

Lectures 6

Schedulability Analysis Part 2

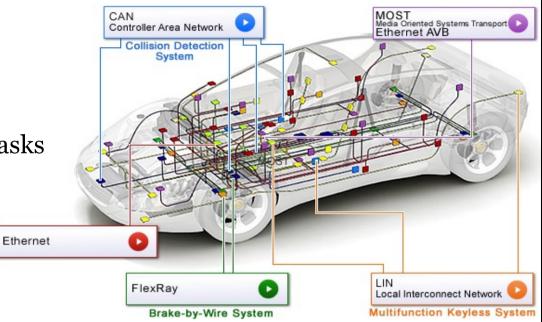
Saad Mubeen





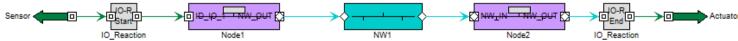
Response-time analysis (RTA) of tasks

- Basic RTA
- RTA with blocking
- RTA with jitter
- RTA with Offsets





- RTA of Controller Area Network (CAN) messages
- Practical limitations in CAN Controllers and their effect on RTA for CAN
- Holistic RTA (for distributed systems)
- Data-propagation delay analysis
 - Single node systems
 - Distributed systems (End-to-end data-propagation delay analysis)



Figures courtesy of: http://www.cvel.clemson.edu/
http://www.renesas.eu

Controller Area Network (CAN): facts and motivation

- Modern premium cars
 - 70-100 ECUs
 - 5 or more networks
- Controller Area Network (CAN)
 - Speed up to 1 Mbit/s
 - ISO 11898 (1-3)
 - According to CiA, more than 2 billion CAN controllers have been sold
 - 80% have been used in the automotive domain
- Modern heavy trucks
 - 45 ECUs
 - 20 CAN buses
 - over 6000 MSGs
- Higher-level protocols for CAN
 - CANopen, MilCAN, HCAN, AUTOSAR

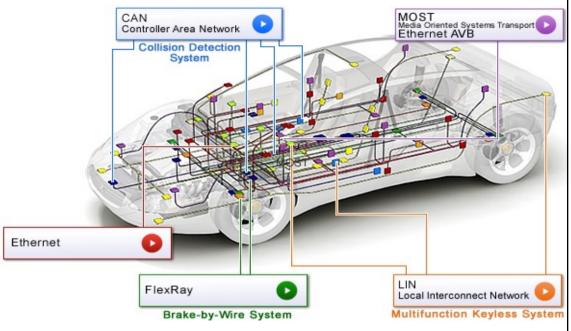


Figure courtesy of www.renesas.eu



Figure courtesy of www.timmo-2-use.org



Response Time of CAN message

 R_i = Worst-case response time of a CAN message "i"

 R_i = Worst-case transmission time of "i"

- + Blocking time due to lower priority messages
- + Interference due to higher priority messages

What about C_i?

- In RTA for tasks, C_i is the "execution time"
 - For a message, C_i is the **transmission time**
- All nodes use the same baudrate
 - Time to send one bit = τ_{bit} (Tau-bit)
- S_i = the maximum size of message payload
 - S_i is 0...8 bytes



SOF (Start Of Frame)	Identifier	RTR (Remote Trans- mission Request)	Contro I	Data	CRC (Cyclic Redun- dancy Check)	CRC Delimiter	ACK Acknow- ledg	ACK Delimit er	EOF (End Of Frame)	IFS (Inter Frame Space)
1	11	1	6	0-8	15	1	1	1	7	min 3
bit	bits	bit	bits	bytes	bits	bit	bit	bit	bits	bits

+ 47 control bits

What about C_i (cont)

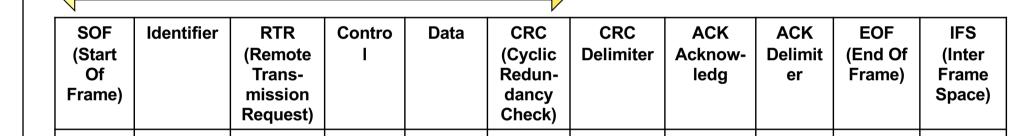
- Other source of variability
 - Bit-stuffing

11

bits

bit

- (i.e. adding extra 1 or 0 to avoid six consecutive equals)
- 34 of 47 control-bits + payload are subjected to bit-stuffing



15

bits

bit

bit

34+8*S_i bits are subjected to bit-stuffing

0-8

bytes

Maximum of |(34+8S_i-1)/4| stuff-bits

6

bits

bit

•
$$C_i = (47 + 8S_i + [(34+8S_i - 1)/4])\tau_{bit}$$

min 3

bits

bits

bit

What about B_i?

- Blocking = bounded interference from *lower* priority messages
 - For CAN: B_i is transmission time from 1 lower priority message
 - $B_i = \forall_{j \in lp(i)} max(C_j)$
- Safe approximation of blocking
 - $B_i = \forall_i \max(C_i)$
 - or even safer/simpler: $B_i = 135 \tau_{bit}$

$$C_i = (47 + 8S_i + [(34+8S_i-1)/4])\tau_{bit}$$

Put $S_i = 8$

What about B_i?

• Blocking = bounded interference from *lower* priority messages

- For CAN: B_i is transmission time from 1 lower priority message
- $B_i = \forall_{j \in lp(i)} max(C_j)$

Safe approximation of blocking

- $B_i = \forall_i \max(C_i)$
- or even safer/simpler: $B_i = 135 \tau_{bit}$

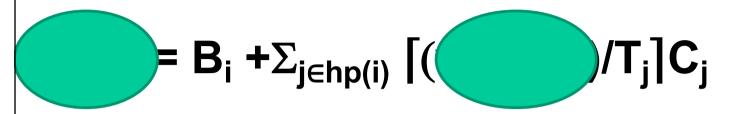
Why approximate?

- Exact $B_i = \forall_{j \in lp(i)} max(C_j)$
- Risk of "unknown" low priority messages
 - E.g. third party equipment
 - Incomplete model for analysis
- Also: Analysis becomes significantly more complex:
 - Controller Area Network (CAN) schedulability analysis: Refuted, revisited and revised. Robert I. Davis, Alan Burns, Reinder J. Bril, Johan J. Lukkien. J. of Real-Time Systems 2007

Asynchronous nodes and signal propagation

- CAN is global priority queue
 - But: Nodes are not synchronized
- Maximum delay to observe common event
 - Bounded by signal propagation delay
 - Remember: Arbitration works since delay is "small enough"
 - i.e. delay is never bigger than $au_{\rm bit}$
- Signal propagation gives "uncertainty" in observation of arrivals at remote nodes
 - This uncertainty is called "Jitter"
 - Jitter increases the time-interval during which a high-priority message can interfere
 - This increase is bounded by $au_{\rm bit}$

Putting in all together I



Jitter:

- Increases the time-interval during which a highpriority message can interfere
- This increase is bounded by $au_{\rm bit}$

$$= B_i + \sum_{j \in hp(i)} \left[\left(+ \tau_{bit} \right) / T_j \right] C_j$$

Putting in all together II

- Non-preemtive transmission
 - "Window of interference" (w_i) does NOT include C_i
 - I.e. hp(i) messages cannot interfere once transmission has started

$$\mathbf{W}^{n+1}_{i} = \mathbf{B}_{i} + \sum_{j \in hp(i)} \left[(\mathbf{w}^{n}_{i} + \tau_{bit}) / \mathbf{T}_{j} \right] \mathbf{C}_{j}$$

- Instead C_i is added to the "window"
 - C_i comes after the window of interference

$$R_i = C_i + w_i$$

RTA for CAN messages with Jitter

Response time analysis must be updated with a release jitter term:

$$J_{i}$$

New equation:

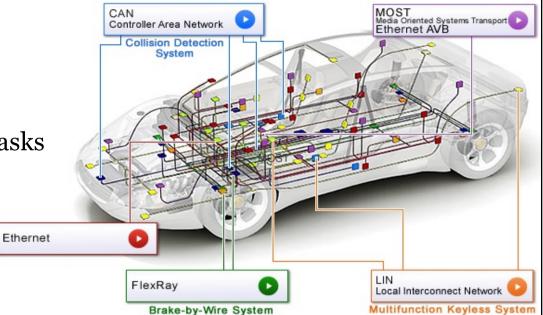
$$w_i^{n+1} = B_i + \sum_{\forall j \in hp(i)} \left[\frac{w_i^n + J_j + \tau_{bit}}{T_j} \right] C_j$$

$$R_i = C_i + J_i + w_i$$

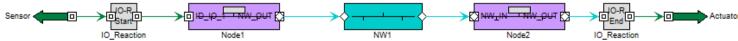


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Response Time Analysis for Controller Area Network (CAN): Practical Limitations

Saad Mubeen

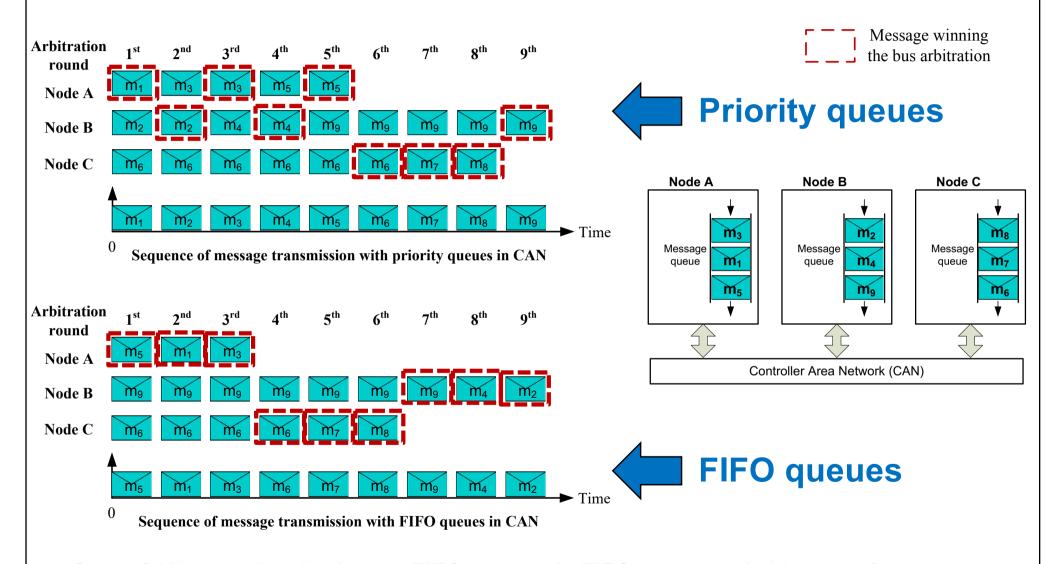


The classical RTA for CAN (that we studied in this lecture) assumes that if there are more than one messages in a node that are ready to be sent then the highest priority message will be sent first.

However, this assumption may be violated due to various types of queuing polices implemented by the CAN device drivers and communications stacks, internal organization and hardware limitations in the CAN controllers.

In addition to these limitations, mixed transmission patterns supported by higher-level protocols can have significant impact on the timing behavior of CAN messages.

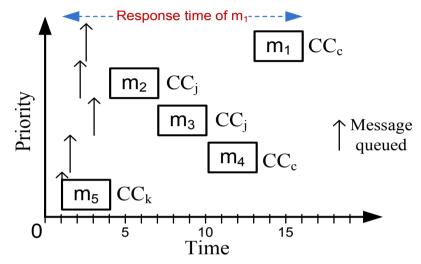
- Queuing policies used in the CAN controllers
 - Priority
 - First In First Out (FIFO)
- Buffer limitations in the CAN controllers
 - Non-abortable transmit buffers
 - Abortable transmit buffers
- Mixed messages implemented by higher-level protocols



Some CAN controllers implement FIFO queues. In FIFO queues, priorities are often not respected. There may be long buffering delays. This may results in longer response times of messages especially those with higher priorities. RTA for CAN should consider these issues.

Buffer limitations in the CAN controllers

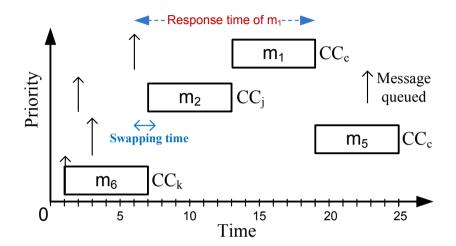
- Non-abortable transmit buffers
- When the highest prio msg m₁ in node CC_c is queued, all trasnsmit buffers in CC_c are occupoed by lower priority messages.
- The CAN controllers do not support transmission abort requests, hence m₁ has to wait in the ready queue before the highest priority message in CC_c (say m₄) is transmitted thereby vacating a space for m₁ in the transmit buffers.



- However, before starting its transmission, m₄ can be interefered by higher priority messages from other nodes whose priorities are smaller than m₁. This results in an extra delay for m₁ other than the normal blocking (e.g., due to m₅) and interference from higher priority messages (if any).
- This extra delay should be considered in the worst-case response time of m₁.

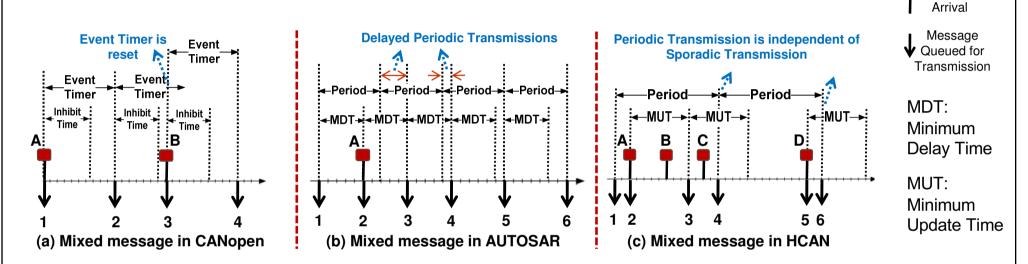
Buffer limitations in the CAN controllers

- Abortable transmit buffers
- When the highest prio msg m₁ in node CC_c is queued, all trasnsmit buffers in CC_c are occupoed by lower priority messages.
- The CAN controllers support transmission abort requests, hence the highest priority message in the transmit buffers of CC_c is swapped with m₁.



- If the swapping time is long, it is possible that a lower priority msg (say m₂) belonging to another node CC_i may win the bus arbitration and start to transmit.
- Since CAN uses non-preemptive sheduling, m₂ cannot be preempted. This results in an extra delay for m₁ other than the normal blocking (e.g., due to m₅) and interference from higher priority messages (if any).
- This extra delay should be considered in the worst-case response time of m₁.

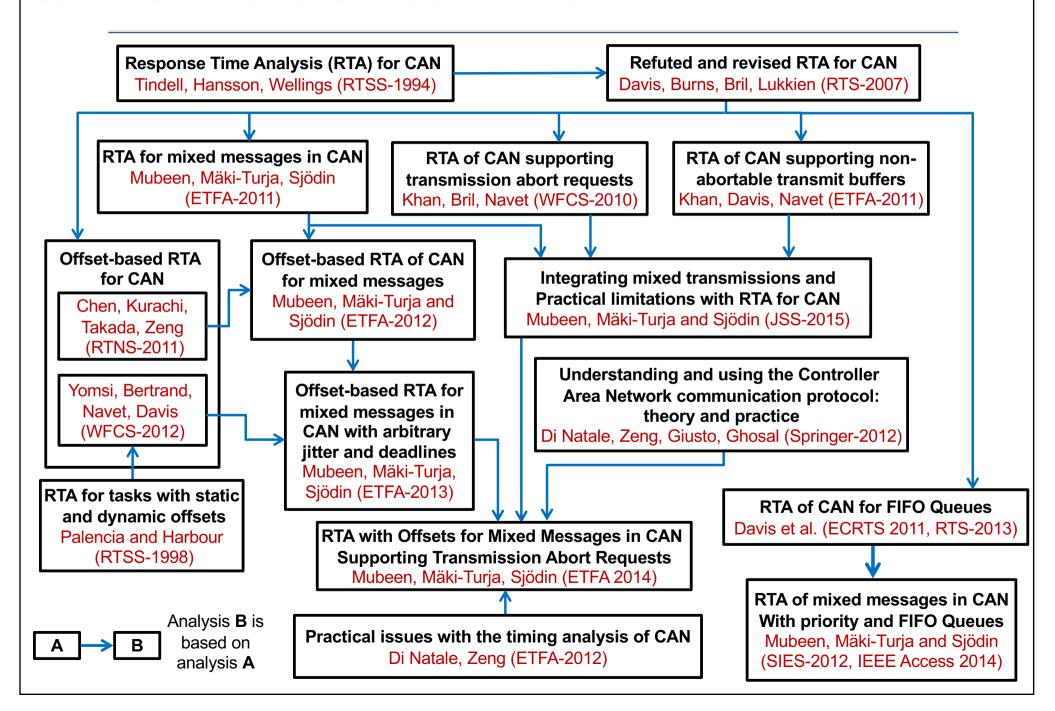
- Mixed messages are both periodic and sporadic
 - Mixed messages implemented by several higher-level protocols



Event

- The implementations of mixed messages in (a) and (b) are supported by the classical RTA for CAN
 - → This is because the worst-case periodicity is bounded by Inhibit time and MDT in the implementations (a) and (b) respectively
- However, some implementations of mixed messages such as implementation (c) are not supported by the classical RTA for CAN
 - → This is because the worst-case periodicity is neither bounded by period nor by minimum inter-arrival time (e.g. MUT in (c))

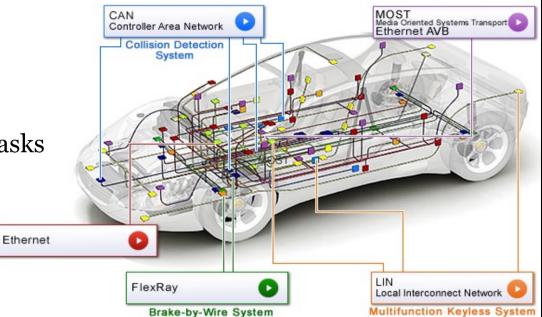
Some recent extensions of RTA for CAN





• Response-time analysis (RTA) of tasks

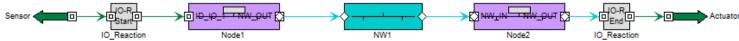
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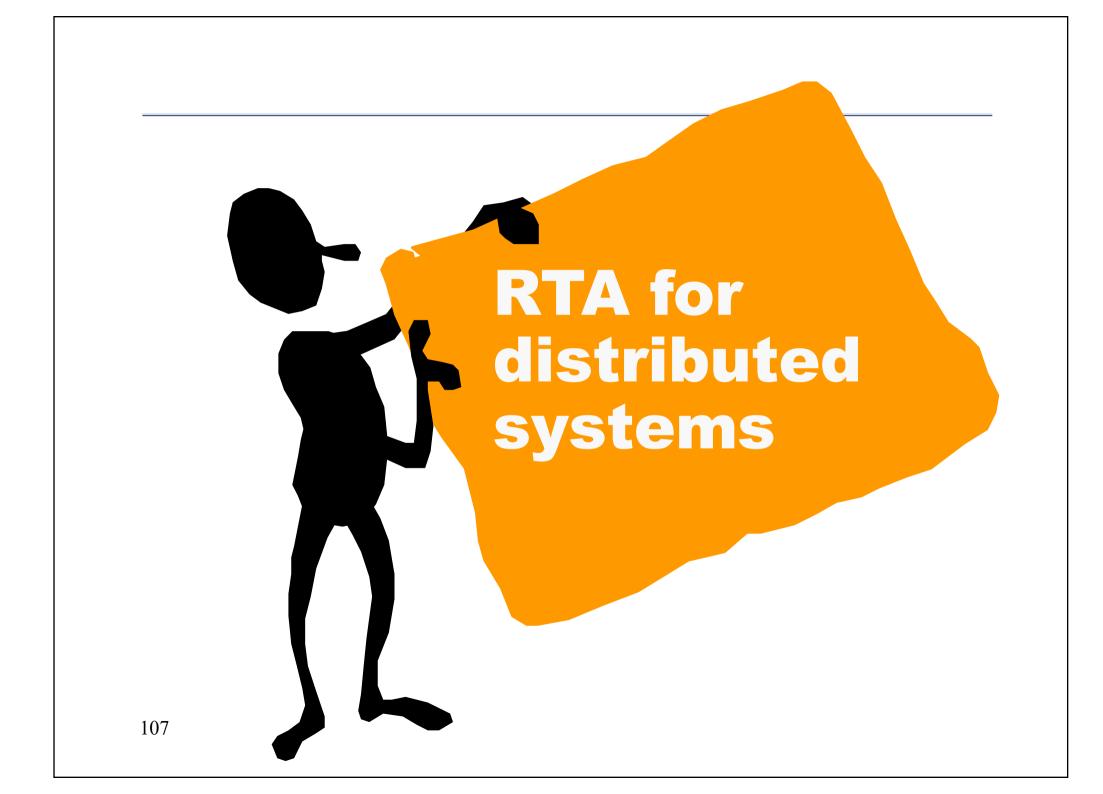
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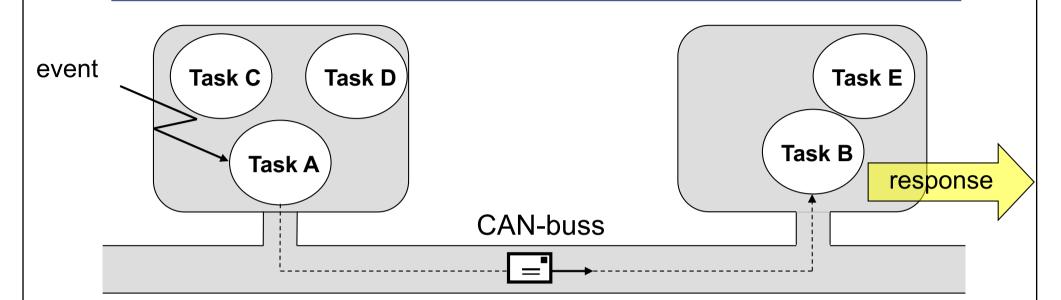
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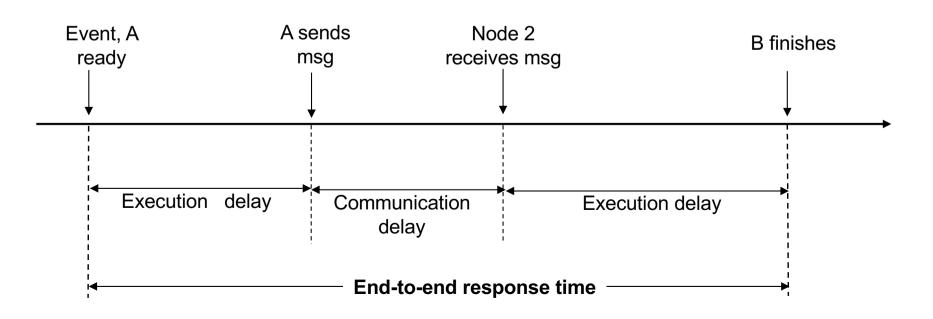


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End-to-end deadline

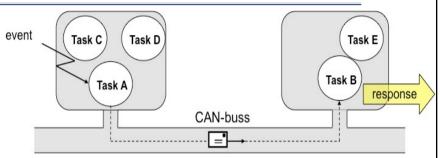




Distributed transaction – response times

Execution delay of A (= R_A) – Caused by other tasks on node 1

Use response time analysis to calculate R_A



Message release jitter (J_{m1}) – Caused by variations in A's execution

Message jitter = difference between R_A^{max} and R_A^{min}

Communication delay (= R_{m1}) – Caused by other messages on the bus

• Use jitter inherited from A (J_{m1}) and apply response time analysis for CAN to calculate R_{m1}

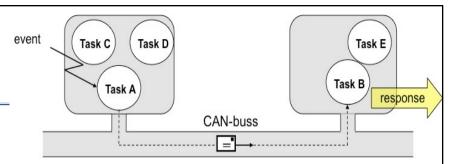
Release jitter for B (J_B) – Caused by variations in m1's transmission

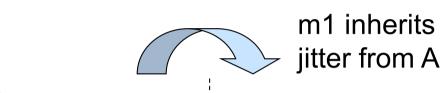
- Release jitter for B = difference between R_{m1}^{max} and R_{m1}^{min}
- $R_{m1}^{min} = 47\tau_{bit}$ (no other messages on the bus) $C_i = (47 + 8S_i + \lfloor (34 + 8S_i 1)/4 \rfloor)\tau_{bit}$

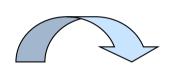
Execution delay of B ($= R_B$) – Caused by other tasks on node 2

Use jitter inherited form m1 (J_B) and calculate response time for B

Distributed transaction -Response times







B inherits jitter from m1

m1

$$w_A^{n+1} = C_A + B_A + \sum_{\forall j \in hp(A)} \left\lceil \frac{w_A^n + J_j}{T_j} \right\rceil C_j$$

$$R_{\scriptscriptstyle A} = w_{\scriptscriptstyle A} + J_{\scriptscriptstyle A}$$

$$J_{m1} = R_A^{\max} - R_A^{\min}$$

$$w_A^{n+1} = C_A + B_A + \sum_{\forall j \in hp(A)} \left\lceil \frac{w_A^n + J_j}{T_j} \right\rceil C_j$$

$$R_A = w_A + J_A$$

$$w_{m1} = B_{m1} + \sum_{\forall j \in hp(m1)} \left\lceil \frac{w_{m1} + J_j + \tau_{bit}}{T_j} \right\rceil C_j$$

$$w_B^{n+1} = C_B + B_B + \sum_{\forall j \in hp(B)} \left\lceil \frac{w_B^n + J_j}{T_j} \right\rceil C_j$$

$$R_{m1} = J_{m1} + W_{m1} + C_{m1}$$

$$J_{B} = R_{m1}^{\text{max}} - R_{m1}^{\text{min}}$$

$$w_B^{n+1} = C_B + B_B + \sum_{\forall j \in hp(B)} \left| \frac{w_B^n + J_j}{T_j} \right| C_j$$

$$R_{B} = W_{B} + J_{B}$$

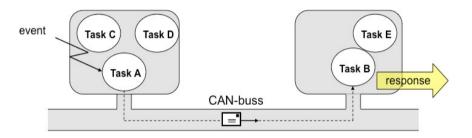
0 = Safeapproximation

47 $\tau_{\rm bit}$ = Safe approximation

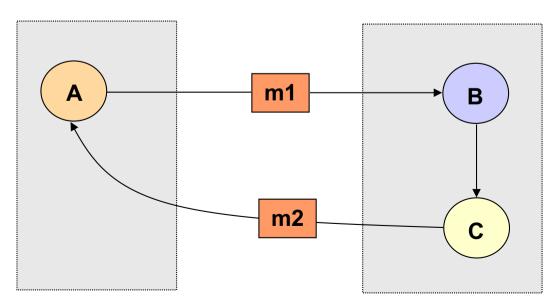
Holistic schedulability analysis

We learned how to handle jitter, as long this inheritance goes in one direction

- Eg., from A via CAN-bus to B, as in previous example



But what happens if we have communication in both directions?



- 1. B inherits jitter from A (via m1)
- 2. C inherits jitter from B
- 3. A inherits jitter from C (via m2)

Holistic schedulability problem!

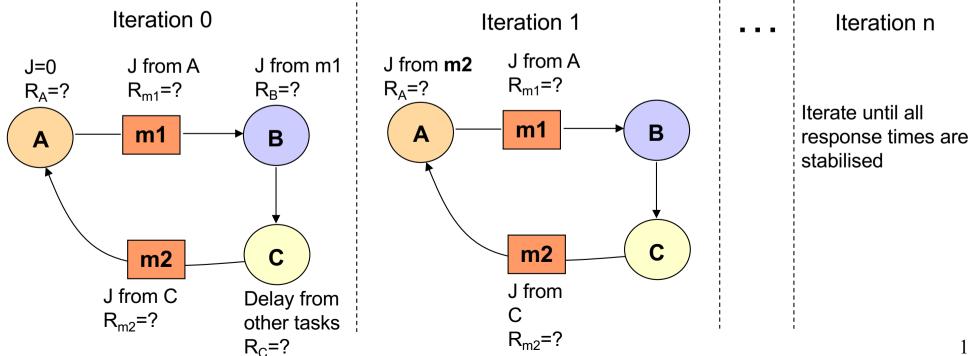
Holistic scheduling

A m1 B B C C

One more iteration level

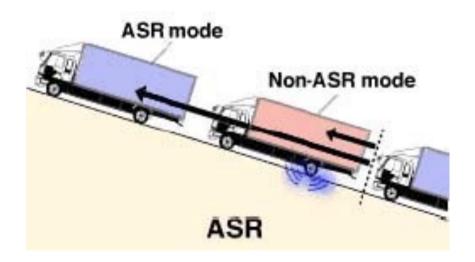
- 1. Initially we assume jitter=0
- 2. We iterate as before on each node and bus until we reach a local fix point
- 3. A fix point on a node gives information about jitter-inheritance
- 4. We must continue until jitter gets unchanged (global fix point)

Example:



Anti-Slip Regulation System for vehicles

- For easier starting, accelerating and climbing on wet surfaces
- Spinnig of the wheels detected by wheels sensors and engine rotation adjusted to mantain the optimum speed



System consist of five nodes and a CAN bus

- 4 wheel nodes (wheel1, wheel2, wheel3, wheel4)
- 1 node for the central unit (central)

Tasks on each wheel node

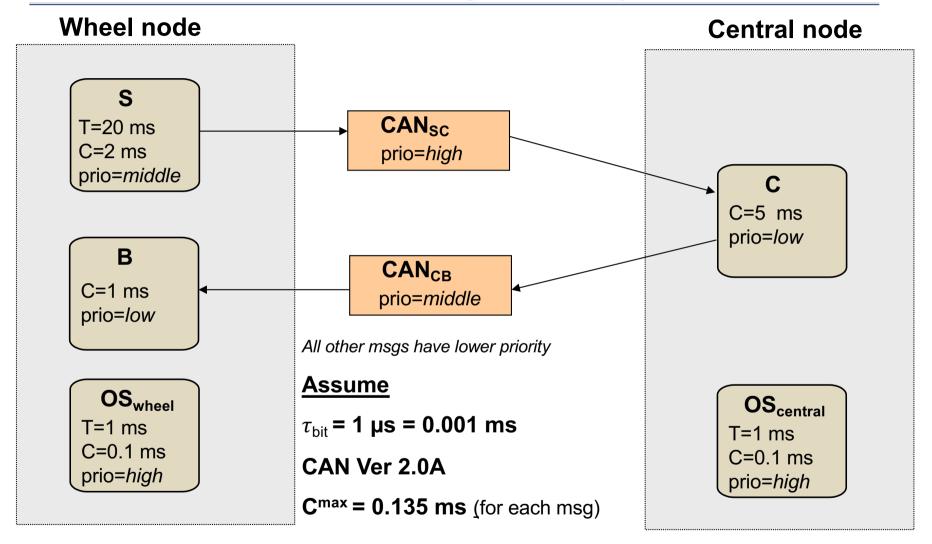
- S sampling of rotation speed of a wheel (time-trigged)
- B brakes a wheel (event-trigged)
- **OS**_{wheel} system task (time-trigged)

Tasks on the node central

- **C** calculates brake power based on the rotation speed (event-trigged)
- OS_{central} system task (time-trigged)

Communication between wheel and central:

- 1. Sample rotation speed of the *wheel*
- 2. Send speed data to *central* via CAN-bus
- 3. Calculate brake power on node *central*
- 4. Send result to wheel



Will the distributed transaction $S \rightarrow CAN_{SC} \rightarrow C \rightarrow CAN_{CB} \rightarrow B$ meet its end-to-end deadline of **18** ms?

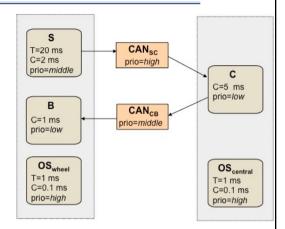
We start by calculating response time of S:

- No blocking (no common resources): B_S=0
- High priority tasks on the same node: hp(S) = {OS_{wheel}}

$$w_{S_V}^0 = C_S = 2$$

$$w_S^1 = C_S + B_S + \left[\frac{w_S^0 + J_{OSwheel}}{T_{OSwheel}} \right] * C_{OSwheel} = 2 + 0 + \left[\frac{2 + 0}{1} \right] * 0.1 = 2.2$$

$$w_S^2 = 2 + 0 + \left[\frac{2.2 + 0}{1} \right] * 0.1 = 2.3$$



$$w_S^3 = 2 + 0 + \left[\frac{2.3 + 0}{1}\right] * 0.1 = 2.3$$
 $w_S^3 = w_S^2 \rightarrow w_S = 2.3$

Incomming jitter in the first itteration step is equal to zero. Response time is:

$$R_S = J_S + w_S = 0 + 2.3 = 2.3 ms$$

We continue by calculating response time for CAN_{SC} :

• Low prio messages: $Ip(CAN_{SC})=\{CAN_{CB}, all other\}$. This gives blocking factor:

$$B_{CAN_{SC}} = \max_{\forall k \in lp(CAN_{SC})} C_k = \max(CAN_{CB}, all_other) = \max(0.135, 0.135, ...) = 0.135ms$$

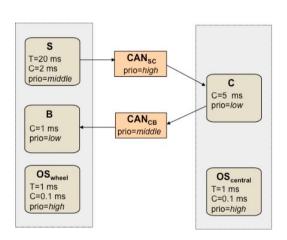
No high prio messages: hp(CAN_{SC}) = {}

$$W_{CAN_{SC}} = B_{CAN_{SC}} + 0 = 0.135 ms$$

 Incomming jitter is equal to response time of the task that sends the message, i.e., task S:

$$J_{CAN_{SC}} = R_S = 2.3ms$$

(Note! We assume Rmin=0, analysis is pessimistic but safe)



Finally, we get the response time for CAN_{SC}:

$$R_{CAN_{SC}} = J_{CAN_{SC}} + w_{CAN_{SC}} + C_{CAN_{SC}} = 2.3 + 0.135 + 0.135 = 2.57 ms$$

Next step is to calculate response time for C (that is activated by CAN_{SC}):

No blocking: dvs B_C=0
$$w_C^0 = C_C = 5$$

• $hp(C) = {OS_{central}}$

$$w_{C}^{1} = C_{C} + B_{C} + \left[\frac{w_{C}^{0} + J_{OScentral}}{T_{OScentral}}\right] * C_{OScentral} = 5 + 0 + \left[\frac{5 + 0}{1}\right] * 0.1 = 5.5$$

$$w_C^2 = 5 + 0 + \left\lceil \frac{5.5 + 0}{1} \right\rceil * 0.1 = 5.6$$

$$w_C^3 = 5 + 0 + \left[\frac{5.6 + 0}{1} \right] * 0.1 = 5.6$$

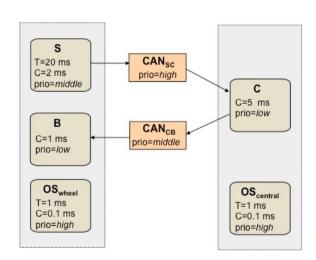
$$w_C^3 = w_C^2 \rightarrow w_C = 5.6$$

The jitter is:

$$J_C = R_{CAN_{SC}} = 2.57 ms$$

Response time is:

$$R_C = J_C + w_C = 2.57 + 5.6 = 8.17 ms$$



Because we are interested in entire transaction $S \rightarrow CAN_{SC} \rightarrow C \rightarrow CAN_{CB} \rightarrow B$ we continue by calculating the response time of CAN_{CB} :

Low priority messages: Ip(CAN_{CB})={all except CAN_{SC}}. Blocking factor is:

$$B_{CAN_{CB}} = \max_{\forall k \in Ip(CAN_{CB})} C_k = 0.135ms$$

• One high priority message: $hp(CAN_{CB}) = \{CAN_{SC}\}$. Period time for CAN_{SC} is equal to the period time of the task that sends it, i.e., task S, T= 20ms. This gives:

$$w_{CAN_{CB}}^{0} = 0.135ms$$

$$w_{CAN_{CB}}^{1} = B_{CAN_{CB}} + \left[\frac{w_{CAN_{CB}}^{0} + J_{CAN_{SC}} + \tau_{bit}}{T_{CAN_{SC}}} \right] * C_{CAN_{SC}} = 0.135 + \left[\frac{0 + 2.3 + 0.001}{20} \right] * 0.135 = 0.27$$

$$w_{CAN_{CB}}^{2} = 0.135 + \left[\frac{1.135 + 2.3 + 0.001}{20} \right] * 0.135 = 0.27 = w_{CAN_{CB}}^{1} \rightarrow w_{CAN_{CB}} = 0.27ms$$

Incoming jitter is equal to response time for the task that is sending the message, i.e.,
 task C:

$$J_{CAN_{CP}} = R_C = 8.17 ms$$

Finally, we get response time for CAN_{CB} as:

$$R_{CAN_{CR}} = J_{CAN_{CR}} + w_{CAN_{CR}} + C_{CAN_{CR}} = 8.17 + 0.27 + 0.135 = 8.575 ms$$

We calculate the response time for the last part of the chain, task B:

•
$$B_B=0$$

•
$$hp(B) = \{S, OS_{wheel}\}$$

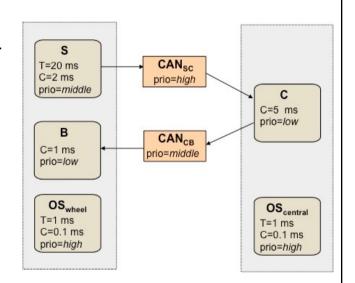
$$\begin{split} w_B^0 &= C_B = 1 \\ w_B^1 &= C_B + B_B + \left\lceil \frac{w_B^0 + J_{OSwheel}}{T_{OSwheel}} \right\rceil * C_{OSwheel} + \left\lceil \frac{w_B^0 + J_S}{T_S} \right\rceil * C_S = 3.1 \\ w_B^2 &= 1 + 0 + \left\lceil \frac{3.1 + 0}{1} \right\rceil * 0.1 + \left\lceil \frac{3.1 + 0}{20} \right\rceil 2 = 1 + 0 + 0.4 + 2 = 3.4 \\ w_B^3 &= 1 + 0 + \left\lceil \frac{3.4 + 0}{1} \right\rceil * 0.1 + \left\lceil \frac{3.4 + 0}{20} \right\rceil 2 = 1 + 0 + 0.4 + 2 = 3.4 \\ w_B^3 &= w_B^2 \longrightarrow w_B = 3.4 ms \end{split}$$

Incomming jitter depends on CAN_{CB} :

$$J_B = R_{CAN_{CR}} = 8.575 ms$$

Response time is:

$$R_B = J_B + w_B = 8.575 + 3.4 = 11.975 ms$$

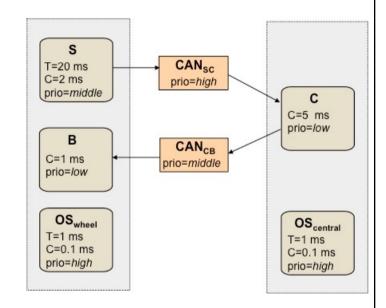


Example holistic scheduling – ASR system

We are done with the first iteration. Do we need to continue iterating?

No, since higher priority tasks to B has not changed their jitter

In other words, nothing will change if we iterate once again



Answer: Transaction's response time is 11.975 ms is less that the deadline 18ms, so it will meet its timing requirement.



Response-time analysis (RTA) of tasks

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- CAN Controller Area Network

 Collision Detection
 System

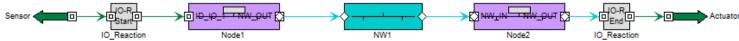
 Collision Detection
 System

 FlexRay

 LIN
 Local Interconnect Network

 Brake-by-Wire System

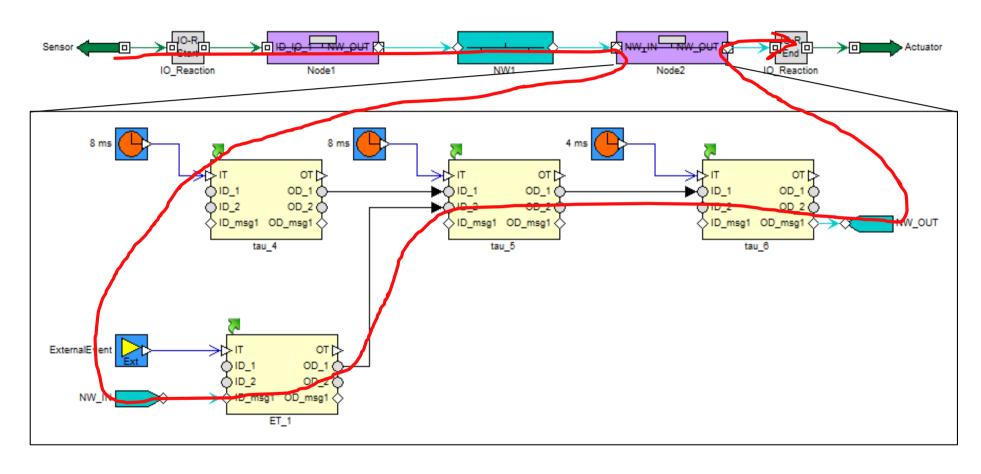
 Multifunction Keyless System
- RTA of Controller Area Network (CAN) messages
- Practical limitations in CAN Controllers and their effect on RTA for CAN
- Holistic RTA (for distributed systems)
- Data-propagation delay analysis
 - Single node systems
 - Distributed systems (End-to-end data-propagation delay analysis)



Figures courtesy of: http://www.cvel.clemson.edu/
http://www.renesas.eu



End-to-end data propagation delay analysis





Register-based Communication

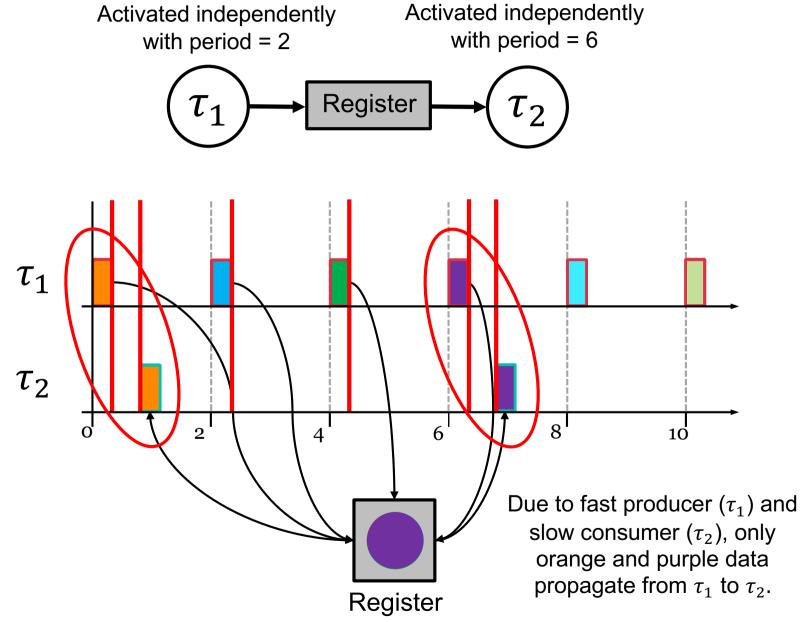
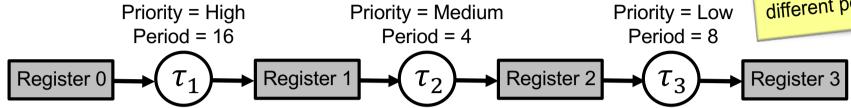


Figure courtesy of Matthias Becker, MDH

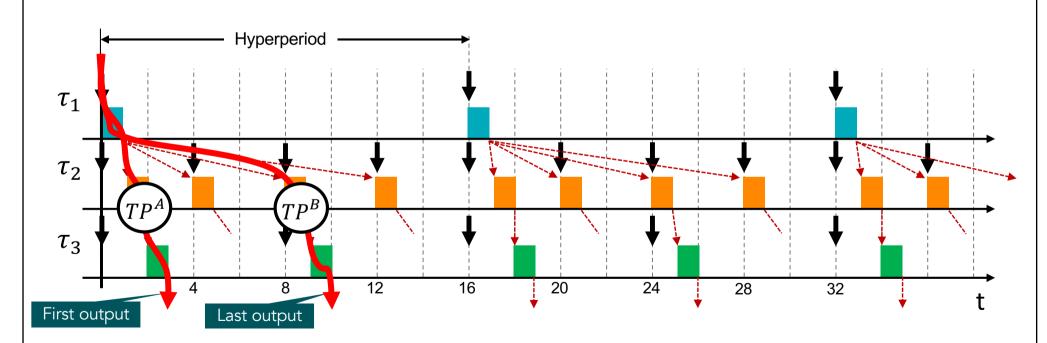


Data Propagation Delays

This is a multi-rate chain as the tasks are activated with different periods.



Tasks are independently activated

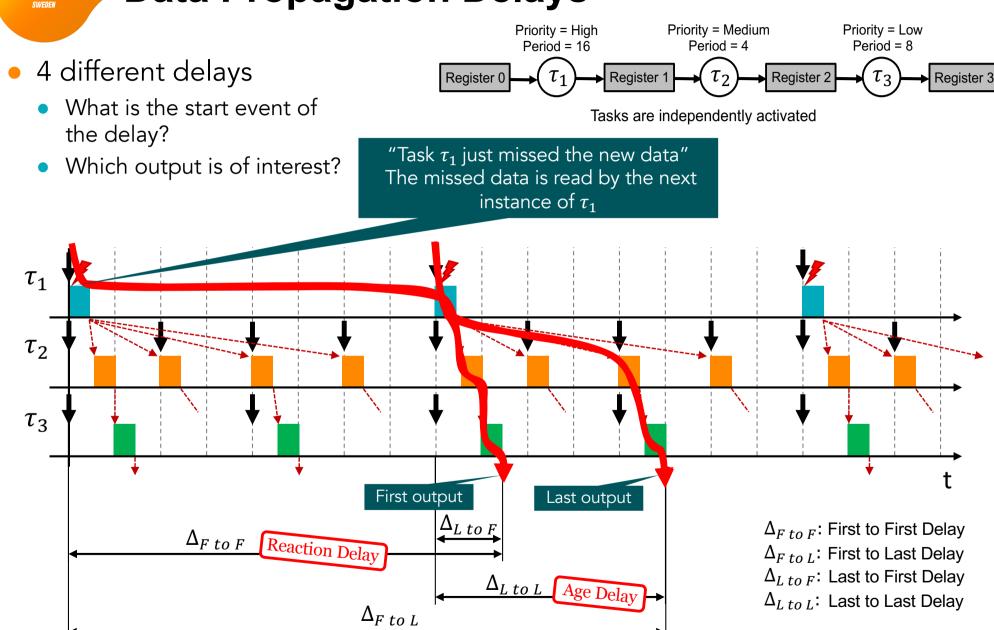


 TP^A : Timed Path A

 TP^B : Timed Path B

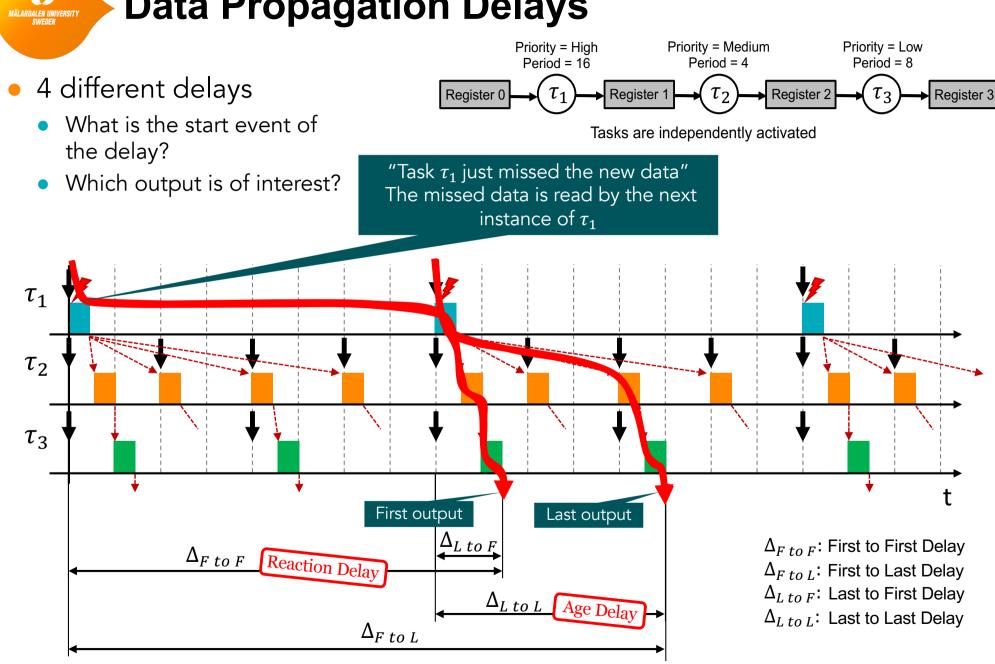


Data Propagation Delays





Data Propagation Delays





Data Propagation Delays: Reaction & Age

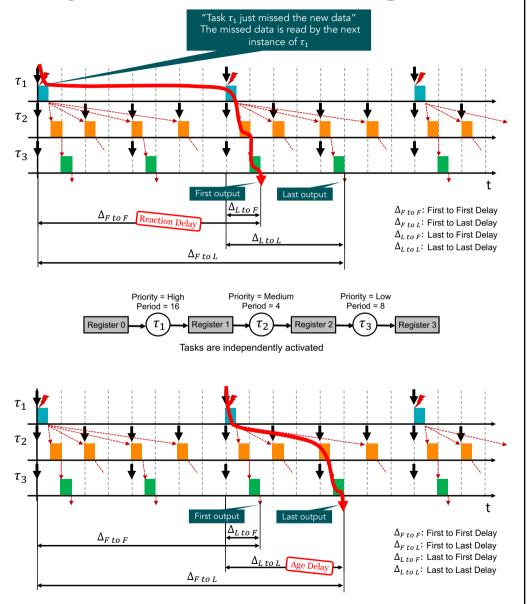
Data Reaction delay

- Applications in the body electronics domain in vehicles
 - Button-to-action delay
 - E.g., electronic door lock in a car

Data Age delay

- Applications in the control systems domain in vehicles
 - Maximum age of the data
 - E.g., control signals driving the actuators do not exceed a maximum age

Note: "last-to-last" (Age) considers the delay between the last input of the chain (that is not overwritten) until the last output of the chain (even in case of duplicates) [1].



[1] N. Feiertag, K. Richter, J. Nordlander, J. Jonsson, A Compositional Framework for End-to-End Path Delay Calculation of Automotive Systems under Different Path Semantics, Workshop on Compositional Theory and Technology for Real-Time Embedded Systems (CRTS), 2008.



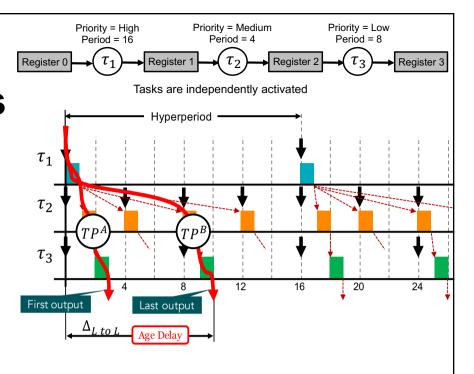
Assumptions, Definitions & Notations

Assumptions (for simplified analysis)

- No offsets
- Within a task chain, the priority of any task is not higher than the priority of its predecessor task

Note that the analysis in III is not restricted by these assumptions.

These assumptions are made to simplify the analysis for the purpose of understanding.

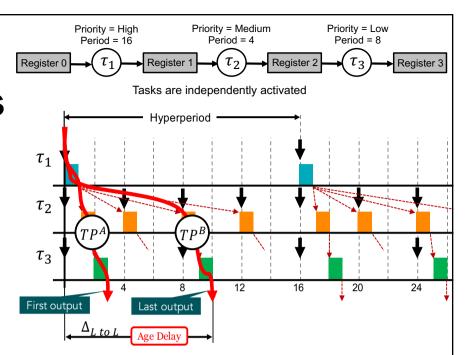


[1] N. Feiertag, K. Richter, J. Nordlander, J. Jonsson, A Compositional Framework for End-to-End Path Delay Calculation of Automotive Systems under Different Path Semantics, Workshop on Compositional Theory and Technology for Real-Time Embedded Systems (CRTS), 2008.

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Assumptions, Definitions & Notations



- **Timed Path (TP)** is a sequence of task instances along a task chain.
- A **reachable timed path** consists of the sequence of task instances along the chain through which the data can actually propagate from the first task to the last task in the chain.
- TP^A and TP^B in the figure are two reachable timed paths.
 - o $TP^A = (\tau_1(1), \tau_2(1), \tau_3(1))$, i.e., timed path from the 1st instance of τ_1 to the 1st instance of τ_2 to the 1st instance of τ_3

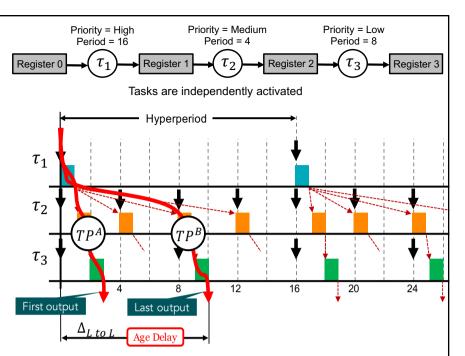
$$\circ$$
 $TP^B =$

• Path: $(\tau_1(1), \tau_2(2), \tau_3(2))$

Note that 'n' in $T_k(n)$ represents the n^{th} instance of task T_k . The first instance of T_k is represented by $T_k(1)$.



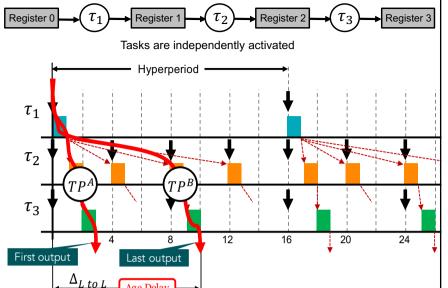
Assumptions, Definitions & Notations



- Let delay in the time path TP^A be represented by $Delay(TP^A)$
- $\alpha_k(n)$ = Activation time of n^{th} instance of task τ_k • $\alpha_1(1) = 0$, $\alpha_2(1) = 0$, $\alpha_3(1) = 0$
- $\alpha_k(TP^B)$ is the activation time of the instance of the kth task which is in path TP^B
 - \circ $\alpha_1 (TP^B) =$
 - \circ $\alpha_2 (TP^B) =$
 - \circ $\alpha_3 (TP^B) =$
- $R_k(n)$ = Response time of n^{th} instance of task τ_k
 - $R_3(1) = 3$, because the first instance of τ_3 experiences interference from the first instances of τ_1 and τ_2 (assuming no other interferences)
 - $R_3(2) =$



Simplified Calculations Register of the Age Delay



Period = 4

Period = 8

Step 1

Identify all reachable timed paths TPs that are initiated in the hyperperiod.

Step 2

Calculate the delay in each path separately

$$Delay(TP^M) =$$

Where,

 $\alpha_{Last}(TP^M)$ is the activation time of the instance of the Last task which is in path TP^M

 $R_{last}(TP^M)$ is the response time of the instance of the Last task which is in path TP^M

 $\alpha_{\text{First}}(TP^M)$ is the activation time of the instance of the First task which is in path TP^M

Step 3

Age delay is equal to the maximum delay among all reachable timed paths

Age delay =
$$Max\{Delay(TP^1), ..., Delay(TP^M)\}$$

Example

Delay
$$(TP^A) = \alpha_3(TP^A) + R_3(TP^A) - \alpha_1(TP^A) = \alpha_3(1) + R_3(1) - \alpha_1(1) =$$

$$Delay(TP^B) = \alpha_3(TP^B) + R_3(TP^B) - \alpha_1(TP^B) = =$$

Age delay = $Max\{Delay(TP^A), Delay(TP^B)\} = Max\{3, 10\} = 10$





Simplified Calculations Register 0 for the Reaction Delay

Register 1 Register 2

First output

'Task just missed the new data' The missed data is read by the

 $\Delta_{F \underline{to F}}$ Reaction Delay

Tasks are independently activated

Period = 4

Period = 8

Register 3

Step 1

Identify all reachable timed paths in the hyperperiod that have **non-duplicate** ("first") output of the chain. Consider the effect of just missing the new data at the input (first task in the chain), e.g., TP^X



Calculate the delay in each such path separately

Reaction
$$(TP^M) = \alpha_{Last}(TP^M) + R_{Last}(TP^M) - \alpha_{First}(Pred(TP^M))$$

Where, $\alpha_{First}(Pred(TP^M))$ is the activation time of the instance that is predecessor to the instance of the First task which is in path TP^{M}

 au_1

 τ_2

 au_3

Step 3

Reaction delay is equal to the maximum delay among all such timed paths

Reaction delay =
$$Max\{Delay(TP^1), ..., Delay(TP^M)\}$$

Example

$$Delay(TP^X) = \alpha_3(TP^X) + R_3(TP^X) - \alpha_1(Pred(TP^X)) =$$

Reaction delay =
$$Max{Delay(TP^X)} = 19$$

There is only one reachable TP in this example that fulfills the criteria in Step 1





Lecture 7: Highlight Group Assignment: End-to-end Data-propagation Delay Analysis of a Distributed Real-time System

- Make Groups. Each group should contain around 4-5 persons
- Give a cool name to your group
- We will solve an assignment in groups during the lecture
- You will write the name of of your group together with the analysis results on the board
- We will compare the results and discuss the solution(s)