Exercises Week 4

① Proving Set Properties (from 2023):

Show that $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$ without using Theorem 3.4.

Solution:

We prove the statement by showing that any element of the left set is also in the right set and vice versa. Let x be arbitrary.

$$
\begin{array}{lll}
 & x \in A \setminus (B \setminus C) & \\
\Leftrightarrow & x \in A \wedge \neg(x \in B \setminus C) & \text{def. } X \setminus Y \\
\Leftrightarrow & x \in A \wedge \neg(x \in B \wedge \neg(x \in C)) & \text{def. } X \setminus Y \\
\Leftrightarrow & x \in A \wedge (\neg(x \in B) \vee \neg\neg(x \in C)) & \text{de Morgan} \\
\Leftrightarrow & x \in A \wedge (\neg(x \in B) \vee x \in C) & \text{double neg.} \\
\Leftrightarrow & (x \in A \wedge \neg(x \in B)) \vee (x \in A \wedge x \in C) & \text{distributivity} \\
\Leftrightarrow & x \in A \setminus B \vee (x \in A \wedge x \in C) & \text{def. } X \setminus Y \\
\Leftrightarrow & x \in A \setminus B \vee x \in A \cap C & \text{def. } \cap \\
\Leftrightarrow & x \in (A \setminus B) \cup (A \cap C) & \text{def. } \cup
\end{array}
$$

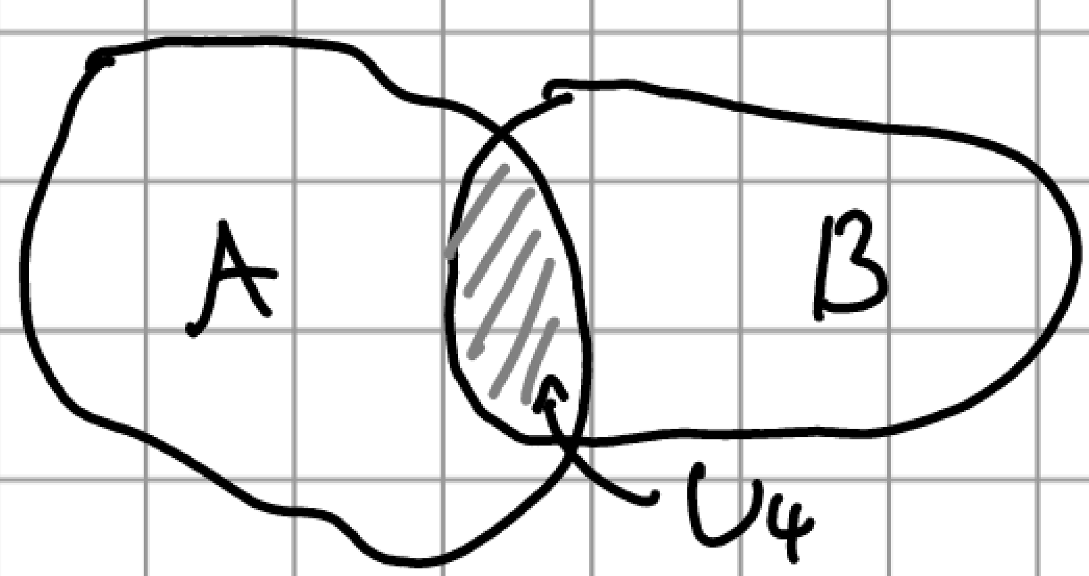
② Set Expressions:

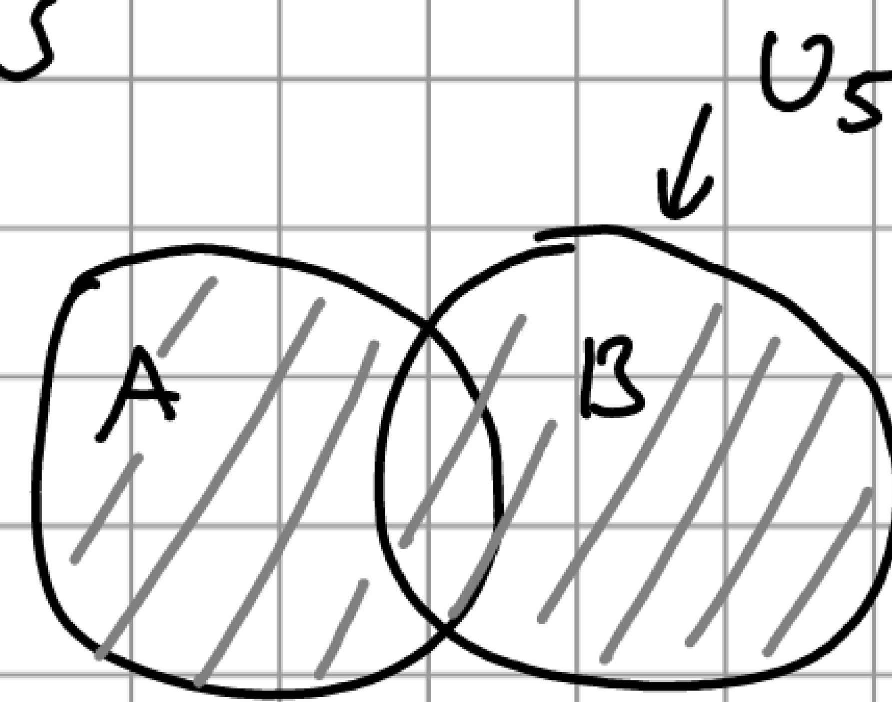
Express the following sets using $A, B, C, \cup, \cap, \setminus, P(A), P(B), P(C)$:

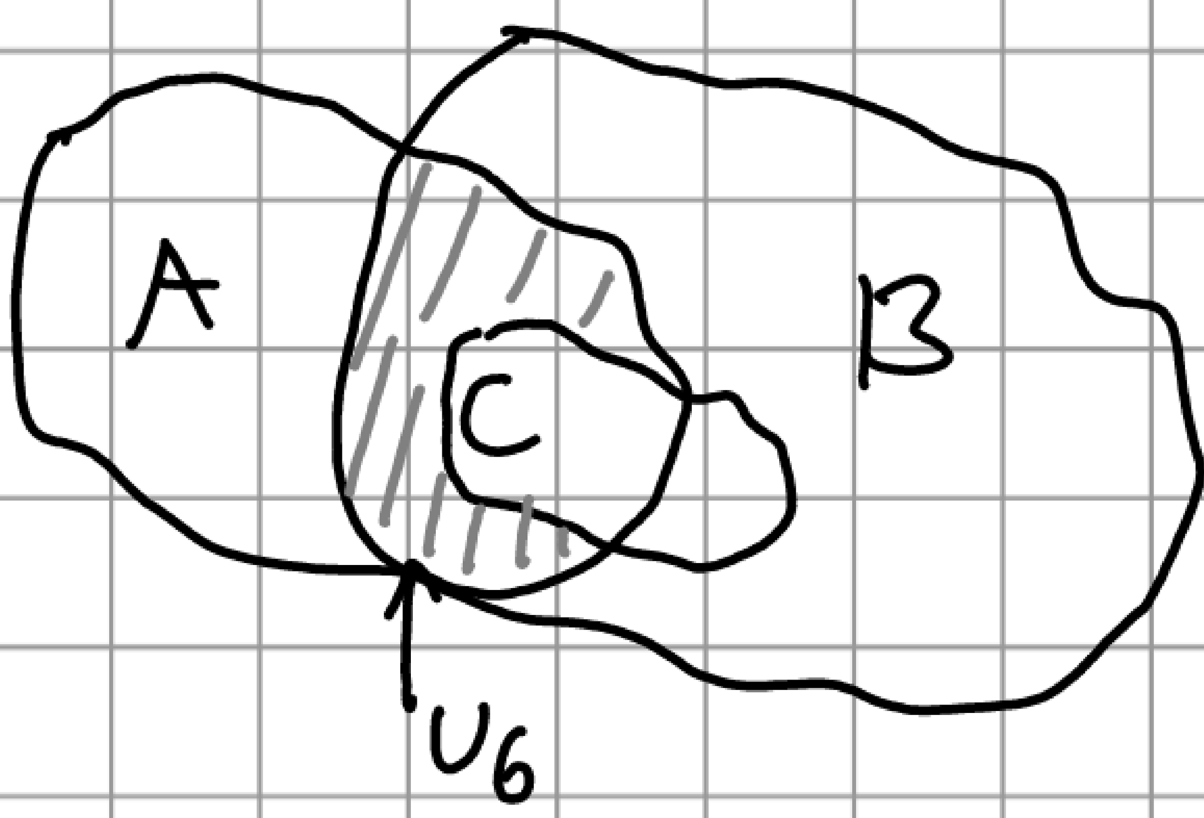
- U_1 contains all elements from A or B. But any set that is a subset of C should not be in U_1.

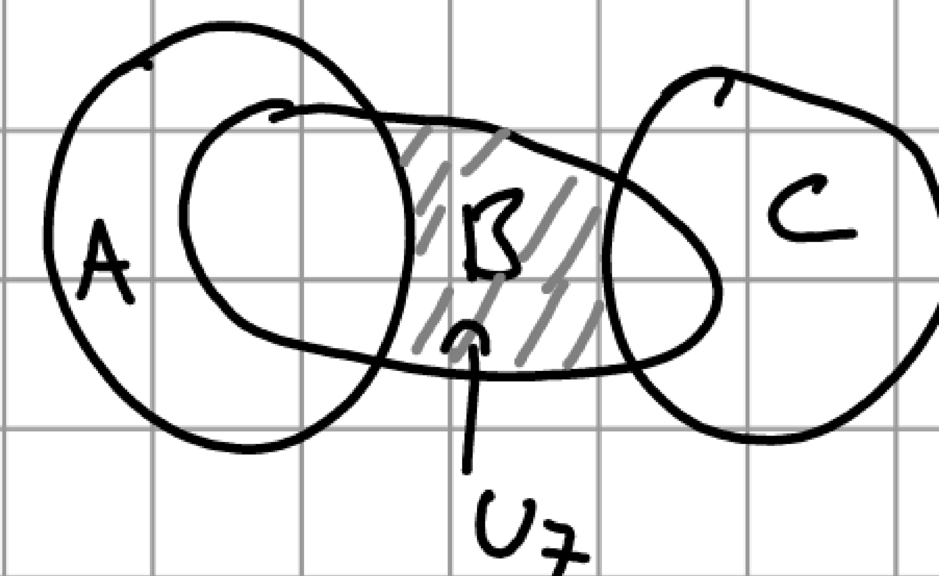
- $x \in U_2 \Leftrightarrow (x \in A \wedge x \in B) \vee x \in C$

- $U_3 = \{x \mid x \in C \land \neg(x \in B)\}$

- 

- 

- 

- 

Solution:

$U_1 = (A \cup B) \setminus P(C)$
$U_2 = (A \cup B) \cap C$
$U_3 = C \setminus B$
$U_4 = A \cap B$

$U_5 = A \cup B$
$U_6 = (A \cap B) \setminus C$
$U_7 = B \setminus (A \cup C)$

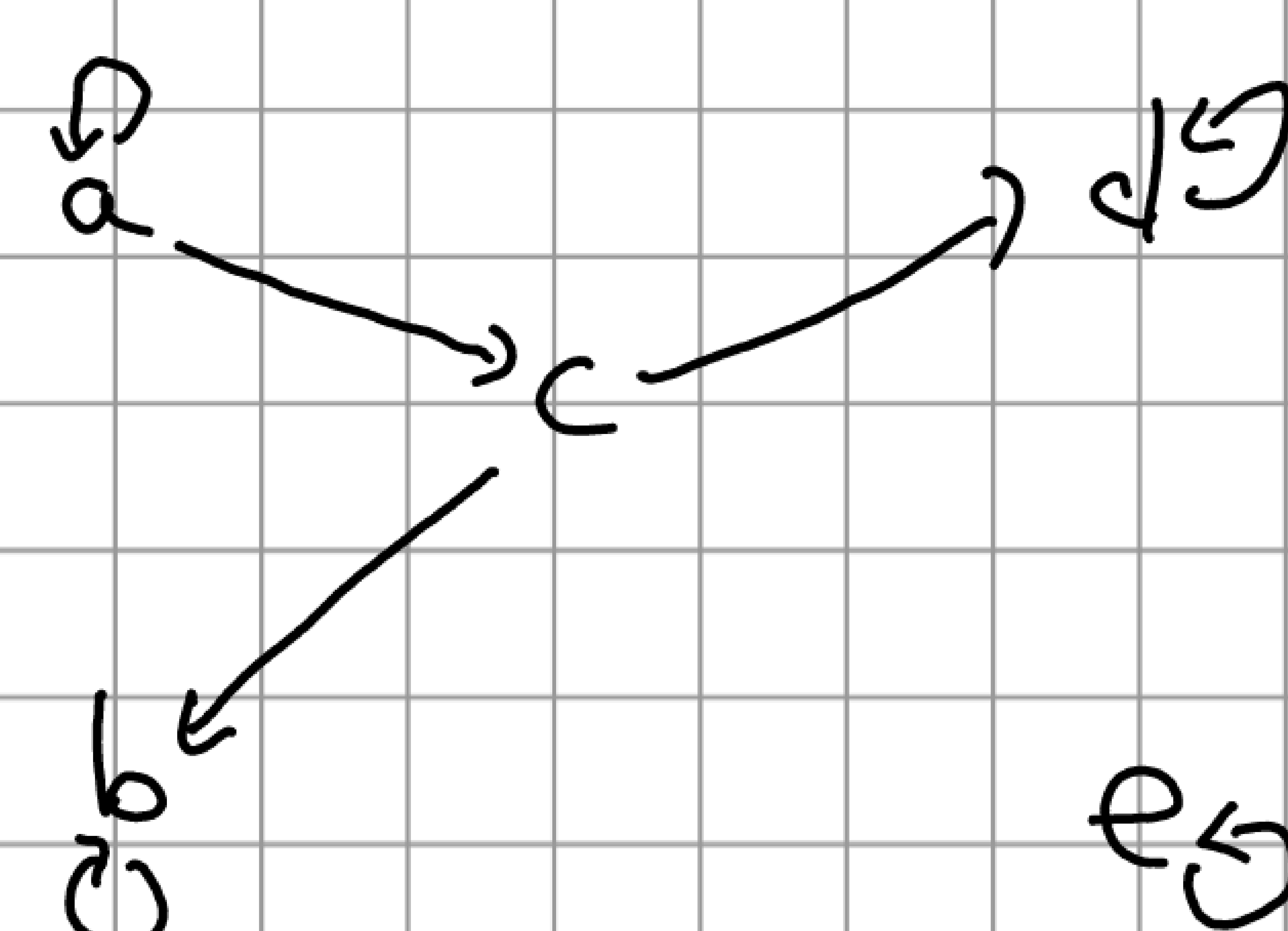
There might be other valid solutions!

3) Relations:

For the following relations, find out whether they are reflexive, transitive, symmetric, antisymmetric.

- p:

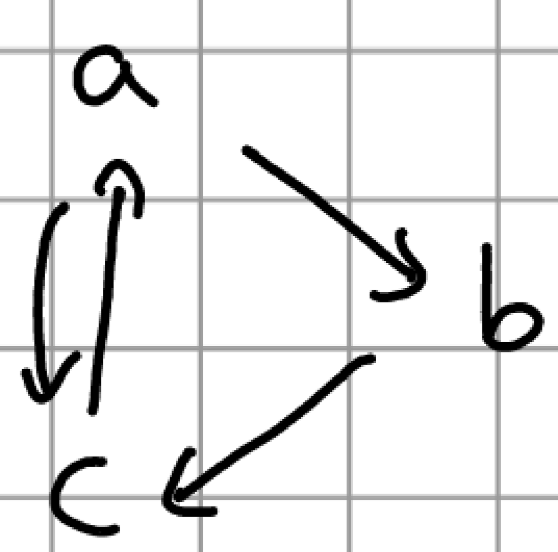
	a	b	c	d
a	1	0	1	0
b	0	1	0	1
c	1	0	1	0
d	0	1	0	1

- q: 

- $r = \{(a,b), (b,c), (a,c), (b,a), (c,b), (c,a), (a,a), (b,b), (c,c)\}$

$r \subseteq \{a, b, c\}^2$

- s:



Solutions:

p:
- ✓ reflexive
- ✓ symmetric
- ✓ transitive
- ✗ antisymmetric

q:
- ✓ reflexive
- ✗ symmetric
- ✗ transitive
- ✓ antisymmetric

r:
- ✓ reflexive
- ✓ symmetric
- ✓ transitive
- ✗ antisymmetric

s:
- ✗ reflexive
- ✗ symmetric
- ✗ antisymmetric
- ✗ transitive (because (a,c) and (c,a) are in s, but not (a,a))

Tipp: Draw relations like r as graphs so you can see the properties more easily.