Supplementary Methods

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Generation of minimum-jerk trajectories

The Minimum Jerk Model (MJM) is designed to maximize movement smoothness. Target trajectories were generated as the optimal paths defined by the MJM, which minimizes the following objective function (Flash & Hogan, 1985):

The MJM is intended to maximize the smoothness of the movement. Target lines were generated as the optimal trajectory of the MJM. The MJM is based on making an objective function its minimum value, as follows (Flash & Hogan, 1985).

$$C_J = \int_0^{t_f} \left\{ \left(\frac{d^3x}{dt^3}\right)^2 + \left(\frac{d^3y}{dt^3}\right)^2 \right\} dt \rightarrow min.$$

where t_f is the total duration of the movement and D_J represents the cost function (where J denotes jerk).

With defined endpoints and via-points, minimum-jerk trajectories (MJTs) can be determined as unique paths with specific speed profiles (Todorov & Jordan, 1998). To generate MJTs on a two-dimensional plane, the following functions of time and position (x, y) were used, incorporating via-points as described by Todorov & Jordan (1998):

For $\tau = t \,/\, t_f$,

$$x(t) = x_0 + (x_0 - x_f)(15\tau^5 - 10\tau^4 - 6\tau^3)$$

$$y(t) = y_0 + (y_0 - y_f)(15\tau^5 - 10\tau^4 - 6\tau^3)$$

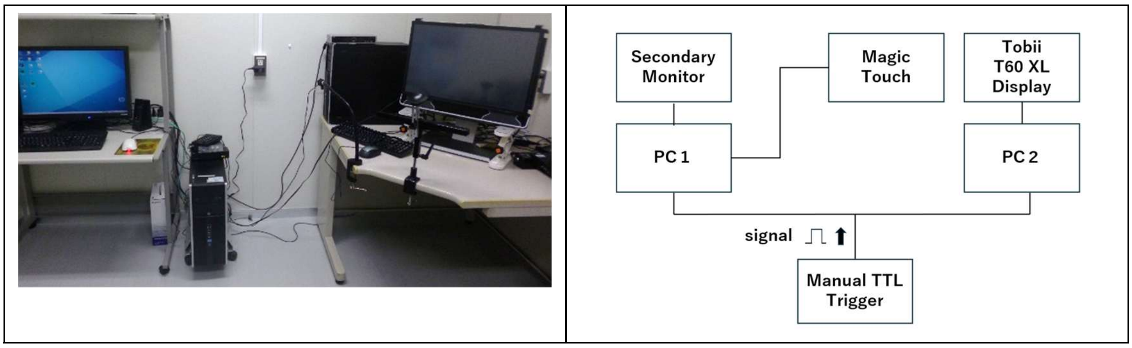
where (x_0, y_0) and (x_f, y_f) represent the start and final positions at t = 0 and t = t_f, respectively.

The target trajectories presented on the display included both straight lines and curves computed based on the minimum-jerk model. The time to pass each viapoint was varied in 1/1000 increments of the total movement duration, and the trajectory minimizing the cost function was selected. To ensure that all target trajectories had the same total length (800 pixels), corresponding to the shortest straight-line path along the x-axis, the curved trajectories were uniformly scaled while keeping the start point fixed. On the display, the total trajectory length was 254.4 mm, and the line width was approximately 8 mm.

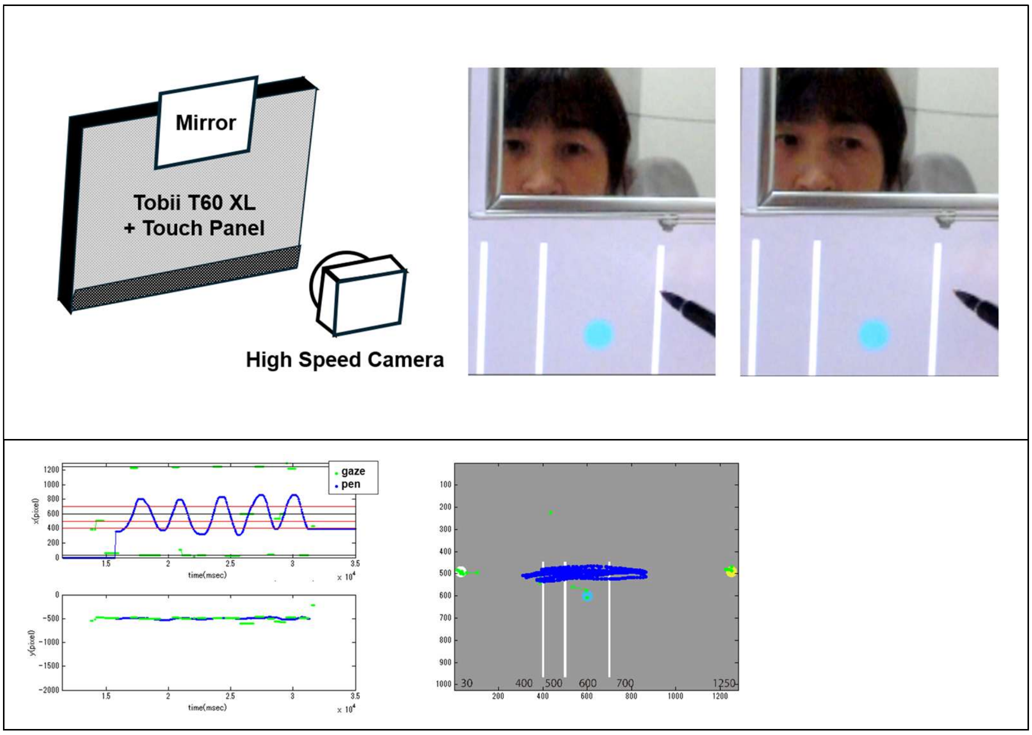
Straight Line: The starting points were fixed at two screen coordinates: (100, 350) and (100, 450). The end points were set at (900, 350), (900, 450), and (900, 550), resulting in six straight-line trajectories from all combinations of start and end points.
Single via-point, curved Line: For single-viapoint curves, the intermediate point was placed at either (500, 250) or (500, 650), forming U-shaped or inverted-U-shaped paths.
Two-viapoint curves: The first viapoint was placed at (350, 250) or (350, 650), and the second at (650, 250) or (650, 650), resulting in horizontally oriented S-shaped trajectories.

Synchronization between devices

For the pen measurements, a capacitive transparent touch panel (Magic Touch, KEYTEC Richardson, USA) was set over the Tobii T60 XL eye tracker with supportive material placed under each side to stabilize the monitor. The participants used a pen with a felt tip to trace directly over the touch panel. Monitors were placed between 50–55 cm from the participants' faces. As the participants traced the line, a thin black line appeared on the monitor for visualize the tracing line.



The eye tracker and touch-panel PC were synchronized by a square wave signal through a circuit constructed in-house. A high-speed camera was used to calibrate the recording time between the eye tracker and touch-panel PC according to the manufacturer's instructions (Tobii Technology, 2010). The high-speed camera revealed a time lag and a variance between the eye tracker and touch-panel PC timestamps.



A mirror was set over the touch-panel/eye-tracker display and captured with a high-speed camera. At the same time, pen X-Y position and gaze X-Y position were recorded. Using the camera image of eye movement and the recorded data to Tobii eye tracker, the time lag between Tobii and touch panel was estimated.

The estimated mean time lag was 88 ms, partly coming data buffer inside Magic Touch. After correction of the time lag, standard deviation of the variance between Tobii eye-tacker and Magic Touch was 6 ms.

We conducted additional experiments to confirm the accuracy of the pen and gaze position in our system; the accuracy of our measurements was equivalent to those reported by the manufacturer.

Practice Trials

Participants performed practice trials to familiarize themselves with the line-tracing task on the monitor. While the number of trials varied by individual, 50 trials were typically sufficient for most participants to achieve proficiency.

Experimental Procedure

The experiment consisted of three parts, each preceded by a set of practice trials.

Experiment 1: Participants traced lines at a natural, self-selected speed.

Experiment 2: Participants traced lines within a specified time limit.

Experiment 3: Participants observed a shape (e.g., "T", "H", or "+") on a white background (600 x 600 pixels; 192 x 192 mm) for 5 seconds. After a 1-second pause with a black screen, they were asked to reproduce the shape.

Note: Only the data from the second experiment was analyzed in this report.

The standard procedure of calibration of Tobii eye tracker was performed at the start of the experiment, and several times during the experiment to ensure the eye tracking is precise. At the beginning of experiment, participants were asked to set their hands in a resting position approximately 40 cm away from the display. After each line tracing, the participants were asked to place their arms back in the resting position. Each set of tracing consisted of 15 target lines. After training session of about 50 line-tracing (3 sets), depending on the participant, the main part of experiment started.

Target line was only shown for a restricted period (3 sec or 4.5 sec) for the speed control conditions, and participants were directed to start with newly shown line if they did not complete the old one. The total length of experiment varied between participants, ranging from 2hr to 4hr, including resting time, however, some participants did not have enough time in one day, and in such a case, they were asked to come on a separate day.

Data preprocessing

For collecting the data with enough quality for analysis, each of 30 target lines was presented multiple times and from them, trials with qualified data was used for analysis, such that the first trial that the gaze data covers more than 80% of the tracing was used for the analysis.

The saccade was defined in two ways. One was using the assuming an angular speed of eyes as described in (Holmqvist, Marcus et al. 2011). However, because of rapid movement of saccades frequently resulted in missing data. Then, if the fixation is shifted from one place to another in a narrow time window, we interpret that as a saccade.

For estimation of pen speed, X-Y pen position data was filtered with averaging filter of 5 points with no weight. We fitted the X-Y position of the measured pen and gaze to the position of original target line by least squares method. Distance is based on the length of path from the start point (left end) of the target line.

For few line cases, there were no trials that matched the criteria. In such a case, data from closely related target lines was replaced with the missing data. Those cases were approximately 1% of the total data and not affecting the interpretation of the result of the current report.

Robust fit generation of linear regression model

The standard Ordinary Least Squares (OLS) is sensitive to outliers, since those pesky data points that deviate significantly from the rest of the set and pull the regression line toward them. In the Statistics and Machine Learning Toolbox of MATLAB, with the robust fit option of *fitlm* function, the OLS is replaced with M-estimation, which is solved using the Iteratively Reweighted Least Squares (IRLS) method. Instead of treating every data point equally, the algorithm assigns a weight to each point based on its residual (the distance from the fitted line). By this algorithm, points far from the line get lower weights, effectively "down weighting" the outliers, so they don't distort the model.

Participants

Seventeen participants (14 females, 3 males; aged 20–56 years) took part in the study. They were categorized into two groups based on their calligraphy expertise:

Professional calligraphers (n = 7): Recruited from a local community of calligraphers, all of whom had professional experience as calligraphy instructors

No formal training (n = 10): Individuals with no specialized background in calligraphy.

Participants were recruited from the local community surrounding the research institute. All participants had no reported mental or neurological conditions and possessed normal or corrected-to-normal vision.

Table 1. Characteristics of the participants in the present study.

Gender	Age (y)	Daily habit
Professional calligrapher (*n* = 7)		
F	50	
F	43	
F	46	Design, ballet
F	56	
F	50	
F	39	Basketball
F	51	Tennis
No formal training (*n* = 10)		
F	45	Computer-based work
F	53	Reading, piano, gardening
M	30	

M	20	Computer games, singing
F	53	Table tennis, reading
F	21	Student
F	50	
M	28	Student, drawing, computer games
M	22	Student, teaching as a schoolteacher, piano, soccer
M	23	Computer games for 30 min every day

For participants where no specific daily habit was reported (e.g., F, 50; F, 43), the "Daily habit" field remains blank as no notable activities were identified during the interview.

References

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