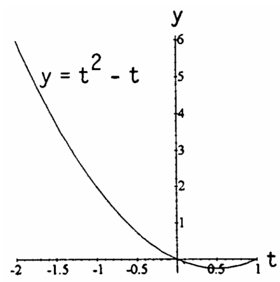
60. $\text{av}(f) = \left(\frac{1}{1-(-2)}\right) \int_{-2}^{1} (t^2 - t)\, dt$

$= \frac{1}{3} \int_{-2}^{1} t^2\, dt - \frac{1}{3} \int_{-2}^{1} t\, dt$

$= \frac{1}{3} \int_{0}^{1} t^2\, dt - \frac{1}{3} \int_{0}^{-2} t^2\, dt - \frac{1}{3}\left(\frac{1^2}{2} - \frac{(-2)^2}{2}\right)$

$= \frac{1}{3}\left(\frac{1^3}{3}\right) - \frac{1}{3}\left(\frac{(-2)^3}{3}\right) + \frac{1}{2} = \frac{3}{2}.$



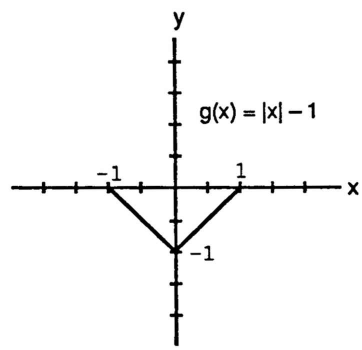
61. (a) $\text{av}(g) = \left(\frac{1}{1-(-1)}\right) \int_{-1}^{1} (|x| - 1)\, dx$

$= \frac{1}{2} \int_{-1}^{0} (-x - 1)\, dx + \frac{1}{2} \int_{0}^{1} (x - 1)\, dx$

$= -\frac{1}{2} \int_{-1}^{0} x\, dx - \frac{1}{2} \int_{-1}^{0} 1\, dx + \frac{1}{2} \int_{0}^{1} x\, dx - \frac{1}{2} \int_{0}^{1} 1\, dx$

$= -\frac{1}{2}\left(\frac{0^2}{2} - \frac{(-1)^2}{2}\right) - \frac{1}{2}(0 - (-1)) + \frac{1}{2}\left(\frac{1^2}{2} - \frac{0^2}{2}\right) - \frac{1}{2}(1 - 0)$

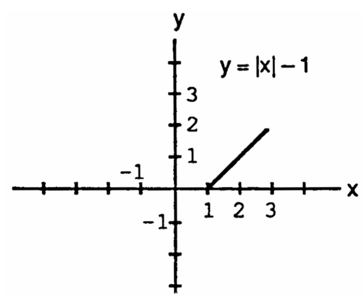
$= -\frac{1}{2}.$



(b) $\text{av}(g) = \left(\frac{1}{3-1}\right) \int_{1}^{3} (|x| - 1)\, dx = \frac{1}{2} \int_{1}^{3} (x - 1)\, dx$

$= \frac{1}{2} \int_{1}^{3} x\, dx - \frac{1}{2} \int_{1}^{3} 1\, dx = \frac{1}{2}\left(\frac{3^2}{2} - \frac{1^2}{2}\right) - \frac{1}{2}(3 - 1)$

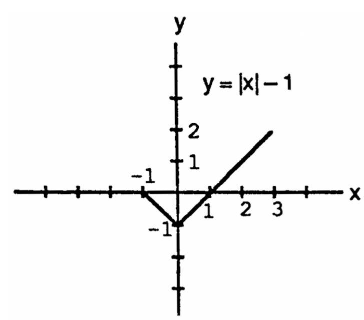
$= 1.$



(c) $\text{av}(g) = \left(\frac{1}{3-(-1)}\right) \int_{-1}^{3} (|x| - 1)\, dx$

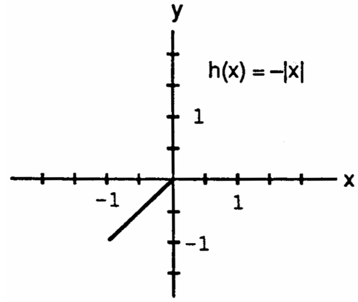
$= \frac{1}{4} \int_{-1}^{1} (|x| - 1)\, dx + \frac{1}{4} \int_{1}^{3} (|x| - 1)\, dx$

$= \frac{1}{4}(-1 + 2) = \frac{1}{4}$ (see parts (a) and (b) above).

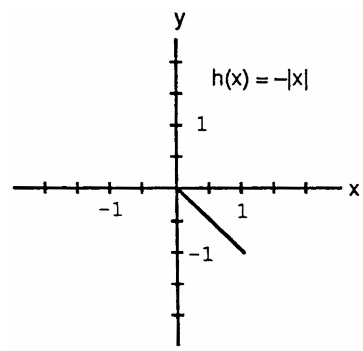


62. (a) $\text{av}(h) = \left(\frac{1}{0-(-1)}\right) \int_{-1}^{0} -|x|\, dx = \int_{-1}^{0} -(-x)\, dx$

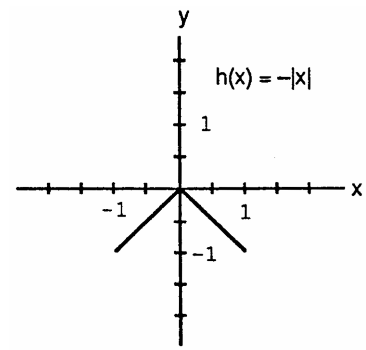
$= \int_{-1}^{0} x\, dx = \frac{0^2}{2} - \frac{(-1)^2}{2} = -\frac{1}{2}.$



(b) $\text{av}(h) = \left(\frac{1}{1-0}\right) \int_0^1 -|x|\, dx = -\int_0^1 x\, dx$
$= -\left(\frac{1^2}{2} - \frac{0^2}{2}\right) = -\frac{1}{2}.$



(c) $\text{av}(h) = \left(\frac{1}{1-(-1)}\right) \int_{-1}^1 -|x|\, dx$
$= \frac{1}{2}\left(\int_{-1}^0 -|x|\, dx + \int_0^1 -|x|\, dx\right)$
$= \frac{1}{2}\left(-\frac{1}{2} + \left(-\frac{1}{2}\right)\right) = -\frac{1}{2}$ (see parts (a) and (b) above).



63. Consider the partition P that subdivides the interval $[a, b]$ into n subintervals of width $\triangle x = \frac{b-a}{n}$ and let c_k be the right endpoint of each subinterval. So the partition is $P = \left\{a, a + \frac{b-a}{n}, a + \frac{2(b-a)}{n}, \ldots, a + \frac{n(b-a)}{n}\right\}$ and $c_k = a + \frac{k(b-a)}{n}$. We get the Riemann sum $\sum_{k=1}^{n} f(c_k)\triangle x = \sum_{k=1}^{n} c \cdot \frac{b-a}{n} = \frac{c(b-a)}{n} \sum_{k=1}^{n} 1 = \frac{c(b-a)}{n} \cdot n = c(b-a)$. As $n \to \infty$ and $\|P\| \to 0$ this expression remains $c(b-a)$. Thus, $\int_a^b c\, dx = c(b-a)$.

64. Consider the partition P that subdivides the interval $[0, 2]$ into n subintervals of width $\triangle x = \frac{2-0}{n} = \frac{2}{n}$ and let c_k be the right endpoint of each subinterval. So the partition is $P = \left\{0, \frac{2}{n}, 2 \cdot \frac{2}{n}, \ldots, n \cdot \frac{2}{n} = 2\right\}$ and $c_k = k \cdot \frac{2}{n} = \frac{2k}{n}$. We get the Riemann sum $\sum_{k=1}^{n} f(c_k)\triangle x = \sum_{k=1}^{n} \left[2\left(\frac{2k}{n}\right) + 1\right] \cdot \frac{2}{n} = \frac{2}{n} \sum_{k=1}^{n} \left(\frac{4k}{n} + 1\right) = \frac{8}{n^2} \sum_{k=1}^{n} k + \frac{2}{n} \sum_{k=1}^{n} 1 = \frac{8}{n^2} \cdot \frac{n(n+1)}{2} + \frac{2}{n} \cdot n = \frac{4(n+1)}{n} + 2.$ As $n \to \infty$ and $\|P\| \to 0$ the expression $\frac{4(n+1)}{n} + 2$ has the value $4 + 2 = 6$. Thus, $\int_0^2 (2x+1)\, dx = 6$.

65. Consider the partition P that subdivides the interval $[a, b]$ into n subintervals of width $\triangle x = \frac{b-a}{n}$ and let c_k be the right endpoint of each subinterval. So the partition is $P = \left\{a, a + \frac{b-a}{n}, a + \frac{2(b-a)}{n}, \ldots, a + \frac{n(b-a)}{n}\right\}$ and $c_k = a + \frac{k(b-a)}{n}$. We get the Riemann sum $\sum_{k=1}^{n} f(c_k)\triangle x = \sum_{k=1}^{n} c_k^2 \left(\frac{b-a}{n}\right) = \frac{b-a}{n} \sum_{k=1}^{n} \left(a + \frac{k(b-a)}{n}\right)^2 = \frac{b-a}{n} \sum_{k=1}^{n} \left(a^2 + \frac{2ak(b-a)}{n} + \frac{k^2(b-a)^2}{n^2}\right)$
$= \frac{b-a}{n}\left(\sum_{k=1}^{n} a^2 + \frac{2a(b-a)}{n} \sum_{k=1}^{n} k + \frac{(b-a)^2}{n^2} \sum_{k=1}^{n} k^2\right) = \frac{b-a}{n} \cdot na^2 + \frac{2a(b-a)^2}{n^2} \cdot \frac{n(n+1)}{2} + \frac{(b-a)^3}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$
$= (b-a)a^2 + a(b-a)^2 \cdot \frac{n+1}{n} + \frac{(b-a)^3}{6} \cdot \frac{(n+1)(2n+1)}{n^2} = (b-a)a^2 + a(b-a)^2 \cdot \frac{1+\frac{1}{n}}{1} + \frac{(b-a)^3}{6} \cdot \frac{2+\frac{3}{n}+\frac{1}{n^2}}{1}$
As $n \to \infty$ and $\|P\| \to 0$ this expression has value $(b-a)a^2 + a(b-a)^2 \cdot 1 + \frac{(b-a)^3}{6} \cdot 2$
$= ba^2 - a^3 + ab^2 - 2a^2b + a^3 + \frac{1}{3}(b^3 - 3b^2a + 3ba^2 - a^3) = \frac{b^3}{3} - \frac{a^3}{3}$. Thus, $\int_a^b x^2 dx = \frac{b^3}{3} - \frac{a^3}{3}$.

66. Consider the partition P that subdivides the interval $[-1, 0]$ into n subintervals of width $\triangle x = \frac{0-(-1)}{n} = \frac{1}{n}$ and let c_k be the right endpoint of each subinterval. So the partition is $P = \left\{-1, -1 + \frac{1}{n}, -1 + 2 \cdot \frac{1}{n}, \ldots, -1 + n \cdot \frac{1}{n} = 0\right\}$ and

$c_k = -1 + k \cdot \frac{1}{n} = -1 + \frac{k}{n}$. We get the Riemann sum $\sum_{k=1}^{n} f(c_k)\triangle x = \sum_{k=1}^{n}\left(\left(-1+\frac{k}{n}\right) - \left(-1+\frac{k}{n}\right)^2\right)\cdot\frac{1}{n}$
$= \frac{1}{n}\sum_{k=1}^{n}\left(-1+\frac{k}{n}-1+\frac{2k}{n}-\left(\frac{k}{n}\right)^2\right) = -\frac{2}{n}\sum_{k=1}^{n}1 + \frac{3}{n^2}\sum_{k=1}^{n}k - \frac{1}{n^3}\sum_{k=1}^{n}k^2 = -\frac{2}{n}\cdot n + \frac{3}{n^2}\cdot\frac{n(n+1)}{2} - \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$
$= -2 + \frac{3(n+1)}{2n} - \frac{(n+1)(2n+1)}{6n^2}$. As $n \to \infty$ and $\|P\| \to 0$ this expression has value $-2 + \frac{3}{2} - \frac{1}{3} = -\frac{5}{6}$. Thus, $\int_{-1}^{0}(x - x^2)dx = -\frac{5}{6}$.

67. Consider the partition P that subdivides the interval $[-1, 2]$ into n subintervals of width $\triangle x = \frac{2-(-1)}{n} = \frac{3}{n}$ and let c_k be the right endpoint of each subinterval. So the partition is $P = \{-1, -1+\frac{3}{n}, -1+2\cdot\frac{3}{n}, \ldots, -1+n\cdot\frac{3}{n} = 2\}$ and $c_k = -1 + k\cdot\frac{3}{n} = -1+\frac{3k}{n}$. We get the Riemann sum $\sum_{k=1}^{n} f(c_k)\triangle x = \sum_{k=1}^{n}\left(3\left(-1+\frac{3k}{n}\right)^2 - 2\left(-1+\frac{3k}{n}\right)+1\right)\cdot\frac{3}{n}$
$= \frac{3}{n}\sum_{k=1}^{n}\left(3 - \frac{18k}{n} + \frac{27k^2}{n^2} + 2 - \frac{6k}{n} + 1\right) = \frac{18}{n}\sum_{k=1}^{n}1 - \frac{72}{n^2}\sum_{k=1}^{n}k + \frac{81}{n^3}\sum_{k=1}^{n}k^2 = \frac{18}{n}\cdot n - \frac{72}{n^2}\cdot\frac{n(n+1)}{2} + \frac{81}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$
$= 18 - \frac{36(n+1)}{n} + \frac{27(n+1)(2n+1)}{2n^2}$. As $n \to \infty$ and $\|P\| \to 0$ this expression has value $18 - 36 + 27 = 9$. Thus, $\int_{-1}^{2}(3x^2 - 2x + 1)dx = 9$.

68. Consider the partition P that subdivides the interval $[-1, 1]$ into n subintervals of width $\triangle x = \frac{1-(-1)}{n} = \frac{2}{n}$ and let c_k be the right endpoint of each subinterval. So the partition is $P = \{-1, -1+\frac{2}{n}, -1+2\cdot\frac{2}{n}, \ldots, -1+n\cdot\frac{2}{n} = 1\}$ and $c_k = -1+k\cdot\frac{2}{n} = -1+\frac{2k}{n}$. We get the Riemann sum $\sum_{k=1}^{n} f(c_k)\triangle x = \sum_{k=1}^{n} c_k^3\left(\frac{2}{n}\right) = \frac{2}{n}\sum_{k=1}^{n}\left(-1+\frac{2k}{n}\right)^3$
$= \frac{2}{n}\sum_{k=1}^{n}\left(-1+\frac{6k}{n} - \frac{12k^2}{n^2} + \frac{8k^3}{n^3}\right) = \frac{2}{n}\left(-\sum_{k=1}^{n}1 + \frac{6}{n}\sum_{k=1}^{n}k - \frac{12}{n^2}\sum_{k=1}^{n}k^2 + \frac{8}{n^3}\sum_{k=1}^{n}k^3\right)$
$= -\frac{2}{n}\cdot n + \frac{12}{n^2}\cdot\frac{n(n+1)}{2} - \frac{24}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{16}{n^4}\cdot\left(\frac{n(n+1)}{2}\right)^2 = -2 + 6\cdot\frac{n+1}{n} - 4\cdot\frac{(n+1)(2n+1)}{n^2} + 4\cdot\frac{(n+1)^2}{n^2}$
$= -2 + 6\cdot\frac{1+\frac{1}{n}}{1} - 4\cdot\frac{2+\frac{3}{n}+\frac{1}{n^2}}{1} + 4\cdot\frac{1+\frac{2}{n}+\frac{1}{n^2}}{1}$. As $n \to \infty$ and $\|P\| \to 0$ this expression has value $-2+6-8+4 = 0$. Thus, $\int_{-1}^{1}x^3dx = 0$.

69. Consider the partition P that subdivides the interval $[a, b]$ into n subintervals of width $\triangle x = \frac{b-a}{n}$ and let c_k be the right endpoint of each subinterval. So the partition is $P = \{a, a+\frac{b-a}{n}, a+\frac{2(b-a)}{n}, \ldots, a+\frac{n(b-a)}{n} = b\}$ and $c_k = a + \frac{k(b-a)}{n}$. We get the Riemann sum $\sum_{k=1}^{n} f(c_k)\triangle x = \sum_{k=1}^{n} c_k^3\left(\frac{b-a}{n}\right) = \frac{b-a}{n}\sum_{k=1}^{n}\left(a+\frac{k(b-a)}{n}\right)^3$
$= \frac{b-a}{n}\sum_{k=1}^{n}\left(a^3 + \frac{3a^2k(b-a)}{n} + \frac{3ak^2(b-a)^2}{n^2} + \frac{k^3(b-a)^3}{n^3}\right) = \frac{b-a}{n}\left(\sum_{k=1}^{n}a^3 + \frac{3a^2(b-a)}{n}\sum_{k=1}^{n}k + \frac{3a(b-a)^2}{n^2}\sum_{k=1}^{n}k^2 + \frac{(b-a)^3}{n^3}\sum_{k=1}^{n}k^3\right)$
$= \frac{b-a}{n}\cdot na^3 + \frac{3a^2(b-a)^2}{n^2}\cdot\frac{n(n+1)}{2} + \frac{3a(b-a)^3}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{(b-a)^4}{n^4}\cdot\left(\frac{n(n+1)}{2}\right)^2$
$= (b-a)a^3 + \frac{3a^2(b-a)^2}{2}\cdot\frac{n+1}{n} + \frac{a(b-a)^3}{2}\cdot\frac{(n+1)(2n+1)}{n^2} + \frac{(b-a)^4}{4}\cdot\frac{(n+1)^2}{n^2}$
$= (b-a)a^3 + \frac{3a^2(b-a)^2}{2}\cdot\frac{1+\frac{1}{n}}{1} + \frac{a(b-a)^3}{2}\cdot\frac{2+\frac{3}{n}+\frac{1}{n^2}}{1} + \frac{(b-a)^4}{4}\cdot\frac{1+\frac{2}{n}+\frac{1}{n^2}}{1}$. As $n \to \infty$ and $\|P\| \to 0$ this expression has value $(b-a)a^3 + \frac{3a^2(b-a)^2}{2} + a(b-a)^3 + \frac{(b-a)^4}{4} = \frac{b^4}{4} - \frac{a^4}{4}$. Thus, $\int_{a}^{b}x^3dx = \frac{b^4}{4} - \frac{a^4}{4}$.

70. Consider the partition P that subdivides the interval $[0, 1]$ into n subintervals of width $\triangle x = \frac{1-0}{n} = \frac{1}{n}$ and let c_k be the right endpoint of each subinterval. So the partition is $P = \{0, 0+\frac{1}{n}, 0+2\cdot\frac{1}{n}, \ldots, 0+n\cdot\frac{1}{n} = 1\}$ and $c_k = 0 + k\cdot\frac{1}{n} = \frac{k}{n}$. We get the Riemann sum $\sum_{k=1}^{n} f(c_k)\triangle x = \sum_{k=1}^{n}(3c_k - c_k^3)\left(\frac{1}{n}\right) = \frac{1}{n}\sum_{k=1}^{n}\left(3\cdot\frac{k}{n} - \left(\frac{k}{n}\right)^3\right) = \frac{1}{n}\left(\frac{3}{n}\sum_{k=1}^{n}k - \frac{1}{n^3}\sum_{k=1}^{n}k^3\right)$

$= \frac{3}{n^2} \cdot \frac{n(n+1)}{2} - \frac{1}{n^4} \cdot \left(\frac{n(n+1)}{2}\right)^2 = \frac{3}{2} \cdot \frac{n+1}{n} - \frac{1}{4} \cdot \frac{(n+1)^2}{n^2} = \frac{3}{2} \cdot \frac{1+\frac{1}{n}}{1} - \frac{1}{4} \cdot \frac{1+\frac{2}{n}+\frac{1}{n^2}}{1}$. As $n \to \infty$ and $\|P\| \to 0$ this expression has value $\frac{3}{2} - \frac{1}{4} = \frac{5}{4}$. Thus, $\int_0^1 (3x - x^3)dx = \frac{5}{4}$.

71. To find where $x - x^2 \ge 0$, let $x - x^2 = 0 \Rightarrow x(1-x) = 0 \Rightarrow x = 0$ or $x = 1$. If $0 < x < 1$, then $0 < x - x^2 \Rightarrow a = 0$ and $b = 1$ maximize the integral.

72. To find where $x^4 - 2x^2 \le 0$, let $x^4 - 2x^2 = 0 \Rightarrow x^2(x^2 - 2) = 0 \Rightarrow x = 0$ or $x = \pm\sqrt{2}$. By the sign graph, $+++++\underset{-\sqrt{2}}{0}--\underset{0}{0}--\underset{\sqrt{2}}{0}+++++++$, we can see that $x^4 - 2x^2 \le 0$ on $\left[-\sqrt{2}, \sqrt{2}\right] \Rightarrow a = -\sqrt{2}$ and $b = \sqrt{2}$ minimize the integral.

73. $f(x) = \frac{1}{1+x^2}$ is decreasing on $[0,1] \Rightarrow$ maximum value of f occurs at $0 \Rightarrow \max f = f(0) = 1$; minimum value of f occurs at $1 \Rightarrow \min f = f(1) = \frac{1}{1+1^2} = \frac{1}{2}$. Therefore, $(1-0)\min f \le \int_0^1 \frac{1}{1+x^2}\,dx \le (1-0)\max f \Rightarrow \frac{1}{2} \le \int_0^1 \frac{1}{1+x^2}\,dx \le 1$. That is, an upper bound $= 1$ and a lower bound $= \frac{1}{2}$.

74. See Exercise 73 above. On $[0, 0.5]$, $\max f = \frac{1}{1+0^2} = 1$, $\min f = \frac{1}{1+(0.5)^2} = 0.8$. Therefore $(0.5-0)\min f \le \int_0^{0.5} f(x)\,dx \le (0.5-0)\max f \Rightarrow \frac{2}{5} \le \int_0^{0.5} \frac{1}{1+x^2}\,dx \le \frac{1}{2}$. On $[0.5, 1]$, $\max f = \frac{1}{1+(0.5)^2} = 0.8$ and $\min f = \frac{1}{1+1^2} = 0.5$. Therefore $(1-0.5)\min f \le \int_{0.5}^1 \frac{1}{1+x^2}\,dx \le (1-0.5)\max f \Rightarrow \frac{1}{4} \le \int_{0.5}^1 \frac{1}{1+x^2}\,dx \le \frac{2}{5}$. Then $\frac{1}{4} + \frac{2}{5} \le \int_0^{0.5} \frac{1}{1+x^2}\,dx + \int_{0.5}^1 \frac{1}{1+x^2}\,dx \le \frac{1}{2} + \frac{2}{5} \Rightarrow \frac{13}{20} \le \int_0^1 \frac{1}{1+x^2}\,dx \le \frac{9}{10}$.

75. $-1 \le \sin(x^2) \le 1$ for all $x \Rightarrow (1-0)(-1) \le \int_0^1 \sin(x^2)\,dx \le (1-0)(1)$ or $\int_0^1 \sin x^2\,dx \le 1 \Rightarrow \int_0^1 \sin x^2\,dx$ cannot equal 2.

76. $f(x) = \sqrt{x+8}$ is increasing on $[0,1] \Rightarrow \max f = f(1) = \sqrt{1+8} = 3$ and $\min f = f(0) = \sqrt{0+8} = 2\sqrt{2}$. Therefore, $(1-0)\min f \le \int_0^1 \sqrt{x+8}\,dx \le (1-0)\max f \Rightarrow 2\sqrt{2} \le \int_0^1 \sqrt{x+8}\,dx \le 3$.

77. If $f(x) \ge 0$ on $[a,b]$, then $\min f \ge 0$ and $\max f \ge 0$ on $[a,b]$. Now, $(b-a)\min f \le \int_a^b f(x)\,dx \le (b-a)\max f$. Then $b \ge a \Rightarrow b - a \ge 0 \Rightarrow (b-a)\min f \ge 0 \Rightarrow \int_a^b f(x)\,dx \ge 0$.

78. If $f(x) \le 0$ on $[a,b]$, then $\min f \le 0$ and $\max f \le 0$. Now, $(b-a)\min f \le \int_a^b f(x)\,dx \le (b-a)\max f$. Then $b \ge a \Rightarrow b - a \ge 0 \Rightarrow (b-a)\max f \le 0 \Rightarrow \int_a^b f(x)\,dx \le 0$.

79. $\sin x \le x$ for $x \ge 0 \Rightarrow \sin x - x \le 0$ for $x \ge 0 \Rightarrow \int_0^1 (\sin x - x)\,dx \le 0$ (see Exercise 78) $\Rightarrow \int_0^1 \sin x\,dx - \int_0^1 x\,dx \le 0 \Rightarrow \int_0^1 \sin x\,dx \le \int_0^1 x\,dx \Rightarrow \int_0^1 \sin x\,dx \le \left(\frac{1^2}{2} - \frac{0^2}{2}\right) \Rightarrow \int_0^1 \sin x\,dx \le \frac{1}{2}$. Thus an upper bound is $\frac{1}{2}$.

80. $\sec x \ge 1 + \frac{x^2}{2}$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow \sec x - \left(1 + \frac{x^2}{2}\right) \ge 0$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow \int_0^1 \left[\sec x - \left(1 + \frac{x^2}{2}\right)\right] dx \ge 0$ (see Exercise 77) since $[0, 1]$ is contained in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow \int_0^1 \sec x\, dx - \int_0^1 \left(1 + \frac{x^2}{2}\right) dx \ge 0 \Rightarrow \int_0^1 \sec x\, dx \ \ge \int_0^1 \left(1 + \frac{x^2}{2}\right)$ $\Rightarrow \int_0^1 \sec x\, dx \ge \int_0^1 1\, dx + \frac{1}{2}\int_0^1 x^2\, dx \Rightarrow \int_0^1 \sec x\, dx \ge (1-0) + \frac{1}{2}\left(\frac{1^3}{3}\right) \Rightarrow \int_0^1 \sec x\, dx \ge \frac{7}{6}$. Thus a lower bound is $\frac{7}{6}$.

81. Yes, for the following reasons: $\text{av}(f) = \frac{1}{b-a}\int_a^b f(x)\, dx$ is a constant K. Thus $\int_a^b \text{av}(f)\, dx = \int_a^b K\, dx = K(b-a)$ $\Rightarrow \int_a^b \text{av}(f)\, dx = (b-a)K = (b-a)\cdot\frac{1}{b-a}\int_a^b f(x)\, dx = \int_a^b f(x)\, dx.$

82. All three rules hold. The reasons: On any interval $[a, b]$ on which f and g are integrable, we have:

(a) $\text{av}(f+g) = \frac{1}{b-a}\int_a^b [f(x)+g(x)]\, dx = \frac{1}{b-a}\left[\int_a^b f(x)\, dx + \int_a^b g(x)\, dx\right] = \frac{1}{b-a}\int_a^b f(x)\, dx + \frac{1}{b-a}\int_a^b g(x)\, dx$ $= \text{av}(f) + \text{av}(g)$

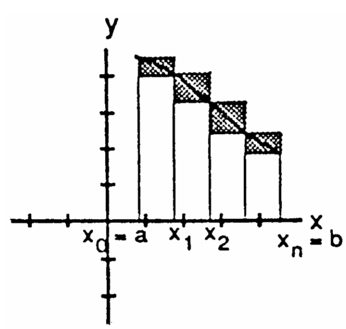
(b) $\text{av}(kf) = \frac{1}{b-a}\int_a^b kf(x)\, dx = \frac{1}{b-a}\left[k\int_a^b f(x)\, dx\right] = k\left[\frac{1}{b-a}\int_a^b f(x)\, dx\right] = k\,\text{av}(f)$

(c) $\text{av}(f) = \frac{1}{b-a}\int_a^b f(x)\, dx \le \frac{1}{b-a}\int_a^b g(x)\, dx$ since $f(x) \le g(x)$ on $[a, b]$, and $\frac{1}{b-a}\int_a^b g(x)\, dx = \text{av}(g)$. Therefore, $\text{av}(f) \le \text{av}(g)$.

83. (a) $U = \max_1 \Delta x + \max_2 \Delta x + \ldots + \max_n \Delta x$ where $\max_1 = f(x_1), \max_2 = f(x_2), \ldots, \max_n = f(x_n)$ since f is increasing on $[a, b]$; $L = \min_1 \Delta x + \min_2 \Delta x + \ldots + \min_n \Delta x$ where $\min_1 = f(x_0), \min_2 = f(x_1), \ldots,$ $\min_n = f(x_{n-1})$ since f is increasing on $[a, b]$. Therefore
$U - L = (\max_1 - \min_1)\Delta x + (\max_2 - \min_2)\Delta x + \ldots + (\max_n - \min_n)\Delta x$
$= (f(x_1) - f(x_0))\Delta x + (f(x_2) - f(x_1))\Delta x + \ldots + (f(x_n) - f(x_{n-1}))\Delta x = (f(x_n) - f(x_0))\Delta x = (f(b) - f(a))\Delta x.$

(b) $U = \max_1 \Delta x_1 + \max_2 \Delta x_2 + \ldots + \max_n \Delta x_n$ where $\max_1 = f(x_1), \max_2 = f(x_2), \ldots, \max_n = f(x_n)$ since f is increasing on$[a, b]$; $L = \min_1 \Delta x_1 + \min_2 \Delta x_2 + \ldots + \min_n \Delta x_n$ where $\min_1 = f(x_0), \min_2 = f(x_1), \ldots, \min_n = f(x_{n-1})$ since f is increasing on $[a, b]$. Therefore
$U - L = (\max_1 - \min_1)\Delta x_1 + (\max_2 - \min_2)\Delta x_2 + \ldots + (\max_n - \min_n)\Delta x_n$
$= (f(x_1) - f(x_0))\Delta x_1 + (f(x_2) - f(x_1))\Delta x_2 + \ldots + (f(x_n) - f(x_{n-1}))\Delta x_n$
$\le (f(x_1) - f(x_0))\Delta x_{max} + (f(x_2) - f(x_1))\Delta x_{max} + \ldots + (f(x_n) - f(x_{n-1}))\Delta x_{max}$. Then
$U - L \le (f(x_n) - f(x_0))\Delta x_{max} = (f(b) - f(a))\Delta x_{max} = |f(b) - f(a)|\,\Delta x_{max}$ since $f(b) \ge f(a)$. Thus
$\lim_{\|P\| \to 0} (U - L) = \lim_{\|P\| \to 0} (f(b) - f(a))\Delta x_{max} = 0$, since $\Delta x_{max} = \|P\|$.

84. (a) $U = \max_1 \Delta x + \max_2 \Delta x + \ldots + \max_n \Delta x$ where $\max_1 = f(x_0), \max_2 = f(x_1), \ldots, \max_n = f(x_{n-1})$ since f is decreasing on $[a, b]$;
$L = \min_1 \Delta x + \min_2 \Delta x + \ldots + \min_n \Delta x$ where $\min_1 = f(x_1), \min_2 = f(x_2), \ldots, \min_n = f(x_n)$ since f is decreasing on $[a, b]$. Therefore
$U - L = (\max_1 - \min_1)\Delta x + (\max_2 - \min_2)\Delta x + \ldots + (\max_n - \min_n)\Delta x$
$= (f(x_0) - f(x_1))\Delta x + (f(x_1) - f(x_2))\Delta x + \ldots + (f(x_{n-1}) - f(x_n))\Delta x = (f(x_0) - f(x_n))\Delta x$
$= (f(a) - f(b))\Delta x.$



(b) $U = \max_1 \Delta x_1 + \max_2 \Delta x_2 + \dots + \max_n \Delta x_n$ where $\max_1 = f(x_0)$, $\max_2 = f(x_1), \dots, \max_n = f(x_{n-1})$ since f is decreasing on[a, b]; $L = \min_1 \Delta x_1 + \min_2 \Delta x_2 + \dots + \min_n \Delta x_n$ where $\min_1 = f(x_1)$, $\min_2 = f(x_2), \dots, \min_n = f(x_n)$ since f is decreasing on [a, b]. Therefore
$U - L = (\max_1 - \min_1)\,\Delta x_1 + (\max_2 - \min_2)\,\Delta x_2 + \dots + (\max_n - \min_n)\,\Delta x_n$
$= (f(x_0) - f(x_1))\,\Delta x_1 + (f(x_1) - f(x_2))\Delta x_2 + \dots + (f(x_{n-1}) - f(x_n))\,\Delta x_n$
$\le (f(x_0) - f(x_n))\,\Delta x_{max} = (f(a) - f(b)\,\Delta x_{max} = |f(b) - f(a)|\,\Delta x_{max}$ since $f(b) \le f(a)$. Thus
$\lim\limits_{\|P\| \to 0} (U - L) = \lim\limits_{\|P\| \to 0} |f(b) - f(a)|\,\Delta x_{max} = 0$, since $\Delta x_{max} = \|P\|$.

85. (a) Partition $\left[0, \frac{\pi}{2}\right]$ into n subintervals, each of length $\Delta x = \frac{\pi}{2n}$ with points $x_0 = 0$, $x_1 = \Delta x$, $x_2 = 2\Delta x, \dots, x_n = n\Delta x = \frac{\pi}{2}$. Since sin x is increasing on $\left[0, \frac{\pi}{2}\right]$, the upper sum U is the sum of the areas of the circumscribed rectangles of areas $f(x_1)\,\Delta x = (\sin \Delta x)\Delta x$, $f(x_2)\,\Delta x = (\sin 2\Delta x)\,\Delta x, \dots, f(x_n)\,\Delta x = (\sin n\Delta x)\,\Delta x$. Then $U = (\sin \Delta x + \sin 2\Delta x + \dots + \sin n\Delta x)\,\Delta x = \left[\frac{\cos \frac{\Delta x}{2} - \cos\left(\left(n + \frac{1}{2}\right)\Delta x\right)}{2 \sin \frac{\Delta x}{2}}\right] \Delta x$

$$= \left[\frac{\cos \frac{\pi}{4n} - \cos\left(\left(n + \frac{1}{2}\right)\frac{\pi}{2n}\right)}{2 \sin \frac{\pi}{4n}}\right]\left(\frac{\pi}{2n}\right) = \frac{\pi\left(\cos \frac{\pi}{4n} - \cos\left(\frac{\pi}{2} + \frac{\pi}{4n}\right)\right)}{4n \sin \frac{\pi}{4n}} = \frac{\cos \frac{\pi}{4n} - \cos\left(\frac{\pi}{2} + \frac{\pi}{4n}\right)}{\left(\frac{\sin \frac{\pi}{4n}}{\frac{\pi}{4n}}\right)}$$

(b) The area is $\int_0^{\pi/2} \sin x\, dx = \lim\limits_{n \to \infty} \frac{\cos \frac{\pi}{4n} - \cos\left(\frac{\pi}{2} + \frac{\pi}{4n}\right)}{\left(\frac{\sin \frac{\pi}{4n}}{\frac{\pi}{4n}}\right)} = \frac{1 - \cos \frac{\pi}{2}}{1} = 1.$

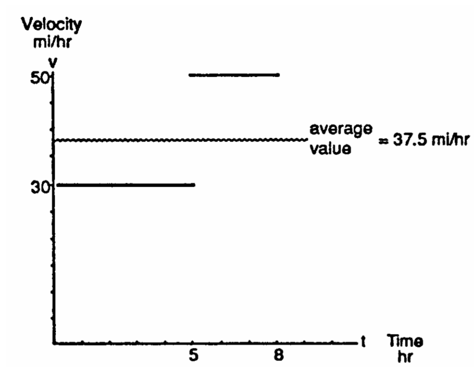
86. (a) The area of the shaded region is $\sum\limits_{i=1}^{n} \triangle x_i \cdot m_i$ which is equal to L.

(b) The area of the shaded region is $\sum\limits_{i=1}^{n} \triangle x_i \cdot M_i$ which is equal to U.

(c) The area of the shaded region is the difference in the areas of the shaded regions shown in the second part of the figure and the first part of the figure. Thus this area is $U - L$.

87. By Exercise 86, $U - L = \sum\limits_{i=1}^{n} \triangle x_i \cdot M_i - \sum\limits_{i=1}^{n} \triangle x_i \cdot m_i$ where $M_i = \max\{f(x) \text{ on the ith subinterval}\}$ and $m_i = \min\{f(x) \text{ on the ith subinterval}\}$. Thus $U - L = \sum\limits_{i=1}^{n} (M_i - m_i)\triangle x_i < \sum\limits_{i=1}^{n} \epsilon \cdot \triangle x_i$ provided $\triangle x_i < \delta$ for each $i = 1, \dots, n$. Since $\sum\limits_{i=1}^{n} \epsilon \cdot \triangle x_i = \epsilon \sum\limits_{i=1}^{n} \triangle x_i = \epsilon(b - a)$ the result, $U - L < \epsilon(b - a)$ follows.

88. The car drove the first 150 miles in 5 hours and the second 150 miles in 3 hours, which means it drove 300 miles in 8 hours, for an average of $\frac{300}{8}$ mi/hr $= 37.5$ mi/hr. In terms of average values of functions, the function whose average value we seek is $v(t) = \begin{cases} 30, & 0 \le t \le 5 \\ 50, & 5 < 1 \le 8 \end{cases}$, and the average value is $\frac{(30)(5) + (50)(3)}{8} = 37.5$.



89-94. Example CAS commands:

Maple:

```
with( plots );
with( Student[Calculus1] );
f := x -> 1-x;
a := 0;
b := 1;
N :=[ 4, 10, 20, 50 ];
P := [seq( RiemannSum( f(x), x=a..b, partition=n, method=random, output=plot ), n=N )]:
display( P, insequence=true );
```

95-98. Example CAS commands:

Maple:

```
with( Student[Calculus1] );
f := x -> sin(x);
a := 0;
b := Pi;
plot( f(x), x=a..b, title="#95(a) (Section 5.3)" );
N := [ 100, 200, 1000 ];                                   # (b)
for n in N do
  Xlist := [ a+1.*(b-a)/n*i $ i=0..n ];
  Ylist := map( f, Xlist );
end do:
for n in N do                                              # (c)
  Avg[n] := evalf(add(y,y=Ylist)/nops(Ylist));
end do;
avg := FunctionAverage( f(x), x=a..b, output=value );
evalf( avg );
FunctionAverage(f(x),x=a..b,output=plot);      # (d)
fsolve( f(x)=avg, x=0.5 );
fsolve( f(x)=avg, x=2.5 );
fsolve( f(x)=Avg[1000], x=0.5 );
fsolve( f(x)=Avg[1000], x=2.5 );
```

89-98. Example CAS commands:

Mathematica: (assigned function and values for a, b, and n may vary)

Sums of rectangles evaluated at left-hand endpoints can be represented and evaluated by this set of commands

```
Clear[x, f, a, b, n]
{a, b}={0, π}; n =10; dx = (b − a)/n;
f = Sin[x]^2;
xvals =Table[N[x], {x, a, b − dx, dx}];
yvals = f /.x → xvals;
boxes = MapThread[Line[{{#1,0},{#1, #3},{#2, #3},{#2, 0}]&,{xvals, xvals + dx, yvals}];
Plot[f, {x, a, b}, Epilog → boxes];
Sum[yvals[[i]] dx, {i, 1, Length[yvals]}]//N
```

Sums of rectangles evaluated at right-hand endpoints can be represented and evaluated by this set of commands.

```
Clear[x, f, a, b, n]
{a, b}={0, π}; n =10; dx = (b − a)/n;
f = Sin[x]^2;
```

```
xvals =Table[N[x], {x, a + dx, b, dx}];
yvals = f /.x → xvals;
boxes = MapThread[Line[{{#1,0},{#1, #3},{#2, #3},{#2, 0}]&,{xvals − dx,xvals, yvals}];
Plot[f, {x, a, b}, Epilog → boxes];
Sum[yvals[[i]] dx, {i, 1,Length[yvals]}]//N
```

Sums of rectangles evaluated at midpoints can be represented and evaluated by this set of commands.

```
Clear[x, f, a, b, n]
{a, b}={0, π}; n =10; dx = (b − a)/n;
f = Sin[x]^2;
xvals =Table[N[x], {x, a + dx/2, b − dx/2, dx}];
yvals = f /.x → xvals;
boxes = MapThread[Line[{{#1,0},{#1, #3},{#2, #3},{#2, 0}]&,{xvals − dx/2, xvals + dx/2, yvals}];
Plot[f, {x, a, b},Epilog → boxes];
Sum[yvals[[i]] dx, {i, 1, Length[yvals]}]//N
```

5.4 THE FUNDAMENTAL THEOREM OF CALCULUS

1. $\int_{-2}^{0}(2x+5)\,dx = \left[x^2+5x\right]_{-2}^{0} = \left(0^2+5(0)\right) - \left((-2)^2+5(-2)\right) = 6$

2. $\int_{-3}^{4}\left(5-\frac{x}{2}\right)dx = \left[5x-\frac{x^2}{4}\right]_{-3}^{4} = \left(5(4)-\frac{4^2}{4}\right) - \left(5(-3)-\frac{(-3)^2}{4}\right) = \frac{133}{4}$

3. $\int_{0}^{2}x(x-3)\,dx = \int_{0}^{2}(x^2-3x)\,dx = \left[\frac{x^3}{3}-\frac{3x^2}{2}\right]_{0}^{2} = \left(\frac{(2)^3}{3}-\frac{3(2)^2}{2}\right) - \left(\frac{(0)^3}{3}-\frac{3(0)^2}{2}\right) = -\frac{10}{3}$

4. $\int_{-1}^{1}(x^2-2x+3)\,dx = \left[\frac{x^3}{3}-x^2+3x\right]_{-1}^{1} = \left(\frac{(1)^3}{3}-(1)^2+3(1)\right) - \left(\frac{(-1)^3}{3}-(-1)^2+3(-1)\right) = \frac{20}{3}$

5. $\int_{0}^{4}\left(3x-\frac{x^3}{4}\right)dx = \left[\frac{3x^2}{2}-\frac{x^4}{16}\right]_{0}^{4} = \left(\frac{3(4)^2}{2}-\frac{4^4}{16}\right) - \left(\frac{3(0)^2}{2}-\frac{(0)^4}{16}\right) = 8$

6. $\int_{-2}^{2}(x^3-2x+3)\,dx = \left[\frac{x^4}{4}-x^2+3x\right]_{-2}^{2} = \left(\frac{2^4}{4}-2^2+3(2)\right) - \left(\frac{(-2)^4}{4}-(-2)^2+3(-2)\right) = 12$

7. $\int_{0}^{1}\left(x^2+\sqrt{x}\right)dx = \left[\frac{x^3}{3}+\frac{2}{3}x^{3/2}\right]_{0}^{1} = \left(\frac{1}{3}+\frac{2}{3}\right) - 0 = 1$

8. $\int_{1}^{32} x^{-6/5}\,dx = \left[-5x^{-1/5}\right]_{1}^{32} = \left(-\frac{5}{2}\right) - (-5) = \frac{5}{2}$

9. $\int_{0}^{\pi/3} 2\sec^2 x\,dx = \left[2\tan x\right]_{0}^{\pi/3} = \left(2\tan\left(\frac{\pi}{3}\right)\right) - (2\tan 0) = 2\sqrt{3} - 0 = 2\sqrt{3}$

10. $\int_{0}^{\pi}(1+\cos x)\,dx = \left[x+\sin x\right]_{0}^{\pi} = (\pi+\sin\pi) - (0+\sin 0) = \pi$

11. $\int_{\pi/4}^{3\pi/4}\csc\theta\cot\theta\,d\theta = \left[-\csc\theta\right]_{\pi/4}^{3\pi/4} = \left(-\csc\left(\frac{3\pi}{4}\right)\right) - \left(-\csc\left(\frac{\pi}{4}\right)\right) = -\sqrt{2} - \left(-\sqrt{2}\right) = 0$

12. $\int_{0}^{\pi/3} 4\sec u\tan u\,du = \left[4\sec u\right]_{0}^{\pi/3} = 4\sec\left(\frac{\pi}{3}\right) - 4\sec 0 = 4(2) - 4(1) = 4$

13. $\int_{\pi/2}^{0} \frac{1+\cos 2t}{2}\,dt = \int_{\pi/2}^{0} \left(\frac{1}{2} + \frac{1}{2}\cos 2t\right) dt = \left[\frac{1}{2}t + \frac{1}{4}\sin 2t\right]_{\pi/2}^{0} = \left(\frac{1}{2}(0) + \frac{1}{4}\sin 2(0)\right) - \left(\frac{1}{2}\left(\frac{\pi}{2}\right) + \frac{1}{4}\sin 2\left(\frac{\pi}{2}\right)\right) = -\frac{\pi}{4}$

14. $\int_{-\pi/3}^{\pi/3} \frac{1-\cos 2t}{2}\,dt = \int_{-\pi/3}^{\pi/3} \left(\frac{1}{2} - \frac{1}{2}\cos 2t\right) dt = \left[\frac{1}{2}t - \frac{1}{4}\sin 2t\right]_{-\pi/3}^{\pi/3}$
$= \left(\frac{1}{2}\left(\frac{\pi}{3}\right) - \frac{1}{4}\sin 2\left(\frac{\pi}{3}\right)\right) - \left(\frac{1}{2}\left(-\frac{\pi}{3}\right) - \frac{1}{4}\sin 2\left(-\frac{\pi}{3}\right)\right) = \frac{\pi}{6} - \frac{1}{4}\sin\frac{2\pi}{3} + \frac{\pi}{6} + \frac{1}{4}\sin\left(\frac{-2\pi}{3}\right) = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$

15. $\int_{0}^{\pi/4} \tan^2 x\,dx = \int_{0}^{\pi/4} (\sec^2 x - 1)dx = [\tan x - x]_0^{\pi/4} = \left(\tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4}\right) - (\tan(0) - 0) = 1 - \frac{\pi}{4}$

16. $\int_{0}^{\pi/6} (\sec x + \tan x)^2\,dx = \int_{0}^{\pi/6} (\sec^2 x + 2\sec x\tan x + \tan^2 x)dx = \int_{0}^{\pi/6} (2\sec^2 x + 2\sec x\tan x - 1)dx$
$= [2\tan x + 2\sec x - x]_0^{\pi/6} = \left(2\tan\left(\frac{\pi}{6}\right) + 2\sec\left(\frac{\pi}{6}\right) - \left(\frac{\pi}{6}\right)\right) - (2\tan 0 + 2\sec 0 - 0) = 2\sqrt{3} - \frac{\pi}{6} - 2$

17. $\int_{0}^{\pi/8} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/8} = \left(-\frac{1}{2}\cos 2\left(\frac{\pi}{8}\right)\right) - \left(-\frac{1}{2}\cos 2(0)\right) = \frac{2-\sqrt{2}}{4}$

18. $\int_{-\pi/3}^{-\pi/4} \left(4\sec^2 t + \frac{\pi}{t^2}\right) dt = \int_{-\pi/3}^{-\pi/4} (4\sec^2 t + \pi t^{-2})\,dt = \left[4\tan t - \frac{\pi}{t}\right]_{-\pi/3}^{-\pi/4}$
$= \left(4\tan\left(-\frac{\pi}{4}\right) - \frac{\pi}{\left(-\frac{\pi}{4}\right)}\right) - \left(4\tan\left(\frac{\pi}{3}\right) - \frac{\pi}{\left(-\frac{\pi}{3}\right)}\right) = (4(-1) + 4) - \left(4\left(-\sqrt{3}\right) + 3\right) = 4\sqrt{3} - 3$

19. $\int_{1}^{-1} (r+1)^2\,dr = \int_{1}^{-1} (r^2 + 2r + 1)\,dr = \left[\frac{r^3}{3} + r^2 + r\right]_1^{-1} = \left(\frac{(-1)^3}{3} + (-1)^2 + (-1)\right) - \left(\frac{1^3}{3} + 1^2 + 1\right) = -\frac{8}{3}$

20. $\int_{-\sqrt{3}}^{\sqrt{3}} (t+1)(t^2+4)\,dt = \int_{-\sqrt{3}}^{\sqrt{3}} (t^3 + t^2 + 4t + 4)\,dt = \left[\frac{t^4}{4} + \frac{t^3}{3} + 2t^2 + 4t\right]_{-\sqrt{3}}^{\sqrt{3}}$
$= \left(\frac{\left(\sqrt{3}\right)^4}{4} + \frac{\left(\sqrt{3}\right)^3}{3} + 2\left(\sqrt{3}\right)^2 + 4\sqrt{3}\right) - \left(\frac{\left(-\sqrt{3}\right)^4}{4} + \frac{\left(-\sqrt{3}\right)^3}{3} + 2\left(-\sqrt{3}\right)^2 + 4\left(-\sqrt{3}\right)\right) = 10\sqrt{3}$

21. $\int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - \frac{1}{u^5}\right) du = \int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - u^{-5}\right) du = \left[\frac{u^8}{16} + \frac{1}{4u^4}\right]_{\sqrt{2}}^{1} = \left(\frac{1^8}{16} + \frac{1}{4(1)^4}\right) - \left(\frac{\left(\sqrt{2}\right)^8}{16} + \frac{1}{4\left(\sqrt{2}\right)^4}\right) = -\frac{3}{4}$

22. $\int_{-3}^{-1} \frac{y^5 - 2y}{y^3}\,dy = \int_{-3}^{-1} (y^2 - 2y^{-2})\,dy = \left[\frac{y^3}{3} + 2y^{-1}\right]_{-3}^{-1} = \left(\frac{(-1)^3}{3} + \frac{2}{(-1)}\right) - \left(\frac{(-3)^3}{3} + \frac{2}{(-3)}\right) = \frac{22}{3}$

23. $\int_{1}^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2}\,ds = \int_{1}^{\sqrt{2}} \left(1 + s^{-3/2}\right) ds = \left[s - \frac{2}{\sqrt{s}}\right]_1^{\sqrt{2}} = \left(\sqrt{2} - \frac{2}{\sqrt{\sqrt{2}}}\right) - \left(1 - \frac{2}{\sqrt{1}}\right) = \sqrt{2} - 2^{3/4} + 1$
$= \sqrt{2} - \sqrt[4]{8} + 1$

24. $\int_{1}^{8} \frac{\left(x^{1/3}+1\right)\left(2 - x^{2/3}\right)}{x^{1/3}}\,dx = \int_{1}^{8} \frac{2x^{1/3} - x + 2 - x^{2/3}}{x^{1/3}}\,dx = \int_{1}^{8} \left(2 - x^{2/3} + 2x^{-1/3} - x^{1/3}\right) dx =$
$\left[2x - \frac{3}{5}x^{5/3} + 3x^{2/3} - \frac{3}{4}x^{4/3}\right]_1^3 = \left(2(8) - \frac{3}{5}(8)^{5/3} + 3(8)^{2/3} - \frac{3}{4}(8)^{4/3}\right) - \left(2(1) - \frac{3}{5}(1)^{5/3} + 3(1)^{2/3} - \frac{3}{4}(1)^{4/3}\right)$
$= -\frac{137}{20}$

25. $\int_{\pi/2}^{\pi} \frac{\sin 2x}{2\sin x}\,dx = \int_{\pi/2}^{\pi} \frac{2\sin x\cos x}{2\sin x}\,dx = \int_{\pi/2}^{\pi} \cos x\,dx = \left[\sin x\right]_{\pi/2}^{\pi} = (\sin(\pi)) - \left(\sin\left(\frac{\pi}{2}\right)\right) = -1$

26. $\int_0^{\pi/3} (\cos x + \sec x)^2\, dx = \int_0^{\pi/3} (\cos^2 x + 2 + \sec^2 x) dx = \int_0^{\pi/3} \left(\frac{\cos 2x + 1}{2} + 2 + \sec^2 x\right) dx$
$= \int_0^{\pi/3} \left(\frac{1}{2}\cos 2x + \frac{5}{2} + \sec^2 x\right) dx = \left[\frac{1}{4}\sin 2x + \frac{5}{2}x + \tan x\right]_0^{\pi/3}$
$= \left(\frac{1}{4}\sin 2\left(\frac{\pi}{3}\right) + \frac{5}{2}\left(\frac{\pi}{3}\right) + \tan\left(\frac{\pi}{3}\right)\right) - \left(\frac{1}{4}\sin 2(0) + \frac{5}{2}(0) + \tan(0)\right) = \frac{5\pi}{6} + \frac{9\sqrt{3}}{8}$

27. $\int_{-4}^{4} |x|\, dx = \int_{-4}^{0} |x|\, dx + \int_0^4 |x|\, dx = -\int_{-4}^{0} x\, dx + \int_0^4 x\, dx = \left[-\frac{x^2}{2}\right]_{-4}^{0} + \left[\frac{x^2}{2}\right]_0^4 = \left(-\frac{0^2}{2} + \frac{(-4)^2}{2}\right) + \left(\frac{4^2}{2} - \frac{0^2}{2}\right) = 16$

28. $\int_0^{\pi} \frac{1}{2}(\cos x + |\cos x|)\, dx = \int_0^{\pi/2} \frac{1}{2}(\cos x + \cos x)\, dx + \int_{\pi/2}^{\pi} \frac{1}{2}(\cos x - \cos x)\, dx = \int_0^{\pi/2} \cos x\, dx = [\sin x]_0^{\pi/2}$
$= \sin\frac{\pi}{2} - \sin 0 = 1$

29. (a) $\int_0^{\sqrt{x}} \cos t\, dt = [\sin t]_0^{\sqrt{x}} = \sin\sqrt{x} - \sin 0 = \sin\sqrt{x} \Rightarrow \frac{d}{dx}\left(\int_0^{\sqrt{x}} \cos t\, dt\right) = \frac{d}{dx}\left(\sin\sqrt{x}\right) = \cos\sqrt{x}\left(\frac{1}{2}x^{-1/2}\right)$
$= \frac{\cos\sqrt{x}}{2\sqrt{x}}$

(b) $\frac{d}{dx}\left(\int_0^{\sqrt{x}} \cos t\, dt\right) = \left(\cos\sqrt{x}\right)\left(\frac{d}{dx}\left(\sqrt{x}\right)\right) = \left(\cos\sqrt{x}\right)\left(\frac{1}{2}x^{-1/2}\right) = \frac{\cos\sqrt{x}}{2\sqrt{x}}$

30. (a) $\int_1^{\sin x} 3t^2\, dt = [t^3]_1^{\sin x} = \sin^3 x - 1 \Rightarrow \frac{d}{dx}\left(\int_1^{\sin x} 3t^2\, dt\right) = \frac{d}{dx}(\sin^3 x - 1) = 3\sin^2 x \cos x$

(b) $\frac{d}{dx}\left(\int_1^{\sin x} 3t^2\, dt\right) = (3\sin^2 x)\left(\frac{d}{dx}(\sin x)\right) = 3\sin^2 x\cos x$

31. (a) $\int_0^{t^4} \sqrt{u}\, du = \int_0^{t^4} u^{1/2}\, du = \left[\frac{2}{3}u^{3/2}\right]_0^{t^4} = \frac{2}{3}(t^4)^{3/2} - 0 = \frac{2}{3}t^6 \Rightarrow \frac{d}{dt}\left(\int_0^{t^4} \sqrt{u}\, du\right) = \frac{d}{dt}\left(\frac{2}{3}t^6\right) = 4t^5$

(b) $\frac{d}{dt}\left(\int_0^{t^4} \sqrt{u}\, du\right) = \sqrt{t^4}\left(\frac{d}{dt}(t^4)\right) = t^2(4t^3) = 4t^5$

32. (a) $\int_0^{\tan\theta} \sec^2 y\, dy = [\tan y]_0^{\tan\theta} = \tan(\tan\theta) - 0 = \tan(\tan\theta) \Rightarrow \frac{d}{d\theta}\left(\int_0^{\tan\theta} \sec^2 y\, dy\right) = \frac{d}{d\theta}(\tan(\tan\theta))$
$= (\sec^2(\tan\theta))\sec^2\theta$

(b) $\frac{d}{d\theta}\left(\int_0^{\tan\theta} \sec^2 y\, dy\right) = (\sec^2(\tan\theta))\left(\frac{d}{d\theta}(\tan\theta)\right) = (\sec^2(\tan\theta))\sec^2\theta$

33. $y = \int_0^x \sqrt{1+t^2}\, dt \Rightarrow \frac{dy}{dx} = \sqrt{1+x^2}$

34. $y = \int_1^x \frac{1}{t}\, dt \Rightarrow \frac{dy}{dx} = \frac{1}{x},\ x > 0$

35. $y = \int_{\sqrt{x}}^{0} \sin t^2\, dt = -\int_0^{\sqrt{x}} \sin t^2\, dt \Rightarrow \frac{dy}{dx} = -\left(\sin\left(\sqrt{x}\right)^2\right)\left(\frac{d}{dx}\left(\sqrt{x}\right)\right) = -(\sin x)\left(\frac{1}{2}x^{-1/2}\right) = -\frac{\sin x}{2\sqrt{x}}$

36. $y = x\int_2^{x^2} \sin t^3\, dt \Rightarrow \frac{dy}{dx} = x \cdot \frac{d}{dx}\left(\int_2^{x^2} \sin t^3\, dt\right) + 1 \cdot \int_2^{x^2} \sin t^3\, dt = x \cdot \sin(x^2)^3 \frac{d}{dx}(x^2) + \int_2^{x^2} \sin t^3\, dt$
$= 2x^2 \sin x^6 + \int_2^{x^2} \sin t^3\, dt$

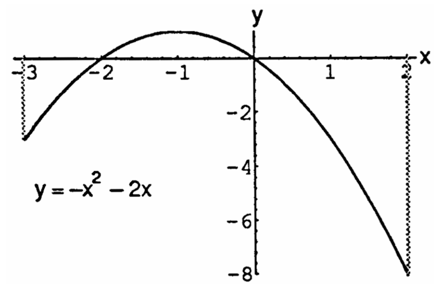
37. $y = \int_{-1}^{x} \frac{t^2}{t^2+4}\, dt - \int_3^x \frac{t^2}{t^2+4}\, dt \Rightarrow \frac{dy}{dx} = \frac{x^2}{x^2+4} - \frac{x^2}{x^2+4} = 0$

38. $y = \left(\int_0^x (t^3+1)^{10} dt\right)^3 \Rightarrow \frac{dy}{dx} = 3\left(\int_0^x (t^3+1)^{10} dt\right) \frac{d}{dx}\left(\int_0^x (t^3+1)^{10} dt\right) = 3(x^3+1)^{10}\left(\int_0^x (t^3+1)^{10} dt\right)$

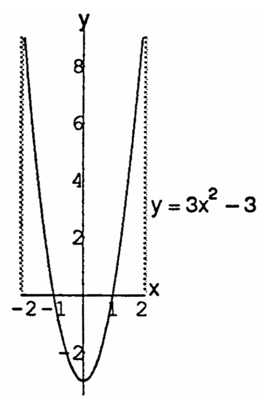
39. $y = \int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}}, |x| < \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-\sin^2 x}}\left(\frac{d}{dx}(\sin x)\right) = \frac{1}{\sqrt{\cos^2 x}}(\cos x) = \frac{\cos x}{|\cos x|} = \frac{\cos x}{\cos x} = 1$ since $|x| < \frac{\pi}{2}$

40. $y = \int_0^{\tan x} \frac{dt}{1+t^2} \Rightarrow \frac{dy}{dx} = \left(\frac{1}{1+\tan^2 x}\right)\left(\frac{d}{dx}(\tan x)\right) = \left(\frac{1}{\sec^2 x}\right)(\sec^2 x) = 1$

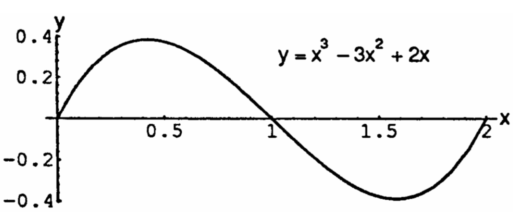
41. $-x^2 - 2x = 0 \Rightarrow -x(x+2) = 0 \Rightarrow x = 0$ or $x = -2$; Area
$= -\int_{-3}^{-2}(-x^2-2x)dx + \int_{-2}^{0}(-x^2-2x)dx - \int_0^2(-x^2-2x)dx$
$= -\left[-\frac{x^3}{3} - x^2\right]_{-3}^{-2} + \left[-\frac{x^3}{3} - x^2\right]_{-2}^{0} - \left[-\frac{x^3}{3} - x^2\right]_0^2$
$= -\left(\left(-\frac{(-2)^3}{3} - (-2)^2\right) - \left(-\frac{(-3)^3}{3} - (-3)^2\right)\right)$
$+ \left(\left(-\frac{0^3}{3} - 0^2\right) - \left(-\frac{(-2)^3}{3} - (-2)^2\right)\right)$
$- \left(\left(-\frac{2^3}{3} - 2^2\right) - \left(-\frac{0^3}{3} - 0^2\right)\right) = \frac{28}{3}$



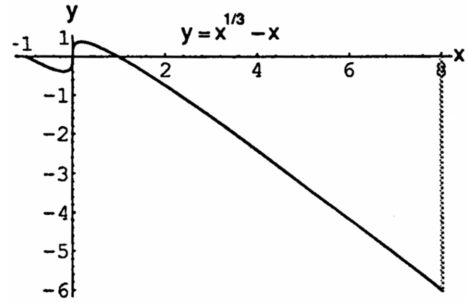
42. $3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$; because of symmetry about the y-axis, Area $= 2\left(-\int_0^1 (3x^2-3)dx + \int_1^2 (3x^2-3)dx\right)$
$2\left(-[x^3-3x]_0^1 + [x^3-3x]_1^2\right) = 2[-((1^3-3(1)) - (0^3-3(0)))$
$+ ((2^3-3(2)) - (1^3-3(1))] = 2(6) = 12$



43. $x^3 - 3x^2 + 2x = 0 \Rightarrow x(x^2 - 3x + 2) = 0$
$\Rightarrow x(x-2)(x-1) = 0 \Rightarrow x = 0, 1,$ or 2;
Area $= \int_0^1 (x^3-3x^2+2x)dx - \int_1^2 (x^3-3x^2+2x)dx$
$= \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 - \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2$
$= \left(\frac{1^4}{4} - 1^3 + 1^2\right) - \left(\frac{0^4}{4} - 0^3 + 0^2\right)$
$- \left[\left(\frac{2^4}{4} - 2^3 + 2^2\right) - \left(\frac{1^4}{4} - 1^3 + 1^2\right)\right] = \frac{1}{2}$



44. $x^{1/3} - x = 0 \Rightarrow x^{1/3}\left(1 - x^{2/3}\right) = 0 \Rightarrow x^{1/3} = 0$ or $1 - x^{2/3} = 0 \Rightarrow x = 0$ or $1 = x^{2/3} \Rightarrow x = 0$ or $1 = x^2 \Rightarrow x = 0$ or ± 1;



$$\text{Area} = -\int_{-1}^{0}\left(x^{1/3} - x\right)dx + \int_{0}^{1}\left(x^{1/3} - x\right)dx - \int_{1}^{8}\left(x^{1/3} - x\right)dx$$
$$= -\left[\tfrac{3}{4}x^{4/3} - \tfrac{x^2}{2}\right]_{-1}^{0} + \left[\tfrac{3}{4}x^{4/3} - \tfrac{x^2}{2}\right]_{0}^{1} - \left[\tfrac{3}{4}x^{4/3} - \tfrac{x^2}{2}\right]_{1}^{8}$$
$$= -\left[\left(\tfrac{3}{4}(0)^{4/3} - \tfrac{0^2}{2}\right) - \left(\tfrac{3}{4}(-1)^{4/3} - \tfrac{(-1)^2}{2}\right)\right]$$
$$+ \left[\left(\tfrac{3}{4}(1)^{4/3} - \tfrac{1^2}{2}\right) - \left(\tfrac{3}{4}(0)^{4/3} - \tfrac{0^2}{2}\right)\right]$$
$$- \left[\left(\tfrac{3}{4}(8)^{4/3} - \tfrac{8^2}{2}\right) - \left(\tfrac{3}{4}(1)^{4/3} - \tfrac{1^2}{2}\right)\right]$$
$$= \tfrac{1}{4} + \tfrac{1}{4} - \left(-20 - \tfrac{3}{4} + \tfrac{1}{2}\right) = \tfrac{83}{4}$$

45. The area of the rectangle bounded by the lines $y = 2$, $y = 0$, $x = \pi$, and $x = 0$ is 2π. The area under the curve $y = 1 + \cos x$ on $[0, \pi]$ is $\int_0^{\pi}(1 + \cos x)\,dx = [x + \sin x]_0^{\pi} = (\pi + \sin\pi) - (0 + \sin 0) = \pi$. Therefore the area of the shaded region is $2\pi - \pi = \pi$.

46. The area of the rectangle bounded by the lines $x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$, $y = \sin\frac{\pi}{6} = \frac{1}{2} = \sin\frac{5\pi}{6}$, and $y = 0$ is $\frac{1}{2}\left(\frac{5\pi}{6} - \frac{\pi}{6}\right) = \frac{\pi}{3}$. The area under the curve $y = \sin x$ on $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$ is $\int_{\pi/6}^{5\pi/6}\sin x\,dx = [-\cos x]_{\pi/6}^{5\pi/6}$ $= \left(-\cos\frac{5\pi}{6}\right) - \left(-\cos\frac{\pi}{6}\right) = -\left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2} = \sqrt{3}$. Therefore the area of the shaded region is $\sqrt{3} - \frac{\pi}{3}$.

47. On $\left[-\frac{\pi}{4}, 0\right]$: The area of the rectangle bounded by the lines $y = \sqrt{2}$, $y = 0$, $\theta = 0$, and $\theta = -\frac{\pi}{4}$ is $\sqrt{2}\left(\frac{\pi}{4}\right)$ $= \frac{\pi\sqrt{2}}{4}$. The area between the curve $y = \sec\theta\tan\theta$ and $y = 0$ is $-\int_{-\pi/4}^{0}\sec\theta\tan\theta\,d\theta = [-\sec\theta]_{-\pi/4}^{0}$ $= (-\sec 0) - \left(-\sec\left(-\frac{\pi}{4}\right)\right) = \sqrt{2} - 1$. Therefore the area of the shaded region on $\left[-\frac{\pi}{4}, 0\right]$ is $\frac{\pi\sqrt{2}}{4} + \left(\sqrt{2} - 1\right)$. On $\left[0, \frac{\pi}{4}\right]$: The area of the rectangle bounded by $\theta = \frac{\pi}{4}$, $\theta = 0$, $y = \sqrt{2}$, and $y = 0$ is $\sqrt{2}\left(\frac{\pi}{4}\right) = \frac{\pi\sqrt{2}}{4}$. The area under the curve $y = \sec\theta\tan\theta$ is $\int_0^{\pi/4}\sec\theta\tan\theta\,d\theta = [\sec\theta]_0^{\pi/4} = \sec\frac{\pi}{4} - \sec 0 = \sqrt{2} - 1$. Therefore the area of the shaded region on $\left[0, \frac{\pi}{4}\right]$ is $\frac{\pi\sqrt{2}}{4} - \left(\sqrt{2} - 1\right)$. Thus, the area of the total shaded region is $\left(\frac{\pi\sqrt{2}}{4} + \sqrt{2} - 1\right) + \left(\frac{\pi\sqrt{2}}{4} - \sqrt{2} + 1\right) = \frac{\pi\sqrt{2}}{2}$.

48. The area of the rectangle bounded by the lines $y = 2$, $y = 0$, $t = -\frac{\pi}{4}$, and $t = 1$ is $2\left(1 - \left(-\frac{\pi}{4}\right)\right) = 2 + \frac{\pi}{2}$. The area under the curve $y = \sec^2 t$ on $\left[-\frac{\pi}{4}, 0\right]$ is $\int_{-\pi/4}^{0}\sec^2 t\,dt = [\tan t]_{-\pi/4}^{0} = \tan 0 - \tan\left(-\frac{\pi}{4}\right) = 1$. The area under the curve $y = 1 - t^2$ on $[0, 1]$ is $\int_0^1 (1 - t^2)\,dt = \left[t - \frac{t^3}{3}\right]_0^1 = \left(1 - \frac{1^3}{3}\right) - \left(0 - \frac{0^3}{3}\right) = \frac{2}{3}$. Thus, the total area under the curves on $\left[-\frac{\pi}{4}, 1\right]$ is $1 + \frac{2}{3} = \frac{5}{3}$. Therefore the area of the shaded region is $\left(2 + \frac{\pi}{2}\right) - \frac{5}{3} = \frac{1}{3} + \frac{\pi}{2}$.

49. $y = \int_{\pi}^{x}\frac{1}{t}\,dt - 3 \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ and $y(\pi) = \int_{\pi}^{\pi}\frac{1}{t}\,dt - 3 = 0 - 3 = -3 \Rightarrow$ (d) is a solution to this problem.

50. $y = \int_{-1}^{x}\sec t\,dt + 4 \Rightarrow \frac{dy}{dx} = \sec x$ and $y(-1) = \int_{-1}^{-1}\sec t\,dt + 4 = 0 + 4 = 4 \Rightarrow$ (c) is a solution to this problem.

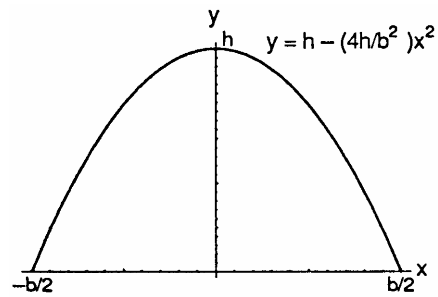
51. $y = \int_{0}^{x}\sec t\,dt + 4 \Rightarrow \frac{dy}{dx} = \sec x$ and $y(0) = \int_{0}^{0}\sec t\,dt + 4 = 0 + 4 = 4 \Rightarrow$ (b) is a solution to this problem.

52. $y = \int_1^x \frac{1}{t}\,dt - 3 \Rightarrow \frac{dy}{dx} = \frac{1}{x}$ and $y(1) = \int_1^1 \frac{1}{t}\,dt - 3 = 0 - 3 = -3 \Rightarrow$ (a) is a solution to this problem.

53. $y = \int_2^x \sec t\,dt + 3$

54. $y = \int_1^x \sqrt{1+t^2}\,dt - 2$

55. Area $= \int_{-b/2}^{b/2} \left(h - \left(\frac{4h}{b^2}\right)x^2\right) dx = \left[hx - \frac{4hx^3}{3b^2}\right]_{-b/2}^{b/2}$
$= \left(h\left(\frac{b}{2}\right) - \frac{4h\left(\frac{b}{2}\right)^3}{3b^2}\right) - \left(h\left(-\frac{b}{2}\right) - \frac{4h\left(-\frac{b}{2}\right)^3}{3b^2}\right)$
$= \left(\frac{bh}{2} - \frac{bh}{6}\right) - \left(-\frac{bh}{2} + \frac{bh}{6}\right) = bh - \frac{bh}{3} = \frac{2}{3}\,bh$



56. $k > 0 \Rightarrow$ one arch of $y = \sin kx$ will occur over the interval $\left[0, \frac{\pi}{k}\right] \Rightarrow$ the area $= \int_0^{\pi/k} \sin kx\,dx = \left[-\frac{1}{k}\cos kx\right]_0^{\pi/k}$
$= -\frac{1}{k}\cos\left(k\left(\frac{\pi}{k}\right)\right) - \left(-\frac{1}{k}\cos(0)\right) = \frac{2}{k}$

57. $\frac{dc}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2} \Rightarrow c = \int_0^x \frac{1}{2}t^{-1/2}dt = \left[t^{1/2}\right]_0^x = \sqrt{x};\; c(100) - c(1) = \sqrt{100} - \sqrt{1} = \$9.00$

58. $r = \int_0^3 \left(2 - \frac{2}{(x+1)^2}\right) dx = 2\int_0^3 \left(1 - \frac{1}{(x+1)^2}\right) dx = 2\left[x - \left(\frac{-1}{x+1}\right)\right]_0^3 = 2\left[\left(3 + \frac{1}{(3+1)}\right) - \left(0 + \frac{1}{(0+1)}\right)\right]$
$= 2\left[3\,\frac{1}{4} - 1\right] = 2\left(2\,\frac{1}{4}\right) = 4.5$ or \$4500

59. (a) $t = 0 \Rightarrow T = 85 - 3\sqrt{25-0} = 70°$ F; $t = 16 \Rightarrow T = 85 - 3\sqrt{25-16} = 76°$ F;
$t = 25 \Rightarrow T = 85 - 3\sqrt{25-25} = 85°$ F

(b) average temperatuve $= \frac{1}{25-0}\int_0^{25}\left(85 - 3\sqrt{25-t}\right) dt = \frac{1}{25}\left[85t + 2(25-t)^{3/2}\right]_0^{25}$
$= \frac{1}{25}\left(85(25) + 2(25-25)^{3/2}\right) - \frac{1}{25}\left(85(0) + 2(25-0)^{3/2}\right) = 75°$ F

60. (a) $t = 0 \Rightarrow H = \sqrt{0+1} + 5(0)^{1/3} = 1$ ft; $t = 4 \Rightarrow H = \sqrt{4+1} + 5(4)^{1/3} = \sqrt{5} + 5\sqrt[3]{4} \approx 10.17$ ft;
$t = 8 \Rightarrow H = \sqrt{8+1} + 5(8)^{1/3} = 13$ ft

(b) average height $= \frac{1}{8-0}\int_0^8 \left(\sqrt{t+1} + 5t^{1/3}\right) dt = \frac{1}{8}\left[\frac{2}{3}(t+1)^{3/2} + \frac{15}{4}t^{4/3}\right]_0^8$
$= \frac{1}{8}\left(\frac{2}{3}(8+1)^{3/2} + \frac{15}{4}(8)^{4/3}\right) - \frac{1}{8}\left(\frac{2}{3}(0+1)^{3/2} + \frac{15}{4}(0)^{4/3}\right) = \frac{29}{3} \approx 9.67$ ft

61. $\int_1^x f(t)\,dt = x^2 - 2x + 1 \Rightarrow f(x) = \frac{d}{dx}\int_1^x f(t)\,dt = \frac{d}{dx}(x^2 - 2x + 1) = 2x - 2$

62. $\int_0^x f(t)\,dt = x\cos\pi x \Rightarrow f(x) = \frac{d}{dx}\int_0^x f(t)\,dt = \cos\pi x - \pi x\sin\pi x \Rightarrow f(4) = \cos\pi(4) - \pi(4)\sin\pi(4) = 1$

63. $f(x) = 2 - \int_2^{x+1} \frac{9}{1+t}\,dt \Rightarrow f'(x) = -\frac{9}{1+(x+1)} = \frac{-9}{x+2} \Rightarrow f'(1) = -3;\; f(1) = 2 - \int_2^{1+1}\frac{9}{1+t}\,dt = 2 - 0 = 2;$
$L(x) = -3(x-1) + f(1) = -3(x-1) + 2 = -3x + 5$

64. $g(x) = 3 + \int_1^{x^2} \sec(t-1)\,dt \Rightarrow g'(x) = (\sec(x^2-1))(2x) = 2x\sec(x^2-1) \Rightarrow g'(-1) = 2(-1)\sec((-1)^2-1)$
$= -2; g(-1) = 3 + \int_1^{(-1)^2} \sec(t-1)\,dt = 3 + \int_1^1 \sec(t-1)\,dt = 3 + 0 = 3; L(x) = -2(x-(-1)) + g(-1)$
$= -2(x+1) + 3 = -2x + 1$

65. (a) True: since f is continuous, g is differentiable by Part 1 of the Fundamental Theorem of Calculus.
(b) True: g is continuous because it is differentiable.
(c) True, since $g'(1) = f(1) = 0$.
(d) False, since $g''(1) = f'(1) > 0$.
(e) True, since $g'(1) = 0$ and $g''(1) = f'(1) > 0$.
(f) False: $g''(x) = f'(x) > 0$, so g'' never changes sign.
(g) True, since $g'(1) = f(1) = 0$ and $g'(x) = f(x)$ is an increasing function of x (because $f'(x) > 0$).

66. Let $a = x_0 < x_1 < x_2 \cdots < x_n = b$ be any partition of [a, b] and let F be any antiderivative of f.
(a) $\sum_{i=1}^{n}\left[F(x_i) - F(x_{i-1})\right]$
$= \left[F(x_1) - F(x_0)\right] + \left[F(x_2) - F(x_1)\right] + \left[F(x_3) - F(x_2)\right] + \cdots + \left[F(x_{n-1}) - F(x_{n-2})\right] + \left[F(x_n) - F(x_{n-1})\right]$
$= -F(x_0) + F(x_1) - F(x_1) + F(x_2) - F(x_2) + \cdots + F(x_{n-1}) - F(x_{n-1}) + F(x_n) = F(x_n) - F(x_0) = F(b) - F(a)$
(b) Since F is any antiderivative of f on [a, b] $\Rightarrow$ F is differentiable on [a, b] $\Rightarrow$ F is continuous on [a, b]. Consider any subinterval $[x_{i-1}, x_i]$ in [a, b], then by the Mean Value Theorem there is at least one number c_i in (x_{i-1}, x_i) such that $\left[F(x_i) - F(x_{i-1})\right] = F'(c_i)(x_i - x_{i-1}) = f(c_i)(x_i - x_{i-1}) = f(c_i)\Delta x_i$. Thus $F(b) - F(a) = \sum_{i=1}^{n}\left[F(x_i) - F(x_{i-1})\right]$
$= \sum_{i=1}^{n} f(c_i)\Delta x_i$.
(c) Taking the limit of $F(b) - F(a) = \sum_{i=1}^{n} f(c_i)\Delta x_i$ we obtain $\lim_{\|P\|\to 0}(F(b) - F(a)) = \lim_{\|P\|\to 0}\left(\sum_{i=1}^{n} f(c_i)\Delta x_i\right)$
$\Rightarrow F(b) - F(a) = \int_a^b f(x)\,dx$

67-70. Example CAS commands:
Maple:
```
with( plots );
f := x -> x^3-4*x^2+3*x;
a := 0;
b := 4;
F := unapply( int(f(t),t=a..x), x );                    # (a)
p1 := plot( [f(x),F(x)], x=a..b, legend=["y = f(x)","y = F(x)"], title="#67(a) (Section 5.4)" ):
p1;
dF := D(F);                                              # (b)
q1 := solve( dF(x)=0, x );
pts1 := [ seq( [x,f(x)], x=remove(has,evalf([q1]),I) ) ];
p2 := plot( pts1, style=point, color=blue, symbolsize=18, symbol=diamond, legend="(x,f(x)) where F '(x)=0" ):
display( [p1,p2], title="81(b) (Section 5.4)" );
incr := solve( dF(x)>0, x );                             # (c)
decr := solve( dF(x)<0, x );
df := D(f);                                              # (d)
p3 := plot( [df(x),F(x)], x=a..b, legend=["y = f '(x)","y = F(x)"], title="#67(d) (Section 5.4)" ):
p3;
q2 := solve( df(x)=0, x );
```

```
pts2 := [ seq( [x,F(x)], x=remove(has,evalf([q2]),I) ) ];
p4 := plot( pts2, style=point, color=blue, symbolsize=18, symbol=diamond, legend="(x,f(x)) where f '(x)=0" ):
display( [p3,p4], title="81(d) (Section 5.4)" );
```

71-74. Example CAS commands:

Maple:

```
a := 1;
u := x -> x^2;
f := x -> sqrt(1-x^2);
F := unapply( int( f(t), t=a..u(x) ), x );
dF := D(F);                # (b)
cp := solve( dF(x)=0, x );
solve( dF(x)>0, x );
solve( dF(x)<0, x );
d2F := D(dF);              # (c)
solve( d2F(x)=0, x );
plot( F(x), x=-1..1, title="#71(d) (Section 5.4)" );
```

75. Example CAS commands:

Maple:

```
f := `f`;
q1 := Diff( Int( f(t), t=a..u(x) ), x );
d1 := value( q1 );
```

76. Example CAS commands:

Maple:

```
f := `f`;
q2 := Diff( Int( f(t), t=a..u(x) ), x,x );
value( q2 );
```

67-76. Example CAS commands:

Mathematica: (assigned function and values for a, and b may vary)

For transcendental functions the FindRoot is needed instead of the Solve command.

The Map command executes FindRoot over a set of initial guesses

Initial guesses will vary as the functions vary.

```
Clear[x, f, F]
{a, b}= {0, 2π}; f[x_] = Sin[2x] Cos[x/3]
F[x_] = Integrate[f[t], {t, a, x}]
Plot[{f[x], F[x]},{x, a, b}]
x/.Map[FindRoot[F'[x]==0, {x, #}] &,{2, 3, 5, 6}]
x/.Map[FindRoot[f'[x]==0, {x, #}] &,{1, 2, 4, 5, 6}]
```

Slightly alter above commands for 75 - 80.

```
Clear[x, f, F, u]
a=0;  f[x_] = x^2 − 2x − 3
u[x_] = 1 − x^2
F[x_] = Integrate[f[t], {t, a, u(x)}]
x/.Map[FindRoot[F'[x]==0,{x,#}] &,{1,2,3,4}]
x/.Map[FindRoot[F''[x]==0,{x,#}] &,{1,2,3,4}]
```

After determining an appropriate value for b, the following can be entered
b = 4;
Plot[{F[x], {x, a, b}]

5.5 INDEFINTE INTEGRALS AND THE SUBSTITUTION RULE

1. Let $u = 2x + 4 \Rightarrow du = 2\,dx \Rightarrow \frac{1}{2}\,du = dx$
$\int 2\left(2x+4\right)^5 dx = \int 2u^5\,\frac{1}{2}\,du = \int u^5\,du = \frac{1}{6}\,u^6 + C = \frac{1}{6}\left(2x+4\right)^6 + C$

2. Let $u = 7x - 1 \Rightarrow du = 7\,dx \Rightarrow \frac{1}{7}\,du = dx$
$\int 7\sqrt{7x-1}\,dx = \int 7(7x-1)^{1/2}\,dx = \int 7u^{1/2}\,\frac{1}{7}du = \int u^{1/2}\,du = \frac{2}{3}\,u^{3/2} + C = \frac{2}{3}\,(7x-1)^{3/2} + C$

3. Let $u = x^2 + 5 \Rightarrow du = 2x\,dx \Rightarrow \frac{1}{2}\,du = x\,dx$
$\int 2x\left(x^2+5\right)^{-4}dx = \int 2\,u^{-4}\,\frac{1}{2}du = \int u^{-4}\,du = -\frac{1}{3}\,u^{-3} + C = -\frac{1}{3}\left(x^2+5\right)^{-3} + C$

4. Let $u = x^4 + 1 \Rightarrow du = 4x^3\,dx \Rightarrow \frac{1}{4}\,du = x^3\,dx$
$\int \frac{4x^3}{(x^4+1)^2}dx = \int 4x^3(x^4+1)^{-2}dx = \int 4\,u^{-2}\,\frac{1}{4}du = \int u^{-2}\,du = -\,u^{-1} + C = \frac{-1}{x^4+1} + C$

5. Let $u = 3x^2 + 4x \Rightarrow du = (6x+4)dx = 2(3x+2)\,dx \Rightarrow \frac{1}{2}\,du = (3x+2)\,dx$
$\int (3x+2)(3\,x^2+4x)^4 dx = \int u^4\,\frac{1}{2}du = \frac{1}{2}\int u^4\,du = \frac{1}{10}\,u^5 + C = \frac{1}{10}\left(3x^2+4x\right)^5 + C$

6. Let $u = 1 + \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx$
$\int \frac{(1+\sqrt{x})^{1/3}}{\sqrt{x}}dx = \int \left(1+\sqrt{x}\right)^{1/3}\frac{1}{\sqrt{x}}\,dx = \int u^{1/3}\,2\,du = 2\int u^{1/3}\,du = 2\cdot\frac{3}{4}\,u^{4/3} + C = \frac{3}{2}\left(1+\sqrt{x}\right)^{4/3} + C$

7. Let $u = 3x \Rightarrow du = 3\,dx \Rightarrow \frac{1}{3}\,du = dx$
$\int \sin 3x\,dx = \int \frac{1}{3}\sin u\,du = -\frac{1}{3}\cos u + C = -\frac{1}{3}\cos 3x + C$

8. Let $u = 2x^2 \Rightarrow du = 4x\,dx \Rightarrow \frac{1}{4}\,du = x\,dx$
$\int x\sin(2x^2)\,dx = \int \frac{1}{4}\sin u\,du = -\frac{1}{4}\cos u + C = -\frac{1}{4}\cos 2x^2 + C$

9. Let $u = 2t \Rightarrow du = 2\,dt \Rightarrow \frac{1}{2}\,du = dt$
$\int \sec 2t\tan 2t\,dt = \int \frac{1}{2}\sec u\tan u\,du = \frac{1}{2}\sec u + C = \frac{1}{2}\sec 2t + C$

10. Let $u = 1 - \cos\frac{t}{2} \Rightarrow du = \frac{1}{2}\sin\frac{t}{2}\,dt \Rightarrow 2\,du = \sin\frac{t}{2}\,dt$
$\int \left(1-\cos\frac{t}{2}\right)^2\left(\sin\frac{t}{2}\right)dt = \int 2u^2\,du = \frac{2}{3}\,u^3 + C = \frac{2}{3}\left(1-\cos\frac{t}{2}\right)^3 + C$

11. Let $u = 1 - r^3 \Rightarrow du = -3r^2\,dr \Rightarrow -3\,du = 9r^2\,dr$
$\int \frac{9r^2\,dr}{\sqrt{1-r^3}} = \int -3u^{-1/2}\,du = -3(2)u^{1/2} + C = -6\left(1-r^3\right)^{1/2} + C$

12. Let $u = y^4 + 4y^2 + 1 \Rightarrow du = (4y^3+8y)\,dy \Rightarrow 3\,du = 12\,(y^3+2y)\,dy$
$\int 12\,(y^4+4y^2+1)^2\,(y^3+2y)\,dy = \int 3u^2\,du = u^3 + C = (y^4+4y^2+1)^3 + C$

13. Let $u = x^{3/2} - 1 \Rightarrow du = \frac{3}{2} x^{1/2}\,dx \Rightarrow \frac{2}{3}\,du = \sqrt{x}\,dx$
$\int \sqrt{x}\,\sin^2\left(x^{3/2} - 1\right) dx = \int \frac{2}{3}\sin^2 u\,du = \frac{2}{3}\left(\frac{u}{2} - \frac{1}{4}\sin 2u\right) + C = \frac{1}{3}\left(x^{3/2} - 1\right) - \frac{1}{6}\sin\left(2x^{3/2} - 2\right) + C$

14. Let $u = -\frac{1}{x} \Rightarrow du = \frac{1}{x^2}\,dx$
$\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right) dx = \int \cos^2(-u)\,du = \int \cos^2(u)\,du = \left(\frac{u}{2} + \frac{1}{4}\sin 2u\right) + C = -\frac{1}{2x} + \frac{1}{4}\sin\left(-\frac{2}{x}\right) + C$
$= -\frac{1}{2x} - \frac{1}{4}\sin\left(\frac{2}{x}\right) + C$

15. (a) Let $u = \cot 2\theta \Rightarrow du = -2\csc^2 2\theta\,d\theta \Rightarrow -\frac{1}{2}\,du = \csc^2 2\theta\,d\theta$
$\int \csc^2 2\theta \cot 2\theta\,d\theta = -\int \frac{1}{2}u\,du = -\frac{1}{2}\left(\frac{u^2}{2}\right) + C = -\frac{u^2}{4} + C = -\frac{1}{4}\cot^2 2\theta + C$
(b) Let $u = \csc 2\theta \Rightarrow du = -2\csc 2\theta\cot 2\theta\,d\theta \Rightarrow -\frac{1}{2}\,du = \csc 2\theta \cot 2\theta\,d\theta$
$\int \csc^2 2\theta \cot 2\theta\,d\theta = \int -\frac{1}{2}u\,du = -\frac{1}{2}\left(\frac{u^2}{2}\right) + C = -\frac{u^2}{4} + C = -\frac{1}{4}\csc^2 2\theta + C$

16. (a) Let $u = 5x + 8 \Rightarrow du = 5\,dx \Rightarrow \frac{1}{5}\,du = dx$
$\int \frac{dx}{\sqrt{5x+8}} = \int \frac{1}{5}\left(\frac{1}{\sqrt{u}}\right) du = \frac{1}{5}\int u^{-1/2}\,du = \frac{1}{5}\left(2u^{1/2}\right) + C = \frac{2}{5}u^{1/2} + C = \frac{2}{5}\sqrt{5x+8} + C$
(b) Let $u = \sqrt{5x+8} \Rightarrow du = \frac{1}{2}(5x+8)^{-1/2}(5)\,dx \Rightarrow \frac{2}{5}\,du = \frac{dx}{\sqrt{5x+8}}$
$\int \frac{dx}{\sqrt{5x+8}} = \int \frac{2}{5}\,du = \frac{2}{5}u + C = \frac{2}{5}\sqrt{5x+8} + C$

17. Let $u = 3 - 2s \Rightarrow du = -2\,ds \Rightarrow -\frac{1}{2}\,du = ds$
$\int \sqrt{3-2s}\,ds = \int \sqrt{u}\left(-\frac{1}{2}\,du\right) = -\frac{1}{2}\int u^{1/2}\,du = \left(-\frac{1}{2}\right)\left(\frac{2}{3}u^{3/2}\right) + C = -\frac{1}{3}(3-2s)^{3/2} + C$

18. Let $u = 5s + 4 \Rightarrow du = 5\,ds \Rightarrow \frac{1}{5}\,du = ds$
$\int \frac{1}{\sqrt{5s+4}}\,ds = \int \frac{1}{\sqrt{u}}\left(\frac{1}{5}\,du\right) = \frac{1}{5}\int u^{-1/2}\,du = \left(\frac{1}{5}\right)\left(2u^{1/2}\right) + C = \frac{2}{5}\sqrt{5s+4} + C$

19. Let $u = 1 - \theta^2 \Rightarrow du = -2\theta\,d\theta \Rightarrow -\frac{1}{2}\,du = \theta\,d\theta$
$\int \theta\,\sqrt[4]{1-\theta^2}\,d\theta = \int \sqrt[4]{u}\left(-\frac{1}{2}\,du\right) = -\frac{1}{2}\int u^{1/4}\,du = \left(-\frac{1}{2}\right)\left(\frac{4}{5}u^{5/4}\right) + C = -\frac{2}{5}\left(1-\theta^2\right)^{5/4} + C$

20. Let $u = 7 - 3y^2 \Rightarrow du = -6y\,dy \Rightarrow -\frac{1}{2}\,du = 3y\,dy$
$\int 3y\sqrt{7-3y^2}\,dy = \int \sqrt{u}\left(-\frac{1}{2}\,du\right) = -\frac{1}{2}\int u^{1/2}\,du = \left(-\frac{1}{2}\right)\left(\frac{2}{3}u^{3/2}\right) + C = -\frac{1}{3}\left(7-3y^2\right)^{3/2} + C$

21. Let $u = 1 + \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx$
$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}\,dx = \int \frac{2\,du}{u^2} = -\frac{2}{u} + C = \frac{-2}{1+\sqrt{x}} + C$

22. Let $u = 3z + 4 \Rightarrow du = 3\,dz \Rightarrow \frac{1}{3}\,du = dz$
$\int \cos(3z+4)\,dz = \int (\cos u)\left(\frac{1}{3}\,du\right) = \frac{1}{3}\int \cos u\,du = \frac{1}{3}\sin u + C = \frac{1}{3}\sin(3z+4) + C$

23. Let $u = 3x + 2 \Rightarrow du = 3\,dx \Rightarrow \frac{1}{3}\,du = dx$
$\int \sec^2(3x+2)\,dx = \int (\sec^2 u)\left(\frac{1}{3}\,du\right) = \frac{1}{3}\int \sec^2 u\,du = \frac{1}{3}\tan u + C = \frac{1}{3}\tan(3x+2) + C$

24. Let $u = \tan x \Rightarrow du = \sec^2 x\,dx$
$\int \tan^2 x \sec^2 x\,dx = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}\tan^3 x + C$

25. Let $u = \sin\left(\frac{x}{3}\right) \Rightarrow du = \frac{1}{3}\cos\left(\frac{x}{3}\right)dx \Rightarrow 3\,du = \cos\left(\frac{x}{3}\right)dx$
$\int \sin^5\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)dx = \int u^5\,(3\,du) = 3\left(\frac{1}{6}u^6\right) + C = \frac{1}{2}\sin^6\left(\frac{x}{3}\right) + C$

26. Let $u = \tan\left(\frac{x}{2}\right) \Rightarrow du = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)dx \Rightarrow 2\,du = \sec^2\left(\frac{x}{2}\right)dx$
$\int \tan^7\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)dx = \int u^7\,(2\,du) = 2\left(\frac{1}{8}u^8\right) + C = \frac{1}{4}\tan^8\left(\frac{x}{2}\right) + C$

27. Let $u = \frac{r^3}{18} - 1 \Rightarrow du = \frac{r^2}{6}dr \Rightarrow 6\,du = r^2\,dr$
$\int r^2\left(\frac{r^3}{18} - 1\right)^5 dr = \int u^5\,(6\,du) = 6\int u^5\,du = 6\left(\frac{u^6}{6}\right) + C = \left(\frac{r^3}{18} - 1\right)^6 + C$

28. Let $u = 7 - \frac{r^5}{10} \Rightarrow du = -\frac{1}{2}r^4\,dr \Rightarrow -2\,du = r^4\,dr$
$\int r^4\left(7 - \frac{r^5}{10}\right)^3 dr = \int u^3\,(-2\,du) = -2\int u^3\,du = -2\left(\frac{u^4}{4}\right) + C = -\frac{1}{2}\left(7 - \frac{r^5}{10}\right)^4 + C$

29. Let $u = x^{3/2} + 1 \Rightarrow du = \frac{3}{2}x^{1/2}\,dx \Rightarrow \frac{2}{3}\,du = x^{1/2}\,dx$
$\int x^{1/2}\sin\left(x^{3/2} + 1\right)dx = \int (\sin u)\left(\frac{2}{3}\,du\right) = \frac{2}{3}\int \sin u\,du = \frac{2}{3}(-\cos u) + C = -\frac{2}{3}\cos\left(x^{3/2} + 1\right) + C$

30. Let $u = \csc\left(\frac{v-\pi}{2}\right) \Rightarrow du = -\frac{1}{2}\csc\left(\frac{v-\pi}{2}\right)\cot\left(\frac{v-\pi}{2}\right)dv \Rightarrow -2\,du = \csc\left(\frac{v-\pi}{2}\right)\cot\left(\frac{v-\pi}{2}\right)dv$
$\int \csc\left(\frac{v-\pi}{2}\right)\cot\left(\frac{v-\pi}{2}\right)dv = \int -2\,du = -2u + C = -2\csc\left(\frac{v-\pi}{2}\right) + C$

31. Let $u = \cos(2t+1) \Rightarrow du = -2\sin(2t+1)\,dt \Rightarrow -\frac{1}{2}\,du = \sin(2t+1)\,dt$
$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}dt = \int -\frac{1}{2}\frac{du}{u^2} = \frac{1}{2u} + C = \frac{1}{2\cos(2t+1)} + C$

32. Let $u = \sec z \Rightarrow du = \sec z\tan z\,dz$
$\int \frac{\sec z\tan z}{\sqrt{\sec z}}dz = \int \frac{1}{\sqrt{u}}du = \int u^{-1/2}\,du = 2u^{1/2} + C = 2\sqrt{\sec z} + C$

33. Let $u = \frac{1}{t} - 1 = t^{-1} - 1 \Rightarrow du = -t^{-2}\,dt \Rightarrow -du = \frac{1}{t^2}dt$
$\int \frac{1}{t^2}\cos\left(\frac{1}{t} - 1\right)dt = \int (\cos u)(-du) = -\int \cos u\,du = -\sin u + C = -\sin\left(\frac{1}{t} - 1\right) + C$

34. Let $u = \sqrt{t} + 3 = t^{1/2} + 3 \Rightarrow du = \frac{1}{2}t^{-1/2}\,dt \Rightarrow 2\,du = \frac{1}{\sqrt{t}}dt$
$\int \frac{1}{\sqrt{t}}\cos\left(\sqrt{t} + 3\right)dt = \int (\cos u)(2\,du) = 2\int \cos u\,du = 2\sin u + C = 2\sin\left(\sqrt{t} + 3\right) + C$

35. Let $u = \sin\frac{1}{\theta} \Rightarrow du = \left(\cos\frac{1}{\theta}\right)\left(-\frac{1}{\theta^2}\right)d\theta \Rightarrow -du = \frac{1}{\theta^2}\cos\frac{1}{\theta}\,d\theta$
$\int \frac{1}{\theta^2}\sin\frac{1}{\theta}\cos\frac{1}{\theta}\,d\theta = \int -u\,du = -\frac{1}{2}u^2 + C = -\frac{1}{2}\sin^2\frac{1}{\theta} + C$

36. Let $u = \csc\sqrt{\theta} \Rightarrow du = \left(-\csc\sqrt{\theta}\cot\sqrt{\theta}\right)\left(\frac{1}{2\sqrt{\theta}}\right)d\theta \Rightarrow -2\,du = \frac{1}{\sqrt{\theta}}\cot\sqrt{\theta}\csc\sqrt{\theta}\,d\theta$
$\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\sin^2\sqrt{\theta}}d\theta = \int \frac{1}{\sqrt{\theta}}\cot\sqrt{\theta}\csc\sqrt{\theta}\,d\theta = \int -2\,du = -2u + C = -2\csc\sqrt{\theta} + C = -\frac{2}{\sin\sqrt{\theta}} + C$

37. Let $u = 1 + t^4 \Rightarrow du = 4t^3\,dt \Rightarrow \frac{1}{4}\,du = t^3\,dt$
$\int t^3\left(1 + t^4\right)^3 dt = \int u^3\left(\frac{1}{4}\,du\right) = \frac{1}{4}\left(\frac{1}{4}u^4\right) + C = \frac{1}{16}\left(1 + t^4\right)^4 + C$

38. Let $u = 1 - \frac{1}{x} \Rightarrow du = \frac{1}{x^2}\,dx$

$$\int \sqrt{\frac{x-1}{x^5}}\,dx = \int \frac{1}{x^2}\sqrt{\frac{x-1}{x}}\,dx = \int \frac{1}{x^2}\sqrt{1-\frac{1}{x}}\,dx = \int \sqrt{u}\,du = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}\left(1-\frac{1}{x}\right)^{3/2} + C$$

39. Let $u = 2 - \frac{1}{x} \Rightarrow du = \frac{1}{x^2}\,dx$

$$\int \frac{1}{x^2}\sqrt{2-\frac{1}{x}}\,dx = \int \sqrt{u}\,du = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C = \frac{2}{3}\left(2-\frac{1}{x}\right)^{3/2} + C$$

40. Let $u = 1 - \frac{1}{x^2} \Rightarrow du = \frac{2}{x^3}\,dx \Rightarrow \frac{1}{2}\,du = \frac{1}{x^3}\,dx$

$$\int \frac{1}{x^3}\sqrt{\frac{x^2-1}{x^2}}\,dx = \int \frac{1}{x^3}\sqrt{1-\frac{1}{x^2}}\,dx = \int \sqrt{u}\,\frac{1}{2}\,du = \frac{1}{2}\int u^{1/2}\,du = \frac{1}{3}u^{3/2} + C = \frac{1}{3}\left(1-\frac{1}{x^2}\right)^{3/2} + C$$

41. Let $u = 1 - \frac{3}{x^3} \Rightarrow du = \frac{9}{x^4}\,dx \Rightarrow \frac{1}{9}\,du = \frac{1}{x^4}\,dx$

$$\int \sqrt{\frac{x^3-3}{x^{11}}}\,dx = \int \frac{1}{x^4}\sqrt{\frac{x^3-3}{x^3}}\,dx = \int \frac{1}{x^4}\sqrt{1-\frac{3}{x^3}}\,dx = \int \sqrt{u}\,\frac{1}{9}\,du = \frac{1}{9}\int u^{1/2}\,du = \frac{2}{27}u^{3/2} + C = \frac{2}{27}\left(1-\frac{3}{x^3}\right)^{3/2} + C$$

42. Let $u = x^3 - 1 \Rightarrow du = 3x^2\,dx \Rightarrow \frac{1}{3}\,du = x^2\,dx$

$$\int \sqrt{\frac{x^4}{x^3-1}}\,dx = \int \frac{x^2}{\sqrt{x^3-1}}\,dx = \int \frac{1}{\sqrt{u}}\,\frac{1}{3}\,du = \frac{1}{3}\int u^{-1/2}\,du = \frac{2}{3}u^{1/2} + C = \frac{2}{3}\left(x^3-1\right)^{3/2} + C$$

43. Let $u = x - 1$. Then $du = dx$ and $x = u + 1$. Thus $\int x(x-1)^{10}\,dx = \int (u+1)u^{10}\,du = \int \left(u^{11} + u^{10}\right)du$
$= \frac{1}{12}u^{12} + \frac{1}{11}u^{11} + C = \frac{1}{12}(x-1)^{12} + \frac{1}{11}(x-1)^{11} + C$

44. Let $u = 4 - x$. Then $du = -1\,dx$ and $(-1)\,du = dx$ and $x = 4 - u$. Thus $\int x\sqrt{4-x}\,dx = \int (4-u)\sqrt{u}\,(-1)du$
$= \int (4-u)\left(-u^{1/2}\right)du = \int \left(u^{3/2} - 4u^{1/2}\right)du = \frac{2}{5}u^{5/2} - \frac{8}{3}u^{3/2} + C = \frac{2}{5}(4-x)^{5/2} - \frac{8}{3}(4-x)^{3/2} + C$

45. Let $u = 1 - x$. Then $du = -1\,dx$ and $(-1)\,du = dx$ and $x = 1 - u$. Thus $\int (x+1)^2(1-x)^5\,dx$
$= \int (2-u)^2 u^5\,(-1)\,du = \int \left(-u^7 + 4u^6 - 4u^5\right)du = -\frac{1}{8}u^8 + \frac{4}{7}u^7 - \frac{2}{3}u^6 + C$
$= -\frac{1}{8}(1-x)^8 + \frac{4}{7}(1-x)^7 - \frac{2}{3}(1-x)^6 + C$

46. Let $u = x - 5$. Then $du = dx$ and $x = u + 5$. Thus $\int (x+5)(x-5)^{1/3}\,dx = \int (u+10)u^{1/3}\,du = \int \left(u^{4/3} + 10u^{1/3}\right)du$
$= \frac{3}{7}u^{7/3} + \frac{15}{2}u^{4/3} + C = \frac{3}{7}(x-5)^{7/3} + \frac{15}{2}(x-5)^{4/3} + C$

47. Let $u = x^2 + 1$. Then $du = 2x\,dx$ and $\frac{1}{2}du = x\,dx$ and $x^2 = u - 1$. Thus $\int x^3\sqrt{x^2+1}\,dx = \int (u-1)\frac{1}{2}\sqrt{u}\,du$
$= \frac{1}{2}\int \left(u^{3/2} - u^{1/2}\right)du = \frac{1}{2}\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right] + C = \frac{1}{5}u^{5/2} - \frac{1}{3}u^{3/2} + C = \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$

48. Let $u = x^3 + 1 \Rightarrow du = 3x^2dx$ and $x^3 = u - 1$. So $\int 3x^5\sqrt{x^3+1}\,dx = \int (u-1)\sqrt{u}\,du = \int \left(u^{3/2} - u^{1/2}\right)du$
$= \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C = \frac{2}{5}(x^3+1)^{5/2} - \frac{2}{3}(x^3+1)^{3/2} + C$

49. Let $u = x^2 - 4 \Rightarrow du = 2x\,dx$ and $\frac{1}{2}\,du = x\,dx$. Thus $\int \frac{x}{(x^2-4)^3}\,dx = \int (x^2-4)^{-3}x\,dx = \int u^{-3}\frac{1}{2}\,du = \frac{1}{2}\int u^{-3}\,du$
$= -\frac{1}{4}u^{-2} + C = -\frac{1}{4}(x^2-4)^{-2} + C$

50. Let $u = x - 4 \Rightarrow du = dx$ and $x = u + 4$. Thus $\int \frac{x}{(x-4)^3}\,dx = \int (x-4)^{-3}x\,dx = \int u^{-3}(u+4)du = \int \left(u^{-2} + 4u^{-3}\right)du$
$= -u^{-1} - 2u^{-2} + C = -(x-4)^{-1} - 2(x-4)^{-2} + C$

51. (a) Let $u = \tan x \Rightarrow du = \sec^2 x\, dx$; $v = u^3 \Rightarrow dv = 3u^2\, du \Rightarrow 6\, dv = 18u^2\, du$; $w = 2 + v \Rightarrow dw = dv$

$\int \frac{18 \tan^2 x \sec^2 x}{(2+\tan^3 x)^2}\, dx = \int \frac{18u^2}{(2+u^3)^2}\, du = \int \frac{6\, dv}{(2+v)^2} = \int \frac{6\, dw}{w^2} = 6 \int w^{-2}\, dw = -6w^{-1} + C = -\frac{6}{2+v} + C$

$= -\frac{6}{2+u^3} + C = -\frac{6}{2+\tan^3 x} + C$

(b) Let $u = \tan^3 x \Rightarrow du = 3 \tan^2 x \sec^2 x\, dx \Rightarrow 6\, du = 18 \tan^2 x \sec^2 x\, dx$; $v = 2 + u \Rightarrow dv = du$

$\int \frac{18 \tan^2 x \sec^2 x}{(2+\tan^3 x)^2}\, dx = \int \frac{6\, du}{(2+u)^2} = \int \frac{6\, dv}{v^2} = -\frac{6}{v} + C = -\frac{6}{2+u} + C = -\frac{6}{2+\tan^3 x} + C$

(c) Let $u = 2 + \tan^3 x \Rightarrow du = 3 \tan^2 x \sec^2 x\, dx \Rightarrow 6\, du = 18 \tan^2 x \sec^2 x\, dx$

$\int \frac{18 \tan^2 x \sec^2 x}{(2+\tan^3 x)^2}\, dx = \int \frac{6\, du}{u^2} = -\frac{6}{u} + C = -\frac{6}{2+\tan^3 x} + C$

52. (a) Let $u = x - 1 \Rightarrow du = dx$; $v = \sin u \Rightarrow dv = \cos u\, du$; $w = 1 + v^2 \Rightarrow dw = 2v\, dv \Rightarrow \frac{1}{2}\, dw = v\, dv$

$\int \sqrt{1 + \sin^2 (x-1)} \sin (x-1) \cos (x-1)\, dx = \int \sqrt{1 + \sin^2 u} \sin u \cos u\, du = \int v \sqrt{1 + v^2}\, dv$

$= \int \frac{1}{2} \sqrt{w}\, dw = \frac{1}{3} w^{3/2} + C = \frac{1}{3} (1 + v^2)^{3/2} + C = \frac{1}{3} (1 + \sin^2 u)^{3/2} + C = \frac{1}{3} (1 + \sin^2 (x-1))^{3/2} + C$

(b) Let $u = \sin (x-1) \Rightarrow du = \cos (x-1)\, dx$; $v = 1 + u^2 \Rightarrow dv = 2u\, du \Rightarrow \frac{1}{2}\, dv = u\, du$

$\int \sqrt{1 + \sin^2 (x-1)} \sin (x-1) \cos (x-1)\, dx = \int u \sqrt{1 + u^2}\, du = \int \frac{1}{2} \sqrt{v}\, dv = \int \frac{1}{2} v^{1/2}\, dv$

$= \left(\frac{1}{2}\left(\frac{2}{3}\right) v^{3/2}\right) + C = \frac{1}{3} v^{3/2} + C = \frac{1}{3} (1 + u^2)^{3/2} + C = \frac{1}{3} (1 + \sin^2 (x-1))^{3/2} + C$

(c) Let $u = 1 + \sin^2 (x-1) \Rightarrow du = 2 \sin (x-1) \cos (x-1)\, dx \Rightarrow \frac{1}{2}\, du = \sin (x-1) \cos (x-1)\, dx$

$\int \sqrt{1 + \sin^2 (x-1)} \sin (x-1) \cos (x-1)\, dx = \int \frac{1}{2} \sqrt{u}\, du = \int \frac{1}{2} u^{1/2}\, du = \frac{1}{2} \left(\frac{2}{3} u^{3/2}\right) + C$

$= \frac{1}{3} (1 + \sin^2 (x-1))^{3/2} + C$

53. Let $u = 3(2r-1)^2 + 6 \Rightarrow du = 6(2r-1)(2)\, dr \Rightarrow \frac{1}{12}\, du = (2r-1)\, dr$; $v = \sqrt{u} \Rightarrow dv = \frac{1}{2\sqrt{u}}\, du \Rightarrow \frac{1}{6}\, dv = \frac{1}{12\sqrt{u}}\, du$

$\int \frac{(2r-1) \cos \sqrt{3(2r-1)^2 + 6}}{\sqrt{3(2r-1)^2 + 6}}\, dr = \int \left(\frac{\cos \sqrt{u}}{\sqrt{u}}\right) \left(\frac{1}{12}\, du\right) = \int (\cos v) \left(\frac{1}{6}\, dv\right) = \frac{1}{6} \sin v + C = \frac{1}{6} \sin \sqrt{u} + C$

$= \frac{1}{6} \sin \sqrt{3(2r-1)^2 + 6} + C$

54. Let $u = \cos \sqrt{\theta} \Rightarrow du = \left(-\sin \sqrt{\theta}\right) \left(\frac{1}{2\sqrt{\theta}}\right) d\theta \Rightarrow -2\, du = \frac{\sin \sqrt{\theta}}{\sqrt{\theta}}\, d\theta$

$\int \frac{\sin \sqrt{\theta}}{\sqrt{\theta \cos^3 \sqrt{\theta}}}\, d\theta = \int \frac{\sin \sqrt{\theta}}{\sqrt{\theta} \sqrt{\cos^3 \sqrt{\theta}}}\, d\theta = \int \frac{-2\, du}{u^{3/2}} = -2 \int u^{-3/2}\, du = -2 \left(-2u^{-1/2}\right) + C = \frac{4}{\sqrt{u}} + C$

$= \frac{4}{\sqrt{\cos \sqrt{\theta}}} + C$

55. Let $u = 3t^2 - 1 \Rightarrow du = 6t\, dt \Rightarrow 2\, du = 12t\, dt$

$s = \int 12t \left(3t^2 - 1\right)^3 dt = \int u^3 (2\, du) = 2 \left(\frac{1}{4} u^4\right) + C = \frac{1}{2} u^4 + C = \frac{1}{2} \left(3t^2 - 1\right)^4 + C$;

$s = 3$ when $t = 1 \Rightarrow 3 = \frac{1}{2} (3-1)^4 + C \Rightarrow 3 = 8 + C \Rightarrow C = -5 \Rightarrow s = \frac{1}{2} \left(3t^2 - 1\right)^4 - 5$

56. Let $u = x^2 + 8 \Rightarrow du = 2x\, dx \Rightarrow 2\, du = 4x\, dx$

$y = \int 4x \left(x^2 + 8\right)^{-1/3} dx = \int u^{-1/3} (2\, du) = 2 \left(\frac{3}{2} u^{2/3}\right) + C = 3u^{2/3} + C = 3 \left(x^2 + 8\right)^{2/3} + C$;

$y = 0$ when $x = 0 \Rightarrow 0 = 3(8)^{2/3} + C \Rightarrow C = -12 \Rightarrow y = 3 \left(x^2 + 8\right)^{2/3} - 12$

57. Let $u = t + \frac{\pi}{12} \Rightarrow du = dt$

$s = \int 8 \sin^2 \left(t + \frac{\pi}{12}\right) dt = \int 8 \sin^2 u\, du = 8 \left(\frac{u}{2} - \frac{1}{4} \sin 2u\right) + C = 4 \left(t + \frac{\pi}{12}\right) - 2 \sin \left(2t + \frac{\pi}{6}\right) + C$;

$s = 8$ when $t = 0 \Rightarrow 8 = 4 \left(\frac{\pi}{12}\right) - 2 \sin \left(\frac{\pi}{6}\right) + C \Rightarrow C = 8 - \frac{\pi}{3} + 1 = 9 - \frac{\pi}{3}$

$\Rightarrow s = 4\left(t + \frac{\pi}{12}\right) - 2 \sin \left(2t + \frac{\pi}{6}\right) + 9 - \frac{\pi}{3} = 4t - 2 \sin \left(2t + \frac{\pi}{6}\right) + 9$

58. Let $u = \frac{\pi}{4} - \theta \Rightarrow -du = d\theta$
$r = \int 3\cos^2\left(\frac{\pi}{4} - \theta\right) d\theta = -\int 3\cos^2 u\, du = -3\left(\frac{u}{2} + \frac{1}{4}\sin 2u\right) + C = -\frac{3}{2}\left(\frac{\pi}{4} - \theta\right) - \frac{3}{4}\sin\left(\frac{\pi}{2} - 2\theta\right) + C;$
$r = \frac{\pi}{8}$ when $\theta = 0 \Rightarrow \frac{\pi}{8} = -\frac{3\pi}{8} - \frac{3}{4}\sin\frac{\pi}{2} + C \Rightarrow C = \frac{\pi}{2} + \frac{3}{4} \Rightarrow r = -\frac{3}{2}\left(\frac{\pi}{4} - \theta\right) - \frac{3}{4}\sin\left(\frac{\pi}{2} - 2\theta\right) + \frac{\pi}{2} + \frac{3}{4}$
$\Rightarrow r = \frac{3}{2}\theta - \frac{3}{4}\sin\left(\frac{\pi}{2} - 2\theta\right) + \frac{\pi}{8} + \frac{3}{4} \Rightarrow r = \frac{3}{2}\theta - \frac{3}{4}\cos 2\theta + \frac{\pi}{8} + \frac{3}{4}$

59. Let $u = 2t - \frac{\pi}{2} \Rightarrow du = 2\,dt \Rightarrow -2\,du = -4\,dt$
$\frac{ds}{dt} = \int -4\sin\left(2t - \frac{\pi}{2}\right) dt = \int (\sin u)(-2\,du) = 2\cos u + C_1 = 2\cos\left(2t - \frac{\pi}{2}\right) + C_1;$
at $t = 0$ and $\frac{ds}{dt} = 100$ we have $100 = 2\cos\left(-\frac{\pi}{2}\right) + C_1 \Rightarrow C_1 = 100 \Rightarrow \frac{ds}{dt} = 2\cos\left(2t - \frac{\pi}{2}\right) + 100$
$\Rightarrow s = \int \left(2\cos\left(2t - \frac{\pi}{2}\right) + 100\right) dt = \int (\cos u + 50)\, du = \sin u + 50u + C_2 = \sin\left(2t - \frac{\pi}{2}\right) + 50\left(2t - \frac{\pi}{2}\right) + C_2;$
at $t = 0$ and $s = 0$ we have $0 = \sin\left(-\frac{\pi}{2}\right) + 50\left(-\frac{\pi}{2}\right) + C_2 \Rightarrow C_2 = 1 + 25\pi$
$\Rightarrow s = \sin\left(2t - \frac{\pi}{2}\right) + 100t - 25\pi + (1 + 25\pi) \Rightarrow s = \sin\left(2t - \frac{\pi}{2}\right) + 100t + 1$

60. Let $u = \tan 2x \Rightarrow du = 2\sec^2 2x\, dx \Rightarrow 2\,du = 4\sec^2 2x\, dx$; $v = 2x \Rightarrow dv = 2\,dx \Rightarrow \frac{1}{2}\,dv = dx$
$\frac{dy}{dx} = \int 4\sec^2 2x \tan 2x\, dx = \int u(2\,du) = u^2 + C_1 = \tan^2 2x + C_1;$
at $x = 0$ and $\frac{dy}{dx} = 4$ we have $4 = 0 + C_1 \Rightarrow C_1 = 4 \Rightarrow \frac{dy}{dx} = \tan^2 2x + 4 = (\sec^2 2x - 1) + 4 = \sec^2 2x + 3$
$\Rightarrow y = \int (\sec^2 2x + 3)\, dx = \int (\sec^2 v + 3)\left(\frac{1}{2}\,dv\right) = \frac{1}{2}\tan v + \frac{3}{2}v + C_2 = \frac{1}{2}\tan 2x + 3x + C_2;$
at $x = 0$ and $y = -1$ we have $-1 = \frac{1}{2}(0) + 0 + C_2 \Rightarrow C_2 = -1 \Rightarrow y = \frac{1}{2}\tan 2x + 3x - 1$

61. Let $u = 2t \Rightarrow du = 2\,dt \Rightarrow 3\,du = 6\,dt$
$s = \int 6\sin 2t\, dt = \int (\sin u)(3\,du) = -3\cos u + C = -3\cos 2t + C;$
at $t = 0$ and $s = 0$ we have $0 = -3\cos 0 + C \Rightarrow C = 3 \Rightarrow s = 3 - 3\cos 2t \Rightarrow s\left(\frac{\pi}{2}\right) = 3 - 3\cos(\pi) = 6$ m

62. Let $u = \pi t \Rightarrow du = \pi\, dt \Rightarrow \pi\, du = \pi^2\, dt$
$v = \int \pi^2 \cos \pi t\, dt = \int (\cos u)(\pi\, du) = \pi \sin u + C_1 = \pi\sin(\pi t) + C_1;$
at $t = 0$ and $v = 8$ we have $8 = \pi(0) + C_1 \Rightarrow C_1 = 8 \Rightarrow v = \frac{ds}{dt} = \pi\sin(\pi t) + 8 \Rightarrow s = \int (\pi\sin(\pi t) + 8)\, dt$
$= \int \sin u\, du + 8t + C_2 = -\cos(\pi t) + 8t + C_2$; at $t = 0$ and $s = 0$ we have $0 = -1 + C_2 \Rightarrow C_2 = 1$
$\Rightarrow s = 8t - \cos(\pi t) + 1 \Rightarrow s(1) = 8 - \cos\pi + 1 = 10$ m

63. All three integrations are correct. In each case, the derivative of the function on the right is the integrand on the left, and each formula has an arbitrary constant for generating the remaining antiderivatives. Moreover, $\sin^2 x + C_1 = 1 - \cos^2 x + C_1 \Rightarrow C_2 = 1 + C_1$; also $-\cos^2 x + C_2 = -\frac{\cos 2x}{2} - \frac{1}{2} + C_2 \Rightarrow C_3 = C_2 - \frac{1}{2} = C_1 + \frac{1}{2}$.

64. (a) $\left(\frac{1}{\frac{1}{60} - 0}\right)\int_0^{1/60} V_{max} \sin 120\pi t\, dt = 60\left[-V_{max}\left(\frac{1}{120\pi}\right)\cos(120\pi t)\right]_0^{1/60} = -\frac{V_{max}}{2\pi}[\cos 2\pi - \cos 0]$
$= -\frac{V_{max}}{2\pi}[1 - 1] = 0$

(b) $V_{max} = \sqrt{2}\,V_{rms} = \sqrt{2}(240) \approx 339$ volts

(c) $\int_0^{1/60} (V_{max})^2 \sin^2 120\pi t\, dt = (V_{max})^2 \int_0^{1/60} \left(\frac{1 - \cos 240\pi t}{2}\right) dt = \frac{(V_{max})^2}{2}\int_0^{1/60} (1 - \cos 240\pi t)\, dt$
$= \frac{(V_{max})^2}{2}\left[t - \left(\frac{1}{240\pi}\right)\sin 240\pi t\right]_0^{1/60} = \frac{(V_{max})^2}{2}\left[\left(\frac{1}{60} - \left(\frac{1}{240\pi}\right)\sin(4\pi)\right) - \left(0 - \left(\frac{1}{240\pi}\right)\sin(0)\right)\right] = \frac{(V_{max})^2}{120}$

5.6 SUBSTITUTION AND AREA BETWEEN CURVES

1. (a) Let $u = y + 1 \Rightarrow du = dy$; $y = 0 \Rightarrow u = 1$, $y = 3 \Rightarrow u = 4$

$$\int_0^3 \sqrt{y+1}\, dy = \int_1^4 u^{1/2}\, du = \left[\tfrac{2}{3} u^{3/2}\right]_1^4 = \left(\tfrac{2}{3}\right)(4)^{3/2} - \left(\tfrac{2}{3}\right)(1)^{3/2} = \left(\tfrac{2}{3}\right)(8) - \left(\tfrac{2}{3}\right)(1) = \tfrac{14}{3}$$

(b) Use the same substitution for u as in part (a); $y = -1 \Rightarrow u = 0$, $y = 0 \Rightarrow u = 1$

$$\int_{-1}^0 \sqrt{y+1}\, dy = \int_0^1 u^{1/2}\, du = \left[\tfrac{2}{3} u^{3/2}\right]_0^1 = \left(\tfrac{2}{3}\right)(1)^{3/2} - 0 = \tfrac{2}{3}$$

2. (a) Let $u = 1 - r^2 \Rightarrow du = -2r\, dr \Rightarrow -\tfrac{1}{2}\, du = r\, dr$; $r = 0 \Rightarrow u = 1$, $r = 1 \Rightarrow u = 0$

$$\int_0^1 r\sqrt{1-r^2}\, dr = \int_1^0 -\tfrac{1}{2}\sqrt{u}\, du = \left[-\tfrac{1}{3} u^{3/2}\right]_1^0 = 0 - \left(-\tfrac{1}{3}\right)(1)^{3/2} = \tfrac{1}{3}$$

(b) Use the same substitution for u as in part (a); $r = -1 \Rightarrow u = 0$, $r = 1 \Rightarrow u = 0$

$$\int_{-1}^1 r\sqrt{1-r^2}\, dr = \int_0^0 -\tfrac{1}{2}\sqrt{u}\, du = 0$$

3. (a) Let $u = \tan x \Rightarrow du = \sec^2 x\, dx$; $x = 0 \Rightarrow u = 0$, $x = \tfrac{\pi}{4} \Rightarrow u = 1$

$$\int_0^{\pi/4} \tan x \sec^2 x\, dx = \int_0^1 u\, du = \left[\tfrac{u^2}{2}\right]_0^1 = \tfrac{1^2}{2} - 0 = \tfrac{1}{2}$$

(b) Use the same substitution as in part (a); $x = -\tfrac{\pi}{4} \Rightarrow u = -1$, $x = 0 \Rightarrow u = 0$

$$\int_{-\pi/4}^0 \tan x \sec^2 x\, dx = \int_{-1}^0 u\, du = \left[\tfrac{u^2}{2}\right]_{-1}^0 = 0 - \tfrac{1}{2} = -\tfrac{1}{2}$$

4. (a) Let $u = \cos x \Rightarrow du = -\sin x\, dx \Rightarrow -du = \sin x\, dx$; $x = 0 \Rightarrow u = 1$, $x = \pi \Rightarrow u = -1$

$$\int_0^{\pi} 3\cos^2 x \sin x\, dx = \int_1^{-1} -3u^2\, du = \left[-u^3\right]_1^{-1} = -(-1)^3 - \left(-(1)^3\right) = 2$$

(b) Use the same substitution as in part (a); $x = 2\pi \Rightarrow u = 1$, $x = 3\pi \Rightarrow u = -1$

$$\int_{2\pi}^{3\pi} 3\cos^2 x \sin x\, dx = \int_1^{-1} -3u^2\, du = 2$$

5. (a) $u = 1 + t^4 \Rightarrow du = 4t^3\, dt \Rightarrow \tfrac{1}{4}\, du = t^3\, dt$; $t = 0 \Rightarrow u = 1$, $t = 1 \Rightarrow u = 2$

$$\int_0^1 t^3\left(1+t^4\right)^3 dt = \int_1^2 \tfrac{1}{4} u^3\, du = \left[\tfrac{u^4}{16}\right]_1^2 = \tfrac{2^4}{16} - \tfrac{1^4}{16} = \tfrac{15}{16}$$

(b) Use the same substitution as in part (a); $t = -1 \Rightarrow u = 2$, $t = 1 \Rightarrow u = 2$

$$\int_{-1}^1 t^3\left(1+t^4\right)^3 dt = \int_2^2 \tfrac{1}{4} u^3\, du = 0$$

6. (a) Let $u = t^2 + 1 \Rightarrow du = 2t\, dt \Rightarrow \tfrac{1}{2}\, du = t\, dt$; $t = 0 \Rightarrow u = 1$, $t = \sqrt{7} \Rightarrow u = 8$

$$\int_0^{\sqrt{7}} t\left(t^2+1\right)^{1/3} dt = \int_1^8 \tfrac{1}{2} u^{1/3}\, du = \left[\left(\tfrac{1}{2}\right)\left(\tfrac{3}{4}\right) u^{4/3}\right]_1^8 = \left(\tfrac{3}{8}\right)(8)^{4/3} - \left(\tfrac{3}{8}\right)(1)^{4/3} = \tfrac{45}{8}$$

(b) Use the same substitution as in part (a); $t = -\sqrt{7} \Rightarrow u = 8$, $t = 0 \Rightarrow u = 1$

$$\int_{-\sqrt{7}}^0 t\left(t^2+1\right)^{1/3} dt = \int_8^1 \tfrac{1}{2} u^{1/3}\, du = -\int_1^8 \tfrac{1}{2} u^{1/3}\, du = -\tfrac{45}{8}$$

7. (a) Let $u = 4 + r^2 \Rightarrow du = 2r\, dr \Rightarrow \tfrac{1}{2}\, du = r\, dr$; $r = -1 \Rightarrow u = 5$, $r = 1 \Rightarrow u = 5$

$$\int_{-1}^1 \tfrac{5r}{\left(4+r^2\right)^2}\, dr = 5\int_5^5 \tfrac{1}{2} u^{-2}\, du = 0$$

(b) Use the same substitution as in part (a); $r = 0 \Rightarrow u = 4$, $r = 1 \Rightarrow u = 5$

$$\int_0^1 \tfrac{5r}{\left(4+r^2\right)^2}\, dr = 5\int_4^5 \tfrac{1}{2} u^{-2}\, du = 5\left[-\tfrac{1}{2} u^{-1}\right]_4^5 = 5\left(-\tfrac{1}{2}(5)^{-1}\right) - 5\left(-\tfrac{1}{2}(4)^{-1}\right) = \tfrac{1}{8}$$

8. (a) Let $u = 1 + v^{3/2} \Rightarrow du = \frac{3}{2} v^{1/2} dv \Rightarrow \frac{20}{3} du = 10\sqrt{v}\, dv$; $v = 0 \Rightarrow u = 1$, $v = 1 \Rightarrow u = 2$

$$\int_0^1 \frac{10\sqrt{v}}{\left(1+v^{3/2}\right)^2}\, dv = \int_1^2 \frac{1}{u^2}\left(\frac{20}{3}\, du\right) = \frac{20}{3}\int_1^2 u^{-2}\, du = -\frac{20}{3}\left[\frac{1}{u}\right]_1^2 = -\frac{20}{3}\left[\frac{1}{2} - \frac{1}{1}\right] = \frac{10}{3}$$

(b) Use the same substitution as in part (a); $v = 1 \Rightarrow u = 2$, $v = 4 \Rightarrow u = 1 + 4^{3/2} = 9$

$$\int_1^4 \frac{10\sqrt{v}}{\left(1+v^{3/2}\right)^2}\, dv = \int_2^9 \frac{1}{u^2}\left(\frac{20}{3}\, du\right) = -\frac{20}{3}\left[\frac{1}{u}\right]_2^9 = -\frac{20}{3}\left(\frac{1}{9} - \frac{1}{2}\right) = -\frac{20}{3}\left(-\frac{7}{18}\right) = \frac{70}{27}$$

9. (a) Let $u = x^2 + 1 \Rightarrow du = 2x\, dx \Rightarrow 2\, du = 4x\, dx$; $x = 0 \Rightarrow u = 1$, $x = \sqrt{3} \Rightarrow u = 4$

$$\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\, dx = \int_1^4 \frac{2}{\sqrt{u}}\, du = \int_1^4 2u^{-1/2}\, du = \left[4u^{1/2}\right]_1^4 = 4(4)^{1/2} - 4(1)^{1/2} = 4$$

(b) Use the same substitution as in part (a); $x = -\sqrt{3} \Rightarrow u = 4$, $x = \sqrt{3} \Rightarrow u = 4$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\, dx = \int_4^4 \frac{2}{\sqrt{u}}\, du = 0$$

10. (a) Let $u = x^4 + 9 \Rightarrow du = 4x^3\, dx \Rightarrow \frac{1}{4}\, du = x^3\, dx$; $x = 0 \Rightarrow u = 9$, $x = 1 \Rightarrow u = 10$

$$\int_0^1 \frac{x^3}{\sqrt{x^4+9}}\, dx = \int_9^{10} \frac{1}{4} u^{-1/2}\, du = \left[\frac{1}{4}(2)u^{1/2}\right]_9^{10} = \frac{1}{2}(10)^{1/2} - \frac{1}{2}(9)^{1/2} = \frac{\sqrt{10}-3}{2}$$

(b) Use the same substitution as in part (a); $x = -1 \Rightarrow u = 10$, $x = 0 \Rightarrow u = 9$

$$\int_{-1}^0 \frac{x^3}{\sqrt{x^4+9}}\, dx = \int_{10}^9 \frac{1}{4} u^{-1/2}\, du = -\int_9^{10} \frac{1}{4} u^{-1/2}\, du = \frac{3-\sqrt{10}}{2}$$

11. (a) Let $u = 1 - \cos 3t \Rightarrow du = 3 \sin 3t\, dt \Rightarrow \frac{1}{3}\, du = \sin 3t\, dt$; $t = 0 \Rightarrow u = 0$, $t = \frac{\pi}{6} \Rightarrow u = 1 - \cos \frac{\pi}{2} = 1$

$$\int_0^{\pi/6} (1 - \cos 3t) \sin 3t\, dt = \int_0^1 \frac{1}{3} u\, du = \left[\frac{1}{3}\left(\frac{u^2}{2}\right)\right]_0^1 = \frac{1}{6}(1)^2 - \frac{1}{6}(0)^2 = \frac{1}{6}$$

(b) Use the same substitution as in part (a); $t = \frac{\pi}{6} \Rightarrow u = 1$, $t = \frac{\pi}{3} \Rightarrow u = 1 - \cos \pi = 2$

$$\int_{\pi/6}^{\pi/3} (1 - \cos 3t) \sin 3t\, dt = \int_1^2 \frac{1}{3} u\, du = \left[\frac{1}{3}\left(\frac{u^2}{2}\right)\right]_1^2 = \frac{1}{6}(2)^2 - \frac{1}{6}(1)^2 = \frac{1}{2}$$

12. (a) Let $u = 2 + \tan \frac{t}{2} \Rightarrow du = \frac{1}{2} \sec^2 \frac{t}{2}\, dt \Rightarrow 2\, du = \sec^2 \frac{t}{2}\, dt$; $t = \frac{-\pi}{2} \Rightarrow u = 2 + \tan\left(\frac{-\pi}{4}\right) = 1$, $t = 0 \Rightarrow u = 2$

$$\int_{-\pi/2}^0 \left(2 + \tan \frac{t}{2}\right) \sec^2 \frac{t}{2}\, dt = \int_1^2 u\,(2\, du) = \left[u^2\right]_1^2 = 2^2 - 1^2 = 3$$

(b) Use the same substitution as in part (a); $t = \frac{-\pi}{2} \Rightarrow u = 1$, $t = \frac{\pi}{2} \Rightarrow u = 3$

$$\int_{-\pi/2}^{\pi/2} \left(2 + \tan \frac{t}{2}\right) \sec^2 \frac{t}{2}\, dt = 2\int_1^3 u\, du = \left[u^2\right]_1^3 = 3^2 - 1^2 = 8$$

13. (a) Let $u = 4 + 3 \sin z \Rightarrow du = 3 \cos z\, dz \Rightarrow \frac{1}{3}\, du = \cos z\, dz$; $z = 0 \Rightarrow u = 4$, $z = 2\pi \Rightarrow u = 4$

$$\int_0^{2\pi} \frac{\cos z}{\sqrt{4 + 3 \sin z}}\, dz = \int_4^4 \frac{1}{\sqrt{u}}\left(\frac{1}{3}\, du\right) = 0$$

(b) Use the same substitution as in part (a); $z = -\pi \Rightarrow u = 4 + 3 \sin(-\pi) = 4$, $z = \pi \Rightarrow u = 4$

$$\int_{-\pi}^{\pi} \frac{\cos z}{\sqrt{4 + 3 \sin z}}\, dz = \int_4^4 \frac{1}{\sqrt{u}}\left(\frac{1}{3}\, du\right) = 0$$

14. (a) Let $u = 3 + 2 \cos w \Rightarrow du = -2 \sin w\, dw \Rightarrow -\frac{1}{2}\, du = \sin w\, dw$; $w = -\frac{\pi}{2} \Rightarrow u = 3$, $w = 0 \Rightarrow u = 5$

$$\int_{-\pi/2}^0 \frac{\sin w}{(3 + 2 \cos w)^2}\, dw = \int_3^5 u^{-2}\left(-\frac{1}{2}\, du\right) = \frac{1}{2}\left[u^{-1}\right]_3^5 = \frac{1}{2}\left(\frac{1}{5} - \frac{1}{3}\right) = -\frac{1}{15}$$

(b) Use the same substitution as in part (a); $w = 0 \Rightarrow u = 5$, $w = \frac{\pi}{2} \Rightarrow u = 3$

$$\int_0^{\pi/2} \frac{\sin w}{(3 + 2 \cos w)^2}\, dw = \int_5^3 u^{-2}\left(-\frac{1}{2}\, du\right) = \frac{1}{2}\int_3^5 u^{-2}\, du = \frac{1}{15}$$

15. Let $u = t^5 + 2t \Rightarrow du = (5t^4 + 2)\,dt$; $t = 0 \Rightarrow u = 0$, $t = 1 \Rightarrow u = 3$
$$\int_0^1 \sqrt{t^5 + 2t}\,(5t^4 + 2)\,dt = \int_0^3 u^{1/2}\,du = \left[\tfrac{2}{3}u^{3/2}\right]_0^3 = \tfrac{2}{3}(3)^{3/2} - \tfrac{2}{3}(0)^{3/2} = 2\sqrt{3}$$

16. Let $u = 1 + \sqrt{y} \Rightarrow du = \frac{dy}{2\sqrt{y}}$; $y = 1 \Rightarrow u = 2$, $y = 4 \Rightarrow u = 3$
$$\int_1^4 \frac{dy}{2\sqrt{y}\,(1+\sqrt{y})^2} = \int_2^3 \tfrac{1}{u^2}\,du = \int_2^3 u^{-2}\,du = \left[-u^{-1}\right]_2^3 = \left(-\tfrac{1}{3}\right) - \left(-\tfrac{1}{2}\right) = \tfrac{1}{6}$$

17. Let $u = \cos 2\theta \Rightarrow du = -2\sin 2\theta\,d\theta \Rightarrow -\frac{1}{2}\,du = \sin 2\theta\,d\theta$; $\theta = 0 \Rightarrow u = 1$, $\theta = \frac{\pi}{6} \Rightarrow u = \cos 2\left(\frac{\pi}{6}\right) = \frac{1}{2}$
$$\int_0^{\pi/6} \cos^{-3} 2\theta \sin 2\theta\,d\theta = \int_1^{1/2} u^{-3}\left(-\tfrac{1}{2}\,du\right) = -\tfrac{1}{2}\int_1^{1/2} u^{-3}\,du = \left[-\tfrac{1}{2}\left(\tfrac{u^{-2}}{-2}\right)\right]_1^{1/2} = \frac{1}{4\left(\frac{1}{2}\right)^2} - \frac{1}{4(1)^2} = \tfrac{3}{4}$$

18. Let $u = \tan\left(\frac{\theta}{6}\right) \Rightarrow du = \frac{1}{6}\sec^2\left(\frac{\theta}{6}\right)d\theta \Rightarrow 6\,du = \sec^2\left(\frac{\theta}{6}\right)d\theta$; $\theta = \pi \Rightarrow u = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$, $\theta = \frac{3\pi}{2} \Rightarrow u = \tan\frac{\pi}{4} = 1$
$$\int_\pi^{3\pi/2} \cot^5\left(\tfrac{\theta}{6}\right)\sec^2\left(\tfrac{\theta}{6}\right)d\theta = \int_{1/\sqrt{3}}^1 u^{-5}\,(6\,du) = \left[6\left(\tfrac{u^{-4}}{-4}\right)\right]_{1/\sqrt{3}}^1 = \left[-\tfrac{3}{2u^4}\right]_{1/\sqrt{3}}^1 = -\frac{3}{2(1)^4} - \left(-\frac{3}{2\left(\frac{1}{\sqrt{3}}\right)^4}\right) = 12$$

19. Let $u = 5 - 4\cos t \Rightarrow du = 4\sin t\,dt \Rightarrow \frac{1}{4}\,du = \sin t\,dt$; $t = 0 \Rightarrow u = 5 - 4\cos 0 = 1$, $t = \pi \Rightarrow u = 5 - 4\cos\pi = 9$
$$\int_0^\pi 5\,(5 - 4\cos t)^{1/4}\sin t\,dt = \int_1^9 5u^{1/4}\left(\tfrac{1}{4}\,du\right) = \tfrac{5}{4}\int_1^9 u^{1/4}\,du = \left[\tfrac{5}{4}\left(\tfrac{4}{5}u^{5/4}\right)\right]_1^9 = 9^{5/4} - 1 = 3^{5/2} - 1$$

20. Let $u = 1 - \sin 2t \Rightarrow du = -2\cos 2t\,dt \Rightarrow -\frac{1}{2}\,du = \cos 2t\,dt$; $t = 0 \Rightarrow u = 1$, $t = \frac{\pi}{4} \Rightarrow u = 0$
$$\int_0^{\pi/4} (1 - \sin 2t)^{3/2}\cos 2t\,dt = \int_1^0 -\tfrac{1}{2}u^{3/2}\,du = \left[-\tfrac{1}{2}\left(\tfrac{2}{5}u^{5/2}\right)\right]_1^0 = \left(-\tfrac{1}{5}(0)^{5/2}\right) - \left(-\tfrac{1}{5}(1)^{5/2}\right) = \tfrac{1}{5}$$

21. Let $u = 4y - y^2 + 4y^3 + 1 \Rightarrow du = (4 - 2y + 12y^2)\,dy$; $y = 0 \Rightarrow u = 1$, $y = 1 \Rightarrow u = 4(1) - (1)^2 + 4(1)^3 + 1 = 8$
$$\int_0^1 \left(4y - y^2 + 4y^3 + 1\right)^{-2/3}(12y^2 - 2y + 4)\,dy = \int_1^8 u^{-2/3}\,du = \left[3u^{1/3}\right]_1^8 = 3(8)^{1/3} - 3(1)^{1/3} = 3$$

22. Let $u = y^3 + 6y^2 - 12y + 9 \Rightarrow du = (3y^2 + 12y - 12)\,dy \Rightarrow \frac{1}{3}\,du = (y^2 + 4y - 4)\,dy$; $y = 0 \Rightarrow u = 9$, $y = 1 \Rightarrow u = 4$
$$\int_0^1 \left(y^3 + 6y^2 - 12y + 9\right)^{-1/2}(y^2 + 4y - 4)\,dy = \int_9^4 \tfrac{1}{3}u^{-1/2}\,du = \left[\tfrac{1}{3}\left(2u^{1/2}\right)\right]_9^4 = \tfrac{2}{3}(4)^{1/2} - \tfrac{2}{3}(9)^{1/2} = \tfrac{2}{3}(2 - 3) = -\tfrac{2}{3}$$

23. Let $u = \theta^{3/2} \Rightarrow du = \frac{3}{2}\theta^{1/2}\,d\theta \Rightarrow \frac{2}{3}\,du = \sqrt{\theta}\,d\theta$; $\theta = 0 \Rightarrow u = 0$, $\theta = \sqrt[3]{\pi^2} \Rightarrow u = \pi$
$$\int_0^{\sqrt[3]{\pi^2}} \sqrt{\theta}\cos^2\left(\theta^{3/2}\right)d\theta = \int_0^\pi \cos^2 u\left(\tfrac{2}{3}\,du\right) = \left[\tfrac{2}{3}\left(\tfrac{u}{2} + \tfrac{1}{4}\sin 2u\right)\right]_0^\pi = \tfrac{2}{3}\left(\tfrac{\pi}{2} + \tfrac{1}{4}\sin 2\pi\right) - \tfrac{2}{3}(0) = \tfrac{\pi}{3}$$

24. Let $u = 1 + \frac{1}{t} \Rightarrow du = -t^{-2}\,dt$; $t = -1 \Rightarrow u = 0$, $t = -\frac{1}{2} \Rightarrow u = -1$
$$\int_{-1}^{-1/2} t^{-2}\sin^2\left(1 + \tfrac{1}{t}\right)dt = \int_0^{-1} -\sin^2 u\,du = \left[-\left(\tfrac{u}{2} - \tfrac{1}{4}\sin 2u\right)\right]_0^{-1} = -\left[\left(-\tfrac{1}{2} - \tfrac{1}{4}\sin(-2)\right) - \left(\tfrac{0}{2} - \tfrac{1}{4}\sin 0\right)\right]$$
$$= \tfrac{1}{2} - \tfrac{1}{4}\sin 2$$

25. Let $u = 4 - x^2 \Rightarrow du = -2x\,dx \Rightarrow -\frac{1}{2}\,du = x\,dx$; $x = -2 \Rightarrow u = 0$, $x = 0 \Rightarrow u = 4$, $x = 2 \Rightarrow u = 0$
$$A = -\int_{-2}^0 x\sqrt{4 - x^2}\,dx + \int_0^2 x\sqrt{4 - x^2}\,dx = -\int_0^4 -\tfrac{1}{2}u^{1/2}\,du + \int_4^0 -\tfrac{1}{2}u^{1/2}\,du = 2\int_0^4 \tfrac{1}{2}u^{1/2}\,du = \int_0^4 u^{1/2}\,du$$
$$= \left[\tfrac{2}{3}u^{3/2}\right]_0^4 = \tfrac{2}{3}(4)^{3/2} - \tfrac{2}{3}(0)^{3/2} = \tfrac{16}{3}$$

26. Let $u = 1 - \cos x \Rightarrow du = \sin x\,dx$; $x = 0 \Rightarrow u = 0$, $x = \pi \Rightarrow u = 2$
$$\int_0^\pi (1 - \cos x)\sin x\,dx = \int_0^2 u\,du = \left[\tfrac{u^2}{2}\right]_0^2 = \tfrac{2^2}{2} - \tfrac{0^2}{2} = 2$$

27. Let $u = 1 + \cos x \Rightarrow du = -\sin x\, dx \Rightarrow -du = \sin x\, dx$; $x = -\pi \Rightarrow u = 1 + \cos(-\pi) = 0$, $x = 0 \Rightarrow u = 1 + \cos 0 = 2$
$A = -\int_{-\pi}^{0} 3(\sin x)\sqrt{1+\cos x}\, dx = -\int_0^2 3u^{1/2}(-du) = 3\int_0^2 u^{1/2}\, du = \left[2u^{3/2}\right]_0^2 = 2(2)^{3/2} - 2(0)^{3/2} = 2^{5/2}$

28. Let $u = \pi + \pi \sin x \Rightarrow du = \pi \cos x\, dx \Rightarrow \frac{1}{\pi}\, du = \cos x\, dx$; $x = -\frac{\pi}{2} \Rightarrow u = \pi + \pi \sin\left(-\frac{\pi}{2}\right) = 0$, $x = 0 \Rightarrow u = \pi$
Because of symmetry about $x = -\frac{\pi}{2}$, $A = 2\int_{-\pi/2}^{0} \frac{\pi}{2}(\cos x)(\sin(\pi + \pi \sin x))\, dx = 2\int_0^{\pi} \frac{\pi}{2}(\sin u)\left(\frac{1}{\pi}\, du\right)$
$= \int_0^{\pi} \sin u\, du = [-\cos u]_0^{\pi} = (-\cos \pi) - (-\cos 0) = 2$

29. For the sketch given, $a = 0$, $b = \pi$; $f(x) - g(x) = 1 - \cos^2 x = \sin^2 x = \frac{1-\cos 2x}{2}$;
$A = \int_0^{\pi} \frac{(1-\cos 2x)}{2}\, dx = \frac{1}{2}\int_0^{\pi}(1 - \cos 2x)\, dx = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi} = \frac{1}{2}[(\pi - 0) - (0 - 0)] = \frac{\pi}{2}$

30. For the sketch given, $a = -\frac{\pi}{3}$, $b = \frac{\pi}{3}$; $f(t) - g(t) = \frac{1}{2}\sec^2 t - (-4\sin^2 t) = \frac{1}{2}\sec^2 t + 4\sin^2 t$;
$A = \int_{-\pi/3}^{\pi/3}\left(\frac{1}{2}\sec^2 t + 4\sin^2 t\right) dt = \frac{1}{2}\int_{-\pi/3}^{\pi/3}\sec^2 t\, dt + 4\int_{-\pi/3}^{\pi/3}\sin^2 t\, dt = \frac{1}{2}\int_{-\pi/3}^{\pi/3}\sec^2 t\, dt + 4\int_{-\pi/3}^{\pi/3}\frac{(1-\cos 2t)}{2}\, dt$
$= \frac{1}{2}\int_{-\pi/3}^{\pi/3}\sec^2 t\, dt + 2\int_{-\pi/3}^{\pi/3}(1 - \cos 2t)\, dt = \frac{1}{2}[\tan t]_{-\pi/3}^{\pi/3} + 2\left[t - \frac{\sin 2t}{2}\right]_{-\pi/3}^{\pi/3} = \sqrt{3} + 4\cdot\frac{\pi}{3} - \sqrt{3} = \frac{4\pi}{3}$

31. For the sketch given, $a = -2$, $b = 2$; $f(x) - g(x) = 2x^2 - (x^4 - 2x^2) = 4x^2 - x^4$;
$A = \int_{-2}^{2}(4x^2 - x^4)\, dx = \left[\frac{4x^3}{3} - \frac{x^5}{5}\right]_{-2}^{2} = \left(\frac{32}{3} - \frac{32}{5}\right) - \left[-\frac{32}{3} - \left(-\frac{32}{5}\right)\right] = \frac{64}{3} - \frac{64}{5} = \frac{320-192}{15} = \frac{128}{15}$

32. For the sketch given, $c = 0$, $d = 1$; $f(y) - g(y) = y^2 - y^3$;
$A = \int_0^1 (y^2 - y^3)\, dy = \int_0^1 y^2\, dy - \int_0^1 y^3\, dy = \left[\frac{y^3}{3}\right]_0^1 - \left[\frac{y^4}{4}\right]_0^1 = \frac{(1-0)}{3} - \frac{(1-0)}{4} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

33. For the sketch given, $c = 0$, $d = 1$; $f(y) - g(y) = (12y^2 - 12y^3) - (2y^2 - 2y) = 10y^2 - 12y^3 + 2y$;
$A = \int_0^1 (10y^2 - 12y^3 + 2y)\, dy = \int_0^1 10y^2\, dy - \int_0^1 12y^3\, dy + \int_0^1 2y\, dy = \left[\frac{10}{3}y^3\right]_0^1 - \left[\frac{12}{4}y^4\right]_0^1 + \left[\frac{2}{2}y^2\right]_0^1$
$= \left(\frac{10}{3} - 0\right) - (3 - 0) + (1 - 0) = \frac{4}{3}$

34. For the sketch given, $a = -1$, $b = 1$; $f(x) - g(x) = x^2 - (-2x^4) = x^2 + 2x^4$;
$A = \int_{-1}^{1}(x^2 + 2x^4)\, dx = \left[\frac{x^3}{3} + \frac{2x^5}{5}\right]_{-1}^{1} = \left(\frac{1}{3} + \frac{2}{5}\right) - \left[-\frac{1}{3} + \left(-\frac{2}{5}\right)\right] = \frac{2}{3} + \frac{4}{5} = \frac{10+12}{15} = \frac{22}{15}$

35. We want the area between the line $y = 1$, $0 \le x \le 2$, and the curve $y = \frac{x^2}{4}$, *minus* the area of a triangle (formed by $y = x$ and $y = 1$) with base 1 and height 1. Thus, $A = \int_0^2 \left(1 - \frac{x^2}{4}\right) dx - \frac{1}{2}(1)(1) = \left[x - \frac{x^3}{12}\right]_0^2 - \frac{1}{2}$
$= \left(2 - \frac{8}{12}\right) - \frac{1}{2} = 2 - \frac{2}{3} - \frac{1}{2} = \frac{5}{6}$

36. We want the area between the x-axis and the curve $y = x^2$, $0 \le x \le 1$ *plus* the area of a triangle (formed by $x = 1$, $x + y = 2$, and the x-axis) with base 1 and height 1. Thus, $A = \int_0^1 x^2\, dx + \frac{1}{2}(1)(1) = \left[\frac{x^3}{3}\right]_0^1 + \frac{1}{2} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$

37. AREA = A1 + A2
A1: For the sketch given, $a = -3$ and we find b by solving the equations $y = x^2 - 4$ and $y = -x^2 - 2x$ simultaneously for x: $x^2 - 4 = -x^2 - 2x \Rightarrow 2x^2 + 2x - 4 = 0 \Rightarrow 2(x+2)(x-1) \Rightarrow x = -2$ or $x = 1$ so $b = -2$: $f(x) - g(x) = (x^2 - 4) - (-x^2 - 2x) = 2x^2 + 2x - 4 \Rightarrow A1 = \int_{-3}^{-2}(2x^2 + 2x - 4)\, dx$

$= \left[\frac{2x^3}{3} + \frac{2x^2}{2} - 4x\right]_{-3}^{-2} = \left(-\frac{16}{3} + 4 + 8\right) - (-18 + 9 + 12) = 9 - \frac{16}{3} = \frac{11}{3};$

A2: For the sketch given, $a = -2$ and $b = 1$: $f(x) - g(x) = (-x^2 - 2x) - (x^2 - 4) = -2x^2 - 2x + 4$

$\Rightarrow A2 = -\int_{-2}^{1}(2x^2 + 2x - 4)\,dx = -\left[\frac{2x^3}{3} + x^2 - 4x\right]_{-2}^{1} = -\left(\frac{2}{3} + 1 - 4\right) + \left(-\frac{16}{3} + 4 + 8\right)$

$= -\frac{2}{3} - 1 + 4 - \frac{16}{3} + 4 + 8 = 9;$

Therefore, AREA $= A1 + A2 = \frac{11}{3} + 9 = \frac{38}{3}$

38. AREA $= A1 + A2$

A1: For the sketch given, $a = -2$ and $b = 0$: $f(x) - g(x) = (2x^3 - x^2 - 5x) - (-x^2 + 3x) = 2x^3 - 8x$

$\Rightarrow A1 = \int_{-2}^{0}(2x^3 - 8x)\,dx = \left[\frac{2x^4}{4} - \frac{8x^2}{2}\right]_{-2}^{0} = 0 - (8 - 16) = 8;$

A2: For the sketch given, $a = 0$ and $b = 2$: $f(x) - g(x) = (-x^2 + 3x) - (2x^3 - x^2 - 5x) = 8x - 2x^3$

$\Rightarrow A2 = \int_{0}^{2}(8x - 2x^3)\,dx = \left[\frac{8x^2}{2} - \frac{2x^4}{4}\right]_{0}^{2} = (16 - 8) = 8;$

Therefore, AREA $= A1 + A2 = 16$

39. AREA $= A1 + A2 + A3$

A1: For the sketch given, $a = -2$ and $b = -1$: $f(x) - g(x) = (-x + 2) - (4 - x^2) = x^2 - x - 2$

$\Rightarrow A1 = \int_{-2}^{-1}(x^2 - x - 2)\,dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-2}^{-1} = \left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(-\frac{8}{3} - \frac{4}{2} + 4\right) = \frac{7}{3} - \frac{1}{2} = \frac{14-3}{6} = \frac{11}{6};$

A2: For the sketch given, $a = -1$ and $b = 2$: $f(x) - g(x) = (4 - x^2) - (-x + 2) = -(x^2 - x - 2)$

$\Rightarrow A2 = -\int_{-1}^{2}(x^2 - x - 2)\,dx = -\left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{-1}^{2} = -\left(\frac{8}{3} - \frac{4}{2} - 4\right) + \left(-\frac{1}{3} - \frac{1}{2} + 2\right) = -3 + 8 - \frac{1}{2} = \frac{9}{2};$

A3: For the sketch given, $a = 2$ and $b = 3$: $f(x) - g(x) = (-x + 2) - (4 - x^2) = x^2 - x - 2$

$\Rightarrow A3 = \int_{2}^{3}(x^2 - x - 2)\,dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_{2}^{3} = \left(\frac{27}{3} - \frac{9}{2} - 6\right) - \left(\frac{8}{3} - \frac{4}{2} - 4\right) = 9 - \frac{9}{2} - \frac{8}{3};$

Therefore, AREA $= A1 + A2 + A3 = \frac{11}{6} + \frac{9}{2} + \left(9 - \frac{9}{2} - \frac{8}{3}\right) = 9 - \frac{5}{6} = \frac{49}{6}$

40. AREA $= A1 + A2 + A3$

A1: For the sketch given, $a = -2$ and $b = 0$: $f(x) - g(x) = \left(\frac{x^3}{3} - x\right) - \frac{x}{3} = \frac{x^3}{3} - \frac{4}{3}x = \frac{1}{3}(x^3 - 4x)$

$\Rightarrow A1 = \frac{1}{3}\int_{-2}^{0}(x^3 - 4x)\,dx = \frac{1}{3}\left[\frac{x^4}{4} - 2x^2\right]_{-2}^{0} = 0 - \frac{1}{3}(4 - 8) = \frac{4}{3};$

A2: For the sketch given, $a = 0$ and we find b by solving the equations $y = \frac{x^3}{3} - x$ and $y = \frac{x}{3}$ simultaneously for x: $\frac{x^3}{3} - x = \frac{x}{3} \Rightarrow \frac{x^3}{3} - \frac{4}{3}x = 0 \Rightarrow \frac{x}{3}(x - 2)(x + 2) = 0 \Rightarrow x = -2, x = 0$, or $x = 2$ so $b = 2$:

$f(x) - g(x) = \frac{x}{3} - \left(\frac{x^3}{3} - x\right) = -\frac{1}{3}(x^3 - 4x) \Rightarrow A2 = -\frac{1}{3}\int_{0}^{2}(x^3 - 4x)\,dx = \frac{1}{3}\int_{0}^{2}(4x - x^3) = \frac{1}{3}\left[2x^2 - \frac{x^4}{4}\right]_{0}^{2}$

$= \frac{1}{3}(8 - 4) = \frac{4}{3};$

A3: For the sketch given, $a = 2$ and $b = 3$: $f(x) - g(x) = \left(\frac{x^3}{3} - x\right) - \frac{x}{3} = \frac{1}{3}(x^3 - 4x)$

$\Rightarrow A3 = \frac{1}{3}\int_{2}^{3}(x^3 - 4x)\,dx = \frac{1}{3}\left[\frac{x^4}{4} - 2x^2\right]_{2}^{3} = \frac{1}{3}\left[\left(\frac{81}{4} - 2\cdot 9\right) - \left(\frac{16}{4} - 8\right)\right] = \frac{1}{3}\left(\frac{81}{4} - 14\right) = \frac{25}{12};$

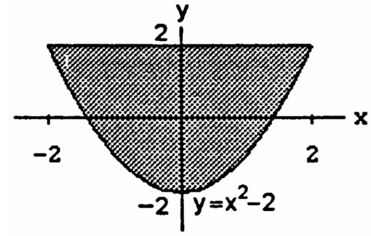
Therefore, AREA $= A1 + A2 + A3 = \frac{4}{3} + \frac{4}{3} + \frac{25}{12} = \frac{32+25}{12} = \frac{19}{4}$

41. $a = -2, b = 2;$

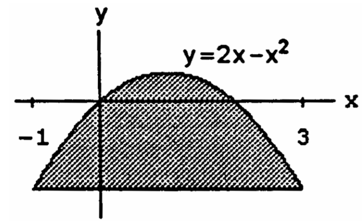
$f(x) - g(x) = 2 - (x^2 - 2) = 4 - x^2$

$\Rightarrow A = \int_{-2}^{2}(4 - x^2)dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$

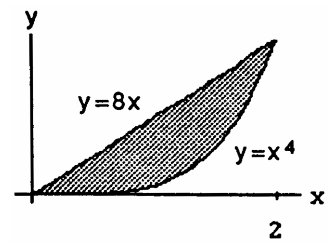
$= 2 \cdot \left(\frac{24}{3} - \frac{8}{3}\right) = \frac{32}{3}$



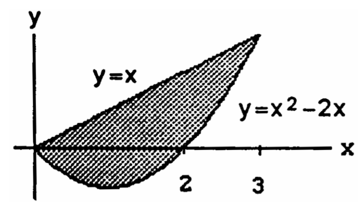
42. $a = -1, b = 3;$
$f(x) - g(x) = (2x - x^2) - (-3) = 2x - x^2 + 3$
$\Rightarrow A = \int_{-1}^{3} (2x - x^2 + 3)\, dx = \left[x^2 - \frac{x^3}{3} + 3x\right]_{-1}^{3}$
$= \left(9 - \frac{27}{3} + 9\right) - \left(1 + \frac{1}{3} - 3\right) = 11 - \frac{1}{3} = \frac{32}{3}$



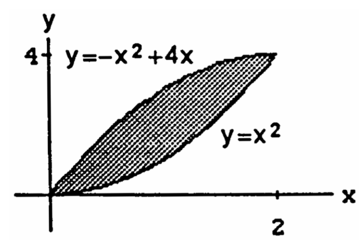
43. $a = 0, b = 2;$
$f(x) - g(x) = 8x - x^4 \Rightarrow A = \int_0^2 (8x - x^4)\, dx$
$= \left[\frac{8x^2}{2} - \frac{x^5}{5}\right]_0^2 = 16 - \frac{32}{5} = \frac{80-32}{5} = \frac{48}{5}$



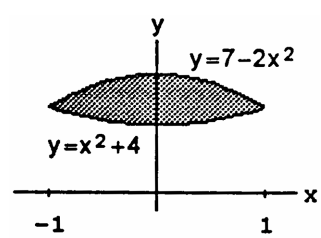
44. Limits of integration: $x^2 - 2x = x \Rightarrow x^2 = 3x$
$\Rightarrow x(x-3) = 0 \Rightarrow a = 0$ and $b = 3;$
$f(x) - g(x) = x - (x^2 - 2x) = 3x - x^2$
$\Rightarrow A = \int_0^3 (3x - x^2)\, dx = \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$
$= \frac{27}{2} - 9 = \frac{27-18}{2} = \frac{9}{2}$



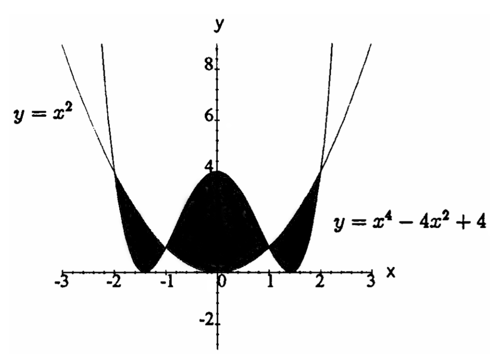
45. Limits of integration: $x^2 = -x^2 + 4x \Rightarrow 2x^2 - 4x = 0$
$\Rightarrow 2x(x-2) = 0 \Rightarrow a = 0$ and $b = 2;$
$f(x) - g(x) = (-x^2 + 4x) - x^2 = -2x^2 + 4x$
$\Rightarrow A = \int_0^2 (-2x^2 + 4x)\, dx = \left[\frac{-2x^3}{3} + \frac{4x^2}{2}\right]_0^2$
$= -\frac{16}{3} + \frac{16}{2} = \frac{-32+48}{6} = \frac{8}{3}$



46. Limits of integration: $7 - 2x^2 = x^2 + 4 \Rightarrow 3x^2 - 3 = 0$
$\Rightarrow 3(x-1)(x+1) = 0 \Rightarrow a = -1$ and $b = 1;$
$f(x) - g(x) = (7 - 2x^2) - (x^2 + 4) = 3 - 3x^2$
$\Rightarrow A = \int_{-1}^{1} (3 - 3x^2)\, dx = 3\left[x - \frac{x^3}{3}\right]_{-1}^{1}$
$= 3\left[\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)\right] = 6\left(\frac{2}{3}\right) = 4$

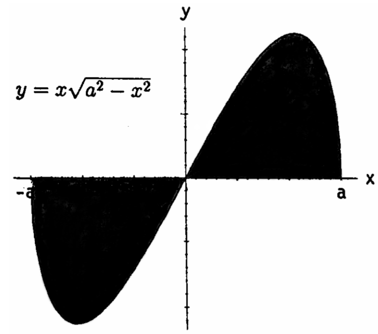


47. Limits of integration: $x^4 - 4x^2 + 4 = x^2$
$\Rightarrow x^4 - 5x^2 + 4 = 0 \Rightarrow (x^2 - 4)(x^2 - 1) = 0$
$\Rightarrow (x+2)(x-2)(x+1)(x-1) = 0 \Rightarrow x = -2, -1, 1, 2;$
$f(x) - g(x) = (x^4 - 4x^2 + 4) - x^2 = x^4 - 5x^2 + 4$ and
$g(x) - f(x) = x^2 - (x^4 - 4x^2 + 4) = -x^4 + 5x^2 - 4$
$\Rightarrow A = \int_{-2}^{-1} (-x^4 + 5x^2 - 4)dx + \int_{-1}^{1} (x^4 - 5x^2 + 4)dx$
$+ \int_1^2 (-x^4 + 5x^2 - 4)dx$

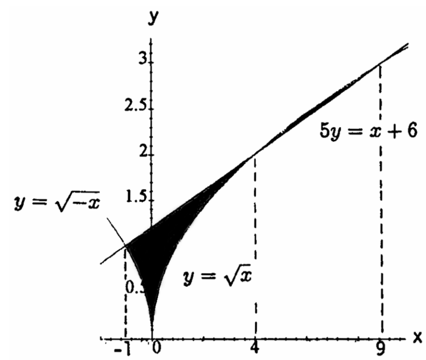


$= \left[-\frac{x^5}{5} + \frac{5x^3}{3} - 4x\right]_{-2}^{-1} + \left[\frac{x^5}{5} - \frac{5x^3}{3} + 4x\right]_{-1}^{1} + \left[\frac{-x^5}{5} + \frac{5x^3}{3} - 4x\right]_1^2$
$= \left(\frac{1}{5} - \frac{5}{3} + 4\right) - \left(\frac{32}{5} - \frac{40}{3} + 8\right) + \left(\frac{1}{5} - \frac{5}{3} + 4\right) - \left(-\frac{1}{5} + \frac{5}{3} - 4\right) + \left(-\frac{32}{5} + \frac{40}{3} - 8\right) - \left(-\frac{1}{5} + \frac{5}{3} - 4\right)$
$= -\frac{60}{5} + \frac{60}{3} = \frac{300-180}{15} = 8$

48. Limits of integration: $x\sqrt{a^2-x^2}=0 \Rightarrow x=0$ or $\sqrt{a^2-x^2}=0 \Rightarrow x=0$ or $a^2-x^2=0 \Rightarrow x=-a, 0, a$;
$A=\int_{-a}^{0}-x\sqrt{a^2-x^2}\,dx+\int_{0}^{a}x\sqrt{a^2-x^2}\,dx$
$=\frac{1}{2}\left[\frac{2}{3}\left(a^2-x^2\right)^{3/2}\right]_{-a}^{0}-\frac{1}{2}\left[\frac{2}{3}\left(a^2-x^2\right)^{3/2}\right]_{0}^{a}$
$=\frac{1}{3}\left(a^2\right)^{3/2}-\left[-\frac{1}{3}\left(a^2\right)^{3/2}\right]=\frac{2a^3}{3}$

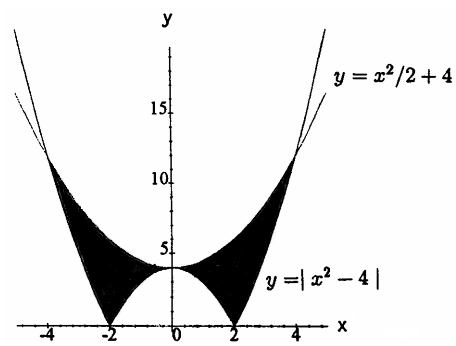


49. Limits of integration: $y=\sqrt{|x|}=\begin{cases}\sqrt{-x}, & x\le 0\\ \sqrt{x}, & x\ge 0\end{cases}$ and
$5y=x+6$ or $y=\frac{x}{5}+\frac{6}{5}$; for $x\le 0$: $\sqrt{-x}=\frac{x}{5}+\frac{6}{5}$
$\Rightarrow 5\sqrt{-x}=x+6 \Rightarrow 25(-x)=x^2+12x+36$
$\Rightarrow x^2+37x+36=0 \Rightarrow (x+1)(x+36)=0$
$\Rightarrow x=-1, -36$ (but $x=-36$ is not a solution);
for $x\ge 0$: $5\sqrt{x}=x+6 \Rightarrow 25x=x^2+12x+36$
$\Rightarrow x^2-13x+36=0 \Rightarrow (x-4)(x-9)=0$
$\Rightarrow x=4, 9$; there are three intersection points and



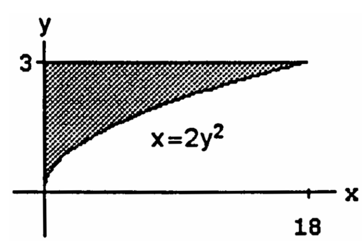
$A=\int_{-1}^{0}\left(\frac{x+6}{5}-\sqrt{-x}\right)dx+\int_{0}^{4}\left(\frac{x+6}{5}-\sqrt{x}\right)dx+\int_{4}^{9}\left(\sqrt{x}-\frac{x+6}{5}\right)dx$
$=\left[\frac{(x+6)^2}{10}+\frac{2}{3}(-x)^{3/2}\right]_{-1}^{0}+\left[\frac{(x+6)^2}{10}-\frac{2}{3}x^{3/2}\right]_{0}^{4}+\left[\frac{2}{3}x^{3/2}-\frac{(x+6)^2}{10}\right]_{4}^{9}$
$=\left(\frac{36}{10}-\frac{25}{10}-\frac{2}{3}\right)+\left(\frac{100}{10}-\frac{2}{3}\cdot 4^{3/2}-\frac{36}{10}+0\right)+\left(\frac{2}{3}\cdot 9^{3/2}-\frac{225}{10}-\frac{2}{3}\cdot 4^{3/2}+\frac{100}{10}\right)=-\frac{50}{10}+\frac{20}{3}=\frac{5}{3}$

50. Limits of integration:
$y=|x^2-4|=\begin{cases}x^2-4, & x\le -2 \text{ or } x\ge 2\\ 4-x^2, & -2\le x\le 2\end{cases}$
for $x\le -2$ and $x\ge 2$: $x^2-4=\frac{x^2}{2}+4$
$\Rightarrow 2x^2-8=x^2+8 \Rightarrow x^2=16 \Rightarrow x=\pm 4$;
for $-2\le x\le 2$: $4-x^2=\frac{x^2}{2}+4 \Rightarrow 8-2x^2=x^2+8$
$\Rightarrow x^2=0 \Rightarrow x=0$; by symmetry of the graph,

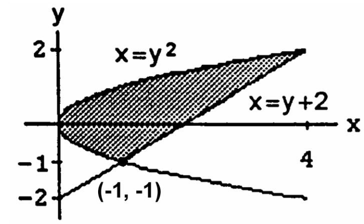


$A=2\int_{0}^{2}\left[\left(\frac{x^2}{2}+4\right)-\left(4-x^2\right)\right]dx+2\int_{2}^{4}\left[\left(\frac{x^2}{2}+4\right)-\left(x^2-4\right)\right]dx=2\left[\frac{x^3}{2}\right]_{0}^{2}+2\left[8x-\frac{x^3}{6}\right]_{2}^{4}$
$=2\left(\frac{8}{2}-0\right)+2\left(32-\frac{64}{6}-16+\frac{8}{6}\right)=40-\frac{56}{3}=\frac{64}{3}$

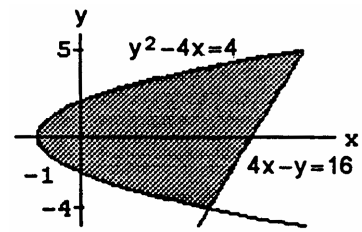
51. Limits of integration: $c=0$ and $d=3$;
$f(y)-g(y)=2y^2-0=2y^2$
$\Rightarrow A=\int_{0}^{3}2y^2\,dy=\left[\frac{2y^3}{3}\right]_{0}^{3}=2\cdot 9=18$



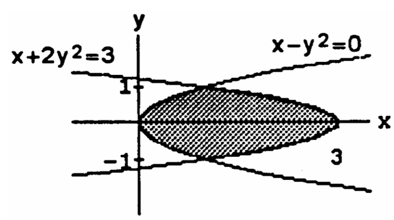
52. Limits of integration: $y^2 = y + 2 \Rightarrow (y+1)(y-2) = 0$
$\Rightarrow c = -1$ and $d = 2$; $f(y) - g(y) = (y + 2) - y^2$
$\Rightarrow A = \int_{-1}^{2} (y + 2 - y^2)\,dy = \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2}$
$= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = 6 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3} = \frac{9}{2}$



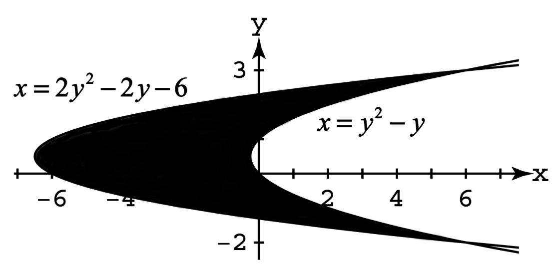
53. Limits of integration: $4x = y^2 - 4$ and $4x = 16 + y$
$\Rightarrow y^2 - 4 = 16 + y \Rightarrow y^2 - y - 20 = 0 \Rightarrow$
$(y-5)(y+4) = 0 \Rightarrow c = -4$ and $d = 5$;
$f(y) - g(y) = \left(\frac{16+y}{4}\right) - \left(\frac{y^2-4}{4}\right) = \frac{-y^2+y+20}{4}$
$\Rightarrow A = \frac{1}{4}\int_{-4}^{5} (-y^2 + y + 20)\,dy$
$= \frac{1}{4}\left[-\frac{y^3}{3} + \frac{y^2}{2} + 20y\right]_{-4}^{5}$
$= \frac{1}{4}\left(-\frac{125}{3} + \frac{25}{2} + 100\right) - \frac{1}{4}\left(\frac{64}{3} + \frac{16}{2} - 80\right)$
$= \frac{1}{4}\left(-\frac{189}{3} + \frac{9}{2} + 180\right) = \frac{243}{8}$



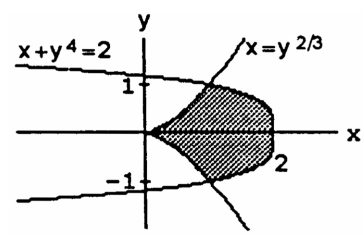
54. Limits of integration: $x = y^2$ and $x = 3 - 2y^2$
$\Rightarrow y^2 = 3 - 2y^2 \Rightarrow 3y^2 = 3 \Rightarrow 3(y-1)(y+1) = 0$
$\Rightarrow c = -1$ and $d = 1$; $f(y) - g(y) = (3 - 2y^2) - y^2$
$= 3 - 3y^2 = 3(1 - y^2) \Rightarrow A = 3\int_{-1}^{1} (1 - y^2)\,dy$
$= 3\left[y - \frac{y^3}{3}\right]_{-1}^{1} = 3\left(1 - \frac{1}{3}\right) - 3\left(-1 + \frac{1}{3}\right)$
$= 3 \cdot 2\left(1 - \frac{1}{3}\right) = 4$



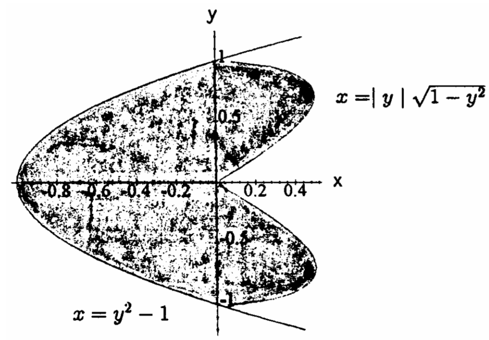
55. Limits of integration: $x = y^2 - y$ and $x = 2y^2 - 2y - 6$
$\Rightarrow y^2 - y = 2y^2 - 2y - 6 \Rightarrow y^2 - y - 6 = 0$
$\Rightarrow (y-3)(y+2) = 0 \Rightarrow c = -2$ and $d = 3$;
$f(y) - g(y) = (y^2 - y) - (2y^2 - 2y - 6) = -y^2 + y + 6$
$\Rightarrow A = \int_{-2}^{3} (-y^2 + y + 6)\,dy = \left[-\frac{y^3}{3} + \frac{1}{2}y^2 + 6y\right]_{-2}^{3}$
$= \left(-9 + \frac{9}{2} + 18\right) - \left(\frac{8}{3} + 2 - 12\right) = \frac{125}{6}$



56. Limits of integration: $x = y^{2/3}$ and $x = 2 - y^4$
$\Rightarrow y^{2/3} = 2 - y^4 \Rightarrow c = -1$ and $d = 1$;
$f(y) - g(y) = (2 - y^4) - y^{2/3}$
$\Rightarrow A = \int_{-1}^{1} \left(2 - y^4 - y^{2/3}\right)dy$
$= \left[2y - \frac{y^5}{5} - \frac{3}{5}y^{5/3}\right]_{-1}^{1}$
$= \left(2 - \frac{1}{5} - \frac{3}{5}\right) - \left(-2 + \frac{1}{5} + \frac{3}{5}\right)$
$= 2\left(2 - \frac{1}{5} - \frac{3}{5}\right) = \frac{12}{5}$

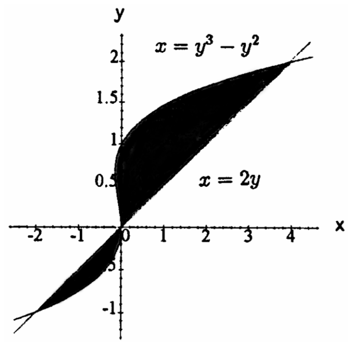


57. Limits of integration: $x = y^2 - 1$ and $x = |y|\sqrt{1-y^2}$
$\Rightarrow\ y^2 - 1 = |y|\sqrt{1-y^2}\ \Rightarrow\ y^4 - 2y^2 + 1 = y^2(1-y^2)$
$\Rightarrow\ y^4 - 2y^2 + 1 = y^2 - y^4\ \Rightarrow\ 2y^4 - 3y^2 + 1 = 0$
$\Rightarrow\ (2y^2-1)(y^2-1) = 0\ \Rightarrow\ 2y^2 - 1 = 0$ or $y^2 - 1 = 0$
$\Rightarrow\ y^2 = \frac{1}{2}$ or $y^2 = 1\ \Rightarrow\ y = \pm\frac{\sqrt{2}}{2}$ or $y = \pm 1$.
Substitution shows that $\frac{\pm\sqrt{2}}{2}$ are not solutions $\Rightarrow\ y = \pm 1$;
for $-1 \le y \le 0$, $f(x) - g(x) = -y\sqrt{1-y^2} - (y^2 - 1)$
$= 1 - y^2 - y(1-y^2)^{1/2}$, and by symmetry of the graph,

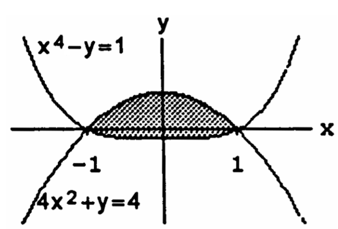


$$A = 2\int_{-1}^{0}\left[1 - y^2 - y(1-y^2)^{1/2}\right] dy$$
$$= 2\int_{-1}^{0}(1-y^2)\, dy - 2\int_{-1}^{0} y(1-y^2)^{1/2}\, dy = 2\left[y - \frac{y^3}{3}\right]_{-1}^{0} + 2\left(\frac{1}{2}\right)\left[\frac{2(1-y^2)^{3/2}}{3}\right]_{-1}^{0}$$
$$= 2\left[(0-0) - \left(-1 + \frac{1}{3}\right)\right] + \left(\frac{2}{3} - 0\right) = 2$$

58. AREA = A1 + A2
Limits of integration: $x = 2y$ and $x = y^3 - y^2\ \Rightarrow$
$y^3 - y^2 = 2y \Rightarrow\ y(y^2 - y - 2) = y(y+1)(y-2) = 0$
$\Rightarrow\ y = -1, 0, 2$:
for $-1 \le y \le 0$, $f(y) - g(y) = y^3 - y^2 - 2y$
$$\Rightarrow\ A1 = \int_{-1}^{0}(y^3 - y^2 - 2y)\, dy = \left[\frac{y^4}{4} - \frac{y^3}{3} - y^2\right]_{-1}^{0}$$
$= 0 - \left(\frac{1}{4} + \frac{1}{3} - 1\right) = \frac{5}{12}$;
for $0 \le y \le 2$, $f(y) - g(y) = 2y - y^3 + y^2$
$$\Rightarrow\ A2 = \int_{0}^{2}(2y - y^3 + y^2)\, dy = \left[y^2 - \frac{y^4}{4} + \frac{y^3}{3}\right]_{0}^{2}$$
$\Rightarrow\ \left(4 - \frac{16}{4} + \frac{8}{3}\right) - 0 = \frac{8}{3}$;
Therefore, $A1 + A2 = \frac{5}{12} + \frac{8}{3} = \frac{37}{12}$



59. Limits of integration: $y = -4x^2 + 4$ and $y = x^4 - 1$
$\Rightarrow\ x^4 - 1 = -4x^2 + 4\ \Rightarrow\ x^4 + 4x^2 - 5 = 0$
$\Rightarrow\ (x^2+5)(x-1)(x+1) = 0\ \Rightarrow\ a = -1$ and $b = 1$;
$f(x) - g(x) = -4x^2 + 4 - x^4 + 1 = -4x^2 - x^4 + 5$
$$\Rightarrow\ A = \int_{-1}^{1}(-4x^2 - x^4 + 5)\, dx = \left[-\frac{4x^3}{3} - \frac{x^5}{5} + 5x\right]_{-1}^{1}$$
$= \left(-\frac{4}{3} - \frac{1}{5} + 5\right) - \left(\frac{4}{3} + \frac{1}{5} - 5\right) = 2\left(-\frac{4}{3} - \frac{1}{5} + 5\right) = \frac{104}{15}$



60. Limits of integration: $y = x^3$ and $y = 3x^2 - 4$
$\Rightarrow\ x^3 - 3x^2 + 4 = 0\ \Rightarrow\ (x^2 - x - 2)(x - 2) = 0$
$\Rightarrow\ (x+1)(x-2)^2 = 0\ \Rightarrow\ a = -1$ and $b = 2$;
$f(x) - g(x) = x^3 - (3x^2 - 4) = x^3 - 3x^2 + 4$
$$\Rightarrow\ A = \int_{-1}^{2}(x^3 - 3x^2 + 4)\, dx = \left[\frac{x^4}{4} - \frac{3x^3}{3} + 4x\right]_{-1}^{2}$$
$= \left(\frac{16}{4} - \frac{24}{3} + 8\right) - \left(\frac{1}{4} + 1 - 4\right) = \frac{27}{4}$