Semnale elementare

Cuprins

- Recapitulare
- De ce semnale elementare?
- Semnalul treaptă unitate
- Impuls Dirac
- Semnal rampă
- Sinus, cosinus
- Exponențială reală, complexă

Recapitulare

Ce este un semnal?
Funcție matematica cu ajutorul căreia modelăm un proces sau care conține informație despre un fenomen.
Orice mărime care variază (în timp, spațiu etc.) poate transmite informație.

Procese continue -> semnale continue
Procese discrete -> semnale discrete

Procese deterministe -> semnale deterministe
Procese aleatoare –> semnale aleatoare
Procese haotice -> semnale haotice

Recapitulare

Transformări elementare in raport cu timpul (t) sau [n]:

- reflexia
- dilatarea si compresia
- intarzierea si avansul

Proprietati generale

- Paritate / imparitate
- Periodicitatea

Asocierea unui semnal periodic unuia neperiodic

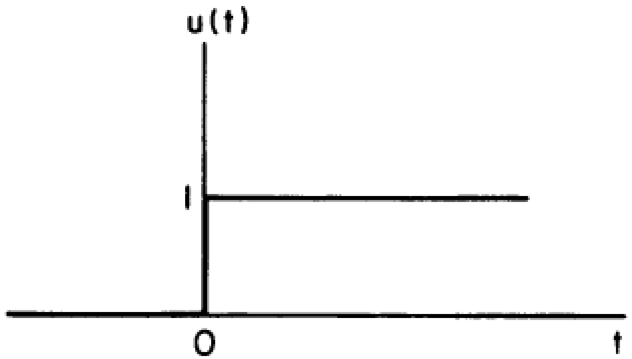
Energia și puterea

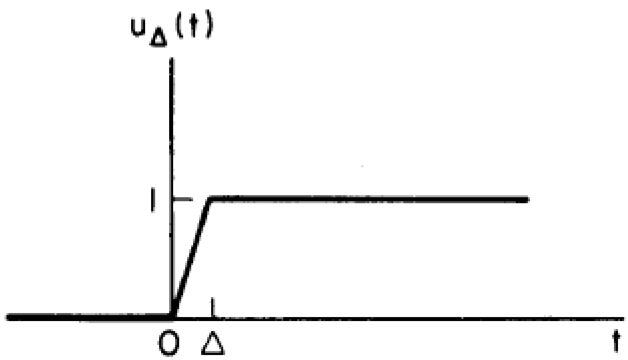
Semnale elementare

Semnale analogice	Semnale discrete
Impuls Dirac/ Funcţie Delta	Impuls unitar/ Impuls Dirac
Treaptă unitate	Treapta unitate
Rampă	Rampă
Cosinus/ Sinus	Cosinus/ Sinus
Exponenţial reală	Exponenţial reală
Exponențial complex	Exponențial complexă

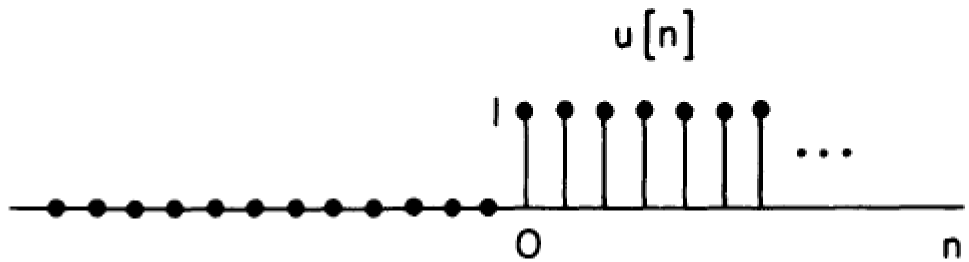
Semnal treaptă unitate

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$



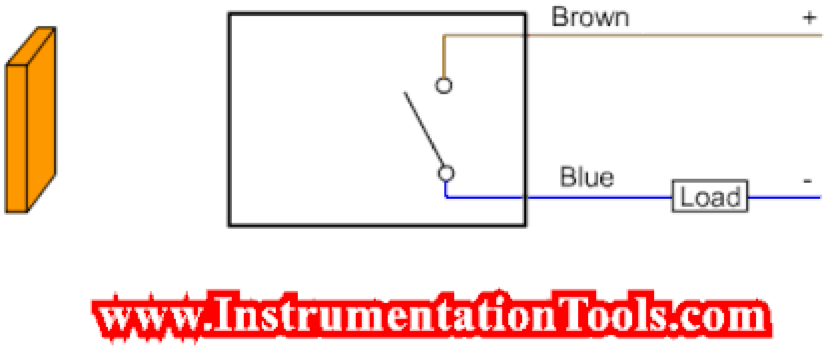


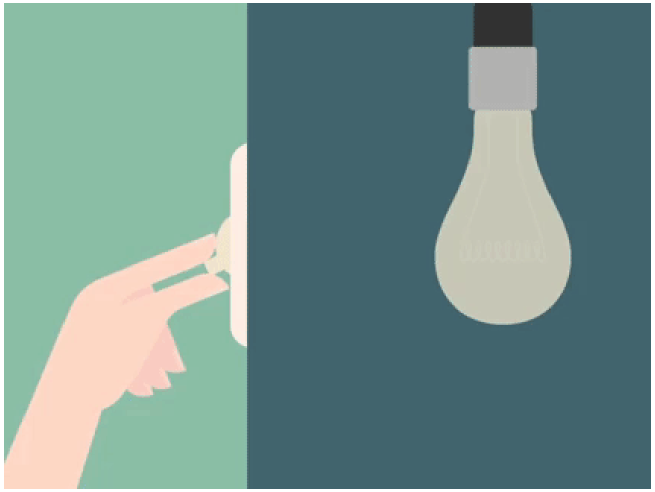
$u(t) = u_\Delta(t)$ as $\Delta \rightarrow 0$



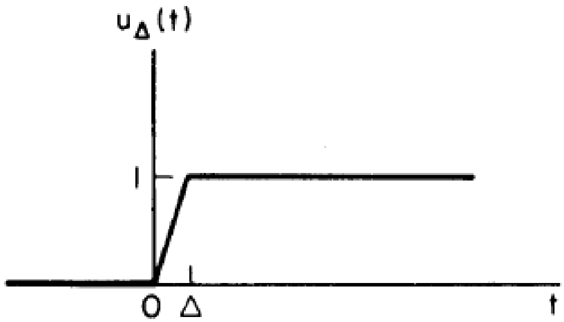
$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Semnal treaptă unitate



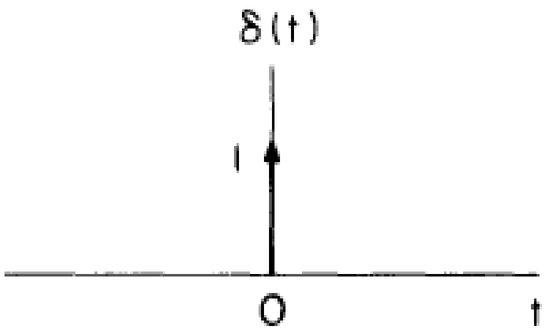


Semnalul impuls



$$\delta_{\triangle}(t) = \frac{du_{\triangle}(t)}{dt}$$

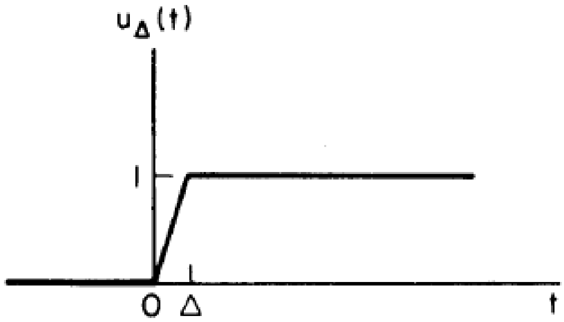
$$\delta(t) = \delta_{\triangle}(t) \text{ as } \triangle \rightarrow 0$$



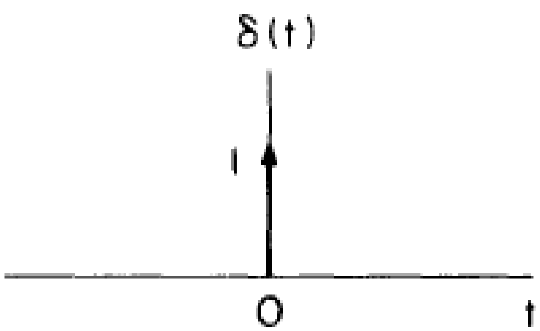
$$\delta(t) = \frac{du(t)}{dt}$$

Interpretarea funcției Delta ca trecerea la limită a unui dreptunghi cu arie unitară.

Semnalul impuls



$u(t) = u_\Delta(t)$ as $\Delta \to 0$



$$\delta_\triangle(t) = \frac{du_\triangle(t)}{dt}$$

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$$\delta(t) = \frac{du(t)}{dt}$$

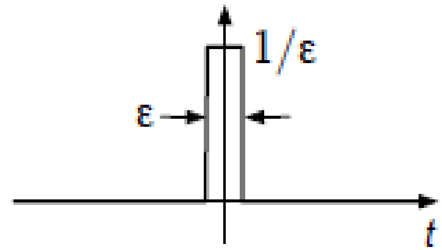
Interpretarea funcției Delta ca trecerea la limită a unui dreptunghi cu arie unitară.

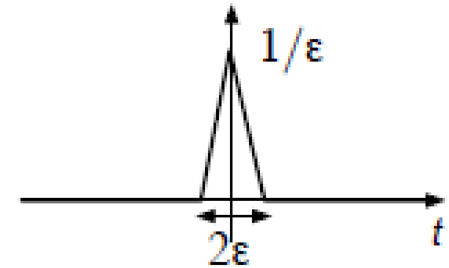
Semnalul impuls

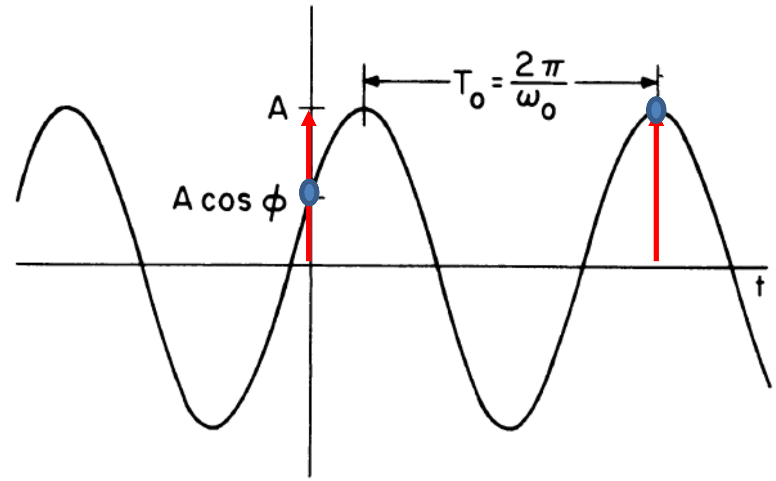
$$s(t), continuu, L2$$

$$s(0) = \int_{-\infty}^{\infty} s(t) \cdot \delta(t) dt$$

$$\int_{-\infty}^{\infty} s(t) \cdot \delta(t - T) dt = s(T)$$



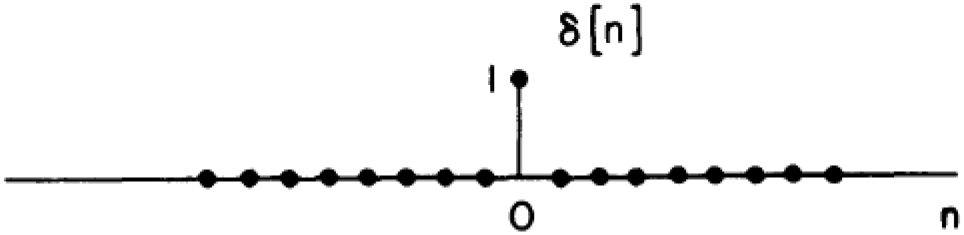




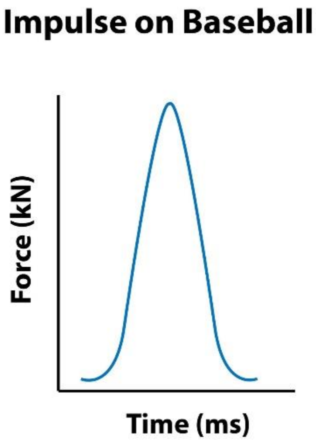
Interpretarea funcției Delta prin comportamentul sub integrală.
Are rolul unei “funcții selector”.
Nu este o funcție. Definiția formală este cea de pe acest slide.
Modelează un semnal foarte mare în origine, foarte mic cu cât se depărtează de origine și care are integrală egală cu 1.

Semnalul impuls unitar

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



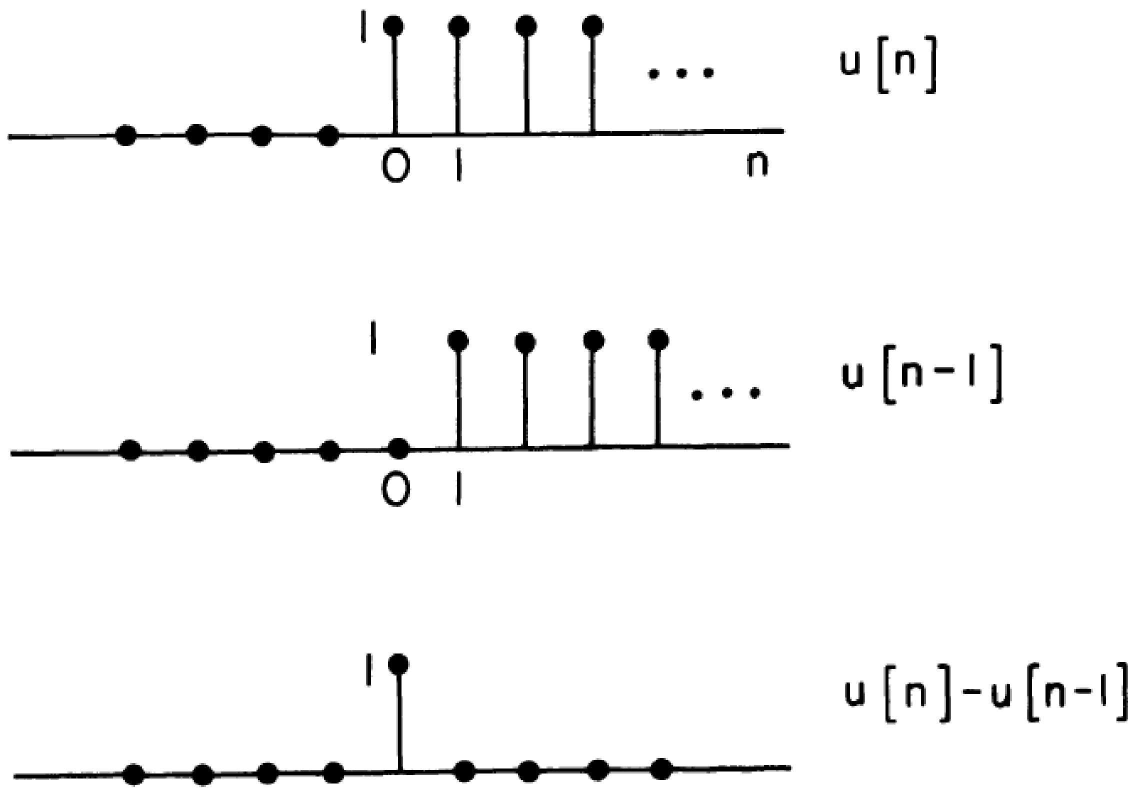
Semnalul impuls



Alte relații impuls treaptă

$$\delta[n] = u[n] - u[n-1]$$

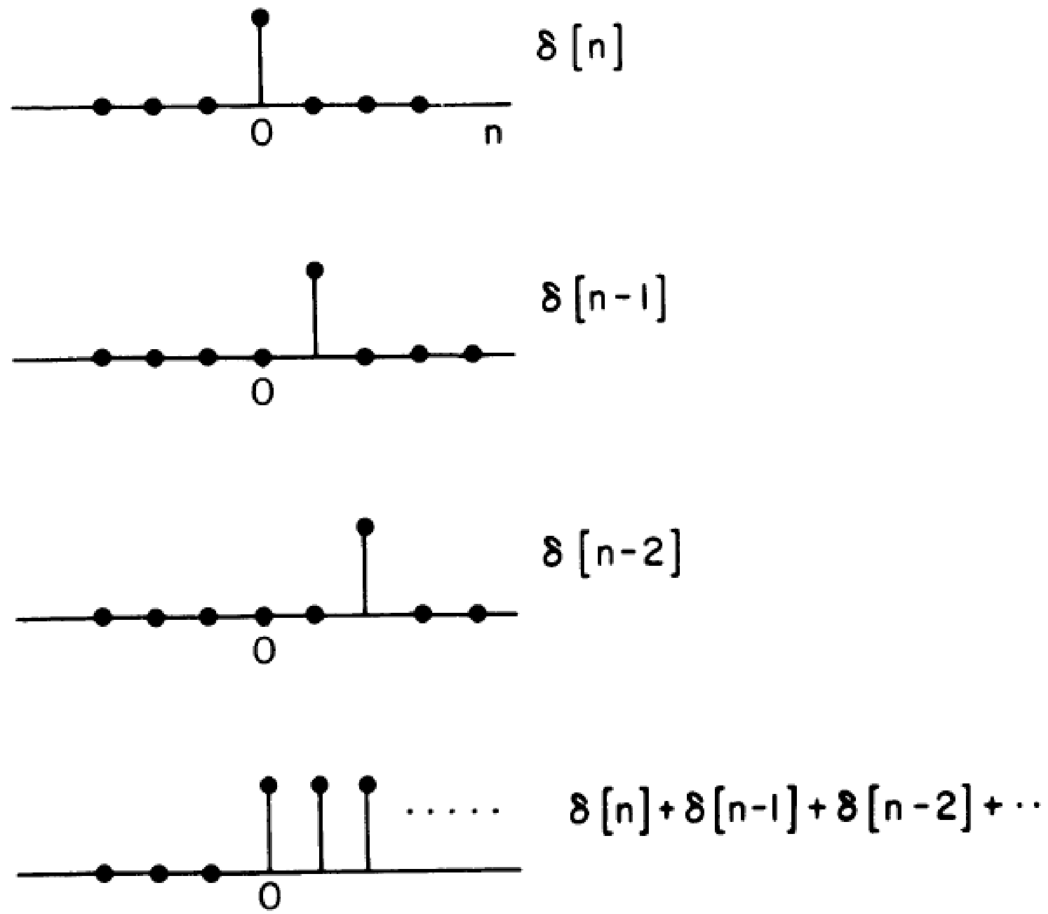
Impulsul Dirac definit ca diferența a două semnale treaptă unitate.



Alte relații între impuls și treaptă

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

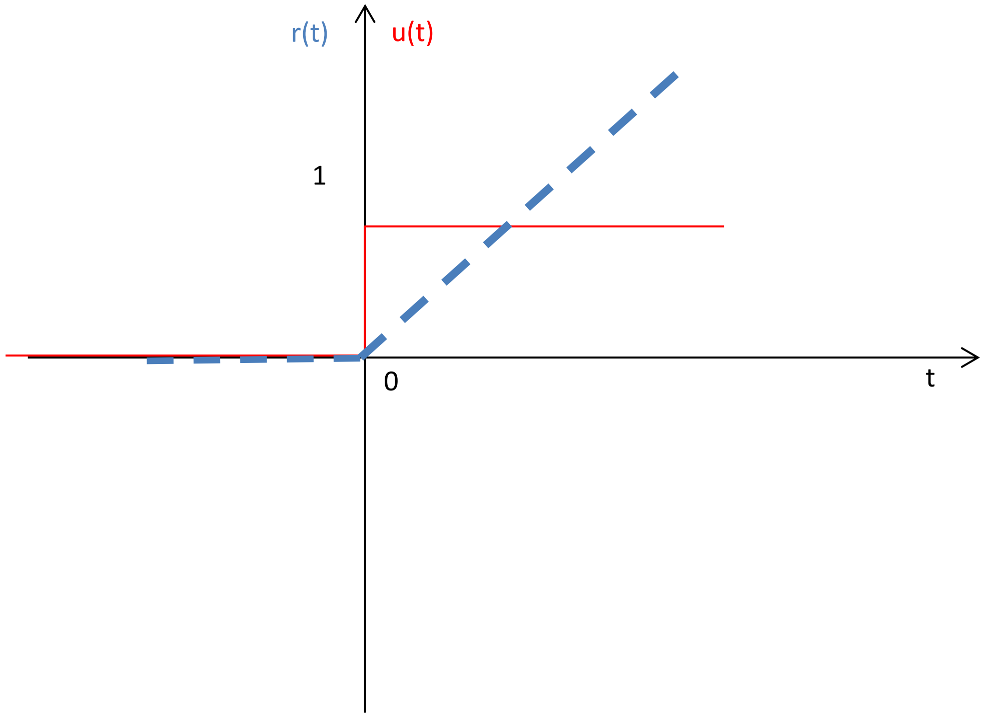
Semnalul treaptă unitate definit ca superpoziția unor impulsuri întârziate.



Semnalul rampă

$$\mathrm{r}(t) = \int_{-\infty}^{\infty} u(t)\,dt = \begin{cases} t + C, t \geq 0 \\ C, t < 0 \end{cases}$$

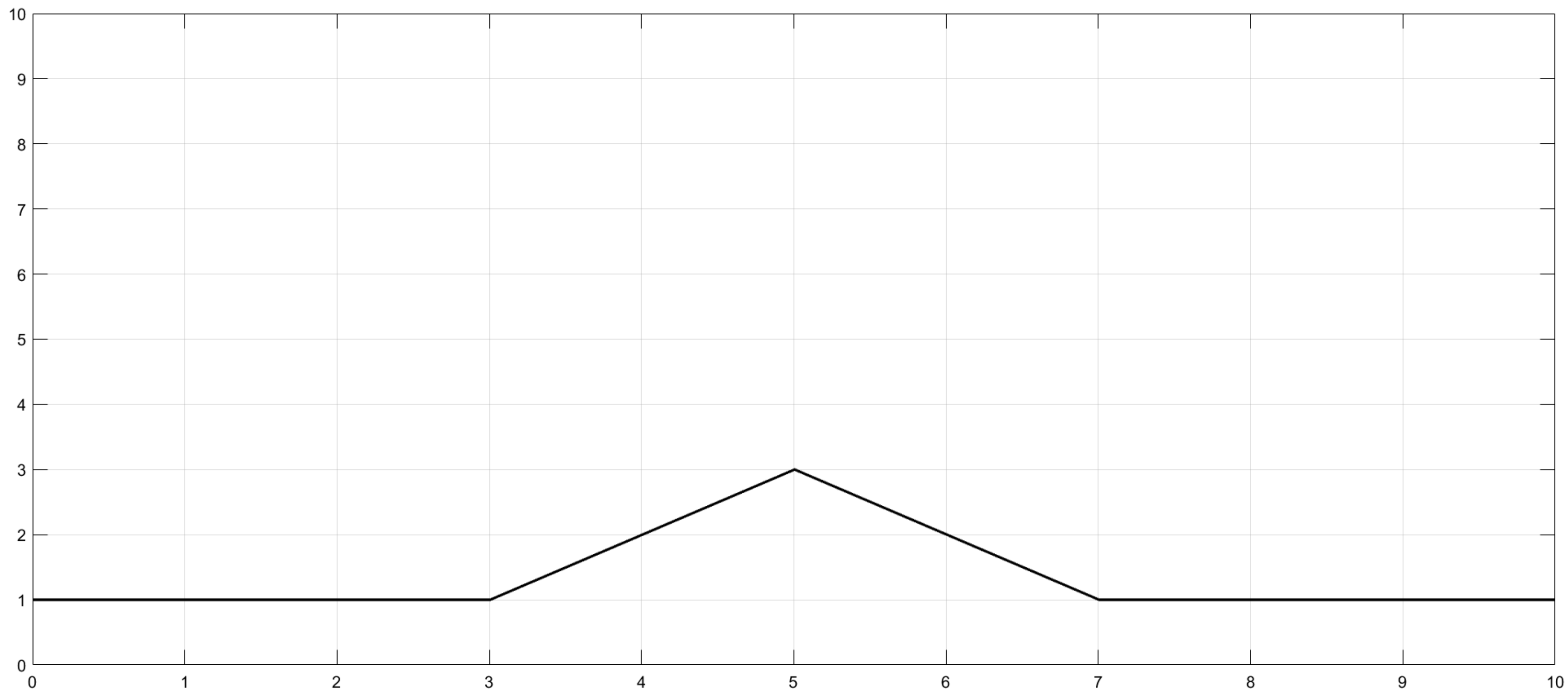
$$\mathrm{r}(t) = \begin{cases} t, t \geq 0 \\ 0, t < 0 \end{cases}$$



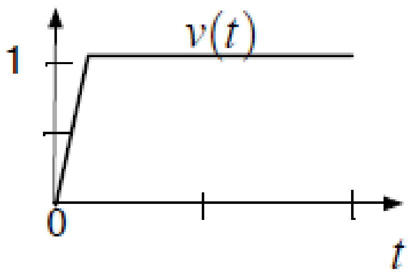
$$\mathrm{r}[n] = \begin{cases} n, n \geq 0 \\ 0, n < 0 \end{cases}$$

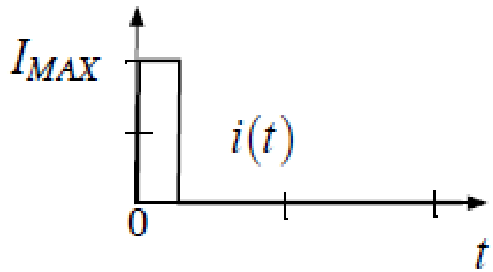
Exemplul 1

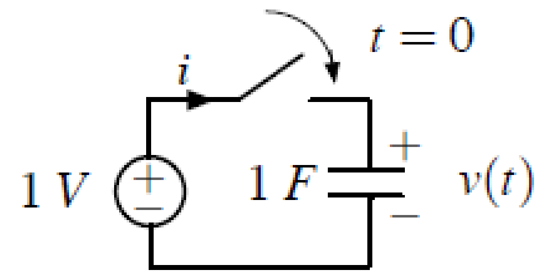
Se dă semnalul din figura de mai jos. Exprimați acest semnal în funcție de semnalele elementare studiate.



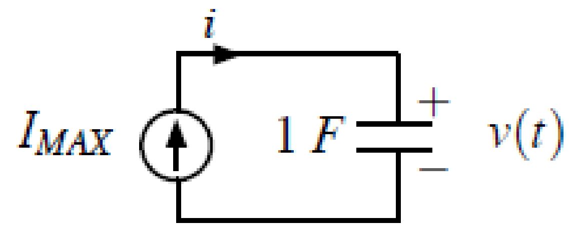
Exemplul 2







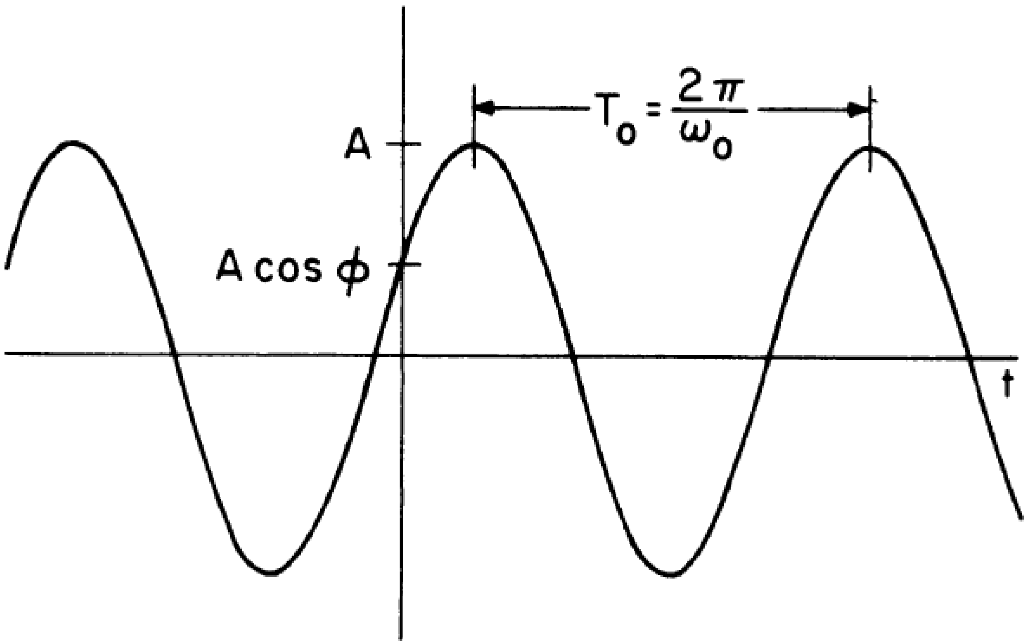
- Curentul I este diferit de 0 pentru un interval scurt.
- Curentul poate fi aproximat cu un impuls cu amplitudine 1 in t=0.



$$i(t) = \frac{dv(t)}{dt} = I_{\max}, \quad v(0) = 0$$

Semnalul cosinus

$$x(t) = A\cos(\omega_o t + \phi)$$



Semnalul cosinus

- Periodic:

$$x(t) = x(t + T_o) \quad \text{period} \triangleq \text{smallest } T_o$$

$$A\cos[\omega_o t + \phi] = A\cos[\omega_o t + \underbrace{\omega_o T_o}_{2\pi m} + \phi]$$

$$T_o = \frac{2\pi m}{\omega_o} \Rightarrow \text{period} = \frac{2\pi}{\omega_o}$$

- Time Shift <=> Phase Change

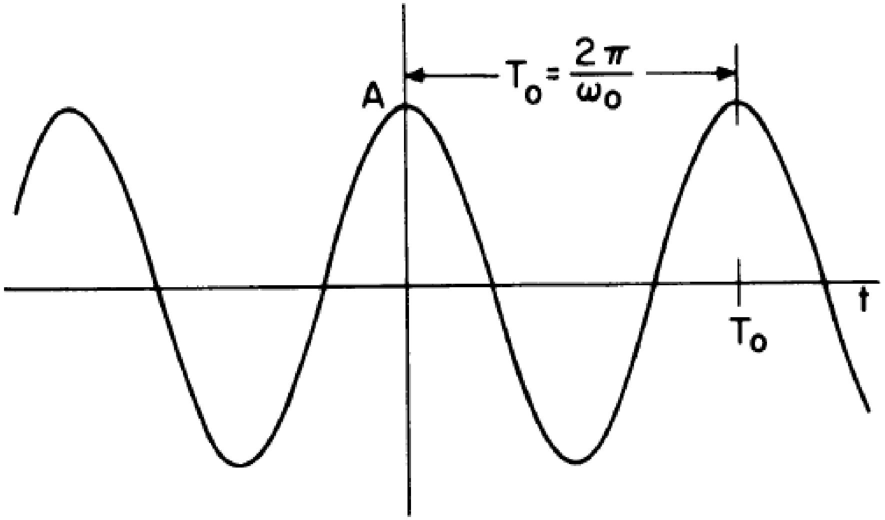
$$A\cos[\omega_o (t + t_o)] = A\cos[\omega_o t + \omega_o t_o]$$

$$A\cos[\omega_o (t + t_o) + \phi] = A\cos[\omega_o t + \omega_o t_o + \phi]$$

Semnalul cosinus

$\phi = 0 \qquad x(t) = A\cos\omega_0 t$

Semnal cosinus



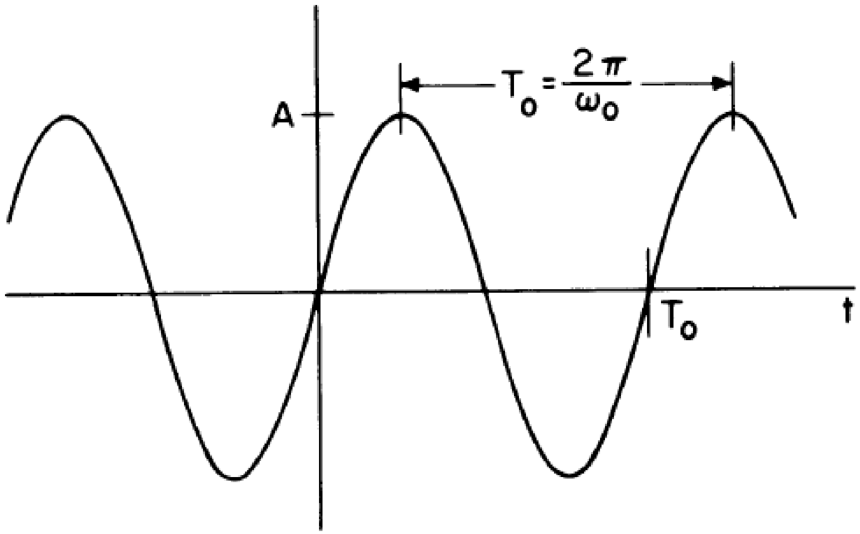
Periodic: $x(t) = x(t + T_0)$

Even: $x(t) = x(-t)$

Semnalul sinus

Semnal sinus

$$\phi = -\frac{\pi}{2} \qquad x(t) = \begin{cases} A\cos\left(\omega_o t - \frac{\pi}{2}\right) \\ A\sin\omega_o t \\ A\cos\left[\omega_o\left(t - \frac{T_o}{4}\right)\right] \end{cases}$$



Periodic: $x(t) = x(t + T_o)$

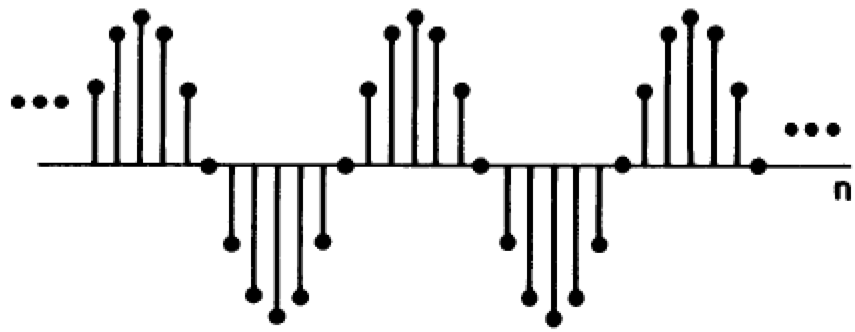
Odd: $x(t) = -x(-t)$

Semnalul cosinus discret

$$x[n] = A\cos(\Omega_o n + \phi)$$

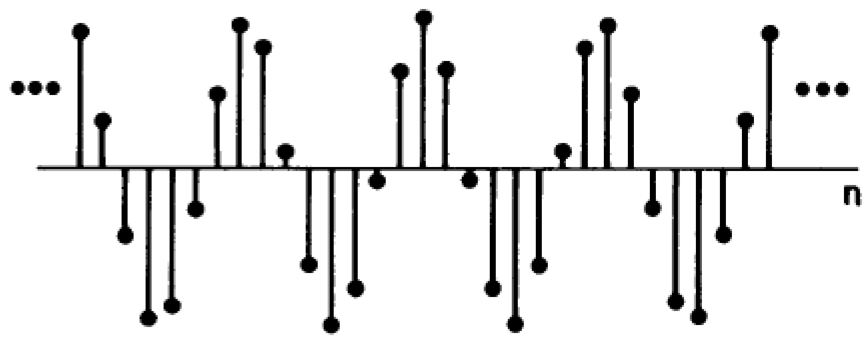
radiani $\Omega_o = \frac{2\pi}{12}$

$\phi = 0$



$\Omega_o = \frac{8\pi}{31}$

$\phi = 0$

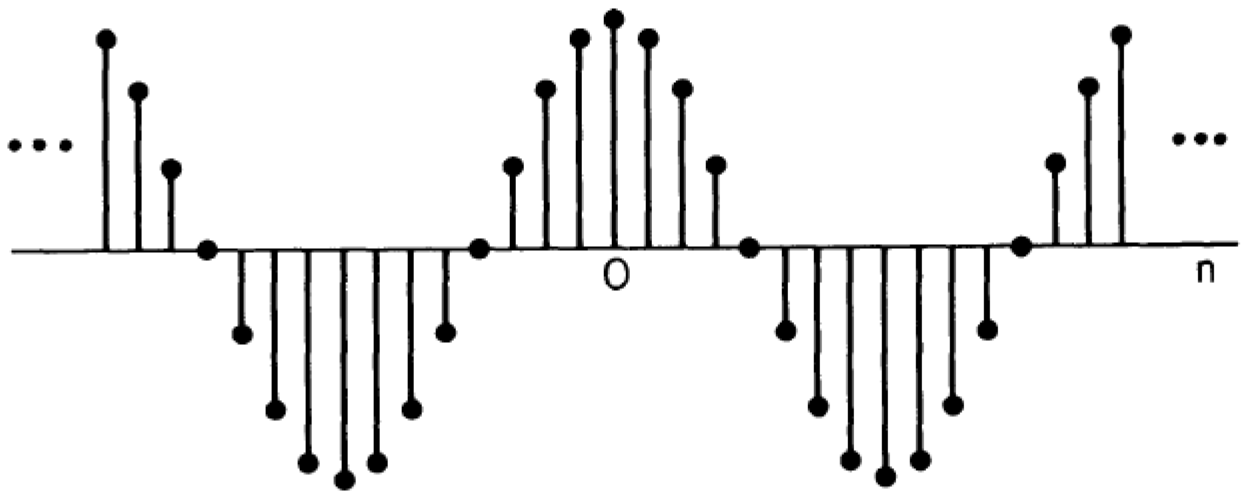


$$x(t) = A\cos\omega_o t$$

Radiani/secunda

Semnalul cosinus discret

$\phi = 0 \qquad x[n] = A \cos \Omega_0 n$



$\Omega_0 = \frac{\pi}{8}$

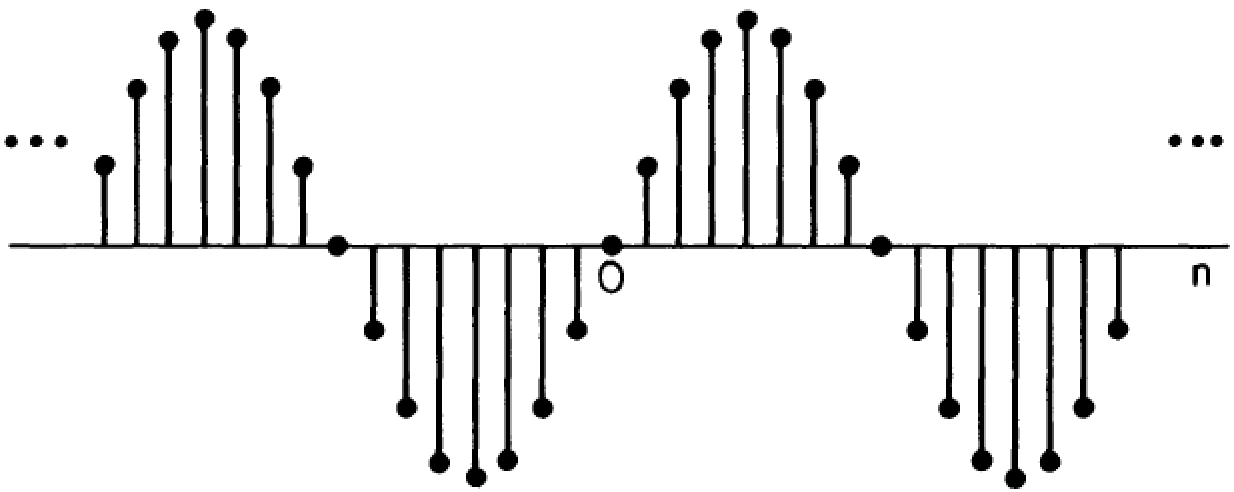
even: $\qquad x[n] = x[-n]$

Semnalul sinus discret

$\phi = -\frac{\pi}{2}$

$$x[n] = \begin{cases} A\cos(\Omega_o n - \frac{\pi}{2}) \\ A\sin \Omega_o n \\ A\cos[\Omega_o(n - n_o)] \end{cases}$$

$n_o = ?$



$\Omega_o = \frac{\pi}{8}$

odd: $x[n] = -x[-n]$

Echivalența deplasare-schimbare de fază

$$x[n] = A\cos(\Omega_o n + \phi)$$

Periodic?

$x[n] = x[n+N]$ smallest integer $N \triangleq$ period

$$A\cos[\Omega_o(n+N)+\phi] = A\cos[\Omega_o n + \underbrace{\Omega_o N}_{} + \phi]$$

integer multiple of 2π ?

Periodic $=> \Omega_o N = 2\pi m$

$$N = \frac{2\pi m}{\Omega_o}$$

N,m must be integers

smallest N (if any) = period

Echivalența deplasare-schimbare de fază

Time Shift => Phase Change

$$A \cos[\Omega_o(n + n_o)] = A \cos[\Omega_o n + \Omega_o n_o]$$

Time Shift $\overset{?}{\Leftarrow}$ Phase Change

$$A \cos[\Omega_o(n + n_o)] \overset{?}{=} A \cos[\Omega_o n + \phi]$$

Cosinus analogic vs. cosinus discret

$A\cos(\omega_o t + \phi)$	$A\cos(\Omega_o n + \phi)$
Distinct signals for distinct values of ω_o	Identical signals for values of Ω_o separated by 2π
Periodic for any choice of ω_o	Periodic only if $$\Omega_o = \frac{2\pi m}{N}$$ for some integers $N > 0$ and m

Cosinus analogic vs. cosinus discret

Continuous time:

$x_1(t) = A\cos(\omega_1 t + \phi)$ If $\omega_2 \neq \omega_1$

$x_2(t) = A\cos(\omega_2 t + \phi)$ Then $x_2(t) \neq x_1(t)$

Discrete time:

$x_1[n] = A\cos[\Omega_1 n + \phi]$ If $\Omega_2 = \Omega_1 + 2\pi m$

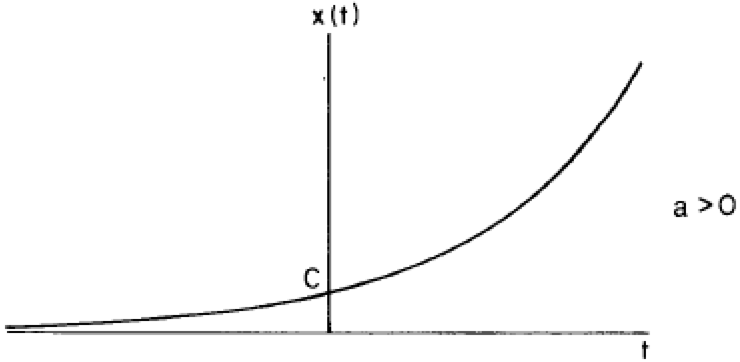
$x_2[n] = A\cos[\Omega_2 n + \phi]$ Then $x_2[n] = x_1[n]$

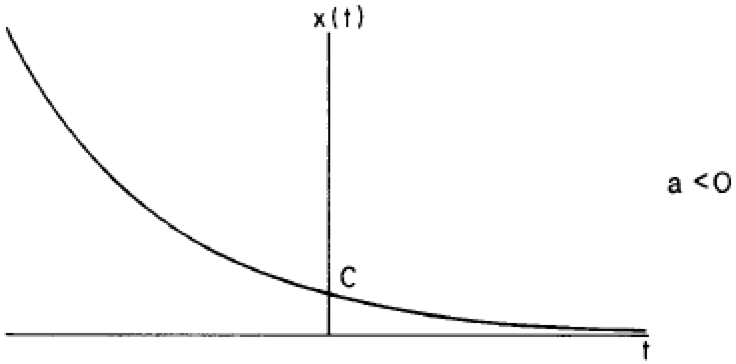

DACĂ NU REUŞESC, MĂ DUC LA PESCUIT DE BĂRCI !
vidu.ro

Semnalul exponențial real

$x(t) = Ce^{at}$

C and a are real numbers





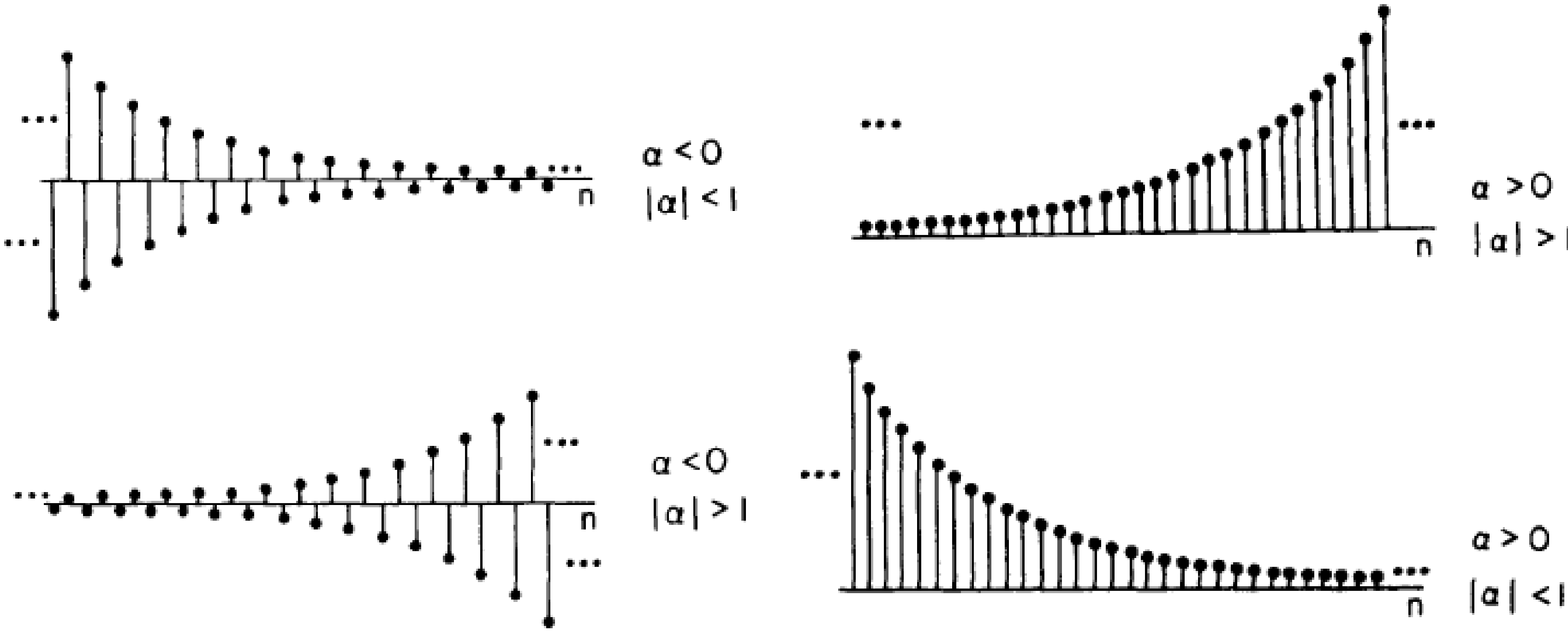
Time Shift <=> Scale Change

$Ce^{a(t+t_0)} = Ce^{at_0}e^{at}$

Semnalul exponențial real discret

$$x[n] = Ce^{\beta n} = C\alpha^n$$

C, α are real numbers



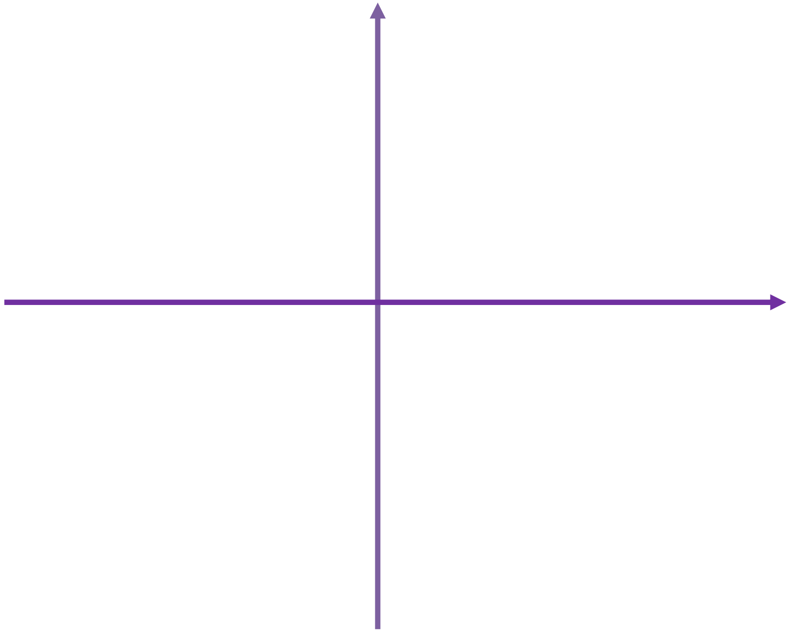
Numere complexe

$$x = x_r + j \cdot x_i$$

x_r - parte reală

x_i - parte imaginară

$$|x| = \sqrt{{x_r}^2 + {x_i}^2}$$



$$x_r = |x| \cos\varphi$$

$$x_i = |x| \sin\varphi$$

$$x = |x|(\cos\varphi + j \cdot \sin\varphi)$$

Semnalul exponențial complex

$$x(t) = Ce^{at}$$

C and a are complex numbers

$$C = |C|\, e^{j\theta}$$

$$a = r + j\omega_o$$

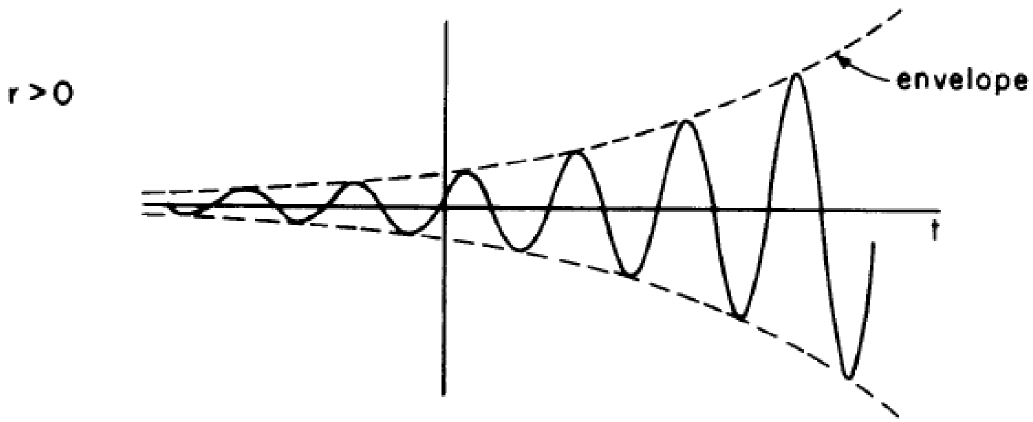
$$x(t) = |C|\, e^{j\theta}\, e^{(r + j\omega_o)t}$$

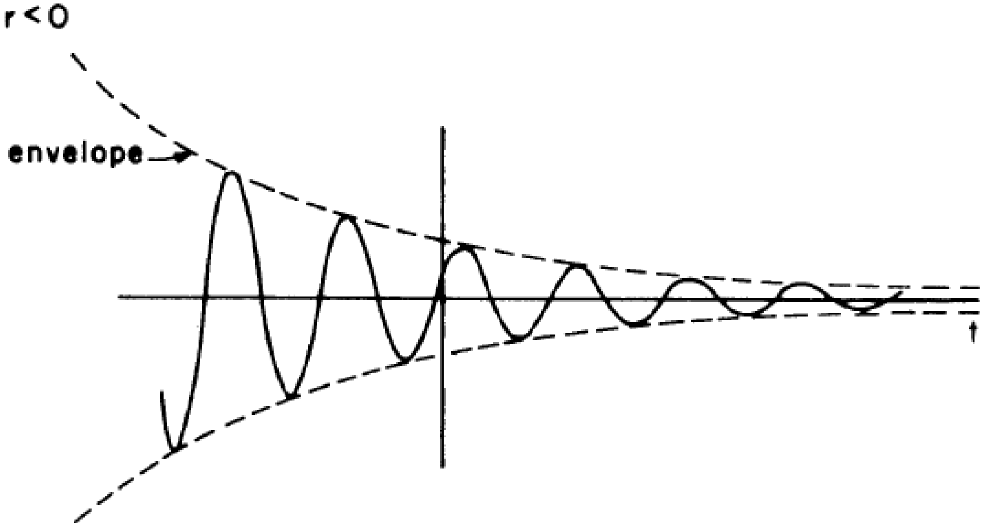
$$= |C|\, e^{rt}\, \underbrace{e^{j(\omega_o t + \theta)}}$$

Euler's Relation: $\cos(\omega_o t + \theta) + j\sin(\omega_o t + \theta) = e^{j(\omega_o t + \theta)}$

$$x(t) = |C|\, e^{rt} \cos(\omega_o t + \theta) + j\, |C|\, e^{rt} \sin(\omega_o t + \theta)$$

Semnalul exponențial complex





Semnalul exponențial complex

$$x[n] = C\alpha^n$$

C and α are complex numbers

$$C = |C|\, e^{j\theta}$$

$$\alpha = |\alpha|\, e^{j\Omega_o}$$

$$x[n] = |C|\, e^{j\theta} \left(|\alpha|\, e^{j\Omega_o}\right)^n$$

$$= |C|\, |\alpha|^n \underbrace{e^{j(\Omega_o n + \theta)}}$$

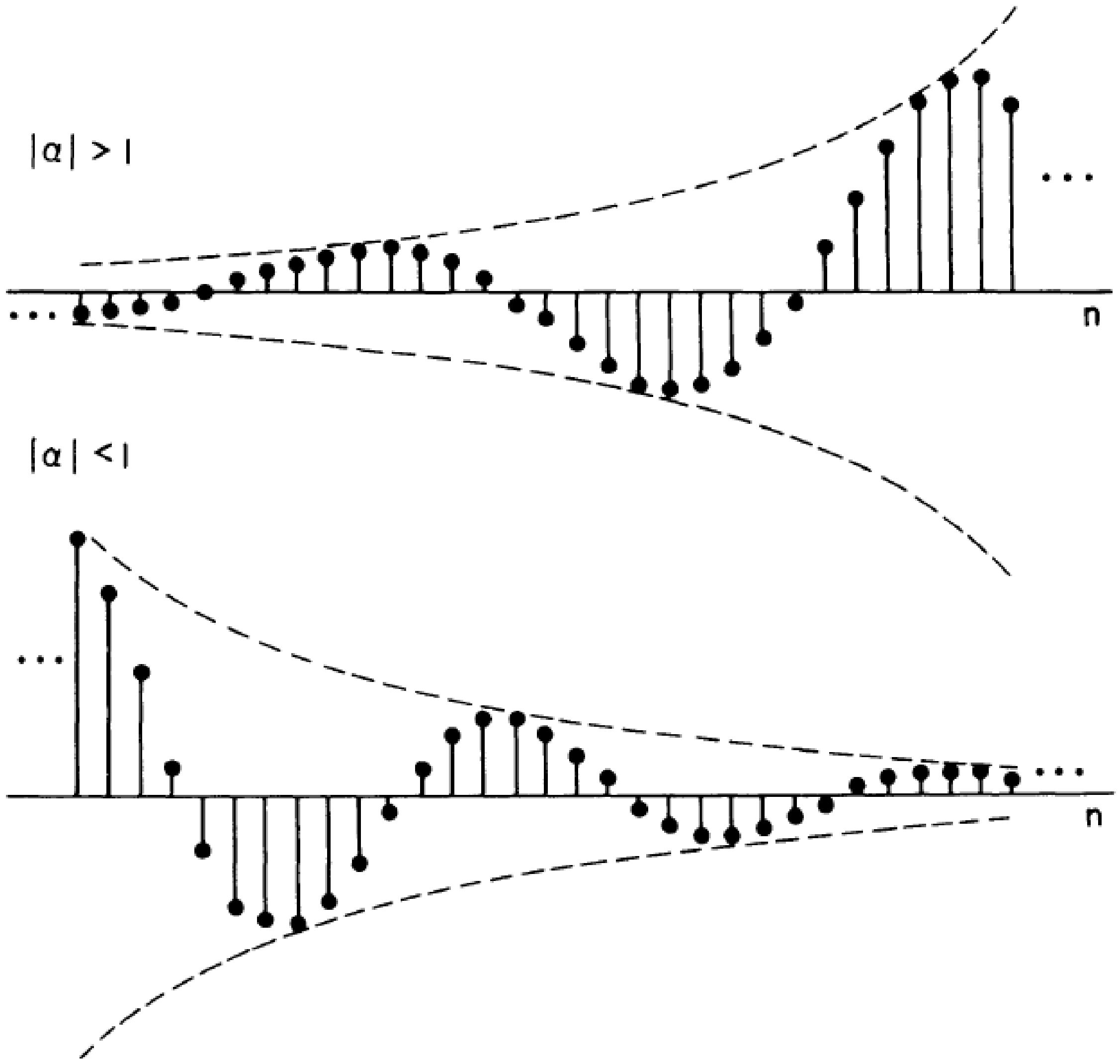
Euler's Relation: $\cos(\Omega_o n + \theta) + j\sin(\Omega_o n + \theta)$

$$x[n] = |C|\,|\alpha|^n \cos(\Omega_o n + \theta) + j\,|C|\,|\alpha|^n \sin(\Omega_o n + \theta)$$

$|\alpha| = 1$ => sinusoidal real and imaginary parts

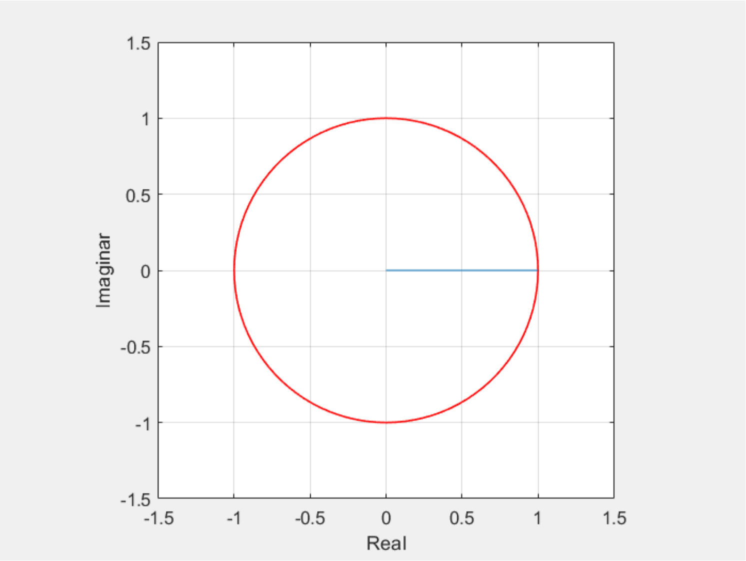
$C e^{j\Omega_o n}$ periodic ?

Semnalul exponențial complex

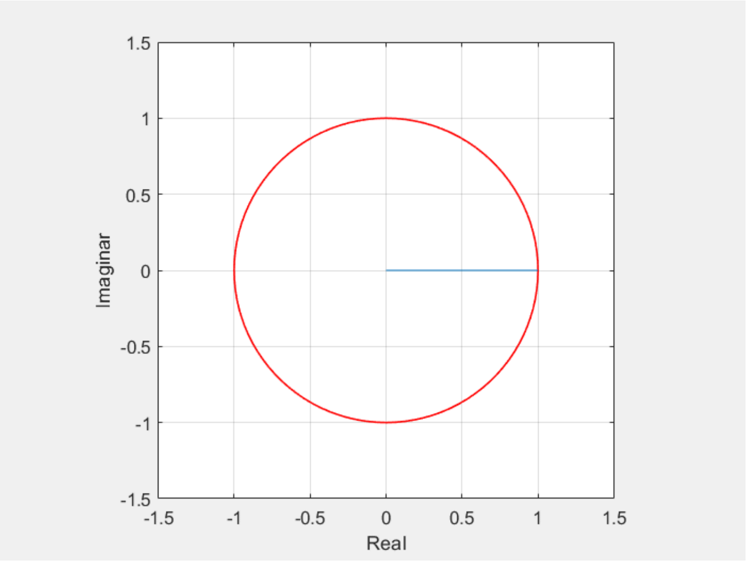


Exemplu

$$s[n] = e^{-j\Omega n}$$



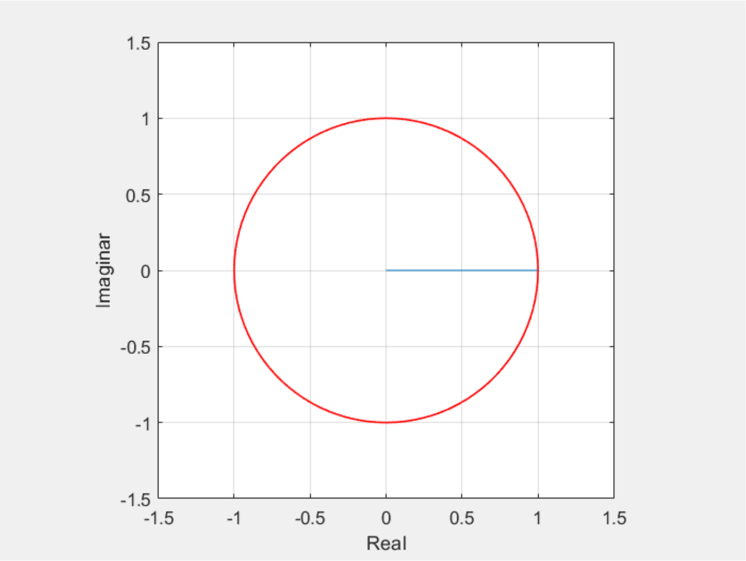
$$s[n] = e^{+j\Omega n}$$

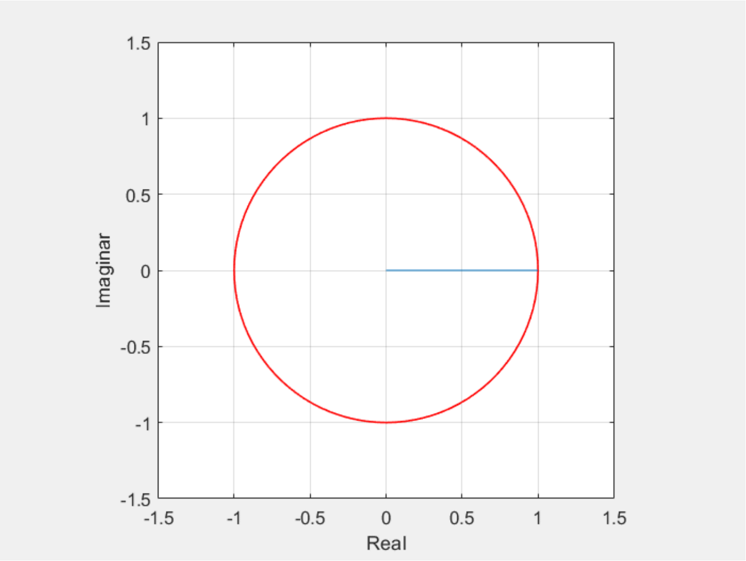


Exemplu

$$s[n] = e^{-j\Omega n}$$

$$s[n] = e^{+j\Omega n}$$





Întrebări