g(x) = |x|$, which is continuous for all x. By Theorem 10 from Chapter 2, $|f'(x)| = \left|\lim_{w \to x} \frac{f(w) - f(x)}{w - x}\right| = \lim_{w \to x} \left|\frac{f(w) - f(x)}{w - x}\right| = \lim_{w \to x} \frac{|f(w) - f(x)|}{|w - x|}$. Since $|f(w) - f(x)| \le |w - x|$ for all w and x $\Rightarrow \frac{|f(w)-f(x)|}{|w-x|} \le 1$ as long as $w \ne x$. By Theorem 5 from Chapter 2, $|f'(x)| = \lim_{w \to x} \frac{|f(w) - f(x)|}{|w - x|} \le \lim_{w \to x} 1 = 1 \Rightarrow |f'(x)| \le 1 \Rightarrow -1 \le f'(x) \le 1$.

67. By the Mean Value Theorem we have $\frac{f(b) - f(a)}{b - a} = f'(c)$ for some point c between a and b. Since $b - a > 0$ and $f(b) < f(a)$, we have $f(b) - f(a) < 0 \Rightarrow f'(c) < 0$.

68. The condition is that f' should be continuous over $[a, b]$. The Mean Value Theorem then guarantees the existence of a point c in (a, b) such that $\frac{f(b) - f(a)}{b - a} = f'(c)$. If f' is continuous, then it has a minimum and maximum value on $[a, b]$, and $\min f' \le f'(c) \le \max f'$, as required.

69. $f'(x) = (1 + x^4 \cos x)^{-1} \Rightarrow f''(x) = -(1 + x^4 \cos x)^{-2}(4x^3 \cos x - x^4 \sin x)$
$= -x^3(1 + x^4 \cos x)^{-2}(4 \cos x - x \sin x) < 0$ for $0 \le x \le 0.1 \Rightarrow f'(x)$ is decreasing when $0 \le x \le 0.1$
$\Rightarrow \min f' \approx 0.9999$ and $\max f' = 1$. Now we have $0.9999 \le \frac{f(0.1) - 1}{0.1} \le 1 \Rightarrow 0.09999 \le f(0.1) - 1 \le 0.1$
$\Rightarrow 1.09999 \le f(0.1) \le 1.1$.

70. $f'(x) = (1 - x^4)^{-1} \Rightarrow f''(x) = -(1 - x^4)^{-2}(-4x^3) = \frac{4x^3}{(1 - x^4)^3} > 0$ for $0 < x \le 0.1 \Rightarrow f'(x)$ is increasing when $0 \le x \le 0.1 \Rightarrow \min f' = 1$ and $\max f' = 1.0001$. Now we have $1 \le \frac{f(0.1) - 2}{0.1} \le 1.0001$
$\Rightarrow 0.1 \le f(0.1) - 2 \le 0.10001 \Rightarrow 2.1 \le f(0.1) \le 2.10001$.

71. (a) Suppose $x < 1$, then by the Mean Value Theorem $\frac{f(x) - f(1)}{x - 1} < 0 \Rightarrow f(x) > f(1)$. Suppose $x > 1$, then by the Mean Value Theorem $\frac{f(x) - f(1)}{x - 1} > 0 \Rightarrow f(x) > f(1)$. Therefore $f(x) \ge 1$ for all x since $f(1) = 1$.
(b) Yes. From part (a), $\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} \le 0$ and $\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} \ge 0$. Since $f'(1)$ exists, these two one-sided limits are equal and have the value $f'(1) \Rightarrow f'(1) \le 0$ and $f'(1) \ge 0 \Rightarrow f'(1) = 0$.

72. From the Mean Value Theorem we have $\frac{f(b) - f(a)}{b - a} = f'(c)$ where c is between a and b. But $f'(c) = 2pc + q = 0$ has only one solution $c = -\frac{q}{2p}$. (Note: $p \ne 0$ since f is a quadratic function.)

4.3 MONOTONIC FUNCTIONS AND THE FIRST DERIVATIVE TEST

1. (a) $f'(x) = x(x - 1) \Rightarrow$ critical points at 0 and 1
(b) $f' = \underset{}{+++} \underset{0}{|} --- \underset{1}{|} +++ \Rightarrow$ increasing on $(-\infty, 0)$ and $(1, \infty)$, decreasing on $(0, 1)$
(c) Local maximum at $x = 0$ and a local minimum at $x = 1$

2. (a) $f'(x) = (x - 1)(x + 2) \Rightarrow$ critical points at -2 and 1
(b) $f' = +++ \underset{-2}{|} --- \underset{1}{|} +++ \Rightarrow$ increasing on $(-\infty, -2)$ and $(1, \infty)$, decreasing on $(-2, 1)$
(c) Local maximum at $x = -2$ and a local minimum at $x = 1$

3. (a) $f'(x) = (x - 1)^2(x + 2) \Rightarrow$ critical points at -2 and 1
(b) $f' = --- \underset{-2}{|} +++ \underset{1}{|} +++ \Rightarrow$ increasing on $(-2, 1)$ and $(1, \infty)$, decreasing on $(-\infty, -2)$

(c) No local maximum and a local minimum at $x = -2$

4. (a) $f'(x) = (x-1)^2(x+2)^2 \Rightarrow$ critical points at -2 and 1
(b) $f' = \underset{-2}{+++\,|}\,\underset{1}{+++\,|}\,+++ \Rightarrow$ increasing on $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$, never decreasing
(c) No local extrema

5. (a) $f'(x) = (x-1)(x+2)(x-3) \Rightarrow$ critical points at -2, 1 and 3
(b) $f' = \underset{-2}{---\,|}\,\underset{1}{+++\,|}\,\underset{3}{---\,|}\,+++ \Rightarrow$ increasing on $(-2, 1)$ and $(3, \infty)$, decreasing on $(-\infty, -2)$ and $(1, 3)$
(c) Local maximum at $x = 1$, local minima at $x = -2$ and $x = 3$

6. (a) $f'(x) = (x-7)(x+1)(x+5) \Rightarrow$ critical points at -5, -1 and 7
(b) $f' = \underset{-5}{---\,|}\,\underset{-1}{+++\,|}\,\underset{7}{---\,|}\,+++ \Rightarrow$ increasing on $(-5, -1)$ and $(7, \infty)$, decreasing on $(-\infty, -5)$ and $(-1, 7)$
(c) Local maximum at $x = -1$, local minima at $x = -5$ and $x = 7$

7. (a) $f'(x) = \frac{x^2(x-1)}{(x+2)} \Rightarrow$ critical points at $x = 0$, $x = 1$ and $x = -2$
(b) $f' = \underset{-2}{+++)(}\,\underset{0}{---\,|}\,\underset{1}{---\,|}\,+++ \Rightarrow$ increasing on $(-\infty, -2)$ and $(1, \infty)$, decreasing on $(-2, 0)$ and $(0, 1)$
(c) Local minimum at $x = 1$

8. (a) $f'(x) = \frac{(x-2)(x+4)}{(x+1)(x-3)} \Rightarrow$ critical points at $x = 2$, $x = -4$, $x = -1$, and $x = 3$
(b) $f' = \underset{-4}{+++\,|}\,\underset{-1}{---\,)(}\,\underset{2}{+++\,|}\,\underset{3}{---)(}+++ \Rightarrow$ increasing on $(-\infty, -4)$, $(-1, 2)$ and $(3, \infty)$, decreasing on $(-4, -1)$ and $(2, 3)$
(c) Local maximum at $x = -4$ and $x = 2$

9. (a) $f'(x) = 1 - \frac{4}{x^2} = \frac{x^2-4}{x^2} \Rightarrow$ critical points at $x = -2$, $x = 2$ and $x = 0$.
(b) $f' = \underset{-2}{+++\,|}\,\underset{0}{---)(}\,\underset{2}{---\,|}\,+++ \Rightarrow$ increasing on $(-\infty, -2)$ and $(2, \infty)$, decreasing on $(-2, 0)$ and $(0, 2)$
(c) Local maximum at $x = -2$, local minimum at $x = 2$

10. (a) $f'(x) = 3 - \frac{6}{\sqrt{x}} = \frac{3\sqrt{x}-6}{\sqrt{x}} \Rightarrow$ critical points at $x = 4$ and $x = 0$
(b) $f' = \underset{0}{(}\,\underset{4}{---\,|}\,+++ \Rightarrow$ increasing on $(4, \infty)$, decreasing on $(0, 4)$
(c) Local minimum at $x = 4$

11. (a) $f'(x) = x^{-1/3}(x+2) \Rightarrow$ critical points at $x = -2$ and $x = 0$
(b) $f' = \underset{-2}{+++\,|}\,\underset{0}{---\,)(}\,+++ \Rightarrow$ increasing on $(-\infty, -2)$ and $(0, \infty)$, decreasing on $(-2, 0)$
(c) Local maximum at $x = -2$, local minimum at $x = 0$

12. (a) $f'(x) = x^{-1/2}(x-3) \Rightarrow$ critical points at $x = 0$ and $x = 3$
(b) $f' = \underset{0}{(}\,\underset{3}{---\,|}\,+++ \Rightarrow$ increasing on $(3, \infty)$, decreasing on $(0, 3)$
(c) No local maximum and a local minimum at $x = 3$

13. (a) $f'(x) = (\sin x - 1)(2\cos x + 1)$, $0 \le x \le 2\pi \Rightarrow$ critical points at $x = \frac{\pi}{2}$, $x = \frac{2\pi}{3}$, and $x = \frac{4\pi}{3}$
(b) $f' = \underset{0}{[}\,\underset{\frac{\pi}{2}}{---\,|}\,\underset{\frac{2\pi}{3}}{---\,|}\,\underset{\frac{4\pi}{3}}{+++\,|}\,\underset{2\pi}{---\,]} \Rightarrow$ increasing on $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$, decreasing on $\left(0, \frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and $\left(\frac{4\pi}{3}, 2\pi\right)$

(c) Local maximum at $x = \frac{4\pi}{3}$ and $x = 0$, local minimum at $x = \frac{2\pi}{3}$ and $x = 2\pi$

14. (a) $f'(x) = (\sin x + \cos x)(\sin x - \cos x),\ 0 \le x \le 2\pi \Rightarrow$ critical points at $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, $x = \frac{5\pi}{4}$, and $x = \frac{7\pi}{4}$
(b) $f' = \underset{0}{[}---\underset{\frac{\pi}{4}}{|}+++\underset{\frac{3\pi}{4}}{|}---\underset{\frac{5\pi}{4}}{|}+++\underset{\frac{7\pi}{4}}{|}---\underset{2\pi}{]} \Rightarrow$ increasing on $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ and $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$, decreasing on $\left(0, \frac{\pi}{4}\right)$, $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ and $\left(\frac{7\pi}{4}, 2\pi\right)$
(c) Local maximum at $x = 0$, $x = \frac{3\pi}{4}$ and $x = \frac{7\pi}{4}$, local minimum at $x = \frac{\pi}{4}$, $x = \frac{5\pi}{4}$ and $x = 2\pi$

15. (a) Increasing on $(-2, 0)$ and $(2, 4)$, decreasing on $(-4, -2)$ and $(0, 2)$
(b) Absolute maximum at $(-4, 2)$, local maximum at $(0, 1)$ and $(4, -1)$; Absolute minimum at $(2, -3)$, local minimum at $(-2, 0)$

16. (a) Increasing on $(-4, -3.25)$, $(-1.5, 1)$, and $(2, 4)$, decreasing on $(-3.25, -1.5)$ and $(1, 2)$
(b) Absolute maximum at $(4, 2)$, local maximum at $(-3.25, 1)$ and $(1, 1)$; Absolute minimum at $(-1.5, -1)$, local minimum at $(-4, 0)$ and $(2, 0)$

17. (a) Increasing on $(-4, -1)$, $(0.5, 2)$, and $(2, 4)$, decreasing on $(-1, 0.5)$
(b) Absolute maximum at $(4, 3)$, local maximum at $(-1, 2)$ and $(2, 1)$; No absolute minimum, local minimum at $(-4, -1)$and $(0.5, -1)$

18. (a) Increasing on $(-4, -2.5)$, $(-1, 1)$, and $(3, 4)$, decreasing on $(-2.5, -1)$ and $(1, 3)$
(b) No absolute maximum, local maximum at $(-2.5, 1)$, $(1, 2)$ and $(4, 2)$; No absolute minimum, local minimum at $(-1, 0)$ and $(3, 1)$

19. (a) $g(t) = -t^2 - 3t + 3 \Rightarrow g'(t) = -2t - 3 \Rightarrow$ a critical point at $t = -\frac{3}{2}$; $g' = +++\underset{-3/2}{|}---$, increasing on $\left(-\infty, -\frac{3}{2}\right)$, decreasing on $\left(-\frac{3}{2}, \infty\right)$
(b) local maximum value of $g\left(-\frac{3}{2}\right) = \frac{21}{4}$ at $t = -\frac{3}{2}$, absolute maximum is $\frac{21}{4}$ at $t = -\frac{3}{2}$

20. (a) $g(t) = -3t^2 + 9t + 5 \Rightarrow g'(t) = -6t + 9 \Rightarrow$ a critical point at $t = \frac{3}{2}$; $g' = +++\underset{3/2}{|}---$, increasing on $\left(-\infty, \frac{3}{2}\right)$, decreasing on $\left(\frac{3}{2}, \infty\right)$
(b) local maximum value of $g\left(\frac{3}{2}\right) = \frac{47}{4}$ at $t = \frac{3}{2}$, absolute maximum is $\frac{47}{4}$ at $t = \frac{3}{2}$

21. (a) $h(x) = -x^3 + 2x^2 \Rightarrow h'(x) = -3x^2 + 4x = x(4 - 3x) \Rightarrow$ critical points at $x = 0, \frac{4}{3}$
$\Rightarrow h' = ---\underset{0}{|}+++\underset{4/3}{|}---$, increasing on $\left(0, \frac{4}{3}\right)$, decreasing on $(-\infty, 0)$ and $\left(\frac{4}{3}, \infty\right)$
(b) local maximum value of $h\left(\frac{4}{3}\right) = \frac{32}{27}$ at $x = \frac{4}{3}$; local minimum value of $h(0) = 0$ at $x = 0$, no absolute extrema

22. (a) $h(x) = 2x^3 - 18x \Rightarrow h'(x) = 6x^2 - 18 = 6\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right) \Rightarrow$ critical points at $x = \pm\sqrt{3}$
$\Rightarrow h' = +++\underset{-\sqrt{3}}{|}---\underset{\sqrt{3}}{|}+++$, increasing on $\left(-\infty, -\sqrt{3}\right)$ and $\left(\sqrt{3}, \infty\right)$, decreasing on $\left(-\sqrt{3}, \sqrt{3}\right)$
(b) a local maximum is $h\left(-\sqrt{3}\right) = 12\sqrt{3}$ at $x = -\sqrt{3}$; local minimum is $h\left(\sqrt{3}\right) = -12\sqrt{3}$ at $x = \sqrt{3}$, no absolute extrema

23. (a) $f(\theta) = 3\theta^2 - 4\theta^3 \Rightarrow f'(\theta) = 6\theta - 12\theta^2 = 6\theta(1 - 2\theta) \Rightarrow$ critical points at $\theta = 0, \frac{1}{2} \Rightarrow f' = \underset{0}{---|}\underset{1/2}{+++|}---$, increasing on $\left(0, \frac{1}{2}\right)$, decreasing on $(-\infty, 0)$ and $\left(\frac{1}{2}, \infty\right)$

(b) a local maximum is $f\left(\frac{1}{2}\right) = \frac{1}{4}$ at $\theta = \frac{1}{2}$, a local minimum is $f(0) = 0$ at $\theta = 0$, no absolute extrema

24. (a) $f(\theta) = 6\theta - \theta^3 \Rightarrow f'(\theta) = 6 - 3\theta^2 = 3\left(\sqrt{2} - \theta\right)\left(\sqrt{2} + \theta\right) \Rightarrow$ critical points at $\theta = \pm\sqrt{2} \Rightarrow$ $f' = \underset{-\sqrt{2}}{---|}\underset{\sqrt{2}}{+++|}---$, increasing on $\left(-\sqrt{2}, \sqrt{2}\right)$, decreasing on $\left(-\infty, -\sqrt{2}\right)$ and $\left(\sqrt{2}, \infty\right)$

(b) a local maximum is $f\left(\sqrt{2}\right) = 4\sqrt{2}$ at $\theta = \sqrt{2}$, a local minimum is $f\left(-\sqrt{2}\right) = -4\sqrt{2}$ at $\theta = -\sqrt{2}$, no absolute extrema

25. (a) $f(r) = 3r^3 + 16r \Rightarrow f'(r) = 9r^2 + 16 \Rightarrow$ no critical points $\Rightarrow f' = +++++$, increasing on $(-\infty, \infty)$, never decreasing

(b) no local extrema, no absolute extrema

26. (a) $h(r) = (r + 7)^3 \Rightarrow h'(r) = 3(r + 7)^2 \Rightarrow$ a critical point at $r = -7 \Rightarrow h' = \underset{-7}{+++|}+++$, increasing on $(-\infty, -7) \cup (-7, \infty)$, never decreasing

(b) no local extrema, no absolute extrema

27. (a) $f(x) = x^4 - 8x^2 + 16 \Rightarrow f'(x) = 4x^3 - 16x = 4x(x + 2)(x - 2) \Rightarrow$ critical points at $x = 0$ and $x = \pm 2$ $\Rightarrow f' = \underset{-2}{---|}\underset{0}{+++|}\underset{2}{---|}+++$, increasing on $(-2, 0)$ and $(2, \infty)$, decreasing on $(-\infty, -2)$ and $(0, 2)$

(b) a local maximum is $f(0) = 16$ at $x = 0$, local minima are $f(\pm 2) = 0$ at $x = \pm 2$, no absolute maximum; absolute minimum is 0 at $x = \pm 2$

28. (a) $g(x) = x^4 - 4x^3 + 4x^2 \Rightarrow g'(x) = 4x^3 - 12x^2 + 8x = 4x(x - 2)(x - 1) \Rightarrow$ critical points at $x = 0, 1, 2$ $\Rightarrow g' = \underset{0}{---|}\underset{1}{+++|}\underset{2}{---|}+++$, increasing on $(0, 1)$ and $(2, \infty)$, decreasing on $(-\infty, 0)$ and $(1, 2)$

(b) a local maximum is $g(1) = 1$ at $x = 1$, local minima are $g(0) = 0$ at $x = 0$ and $g(2) = 0$ at $x = 2$, no absolute maximum; absolute minimum is 0 at $x = 0, 2$

29. (a) $H(t) = \frac{3}{2}t^4 - t^6 \Rightarrow H'(t) = 6t^3 - 6t^5 = 6t^3(1 + t)(1 - t) \Rightarrow$ critical points at $t = 0, \pm 1$ $\Rightarrow H' = \underset{-1}{+++|}\underset{0}{---|}\underset{1}{+++|}---$, increasing on $(-\infty, -1)$ and $(0, 1)$, decreasing on $(-1, 0)$ and $(1, \infty)$

(b) the local maxima are $H(-1) = \frac{1}{2}$ at $t = -1$ and $H(1) = \frac{1}{2}$ at $t = 1$, the local minimum is $H(0) = 0$ at $t = 0$, absolute maximum is $\frac{1}{2}$ at $t = \pm 1$; no absolute minimum

30. (a) $K(t) = 15t^3 - t^5 \Rightarrow K'(t) = 45t^2 - 5t^4 = 5t^2(3 + t)(3 - t) \Rightarrow$ critical points at $t = 0, \pm 3$ $\Rightarrow K' = \underset{-3}{---|}\underset{0}{+++|}\underset{3}{+++|}---$, increasing on $(-3, 0) \cup (0, 3)$, decreasing on $(-\infty, -3)$ and $(3, \infty)$

(b) a local maximum is $K(3) = 162$ at $t = 3$, a local minimum is $K(-3) = -162$ at $t = -3$, no absolute extrema

31. (a) $f(x) = x - 6\sqrt{x - 1} \Rightarrow f'(x) = 1 - \frac{3}{\sqrt{x - 1}} = \frac{\sqrt{x - 1} - 3}{\sqrt{x - 1}} \Rightarrow$ critical points at $x = 1$ and $x = 10$ $\Rightarrow f' = \underset{1}{(}\ \underset{10}{---|}+++$, increasing on $(10, \infty)$, decreasing on $(1, 10)$

(b) a local minimum is $f(10) = -8$, a local and absolute maximum is $f(1) = 1$, absolute minimum of -8 at $x = 10$

32. (a) $g(x) = 4\sqrt{x} - x^2 + 3 \Rightarrow g'(x) = \frac{2}{\sqrt{x}} - 2x = \frac{2-2x^{3/2}}{\sqrt{x}} \Rightarrow$ critical points at $x = 1$ and $x = 0$

$\Rightarrow g' = \underset{0}{(} +++ \underset{1}{|} ---$, increasing on $(0, 1)$, decreasing on $(1, \infty)$

(b) a local minimum is $f(0) = 3$, a local maximum is $f(1) = 6$, absolute maximum of 6 at $x = 1$

33. (a) $g(x) = x\sqrt{8-x^2} = x\left(8-x^2\right)^{1/2} \Rightarrow g'(x) = \left(8-x^2\right)^{1/2} + x\left(\frac{1}{2}\right)\left(8-x^2\right)^{-1/2}(-2x) = \frac{2(2-x)(2+x)}{\sqrt{\left(2\sqrt{2}-x\right)\left(2\sqrt{2}+x\right)}}$

$\Rightarrow$ critical points at $x = \pm 2, \pm 2\sqrt{2} \Rightarrow g' = \underset{-2\sqrt{2}}{(} --- \underset{-2}{|} +++ \underset{2}{|} --- \underset{2\sqrt{2}}{)}$, increasing on $(-2, 2)$, decreasing on $\left(-2\sqrt{2}, -2\right)$ and $\left(2, 2\sqrt{2}\right)$

(b) local maxima are $g(2) = 4$ at $x = 2$ and $g\left(-2\sqrt{2}\right) = 0$ at $x = -2\sqrt{2}$, local minima are $g(-2) = -4$ at $x = -2$ and $g\left(2\sqrt{2}\right) = 0$ at $x = 2\sqrt{2}$, absolute maximum is 4 at $x = 2$; absolute minimum is -4 at $x = -2$

34. (a) $g(x) = x^2\sqrt{5-x} = x^2(5-x)^{1/2} \Rightarrow g'(x) = 2x(5-x)^{1/2} + x^2\left(\frac{1}{2}\right)(5-x)^{-1/2}(-1) = \frac{5x(4-x)}{2\sqrt{5-x}} \Rightarrow$ critical points at $x = 0, 4$ and $5 \Rightarrow g' = --- \underset{0}{|} +++ \underset{4}{|} --- \underset{5}{)}$, increasing on $(0, 4)$, decreasing on $(-\infty, 0)$ and $(4, 5)$

(b) a local maximum is $g(4) = 16$ at $x = 4$, a local minimum is 0 at $x = 0$ and $x = 5$, no absolute maximum; absolute minimum is 0 at $x = 0, 5$

35. (a) $f(x) = \frac{x^2-3}{x-2} \Rightarrow f'(x) = \frac{2x(x-2)-(x^2-3)(1)}{(x-2)^2} = \frac{(x-3)(x-1)}{(x-2)^2} \Rightarrow$ critical points at $x = 1, 3$

$\Rightarrow f' = +++ \underset{1}{|} --- \underset{2}{)(} --- \underset{3}{|} +++$, increasing on $(-\infty, 1)$ and $(3, \infty)$, decreasing on $(1, 2)$ and $(2, 3)$, discontinuous at $x = 2$

(b) a local maximum is $f(1) = 2$ at $x = 1$, a local minimum is $f(3) = 6$ at $x = 3$, no absolute extrema

36. (a) $f(x) = \frac{x^3}{3x^2+1} \Rightarrow f'(x) = \frac{3x^2(3x^2+1)-x^3(6x)}{(3x^2+1)^2} = \frac{3x^2(x^2+1)}{(3x^2+1)^2} \Rightarrow$ a critical point at $x = 0$

$\Rightarrow f' = +++ \underset{0}{|} +++$, increasing on $(-\infty, 0) \cup (0, \infty)$, and never decreasing

(b) no local extrema, no absolute extrema

37. (a) $f(x) = x^{1/3}(x+8) = x^{4/3} + 8x^{1/3} \Rightarrow f'(x) = \frac{4}{3}x^{1/3} + \frac{8}{3}x^{-2/3} = \frac{4(x+2)}{3x^{2/3}} \Rightarrow$ critical points at $x = 0, -2$

$\Rightarrow f' = --- \underset{-2}{|} +++ \underset{0}{)(} +++$, increasing on $(-2, 0) \cup (0, \infty)$, decreasing on $(-\infty, -2)$

(b) no local maximum, a local minimum is $f(-2) = -6\sqrt[3]{2} \approx -7.56$ at $x = -2$, no absolute maximum; absolute minimum is $-6\sqrt[3]{2}$ at $x = -2$

38. (a) $g(x) = x^{2/3}(x+5) = x^{5/3} + 5x^{2/3} \Rightarrow g'(x) = \frac{5}{3}x^{2/3} + \frac{10}{3}x^{-1/3} = \frac{5(x+2)}{3\sqrt[3]{x}} \Rightarrow$ critical points at $x = -2$ and $x = 0 \Rightarrow g' = +++ \underset{-2}{|} --- \underset{0}{)(} +++$, increasing on $(-\infty, -2)$ and $(0, \infty)$, decreasing on $(-2, 0)$

(b) local maximum is $g(-2) = 3\sqrt[3]{4} \approx 4.762$ at $x = -2$, a local minimum is $g(0) = 0$ at $x = 0$, no absolute extrema

39. (a) $h(x) = x^{1/3}\left(x^2-4\right) = x^{7/3} - 4x^{1/3} \Rightarrow h'(x) = \frac{7}{3}x^{4/3} - \frac{4}{3}x^{-2/3} = \frac{\left(\sqrt{7}x+2\right)\left(\sqrt{7}x-2\right)}{3\sqrt[3]{x^2}} \Rightarrow$ critical points at $x = 0, \frac{\pm 2}{\sqrt{7}} \Rightarrow h' = +++ \underset{-2/\sqrt{7}}{|} --- \underset{0}{)(} --- \underset{2/\sqrt{7}}{|} +++$, increasing on $\left(-\infty, \frac{-2}{\sqrt{7}}\right)$ and $\left(\frac{2}{\sqrt{7}}, \infty\right)$, decreasing on $\left(\frac{-2}{\sqrt{7}}, 0\right)$ and $\left(0, \frac{2}{\sqrt{7}}\right)$

(b) local maximum is $h\left(\frac{-2}{\sqrt{7}}\right) = \frac{24\sqrt[3]{2}}{7^{7/6}} \approx 3.12$ at $x = \frac{-2}{\sqrt{7}}$, the local minimum is $h\left(\frac{2}{\sqrt{7}}\right) = -\frac{24\sqrt[3]{2}}{7^{7/6}} \approx -3.12$, no absolute extrema

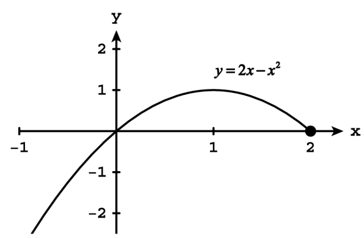
40. (a) $k(x) = x^{2/3}(x^2-4) = x^{8/3} - 4x^{2/3} \Rightarrow k'(x) = \frac{8}{3}x^{5/3} - \frac{8}{3}x^{-1/3} = \frac{8(x+1)(x-1)}{3\sqrt[3]{x}} \Rightarrow$ critical points at $x = 0, \pm 1 \Rightarrow k' = \underset{-1}{---|}\ \underset{0}{+++)(}\ \underset{1}{---|}\ +++$, increasing on $(-1, 0)$ and $(1, \infty)$, decreasing on $(-\infty, -1)$ and $(0, 1)$

(b) local maximum is $k(0) = 0$ at $x = 0$, local minima are $k(\pm 1) = -3$ at $x = \pm 1$, no absolute maximum; absolute minimum is -3 at $x = \pm 1$

41. (a) $f(x) = 2x - x^2 \Rightarrow f'(x) = 2 - 2x \Rightarrow$ a critical point at $x = 1 \Rightarrow f' = \underset{1}{+++|}\ \underset{2}{---]}$ and $f(1) = 1$ and $f(2) = 0$ a local maximum is 1 at $x = 1$, a local minimum is 0 at $x = 2$.

(b) There is an absolute maximum of 1 at $x = 1$; no absolute minimum.

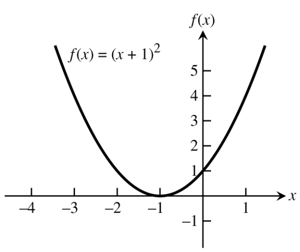
(c)



42. (a) $f(x) = (x+1)^2 \Rightarrow f'(x) = 2(x+1) \Rightarrow$ a critical point at $x = -1 \Rightarrow f' = \underset{-1}{---|}\ \underset{0}{+++]}$ and $f(-1) = 0$, $f(0) = 1$ $\Rightarrow$ a local maximum is 1 at $x = 0$, a local minimum is 0 at $x = -1$

(b) no absolute maximum; absolute minimum is 0 at $x = -1$

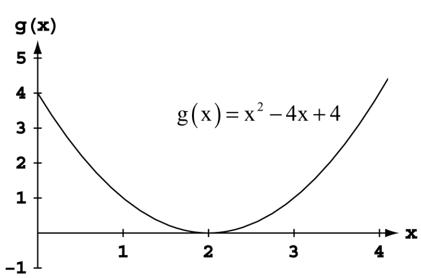
(c)



43. (a) $g(x) = x^2 - 4x + 4 \Rightarrow g'(x) = 2x - 4 = 2(x-2) \Rightarrow$ a critical point at $x = 2 \Rightarrow g' = \underset{1}{[}\ \underset{2}{---|}\ +++$ and $g(1) = 1$, $g(2) = 0 \Rightarrow$ a local maximum is 1 at $x = 1$, a local minimum is $g(2) = 0$ at $x = 2$

(b) no absolute maximum; absolute minimum is 0 at $x = 2$

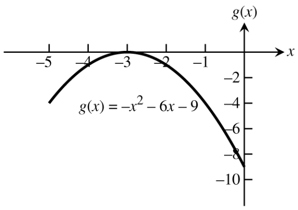
(c)



44. (a) $g(x) = -x^2 - 6x - 9 \Rightarrow g'(x) = -2x - 6 = -2(x+3) \Rightarrow$ a critical point at $x = -3 \Rightarrow g' = \underset{-4}{[}\ \underset{-3}{+++|}\ ---$ and $g(-4) = -1$, $g(-3) = 0 \Rightarrow$ a local maximum is 0 at $x = -3$, a local minimum is -1 at $x = -4$

(b) absolute maximum is 0 at $x = -3$; no absolute minimum

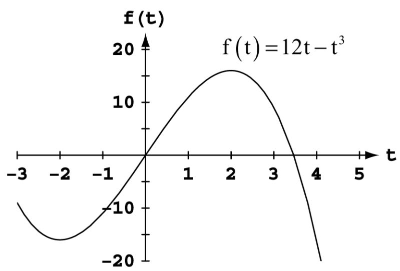
(c)



45. (a) $f(t) = 12t - t^3 \Rightarrow f'(t) = 12 - 3t^2 = 3(2+t)(2-t) \Rightarrow$ critical points at $t = \pm 2 \Rightarrow f' = \underset{-3}{[}\;---\;\underset{-2}{|}\;+++\;\underset{2}{|}\;---$ and $f(-3) = -9$, $f(-2) = -16$, $f(2) = 16 \Rightarrow$ local maxima are -9 at $t = -3$ and 16 at $t = 2$, a local minimum is -16 at $t = -2$

(b) absolute maximum is 16 at $t = 2$; no absolute minimum

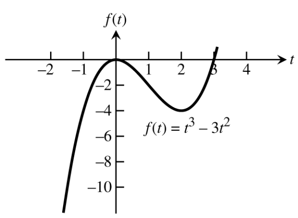
(c)



46. (a) $f(t) = t^3 - 3t^2 \Rightarrow f'(t) = 3t^2 - 6t = 3t(t-2) \Rightarrow$ critical points at $t = 0$ and $t = 2$ $\Rightarrow f' = +++\;\underset{0}{|}\;---\;\underset{2}{|}\;+++\;\underset{3}{]}$ and $f(0) = 0$, $f(2) = -4$, $f(3) = 0 \Rightarrow$ a local maximum is 0 at $t = 0$ and $t = 3$, a local minimum is -4 at $t = 2$

(b) absolute maximum is 0 at $t = 0, 3$; no absolute minimum

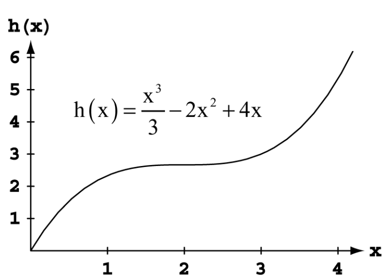
(c)



47. (a) $h(x) = \frac{x^3}{3} - 2x^2 + 4x \Rightarrow h'(x) = x^2 - 4x + 4 = (x-2)^2 \Rightarrow$ a critical point at $x = 2 \Rightarrow h' = \underset{0}{[}\;+++\;\underset{2}{|}\;+++$ and $h(0) = 0 \Rightarrow$ no local maximum, a local minimum is 0 at $x = 0$

(b) no absolute maximum; absolute minimum is 0 at $x = 0$

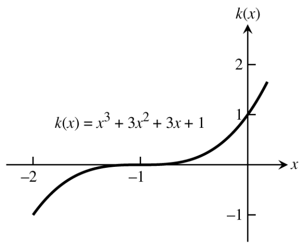
(c)



48. (a) $k(x) = x^3 + 3x^2 + 3x + 1 \Rightarrow k'(x) = 3x^2 + 6x + 3 = 3(x+1)^2 \Rightarrow$ a critical point at $x = -1$
$\Rightarrow k' = \underset{-1}{+++\,|}\,\underset{0}{+++\,]}$ and $k(-1) = 0$, $k(0) = 1 \Rightarrow$ a local maximum is 1 at $x = 0$, no local minimum

(b) absolute maximum is 1 at $x = 0$; no absolute minimum

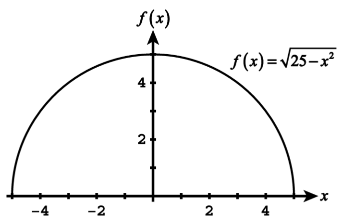
(c)



49. (a) $f(x) = \sqrt{25 - x^2} \Rightarrow f'(x) = \frac{-x}{\sqrt{25 - x^2}} \Rightarrow$ critical points at $x = 0$, $x = -5$, and $x = 5$
$\Rightarrow f' = \underset{-5}{(}\ \underset{0}{+++\,|}\,\underset{5}{---\,)}$, $f(-5) = 0$, $f(0) = 5$, $f(5) = 0 \Rightarrow$ local maximum is 5 at $x = 0$; local minimum of 0 at $x = -5$ and $x = 5$

(b) absolute maximum is 5 at $x = 0$; absolute minimum of 0 at $x = -5$ and $x = 5$

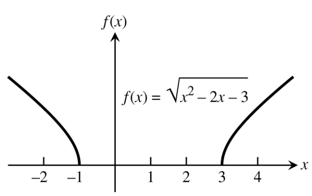
(c)



50. (a) $f(x) = \sqrt{x^2 - 2x - 3}$, $3 \le x < \infty \Rightarrow f'(x) = \frac{2x - 2}{\sqrt{x^2 - 2x - 3}} \Rightarrow$ only critical point in $3 \le x < \infty$ is at $x = 3$
$\Rightarrow f' = \underset{3}{[}\ +++$, $f(3) = 0 \Rightarrow$ local minimum of 0 at $x = 3$, no local maximum

(b) absolute minimum of 0 at $x = 3$, no absolute maximum

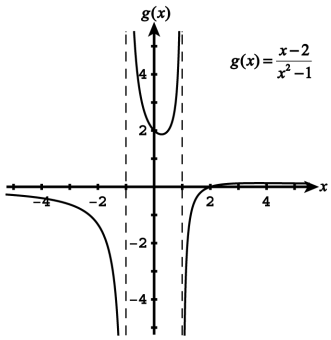
(c)



51. (a) $g(x) = \frac{x-2}{x^2-1}$, $0 \le x < 1 \Rightarrow g'(x) = \frac{-x^2 + 4x - 1}{(x^2-1)^2} \Rightarrow$ only critical point in $0 \le x < 1$ is $x = 2 - \sqrt{3} \approx 0.268$
$\Rightarrow g' = \underset{0}{[}\ \underset{0.268}{---\ |}\ \underset{1}{+++\,)}$, $g\left(2 - \sqrt{3}\right) = \frac{\sqrt{3}}{4\sqrt{3} - 6} \approx 1.866 \Rightarrow$ local minimum of $\frac{\sqrt{3}}{4\sqrt{3} - 6}$ at $x = 2 - \sqrt{3}$, local maximum at $x = 0$.

(b) absolute minimum of $\frac{\sqrt{3}}{4\sqrt{3} - 6}$ at $x = 2 - \sqrt{3}$, no absolute maximum

(c)

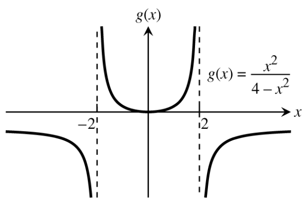


52. (a) $g(x)=\frac{x^2}{4-x^2}$, $-2<x\le 1 \Rightarrow g'(x)=\frac{8x}{(4-x^2)^2} \Rightarrow$ only critical point in $-2<x\le 1$ is $x=0$

$\Rightarrow g' = \underset{-2}{(}\ ---\ \underset{0}{|}\ +++\ \underset{1}{]}$, $g(0)=0 \Rightarrow$ local minimum of 0 at $x=0$, local maximum of $\frac{1}{3}$ at $x=1$.

(b) absolute minimum of 0 at $x=0$, no absolute maximum

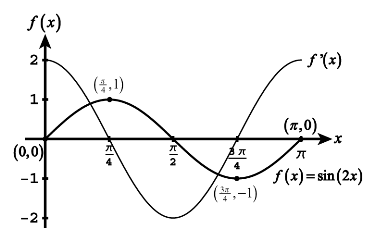
(c)



53. (a) $f(x)=\sin 2x$, $0\le x\le \pi \Rightarrow f'(x)=2\cos 2x$, $f'(x)=0 \Rightarrow \cos 2x=0 \Rightarrow$ critical points are $x=\frac{\pi}{4}$ and $x=\frac{3\pi}{4}$

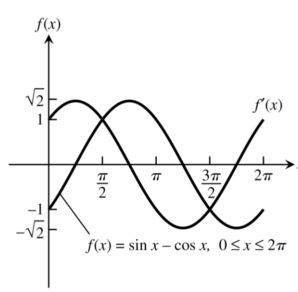
$\Rightarrow f' = \underset{0}{[}\ +++\ \underset{\frac{\pi}{4}}{|}\ ---\ \underset{\frac{3\pi}{4}}{|}\ +++\ \underset{\pi}{]}$, $f(0)=0$, $f\left(\frac{\pi}{4}\right)=1$, $f\left(\frac{3\pi}{4}\right)=-1$, $f(\pi)=0 \Rightarrow$ local maxima are 1 at $x=\frac{\pi}{4}$ and 0 at $x=\pi$, and local minima are -1 at $x=\frac{3\pi}{4}$ and 0 at $x=0$.

(b) The graph of f rises when $f'>0$, falls when $f'<0$, and has local extreme values where $f'=0$. The function f has a local minimum value at $x=0$ and $x=\frac{3\pi}{4}$, where the values of f' change from negative to positive. The function f has a local maximum value at $x=\pi$ and $x=\frac{\pi}{4}$, where the values of f'change from positive to negative.



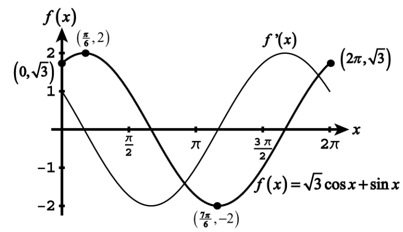
54. (a) $f(x)=\sin x-\cos x$, $0\le x\le 2\pi \Rightarrow f'(x)=\cos x+\sin x$, $f'(x)=0 \Rightarrow \tan x=-1 \Rightarrow$ critical points are $x=\frac{3\pi}{4}$ and $x=\frac{7\pi}{4} \Rightarrow f' = \underset{0}{[}\ +++\ \underset{\frac{3\pi}{4}}{|}\ ---\ \underset{\frac{7\pi}{4}}{|}\ +++\ \underset{2\pi}{]}$, $f(0)=-1$, $f\left(\frac{3\pi}{4}\right)=\sqrt{2}$, $f\left(\frac{7\pi}{4}\right)=-\sqrt{2}$, $f(2\pi)=-1 \Rightarrow$ local maxima are $\sqrt{2}$ at $x=\frac{3\pi}{4}$ and -1 at $x=2\pi$, and local minima are $-\sqrt{2}$ at $x=\frac{7\pi}{4}$ and -1 at $x=0$.

(b) The graph of f rises when $f'>0$, falls when $f'<0$, and has local extreme values where $f'=0$. The function f has a local minimum value at $x=0$ and $x=\frac{7\pi}{4}$, where the values of f' change from negative to positive. The function f has a local maximum value at $x=2\pi$ and $x=\frac{3\pi}{4}$, where the values of f'change from positive to negative.



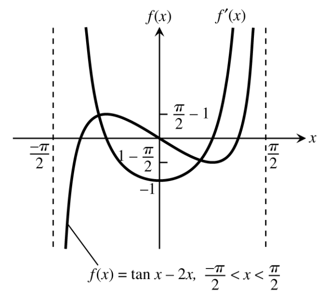
55. (a) $f(x) = \sqrt{3}\cos x + \sin x,\ 0 \le x \le 2\pi \Rightarrow f'(x) = -\sqrt{3}\sin x + \cos x,\ f'(x) = 0 \Rightarrow \tan x = \frac{1}{\sqrt{3}} \Rightarrow$ critical points are $x = \frac{\pi}{6}$ and $x = \frac{7\pi}{6} \Rightarrow f' = \underset{0}{[}+++\underset{\frac{\pi}{6}}{|}---\underset{\frac{7\pi}{6}}{|}+++\underset{2\pi}{]}$, $f(0) = \sqrt{3},\ f\left(\frac{\pi}{6}\right) = 2,\ f\left(\frac{7\pi}{6}\right) = -2,\ f(2\pi) = \sqrt{3} \Rightarrow$ local maxima are 2 at $x = \frac{\pi}{6}$ and $\sqrt{3}$ at $x = 2\pi$, and local minima are -2 at $x = \frac{7\pi}{6}$ and $\sqrt{3}$ at $x = 0$.

(b) The graph of f rises when $f' > 0$, falls when $f' < 0$, and has local extreme values where $f' = 0$. The function f has a local minimum value at $x = 0$ and $x = \frac{7\pi}{6}$, where the values of f' change from negative to positive. The function f has a local maximum value at $x = 2\pi$ and $x = \frac{\pi}{6}$, where the values of f'change from positive to negative.



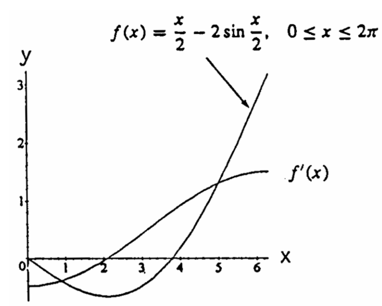
56. (a) $f(x) = -2x + \tan x,\ -\frac{\pi}{2} < x < \frac{\pi}{2} \Rightarrow f'(x) = -2 + \sec^2 x,\ f'(x) = 0 \Rightarrow \sec^2 x = 2 \Rightarrow$ critical points are $x = -\frac{\pi}{4}$ and $x = \frac{\pi}{4} \Rightarrow f' = \underset{-\frac{\pi}{2}}{(}+++\underset{-\frac{\pi}{4}}{|}---\underset{\frac{\pi}{4}}{|}+++\underset{\frac{\pi}{2}}{)}$, $f\left(-\frac{\pi}{4}\right) = \frac{\pi}{2} - 1,\ f\left(\frac{\pi}{4}\right) = 1 - \frac{\pi}{2} \Rightarrow$ local maximum is $\frac{\pi}{2} - 1$ at $x = -\frac{\pi}{4}$, and local minimum is $1 - \frac{\pi}{2}$ at $x = \frac{\pi}{4}$.

(b) The graph of f rises when $f' > 0$, falls when $f' < 0$, and has local extreme values where $f' = 0$. The function f has a local minimum value at $x = \frac{\pi}{4}$, where the values of f' change from negative to positive. The function f has a local maximum value at $x = -\frac{\pi}{4}$, where the values of f'change from positive to negative.



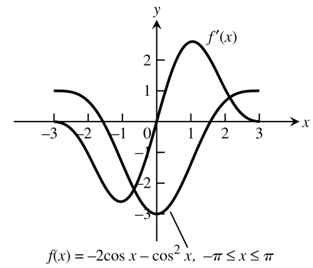
57. (a) $f(x) = \frac{x}{2} - 2\sin\left(\frac{x}{2}\right) \Rightarrow f'(x) = \frac{1}{2} - \cos\left(\frac{x}{2}\right),\ f'(x) = 0 \Rightarrow \cos\left(\frac{x}{2}\right) = \frac{1}{2} \Rightarrow$ a critical point at $x = \frac{2\pi}{3}$ $\Rightarrow f' = \underset{0}{[}---\underset{2\pi/3}{|}+++\underset{2\pi}{]}$ and $f(0) = 0,\ f\left(\frac{2\pi}{3}\right) = \frac{\pi}{3} - \sqrt{3},\ f(2\pi) = \pi \Rightarrow$ local maxima are 0 at $x = 0$ and π at $x = 2\pi$, a local minimum is $\frac{\pi}{3} - \sqrt{3}$ at $x = \frac{2\pi}{3}$

(b) The graph of f rises when $f' > 0$, falls when $f' < 0$, and has a local minimum value at the point where f' changes from negative to positive.



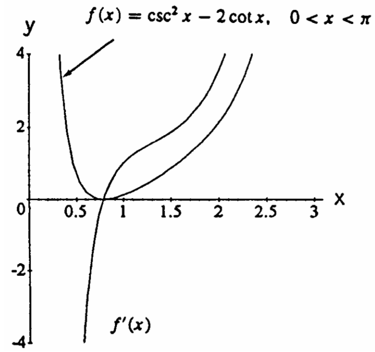
58. (a) $f(x) = -2\cos x - \cos^2 x \Rightarrow f'(x) = 2\sin x + 2\cos x \sin x = 2(\sin x)(1 + \cos x) \Rightarrow$ critical points at $x = -\pi, 0, \pi$ $\Rightarrow f' = \underset{-\pi}{[}---\underset{0}{|}+++\underset{\pi}{]}$ and $f(-\pi) = 1,\ f(0) = -3,\ f(\pi) = 1 \Rightarrow$ a local maximum is 1 at $x = \pm\pi$, a local minimum is -3 at $x = 0$

(b) The graph of f rises when $f' > 0$, falls when $f' < 0$, and has local extreme values where $f' = 0$. The function f has a local minimum value at $x = 0$, where the values of f' change from negative to positive.



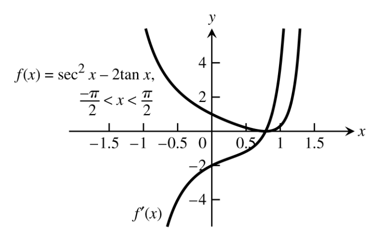
59. (a) $f(x) = \csc^2 x - 2\cot x \Rightarrow f'(x) = 2(\csc x)(-\csc x)(\cot x) - 2\left(-\csc^2 x\right) = -2\left(\csc^2 x\right)(\cot x - 1) \Rightarrow$ a critical point at $x = \frac{\pi}{4} \Rightarrow f' = \underset{0}{(}\ -\!-\!-\ \underset{\pi/4}{|}\ +\!+\!+\ \underset{\pi}{)}$ and $f\left(\frac{\pi}{4}\right) = 0 \Rightarrow$ no local maximum, a local minimum is 0 at $x = \frac{\pi}{4}$

(b) The graph of f rises when $f' > 0$, falls when $f' < 0$, and has a local minimum value at the point where $f' = 0$ and the values of f' change from negative to positive. The graph of f steepens as $f'(x) \to \pm\infty$.



60. (a) $f(x) = \sec^2 x - 2\tan x \Rightarrow f'(x) = 2(\sec x)(\sec x)(\tan x) - 2\sec^2 x = \left(2\sec^2 x\right)(\tan x - 1) \Rightarrow$ a critical point at $x = \frac{\pi}{4} \Rightarrow f' = \underset{-\pi/2}{(}\ -\!-\!-\ \underset{\pi/4}{|}\ +\!+\!+\ \underset{\pi/2}{)}$ and $f\left(\frac{\pi}{4}\right) = 0 \Rightarrow$ no local maximum, a local minimum is 0 at $x = \frac{\pi}{4}$

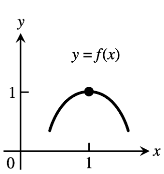
(b) The graph of f rises when $f' > 0$, falls when $f' < 0$, and has a local minimum value where $f' = 0$ and the values of f' change from negative to positive.



61. $h(\theta) = 3\cos\left(\frac{\theta}{2}\right) \Rightarrow h'(\theta) = -\frac{3}{2}\sin\left(\frac{\theta}{2}\right) \Rightarrow h' = \underset{0}{[}\ -\!-\!-\ \underset{2\pi}{]}$, $(0, 3)$ and $(2\pi, -3) \Rightarrow$ a local maximum is 3 at $\theta = 0$, a local minimum is -3 at $\theta = 2\pi$

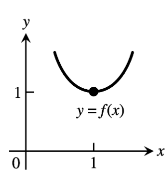
62. $h(\theta) = 5\sin\left(\frac{\theta}{2}\right) \Rightarrow h'(\theta) = \frac{5}{2}\cos\left(\frac{\theta}{2}\right) \Rightarrow h' = \underset{0}{[}\ +\!+\!+\ \underset{\pi}{]}$, $(0, 0)$ and $(\pi, 5) \Rightarrow$ a local maximum is 5 at $\theta = \pi$, a local minimum is 0 at $\theta = 0$

63. (a)



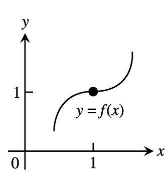
(a)

(b)



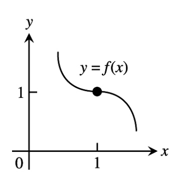
(b)

(c)



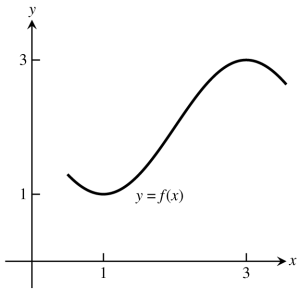
(c)

(d)

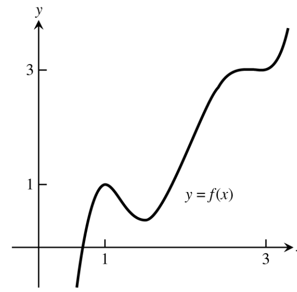


(d)

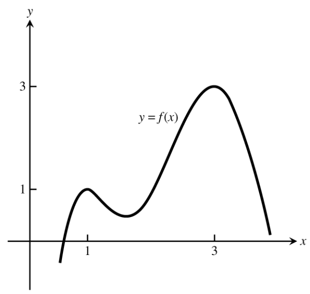
64. (a)



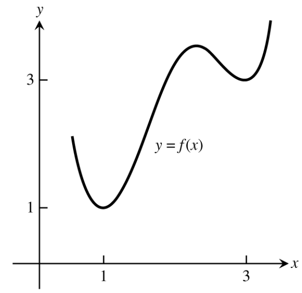
(b)



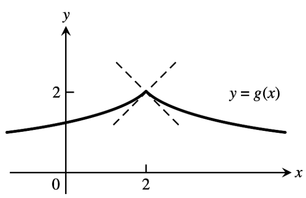
(c)



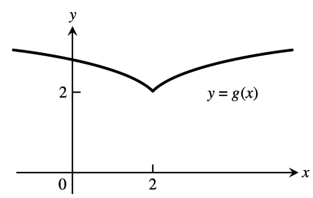
(d)



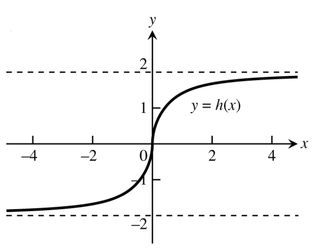
65. (a)



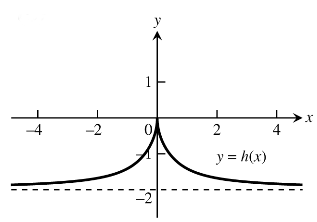
(b)



66. (a)



(b)



67. The function $f(x) = x\sin\left(\frac{1}{x}\right)$ has an infinite number of local maxima and minima. The function $\sin x$ has the following properties: a) it is continuous on $(-\infty, \infty)$; b) it is periodic; and c) its range is $[-1, 1]$. Also, for $a > 0$, the function $\frac{1}{x}$ has a range of $(-\infty, -a] \cup [a, \infty)$ on $\left[-\frac{1}{a}, \frac{1}{a}\right]$. In particular, if $a = 1$, then $\frac{1}{x} \le -1$ or $\frac{1}{x} \ge 1$ when x is in $[-1, 1]$. This means $\sin\left(\frac{1}{x}\right)$ takes on the values of 1 and -1 infinitely many times in times on the interval $[-1, 1]$, which occur when $\frac{1}{x} = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots \Rightarrow x = \pm\frac{2}{\pi}, \pm\frac{2}{3\pi}, \pm\frac{2}{5\pi}, \ldots$. Thus $\sin\left(\frac{1}{x}\right)$ has infinitely many local maxima and minima in the interval $[-1, 1]$. On the interval $[0, 1]$, $-1 \le \sin\left(\frac{1}{x}\right) \le 1$ and since $x > 0$ we have $-x \le x\sin\left(\frac{1}{x}\right) \le x$. On the interval $[-1, 0]$, $-1 \le \sin\left(\frac{1}{x}\right) \le 1$ and since $x < 0$ we have $-x \ge x\sin\left(\frac{1}{x}\right) \ge x$. Thus $f(x)$ is bounded by the lines $y = x$ and $y = -x$. Since $\sin\left(\frac{1}{x}\right)$ oscillates between 1 and -1 infinitely many times on $[-1, 1]$ then f will oscillate between $y = x$ and $y = -x$ infinitely many times. Thus f has infinitely many local maxima and minima. We can see from the graph (and verify later in Chapter 7) that $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) = 1$ and $\lim_{x\to-\infty} x\sin\left(\frac{1}{x}\right) = 1$. The graph of f does not have any absolute maxima., but it does have two absolute minima.

68. $f(x) = a\,x^2 + b\,x + c = a\left(x^2 + \frac{b}{a}\,x\right) + c = a\left(x^2 + \frac{b}{a}\,x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$, a parabola whose vertex is at $x = -\frac{b}{2a}$. Thus when $a > 0$, f is increasing on $\left(\frac{-b}{2a}, \infty\right)$ and decreasing on $\left(-\infty, \frac{-b}{2a}\right)$; when $a < 0$, f is increasing on $\left(-\infty, \frac{-b}{2a}\right)$ and decreasing on $\left(\frac{-b}{2a}, \infty\right)$. Also note that $f'(x) = 2ax + b = 2a\left(x + \frac{b}{2a}\right) \Rightarrow$ for $a > 0$, $f' = \underset{-b/2a}{---\,|\,+++}$; for $a < 0$, $f' = \underset{-b/2a}{+++\,|\,---}$.

69. $f(x) = a\,x^2 + b\,x \Rightarrow f'(x) = 2a\,x + b$, $f(1) = 2 \Rightarrow a + b = 2$, $f'(1) = 0 \Rightarrow 2a + b = 0 \Rightarrow a = -2$, $b = 4$ $\Rightarrow f(x) = -2x^2 + 4x$

70. $f(x) = a\,x^3 + b\,x^2 + c\,x + d \Rightarrow f'(x) = 3a\,x^2 + 2b\,x + c$, $f(0) = 0 \Rightarrow d = 0$, $f(1) = -1 \Rightarrow a + b + c + d = -1$, $f'(0) = 0 \Rightarrow c = 0$, $f'(1) = 0 \Rightarrow 3a + 2b + c = 0 \Rightarrow a = 2$, $b = -3$, $c = 0$, $d = 0 \Rightarrow f(x) = 2x^3 - 3x^2$

4.4 CONCAVITY AND CURVE SKETCHING

1. $y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{3} \Rightarrow y' = x^2 - x - 2 = (x-2)(x+1) \Rightarrow y'' = 2x - 1 = 2\left(x - \frac{1}{2}\right)$. The graph is rising on $(-\infty, -1)$ and $(2, \infty)$, falling on $(-1, 2)$, concave up on $\left(\frac{1}{2}, \infty\right)$ and concave down on $\left(-\infty, \frac{1}{2}\right)$. Consequently, a local maximum is $\frac{3}{2}$ at $x = -1$, a local minimum is -3 at $x = 2$, and $\left(\frac{1}{2}, -\frac{3}{4}\right)$ is a point of inflection.

2. $y = \frac{x^4}{4} - 2x^2 + 4 \Rightarrow y' = x^3 - 4x = x\,(x^2 - 4) = x(x+2)(x-2) \Rightarrow y'' = 3x^2 - 4 = \left(\sqrt{3}x + 2\right)\left(\sqrt{3}x - 2\right)$. The graph is rising on $(-2, 0)$ and $(2, \infty)$, falling on $(-\infty, -2)$ and $(0, 2)$, concave up on $\left(-\infty, -\frac{2}{\sqrt{3}}\right)$ and $\left(\frac{2}{\sqrt{3}}, \infty\right)$ and concave down on $\left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$. Consequently, a local maximum is 4 at $x = 0$, local minima are 0 at $x = \pm 2$, and $\left(-\frac{2}{\sqrt{3}}, \frac{16}{9}\right)$ and $\left(\frac{2}{\sqrt{3}}, \frac{16}{9}\right)$ are points of inflection.

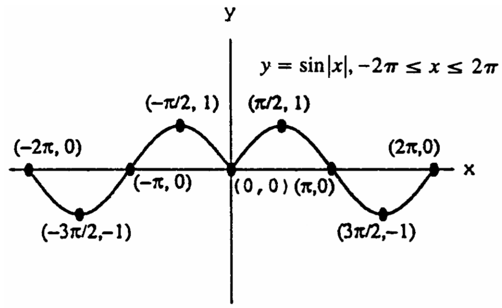
3. $y = \frac{3}{4}\,(x^2 - 1)^{2/3} \Rightarrow y' = \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)(x^2 - 1)^{-1/3}(2x) = x\,(x^2 - 1)^{-1/3}$, $y' = \underset{-1}{---\,)}\,\underset{0}{(+++\,|}\,\underset{1}{---\,)}(+++$ $\Rightarrow$ the graph is rising on $(-1, 0)$ and $(1, \infty)$, falling on $(-\infty, -1)$ and $(0, 1)$ $\Rightarrow$ a local maximum is $\frac{3}{4}$ at $x = 0$, local minima are 0 at $x = \pm 1$; $y'' = (x^2 - 1)^{-1/3} + (x)\left(-\frac{1}{3}\right)(x^2 - 1)^{-4/3}(2x) = \frac{x^2 - 3}{3\sqrt[3]{(x^2-1)^4}}$,
$y'' = \underset{-\sqrt{3}}{+++\,|}\,\underset{-1}{---\,)}\,\underset{1}{(---\,)}\underset{\sqrt{3}}{(---\,|}\,+++$ $\Rightarrow$ the graph is concave up on $\left(-\infty, -\sqrt{3}\right)$ and $\left(\sqrt{3}, \infty\right)$, concave down on $\left(-\sqrt{3}, \sqrt{3}\right) \Rightarrow$ points of inflection at $\left(\pm\sqrt{3}, \frac{3\sqrt[3]{4}}{4}\right)$

4. $y = \frac{9}{14}\,x^{1/3}\,(x^2 - 7) \Rightarrow y' = \frac{3}{14}\,x^{-2/3}\,(x^2 - 7) + \frac{9}{14}\,x^{1/3}(2x) = \frac{3}{2}\,x^{-2/3}\,(x^2 - 1)$, $y' = \underset{-1}{+++\,|}\,\underset{0}{---\,)}\underset{1}{(---\,|}\,+++$ $\Rightarrow$ the graph is rising on $(-\infty, -1)$ and $(1, \infty)$, falling on $(-1, 1) \Rightarrow$ a local maximum is $\frac{27}{7}$ at $x = -1$, a local minimum is $-\frac{27}{7}$ at $x = 1$; $y'' = -x^{-5/3}\,(x^2 - 1) + 3x^{1/3} = 2x^{1/3} + x^{-5/3} = x^{-5/3}\,(2x^2 + 1)$,
$y'' = \underset{0}{---\,)}(+++$ $\Rightarrow$ the graph is concave up on $(0, \infty)$, concave down on $(-\infty, 0) \Rightarrow$ a point of inflection at $(0, 0)$.

5. $y = x + \sin 2x \Rightarrow y' = 1 + 2\cos 2x$, $y' = \underset{-2\pi/3}{[}\,\underset{-\pi/3}{---\,|}\,\underset{\pi/3}{+++\,|}\,\underset{2\pi/3}{---\,]}$ $\Rightarrow$ the graph is rising on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$, falling on $\left(-\frac{2\pi}{3}, -\frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right) \Rightarrow$ local maxima are $-\frac{2\pi}{3} + \frac{\sqrt{3}}{2}$ at $x = -\frac{2\pi}{3}$ and $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$ at $x = \frac{\pi}{3}$, local minima are $-\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ at $x = -\frac{\pi}{3}$ and $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ at $x = \frac{2\pi}{3}$; $y'' = -4\sin 2x$, $y'' = \underset{-2\pi/3}{[}\,\underset{-\pi/2}{---\,|}\,\underset{0}{+++\,|}\,\underset{\pi/2}{---\,|}\,\underset{2\pi/3}{+++\,]}$ $\Rightarrow$ the graph is concave up on $\left(-\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$, concave down on $\left(-\frac{2\pi}{3}, -\frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right) \Rightarrow$ points of inflection at $\left(-\frac{\pi}{2}, -\frac{\pi}{2}\right)$, $(0, 0)$, and $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

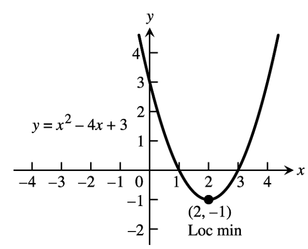
6. $y = \tan x - 4x \Rightarrow y' = \sec^2 x - 4$, $y' = \underset{-\pi/2}{(}\ +++\ \underset{-\pi/3}{|}\ ---\ \underset{\pi/3}{|}\ +++\ \underset{\pi/2}{)}$ $\Rightarrow$ the graph is rising on $\left(-\frac{\pi}{2}, -\frac{\pi}{3}\right)$ and $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$, falling on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$ $\Rightarrow$ a local maximum is $-\sqrt{3} + \frac{4\pi}{3}$ at $x = -\frac{\pi}{3}$, a local minimum is $\sqrt{3} - \frac{4\pi}{3}$ at $x = \frac{\pi}{3}$; $y'' = 2(\sec x)(\sec x)(\tan x) = 2(\sec^2 x)(\tan x)$, $y'' = \underset{-\pi/2}{(}\ ---\ \underset{0}{|}\ +++\ \underset{\pi/2}{)}$ $\Rightarrow$ the graph is concave up on $\left(0, \frac{\pi}{2}\right)$, concave down on $\left(-\frac{\pi}{2}, 0\right)$ $\Rightarrow$ a point of inflection at $(0, 0)$

7. If $x \ge 0$, $\sin|x| = \sin x$ and if $x < 0$, $\sin|x| = \sin(-x) = -\sin x$. From the sketch the graph is rising on $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right)$, $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$, falling on $\left(-2\pi, -\frac{3\pi}{2}\right)$, $\left(-\frac{\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$; local minima are -1 at $x = \pm\frac{3\pi}{2}$ and 0 at $x = 0$; local maxima are 1 at $x = \pm\frac{\pi}{2}$ and 0 at $x = \pm 2\pi$; concave up on $(-2\pi, -\pi)$ and $(\pi, 2\pi)$, and concave down on $(-\pi, 0)$ and $(0, \pi)$ $\Rightarrow$ points of inflection are $(-\pi, 0)$ and $(\pi, 0)$

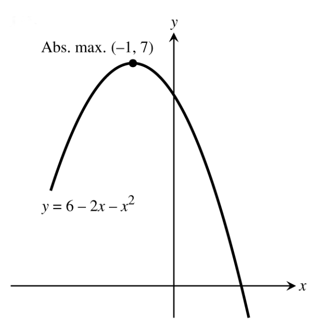


8. $y = 2\cos x - \sqrt{2}\,x \Rightarrow y' = -2\sin x - \sqrt{2}$, $y' = \underset{-\pi}{[}\ ---\ \underset{-3\pi/4}{|}\ +++\ \underset{-\pi/4}{|}\ ---\ \underset{5\pi/4}{|}\ +++\ \underset{3\pi/2}{]}$ $\Rightarrow$ rising on $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right)$and $\left(\frac{5\pi}{4}, \frac{3\pi}{2}\right)$, falling on $\left(-\pi, -\frac{3\pi}{4}\right)$ and $\left(-\frac{\pi}{4}, \frac{5\pi}{4}\right)$ $\Rightarrow$ local maxima are $-2 + \pi\sqrt{2}$ at $x = -\pi$, $\sqrt{2} + \frac{\pi\sqrt{2}}{4}$ at $x = -\frac{\pi}{4}$ and $-\frac{3\pi\sqrt{2}}{2}$ at $x = \frac{3\pi}{2}$, and local minima are $-\sqrt{2} + \frac{3\pi\sqrt{2}}{4}$ at $x = -\frac{3\pi}{4}$ and $-\sqrt{2} - \frac{5\pi\sqrt{2}}{4}$ at $x = \frac{5\pi}{4}$; $y'' = -2\cos x$, $y'' = \underset{-\pi}{[}\ +++\ \underset{-\pi/2}{|}\ ---\ \underset{\pi/2}{|}\ +++\ \underset{3\pi/2}{]}$ $\Rightarrow$ concave up on $\left(-\pi, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, concave down on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\Rightarrow$ points of inflection at $\left(-\frac{\pi}{2}, \frac{\sqrt{2}\pi}{2}\right)$ and $\left(\frac{\pi}{2}, -\frac{\sqrt{2}\pi}{2}\right)$

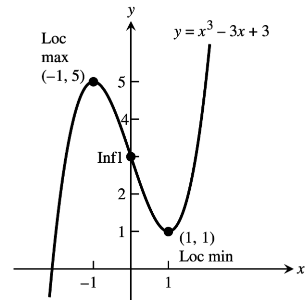
9. When $y = x^2 - 4x + 3$, then $y' = 2x - 4 = 2(x-2)$ and $y'' = 2$. The curve rises on $(2, \infty)$ and falls on $(-\infty, 2)$. At $x = 2$ there is a minimum. Since $y'' > 0$, the curve is concave up for all x.



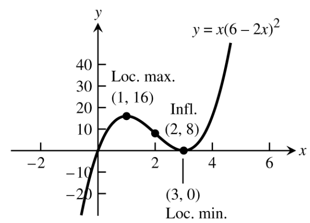
10. When $y = 6 - 2x - x^2$, then $y' = -2 - 2x = -2(1+x)$ and $y'' = -2$. The curve rises on $(-\infty, -1)$ and falls on $(-1, \infty)$. At $x = -1$ there is a maximum. Since $y'' < 0$, the curve is concave down for all x.



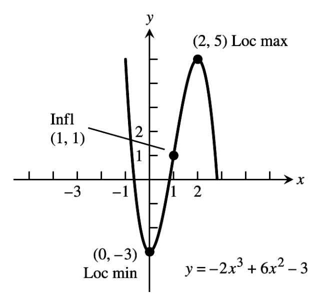
11. When $y = x^3 - 3x + 3$, then $y' = 3x^2 - 3 = 3(x-1)(x+1)$ and $y'' = 6x$. The curve rises on $(-\infty, -1) \cup (1, \infty)$ and falls on $(-1, 1)$. At $x = -1$ there is a local maximum and at $x = 1$ a local minimum. The curve is concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$. There is a point of inflection at $x = 0$.



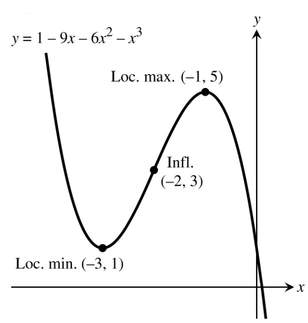
12. When $y = x(6-2x)^2$, then $y' = -4x(6-2x) + (6-2x)^2$ $= 12(3-x)(1-x)$ and $y'' = -12(3-x) - 12(1-x)$ $= 24(x-2)$. The curve rises on $(-\infty, 1) \cup (3, \infty)$ and falls on $(1, 3)$. The curve is concave down on $(-\infty, 2)$ and concave up on $(2, \infty)$. At $x = 2$ there is a point of inflection.



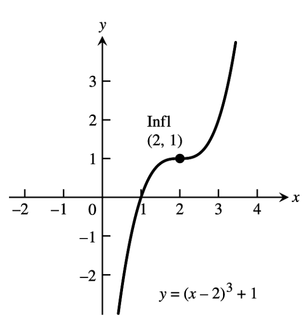
13. When $y = -2x^3 + 6x^2 - 3$, then $y' = -6x^2 + 12x$ $= -6x(x-2)$ and $y'' = -12x + 12 = -12(x-1)$. The curve rises on $(0, 2)$ and falls on $(-\infty, 0)$ and $(2, \infty)$. At $x = 0$ there is a local minimum and at $x = 2$ a local maximum. The curve is concave up on $(-\infty, 1)$ and concave down on $(1, \infty)$. At $x = 1$ there is a point of inflection.



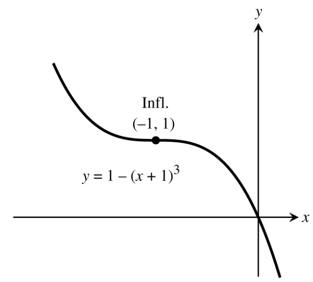
14. When $y = 1 - 9x - 6x^2 - x^3$, then $y' = -9 - 12x - 3x^2$ $= -3(x+3)(x+1)$ and $y'' = -12 - 6x = -6(x+2)$. The curve rises on $(-3, -1)$ and falls on $(-\infty, -3)$ and $(-1, \infty)$. At $x = -1$ there is a local maximum and at $x = -3$ a local minimum. The curve is concave up on $(-\infty, -2)$ and concave down on $(-2, \infty)$. At $x = -2$ there is a point of inflection.



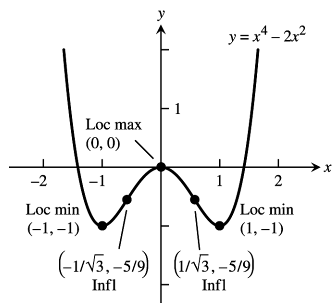
15. When $y = (x-2)^3 + 1$, then $y' = 3(x-2)^2$ and $y'' = 6(x-2)$. The curve never falls and there are no local extrema. The curve is concave down on $(-\infty, 2)$ and concave up on $(2, \infty)$. At $x = 2$ there is a point of inflection.



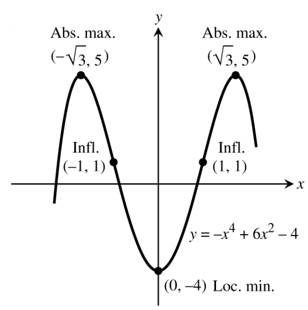
16. When $y = 1 - (x + 1)^3$, then $y' = -3(x + 1)^2$ and $y'' = -6(x + 1)$. The curve never rises and there are no local extrema. The curve is concave up on $(-\infty, -1)$ and concave down on $(-1, \infty)$. At $x = -1$ there is a point of inflection.



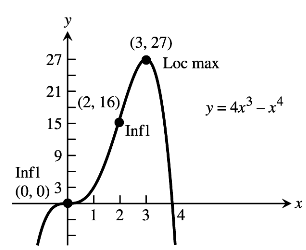
17. When $y = x^4 - 2x^2$, then $y' = 4x^3 - 4x = 4x(x + 1)(x - 1)$ and $y'' = 12x^2 - 4 = 12\left(x + \frac{1}{\sqrt{3}}\right)\left(x - \frac{1}{\sqrt{3}}\right)$. The curve rises on $(-1, 0)$ and $(1, \infty)$ and falls on $(-\infty, -1)$ and $(0, 1)$. At $x = \pm 1$ there are local minima and at $x = 0$ a local maximum. The curve is concave up on $\left(-\infty, -\frac{1}{\sqrt{3}}\right)$ and $\left(\frac{1}{\sqrt{3}}, \infty\right)$ and concave down on $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$. At $x = \frac{\pm 1}{\sqrt{3}}$ there are points of inflection.



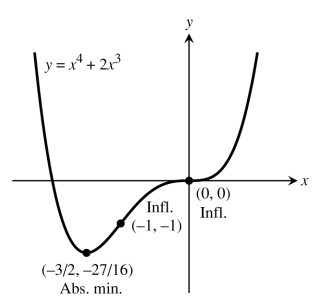
18. When $y = -x^4 + 6x^2 - 4$, then $y' = -4x^3 + 12x$ $= -4x\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$ and $y'' = -12x^2 + 12$ $= -12(x + 1)(x - 1)$. The curve rises on $\left(-\infty, -\sqrt{3}\right)$ and $\left(0, \sqrt{3}\right)$, and falls on $\left(-\sqrt{3}, 0\right)$ and $\left(\sqrt{3}, \infty\right)$. At $x = \pm\sqrt{3}$there are local maxima and at $x = 0$ a local minimum. The curve is concave up on $(-1, 1)$ and concave down on $(-\infty, -1)$ and $(1, \infty)$. At $x = \pm 1$ there are points of inflection.



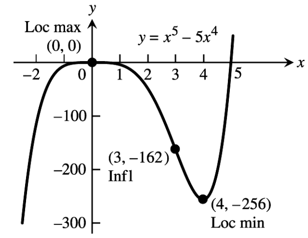
19. When $y = 4x^3 - x^4$, then $y' = 12x^2 - 4x^3 = 4x^2(3 - x)$ and $y'' = 24x - 12x^2 = 12x(2 - x)$. The curve rises on $(-\infty, 3)$ and falls on $(3, \infty)$. At $x = 3$ there is a local maximum, but there is no local minimum. The graph is concave up on $(0,\ 2)$ and concave down on $(-\infty,\ 0)$ and $(2,\ \infty)$. There are inflection points at $x = 0$ and $x = 2$.



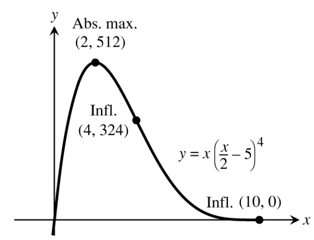
20. When $y = x^4 + 2x^3$, then $y' = 4x^3 + 6x^2 = 2x^2(2x + 3)$ and $y'' = 12x^2 + 12x = 12x(x + 1)$. The curve rises on $\left(-\frac{3}{2}, \infty\right)$ and falls on $\left(-\infty, -\frac{3}{2}\right)$. There is a local minimum at $x = -\frac{3}{2}$, but no local maximum. The curve is concave up on $(-\infty, -1)$ and $(0, \infty)$, and concave down on $(-1, 0)$. At $x = -1$ and $x = 0$ there are points of inflection.



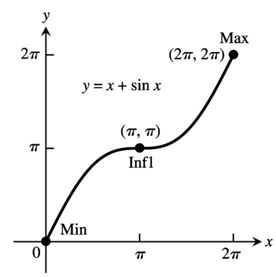
21. When $y = x^5 - 5x^4$, then $y' = 5x^4 - 20x^3 = 5x^3(x - 4)$ and $y'' = 20x^3 - 60x^2 = 20x^2(x - 3)$. The curve rises on $(-\infty, 0)$ and $(4, \infty)$, and falls on $(0, 4)$. There is a local maximum at $x = 0$, and a local minimum at $x = 4$. The curve is concave down on $(-\infty, 3)$ and concave up on $(3, \infty)$. At $x = 3$ there is a point of inflection.



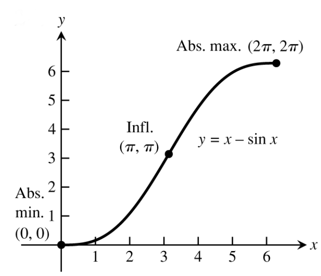
22. When $y = x\left(\frac{x}{2} - 5\right)^4$, then $y' = \left(\frac{x}{2} - 5\right)^4 + x(4)\left(\frac{x}{2} - 5\right)^3\left(\frac{1}{2}\right)$ $= \left(\frac{x}{2} - 5\right)^3\left(\frac{5x}{2} - 5\right)$, and $y'' = 3\left(\frac{x}{2} - 5\right)^2\left(\frac{1}{2}\right)\left(\frac{5x}{2} - 5\right)$ $+ \left(\frac{x}{2} - 5\right)^3\left(\frac{5}{2}\right) = 5\left(\frac{x}{2} - 5\right)^2(x - 4)$. The curve is rising on $(-\infty, 2)$ and $(10, \infty)$, and falling on $(2, 10)$. There is a local maximum at $x = 2$ and a local minimum at $x = 10$. The curve is concave down on $(-\infty, 4)$ and concave up on $(4, \infty)$. At $x = 4$ there is a point of inflection.



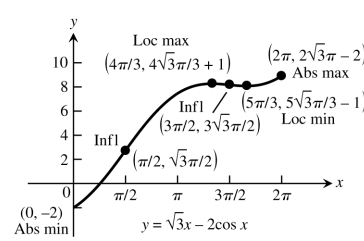
23. When $y = x + \sin x$, then $y' = 1 + \cos x$ and $y'' = -\sin x$. The curve rises on $(0, 2\pi)$. At $x = 0$ there is a local and absolute minimum and at $x = 2\pi$ there is a local and absolute maximum. The curve is concave down on $(0, \pi)$ and concave up on $(\pi, 2\pi)$. At $x = \pi$ there is a point of inflection.



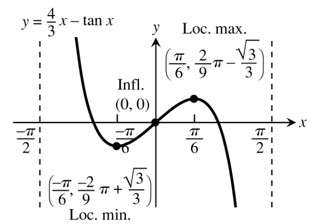
24. When $y = x - \sin x$, then $y' = 1 - \cos x$ and $y'' = \sin x$. The curve rises on $(0, 2\pi)$. At $x = 0$ there is a local and absolute minimum and at $x = 2\pi$ there is a local and absolute maximum. The curve is concave up on $(0, \pi)$ and concave down on $(\pi, 2\pi)$. At $x = \pi$ there is a point of inflection.



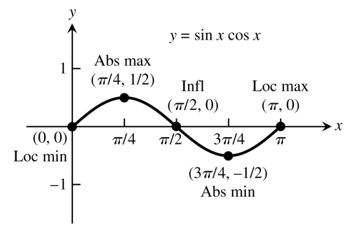
25. When $y = \sqrt{3}x - 2\cos x$, then $y' = \sqrt{3} + 2\sin x$ and $y'' = 2\cos x$. The curve is increasing on $\left(0, \frac{4\pi}{3}\right)$ and $\left(\frac{5\pi}{3}, 2\pi\right)$, and decreasing on $\left(\frac{4\pi}{3}, \frac{5\pi}{3}\right)$. At $x = 0$ there is a local and absolute minimum, at $x = \frac{4\pi}{3}$ there is a local maximum, at $x = \frac{5\pi}{3}$ there is a local minimum, and and at $x = 2\pi$ there is a local and absolute maximum. The curve is concave up on $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$, and is concave down on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. At $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$ there are points of inflection.



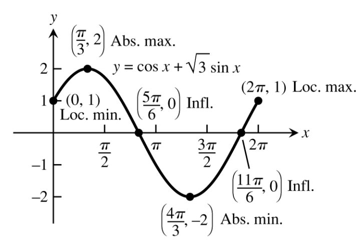
26. When $y = \frac{4}{3}x - \tan x$, then $y' = \frac{4}{3} - \sec^2 x$ and $y'' = -2\sec^2 x \tan x$. The curve is increasing on $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$, and decreasing on $\left(-\frac{\pi}{2}, -\frac{\pi}{6}\right)$ and $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$. At $x = -\frac{\pi}{6}$ there is a local minimum, at $x = \frac{\pi}{6}$ there is a local maximum,there are no absolute maxima or absolute minima. The curve is concave up on $\left(-\frac{\pi}{2}, 0\right)$, and is concave down on$\left(0, \frac{\pi}{2}\right)$. At $x = 0$ there is a point of inflection.



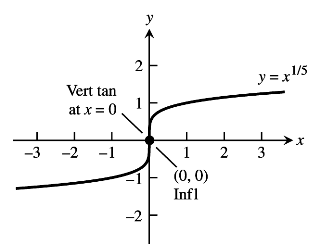
27. When $y = \sin x \cos x$, then $y' = -\sin^2 x + \cos^2 x = \cos 2x$ and $y'' = -2\sin 2x$. The curve is increasing on $\left(0, \frac{\pi}{4}\right)$ and $\left(\frac{3\pi}{4}, \pi\right)$, and decreasing on $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$. At $x = 0$ there is a local minimum, at $x = \frac{\pi}{4}$ there is a local and absolute maximum, at $x = \frac{3\pi}{4}$ there is a local and absolute minimum, and at $x = \pi$ there is a local maximum. The curve is concave down on $\left(0, \frac{\pi}{2}\right)$, and is concave up on$\left(\frac{\pi}{2}, \pi\right)$. At $x = \frac{\pi}{2}$ there is a point of inflection.



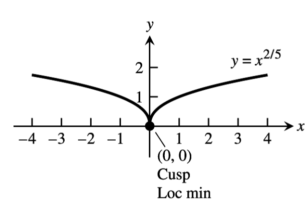
28. When $y = \cos x + \sqrt{3}\sin x$, then $y' = -\sin x + \sqrt{3}\cos x$ and $y'' = -\cos x - \sqrt{3}\sin x$. The curve is increasing on $\left(0, \frac{\pi}{3}\right)$ and $\left(\frac{4\pi}{3}, 2\pi\right)$, and decreasing on $\left(\frac{\pi}{3}, \frac{4\pi}{3}\right)$. At $x = 0$ there is a local minimum, at $x = \frac{\pi}{3}$ there is a local and absolute maximum, at $x = \frac{4\pi}{3}$ there is a local and absolute minimum, and at $x = 2\pi$ there is a local maximum. The curve is concave down on $\left(0, \frac{5\pi}{6}\right)$ and $\left(\frac{11\pi}{6}, 2\pi\right)$, and is concave up on $\left(\frac{5\pi}{6}, \frac{11\pi}{6}\right)$. At $x = \frac{5\pi}{6}$ and $x = \frac{11\pi}{6}$ there are points of inflection.



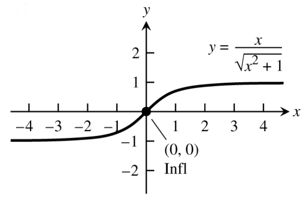
29. When $y = x^{1/5}$, then $y' = \frac{1}{5}x^{-4/5}$ and $y'' = -\frac{4}{25}x^{-9/5}$. The curve rises on $(-\infty, \infty)$ and there are no extrema. The curve is concave up on $(-\infty, 0)$ and concave down on $(0, \infty)$. At $x = 0$ there is a point of inflection.



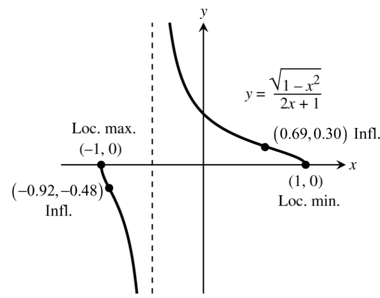
30. When $y = x^{2/5}$, then $y' = \frac{2}{5}x^{-3/5}$ and $y'' = -\frac{6}{25}x^{-8/5}$. The curve is rising on $(0, \infty)$ and falling on $(-\infty, 0)$. At $x = 0$ there is a local and absolute minimum. There is no local or absolute maximum. The curve is concave down on $(-\infty, 0)$ and $(0, \infty)$. There are no points of inflection, but a cusp exists at $x = 0$.



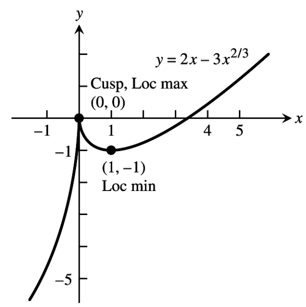
31. When $y = \frac{x}{\sqrt{x^2+1}}$, then $y' = \frac{1}{(x^2+1)^{3/2}}$ and $y'' = \frac{-3x}{(x^2+1)^{5/2}}$. The curve is increasing on $(-\infty, \infty)$. There are no local or absolute extrema. The curve is concave up on $(-\infty, 0)$ and concave down on $(0, \infty)$. At $x = 0$ there is a point of inflection.



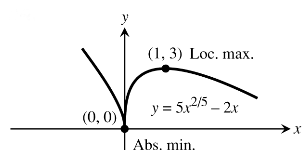
32. When $y = \frac{\sqrt{1-x^2}}{2x+1}$, then $y' = \frac{-(x+2)}{(2x+1)^2\sqrt{1-x^2}}$ and $y'' = \frac{-4x^3-12x^2+7}{(2x+1)^3(1-x^2)^{3/2}}$. The curve is decreasing on $\left(-1, -\frac{1}{2}\right)$ and $\left(-\frac{1}{2}, 1\right)$. There are no absolute extrrema, there is a local maximum at $x = -1$ and a local minimum at $x = 1$. The curve is concave up on $(-1, -0.92)$ and $\left(-\frac{1}{2}, 0.69\right)$, and concave down on $\left(-0.92, -\frac{1}{2}\right)$ and $(0.69, 1)$. At $x \approx -0.92$ and $x \approx 0.69$ there are points of inflection.



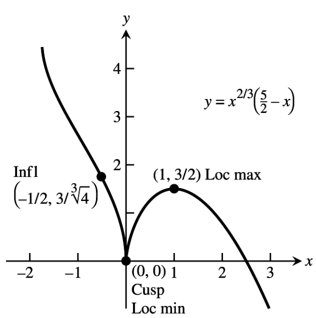
33. When $y = 2x - 3x^{2/3}$, then $y' = 2 - 2x^{-1/3}$ and $y'' = \frac{2}{3}x^{-4/3}$. The curve is rising on $(-\infty, 0)$ and $(1, \infty)$, and falling on $(0, 1)$. There is a local maximum at $x = 0$ and a local minimum at $x = 1$. The curve is concave up on $(-\infty, 0)$ and $(0, \infty)$. There are no points of inflection, but a cusp exists at $x = 0$.



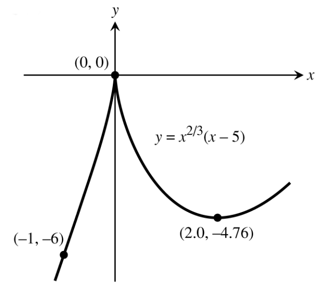
34. When $y = 5x^{2/5} - 2x$, then $y' = 2x^{-3/5} - 2 = 2\left(x^{-3/5} - 1\right)$ and $y'' = -\frac{6}{5}x^{-8/5}$. The curve is rising on $(0, 1)$ and falling on $(-\infty, 0)$ and $(1, \infty)$. There is a local minimum at $x = 0$ and a local maximum at $x = 1$. The curve is concave down on $(-\infty, 0)$ and $(0, \infty)$. There are no points of inflection, but a cusp exists at $x = 0$.



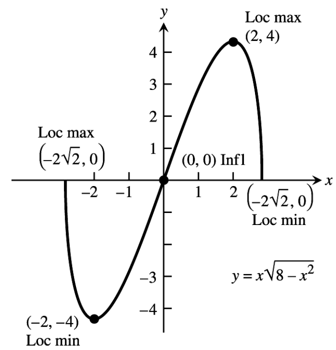
35. When $y = x^{2/3}\left(\frac{5}{2} - x\right) = \frac{5}{2}x^{2/3} - x^{5/3}$, then $y' = \frac{5}{3}x^{-1/3} - \frac{5}{3}x^{2/3} = \frac{5}{3}x^{-1/3}(1 - x)$ and $y'' = -\frac{5}{9}x^{-4/3} - \frac{10}{9}x^{-1/3} = -\frac{5}{9}x^{-4/3}(1 + 2x)$. The curve is rising on $(0, 1)$ and falling on $(-\infty, 0)$ and $(1, \infty)$. There is a local minimum at $x = 0$ and a local maximum at $x = 1$. The curve is concave up on $\left(-\infty, -\frac{1}{2}\right)$ and concave down on $\left(-\frac{1}{2}, 0\right)$ and $(0, \infty)$. There is a point of inflection at $x = -\frac{1}{2}$ and a cusp at $x = 0$.



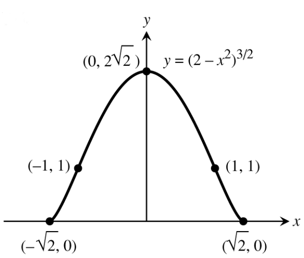
36. When $y = x^{2/3}(x-5) = x^{5/3} - 5x^{2/3}$, then $y' = \frac{5}{3}x^{2/3} - \frac{10}{3}x^{-1/3} = \frac{5}{3}x^{-1/3}(x-2)$ and $y'' = \frac{10}{9}x^{-1/3} + \frac{10}{9}x^{-4/3} = \frac{10}{9}x^{-4/3}(x+1)$. The curve is rising on $(-\infty, 0)$ and $(2, \infty)$, and falling on $(0, 2)$. There is a local minimum at $x = 2$ and a local maximum at $x = 0$. The curve is concave up on $(-1, 0)$ and $(0, \infty)$, and concave down on $(-\infty, -1)$. There is a point of inflection at $x = -1$ and a cusp at $x = 0$.



37. When $y = x\sqrt{8-x^2} = x\left(8-x^2\right)^{1/2}$, then $y' = \left(8-x^2\right)^{1/2} + (x)\left(\frac{1}{2}\right)\left(8-x^2\right)^{-1/2}(-2x)$ $= \left(8-x^2\right)^{-1/2}\left(8-2x^2\right) = \frac{2(2-x)(2+x)}{\sqrt{\left(2\sqrt{2}+x\right)\left(2\sqrt{2}-x\right)}}$ and $y'' = \left(-\frac{1}{2}\right)\left(8-x^2\right)^{-\frac{3}{2}}(-2x)\left(8-2x^2\right) + \left(8-x^2\right)^{-\frac{1}{2}}(-4x)$ $= \frac{2x\left(x^2-12\right)}{\sqrt{\left(8-x^2\right)^3}}$. The curve is rising on $(-2, 2)$, and falling on $\left(-2\sqrt{2}, -2\right)$ and $\left(2, 2\sqrt{2}\right)$. There are local minima $x = -2$ and $x = 2\sqrt{2}$, and local maxima at $x = -2\sqrt{2}$ and $x = 2$. The curve is concave up on $\left(-2\sqrt{2}, 0\right)$ and concave down on $\left(0, 2\sqrt{2}\right)$. There is a point of inflection at $x = 0$.



38. When $y = \left(2-x^2\right)^{3/2}$, then $y' = \left(\frac{3}{2}\right)\left(2-x^2\right)^{1/2}(-2x)$ $= -3x\sqrt{2-x^2} = -3x\sqrt{\left(\sqrt{2}-x\right)\left(\sqrt{2}+x\right)}$ and $y'' = (-3)\left(2-x^2\right)^{1/2} + (-3x)\left(\frac{1}{2}\right)\left(2-x^2\right)^{-1/2}(-2x)$ $= \frac{-6(1-x)(1+x)}{\sqrt{\left(\sqrt{2}-x\right)\left(\sqrt{2}+x\right)}}$. The curve is rising on $\left(-\sqrt{2}, 0\right)$ and falling on $\left(0, \sqrt{2}\right)$. There is a local maximum at $x = 0$, and local minima at $x = \pm\sqrt{2}$. The curve is concave down on $(-1, 1)$ and concave up on $\left(-\sqrt{2}, -1\right)$ and $\left(1, \sqrt{2}\right)$. There are points of inflection at $x = \pm 1$.



39. When $y = \sqrt{16-x^2}$, then $y' = \frac{-x}{\sqrt{16-x^2}}$ and $y'' = \frac{-16}{\left(16-x^2\right)^{3/2}}$. The curve is rising on $(-4, 0)$ and falling on $(0, 4)$. There is a local and absolute maximum at $x = 0$ and local and absolute minima at $x = -4$ and $x = 4$. The curve is concave down on $(-4, 4)$. There are no points of inflection.