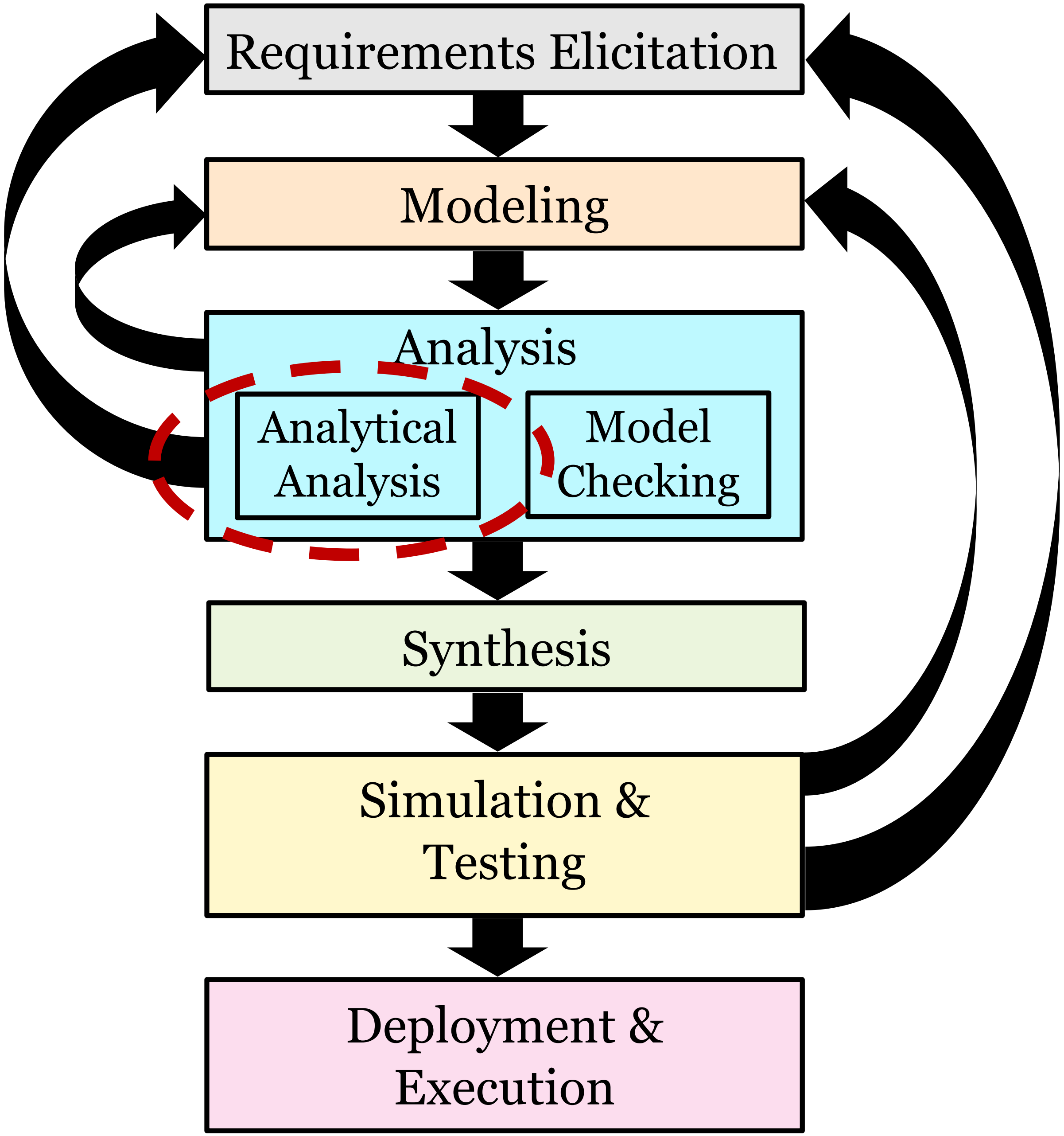
Lectures 5

# Schedulability Analysis
## Part 1

*Saad Mubeen*

# Model-based Software Development Process

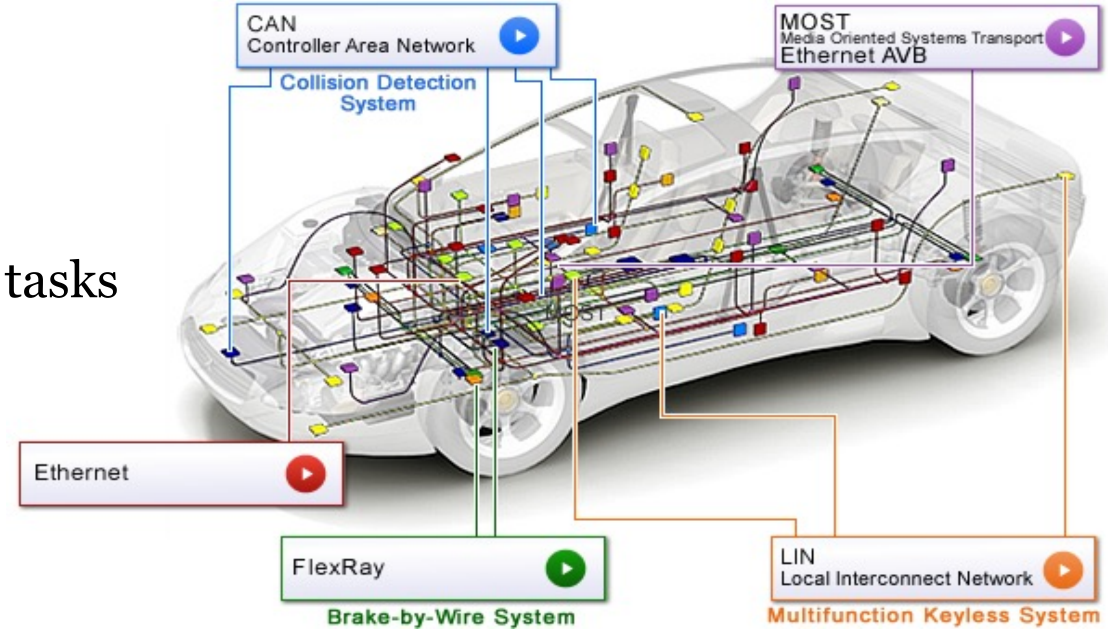


# Learning Objectives of this Lecture

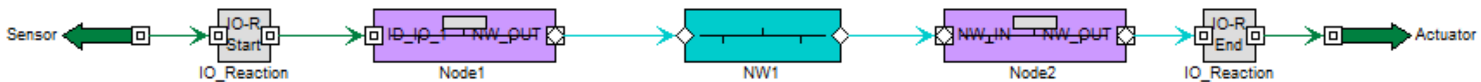
After this set of lectures you should be able to explain and reflect on

- Predicting the timing behavior of a real-time system <u>without actually running or executing it (i.e., in pre-runtime)</u>?

# Outline




- Response-time analysis (RTA) of tasks
  - Basic RTA
  - RTA with blocking
  - RTA with jitter
  - RTA with Offsets
- RTA of Controller Area Network (CAN) messages
- Practical limitations in CAN Controllers and their effect on RTA for CAN
- Holistic RTA (for distributed systems)
- Data-propagation delay analysis
  - Single node systems
  - Distributed systems (End-to-end data-propagation delay analysis)

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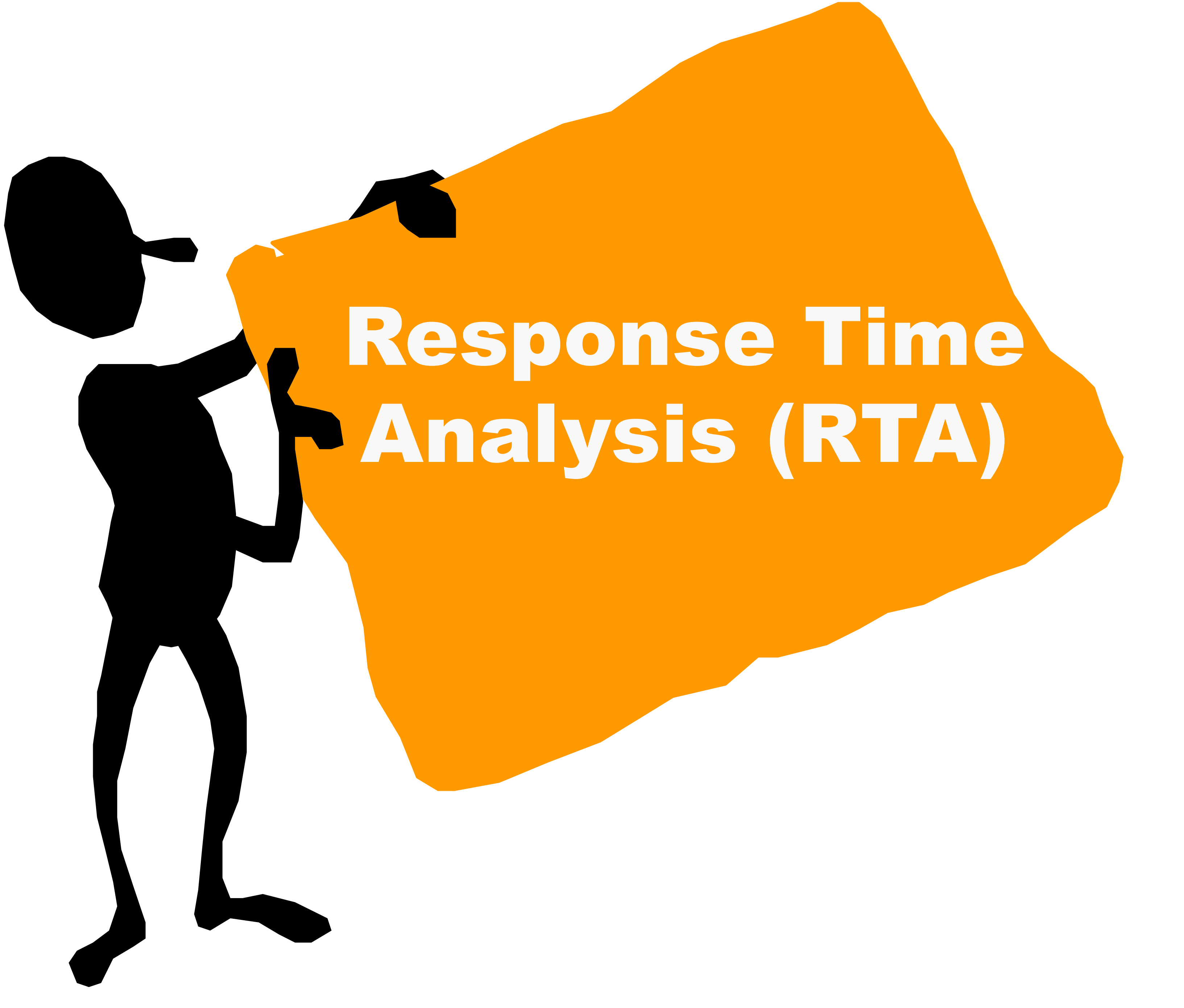

Response Time
Analysis (RTA)

# Response time analysis

**The CPU utilization analysis is not exact!**

- Remember for RM: if $U \leq n(2^{1/n}-1)$ then we cannot conclude if the task set is schedulable or not
- Also does not apply when deadlines are less than periods
  - Deadline Monotonic (DM) has no U-based schedulability test
- We need something else….

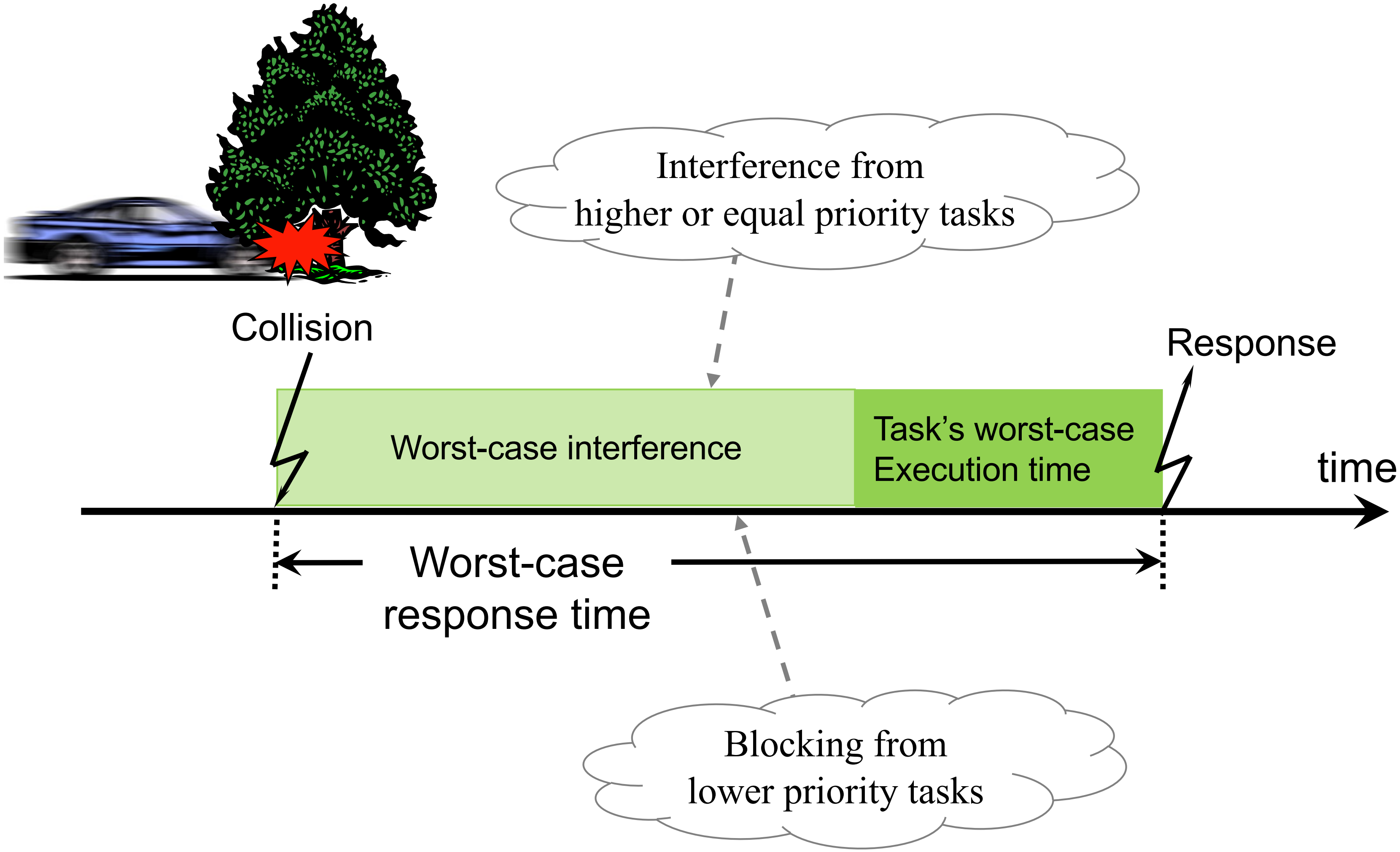
**Response time analysis**

- Predicts if all tasks will meet their deadlines
- Works for any static priority assignment (not only RM)
- Exact analysis – **both sufficient and necessary**

## Response times

**Schedulability with Response-Time Analysis (RTA)**

- For each task check if response time ≤ deadline



# Tools supporting RTA

Rubus-ICE

CANoe

Network Designer CAN

MPS-CAN Analyzer

SymTA/S

CANalyzer

VNA

RTaW-Sim

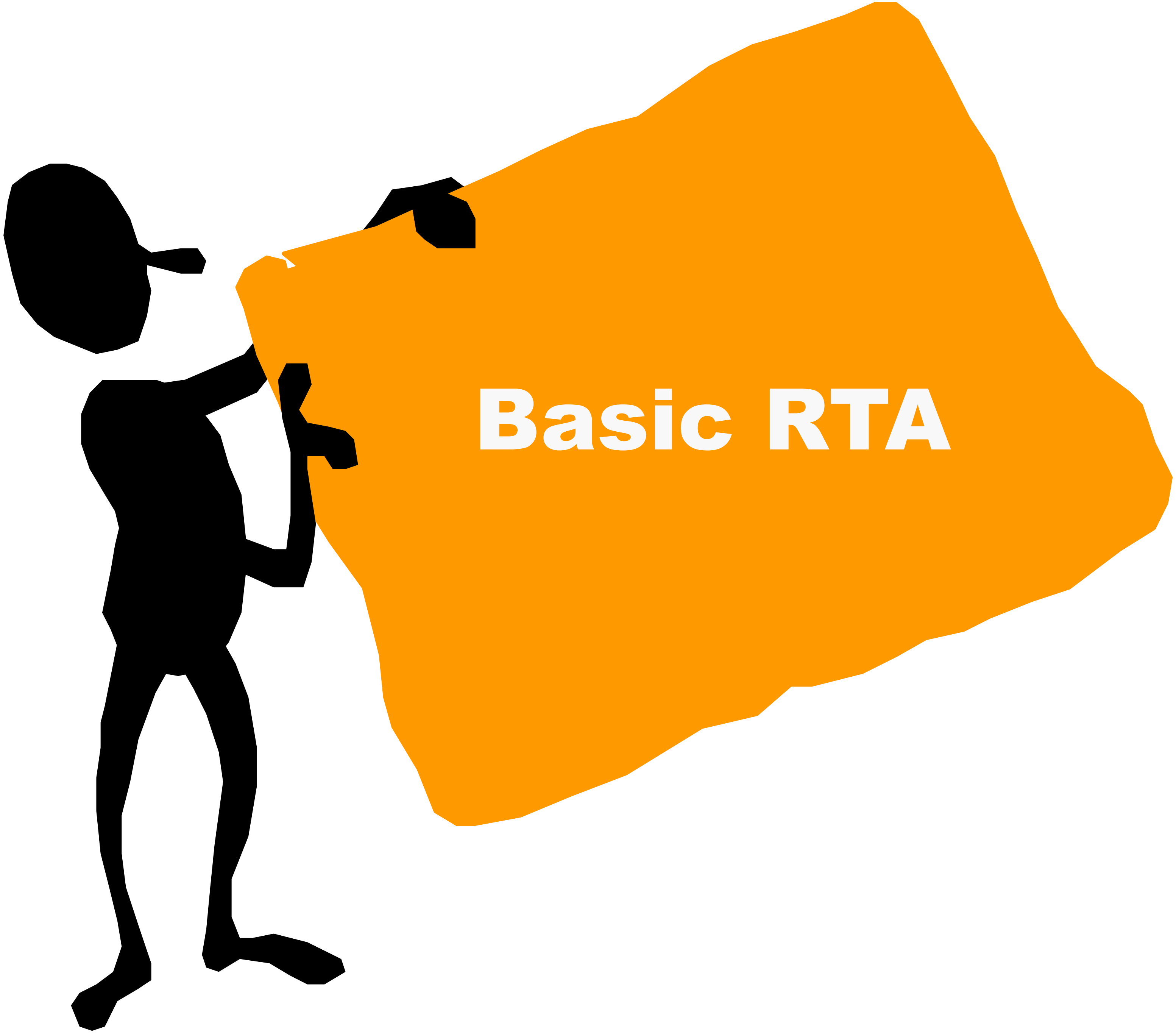
VOLVO

BAE SYSTEMS

KNORR-BREMSE

HOERBIGER

Haldex


Basic RTA

# Task set assumptions

**Preemptive priority scheduling**

**Independent tasks**

- No resource sharing
- No task synchronisation
- Given parameters: T, C, D, and P (priority)
- Periodic activation

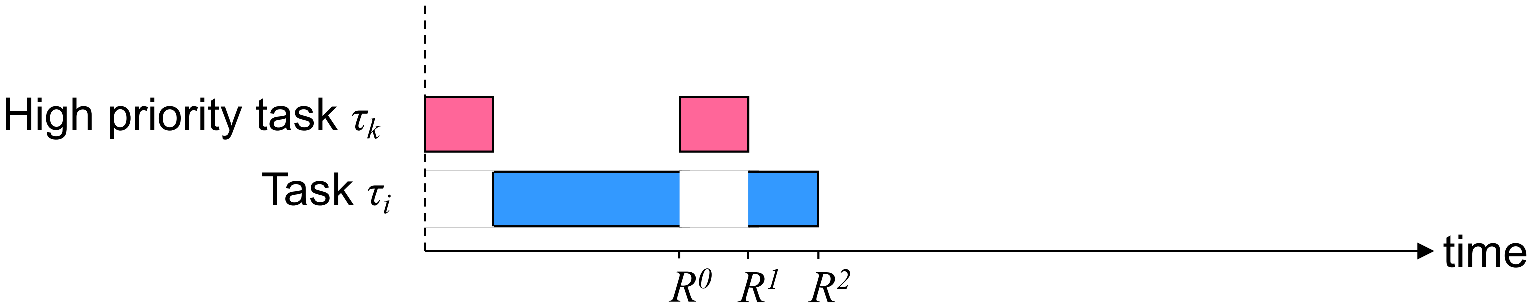
**No OS overhead**

## Response time analysis

**Response time for a task $\tau_i$ :**

1. Critical instant, '

2. Compute how much task $\tau_i$ will be delayed by high priority tasks (response time) and compare it to the deadline of $\tau_i$



# Remember from Lecture 2

## Critical Instant and Critical Time Zone

**Theorem1**: A critical instant for an arbitrary task is the time at which all higher priority tasks are released at the same time.

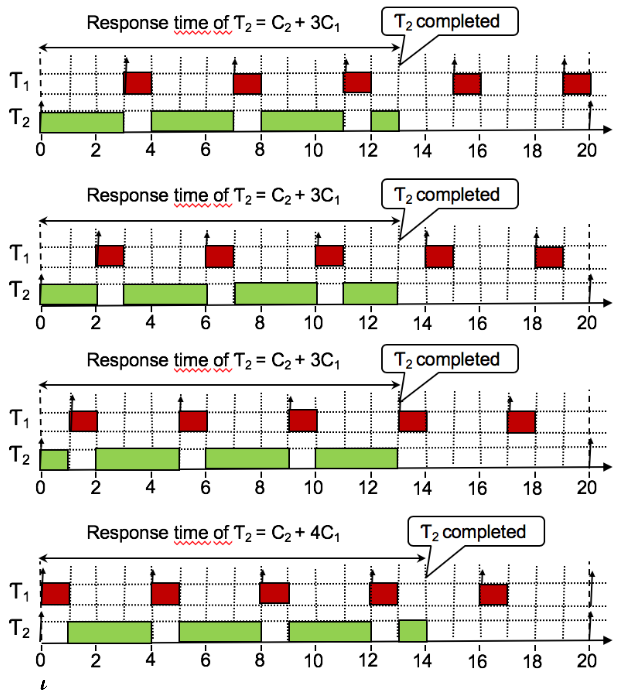
Consider a periodic task set
$\Gamma = \{T_1, T_2, T_3, \ldots, T_n\}$ ;
such that $T_2 > T_1$; $T_n > T_{n-1}$

| Task | T | C | |
|---|---|---|---|
| $T_1$ | 4 | $C_1$=1 | High priority |
| $T_2$ | 20 | $C_2$=10 | Low priority |

*Hyper period*: LCM(4,20)=20

Response time of $T_2$ is largest when $T_1$ is realeased at the same time.

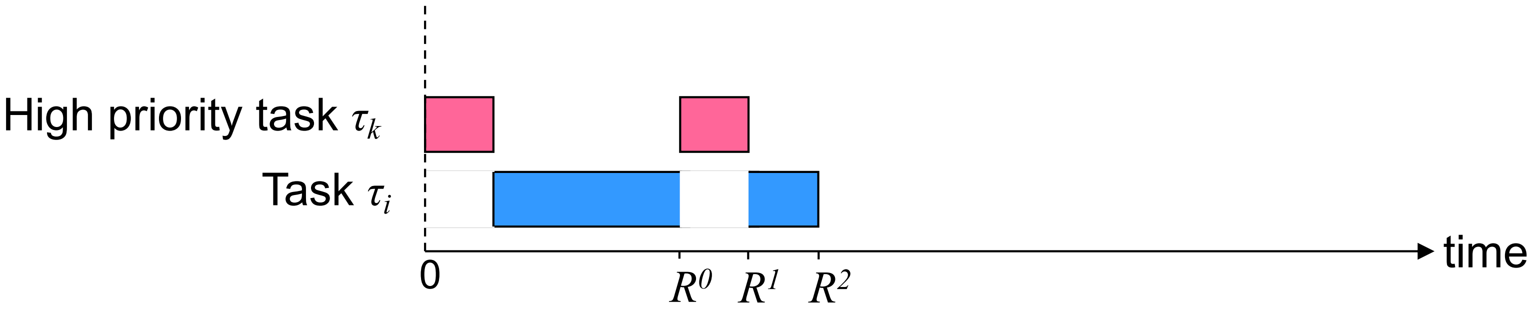
Consequently, response time of $T_n$ is when $T_{n-1}, T_{n-2}, \ldots, T_2, T_1$ are released at the same time.




# Response time analysis

**Response time for a task $\tau_i$ :**

1. Critical instant, for a task, is worst-case response time
   - Critical instant is assumed to the release of all higher priorit
2. Compute how much task $\tau_i$ (response time) and compa



## RTA equation

$$R_i^{n+1} = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i^n}{T_j} \right\rceil C_j$$

Iteration steps:

1. Assume $R^0 = C_i$
2. Iterate until $R^{n+1} = R^n$, or $R^{n+1} > D_i$

$R_i$ = (worst-case) response time

$\tau_j$ = higher priority task

*hp* = a set of tasks that have higher priority than $\tau_i$

$\lceil R_i/T_j \rceil$ = ***maximum* number of pre-emptions** of $\tau_i$ in an interval $[0, R_i)$ by task $\tau_j$

$\lceil R_i/T_j \rceil C_j$ = ***maximal* pre-emption time** of $\tau_i$ in an interval $[0, x)$ by task $\tau_j$

$$R_i^{n+1} = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i^n}{T_j} \right\rceil C_j$$

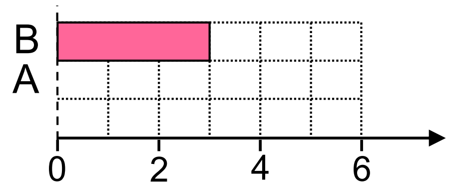
Assume $R^0 = C_i$
Iterate until
$R^{n+1} = R^n$, or $R^{n+1} > D_i$

# RTA – Example 1

Assume 2 tasks A and B with periods 2 and 6, and execution times 1 and 3. Task A has higher priority than B. Calculate B's response time.

**Step 1**: What is B:s response time if only B executes?

$$R_B^0 = C_B = 3$$

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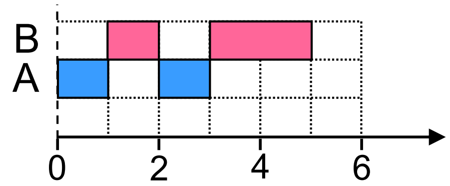
**Step 2**: How much the execution of A within time 0-3 will delay B?

$$R_B^1 = C_B + \left\lceil \frac{R_B^0}{T_A} \right\rceil C_A = 3 + \left\lceil \frac{3}{2} \right\rceil 1 = 3 + 2 = 5$$

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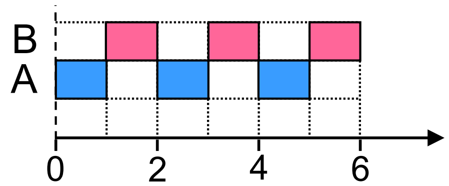
**Step 3**: How much the execution of A within time 0-5 will delay B?

$$R_B^2 = C_B + \left\lceil \frac{R_B^1}{T_A} \right\rceil C_A = 3 + \left\lceil \frac{5}{2} \right\rceil 1 = 3 + 3 = 6$$

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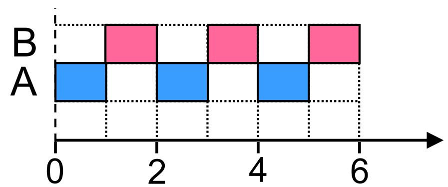
**Steg 4**: How much the execution of A within time 0-6 will delay B?

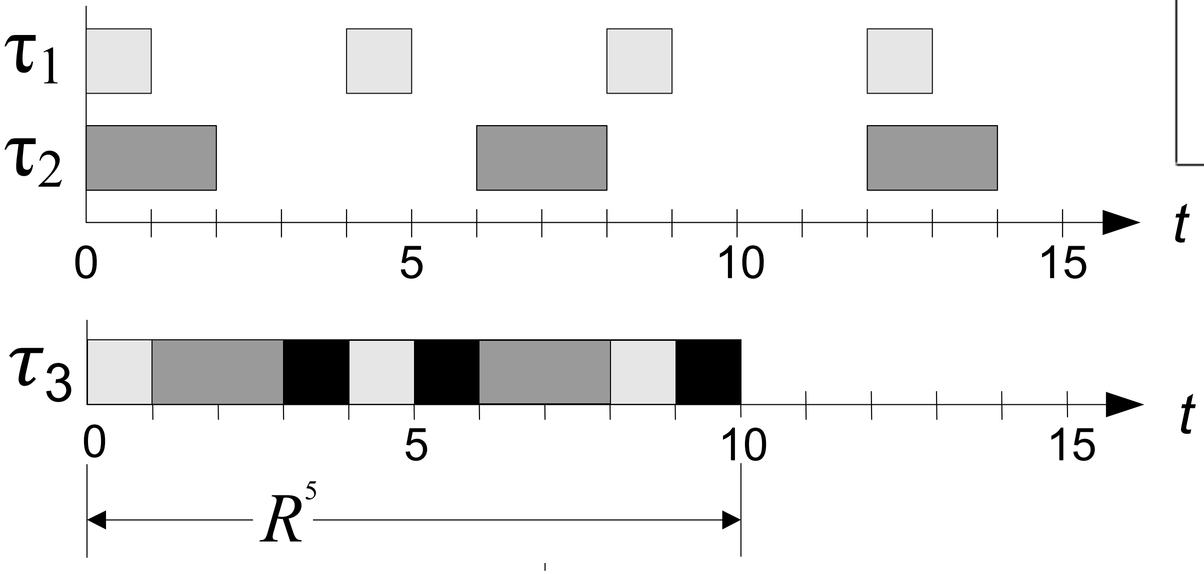
$$R_B^3 = C_B + \left\lceil \frac{R_B^2}{T_A} \right\rceil C_A = 3 + \left\lceil \frac{6}{2} \right\rceil 1 = 3 + 3 = 6$$




**The same result as in previous step → done! ($R_B$=6)**

# RTA – Example 2

| Task | $T_i$ | $C_i$ | $D_i$ | $P_i$ |
|---|---|---|---|---|
| $\tau_1$ | 4 | 1 | 4 | H |
| $\tau_2$ | 6 | 2 | 6 | M |
| $\tau_3$ | 10 | 3 | 10 | L |

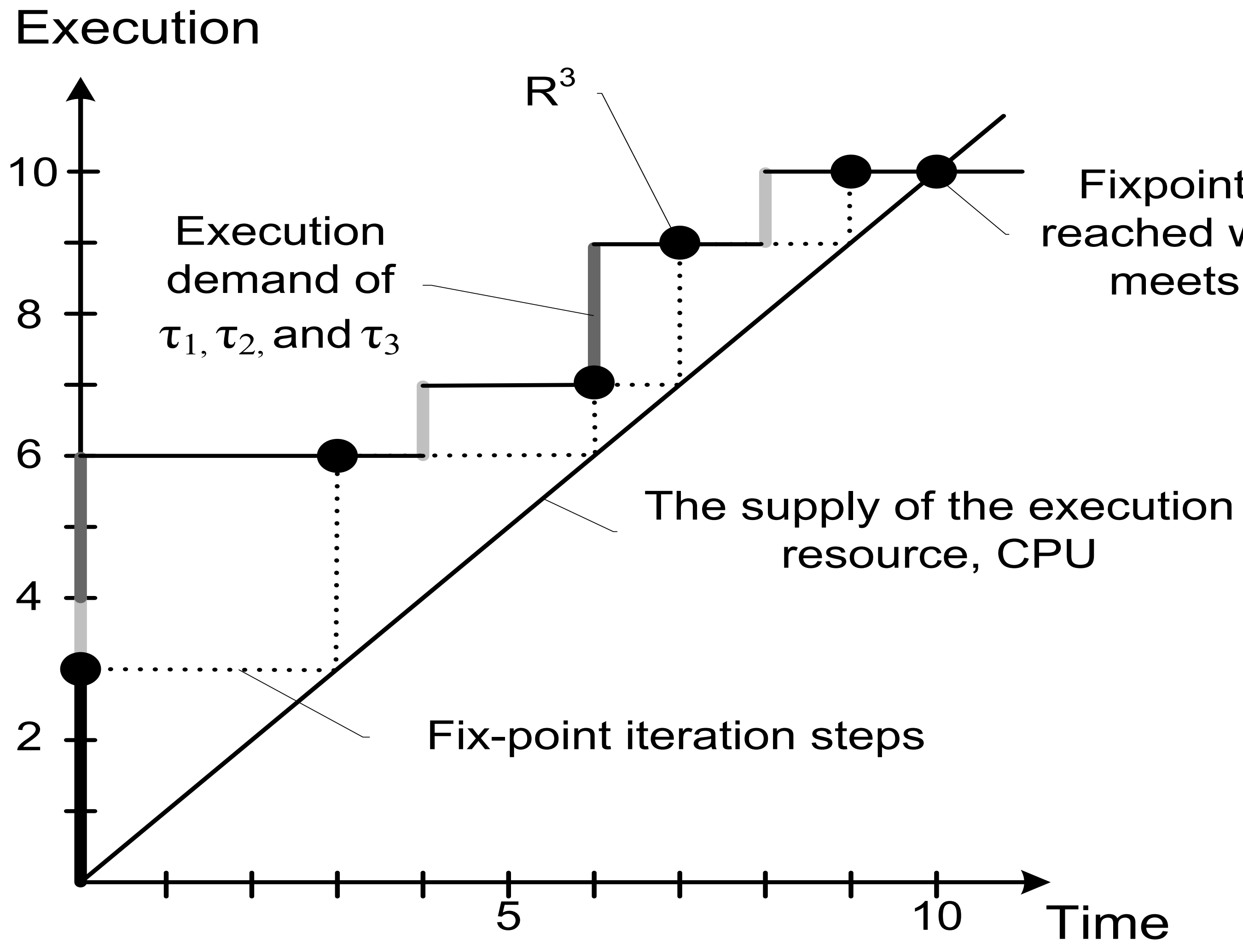



$$R_3^0 = 3$$
$$R_3^1 = 3 + 1 + 2 = 6$$
$$R_3^2 = 3 + 2 + 2 = 7$$
$$R_3^3 = 3 + 2 + 4 = 9$$
$$R_3^4 = 3 + 3 + 4 = 10$$
$$R_3^5 = 3 + 3 + 4 = 10 = R_3^4$$

$$R_i^{n+1} = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i^n}{T_j} \right\rceil C_j$$

# RTA – Example 2 (supply vs. demand)

| Task | $T_i$ | $C_i$ | $D_i$ | $P_i$ |
|---|---|---|---|---|
| $\tau_1$ | 4 | 1 | 4 | H |
| $\tau_2$ | 6 | 2 | 6 | M |
| $\tau_3$ | 10 | 3 | 10 | L |




$$R_3^0 = 3$$

$$R_3^1 = 3 + 1 + 2 = 6$$

$$R_3^2 = 3 + 2 + 2 = 7$$

$$R_3^3 = 3 + 2 + 4 = 9$$

$$R_3^4 = 3 + 3 + 4 = 10$$

$$R_3^5 = 3 + 3 + 4 = 10 = R_3^4$$

# RTA – Example 3

Assume the following task set with priorities according to RM:

| Task | T | C |
|---|---|---|
| A | 3 | 1 |
| B | 6 | 1 |
| C | 5 | 1 |
| D | 10 | 2 |

Can we decide if the set is schedulable by using the **approximate** analysis for RM? If not, use the **exact** analysis.

U=0.9, n(..)=0.75

# RTA – Example 3

Using the approximate analysis for Rate Monotonic

| Task | T | C |
|---|---|---|
| A | 3 | 1 |
| B | 6 | 1 |
| C | 5 | 1 |
| D | 10 | 2 |

$$U = \frac{1}{3} + \frac{1}{6} + \frac{1}{5} + \frac{2}{10} = \frac{27}{30} = 0.9$$

$$n(2^{1/n} - 1) = 4(2^{1/4} - 1) = 0.75$$

$$U \leq n(2^{1/n} - 1)$$ **false!**

The condition 0.9 < 0.75 is false.

Inconclusive: nothing can be said about the schedulability of the task set with RM. The task set may or may not be schedulable

Remember, the condition is **sufficient** but **not neccessary**!

# RTA – Example 3

Using the exact analysis (RTA)

$$R_i^{n+1} = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i^n}{T_j} \right\rceil C_j$$

| Priority | Task | T | C |
|---|---|---|---|
| Priority = 4 (highest) | A | 3 | 1 |
| Priority = 2 | B | 6 | 1 |
| Priority = 3 | C | 5 | 1 |
| Priority =1 (lowest) | D | 10 | 2 |

**Response time for task *A*:**

$$R_A^0 = C_A = 1$$

$$R_A^1 = C_A + \sum_{\forall j \in hp(A)} \left\lceil \frac{R_A^0}{T_j} \right\rceil C_j$$

$$R_A^1 = C_A + 0 = 1 + 0 = 1$$

$$R_A^1 = R_A^0$$

$$\boxed{R_A = 1}$$

**Response time for task *C*:**

$$R_C^0 = C_C = 1$$

$$R_C^1 = C_C + \sum_{\forall j \in hp(C)} \left\lceil \frac{R_C^0}{T_j} \right\rceil C_j$$

$$R_C^1 = C_C + \left\lceil \frac{R_C^0}{T_A} \right\rceil C_A$$

$$R_C^1 = 1 + \left\lceil \frac{1}{3} \right\rceil 1 = 2$$

$$R_C^1 \neq R_C^0$$

$$R_C^2 = C_C + \left\lceil \frac{R_C^1}{T_A} \right\rceil C_A$$

$$R_C^2 = 1 + \left\lceil \frac{2}{3} \right\rceil 1 = 2$$

$$R_C^2 = R_C^1$$

$$\boxed{R_C = 2}$$

**Response time for task *B*:**

$$R_B^0 = C_B = 1$$

$$R_B^1 = C_B + \sum_{\forall j \in hp(B)} \left\lceil \frac{R_B^0}{T_j} \right\rceil C_j$$

$$R_B^1 = C_B + \left\lceil \frac{R_B^0}{T_A} \right\rceil C_A + \left\lceil \frac{R_B^0}{T_C} \right\rceil C_C$$

$$R_B^1 = 1 + \left\lceil \frac{1}{3} \right\rceil 1 + \left\lceil \frac{1}{5} \right\rceil 1 = 3$$

$$R_B^1 \neq R_B^0$$

$$R_B^2 = C_B + \left\lceil \frac{R_B^1}{T_A} \right\rceil C_A + \left\lceil \frac{R_B^1}{T_C} \right\rceil C_C$$

$$R_B^2 = 1 + \left\lceil \frac{3}{3} \right\rceil 1 + \left\lceil \frac{3}{5} \right\rceil 1 = 3$$

$$R_B^2 = R_B^1$$

$$\boxed{R_B = 3}$$

# RTA – Example 3

**Response time for task *D*:**

$R_D^0 = C_D = 2$

$$R_D^1 = C_D + \sum_{\forall j \in hp(D)} \left\lceil \frac{R_D^0}{T_j} \right\rceil C_j$$

$$R_D^1 = C_D + \left\lceil \frac{R_D^0}{T_A} \right\rceil C_A + \left\lceil \frac{R_D^0}{T_B} \right\rceil C_B + \left\lceil \frac{R_D^0}{T_C} \right\rceil C_C$$

$$R_D^1 = 2 + \left\lceil \frac{2}{3} \right\rceil 1 + \left\lceil \frac{2}{6} \right\rceil 1 + \left\lceil \frac{2}{5} \right\rceil 1 = 5$$

$R_D^1 \neq R_D^0$

$$R_D^2 = C_D + \left\lceil \frac{R_D^1}{T_A} \right\rceil C_A + \left\lceil \frac{R_D^1}{T_B} \right\rceil C_B + \left\lceil \frac{R_D^1}{T_C} \right\rceil C_C$$

$$R_D^2 = 2 + \left\lceil \frac{5}{3} \right\rceil 1 + \left\lceil \frac{5}{6} \right\rceil 1 + \left\lceil \frac{5}{5} \right\rceil 1 = 6$$

$R_D^2 \neq R_D^1$

$$R_D^3 = C_D + \left\lceil \frac{R_D^2}{T_A} \right\rceil C_A + \left\lceil \frac{R_D^2}{T_B} \right\rceil C_B + \left\lceil \frac{R_D^2}{T_C} \right\rceil C_C$$

$$R_D^3 = 2 + \left\lceil \frac{6}{3} \right\rceil 1 + \left\lceil \frac{6}{6} \right\rceil 1 + \left\lceil \frac{6}{5} \right\rceil 1 = 7$$

$R_D^3 \neq R_D^2$

$$R_D^4 = C_D + \left\lceil \frac{R_D^3}{T_A} \right\rceil C_A + \left\lceil \frac{R_D^3}{T_B} \right\rceil C_B + \left\lceil \frac{R_D^3}{T_C} \right\rceil C_C$$

$$R_D^4 = 2 + \left\lceil \frac{7}{3} \right\rceil 1 + \left\lceil \frac{7}{6} \right\rceil 1 + \left\lceil \frac{7}{5} \right\rceil 1 = 9$$

$R_D^4 \neq R_D^3$

$$R_D^5 = C_D + \left\lceil \frac{R_D^4}{T_A} \right\rceil C_A + \left\lceil \frac{R_D^4}{T_B} \right\rceil C_B + \left\lceil \frac{R_D^4}{T_C} \right\rceil C_C$$

$$R_D^5 = 2 + \left\lceil \frac{9}{3} \right\rceil 1 + \left\lceil \frac{9}{6} \right\rceil 1 + \left\lceil \frac{9}{5} \right\rceil 1 = 9$$

$R_D^5 = R_D^4$

$$\boxed{R_D = 9}$$

$R_A\ (1) \leq D_A\ (3)$

$R_B\ (3) \leq D_B\ (6)$

$R_C\ (2) \leq D_C\ (5)$

$R_D\ (9) \leq D_D\ (10)$

**The task set is schedulable**

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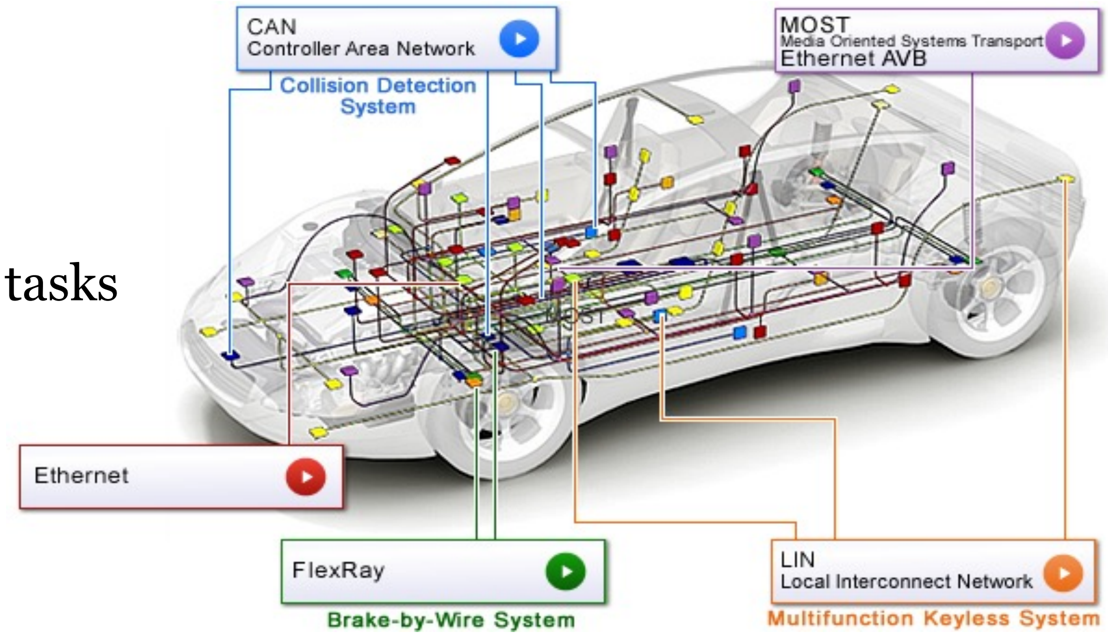


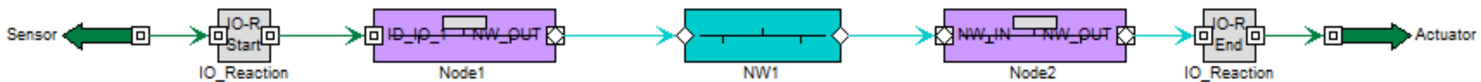


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## RTA with Shared Resources

**Blocking has a negative effect on schedulability**

- We need to include the blocking times in the analysis
- Blocking time $B_i$ of a task $\tau_i$ is the longest time task $\tau_i$ can be blocked by lower priority tasks

Reminder: critical sections are **properly nested**, so duration of a critical section equals the **outmost** critical section.

**Hence, the blocking time is defined in PCP as:**

$$B_i = \max(CS_j, .., CS_k)$$

Where $CS_j..CS_k$ are the critical sections of all tasks that can block task $\tau_i$

# Example blocking time

| Task | Prio | Semaphore | Critical sect. |
|---|---|---|---|
| A | 1 | S1 | 1 |
| B | 3 | S1 | 2 |
| C | 2 | S2 | 3 |
| D | 4 | S2 | 4 |

1 = lowest prio

4 = highest prio

**ceil(S1)** = max( P(A), P(B) ) = max (1,3) = **3**

**ceil(S2)** = max( P(C), P(D) ) = max (2, 4) = **4**

$B_A$=? A has the lowest priority → $\mathbf{B_A=0}$

$B_B$=? lp(B)={A, C} *(lp(B) = all tasks with lower priority than B)*

A uses S1, prio(B)>ceil(S1)? No → A can block B!

C uses S2, prio(B)>ceil(S2)? No → C can block B!

$\mathbf{B_B}$ **= max( CS(A,S1), CS(C,S2) ) = max(1,3) = 3**

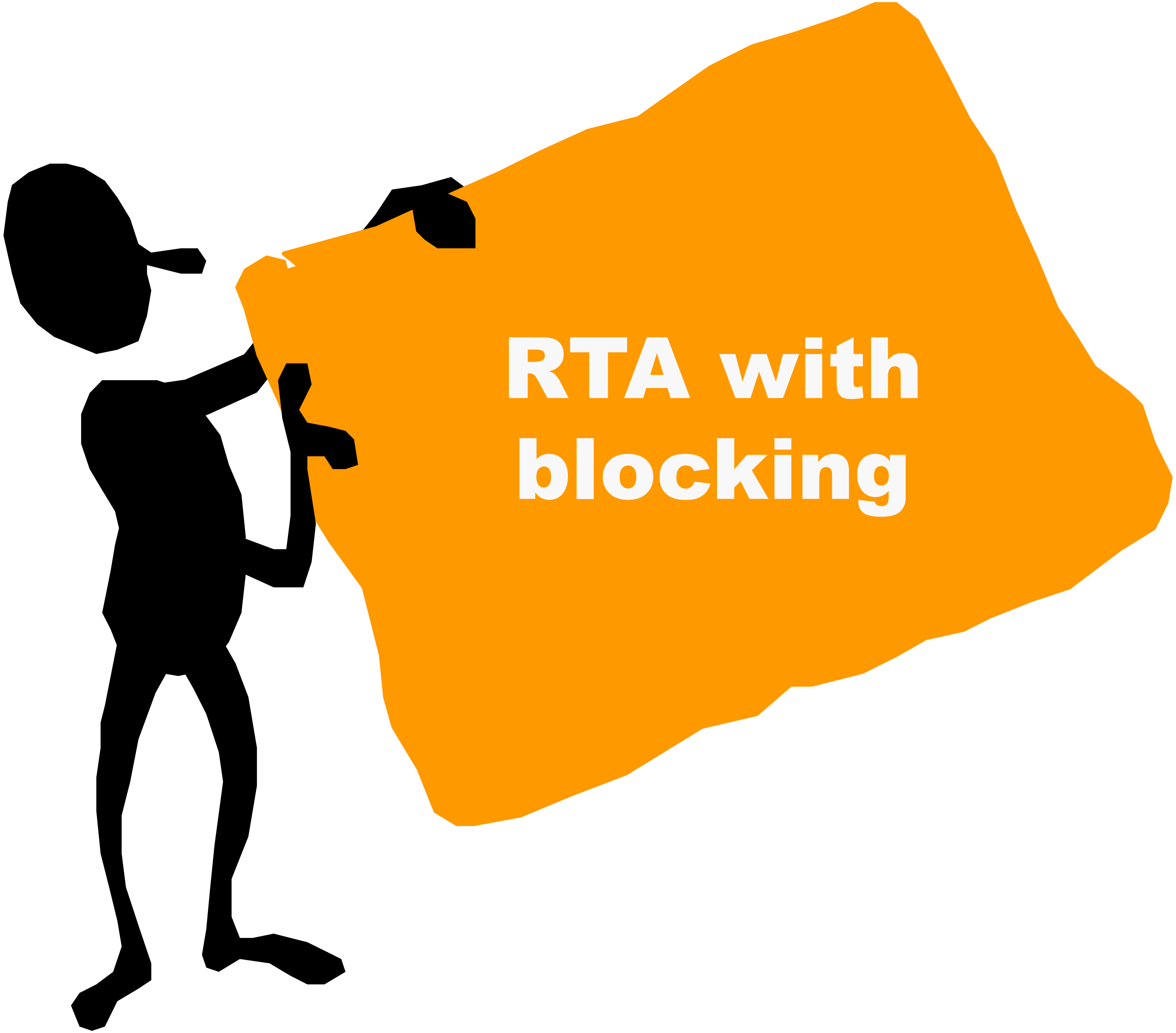
$B_C$=? lp(C)={A}

A uses S1, prio(C)>ceil(S1)? No → A can block C → $\mathbf{B_C}$**= CS(A,S1) = 1**

$B_D$=? lp(D)={A, B, C}

A uses S1, prio(D)>ceil(S1)? Yes → A <u>cannot</u> block D
B uses S1, prio(D)>ceil(S1)? Yes → B <u>cannot</u> block D
C uses S2, prio(D)>ceil(S2)? No → C can block D

$\mathbf{B_D}$ **= CS(C,S2) = 3**


RTA with blocking

## Response Time Analysis with blockings

**<u>Reminder</u>: Response Time Analysis is a method to test the schedulability of static-priority based real-time systems**

1. Assume all tasks with the higher priority than $\tau_i$ are ready at the same time as $\tau_i$ (critical instant)
2. Compute how much task $\tau_i$ will be delayed by high priority tasks (response time) and compare it to the $\tau_i$'s deadline to see if it can make it or not

**Response Time Analysis with blocking time included:**

$$R_i^{n+1} = C_i + B_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i^n}{T_j} \right\rceil C_j$$

# Response Time Analysis with blockings

Assume the following tasks with respective semaphore accesses:

| Task | Period (T) | Execution time (C) | Semaphore | CS |
|---|---|---|---|---|
| A | 100 | 10 | S1 | 1 |
| B | 40 | 12 | S1 | 2 |
| | | | S2 | 1 |
| C | 50 | 6 | S1 | 1 |

Assume Rate Monotonic priority assignment for the tasks and PCP.

a) Calculate priority ceiling for each of the semaphores

b) Calculate blocking factors for each of the tasks

c) Calculate response times for the tasks

**[ Home Practice]**

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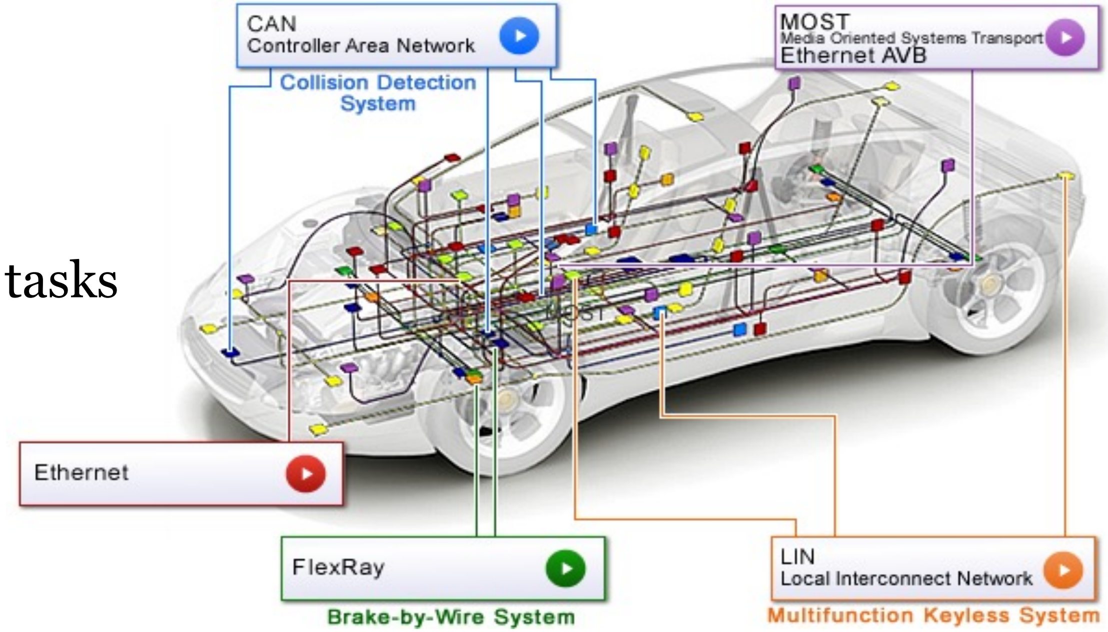
  - RTA with Offsets

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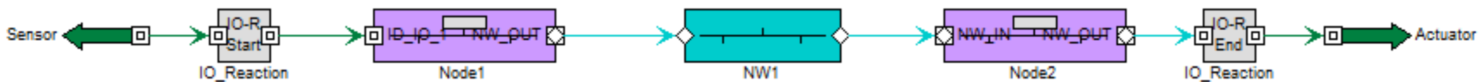


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# Response Time Analysis with release jitter

**We assumed until now that tasks get activated with perfect periodicity**

**Release Jitter (J)**

- The difference between the earliest and the latest point in time a task starts to execute (relatively to the nominal start time)

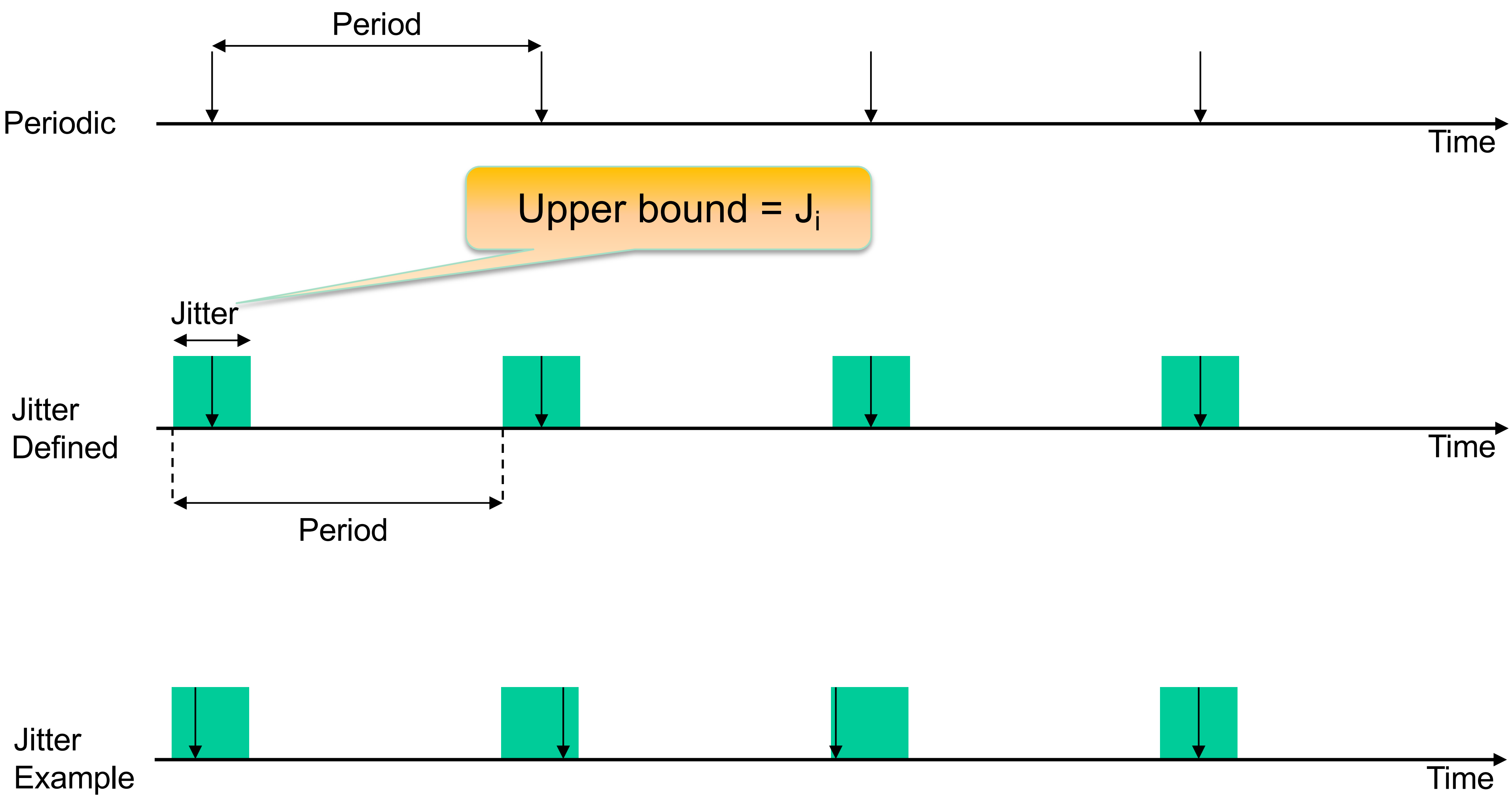
**Jitter may have different causes**

- Task B is triggered by task A, which means that B's activation time may vary (e.g. A might not use its WCET)
- A sensor at node A triggers a task at node B. The message is sent over a network, and may take a varying amount of time to arrive
- Operating systems: Clock tick at 10ms. Want T=23
  - We want activation at 0, 23, 46, …
  - OS is able to activate at 0, 30, 50, …
  - The variation of the actual release times is jitter (0, 7, 4, ...).

**Problem**

- Temporarily shorter periods → low priority tasks can miss deadline

# Jitter visualized



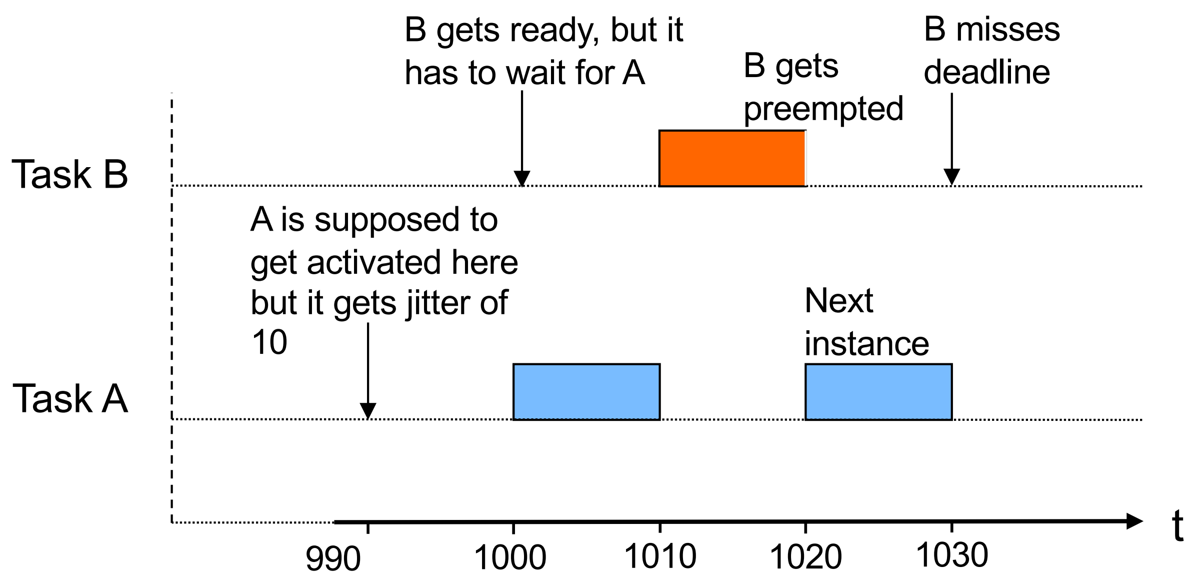
# Response Time Analysis with release jitter

**Example:**

**Assume the following two tasks with calculated response times:**

| Task | T | D | C | R | |
|---|---|---|---|---|---|
| A | 30 | 20 | 10 | 10 | High priority |
| B | 1000 | 25 | 15 | 25 | Low priority |

What happens if A has a release jitter of 10?




RTA with
release
jitter

# Response Time Analysis with release jitter

**Response time analysis must be updated with a release jitter term:**

$$J_i = J_i^{\max} - J_i^{\min}$$

New equation:

$$w_i^{n+1} = C_i + B_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{w_i^n + J_j}{T_j} \right\rceil C_j$$

$$R_i = w_i + J_i^{\max}$$

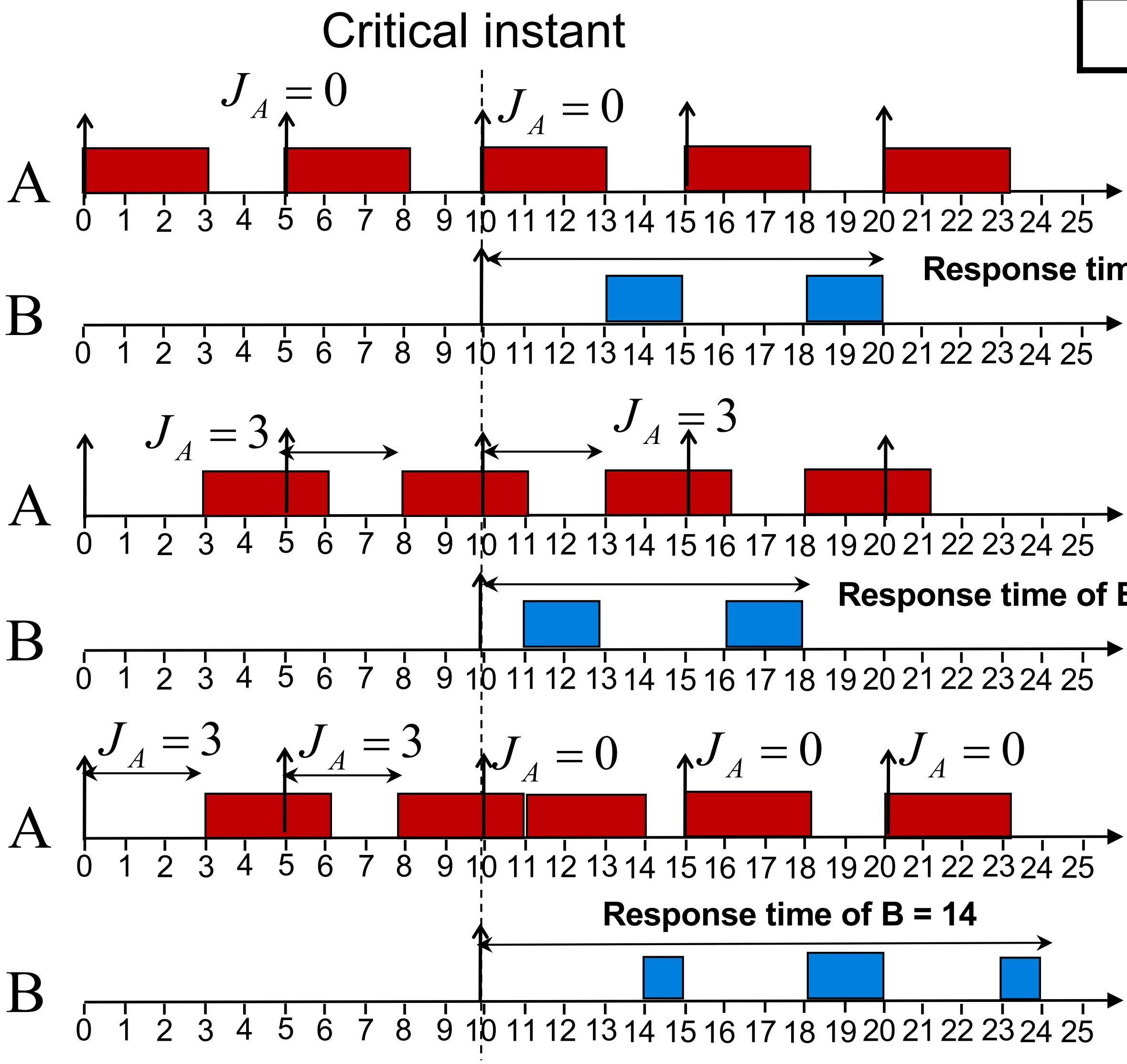
Original version of RM:

- Independent tasks → $J^{max}=J^{min}$ → no release jitter!

# Response Time Analysis with release jitter

**What is the safe assumption about the critical instant?**

| Task | P | C | T | $J^{max}$ | $J^{min}$ |
|---|---|---|---|---|---|
| A | High | 3 | 5 | 3 | 0 |
| B | Low | 4 | 50 | 0 | 0 |




**Response time of B = 10**

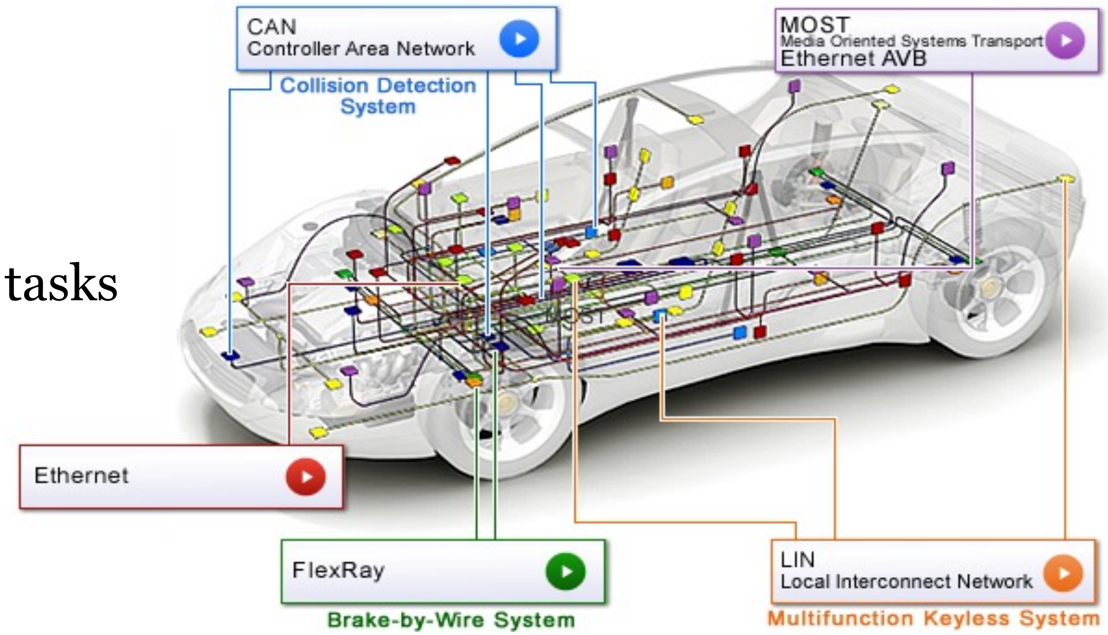
All instances of task A experience $J^{min}$

All instances of task A experience $J^{max}$

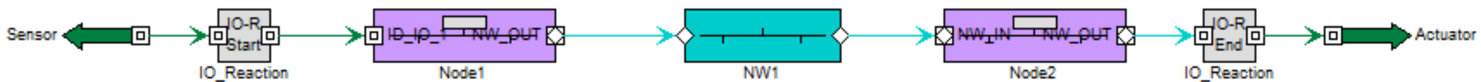
**Response time of B = 8**

- All instances of task A that occur before the critical instant experience $J^{max}$
- All instances of task A that occur at or after the critical instant experience $J^{min}$

# Outline




- Response-time analysis (RTA) of tasks
  - Basic RTA
  - RTA with blocking
  - RTA with jitter
  - RTA with Offsets
- RTA of Controller Area Network (CAN) messages
- Practical limitations in CAN Controllers and their effect on RTA for CAN
- Holistic RTA (for distributed systems)
- Data-propagation delay analysis
  - Single node systems
  - Distributed systems (End-to-end data-propagation delay analysis)

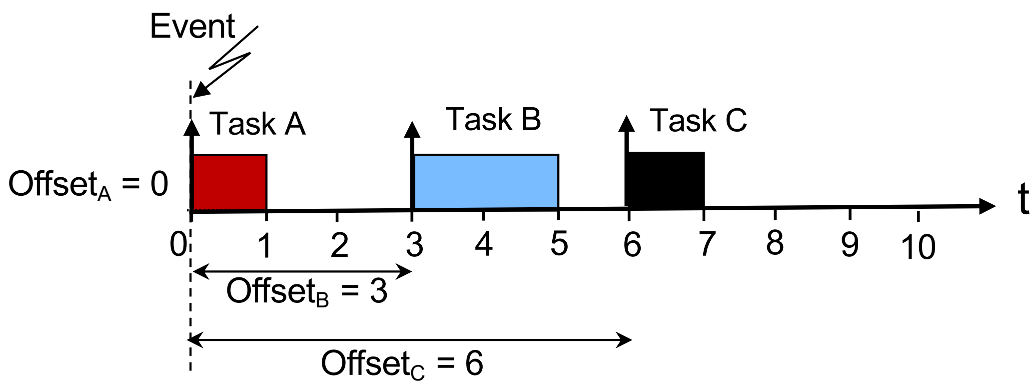



Figures courtesy of: http://www.cvel.clemson.edu/
http:// www.renesas.eu

# Offsets in real-time scheduling

## What is an offset?

- Externally imposed time interval between the arrival of the triggering event and release of the task.
- A fixed time interval between the arrival of sets of tasks.
  - An offset specifies temporal dependency between releases of tasks
- Example: some task activations are statically scheduled: Time triggered



# Offsets in real-time scheduling

## Why offsets?

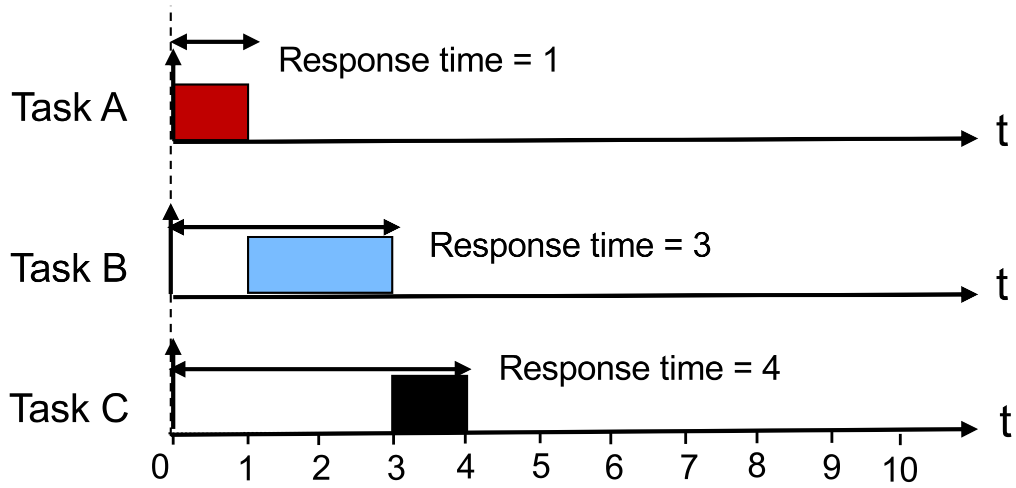
- What if tasks cannot be released simultaneously?
  - Example: the release of a task $\tau_i$ will occur $O$ time units after the release of task $\tau_j$
  - Everything in the system may not happen at once.
- Response times of tasks (especially with lower priority) increase with the increase in the system load.
  - Response times of lower priority tasks can be reduced if the tasks are scheduled with offsets.
- In order to avoid deadlines violations due to high transient loads, embedded systems are often scheduled with offsets.

# Offsets in real-time scheduling

## Why offsets?

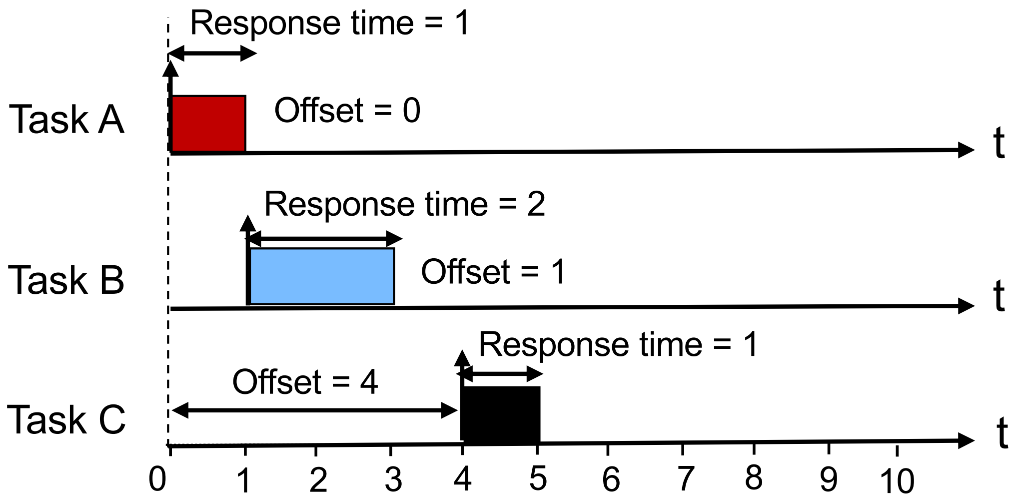
**Example:**

**No offsets**




| Task | Prio | T | C |
|---|---|---|---|
| A | High | 10 | 1 |
| B | Medium | 10 | 2 |
| C | Low | 10 | 1 |

**With offsets**



## Task model with offsets

- Introducing temporal dependencies between tasks
  - Tasks are grouped into transactions and assigned offsets

$$\Gamma = \{\Gamma_1, \Gamma_2, \cdots, \Gamma_n\} \qquad \Gamma_i = \langle \{\tau_{i1}, \tau_{i2}, \ldots, \tau_{in}\}, T_i \rangle \qquad \tau_{ij} = \langle C_{ij}, O_{ij}, P_{ij} \rangle$$

**Notations**

$\Gamma$ -- System $\qquad \Gamma_i$ -- Transaction "i" $\qquad T_i$ -- Transaction period

$\tau_{ij}$ -- Task "j" belonging to transaction $\Gamma_i$

$C_{ij}$ -- Worst-case execution time of $\tau_{ij}$

$O_{ij}$ -- Offset of $\tau_{ij}$ $\qquad P_{ij}$ -- Priority of $\tau_{ij}$

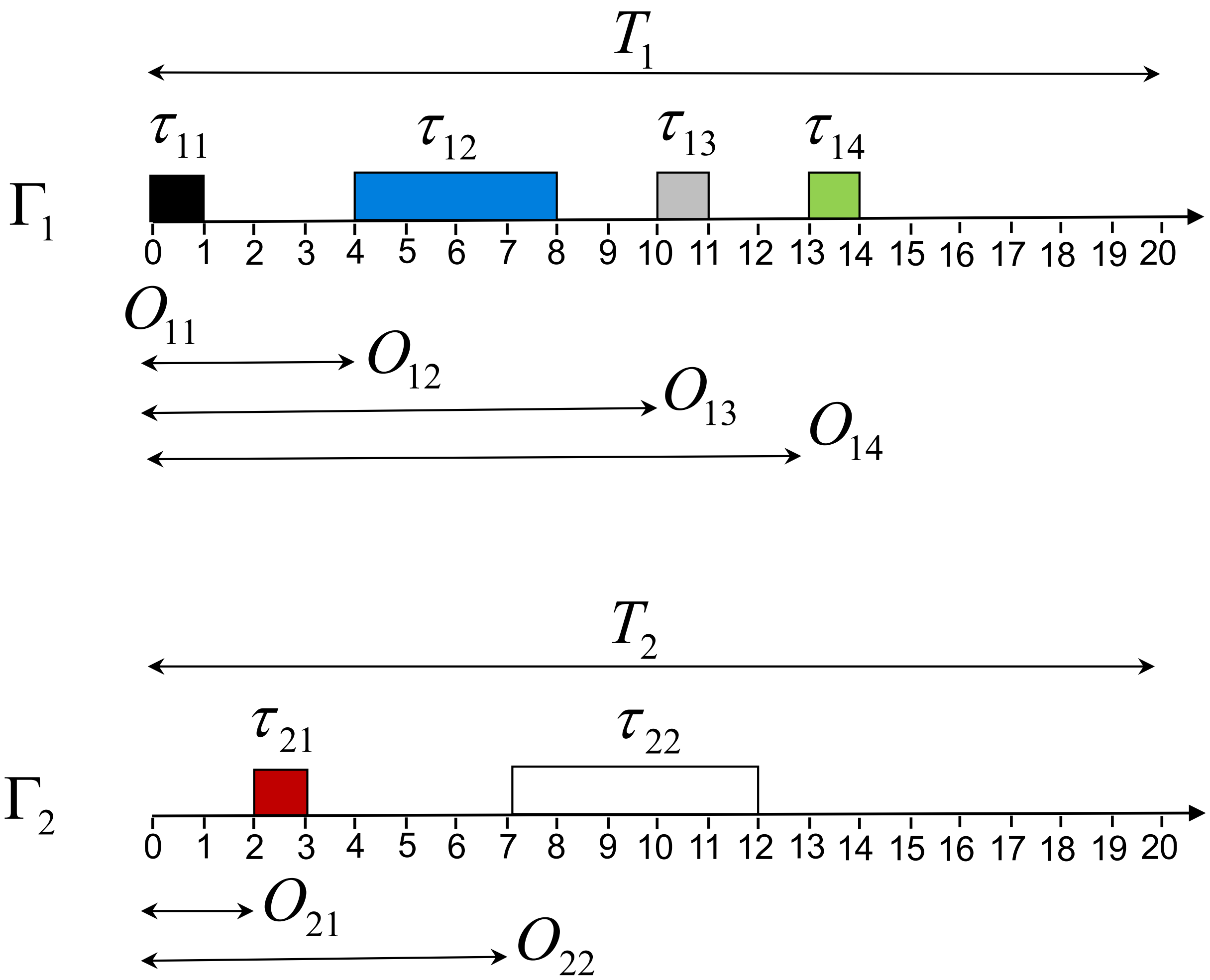
$\tau_{ua}$ -- Task "a" belonging to transaction $\Gamma_u$

We overlaod this notation to represent "task under analysis"

$hp_i(\tau_{ua})$ -- All higher priority tasks to $\tau_{ua}$ in transaction $\Gamma_i$

# Task model with offsets

## Example of a system with two transactions




$\Gamma = \{\Gamma_1, \Gamma_2, \cdots, \Gamma_n\}$ $\Gamma_i = \langle \{\tau_{i1}, \tau_{i2}, ..., \tau_{in}\}, T_i \rangle$ $\tau_{ij} = \langle C_{ij}, O_{ij}, P_{ij} \rangle$

**Notations**

$\Gamma$ -- System $\Gamma_i$ -- Transaction "i" $T_i$ -- Transaction period

$\tau_{ij}$ -- Task "j" belonging to transaction $\Gamma_i$

$C_{ij}$ -- Worst-case execution time of $\tau_{ij}$

$O_{ij}$ -- Offset of $\tau_{ij}$ $P_{ij}$ -- Priority of $\tau_{ij}$

$\tau_{ua}$ -- Task "a" belonging to transaction $\Gamma_u$

We overlaod this notation to represent "task under analysis"

$hp_i(\tau_{ua})$ -- All higher priority tasks to $\tau_{ua}$ in transaction $\Gamma_i$ 5


RTA with
Offsets

# Critical Instant assumption in RTA <u>without</u> offsets

**Response time for a task $\tau_i$ :**

1. <u>Critical instant</u>: Assume all tasks with the higher priority than $\tau_i$ are ***ready at the same time*** as $\tau_i$
2. Critical instant will occur when a task is **released simultaneously** with the release of all higher priority tasks

$$R_i^{n+1} = C_i + \text{interference due to all higher priority tasks}$$

$$R_i^{n+1} = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{R_i^n}{T_j} \right\rceil C_j$$

## Critical Instant assumption in RTA with offsets

**Is the critical instant assumption still valid?**

- All tasks cannot be released simultaneosly

**Assumption in RTA with offsets**

- At least one task out of every transaction coincides with the critical instant of $\tau_{ua}$
- Only tasks with priority higher than $\tau_{ua}$ are considered

**However, it is not known which task coincides with (released at) the critical instant**

- Every higher priority task in a transaction must be treated as a candidate to coincide with the critical instant
- Every possible combination of critical instant candidates must be considered including $\tau_{ua}$

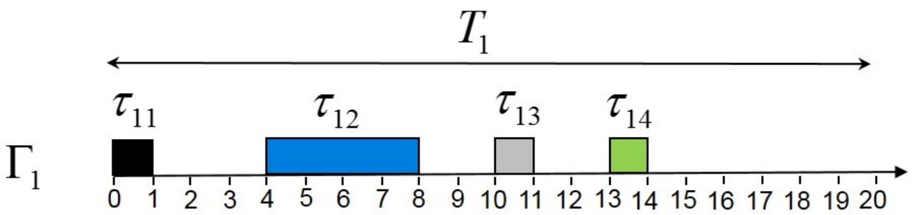
$$R_{uac}^{n+1} = C_i + \text{interference due to all higher priority tasks}$$

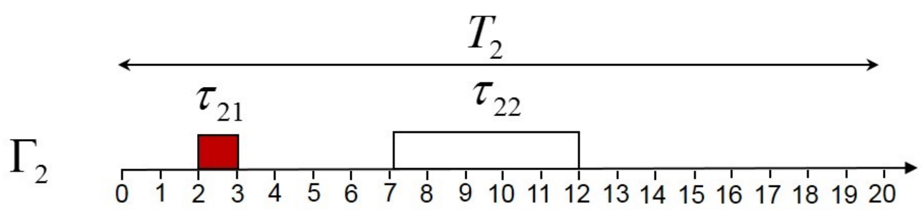
$$R_{ua} = \max_{\forall c \ \ combinations} (R_{uac})$$

## Critical Instant assumption in RTA with offsets

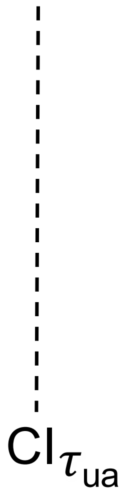
**Example:** Assume priority of $\tau_{1j}$ are high, priority of $\tau_{2j}$ are medium

Assume $\tau_{ua}$ be the task under analysis having the lowest priority.



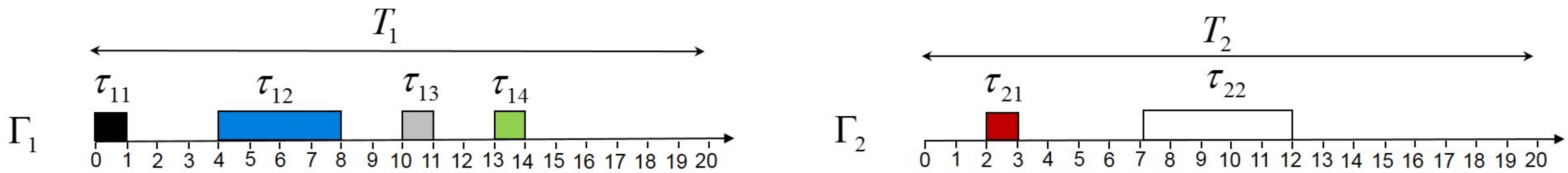

**<u>Critical Instant (CI) combination 1</u>:** Let $\tau_{11}, \tau_{21}$ coincide with CI of $\boldsymbol{\tau_{ua}}$



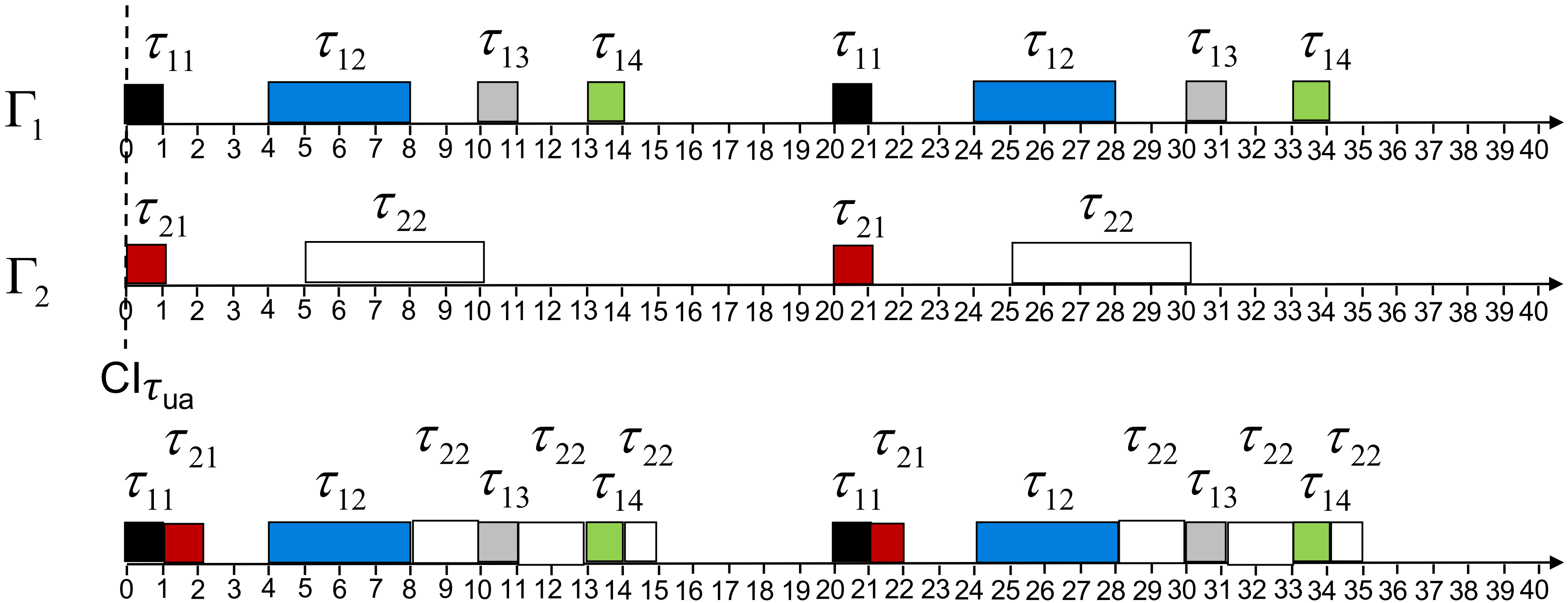
# Critical Instant assumption in RTA with offsets

**Example:** Assume priority of $\tau_{1j}$ are high, priority of $\tau_{2j}$ are medium

Assume $\tau_{ua}$ be the task under analysis having the lowest priority.



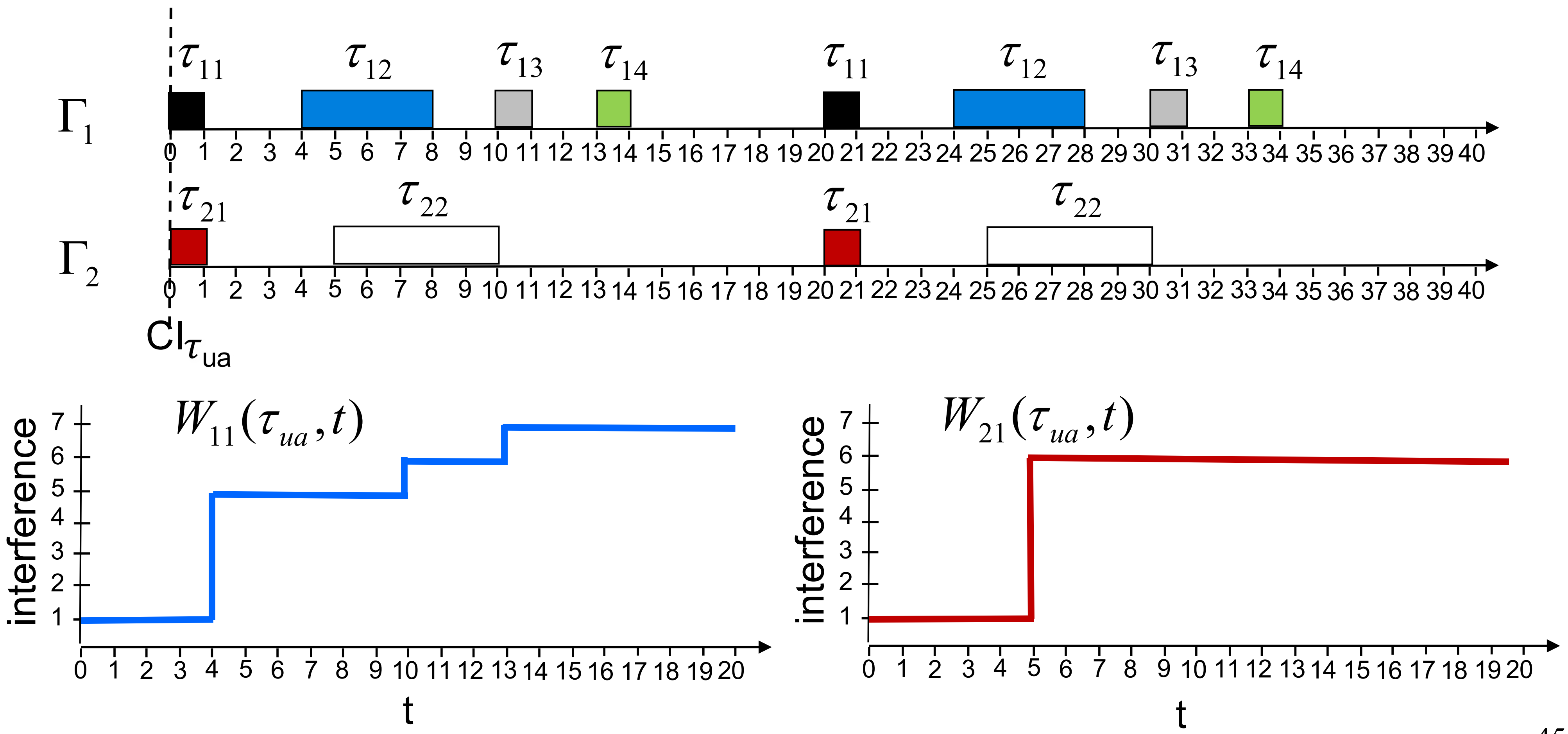

**<u>Critical Instant (CI) combination 1</u>:** Let $\tau_{11}, \tau_{21}$ coincide with CI of $\tau_{ua}$



# Critical Instant assumption in RTA with offsets

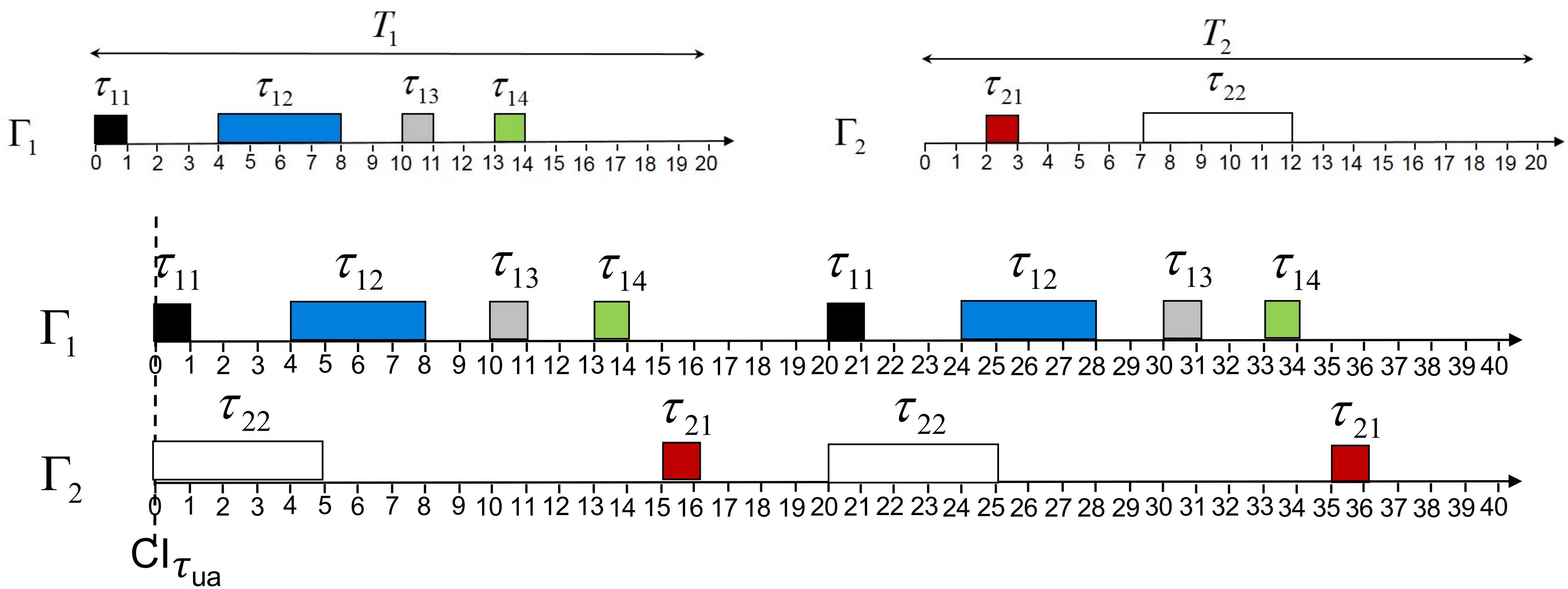
**<u>Critical Instant (CI) combination 1</u>:** Let $\tau_{11}, \tau_{21}$ coincide with CI of $\boldsymbol{\tau_{ua}}$

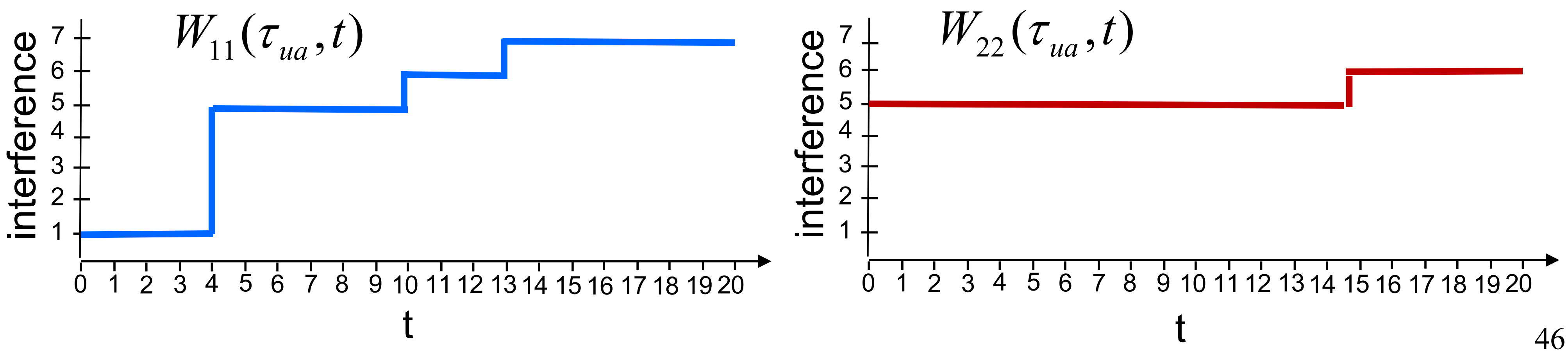
$$W_{ic}(\tau_{ua}, t) = \begin{array}{l}\text{interference from } \Gamma_i \text{ for critical instant candidate } \tau_{ic} \\ \text{experienced by a low priority task } \tau_{ua} \text{ belonging to } \Gamma_u\end{array}$$



## Critical Instant assumption in RTA with offsets

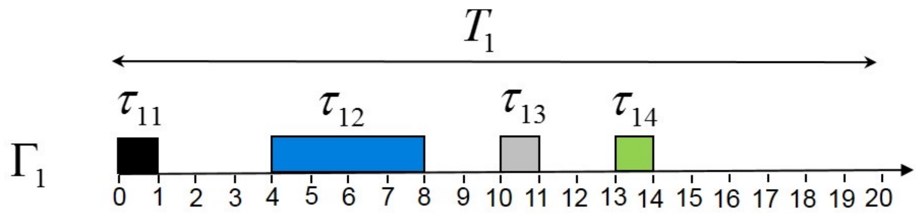
**Critical Instant (CI) combination 2**: Let $\tau_{11}, \tau_{22}$ coincide with CI of $\tau_{ua}$

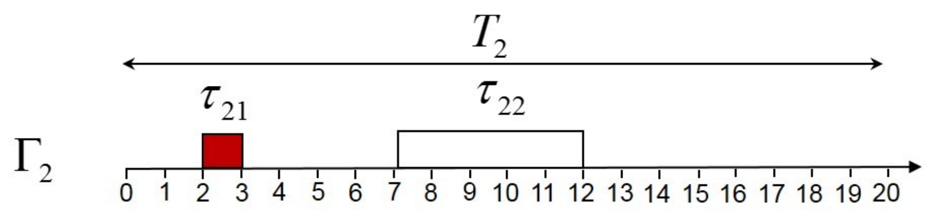


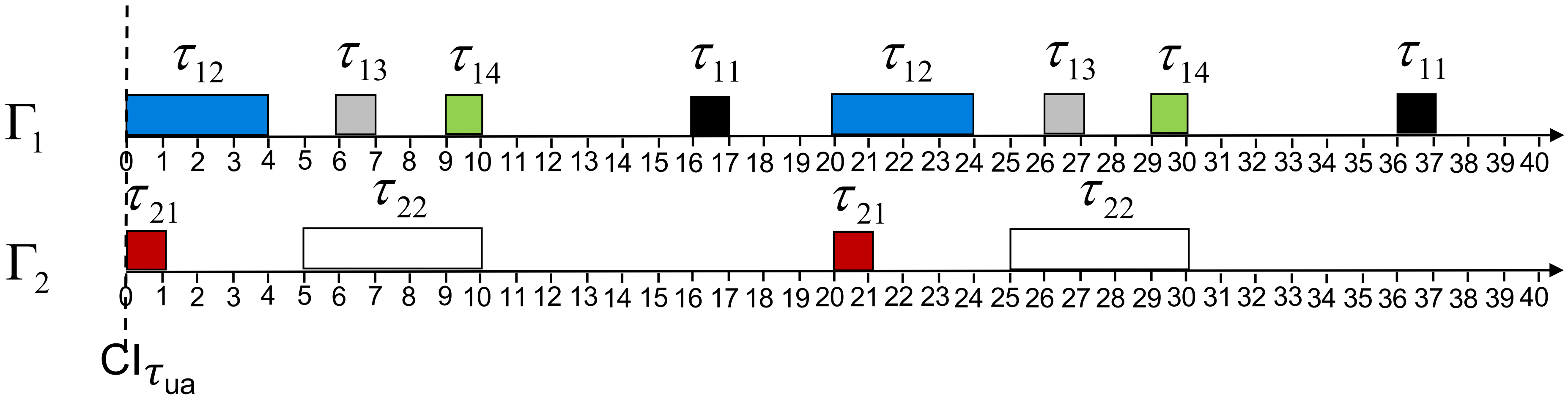



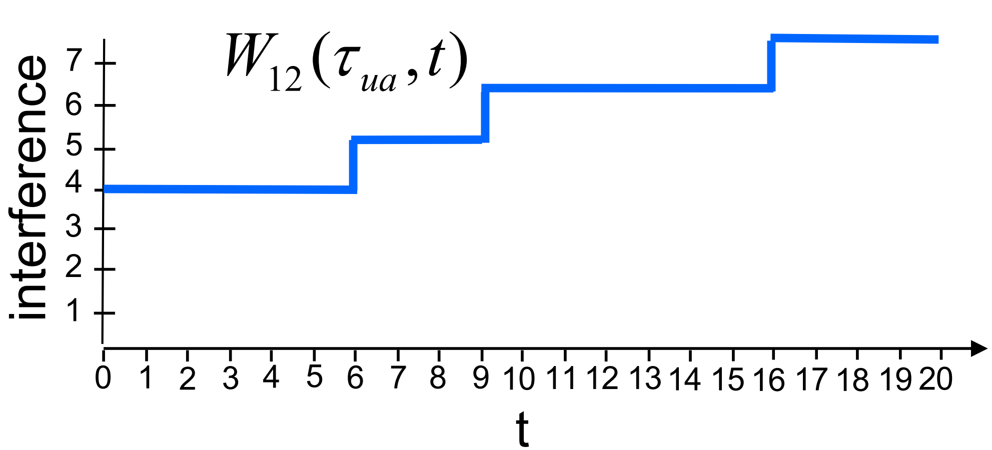
## Critical Instant assumption in RTA with offsets

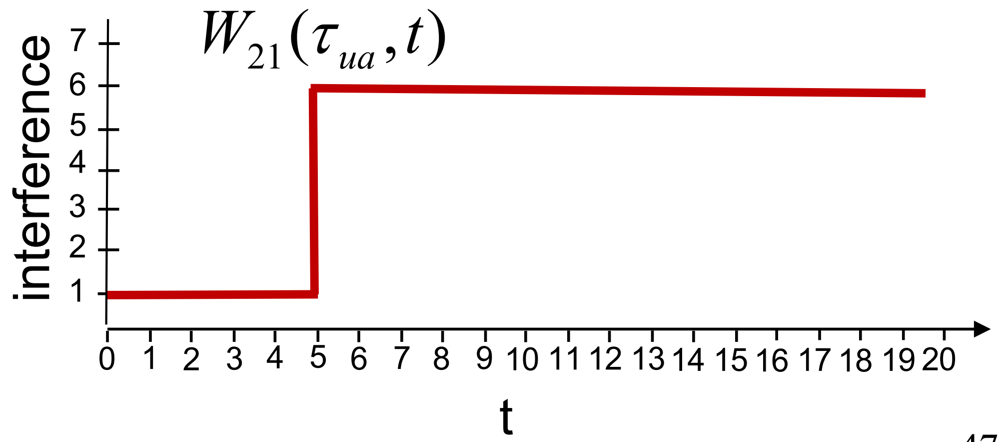
**<u>Critical Instant (CI) combination 3</u>:** Let $\tau_{12}, \tau_{21}$ coincide with CI of $\tau_{ua}$



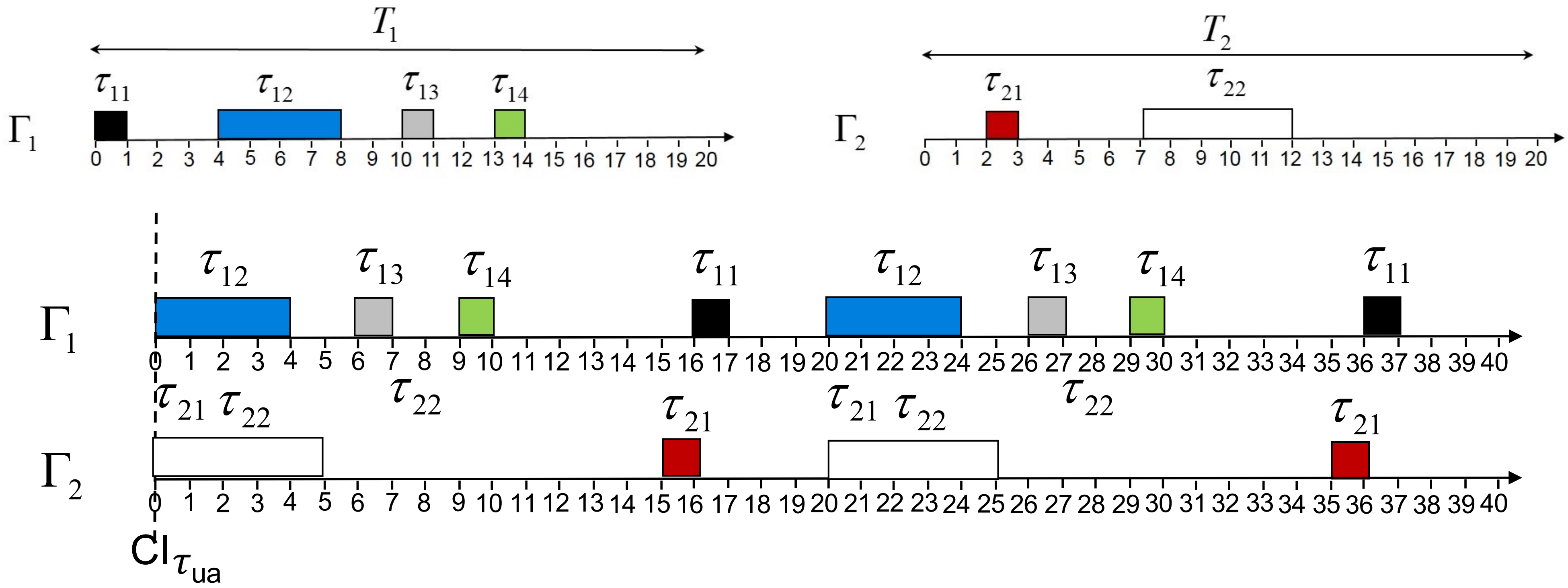



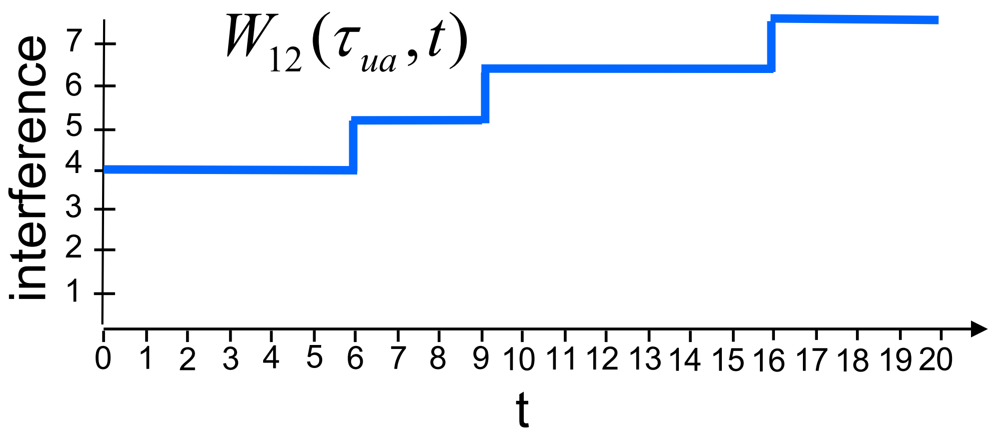



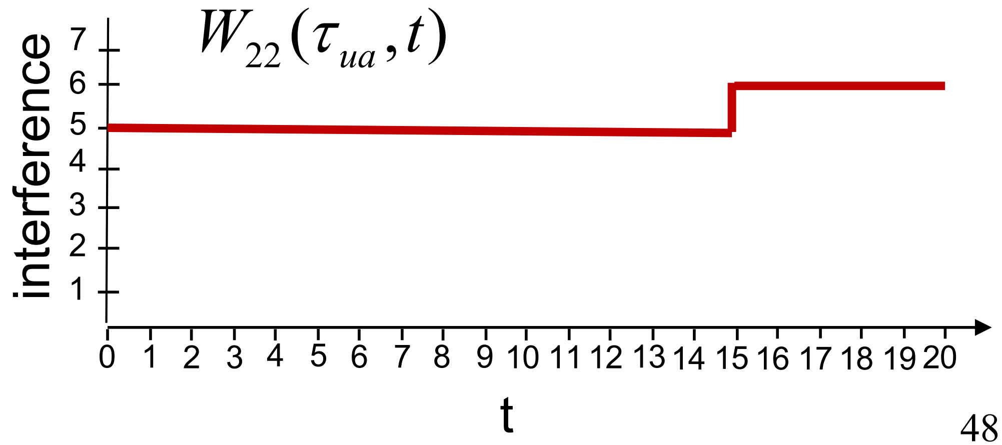



## Critical Instant assumption in RTA with offsets

**Critical Instant (CI) combination 4**: Let $\tau_{12}, \tau_{22}$ coincide with CI of $\tau_{ua}$

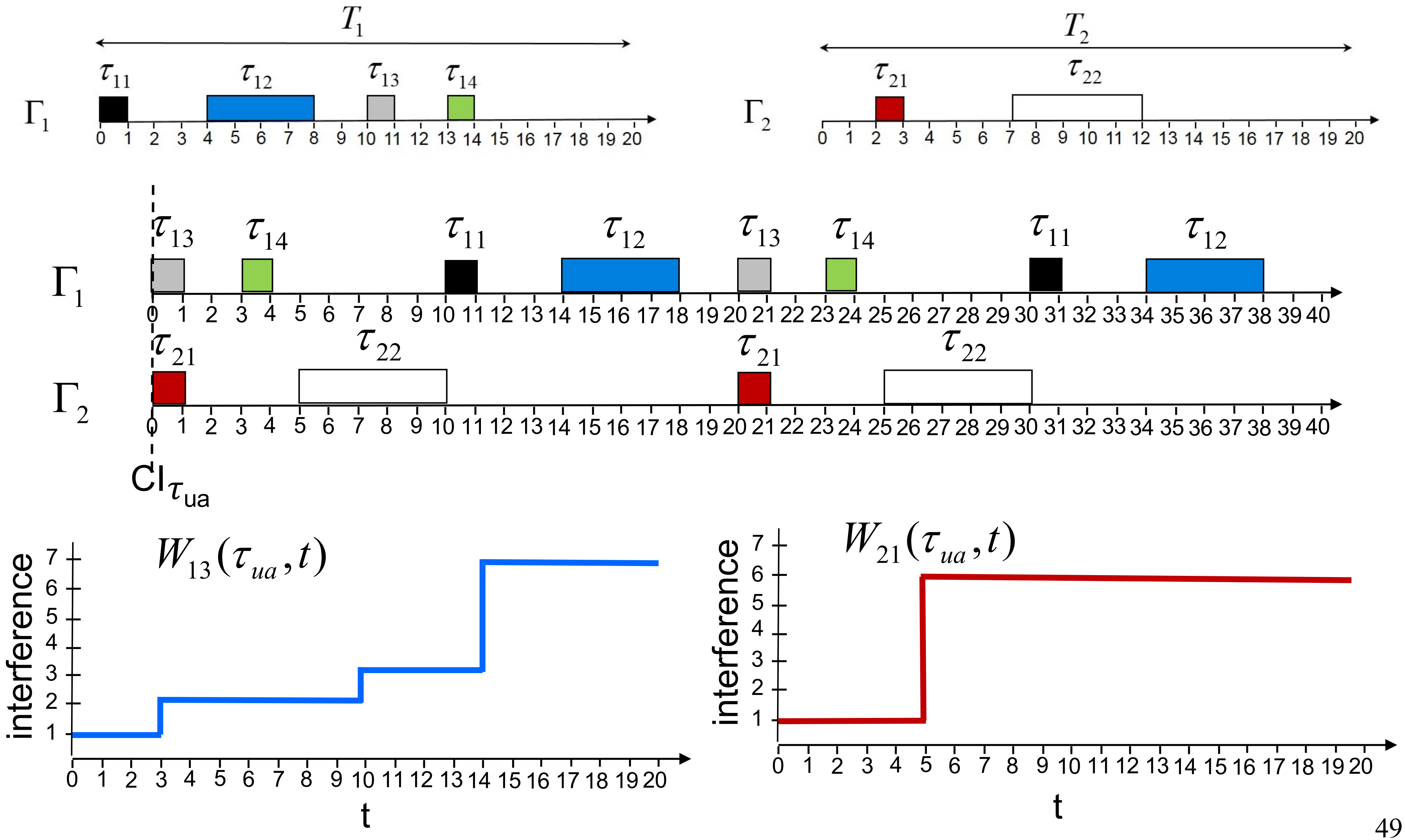


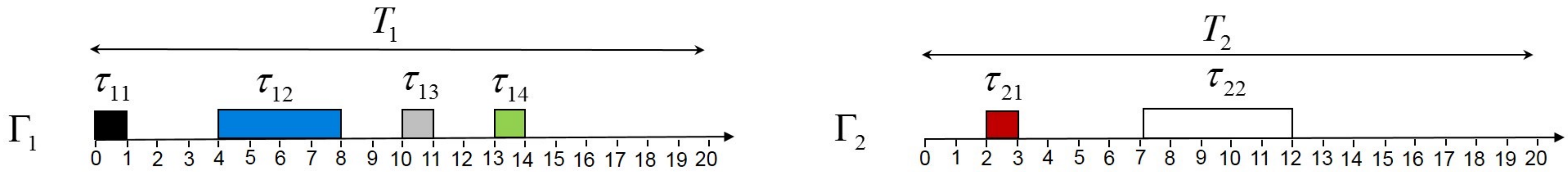
## Critical Instant assumption in RTA with offsets

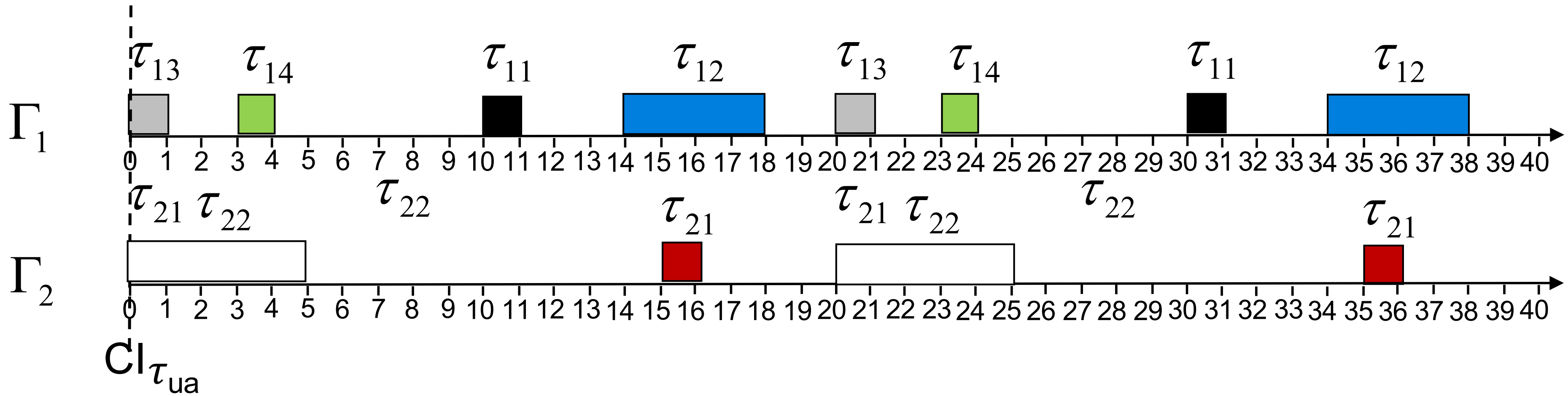
**<u>Critical Instant (CI) combination 5</u>**: Let $\tau_{13}, \tau_{21}$ coincide with CI of $\tau_{\mathbf{ua}}$

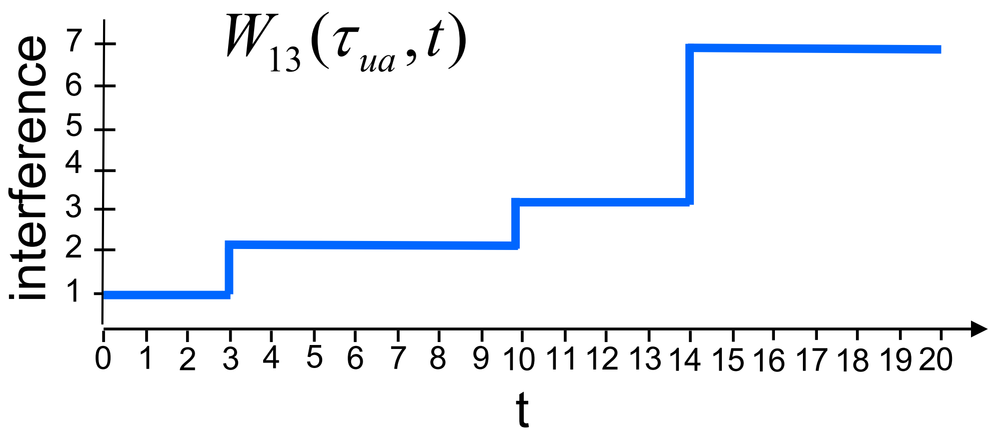


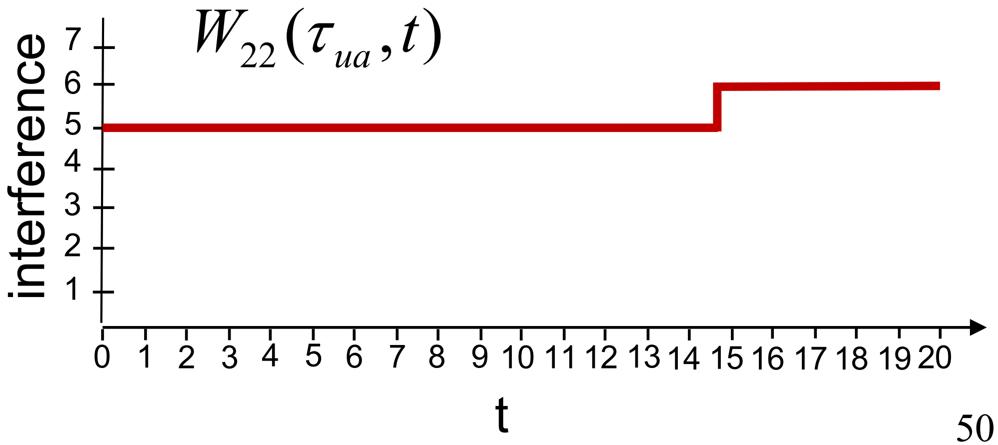
## Critical Instant assumption in RTA with offsets

**<u>Critical Instant (CI) combination 6</u>:** Let $\tau_{13}, \tau_{22}$ coincide with CI of $\tau_{ua}$



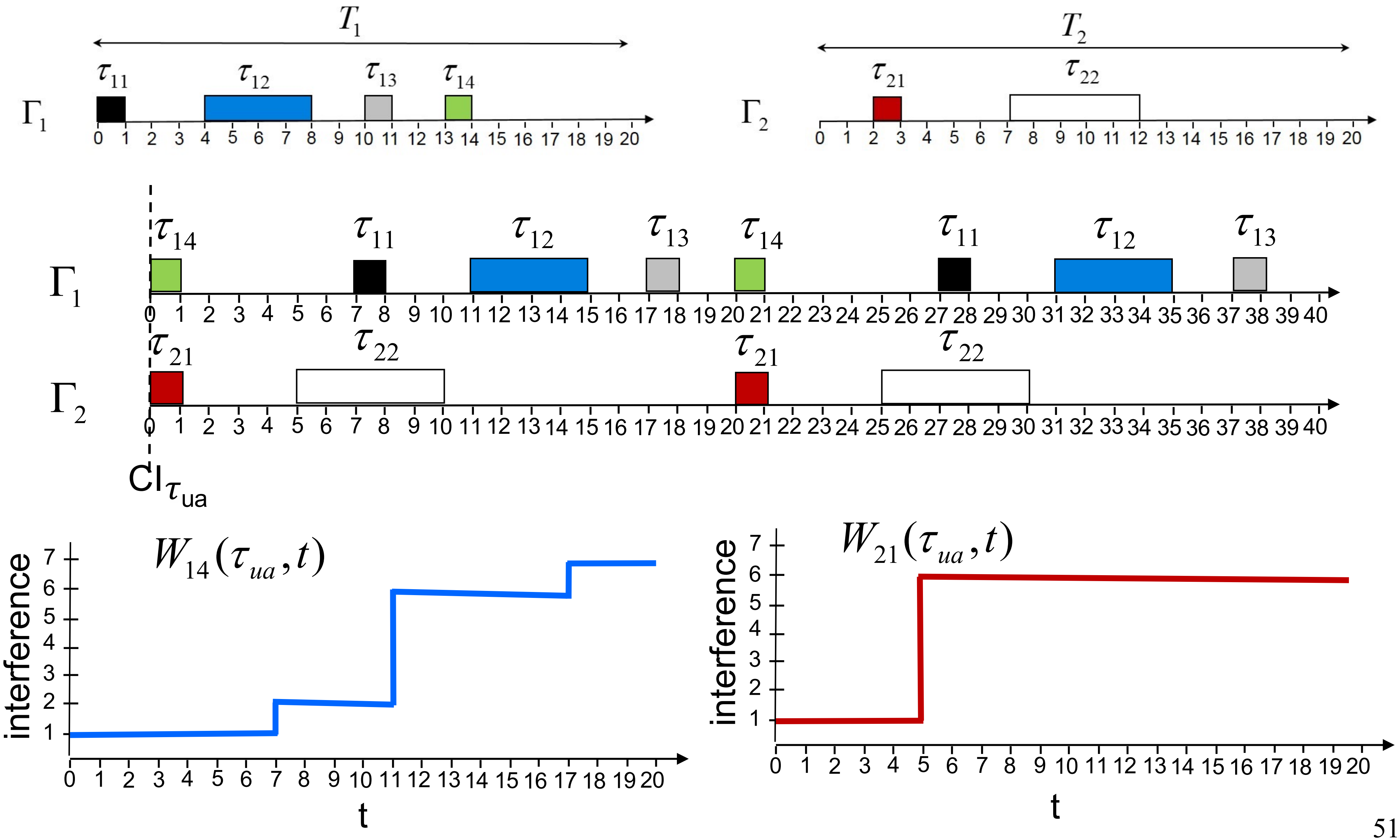



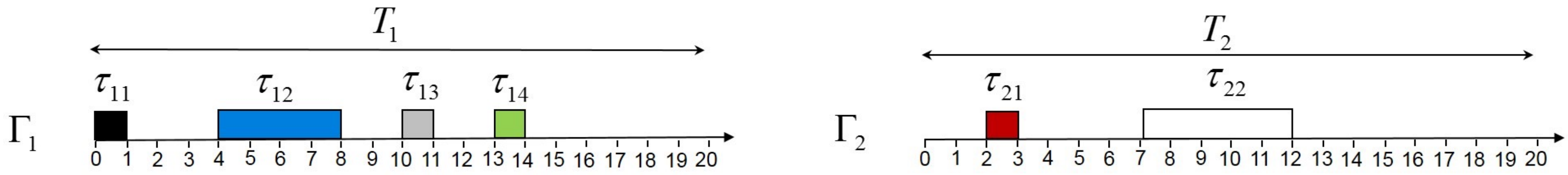
# Critical Instant assumption in RTA with offsets

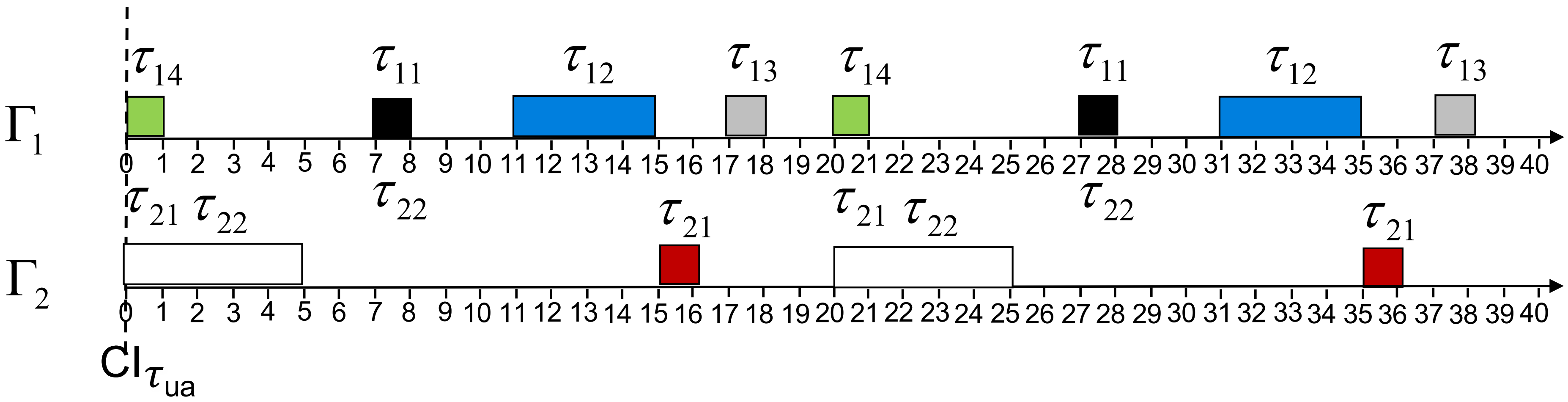
**Critical Instant (CI) combination 7**: Let $\tau_{14}, \tau_{21}$ coincide with CI of $\tau_{ua}$

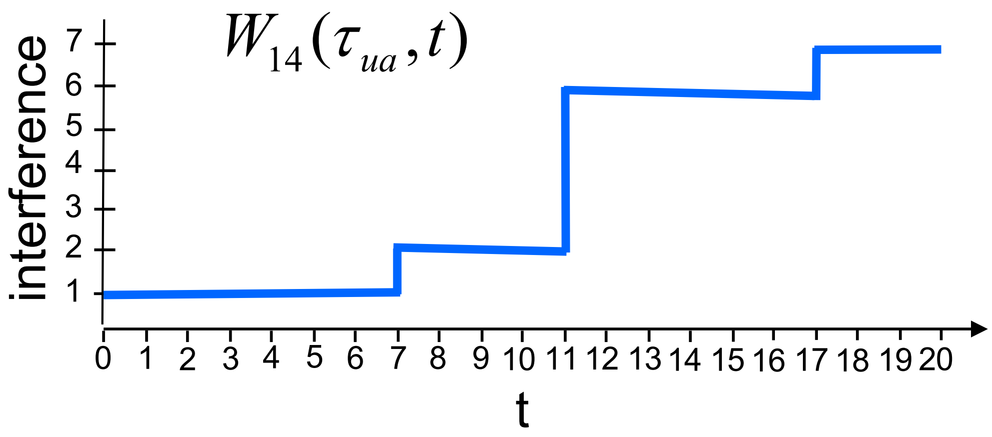


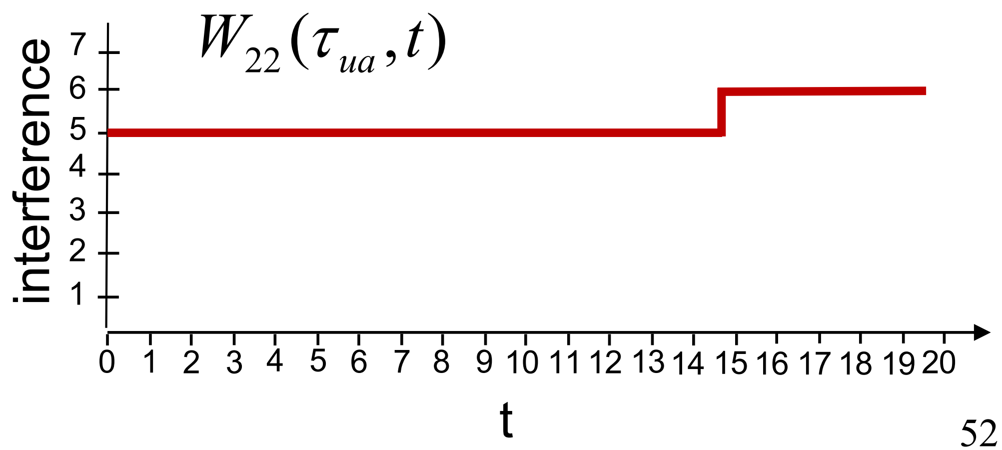
# Critical Instant assumption in RTA with offsets

**<u>Critical Instant (CI) combination 8</u>:** Let $\tau_{14}, \tau_{22}$ coincide with CI of $\tau_{ua}$



## Calculations for the RTA with offsets

Calculations for response time of $\tau_{ua}$ denoted by $R_{uac}$ for the Critical Instant combination

$$R_{uac}^{n+1} = C_{ua} + \sum_{\forall i} W_{ic}(\tau_{ua}, R_{uac}^{n})$$

$$W_{\mathrm{ic}}(\tau_{ua}, \mathrm{t}) = \sum_{\forall j \in hp_i(\tau_{ua})} \left\lceil \frac{t - \Phi_{ijc}}{T_i} \right\rceil C_{ij}$$

$$\Phi_{ijc} = (O_{ij} - O_{ic}) \bmod T_i$$

$$R_{ua} = \max_{\forall c\ \ combinations} (R_{uac})$$

# Exact RTA with offsets VS Approximate RTA with offsets

$$R_{ua} = \max_{\forall c \ \ combinations} (R_{uac})$$

That is the response time of $\tau_{ua}$ is the maximum of calculated response times for all critical instant combinations.

**Exact analysis tries every potential critical instant: intractable!**

- In this previous example, 2 transactions with 4 and 2 tasks results in 4*2= 8 combinations
- Example: 4 transactions with 3, 10, 5, and10 tasks results in 3*10*5*10 = 1500 combinations

**Approximate approach: build abstraction of each transaction**

$$R_{ua}^{n+1} = C_{ua} + \sum_{\forall i} W_i^*(\tau_{ua}, R_{ua}^n)$$

$$W_i^*(\tau_{ua}, R_{ua}^n) = \max_{\forall c}(W_{ic}(\tau_{ua}, R_{ua}^n))$$

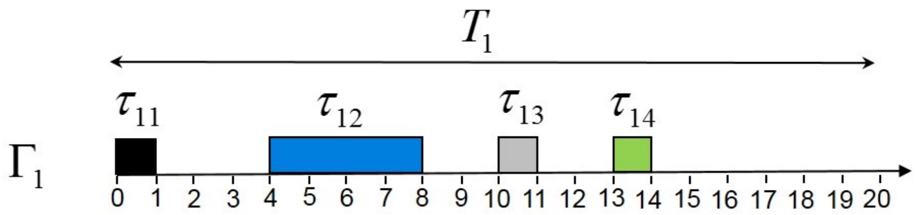
$$\Phi_{ijc} = (O_{ij} - O_{ic}) \bmod T_i$$

$$W_{ic}(\tau_{ua}, R_{ua}^n) = \sum_{\forall j \in hpi(\tau_{ua})} \left\lceil \frac{R_{ua}^n - \Phi_{ijc}}{T_i} \right\rceil C_{ij}$$

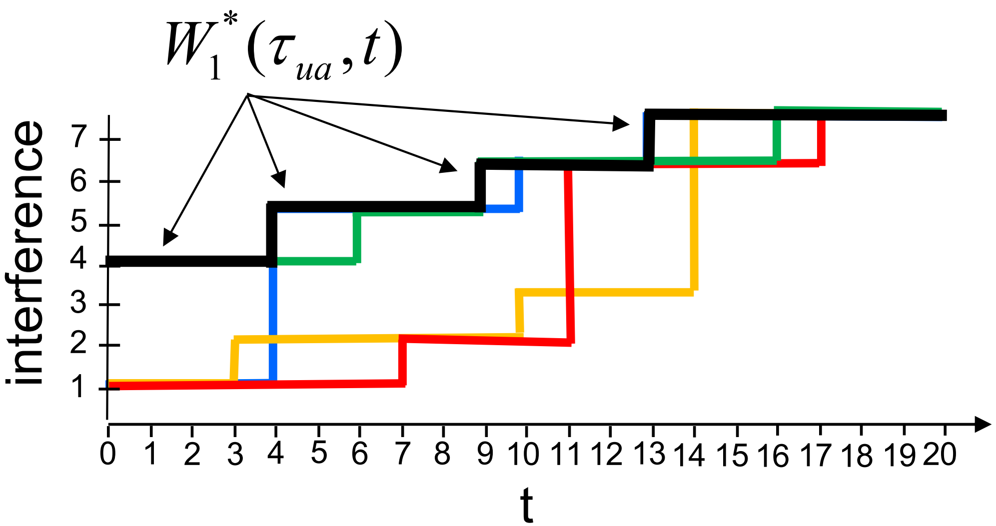
# Approximate RTA with offsets

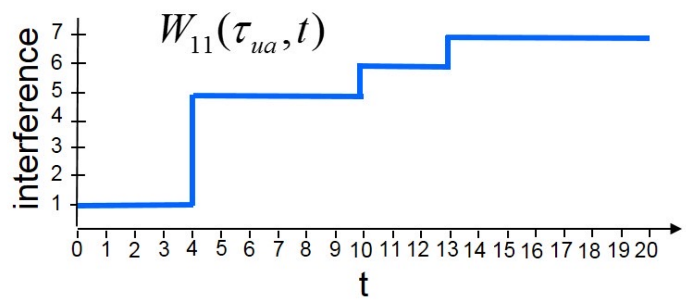
**Approximate analysis considers maximum interference from each transaction**

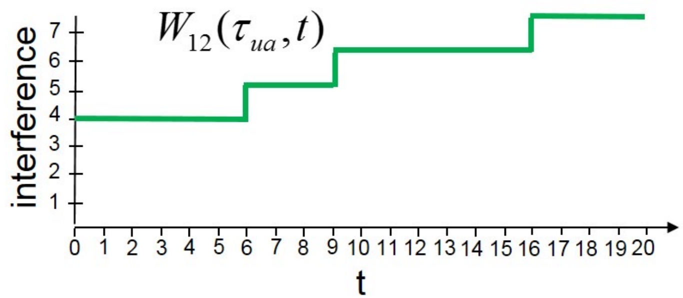
Consider the same example

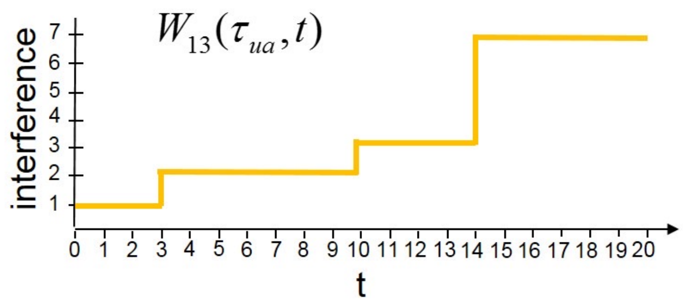


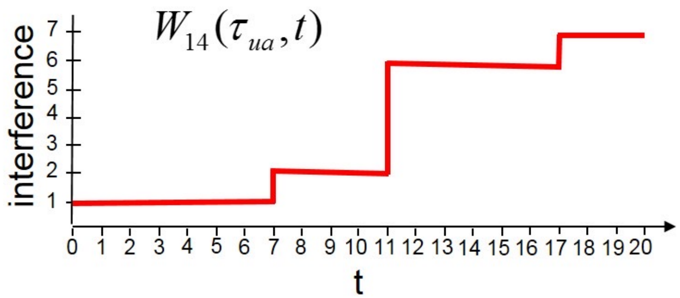

Maximum interference function due to $\Gamma_1$



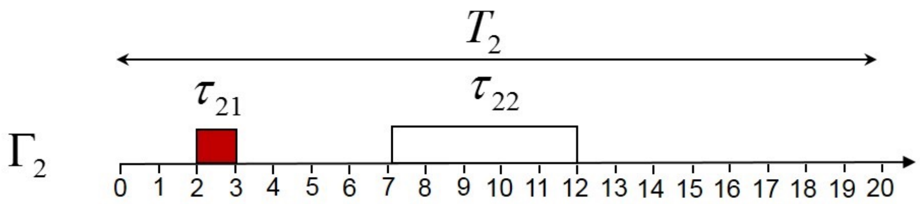



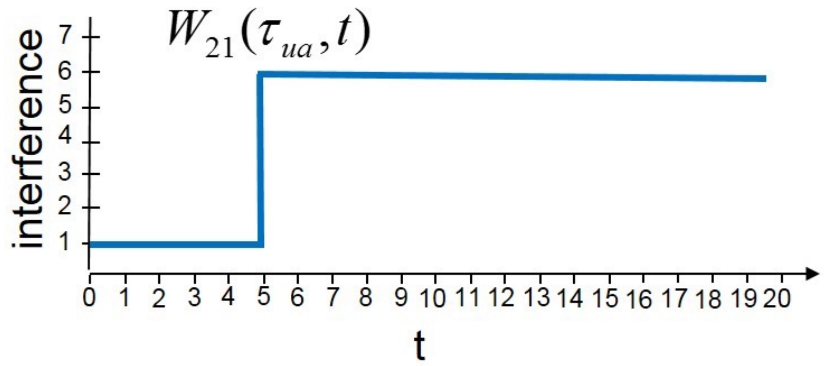



# Approximate RTA with offsets

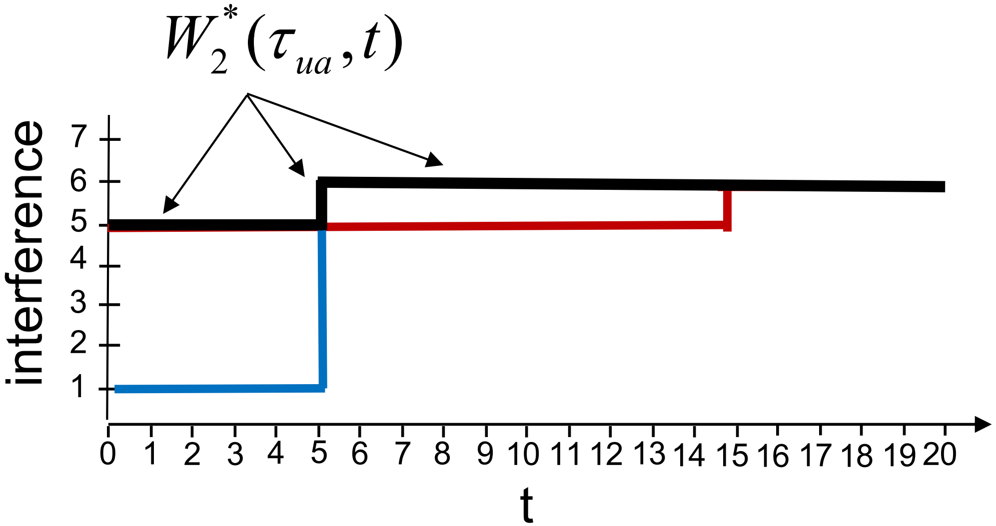
**Approximate analysis considers maximum interference from each transaction**

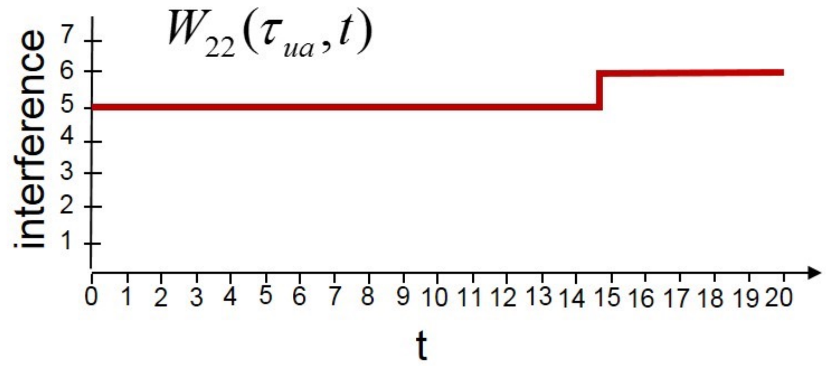
Consider the same example




Maximum interference function due to $\Gamma_2$



# Example of Response Time Analysis with Offsets

*Saad Mubeen*

## Appendix: Response Time Formulae

**Exact Analysis**

Calculations for response time of $\tau_{ua}$ denoted by $R_{uac}$ for the Critical Instant combination

$$R_{uac}^{n+1} = C_{ua} + \sum_{\forall i} W_{ic}(\tau_{ua}, R_{uac}^{n})$$

$$W_{\mathrm{ic}}(\tau_{ua}, \mathrm{t}) = \sum_{\forall j \in hp_i(\tau_{ua})} \left\lceil \frac{t - \Phi_{ijc}}{T_i} \right\rceil C_{ij}$$

$$\Phi_{ijc} = (O_{ij} - O_{ic}) \bmod T_i$$

# Appendix: Response Time Formulae

## Approximate Analysis

Calculations for response time of $\tau_{ua}$ denoted by $R_{ua}$

$$R_{ua}^{n+1} = C_{ua} + \sum_{\forall i} W_i^*(\tau_{ua}, R_{ua}^n)$$

$$W_i^*(\tau_{ua}, R_{ua}^n) = \max_{\forall c}(W_{ic}(\tau_{ua}, R_{ua}^n))$$

$$W_{ic}(\tau_{ua}, R_{ua}^n) = \sum_{\forall j \in hpi(\tau_{ua})} \left\lceil \frac{R_{ua}^n - \Phi_{ijc}}{T_i} \right\rceil C_{ij}$$

$$\Phi_{ijc} = (O_{ij} - O_{ic}) \bmod T_i$$

# Example: Response Time Analysis (RTA) with offsets

$$\Gamma = \{\Gamma_1, \Gamma_2, \Gamma_3\} \qquad \Gamma_i = \langle\{\tau_{i1}, \tau_{i2}, \cdots, \tau_{i2n}\}, T_i\rangle \qquad \tau_{ij} = \langle C_{ij}, O_{ij}, P_{ij}\rangle$$

$$\Gamma_1 = \langle\{\tau_{11}, \tau_{12}\}, 10\rangle \qquad \tau_{11} = \langle 2,2,5\rangle \qquad \tau_{12} = \langle 1,5,4\rangle$$

$$\Gamma_2 = \langle\{\tau_{21}, \tau_{22}\}, 10\rangle \qquad \tau_{21} = \langle 2,1,3\rangle \qquad \tau_{22} = \langle 1,7,2\rangle$$

$$\Gamma_3 = \langle\{\tau_{31}\}, 20\rangle \qquad \tau_{31} = \langle 5,0,1\rangle$$

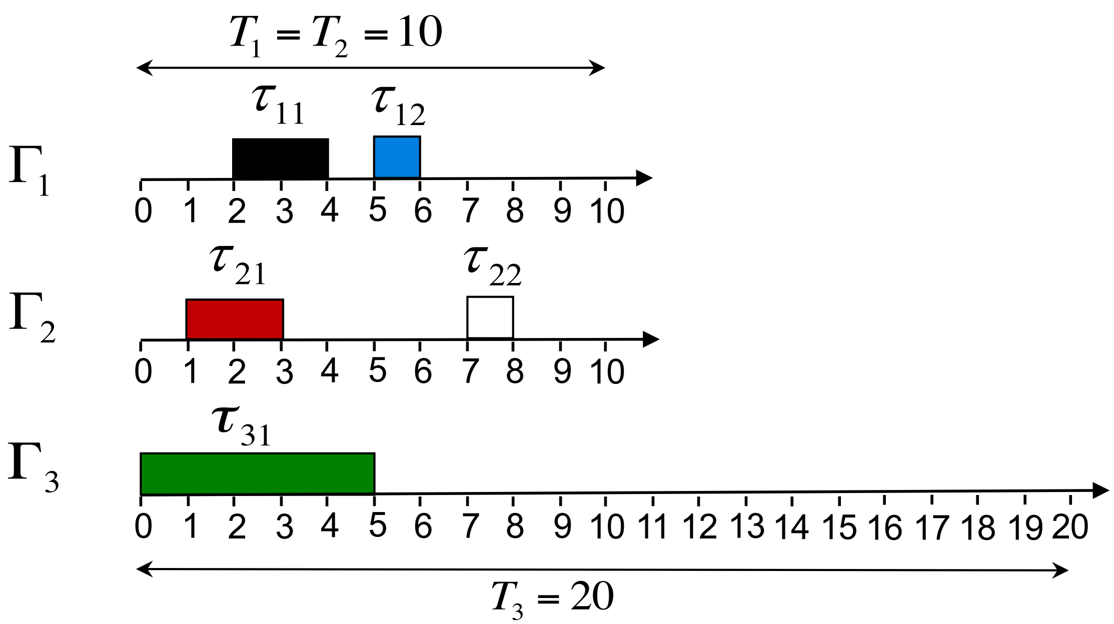
Prio (5) = Highest

Prio (1) = Lowest

**Calculate the response times of all tasks using**

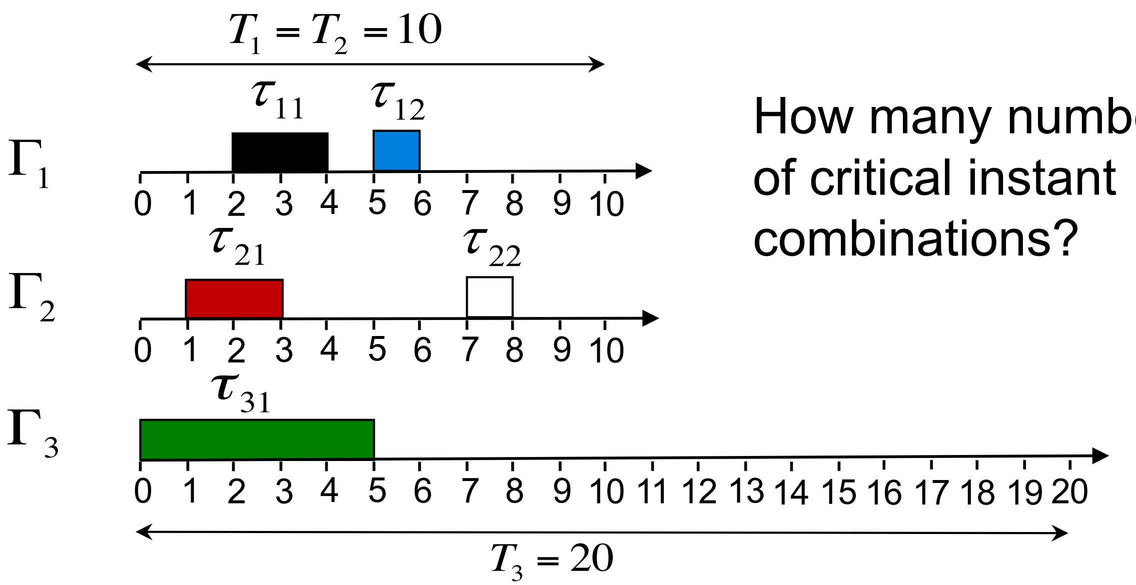
**(a) Exact analysis**

**(b) Approximate analysis**

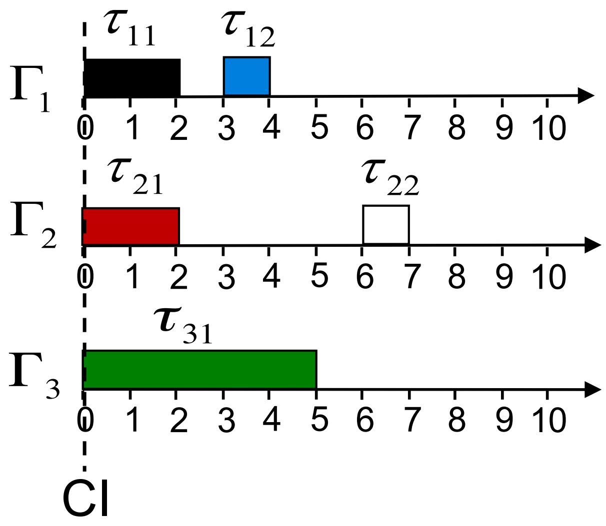


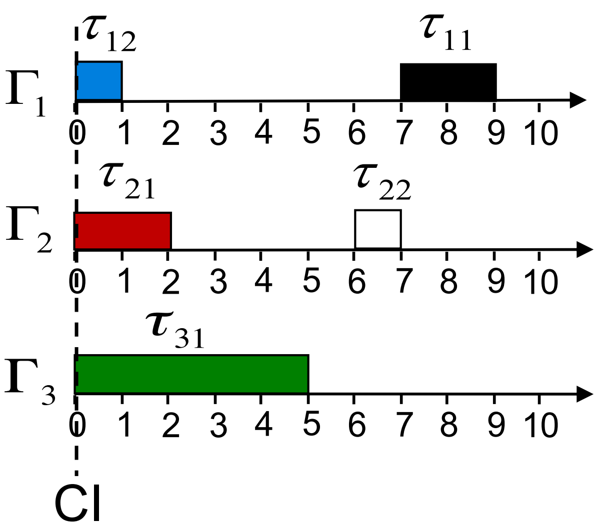
# Example: Response Time Analysis (RTA) with offsets

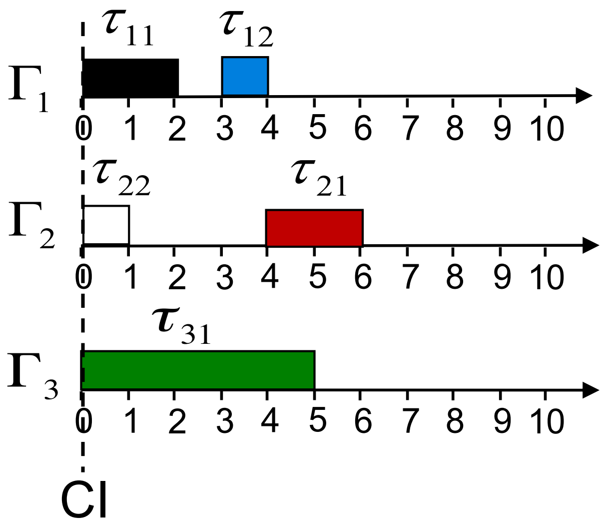
Let $\tau_{31}$ be the task under analysis, i.e., $\tau_{ua}$

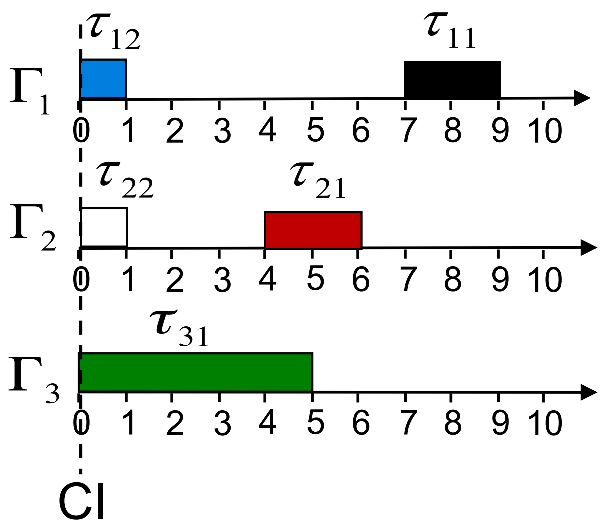



How many number of critical instant combinations?



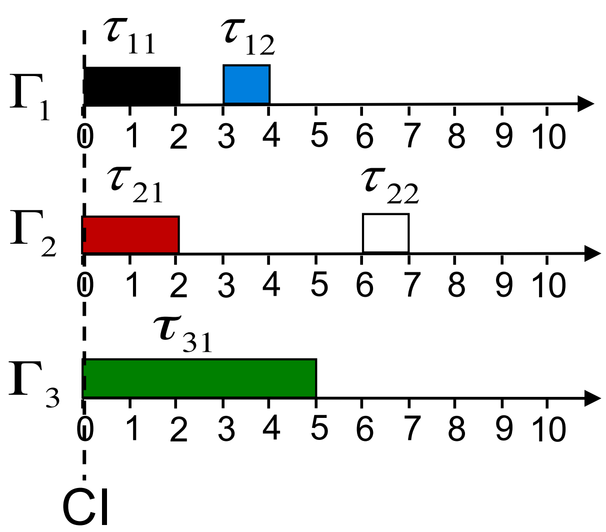



## Example: RTA with offsets using Exact Analysis

$$R_{uac}^{n+1} = C_{ua} + \sum_{\forall i} W_{ic}(\tau_{ua}, R_{uac}^{n})$$

Let $\tau_{31}$ be the task under analysis, i.e., $\tau_{ua}$




**<u>Critical Instant (CI) combination 1</u>:**

Let $\tau_{11}, \tau_{21}, \tau_{31}$ coincide with CI.

$$R_{uac}^{n+1} = C_{ua} + \sum_{\forall i} W_{ic}(\tau_{ua}, R_{uac}^{n})$$

$$R_{uac}^{n+1} = C_{ua} + W_{11}(\tau_{ua}, R_{uac}^{n}) + W_{21}(\tau_{ua}, R_{uac}^{n})$$

$$R_{uac}^{n+1} = C_{ua} + \sum_{\forall j \in hp_1(\tau_{ua})} \left\lceil \frac{R_{uac}^{n} - \Phi_{1j1}}{T_1} \right\rceil C_{1j} + \sum_{\forall j \in hp_2(\tau_{ua})} \left\lceil \frac{R_{uac}^{n} - \Phi_{2j1}}{T_2} \right\rceil C_{2j}$$

$$R_{uac}^{n+1} = C_{ua} + \left\lceil \frac{R_{uac}^{n} - \Phi_{111}}{T_1} \right\rceil C_{11} + \left\lceil \frac{R_{uac}^{n} - \Phi_{121}}{T_1} \right\rceil C_{12} + \left\lceil \frac{R_{uac}^{n} - \Phi_{211}}{T_2} \right\rceil C_{21} + \left\lceil \frac{R_{uac}^{n} - \Phi_{221}}{T_2} \right\rceil C_{22} \cdots\cdots(1)$$

# Example: RTA with offsets using Exact Analysis

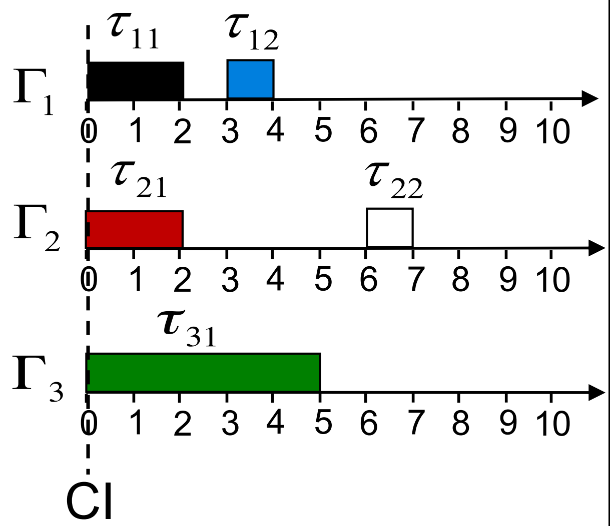
$$\Phi_{ijc} = (O_{ij} - O_{ic}) \bmod T_i$$

$$\Phi_{111} = (O_{11} - O_{11}) \bmod 10 = (2-2) \bmod 10 = 0$$

$$\Phi_{121} = (O_{12} - O_{11}) \bmod 10 = (5-2) \bmod 10 = 3$$

$$\Phi_{211} = (O_{21} - O_{21}) \bmod 10 = (1-1) \bmod 10 = 0$$

$$\Phi_{221} = (O_{22} - O_{21}) \bmod 10 = (7-1) \bmod 10 = 6$$




Using all these values in (1)

$$R^0_{uac} = C_{ua} = 5$$

$$R^1_{uac} = 5 + \left\lceil \frac{5-0}{10} \right\rceil 2 + \left\lceil \frac{5-3}{10} \right\rceil 1 + \left\lceil \frac{5-0}{10} \right\rceil 2 + \left\lceil \frac{5-6}{10} \right\rceil 1$$

$$R^1_{uac} = 5 + 2 + 1 + 2 + 0$$

$$R^1_{uac} = 10$$

# Example: RTA with offsets using Exact Analysis

Next iteration

$$R_{uac}^{2} = 5 + \left\lceil \frac{10-0}{10} \right\rceil 2 + \left\lceil \frac{10-3}{10} \right\rceil 1 + \left\lceil \frac{10-0}{10} \right\rceil 2 + \left\lceil \frac{10-6}{10} \right\rceil 1$$

$$R_{uac}^{2} = 5+2+1+2+1$$

$$R_{uac}^{2} = 11$$

Next iteration

$$R_{uac}^{3} = 5 + \left\lceil \frac{11-0}{10} \right\rceil 2 + \left\lceil \frac{11-3}{10} \right\rceil 1 + \left\lceil \frac{11-0}{10} \right\rceil 2 + \left\lceil \frac{11-6}{10} \right\rceil 1$$

$$R_{uac}^{3} = 5+4+1+4+1$$

$$R_{uac}^{3} = 15$$

# Example: RTA with offsets using Exact Analysis

Next iteration

$$R_{uac}^{4} = 5 + \left\lceil \frac{15-0}{10} \right\rceil 2 + \left\lceil \frac{15-3}{10} \right\rceil 1 + \left\lceil \frac{15-0}{10} \right\rceil 2 + \left\lceil \frac{15-6}{10} \right\rceil 1$$

$$R_{uac}^{4} = 5 + 4 + 2 + 4 + 1$$

$$R_{uac}^{4} = 16$$

Next iteration

$$R_{uac}^{5} = 5 + \left\lceil \frac{16-0}{10} \right\rceil 2 + \left\lceil \frac{16-3}{10} \right\rceil 1 + \left\lceil \frac{16-0}{10} \right\rceil 2 + \left\lceil \frac{16-6}{10} \right\rceil 1$$

$$R_{uac}^{5} = 5 + 4 + 2 + 4 + 1$$

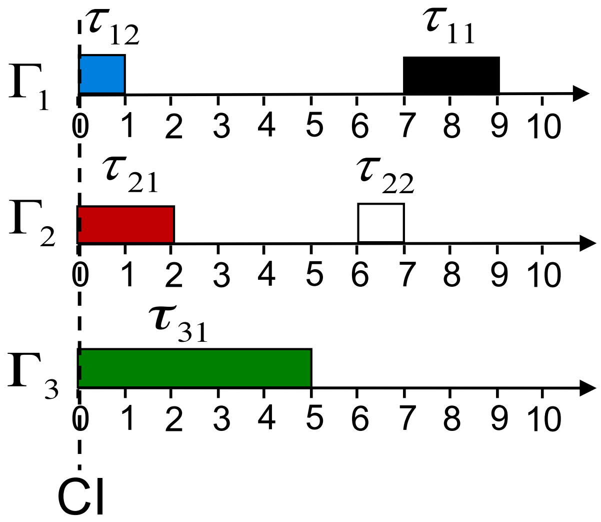
$$R_{uac}^{5} = 16 \qquad \Rightarrow R_{uac}^{5} = R_{uac}^{4} = 16$$

$$\Rightarrow \quad \boxed{R_{uac} = 16}$$

## Example: RTA with offsets using Exact Analysis

**Critical Instant (CI) combination 2:**

Let $\tau_{12}, \tau_{21}, \tau_{31}$ coincide with CI.




$$R_{uac}^{n+1} = C_{ua} + \sum_{\forall i} W_{ic}(\tau_{ua}, R_{uac}^{n})$$

$$R_{uac}^{n+1} = C_{ua} + W_{12}(\tau_{ua}, R_{uac}^{n}) + W_{21}(\tau_{ua}, R_{uac}^{n})$$

$$R_{uac}^{n+1} = C_{ua} + \sum_{\forall j \in hp_1(\tau_{ua})} \left\lceil \frac{R_{uac}^{n} - \Phi_{1j2}}{T_1} \right\rceil C_{1j} + \sum_{\forall j \in hp_2(\tau_{ua})} \left\lceil \frac{R_{uac}^{n} - \Phi_{2j1}}{T_2} \right\rceil C_{2j}$$

$$R_{uac}^{n+1} = C_{ua} + \left\lceil \frac{R_{uac}^{n} - \Phi_{112}}{T_1} \right\rceil C_{11} + \left\lceil \frac{R_{uac}^{n} - \Phi_{122}}{T_1} \right\rceil C_{12} + \left\lceil \frac{R_{uac}^{n} - \Phi_{211}}{T_2} \right\rceil C_{21} + \left\lceil \frac{R_{uac}^{n} - \Phi_{221}}{T_2} \right\rceil C_{22} \cdots\cdots(2)$$

# Example: RTA with offsets using Exact Analysis

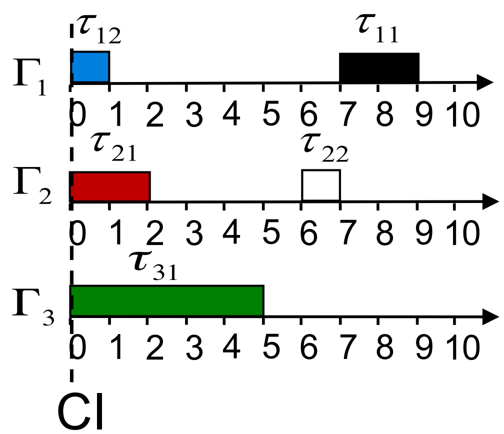
$$\Phi_{ijc} = (O_{ij} - O_{ic}) \bmod T_i$$

$$\Phi_{112} = (O_{11} - O_{12}) \bmod 10 = (2-5) \bmod 10 = 10 - 3 = 7$$

$$\Phi_{122} = (O_{12} - O_{12}) \bmod 10 = (5-5) \bmod 10 = 0$$

$$\Phi_{211} = (O_{21} - O_{21}) \bmod 10 = (1-1) \bmod 10 = 0$$

$$\Phi_{221} = (O_{22} - O_{21}) \bmod 10 = (7-1) \bmod 10 = 6$$




Using all these values in (2)

$$R_{uac}^{0} = C_{ua} = 5$$

$$R_{uac}^{1} = 5 + \left\lceil \frac{5-7}{10} \right\rceil 2 + \left\lceil \frac{5-0}{10} \right\rceil 1 + \left\lceil \frac{5-0}{10} \right\rceil 2 + \left\lceil \frac{5-6}{10} \right\rceil 1$$

$$R_{uac}^{1} = 5 + 0 + 1 + 2 + 0$$

$$R_{uac}^{1} = 8$$

## Example: RTA with offsets using Exact Analysis

Next iteration

$$R_{uac}^{2} = 5 + \left\lceil \frac{8-7}{10} \right\rceil 2 + \left\lceil \frac{8-0}{10} \right\rceil 1 + \left\lceil \frac{8-0}{10} \right\rceil 2 + \left\lceil \frac{8-6}{10} \right\rceil 1$$

$$R_{uac}^{2} = 5+2+1+2+1$$

$$R_{uac}^{2} = 11$$

Next iteration

$$R_{uac}^{3} = 5 + \left\lceil \frac{11-7}{10} \right\rceil 2 + \left\lceil \frac{11-0}{10} \right\rceil 1 + \left\lceil \frac{11-0}{10} \right\rceil 2 + \left\lceil \frac{11-6}{10} \right\rceil 1$$

$$R_{uac}^{3} = 5+2+2+4+1$$

$$R_{uac}^{3} = 14$$

# Example: RTA with offsets using Exact Analysis

Next iteration

$$R_{uac}^{4} = 5 + \left\lceil \frac{14-7}{10} \right\rceil 2 + \left\lceil \frac{14-0}{10} \right\rceil 1 + \left\lceil \frac{14-0}{10} \right\rceil 2 + \left\lceil \frac{14-6}{10} \right\rceil 1$$

$$R_{uac}^{4} = 5 + 2 + 2 + 4 + 1$$

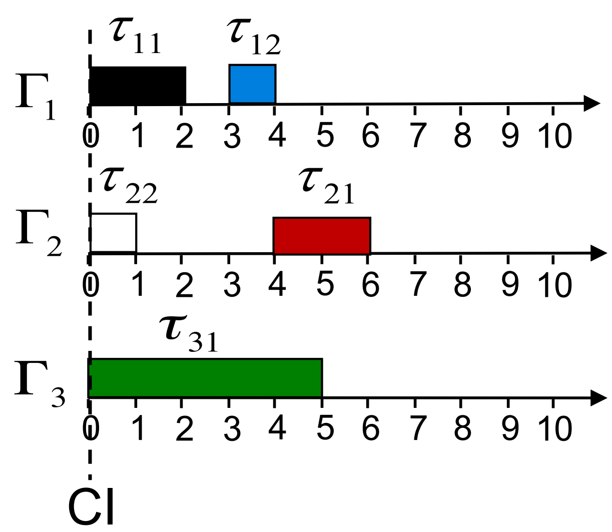
$$R_{uac}^{4} = 14 \qquad \Rightarrow R_{uac}^{4} = R_{uac}^{3} = 14$$

$$\Rightarrow \quad \boxed{R_{uac} = 14}$$

## Example: RTA with offsets using Exact Analysis

**Critical Instant (CI) combination 3:**

Let $\tau_{11}, \tau_{22}, \tau_{31}$ coincide with CI.




$$R_{uac}^{n+1} = C_{ua} + \sum_{\forall i} W_{ic}(\tau_{ua}, R_{uac}^{n})$$

$$R_{uac}^{n+1} = C_{ua} + W_{11}(\tau_{ua}, R_{uac}^{n}) + W_{22}(\tau_{ua}, R_{uac}^{n})$$

$$R_{uac}^{n+1} = C_{ua} + \sum_{\forall j \in hp_1(\tau_{ua})} \left\lceil \frac{R_{uac}^{n} - \Phi_{1j1}}{T_1} \right\rceil C_{1j} + \sum_{\forall j \in hp_2(\tau_{ua})} \left\lceil \frac{R_{uac}^{n} - \Phi_{2j2}}{T_2} \right\rceil C_{2j}$$

$$R_{uac}^{n+1} = C_{ua} + \left\lceil \frac{R_{uac}^{n} - \Phi_{111}}{T_1} \right\rceil C_{11} + \left\lceil \frac{R_{uac}^{n} - \Phi_{121}}{T_1} \right\rceil C_{12} + \left\lceil \frac{R_{uac}^{n} - \Phi_{212}}{T_2} \right\rceil C_{21} + \left\lceil \frac{R_{uac}^{n} - \Phi_{222}}{T_2} \right\rceil C_{22} \cdots\cdots(3)$$

## Example: RTA with offsets using Exact Analysis

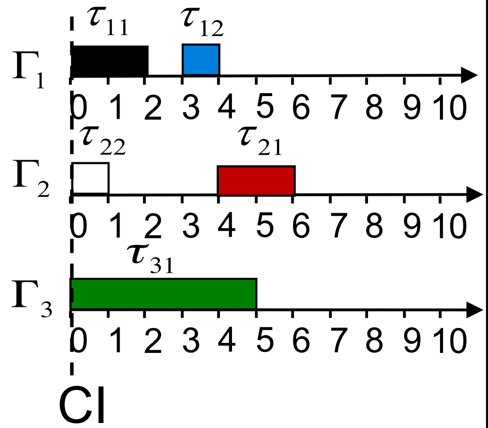
$$\Phi_{ijc} = (O_{ij} - O_{ic}) \bmod T_i$$

$$\Phi_{111} = (O_{11} - O_{11}) \bmod 10 = (2-2) \bmod 10 = 0$$

$$\Phi_{121} = (O_{12} - O_{11}) \bmod 10 = (5-2) \bmod 10 = 3$$

$$\Phi_{212} = (O_{21} - O_{22}) \bmod 10 = (1-7) \bmod 10 = 10 - 6 = 4$$

$$\Phi_{222} = (O_{22} - O_{22}) \bmod 10 = (7-7) \bmod 10 = 0$$




Using all these values in (3)

$$R^0_{uac} = C_{ua} = 5$$

$$R^1_{uac} = 5 + \left\lceil \frac{5-0}{10} \right\rceil 2 + \left\lceil \frac{5-3}{10} \right\rceil 1 + \left\lceil \frac{5-4}{10} \right\rceil 2 + \left\lceil \frac{5-0}{10} \right\rceil 1$$

$$R^1_{uac} = 5 + 2 + 1 + 2 + 1$$

$$R^1_{uac} = 11$$

# Example: RTA with offsets using Exact Analysis

Next iteration

$$R_{uac}^{2} = 5 + \left\lceil \frac{11-0}{10} \right\rceil 2 + \left\lceil \frac{11-3}{10} \right\rceil 1 + \left\lceil \frac{11-4}{10} \right\rceil 2 + \left\lceil \frac{11-0}{10} \right\rceil 1$$

$$R_{uac}^{2} = 5 + 4 + 1 + 2 + 2$$

$$R_{uac}^{2} = 14$$

Next iteration

$$R_{uac}^{3} = 5 + \left\lceil \frac{14-0}{10} \right\rceil 2 + \left\lceil \frac{14-3}{10} \right\rceil 1 + \left\lceil \frac{14-4}{10} \right\rceil 2 + \left\lceil \frac{14-0}{10} \right\rceil 1$$

$$R_{uac}^{3} = 5 + 4 + 2 + 2 + 2$$

$$R_{uac}^{3} = 15$$

## Example: RTA with offsets using Exact Analysis

Next iteration

$$R_{uac}^{4} = 5 + \left\lceil \frac{15-0}{10} \right\rceil 2 + \left\lceil \frac{15-3}{10} \right\rceil 1 + \left\lceil \frac{15-4}{10} \right\rceil 2 + \left\lceil \frac{15-0}{10} \right\rceil 1$$

$$R_{uac}^{4} = 5 + 4 + 2 + 4 + 2$$

$$R_{uac}^{4} = 17$$

Next iteration

$$R_{uac}^{5} = 5 + \left\lceil \frac{17-0}{10} \right\rceil 2 + \left\lceil \frac{17-3}{10} \right\rceil 1 + \left\lceil \frac{17-4}{10} \right\rceil 2 + \left\lceil \frac{17-0}{10} \right\rceil 1$$

$$R_{uac}^{5} = 5 + 4 + 2 + 4 + 2$$

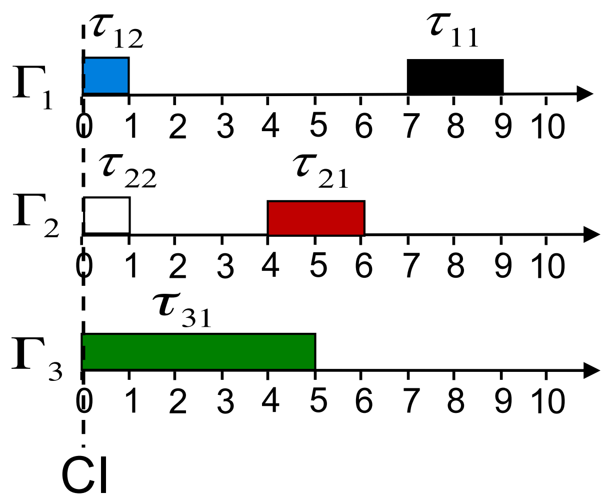
$$R_{uac}^{5} = 17 \qquad \Rightarrow R_{uac}^{5} = R_{uac}^{4} = 17$$

$$\Rightarrow \quad \boxed{R_{uac} = 17}$$

# Example: RTA with offsets using Exact Analysis

**<u>Critical Instant (CI) combination 4</u>:**

Let $\tau_{12}, \tau_{22}, \tau_{31}$ coincide with CI.




$$R_{uac}^{n+1} = C_{ua} + \sum_{\forall i} W_{ic}(\tau_{ua}, R_{uac}^{n})$$

$$R_{uac}^{n+1} = C_{ua} + W_{12}(\tau_{ua}, R_{uac}^{n}) + W_{22}(\tau_{ua}, R_{uac}^{n})$$

$$R_{uac}^{n+1} = C_{ua} + \sum_{\forall j \in hp_1(\tau_{ua})} \left\lceil \frac{R_{uac}^{n} - \Phi_{1j2}}{T_1} \right\rceil C_{1j} + \sum_{\forall j \in hp_2(\tau_{ua})} \left\lceil \frac{R_{uac}^{n} - \Phi_{2j2}}{T_2} \right\rceil C_{2j}$$

$$R_{uac}^{n+1} = C_{ua} + \left\lceil \frac{R_{uac}^{n} - \Phi_{112}}{T_1} \right\rceil C_{11} + \left\lceil \frac{R_{uac}^{n} - \Phi_{122}}{T_1} \right\rceil C_{12} + \left\lceil \frac{R_{uac}^{n} - \Phi_{212}}{T_2} \right\rceil C_{21} + \left\lceil \frac{R_{uac}^{n} - \Phi_{222}}{T_2} \right\rceil C_{22} \cdots\cdots(4)$$

## Example: RTA with offsets using Exact Analysis

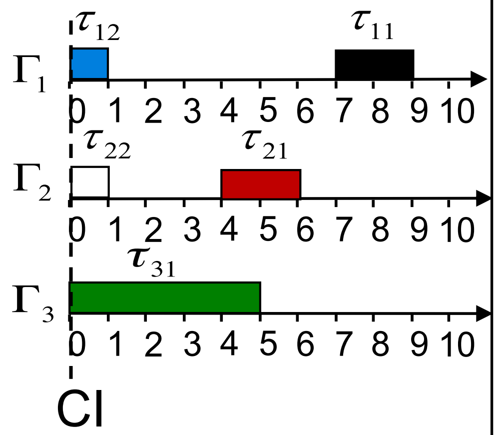
$$\Phi_{ijc} = (O_{ij} - O_{ic}) \bmod T_i$$

$$\Phi_{112} = (O_{11} - O_{12}) \bmod 10 = (2-5) \bmod 10 = 10 - 3 = 7$$

$$\Phi_{122} = (O_{12} - O_{12}) \bmod 10 = (5-5) \bmod 10 = 0$$

$$\Phi_{212} = (O_{21} - O_{22}) \bmod 10 = (1-7) \bmod 10 = 10 - 6 = 4$$

$$\Phi_{222} = (O_{22} - O_{22}) \bmod 10 = (7-7) \bmod 10 = 0$$




Using all these values in (4)

$$R_{uac}^{0} = C_{ua} = 5$$

$$R_{uac}^{1} = 5 + \left\lceil \frac{5-7}{10} \right\rceil 2 + \left\lceil \frac{5-0}{10} \right\rceil 1 + \left\lceil \frac{5-4}{10} \right\rceil 2 + \left\lceil \frac{5-0}{10} \right\rceil 1$$

$$R_{uac}^{1} = 5 + 0 + 1 + 2 + 1$$

$$R_{uac}^{1} = 9$$

## Example: RTA with offsets using Exact Analysis

Next iteration

$$R_{uac}^{2} = 5 + \left\lceil \frac{9-7}{10} \right\rceil 2 + \left\lceil \frac{9-0}{10} \right\rceil 1 + \left\lceil \frac{9-4}{10} \right\rceil 2 + \left\lceil \frac{9-0}{10} \right\rceil 1$$

$$R_{uac}^{2} = 5 + 2 + 1 + 2 + 1$$

$$R_{uac}^{2} = 11$$

Next iteration

$$R_{uac}^{3} = 5 + \left\lceil \frac{11-7}{10} \right\rceil 2 + \left\lceil \frac{11-0}{10} \right\rceil 1 + \left\lceil \frac{11-4}{10} \right\rceil 2 + \left\lceil \frac{11-0}{10} \right\rceil 1$$

$$R_{uac}^{3} = 5 + 2 + 2 + 2 + 2$$

$$R_{uac}^{3} = 13$$

# Example: RTA with offsets using Exact Analysis

Next iteration

$$R_{uac}^{4} = 5 + \left\lceil \frac{13-7}{10} \right\rceil 2 + \left\lceil \frac{13-0}{10} \right\rceil 1 + \left\lceil \frac{13-4}{10} \right\rceil 2 + \left\lceil \frac{13-0}{10} \right\rceil 1$$

$$R_{uac}^{4} = 5 + 2 + 2 + 2 + 2$$

$$R_{uac}^{4} = 13 \qquad \Rightarrow R_{uac}^{4} = R_{uac}^{3} = 13 \qquad \Rightarrow \qquad \boxed{R_{uac} = 13}$$

$$R_{ua} = \max_{\forall c \ \ combinations} (R_{uac})$$

$$R_{ua} = \max(16,14,17,13) = 17$$

$$\boxed{R_{ua} = R_{31} = 17}$$

Response times of other tasks can be calculated in a similar way

# Example: RTA with offsets using Approximate Analysis

$$R_{ua}^{n+1} = C_{ua} + \sum_{\forall i} W_i^*(\tau_{ua}, R_{ua}^n)$$

Where, $W_i^*(\tau_{ua}, R_{ua}^n) = \max_{\forall c}(W_{ic}(\tau_{ua}, R_{ua}^n))$

$$W_{ic}(\tau_{ua}, R_{ua}^n) = \sum_{\forall j \in hpi(\tau_{ua})} \left\lceil \frac{R_{ua}^n - \Phi_{ijc}}{T_i} \right\rceil C_{ij}$$

$$\mathrm{R}_{\mathrm{ua}}^{\mathrm{n+1}} = C_{ua} + W_1^*(\tau_{ua}, R_{ua}^n) + W_2^*(\tau_{ua}, R_{ua}^n)$$

$$R_{ua}^{n+1} = C_{ua} + \max_{\forall c}(W_{1c}(\tau_{ua}, R_{ua}^n)) + \max_{\forall c}(W_{2c}(\tau_{ua}, R_{ua}^n))$$

$$R_{ua}^{n+1} = C_{ua} + \max((W_{11}(\tau_{ua}, R_{ua}^n), \quad (W_{12}(\tau_{ua}, R_{ua}^n))$$

$$+ \max((W_{21}(\tau_{ua}, R_{ua}^n), \quad (W_{22}(\tau_{ua}, R_{ua}^n))$$

# Example: RTA with offsets using Approximate Analysis

$$R_{ua}^{n+1} = C_{ua} + \max\left( \sum_{\forall j \in hp_1(\tau_{ua})} \left\lceil \frac{R_{ua}^n - \Phi_{1j1}}{T_1} \right\rceil C_{1j} \;,\; \sum_{\forall j \in hp_1(\tau_{ua})} \left\lceil \frac{R_{ua}^n - \Phi_{1j2}}{T_1} \right\rceil C_{1j} \right)$$

$$+ \max\left( \sum_{\forall j \in hp_2(\tau_{ua})} \left\lceil \frac{R_{ua}^n - \Phi_{2j1}}{T_2} \right\rceil C_{2j} \;,\; \sum_{\forall j \in hp_2(\tau_{ua})} \left\lceil \frac{R_{ua}^n - \Phi_{2j2}}{T_2} \right\rceil C_{2j} \right)$$

$$R_{ua}^{n+1} = C_{ua} + \max\left( \left\lceil \frac{R_{ua}^n - \Phi_{111}}{T_1} \right\rceil C_{11} + \left\lceil \frac{R_{ua}^n - \Phi_{121}}{T_1} \right\rceil C_{12} \;,\; \left\lceil \frac{R_{ua}^n - \Phi_{112}}{T_1} \right\rceil C_{11} + \left\lceil \frac{R_{ua}^n - \Phi_{122}}{T_1} \right\rceil C_{12} \right)$$

$$+ \max\left( \left\lceil \frac{R_{ua}^n - \Phi_{211}}{T_2} \right\rceil C_{21} + \left\lceil \frac{R_{ua}^n - \Phi_{221}}{T_2} \right\rceil C_{22} \;,\; \left\lceil \frac{R_{ua}^n - \Phi_{212}}{T_2} \right\rceil C_{21} + \left\lceil \frac{R_{ua}^n - \Phi_{222}}{T_2} \right\rceil C_{22} \right) \cdots\cdots (5)$$

## Example: RTA with offsets using Approximate Analysis

$$\Phi_{ijc} = (O_{ij} - O_{ic}) \bmod T_i$$

$$\Phi_{111} = (O_{11} - O_{11}) \bmod 10 = (2-2) \bmod 10 = 0$$

$$\Phi_{121} = (O_{12} - O_{11}) \bmod 10 = (5-2) \bmod 10 = 3$$

$$\Phi_{112} = (O_{11} - O_{12}) \bmod 10 = (2-5) \bmod 10 = 10 - 3 = 7$$

$$\Phi_{122} = (O_{12} - O_{12}) \bmod 10 = (5-5) \bmod 10 = 0$$

$$\Phi_{211} = (O_{21} - O_{21}) \bmod 10 = (1-1) \bmod 10 = 0$$

$$\Phi_{221} = (O_{22} - O_{21}) \bmod 10 = (7-1) \bmod 10 = 6$$

$$\Phi_{212} = (O_{21} - O_{22}) \bmod 10 = (1-7) \bmod 10 = 10 - 6 = 4$$

$$\Phi_{222} = (O_{22} - O_{22}) \bmod 10 = (7-7) \bmod 10 = 0$$

Using all these values in (5)

# Example: RTA with offsets using Approximate Analysis

$$R^0_{uac} = C_{ua} = 5$$

$$R^1_{ua} = 5 + \max\left(\left\lceil \frac{5-0}{10} \right\rceil 2 + \left\lceil \frac{5-3}{10} \right\rceil 1 \ , \ \left\lceil \frac{5-7}{10} \right\rceil 2 + \left\lceil \frac{5-0}{10} \right\rceil 1\right)$$

$$+ \max\left(\left\lceil \frac{5-0}{10} \right\rceil 2 + \left\lceil \frac{5-6}{10} \right\rceil 1 \ , \ \left\lceil \frac{5-4}{10} \right\rceil 2 + \left\lceil \frac{5-0}{10} \right\rceil 1\right)$$

$$R^1_{ua} = 5 + \max(2+1 \ , \ 0+1) + \max(2+0 \ , \ 2+1)$$

$$R^1_{ua} = 5 + \max(3 \ , \ 1) + \max(2 \ , \ 3)$$

$$R^1_{ua} = 5 + 3 + 3$$

$$R^1_{ua} = 11$$

## Example: RTA with offsets using Approximate Analysis

Next iteration

$$R_{ua}^{2} = 5 + \max\left(\left\lceil \frac{11-0}{10} \right\rceil 2 + \left\lceil \frac{11-3}{10} \right\rceil 1 \ , \ \left\lceil \frac{11-7}{10} \right\rceil 2 + \left\lceil \frac{11-0}{10} \right\rceil 1\right)$$

$$+ \max\left(\left\lceil \frac{11-0}{10} \right\rceil 2 + \left\lceil \frac{11-6}{10} \right\rceil 1 \ , \ \left\lceil \frac{11-4}{10} \right\rceil 2 + \left\lceil \frac{11-0}{10} \right\rceil 1\right)$$

$$R_{ua}^{2} = 5 + \max(4 + 1 \ , \ 2 + 2) + \max(4 + 1 \ , \ 2 + 2)$$

$$R_{ua}^{2} = 5 + \max(5 \ , \ 4) + \max(5 \ , \ 4)$$

$$R_{ua}^{2} = 5 + 5 + 5$$

$$R_{ua}^{2} = 15$$

# Example: RTA with offsets using Approximate Analysis

Next iteration

$$R_{ua}^{3} = 5 + \max\left(\left\lceil \frac{15-0}{10} \right\rceil 2 + \left\lceil \frac{15-3}{10} \right\rceil 1 \ , \ \left\lceil \frac{15-7}{10} \right\rceil 2 + \left\lceil \frac{15-0}{10} \right\rceil 1\right)$$

$$+ \max\left(\left\lceil \frac{15-0}{10} \right\rceil 2 + \left\lceil \frac{15-6}{10} \right\rceil 1 \ , \ \left\lceil \frac{15-4}{10} \right\rceil 2 + \left\lceil \frac{15-0}{10} \right\rceil 1\right)$$

$$R_{ua}^{3} = 5 + \max(4+2 \ , \ 2+2) + \max(4+1 \ , \ 4+2)$$

$$R_{ua}^{3} = 5 + \max(6 \ , \ 4) + \max(5 \ , \ 6)$$

$$R_{ua}^{3} = 5 + 6 + 6$$

$$R_{ua}^{3} = 17$$

# Example: RTA with offsets using Approximate Analysis

Next iteration

$$R_{ua}^{4} = 5 + \max\left(\left\lceil \frac{17-0}{10} \right\rceil 2 + \left\lceil \frac{17-3}{10} \right\rceil 1 \ , \ \left\lceil \frac{17-7}{10} \right\rceil 2 + \left\lceil \frac{17-0}{10} \right\rceil 1\right)$$

$$+ \max\left(\left\lceil \frac{17-0}{10} \right\rceil 2 + \left\lceil \frac{17-6}{10} \right\rceil 1 \ , \ \left\lceil \frac{17-4}{10} \right\rceil 2 + \left\lceil \frac{17-0}{10} \right\rceil 1\right)$$

$$R_{ua}^{4} = 5 + \max(4+2 \ , \ 2+2) + \max(4+2 \ , \ 4+2)$$

$$R_{ua}^{4} = 5 + \max(6 \ , \ 4) + \max(6 \ , \ 6)$$

$$R_{ua}^{4} = 5 + 6 + 6$$

$$R_{ua}^{4} = 17 = R_{ua}^{3}$$

$$\boxed{R_{ua} = R_{31} = 17}$$

Response times of other tasks can be calculated in a similar way