3.2 Velocity-based motion model

In the remainder of this chapter we will describe two probabilistic motion models for planar movement: the **velocity motion model** and the **odometry motion model**, the former being the main topic of this section. Remember that when a movement command is given to a robot, there are different factors that affect such movement (*e.g.* wheel slippage, unequal floor, inaccurate calibration, motors response, etc.), adding uncertainty to the actual move done. This results in a need for characterizing the robot motion in *probabilistic terms*, that is:

$$p(x_t|u_t, x_{t-1})$$

being:

- x_t the robot pose at time instant t,
- u_t the motion command (also called control action) at t, and
- x_{t-1} the robot pose at the previous time instant $t-1$.

So basically this probability models the probability distribution over robot poses when executing the motion command u_t, having the robot the previous pose x_{t-1}. In other words, we are considering a function $g(\cdot)$ that performs $x_t = g(x_{t-1}, u_t)$ and outputs $x_t \sim p(x_t|u_t, x_{t-1})$.

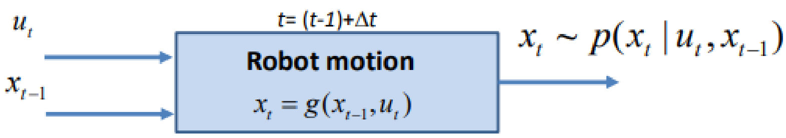


Fig. 1: Inputs and outputs of a probabilistic motion model.

Different definitions for the $g(\cdot)$ function lead to different probabilistic motion models, like the velocity motion model explored here.

3.2.1 The model

Usage: The ***velocity motion model*** is mainly used for motion planning, where the details of the robot's movement are of importance and odometry information is not available (*e.g.* no wheel encoders are available).

This motion model is characterized by the use of two velocities to control the robot's movement: **linear velocity** v and **angular velocity** w. Therefore, during the following sections, the movement commands will be of the form:

$$u_t = \begin{bmatrix} v_t \\ w_t \end{bmatrix}, \quad u_t \sim N(\overline{u}, \Sigma_{u_t})$$

The velocity motion model defines the function $g(\cdot)$ as:

$$g(x_{t-1}, u_t) = x_{t-1} \oplus \Delta x_t, \;\; x_{t-1} \sim N(\overline{x}_{t-1}, \Sigma_{x_{t-1}})$$

being $\Delta_{x_t} = [\Delta_{x_t}, \Delta_{y_t}, \Delta_{\theta_t}]$ (assuming w and v constant):

- $\Delta x_t = \frac{v}{w} \sin(w\Delta t)$
- $\Delta y_t = \frac{v}{w}[1 - \cos(w\Delta t)]$
- $\Delta \theta_t = w\Delta t$

Note that $g(x_{t-1}, u_t) = x_{t-1} \oplus \Delta x_t$ **is not a linear operation!**

In this way, this motion model is characterized by the following equations, depending on the value of the angular velocity w (note that a division by zero would appear in the first case with $w = 0$):

- If $w \neq 0$:

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + \begin{bmatrix} -R\sin\theta_{t-1} + R\sin(\theta_{t-1} + \Delta\theta) \\ R\cos\theta_{t-1} - R\cos(\theta_{t-1} + \Delta\theta) \\ \Delta\theta \end{bmatrix}$$

- If $w = 0$:

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + v \cdot \Delta t \begin{bmatrix} \cos\theta_{t-1} \\ \sin\theta_{t-1} \\ 0 \end{bmatrix}$$

with:

- $v = w \cdot R$ *(R is also called the curvature radius)*
- $\Delta\theta = w \cdot \Delta t$

In [1]:

```
%matplotlib widget

# IMPORTS
import numpy as np
from numpy import random
import matplotlib.pyplot as plt
from IPython.display import display, clear_output
import time


import sys
sys.path.append("..")
from utils.DrawRobot import DrawRobot
from utils.PlotEllipse import PlotEllipse
```

ASSIGNMENT 1: The model in action

Modify the following `next_pose()` function, used in the `VelocityRobot` class below, which computes the next pose x_t of a robot given:

- its previous pose x_{t-1},

- the velocity movement command $u = [v, w]^T$, and
- a lapse of time Δt.

Concretly you have to complete the if-else statement that takes into account when the robot moves in an straight line so $w = 0$. *Note: you don't have to modify the* `None` *in the function header nor in the* `if cov is not None:` *condition.*

Remark that at this point **we are not taking into account uncertainty in the system**: neither from the initial pose ($\Sigma_{x_{t-1}}$) nor the movement (v, w) (Σ_{u_t}).

Example

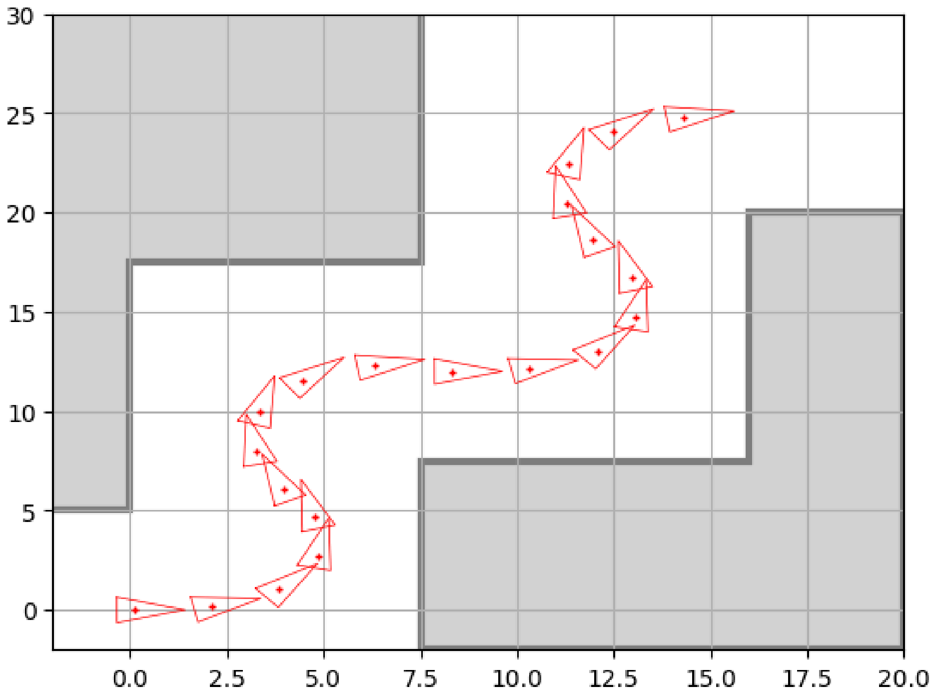


Fig. 2: Route of our robot.

In [2]:
```
def next_pose(x, u, dt, cov=None):
    ''' This function takes pose x and transform it according to the motion u=[v
        applying the differential drive model.

        Args:
            x: current pose
            u: differential command as a vector [v, w]'
            dt: Time interval in which the movement occurs
            cov: covariance of our movement. If not None, then add gaussian nois
    '''
    if cov is not None:
        u += np.sqrt(cov) @ random.randn(2, 1)
        #u = np.random.multivariate_normal(u.flatten(),cov)

    if u[1] == 0: #linear motion w=0
        theta = x[2]
        v = u[0]
        next_x = x + v*dt*np.vstack([np.cos(theta),
                           np.sin(theta),
                           0])
    else: #Non-linear motion w=!0
        R = u[0]/u[1] #v/w=r is the curvature radius
        theta = x[2]
```

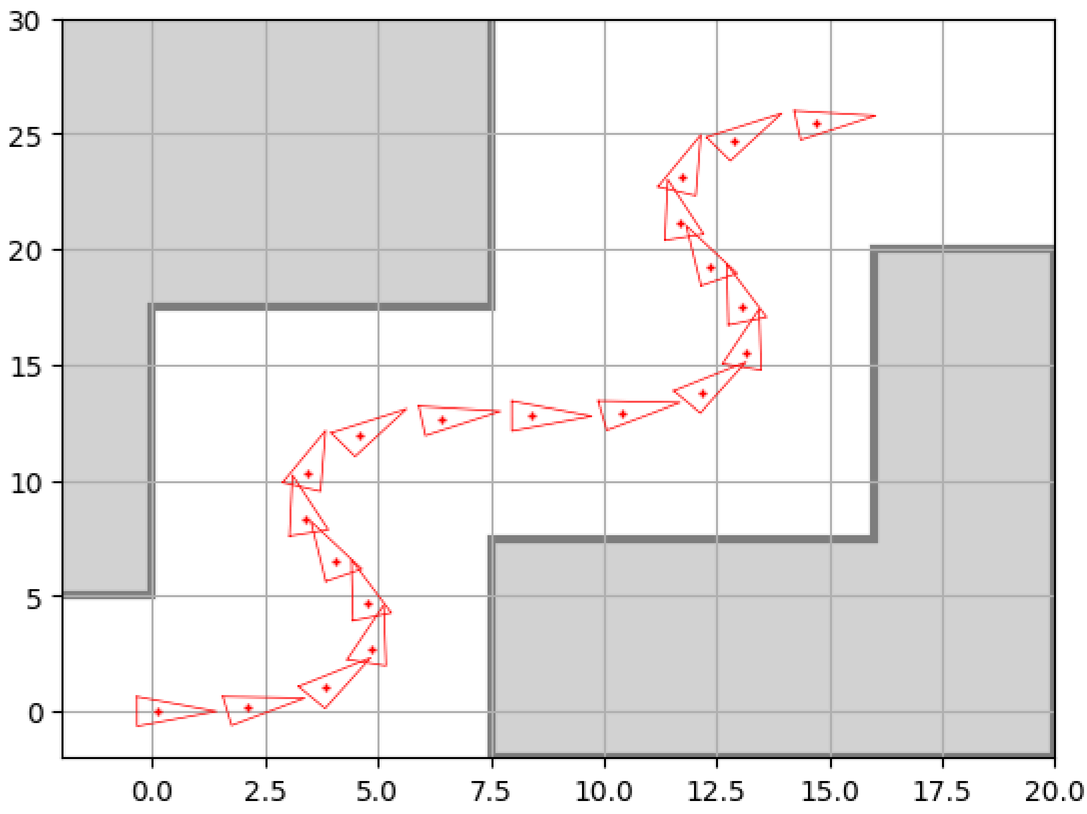
```
        var_theta = u[1]*dt
        next_x = x + np.vstack([-R*np.sin(theta) + R*np.sin(theta + var_theta),
                               R*np.cos(theta) - R*np.cos(theta + var_theta),
                               var_theta])

    return next_x
```

In [3]:
```
class VelocityRobot(object):
    """ Mobile robot implementation that uses velocity commands.

        Attr:
            pose: expected pose of the robot in the real world (without taking a
            dt: Duration of each step in seconds
    """
    def __init__(self, mean, dt):
        self.pose = mean
        self.dt = dt

    def step(self, u):
        self.pose = next_pose(self.pose, u, self.dt)

    def draw(self, fig, ax):
        DrawRobot(fig, ax, self.pose)
```

Test the movement of your robot using the demo below.

In [4]:
```
def main(robot, nSteps):

    v = 1 # Linear Velocity
    l = 0.5 #Half the width of the robot

    # MATPLOTLIB
    fig, ax = plt.subplots()
    plt.ion()
    fig.canvas.draw()
    plt.xlim((-2, 20))
    plt.ylim((-2, 30))
    plt.fill([7.5, 7.5, 16, 16, 20, 20],[-2, 7.5, 7.5, 20, 20, -2],
             facecolor='lightgray', edgecolor='gray', linewidth=3)
    plt.fill([-3, 0, 0, 7.5, 7.5, -3],[5, 5, 17.5, 17.5, 32, 32],
             facecolor='lightgray', edgecolor='gray', linewidth=3)

    plt.grid()

    # MAIN LOOP
    for k in range(1, nSteps + 1):
        #control is a wiggle with constant linear velocity
        u = np.vstack((v, np.pi / 10 * np.sin(4 * np.pi * k/nSteps)))

        robot.step(u)

        #draw occasionally
        if (k-1)%20 == 0:
            robot.draw(fig, ax)
            clear_output(wait=True)
            display(fig)
            time.sleep(0.1)

    plt.close()
```

In [5]:
```
# RUN
dT = 0.1 # time steps size
pose = np.vstack([0., 0., 0.])

robot = VelocityRobot(pose, dT)
main(robot, nSteps=400)
```



Thinking about it (1)

Now that you have some experience with robot motion and the velocity motion model, **answer the following questions**:

- Why do we need to consider two different cases when applying the $g(\cdot)$ function, that is, calculating the new robot pose?

  Necesitamos considerar dos casos diferentes porque el robot puede realizar un movimiento lineal o un movimiento no lineal. En el caso en que el robot realiza un movimiento lineal, la velocidad angular es nula y, si usáramos la misma función g, habría una división entre cero, cosa que no es posible. Para ello, debe definirse un caso diferente.

- How many parameters compound the motion command u_t in this model?

  Está compuesto por dos parámetros, v y w, que son la velocidad lineal y la velocidad angular, respectivamente.

```
In [ ]:
```