Software-Defined Networking and Advanced Network Control Programming

5 Mathematical Tools

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Recap

Relation Between Automata and Languages

Different automatons (自动机) represents different languages

Automaton	Language
finite state machine (NFA & DFA)	regular language
确定性/非确定性有限状态机	正则语言
pushdown automaton (PDA)	context-free language
下推自动机	上下文无关语言
linear-bounded automaton (LBA)	context-sensitive language
线性有界自动机	上下文有关语言
Turing machine	recursively enumerable language
图灵机	递归可枚举语言

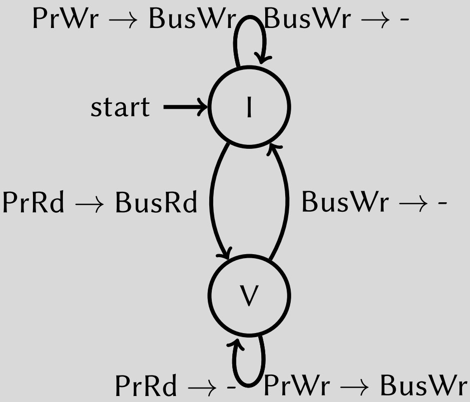
Write Invalidate Protocol

Each cache block is in one of two potential states:

- **Valid** (V): the cache block is up-to-date
- **Invalid** (I): the cache block is not valid

Events (transitions):

- Processor: read (PrRd), write (PrWr)
- Bus: read signal (BusRd), write signal (BusWr)

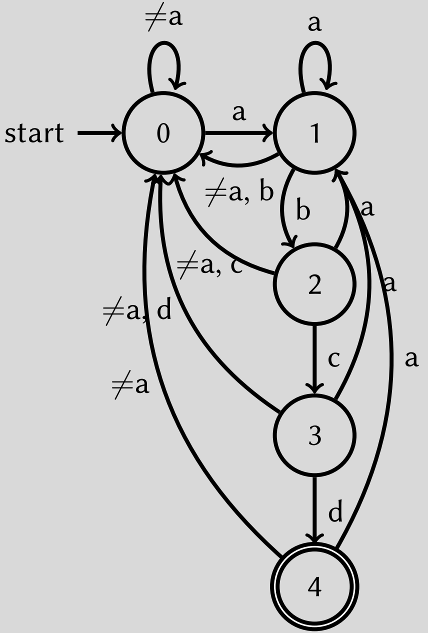


String Matching

Example: Can a string `aaababcabcdefg` be matched by `abcd`?

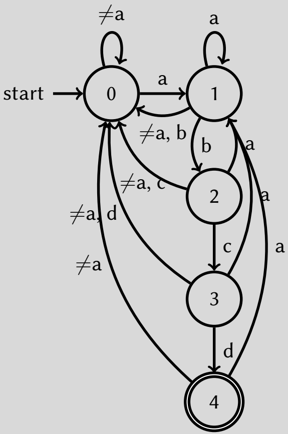
Build an automata as the right:

- Five states:
  - 0: Match nothing
  - 1: Match `a`
  - 2: Match `ab`
  - 3: Match `abc`
  - 4: Match `abcd`: the accepting state
- If the accepting state is reached, a match of `abcd` is found



Representations of Automatons

Graph representation:

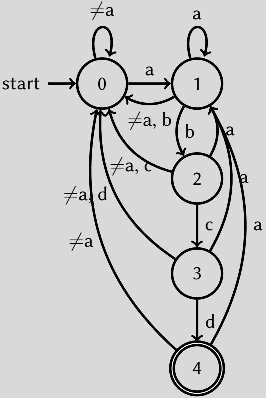


Tabular representation:

	a	b	c	d	x
→ 0	1	0	0	0	0
1	1	2	0	0	0
2	1	0	3	0	0
3	1	0	0	4	0
* 4	4	4	4	4	4

Product of Automata

Automaton for abcd



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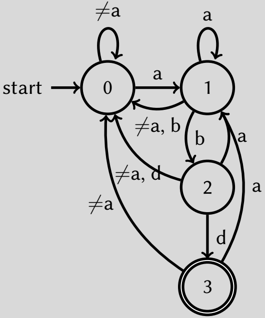
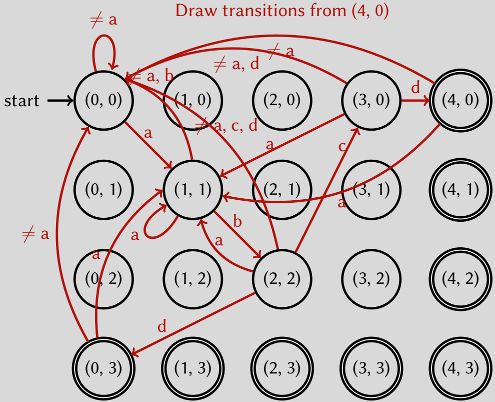


Product Automaton:



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In the coming lectures, we cover the following topics:

- Automata theory
- Linear programming
- Integer linear programming

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In the coming lectures, we cover the following topics:

- Automata theory
- Linear programming
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In this lecture, you should

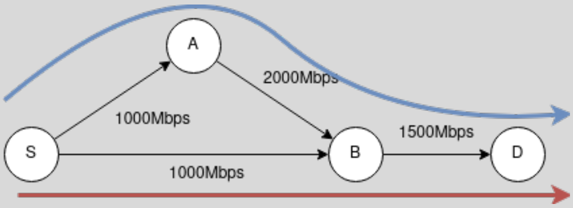
- understand linear programming (线性规划, LP) and its standard form (标准型)
- understand integer linear program (整数线性规划, ILP)

Linear Programming

An Example Problem

Consider the following problem:

- A large file is to be transferred from S to D



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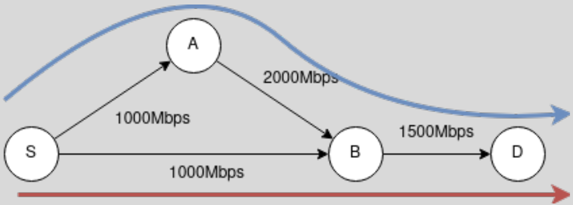

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Source	*Destination*	*Capacity*
S	A	1000
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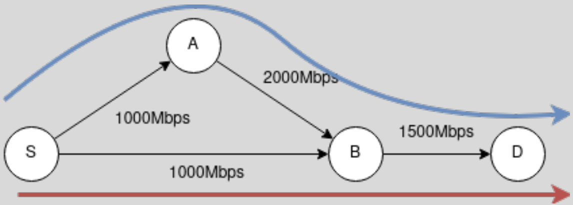
A	B	2000
B	D	1500



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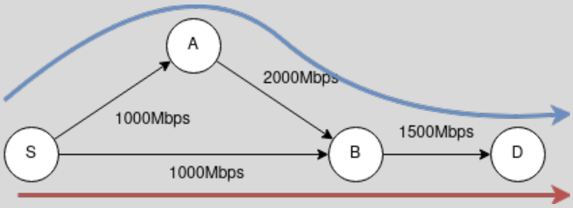
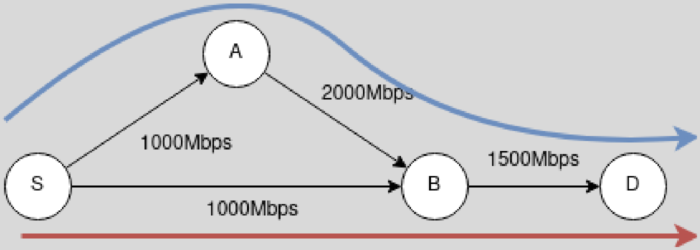
- Question: What is the maximum transfer speed?



Formulate the Problem



- **Decision variables (决策变量):**
  - the transfer speed on path $S \to A \to B \to D$: x_1
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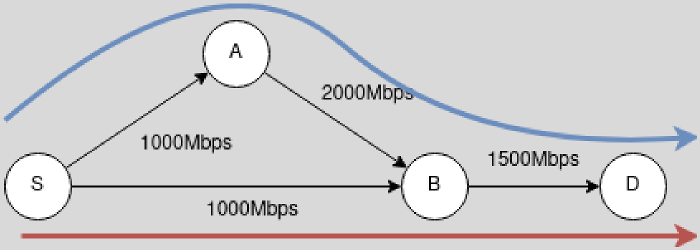

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  - $S \to A$: $x_1 \leq 1000$
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  - $B \to D$: $x_1 + x_2 \leq 1500$

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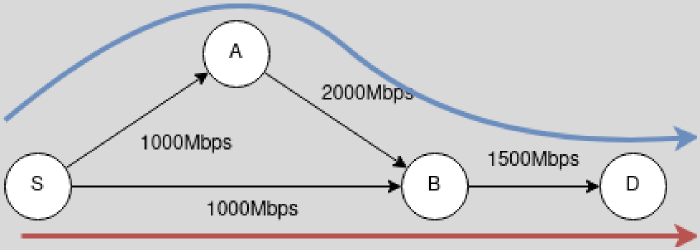

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- **Objective (优化目标):** maximizing the total transfer speed $z = x_1 + x_2$

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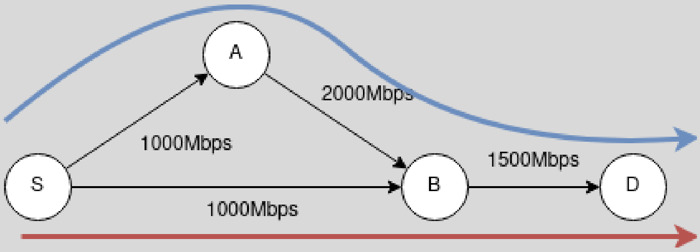

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$$\max x_1 + x_2$$

subject to:

$$\left.\begin{array}{rcrl} x_1 & & & \leq 1000 \\ & & x_2 & \leq 1000 \\ x_1 & & & \leq 2000 \\ x_1 & + & x_2 & \leq 1500 \end{array}\right\}$$

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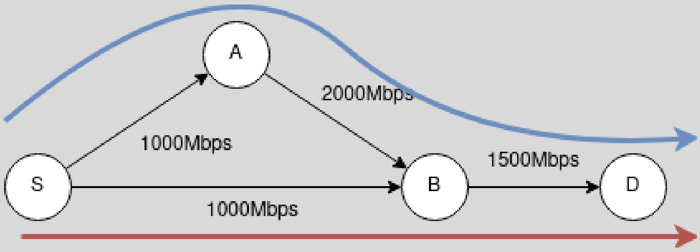

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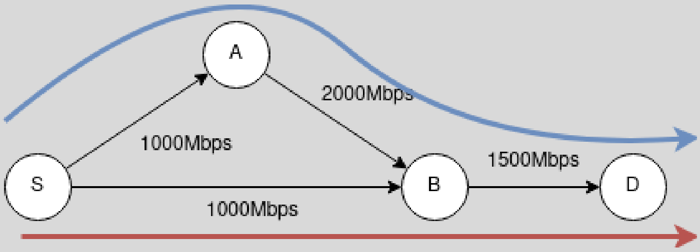

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Optimization Problem in General

An optimization problem in general consists of:

- a set of continuous decision variables (连续变量): $x_1, \ldots, x_N$
- an objective function (目标函数) of the decision variables: $f : X_1 \times \cdots \times X_N \mapsto R$
- a set of constraint functions (约束函数) of the decision variables: $g_1, \ldots, g_M$, $\forall i$, $g_i : X_1 \times \cdots \times X_N \mapsto R$

The standard form of such an optimization problem can be written as:

$$\max f(x_1, \ldots, x_N)$$

subject to:

$$g_1(x_1, \ldots, x_N) = b_1$$
$$\vdots$$
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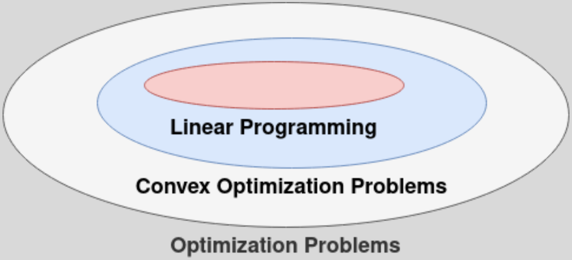
NOTE: There are other variants of standard forms.

Types of Optimization Problems

- Convex optimization problem (COP, 凸优化问题): the objective function and constraint functions are convex



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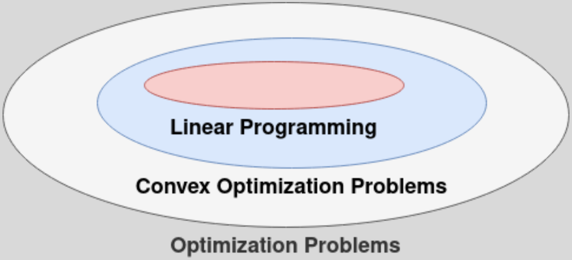

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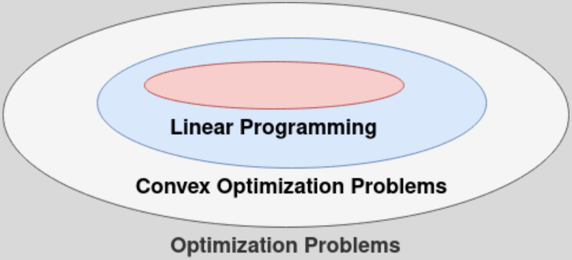
- Linear programming problem (LP, 线性规划问题): the objective function and constraint functions are linear
- The smaller the scope is, the more restricted the objective and constraint functions are, and more efficient algorithms exist



Standard Form of LP in the Matrix Representation

- decision variables:

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

- coefficients of the objective function:

$$\boldsymbol{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_N \end{bmatrix}$$

- coefficients of the constraint functions:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & \dots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{M1} & \dots & a_{MN} \end{bmatrix}$$

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The problem can be formulated as:

$$\max \boldsymbol{c}^T \boldsymbol{x}$$

$$f(\boldsymbol{x}) = \begin{bmatrix} c_1, & \dots, & c_N \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}$$

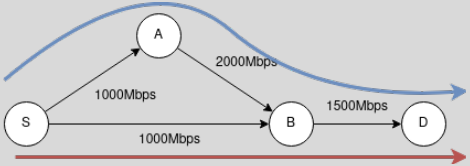
subject to:

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$$

$$\begin{bmatrix} a_{11} & \dots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{M1} & \dots & a_{MN} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_M \end{bmatrix}$$

Transforming Non-standard Form into Standard Form

Inequalities with $\leq$ sign



Original formulation:

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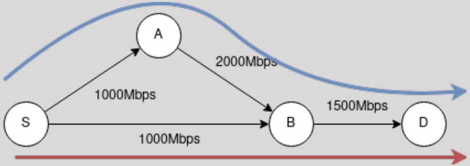

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Standard form:

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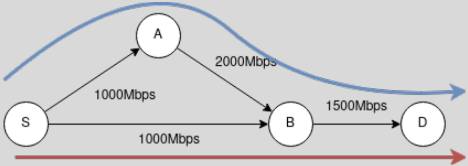

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s_1, s_2, s_3 and s_4 are known as **slack variables** (松弛变量)

Transforming Non-standard Form to Standard Form

Inequalities with $\geq$ sign

Slack variables can transform an inequality with $\leq$, for inequalities with $\geq$, the coefficient of the slack variable should be -1

Example:

$$x_1 + x_2 \geq 50 \Rightarrow x_1 + x_2 - s_1 = 50$$

Sometimes referred to as surplus variable (剩余变量)

Transforming Non-standard Form to Standard Form

Negative or Free Variables

The standard form requires variables to be non-negative. Negative or free variables can be transformed by substitution

Example:

$$\max(x_1 + x_2)$$

subject to:

$$\begin{aligned} x_1 &\leq 0 \\ -2 \leq x_2 &\leq 5 \\ 2x_1 + 3x_2 &\leq 10 \end{aligned}$$

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Let $x_1 = -y_1$, so $y_1 \geq 0$. Let $x_2 = y_2 - y_3$ and $y_2 \geq 0$, $y_3 \geq 0$

Standard form:

$$\max(-y_1) + (y_2 - y_3)$$

subject to:

$$y_2 - y_3 - s_1 = -2$$
$$y_2 - y_3 + s_2 = 5$$
$$-2y_1 + 3y_2 - 3y_3 + s_3 = 10$$

Note on Transforming to Standard Form

- The use of slack variables and substitution are also useful in other optimization techniques, e.g., the Lagrange multiplier (拉格朗日乘子) method
- The transformation is used internally in LP solvers
- Modern LP libraries provide supports for inequality constraints and free variables

Solving Linear Programming

LP algorithms:

- Geometric method: for 2-variable LP
- Simplex method (单纯形法): practically fast, worst-case exponential
- Khachiyan algorithm (1979): polynomial algorithm based on ellipsoid method (椭圆法)
- Karmarkar algorithm (1984): polynomial algorithm based on interior point method (内点法)

Convex optimization algorithms:

- Alternating direction method of multipliers (交替方向乘子法, ADMM)
- Gradient descent algorithm (梯度下降法)

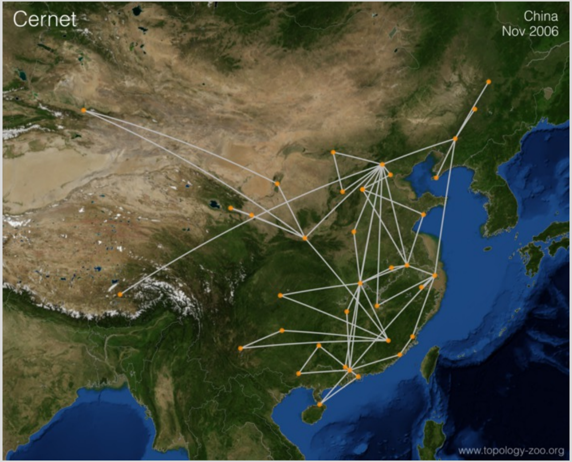
Leonid Genrikhovich Khachiyan. "A Polynomial Algorithm in Linear Programming". In: *Doklady Akademii Nauk*. Vol. 244. Russian Academy of Sciences, 1979, pp. 1093–1096

Narendra Karmarkar. "A New Polynomial-Time Algorithm for Linear Programming". In: *Proceedings of the Sixteenth Annual ACM Symposium on Theory of Computing*. ACM, 1984, pp. 302–311

Traffic Engineering (I)

Traffic engineering (TE, 流量工程) is a real networking problem in ISP networks. We consider a simplified problem:

- Assume there are N nodes and M uni-directional links, the i-th link has a source s_i and the destination d_i



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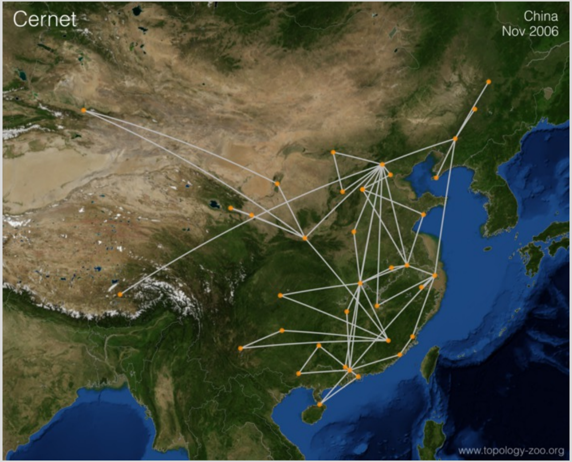

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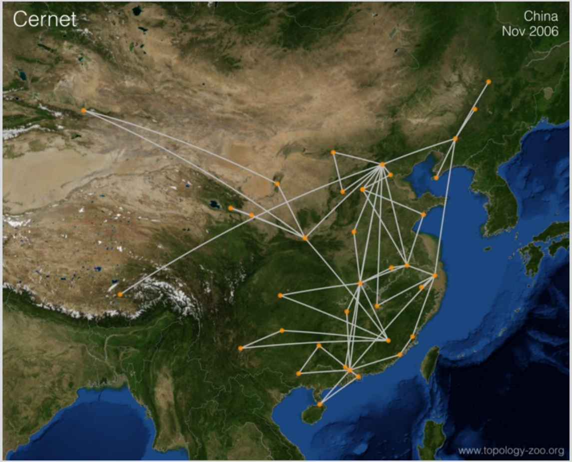

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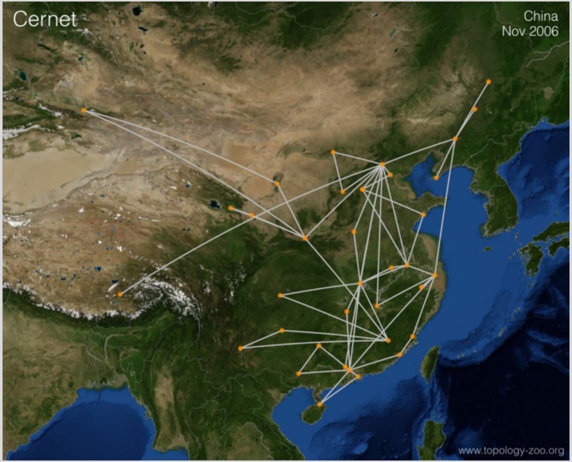

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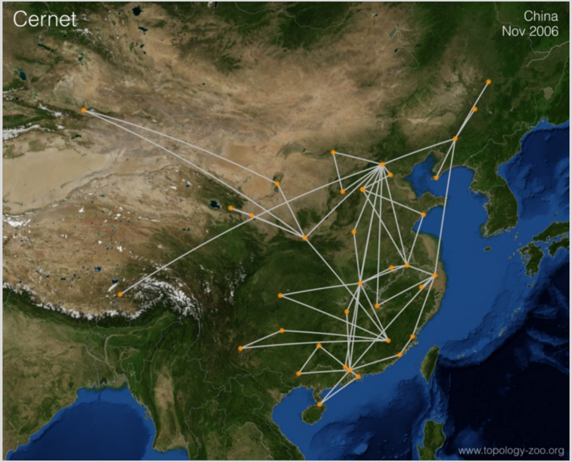
- Objective: to minimize the maximum link utilization



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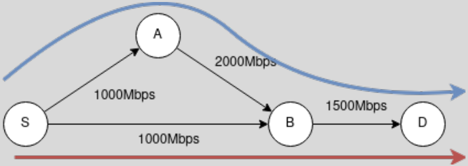
Example



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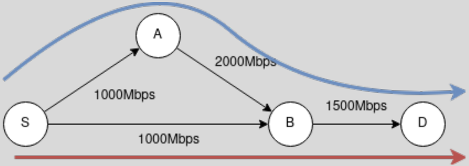

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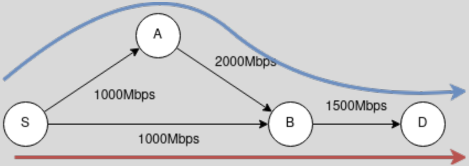
- Links: $\{l_1 = (S, A), l_2 = (A, S), l_3 = (S, B), l_4 = (B, S), l_5 = (A, B), l_6 = (B, A), l_7 = (B, D), l_8 = (D, B)\}$

Example



- Nodes: $\{v_1 = S, v_2 = A, v_3 = B, v_4 = D\}$
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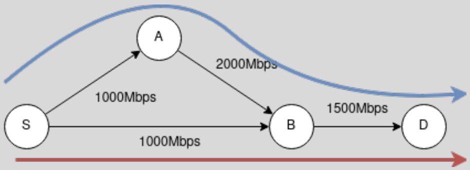
- Link capacity: $c_1 = c_2 = 1000$, $c_3 = c_4 = 1000$, $c_5 = c_6 = 2000$, $c_7 = c_8 = 1500$

Example

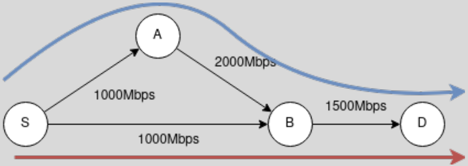


- Nodes: $\{v_1 = S, v_2 = A, v_3 = B, v_4 = D\}$
- Links: $\{l_1 = (S, A), l_2 = (A, S), l_3 = (S, B), l_4 = (B, S), l_5 = (A, B), l_6 = (B, A), l_7 = (B, D), l_8 = (D, B)\}$
- Link capacity: $c_1 = c_2 = 1000$, $c_3 = c_4 = 1000$, $c_5 = c_6 = 2000$, $c_7 = c_8 = 1500$
- Traffic matrix: assume we need 500 Mbps between S and B, and 1000 Mbps between S and D

$$T = \begin{bmatrix} 0 & 500 & 0 & 1000 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Example (cont.)

Understanding Link Utilization (链路利用率)



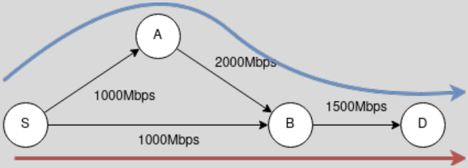
$S \rightarrow B$: 500 Mbps

$S \rightarrow D$: 1000 Mbps

Assume the traffic from S to B is using path $S \rightarrow B$, and the traffic from S to D is using path $S \rightarrow A \rightarrow B \rightarrow D$:

Example (cont.)

Understanding Link Utilization (链路利用率)



$S \to B$: 500 Mbps

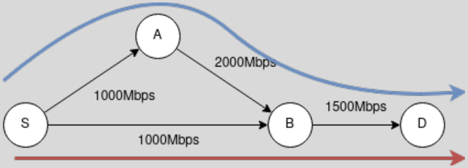
$S \to D$: 1000 Mbps

Assume the traffic from S to B is using path $S \to B$, and the traffic from S to D is using path $S \to A \to B \to D$:

- Total traffic on link $S \to A$: 1000 Mbps, link utilization $u_1 = 1000/1000 = 1$

Example (cont.)

Understanding Link Utilization (链路利用率)



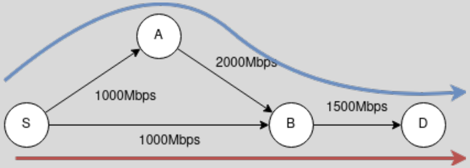
$S \rightarrow B$: 500 Mbps
$S \rightarrow D$: 1000 Mbps

Assume the traffic from S to B is using path $S \rightarrow B$, and the traffic from S to D is using path $S \rightarrow A \rightarrow B \rightarrow D$:

- Total traffic on link $S \rightarrow A$: 1000 Mbps, link utilization $u_1 = 1000/1000 = 1$
- Total traffic on link $S \rightarrow B$: 500 Mbps, link utilization $u_3 = 500/1000 = 0.5$

Example (cont.)

Understanding Link Utilization (链路利用率)



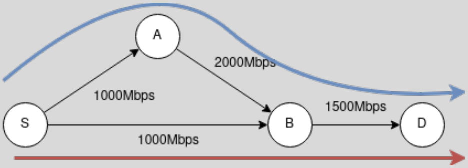
$S \to B$: 500 Mbps
$S \to D$: 1000 Mbps

Assume the traffic from S to B is using path $S \to B$, and the traffic from S to D is using path $S \to A \to B \to D$:

- Total traffic on link $S \to A$: 1000 Mbps, link utilization $u_1 = 1000/1000 = 1$
- Total traffic on link $S \to B$: 500 Mbps, link utilization $u_3 = 500/1000 = 0.5$
- Total traffic on link $A \to B$: 1000 Mbps, link utilization $u_5 = 1000/2000 = 0.5$

Example (cont.)

Understanding Link Utilization (链路利用率)



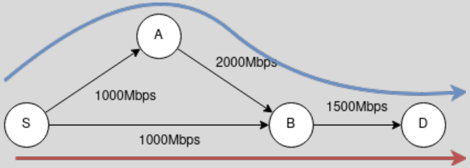
$S \to B$: 500 Mbps
$S \to D$: 1000 Mbps

Assume the traffic from S to B is using path $S \to B$, and the traffic from S to D is using path $S \to A \to B \to D$:

- Total traffic on link $S \to A$: 1000 Mbps, link utilization $u_1 = 1000/1000 = 1$
- Total traffic on link $S \to B$: 500 Mbps, link utilization $u_3 = 500/1000 = 0.5$
- Total traffic on link $A \to B$: 1000 Mbps, link utilization $u_5 = 1000/2000 = 0.5$
- Total traffic on link $B \to D$: 1000 Mbps, link utilization $u_7 = 1000/1500 = 0.67$

Example (cont.)

Understanding Link Utilization (链路利用率)



$S \to B$: 500 Mbps
$S \to D$: 1000 Mbps

Assume the traffic from S to B is using path $S \to B$, and the traffic from S to D is using path $S \to A \to B \to D$:

- Total traffic on link $S \to A$: 1000 Mbps, link utilization $u_1 = 1000/1000 = 1$
- Total traffic on link $S \to B$: 500 Mbps, link utilization $u_3 = 500/1000 = 0.5$
- Total traffic on link $A \to B$: 1000 Mbps, link utilization $u_5 = 1000/2000 = 0.5$
- Total traffic on link $B \to D$: 1000 Mbps, link utilization $u_7 = 1000/1500 = 0.67$
- Maximum link utilization
  $u_{\max} = \max\{u_1 = 1, u_3 = 0.5, u_5 = 0.5, u_7 = 0.67, u_2 = u_4 = u_6 = u_8 = 0\} = 1$

Problem Formulation

- Decision variables:

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  - Traffic split ratio: let x_{ijk} denote the traffic from node i to node j on link k

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- Constraints:

Problem Formulation

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- Constraints:
  - Traffic source:

$$\forall i, j, \sum_{s_k=i} x_{ijk} = t_{ij} \qquad \forall i, j, \sum_{d_k=i} x_{ijk} = 0$$

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  - Traffic sink:

$$\forall i, j, \sum_{d_k=j} x_{ijk} = t_{ij} \qquad \forall i, j, \sum_{s_k=j} x_{ijk} = 0$$

Problem Formulation

- Decision variables:
  - Traffic split ratio: let x_{ijk} denote the traffic from node i to node j on link k
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  - Traffic source:

$$\forall i, j, \sum_{s_k=i} x_{ijk} = t_{ij} \qquad \forall i, j, \sum_{d_k=i} x_{ijk} = 0$$

  - Traffic sink:

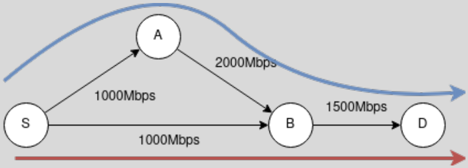
$$\forall i, j, \sum_{d_k=j} x_{ijk} = t_{ij} \qquad \forall i, j, \sum_{s_k=j} x_{ijk} = 0$$

  - Traffic on intermediate node

$$\forall i, j, p \neq i, p \neq j, \sum_{s_k=p} x_{ijk} = \sum_{d_k=p} x_{ijk}$$

Constraint Example

Traffic at the Source Node



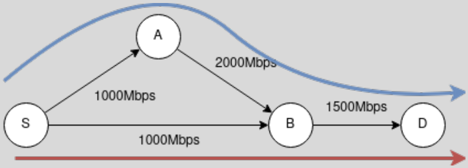
$S \rightarrow B$: 500 Mbps
$S \rightarrow D$: 1000 Mbps

Consider the traffic from S to D:

- At node S, there are two potential links $l_1 = (S, A)$ and $l_3 = (S, B)$ whose source is S, and $l_2 = (A, S)$ and $l_4 = (B, S)$ whose destination is S

Constraint Example

Traffic at the Source Node



$S \rightarrow B$: 500 Mbps
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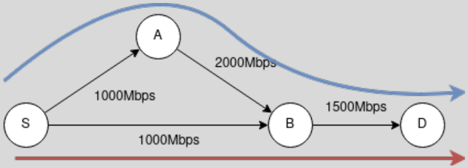
Consider the traffic from S to D:

- At node S, there are two potential links $l_1 = (S, A)$ and $l_3 = (S, B)$ whose source is S, and $l_2 = (A, S)$ and $l_4 = (B, S)$ whose destination is S
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$

Constraint Example

Traffic at the Source Node



$S \rightarrow B$: 500 Mbps
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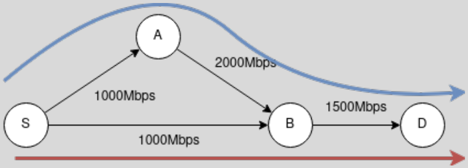
Consider the traffic from S to D:

- At node S, there are two potential links $l_1 = (S, A)$ and $l_3 = (S, B)$ whose source is S, and $l_2 = (A, S)$ and $l_4 = (B, S)$ whose destination is S
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$
  - $x_{S,D,3}$: Traffic from S to D that uses link $l_3 = (S, B)$

Constraint Example

Traffic at the Source Node



$S \to B$: 500 Mbps
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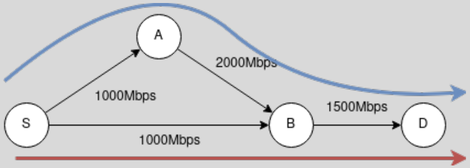
Consider the traffic from S to D:

- At node S, there are two potential links $l_1 = (S, A)$ and $l_3 = (S, B)$ whose source is S, and $l_2 = (A, S)$ and $l_4 = (B, S)$ whose destination is S
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$
  - $x_{S,D,3}$: Traffic from S to D that uses link $l_3 = (S, B)$
  - $x_{S,D,1} + x_{S,D,3} = t_{S,D} = 1000$

Constraint Example

Traffic at the Source Node



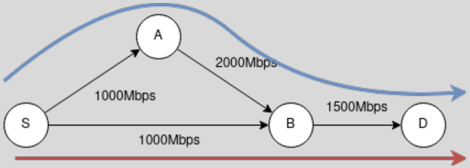
$S \to B$: 500 Mbps
$S \to D$: 1000 Mbps

Consider the traffic from S to D:

- At node S, there are two potential links $l_1 = (S, A)$ and $l_3 = (S, B)$ whose source is S, and $l_2 = (A, S)$ and $l_4 = (B, S)$ whose destination is S
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$
  - $x_{S,D,3}$: Traffic from S to D that uses link $l_3 = (S, B)$
  - $x_{S,D,1} + x_{S,D,3} = t_{S,D} = 1000$
  - $x_{S,D,2}$: Traffic from S to D that uses link $l_2 = (A, S)$

Constraint Example

Traffic at the Source Node



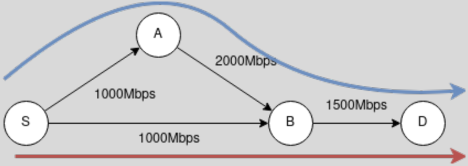
$S \to B$: 500 Mbps
$S \to D$: 1000 Mbps

Consider the traffic from S to D:

- At node S, there are two potential links $l_1 = (S, A)$ and $l_3 = (S, B)$ whose source is S, and $l_2 = (A, S)$ and $l_4 = (B, S)$ whose destination is S
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$
  - $x_{S,D,3}$: Traffic from S to D that uses link $l_3 = (S, B)$
  - $x_{S,D,1} + x_{S,D,3} = t_{S,D} = 1000$
  - $x_{S,D,2}$: Traffic from S to D that uses link $l_2 = (A, S)$
  - $x_{S,D,4}$: Traffic from S to D that uses link $l_4 = (B, S)$

Constraint Example

Traffic at the Source Node



$S \to B$: 500 Mbps
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Consider the traffic from S to D:

- At node S, there are two potential links $l_1 = (S, A)$ and $l_3 = (S, B)$ whose source is S, and $l_2 = (A, S)$ and $l_4 = (B, S)$ whose destination is S
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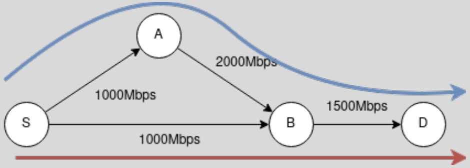
  - $x_{S,D,1} + x_{S,D,3} = t_{S,D} = 1000$
  - $x_{S,D,2}$: Traffic from S to D that uses link $l_2 = (A, S)$
  - $x_{S,D,4}$: Traffic from S to D that uses link $l_4 = (B, S)$
  - $x_{S,D,2} + x_{S,D,4} = 0$

Constraint Example (cont.)

Traffic at the Destination Node



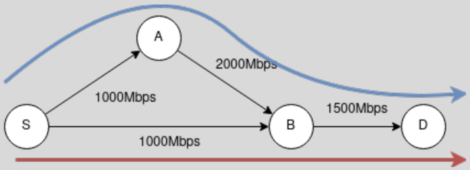
$S \to B$: 500 Mbps
$S \to D$: 1000 Mbps

Consider the traffic from S to D:

- At node D, there are one potential link $l_7 = (B, D)$ whose destination is D, and $l_8 = (D, B)$ whose source is D

Constraint Example (cont.)

Traffic at the Destination Node



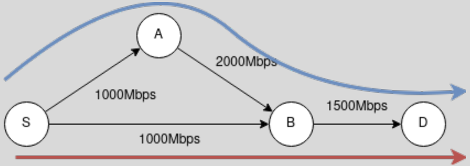
$S \to B$: 500 Mbps
$S \to D$: 1000 Mbps

Consider the traffic from S to D:

- At node D, there are one potential link $l_7 = (B, D)$ whose destination is D, and $l_8 = (D, B)$ whose source is D
  - $x_{S,D,7}$: Traffic from S to D that uses link $l_7 = (B, D)$

Constraint Example (cont.)

Traffic at the Destination Node



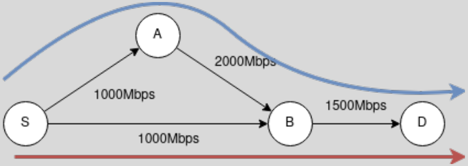
$S \rightarrow B$: 500 Mbps
$S \rightarrow D$: 1000 Mbps

Consider the traffic from S to D:

- At node D, there are one potential link $l_7 = (B, D)$ whose destination is D, and $l_8 = (D, B)$ whose source is D
  - $x_{S,D,7}$: Traffic from S to D that uses link $l_7 = (B, D)$
  - $x_{S,D,7} = t_{S,D} = 1000$

Constraint Example (cont.)

Traffic at the Destination Node



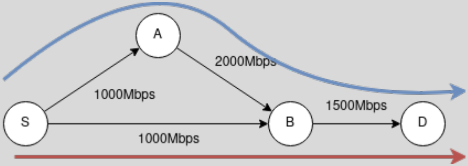
$S \rightarrow B$: 500 Mbps
$S \rightarrow D$: 1000 Mbps

Consider the traffic from S to D:

- At node D, there are one potential link $l_7 = (B, D)$ whose destination is D, and $l_8 = (D, B)$ whose source is D
  - $x_{S,D,7}$: Traffic from S to D that uses link $l_7 = (B, D)$
  - $x_{S,D,7} = t_{S,D} = 1000$
  - $x_{S,D,8}$: Traffic from S to D that uses link $l_8 = (D, B)$

Constraint Example (cont.)

Traffic at the Destination Node



$S \to B$: 500 Mbps
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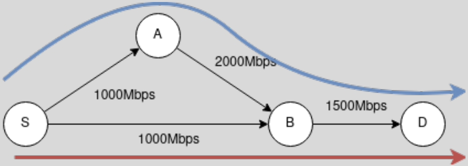
Consider the traffic from S to D:

- At node D, there are one potential link $l_7 = (B, D)$ whose destination is D, and $l_8 = (D, B)$ whose source is D
  - $x_{S,D,7}$: Traffic from S to D that uses link $l_7 = (B, D)$
  - $x_{S,D,7} = t_{S,D} = 1000$
  - $x_{S,D,8}$: Traffic from S to D that uses link $l_8 = (D, B)$
  - $x_{S,D,8} = 0$

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



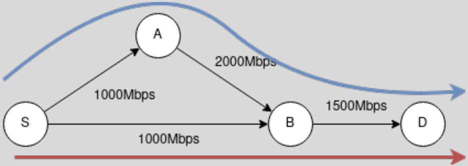
$S \to B$: 500 Mbps
$S \to D$: 1000 Mbps

Consider the traffic from S to D:

- At node A, there are two potential links $l_1 = (S, A)$ and $l_6 = (B, A)$ whose destination is A, and $l_2 = (A, S)$ and $l_5 = (A, B)$ whose source is A

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



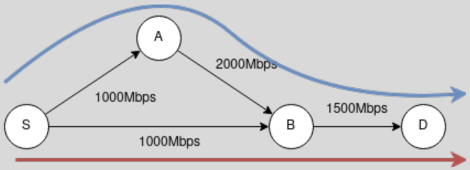
$S \rightarrow B$: 500 Mbps
$S \rightarrow D$: 1000 Mbps

Consider the traffic from S to D:

- At node A, there are two potential links $l_1 = (S, A)$ and $l_6 = (B, A)$ whose destination is A, and $l_2 = (A, S)$ and $l_5 = (A, B)$ whose source is A
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



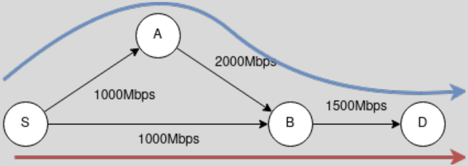
$S \rightarrow B$: 500 Mbps
$S \rightarrow D$: 1000 Mbps

Consider the traffic from S to D:

- At node A, there are two potential links $l_1 = (S, A)$ and $l_6 = (B, A)$ whose destination is A, and $l_2 = (A, S)$ and $l_5 = (A, B)$ whose source is A
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$
  - $x_{S,D,6}$: Traffic from S to D that uses link $l_6 = (B, A)$

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



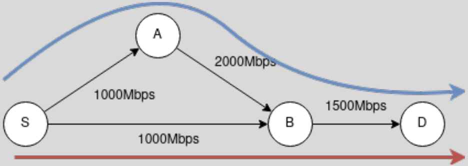
$S \to B$: 500 Mbps
$S \to D$: 1000 Mbps

Consider the traffic from S to D:

- At node A, there are two potential links $l_1 = (S, A)$ and $l_6 = (B, A)$ whose destination is A, and $l_2 = (A, S)$ and $l_5 = (A, B)$ whose source is A
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$
  - $x_{S,D,6}$: Traffic from S to D that uses link $l_6 = (B, A)$
  - $x_{S,D,2}$: Traffic from S to D that uses link $l_2 = (A, S)$

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



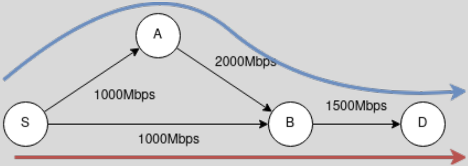
$S \to B$: 500 Mbps
$S \to D$: 1000 Mbps

Consider the traffic from S to D:

- At node A, there are two potential links $l_1 = (S, A)$ and $l_6 = (B, A)$ whose destination is A, and $l_2 = (A, S)$ and $l_5 = (A, B)$ whose source is A
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$
  - $x_{S,D,6}$: Traffic from S to D that uses link $l_6 = (B, A)$
  - $x_{S,D,2}$: Traffic from S to D that uses link $l_2 = (A, S)$
  - $x_{S,D,5}$: Traffic from S to D that uses link $l_5 = (A, B)$

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



$S \rightarrow B$: 500 Mbps
$S \rightarrow D$: 1000 Mbps

Consider the traffic from S to D:

- At node A, there are two potential links $l_1 = (S, A)$ and $l_6 = (B, A)$ whose destination is A, and $l_2 = (A, S)$ and $l_5 = (A, B)$ whose source is A
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$
  - $x_{S,D,6}$: Traffic from S to D that uses link $l_6 = (B, A)$
  - $x_{S,D,2}$: Traffic from S to D that uses link $l_2 = (A, S)$
  - $x_{S,D,5}$: Traffic from S to D that uses link $l_5 = (A, B)$
  - $x_{S,D,1} + x_{S,D,6} = x_{S,D,2} + x_{S,D,5}$

Problem Formulation (Cont.)

- Constraints:

Problem Formulation (Cont.)

- Constraints:
  - Link capacity:

$$\forall k, \sum_{i,j} x_{ijk} \leq c_k$$

Problem Formulation (Cont.)

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$$\forall k,\ u_k = \frac{1}{c_k} \sum_{i,j} x_{ijk}$$

Problem Formulation (Cont.)

- Constraints:
  - Link capacity:

$$\forall k, \sum_{i,j} x_{ijk} \leq c_k$$

  - Link utilization:

$$\forall k, u_k = \frac{1}{c_k} \sum_{i,j} x_{ijk}$$

  - Maximum link utilization:

$$\forall k, u_k \leq u$$

Problem Formulation (Cont.)

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  - Link utilization:

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  - Maximum link utilization:

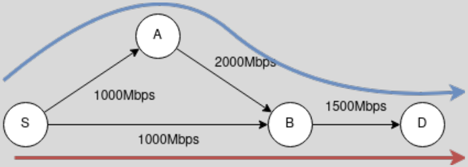
$$\forall k, u_k \leq u$$

- Objective: minimize the maximum link utilization

$$\min u$$

Constraint Example (cont.)

Link Capacity



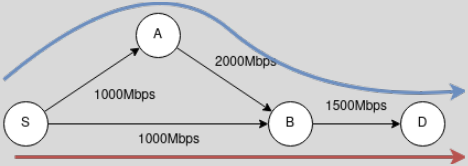
$S \rightarrow B$: 500 Mbps
$S \rightarrow D$: 1000 Mbps

Consider link $l_3 = (S, B)$:

- $S \rightarrow B$ can take path $S \rightarrow B$ and $S \rightarrow D$ can take path $S \rightarrow B \rightarrow D$

Constraint Example (cont.)

Link Capacity



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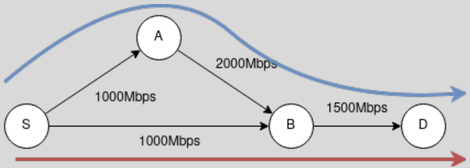
Consider link $l_3 = (S, B)$:

- $S \rightarrow B$ can take path $S \rightarrow B$ and $S \rightarrow D$ can take path $S \rightarrow B \rightarrow D$
- $x_{S,B,3}$: Traffic from S to B that uses link $l_3 = (S, B)$

Constraint Example (cont.)

Link Capacity



$S \rightarrow B$: 500 Mbps
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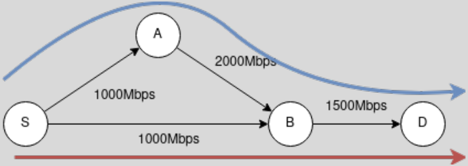
Consider link $l_3 = (S, B)$:

- $S \rightarrow B$ can take path $S \rightarrow B$ and $S \rightarrow D$ can take path $S \rightarrow B \rightarrow D$
- $x_{S,B,3}$: Traffic from S to B that uses link $l_3 = (S, B)$
- $x_{S,D,3}$: Traffic from S to D that uses link $l_3 = (S, B)$

Constraint Example (cont.)

Link Capacity



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- $x_{S,B,3}$: Traffic from S to B that uses link $l_3 = (S, B)$
- $x_{S,D,3}$: Traffic from S to D that uses link $l_3 = (S, B)$
- $x_{S,B,3} + x_{S,D,3} \leq c_3 = 1000$

Integer Linear Programming

Integer Linear Programming

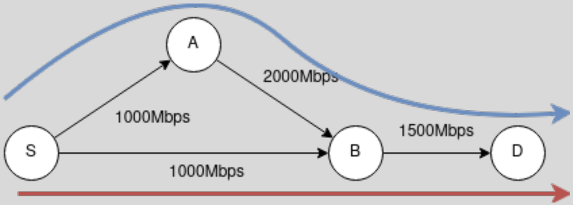
Integer linear programming (ILP) has the same format as an LP with an additional constraint that some decision variables must be integers.

- ILP is NP-hard.
- If some variables are continuous, the problem is known as mixed integer linear programming (MILP)
- If all variables are either 0 or 1, the problem is known as binary linear programming

ILP Example

Consider the following problem:

- There are two available paths:
  $S \to A \to B \to D$, and $S \to B \to D$

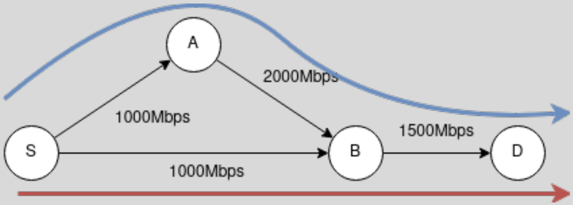


ILP Example

Consider the following problem:

- There are two available paths: $S \rightarrow A \rightarrow B \rightarrow D$, and $S \rightarrow B \rightarrow D$
- The links have different link capacities (unit: Mbps)

Source	*Destination*	*Capacity*
S	A	1000
S	B	1000
A	B	2000
B	D	1500



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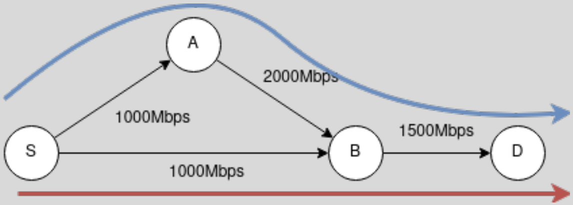
- Video streaming speed is either 3 Mbps (for FHD) and 15 Mbps (for 4K)



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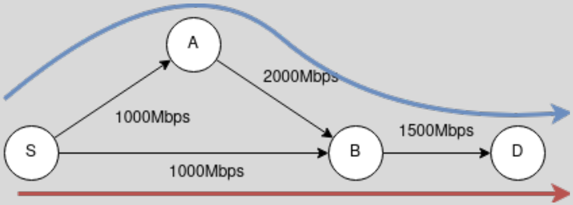

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S	B	1000
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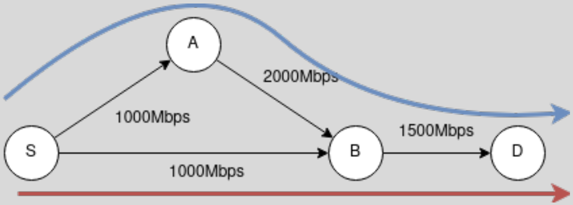
- Video streaming speed is either 3 Mbps (for FHD) and 15 Mbps (for 4K)
- Assume subscribers will pay \$1 for an FHD live streaming and \$10 for 4K live streaming
- Question: What is the maximum profit?



Formulating the Problem as an ILP

- **Decision variables**:
  - the number of FHD subscribers on path $S \to A \to B \to D$: $x_1 \in \mathbb{N}$
  - the number of 4K subscribers on path $S \to A \to B \to D$: $x_2 \in \mathbb{N}$
  - the number of FHD subscribers on path $S \to B \to D$: $x_3 \in \mathbb{N}$
  - the number of 4K subscribers on path $S \to B \to D$: $x_4 \in \mathbb{N}$

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- **Constraints**: the transfer speed on a link must not exceed the link capacity
  - $S \to A$: $3x_1 + 15x_2 \leq 1000$
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  - $A \to B$: $3x_1 + 15x_2 \leq 2000$
  - $B \to D$: $3x_1 + 15x_2 + 3x_3 + 15x_4 \leq 1500$

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  - $B \to D$: $3x_1 + 15x_2 + 3x_3 + 15x_4 \leq 1500$
- **Objective**: maximizing the total profit $z = x_1 + 10x_2 + x_3 + 10x_4$

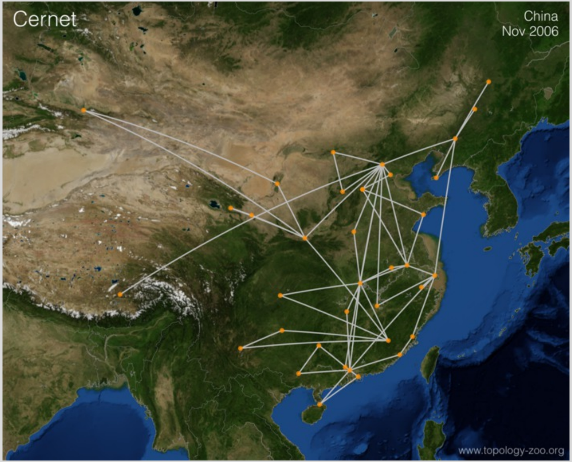
Solving Integer Linear Programming

- No polynomial algorithms exist for ILP: NP-hard
- Rounding method (no guarantee for optimality 最优性 or feasibility 可行性)
- Branch and bound

Traffic Engineering (II)

Now consider the traffic engineering problem using tunnels

- Assume there are N nodes and M uni-directional links, the i-th link has a source s_i and the destination d_i



http://www.topology-zoo.org/gallery.html

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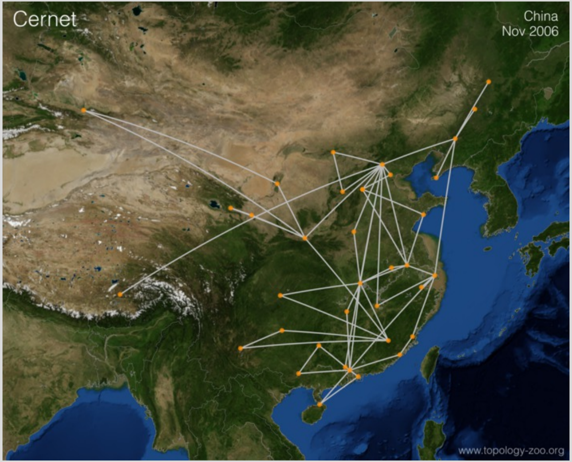

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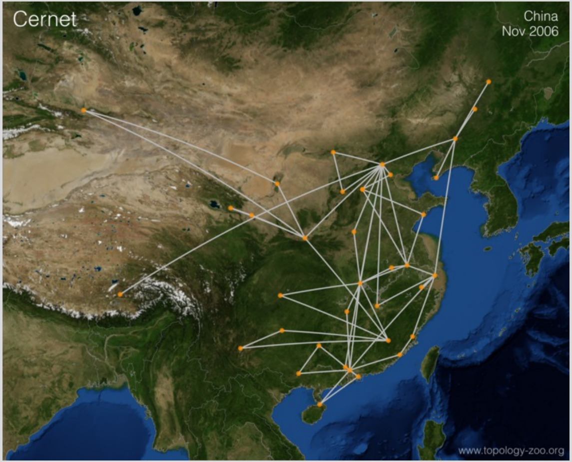

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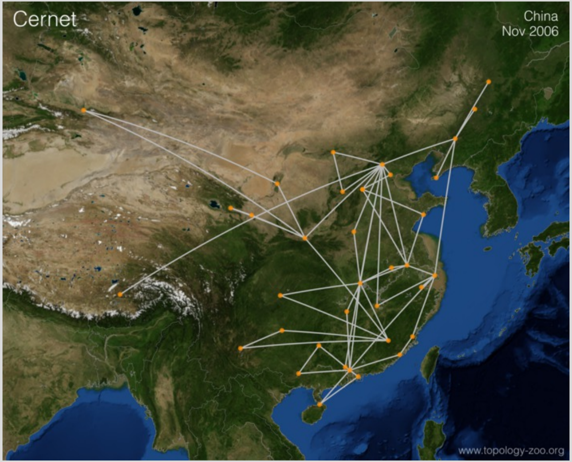

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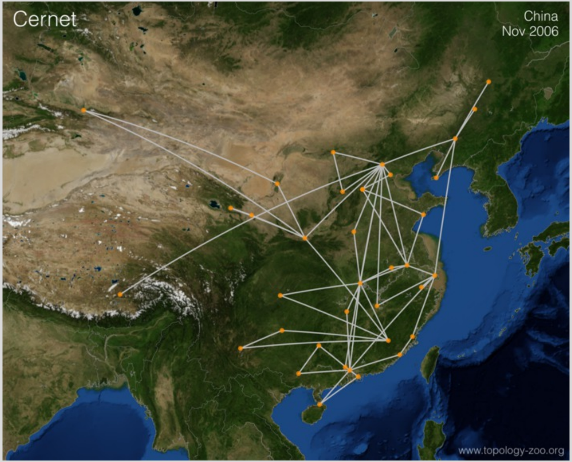

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Problem Formulation

- Decision variables:
  - Traffic path: let x_{ijk} denote whether the traffic from node i to node j uses link k

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Problem Formulation

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Problem Formulation

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  - Traffic source:

$$\forall i, j, \sum_{s_k = i} x_{ijk} = 1 \qquad \forall i, j, \sum_{d_k = i} x_{ijk} = 0$$

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$$\forall i, j, \sum_{d_k = j} x_{ijk} = 1 \qquad \forall i, j, \sum_{s_k = j} x_{ijk} = 0$$

Problem Formulation

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  - Traffic sink:

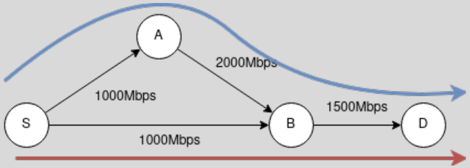
$$\forall i, j, \sum_{d_k=j} x_{ijk} = 1 \qquad \forall i, j, \sum_{s_k=j} x_{ijk} = 0$$

  - Traffic on intermediate node

$$\forall i, j, p \neq i, p \neq j, \sum_{s_k=p} x_{ijk} = \sum_{d_k=p} x_{ijk}$$

Constraint Example

Path Selection at the Source Node



$S \to B$: 500 Mbps
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Consider the traffic from S to D:

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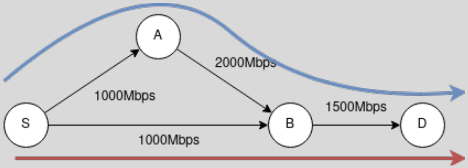
Constraint Example

Path Selection at the Source Node



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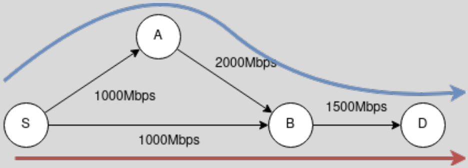
  - $x_{S,D,1}$: Traffic from S to D uses link $l_1 = (S, A)$

Constraint Example

Path Selection at the Source Node



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Consider the traffic from S to D:

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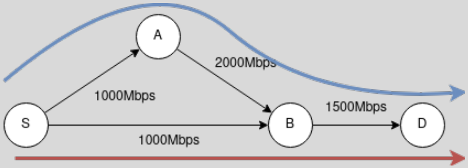
Constraint Example

Path Selection at the Source Node



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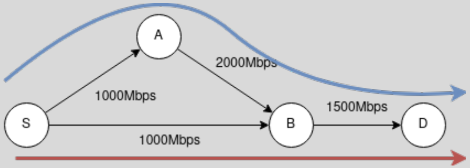
Constraint Example

Path Selection at the Source Node



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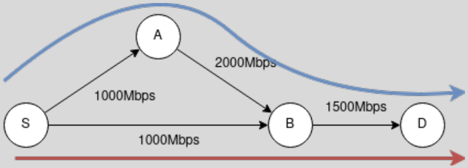
Constraint Example

Path Selection at the Source Node



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  - One and only one link will be used: $x_{S,D,1} + x_{S,D,3} = 1$
  - $x_{S,D,2}$: Traffic from S to D uses link $l_2 = (A, S)$
  - $x_{S,D,4}$: Traffic from S to D uses link $l_4 = (B, S)$

Constraint Example

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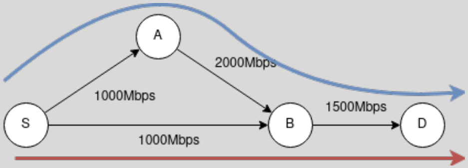

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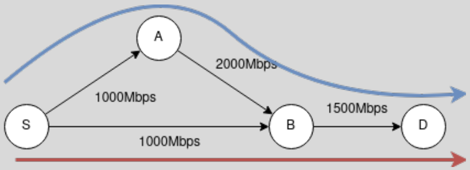
  - $x_{S,D,2} + x_{S,D,4} = 0$

Constraint Example (cont.)

Path Selection at the Destination Node



$S \to B$: 500 Mbps
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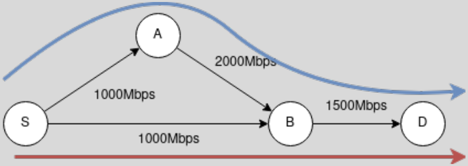
Consider the traffic from S to D:

- At node D, there are one potential link $l_7 = (B, D)$ whose destination is D, and $l_8 = (D, B)$ whose source is D

Constraint Example (cont.)

Path Selection at the Destination Node



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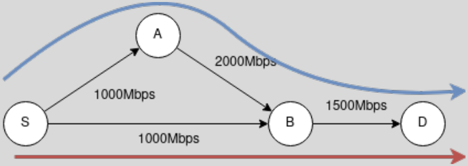
Consider the traffic from S to D:

- At node D, there are one potential link $l_7 = (B, D)$ whose destination is D, and $l_8 = (D, B)$ whose source is D
  - $x_{S,D,7}$: Traffic from S to D uses link $l_7 = (B, D)$

Constraint Example (cont.)

Path Selection at the Destination Node



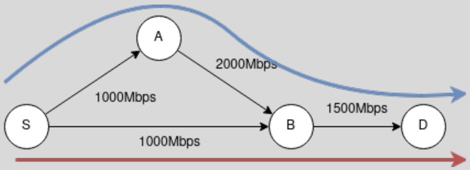
$S \to B$: 500 Mbps
$S \to D$: 1000 Mbps

Consider the traffic from S to D:

- At node D, there are one potential link $l_7 = (B, D)$ whose destination is D, and $l_8 = (D, B)$ whose source is D
  - $x_{S,D,7}$: Traffic from S to D uses link $l_7 = (B, D)$
  - $x_{S,D,7} = 1$

Constraint Example (cont.)

Path Selection at the Destination Node



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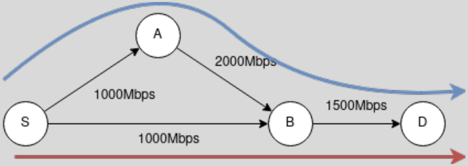
Constraint Example (cont.)

Path Selection at the Destination Node



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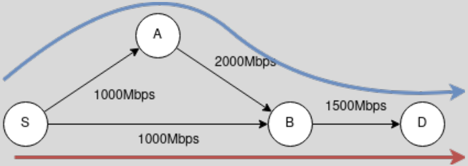
  - $x_{S,D,8}$: Traffic from S to D uses link $l_8 = (D, B)$
  - $x_{S,D,8} = 0$

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



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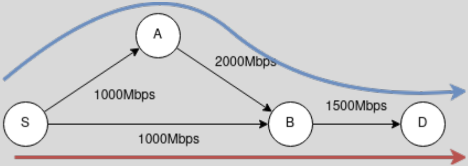
Consider the traffic from S to D:

- At node A, there are two potential links $l_1 = (S, A)$ and $l_6 = (B, A)$ whose destination is A, and $l_2 = (A, S)$ and $l_5 = (A, B)$ whose source is A

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



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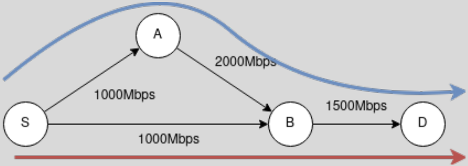
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



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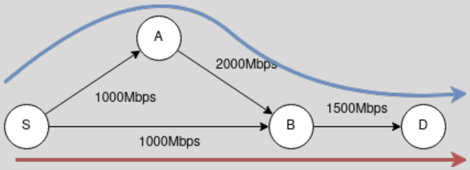
  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$
  - $x_{S,D,6}$: Traffic from S to D that uses link $l_6 = (B, A)$

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



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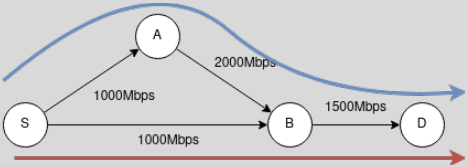
  - $x_{S,D,2}$: Traffic from S to D that uses link $l_2 = (A, S)$

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



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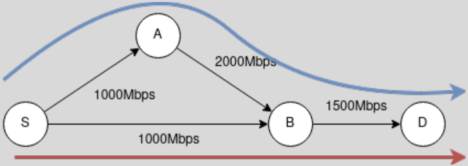
  - $x_{S,D,2}$: Traffic from S to D that uses link $l_2 = (A, S)$
  - $x_{S,D,5}$: Traffic from S to D that uses link $l_5 = (A, B)$

Constraint Example (cont.)

Traffic at the Intermediate Node(s)



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  - $x_{S,D,1}$: Traffic from S to D that uses link $l_1 = (S, A)$
  - $x_{S,D,6}$: Traffic from S to D that uses link $l_6 = (B, A)$
  - $x_{S,D,2}$: Traffic from S to D that uses link $l_2 = (A, S)$
  - $x_{S,D,5}$: Traffic from S to D that uses link $l_5 = (A, B)$
  - $x_{S,D,1} + x_{S,D,6} = x_{S,D,2} + x_{S,D,5}$

Problem Formulation (Cont.)

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- Constraints:
  - Link capacity:

$$\forall k, \sum_{i,j} t_{ij} x_{ijk} \leq c_k$$

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  - Maximum link utilization:

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Problem Formulation (Cont.)

- Constraints:
  - Link capacity:

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$$\forall k, u_k = \frac{1}{c_k} \left(\sum_{i,j} t_{ij} x_{ijk} \right)$$

  - Maximum link utilization:

$$\forall k, u_k \leq u$$

- Objective: minimize the maximum link utilization

$$\min u$$

The End

Summary

In the coming lectures, we cover the following topics:

- Automata theory
- Linear programming
- Integer linear programming

In this lecture, you should

- understand linear programming (线性规划, LP) and its standard form (标准型)
- understand integer linear program (整数线性规划, ILP)

Thanks!

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[1] Narendra Karmarkar. "A New Polynomial-Time Algorithm for Linear Programming". In: *Proceedings of the Sixteenth Annual ACM Symposium on Theory of Computing*. ACM, 1984, pp. 302–311.

[2] Leonid Genrikhovich Khachiyan. "A Polynomial Algorithm in Linear Programming". In: *Doklady Akademii Nauk*. Vol. 244. Russian Academy of Sciences, 1979, pp. 1093–1096.