Модели на реални процеси

Курсова работа 1 – ОДУ от I ред

спец.“Информатика“ 2021/2022

Изготвил : Емил Григоров Медаров

ФН : 2001262013

МОДЕЛИ НА РЕАЛНИ ПРОЦЕСИ

КУРСОВА РАБОТА 1 – ОДУ от I ред

спец. Информатика, 2021/2022

Да се реши **по един пример** от всяка задача.

Задача 1.

а) $y' \cos x + y(1+y) \sin x = 0$

б) $(x^2+1)y' - (2x+1)y = 0$

в) $ye^x \, dy + xe^{y^2} \, dx = 0$

г) $(1+y^2) \sin x \, dx - (1+\cos x) y \, dy = 0.$

Задача 2.

а) $y' = (x+y-1)^2$

б) $y' = \sin(x-y+1)$

в) $(y-3x+2) \, dx + (3x-y-1) \, dy = 0$

г) $(2y-x+1) \, dx + (4y-2x+6) \, dy = 0.$

Задача 3.

а) $xy' = y\left(1 + \ln \dfrac{y}{x}\right)$.

б) $xy' = \dfrac{x^2+y^2}{x+y}$

в) $x \, dy = \left(y - \sqrt{x^2+y^2}\right) dx, \quad x \geq 0$

г) $xy \, dx = (x^2-y^2) \, dy.$

Задача 4.

а) $(2x+y)y' = x+2y$

б) $(y-x)y' = x+y$

в) $(x+y-2)y' + x - y = 0$

г) $(2x-y-2) \, dx + (x+y-4) \, dy = 0.$

Задача 5.

а) $2x^3y' = 2x^2y - 3$

б) $y \, dx = (3x - y^2) \, dy$

в) $y' = y + 2xe^x$

г) $x^3y' + 2x^2y = 2 \ln x.$

Задача 6.

а) $4xy' + (4x+1)y^2 - 4y = 0$

б) $2xy' = 3y - 4xy^3$

в) $5xy^4y' = y^5 + 4$

г) $y\,dx + (2x^2y - 3x)\,dy = 0.$

Задача 7.

а) $y' = \ln \dfrac{y}{y'-1}$

б) $y = y' + \dfrac{1}{2}(x - \ln y')$

в) $\dfrac{1}{4}y'^2 - y' + y = 2x - 3$

г) $xy^4y' + 3y^5 + y'^4 = 0.$

Задача 8.

а) $y = xy' - e^{y'}$

б) $y - xy' = y'^4$

в) $y - xy' + y'\ln y' = 0$

г) $y = xy' - 3\sin^2 y'.$

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Да се реши по един пример от всяка задача.

Зад. 1.

а) $y' \cdot \cos x + y(1+y)\sin x = 0$

$\frac{dy}{dx} \cdot \cos x = -y(1+y) \cdot \sin x$

$\frac{dy}{y(1+y)} = -\frac{\sin x}{\cos x} dx$

$\int \frac{dy}{y(1+y)} = -\int \frac{\sin x}{\cos x} dx + C$

$\frac{1}{y(1+y)} = \frac{A}{y} + \frac{B}{1+y}$

С метод на неопр. коеф.

$1 = A + Ay + By$

$\begin{cases} A + B = 0 \\ A = 1 \end{cases} \quad B = -1$

$\frac{1}{y(1+y)} = \frac{1}{y} - \frac{1}{y+1}$

$$\int\left(\frac{1}{y}-\frac{1}{y+1}\right)dy = \int\frac{d(\cos x)}{\cos x} + C_1$$

$$\int\frac{dy}{y} - \int\frac{d(y+1)}{y+1} = \ln|\cos x| + \ln C$$

$$\ln|y| - \ln|y+1| = \ln C|\cos x|$$

$$\ln\frac{|y|}{|y+1|} = \ln C|\cos x|$$

$$\frac{|y|}{|y+1|} = C|\cos x|$$

б) $(x^2+1)y' - (2x+1)y = 0$

$$(x^2+1)\frac{dy}{dx} - (2x+1)y = 0$$

$$(x^2+1)dy = (2x+1)y \cdot dx$$

$$\frac{dy}{y} = \frac{2x+1}{x^2+1}dx$$

$$\int\frac{dy}{y} = \int\frac{2x+1}{x^2+1}dx + C$$

$$\ln|y| = \int\frac{2x}{x^2+1}dx + \int\frac{1}{x^2+1}dx + C$$

$$\ln|y| = \frac{2}{2}\cdot\int\frac{d(x^2+1)}{x^2+1} + \operatorname{arctg}x + C$$

$$\ln|y| = \ln(x^2+1) + \operatorname{arctg}x + \ln C$$

$$\ln|y| - \ln(x^2+1) - \ln C = \operatorname{arctg}x$$

$$\ln\frac{|y|}{C(x^2+1)} = \operatorname{arctg}x$$

$$\frac{|y|}{C(x^2+1)} = e^{\operatorname{arctg}x}$$

$$\frac{|y|}{e^{\operatorname{arctg}x}} = C(x^2+1),$$

Зад. 2

в) $(y-3x+2)dx + (3x-y-1)dy = 0$

$(3x-y-1)dy = -(y-3x+2)dx$

$$\frac{dy}{dx} = \frac{3x-y-2}{3x-y-1}$$

$$\Delta = \begin{vmatrix} 3 & -1 \\ 3 & -1 \end{vmatrix} = -3-3.(-1) = 0$$

Полагаме $3x - y = z$

$y = 3x - z$

$y' = 3 - z'$

$$y' = \frac{z-2}{z-1}$$

$$3 - z' = \frac{z-2}{z-1}$$

$$-z' = \frac{z-2}{z-1} - 3$$

$$-z' = \frac{z-2-3z+3}{z-1}$$

$$-z' = \frac{-2z+1}{z-1} \quad \Big| \cdot (-1)$$

$$z' = \frac{2z-1}{z-1}$$

$$\frac{dz}{dx} = \frac{2z-1}{z-1}, \quad \frac{(z-1)}{2z-1}dz = dx$$

$$\int \frac{z-1}{2z-1}dz = \int dx + C$$

1).

$$I = \frac{1}{2}\int \frac{2z \overset{-1-1}{+ 2}}{2z-1}\,dz = \frac{1}{2}\int \frac{\cancel{2z-1}}{\cancel{2z-1}}\,dz -$$

$$- \frac{1}{2}\int \frac{dz}{2z-1} = \frac{1}{2}z - \frac{1}{4}\int \frac{d(2z-1)}{2z-1} =$$

$$= \frac{1}{2}z - \frac{1}{4}\ln|2z-1|$$

$$\frac{1}{2}z - \frac{1}{4}\ln|2z-1| = x + C$$

$$\frac{1}{2}(3x-y) - \frac{1}{4}\ln|2(3x-y)-1| = x + C$$

Zad. 3

a) $xy' = y\left(1 + \ln\frac{y}{x}\right)$

$y' = \frac{y}{x}\cdot\left(1 + \ln\frac{y}{x}\right)$

$z = \frac{y}{x}\ ,\quad y = z\cdot x$

$y' = z + x\cdot z'$

$z + xz' = z(1+\ln z)$

$\cancel{z} + xz' = \cancel{z} + z\cdot\ln z$

$x\frac{dz}{dx} = z\cdot\ln z$

$\frac{dz}{z\cdot\ln z} = \frac{dx}{x}$

$\int\frac{dz}{z\ln z} = \int\frac{dx}{x} + C$

$\int\frac{d(\ln z)}{\ln z} = \ln x + \ln C$

$\ln(\ln z) = \ln Cx$

$\ln z = Cx$

$z = e^{Cx}$

$\frac{y}{x} = e^{Cx}$

$\underline{\underline{y = x\cdot e^{Cx}}}$

Зад. 4

г) $(2x-y-2)dx+(x+y-4)dy=0$

$$\frac{dy}{dx}=\frac{-(2x-y-2)}{x+y-4}$$

$$\Delta=\begin{vmatrix}2 & -1\\ 1 & 1\end{vmatrix}=2+1=3\neq 0$$

$$\begin{cases}2x-y-2=0\\ x+y-4=0\end{cases}$$

$3x-6=0$

$x=2$ $\qquad$ $y=2$

$$(2;2)$$
$$\downarrow\quad\downarrow$$
$$x_0\quad y_0$$

$x=u+x_0=u+2$

$y=v+y_0=v+2$

$$\frac{dy}{dx}=\frac{d(v+2)}{d(u+2)}=\frac{dv}{du}=v'$$

$$\frac{dv}{du}=-\frac{(2u+4-v-2-2)}{u+2+v+2-4}$$

$$\frac{dv}{du}=\frac{-2u+v}{u+v}$$

$$\frac{dv}{du}=\frac{\cancel{u}\left(-2+\frac{v}{u}\right)}{\cancel{u}\left(1+\frac{v}{u}\right)}$$

Полагаме $\frac{v}{u} = z$ $\quad v = u \cdot z$

$v' = z' \cdot u + z \cdot u'$

$v' = z' \cdot u + z$

$z' \cdot u + z = \frac{-2 + z}{1 + z}$

$z' \cdot u = \frac{z-2}{z+1} - z$

$z+1$

$z' \cdot u = \frac{\cancel{z} - 2 - z^2 - \cancel{z}}{z-1}$

$z' \cdot u = -\frac{z^2+2}{z+1}$

$\frac{dz}{du} = -\frac{z^2+2}{z+1} \cdot \frac{1}{u}$

$\int \frac{z+1}{z^2+2}\,dz = -\int \frac{du}{u} + \ln C$

$\int \frac{z\,dz}{z^2+2} + \int \frac{1}{z^2+2}\,dz = -\ln u + \ln C$

$\frac{1}{2}\int \frac{d(z^2+2)}{z^2+2} + \int \frac{dz}{z^2+2} = \ln \frac{C}{u}$

$\frac{1}{2}\ln(z^2+2) + \frac{1}{\sqrt{2}}\,\mathrm{arctg}\frac{z}{\sqrt{2}} = \ln\frac{C}{u}$

$z = \frac{v}{u} = \frac{y-2}{x-2}$

$\frac{1}{2}\ln\left(\left(\frac{y-2}{x-2}\right)^2 + 2\right) + \frac{1}{\sqrt{2}}\,\mathrm{arctg}\frac{y-2}{\sqrt{2}(x-2)} = \ln\frac{C}{x-2}$

Зад. 5

a) $2x^3y' = 2x^2y - 3$

$2x^3y' - 2x^2y = -3 \mid : 2x^3$

$y' - \frac{1}{x}y = -\frac{3}{x^3}$

$y = e^{-\int -\frac{1}{x}dx}\left(C + \int \frac{-3}{x^3} e^{\int -\frac{1}{x}dx} dx\right)$

$y = e^{\ln x}\left(C - 3\int \frac{1}{x^3} \cdot e^{-\ln x} dx\right)$

$y = x\left(C - 3\int \frac{1}{x^3} \cdot x^{-1} dx\right.$

$y = x\left(C - 3\int \frac{1}{x^4} dx\right) = x\left(C - 3\int x^{-4} dx\right) =$

$= x\left(C - 3 \cdot \frac{x^{-3}}{-3}\right) = x\left(C + \frac{1}{x^3}\right)$

б) $y' = y + 2x \cdot e^x$

$y' - \underbrace{1}_{a} \cdot y = \underbrace{2x \cdot e^x}_{b}$

$y = e^{-\int(-1)dx} \cdot \left[C + \int 2x \cdot e^x \cdot e^{\int(-1)dx} dx\right]$

$y = e^x \cdot \left[C + \int 2x \cdot e^x \cdot e^{-x} dx\right]$

$y = e^x\left[C + \int 2x\, dx\right]$

$y = e^x\left[C + \cancel{2} \cdot \frac{x^2}{\cancel{2}}\right)$

$\ln \frac{C}{x-2}$

$y = e^x\left(C + x^2\right)$

Зад. 6

б) $2xy' = 3y - 4xy^3$

$2xy' = 3y - 4xy^3 \; | : 2x$

$y' = \frac{3}{2x} \cdot y - 2y^3$ $\qquad m = 3$

$y = 0$ е решение

$\frac{y'}{y^3} = \frac{3}{2x} \cdot \frac{y}{y^3} - 2$

$\frac{\cancel{y}}{y^3} = z \quad \Big| \quad z' = (y^{-2})' = -2y^{-3} y'$

$\frac{1}{y^2} = z \quad \Big| \quad z' = \frac{-2}{y^3} y'$

$\frac{y'}{y^3} = \frac{-z'}{2}$

$\frac{-z'}{2} = \frac{3}{2x} z - 2 \; | \cdot 2$

$-z' = \frac{3}{x} \cdot z - 1 \; | \cdot (-1)$

$z' = \underbrace{-\frac{3}{x}}_{A} \cdot z + \underbrace{1}_{B}$

$z = e^{\int -\frac{3}{x} dx} \left(C + \int 1 \cdot e^{-\int -\frac{3}{x} dx} dx \right)$

(Продължение)

Зад. 6 б)

$$z = e^{-3\ln|x|}\left(C + \int e^{3\ln|x|}dx\right)$$

$$= e^{\ln|x|^{-3}}\left(C + \int e^{\ln|x|^{3}}dx\right)$$

$$= |x|^{-3}\left(C + \int |x|^3 dx\right)$$

$$= \frac{1}{|x|^3}\left(C + \frac{|x|^4}{4}\right)$$

$$\frac{1}{y^2} = \frac{1}{|x|^3}\left(C + \frac{x^4}{4}\right)$$

Зад. 7

a) $y' = \ln\frac{y'}{y'-1}$

б) $y = y' - \frac{1}{2}(x - \ln y')$

Полагаме $y' = p(x)$

$\Rightarrow y = p + \frac{1}{2}x - \frac{1}{2}\ln p$

$y' = p' + \frac{1}{2} - \frac{1}{2}\frac{1}{p}p'$

$p - \frac{1}{2} = p'\left(1 - \frac{1}{2p}\right)$

$\frac{2p-1}{\cancel{2}} = \frac{2p-1}{\cancel{2}p}\frac{dp}{dx}$

Ако $2p - 1 \neq 0$ $\quad p \neq \frac{1}{2}$

$dx = \frac{dp}{p}$

$$\int \frac{dp}{p} = \int dx + C$$

$$\ln p = x + C$$

$$p = e^{x+C}$$

$$y' = e^{x+C}$$

$$\frac{dy}{dx} = e^{x+C}$$

$$dy = e^{x+C} dx$$

$$\int dy = \int e^{x+C} d(x+C)$$

$$\underline{\underline{y = e^{x+C}}}$$

Ако $2p - 1 = 0$

$$p = \frac{1}{2}$$

$$y' = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

$$dy = \frac{1}{2} dx$$

$$\int dy = \int \frac{1}{2} dx$$

$$\underline{\underline{y = \frac{1}{2} x + C}}$$

Зад. 8

а) $x y' - \underbrace{e^{y'}}_{f(p)}$ – у-е на Клеро

Полагаме $y' = p \quad p = p(x)$

$\Rightarrow y = x.p - e^{p}$

$y' = x'p + xp' + e^{p}.p'$

$\cancel{p} = 1\cancel{p} + xp' - e^{p}.p'$

$0 = p'(x - e^{p})$

$p' = 0$

$p = C$

$y = Cx + f(C)$

$y = Cx + (-e^{C})$

$\underline{y = Cx - e^{C}}$

Общо реш.

$x - e^{p} = 0$

$x = e^{p} \Rightarrow p = \ln x$

$y = xp + f(p)$

$y = xp + (-e^{p})$

$y = x.\ln x - e^{\ln x}$

$\underline{y = x.\ln x - x}$

особено реш.

б) $y - xy' = y'^4$

$y = xy' + \underbrace{y'^4}_{f} \quad y' = p$

$y = xp + p^4 \; ; \; y' = x'p + xp' + 4p^3p'$

$$y' = p + xp' + 4p^3p'$$

$$\cancel{p} = \cancel{p} + xp' + 4p^3p'$$

$$0 = p'(x + 4p^3)$$

$$\swarrow \qquad \searrow$$

$$p' = 0$$

$$p = c$$

$$y = cx + f(c)$$

$$\underline{\underline{y = cx + c^4}}$$

$$x + 4p^3 = 0$$

$$x = -4p^3$$

$$y = xp + f(p)$$

$$p^3 = -\frac{x}{4}$$

$$p = -\sqrt[3]{\frac{x}{4}}$$

$$y = x\left(-\sqrt[3]{\frac{x}{4}}\right) + \left(-\sqrt[3]{\frac{x}{4}}\right)^4$$

$$y = -x\sqrt[3]{\frac{x}{4}} + \frac{x}{4}\sqrt[3]{\frac{x}{4}}$$

$$= \left(-x + \frac{x}{4}\right)\sqrt[3]{\frac{x}{4}} =$$

$$\underline{\underline{y = -\frac{3x}{4}\sqrt[3]{\frac{x}{4}}}}$$