Изпит по Математически анализ
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Зад. 1

а) $\lim\limits_{n\to\infty} \frac{4^n+5^{n+1}}{6^n+7^{n-1}} = \lim\limits_{n\to\infty} \frac{4^n+5^n.5}{6^n+7^n.7} =$

$= \lim\limits_{n\to\infty} \frac{5^n\left(\frac{4^n}{5^n}+1.5\right)}{7^n\left(\frac{6^n}{7^n}+1.7\right)} = \lim\limits_{n\to\infty} \left(\frac{5}{7}\right)^n . \frac{\frac{4^n}{5^n}+5}{\frac{6^n}{7^n}+7} =$

$= \lim\limits_{n\to\infty} \frac{5^n}{7^n} . \frac{\frac{4^n}{5^n}+5}{\frac{6^n}{7^n}+7} = \frac{\infty}{\infty} . \frac{5}{7} = 0$

б) $\lim\limits_{n\to\infty} \left(\frac{n+4}{n+3}\right)^n = \lim\limits_{n\to\infty}$

$= \left(\frac{n+4}{n+4-1}\right)^n = \lim\limits_{n\to\infty} \left(1+\frac{1}{n+4}\right) = e^1$

в) $\lim\limits_{x\to 1} \frac{\sin \pi x}{x-1} = \left[\frac{0}{0}\right] = \left[y = \pi x \to x = \frac{y}{\pi}\right]$

$= \lim\limits_{y\to 0} \frac{\sin y}{\frac{y}{\pi}-1} = \frac{1}{\frac{y}{\pi}-1}$

в)

Зад. 2

$\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{2^n} =$

$= \lim\limits_{n\to\infty} \left(\left(\frac{-1}{2}\right)\right)^{\frac{1}{n}}$

Използваме признака на ~~Коши~~ Раабе-Дюамел

Зад. 3

а) $y = -2\arccos\sqrt{x}$

$y' = (-2)\cdot(\arccos\sqrt{x})' = -2 \cdot \frac{1}{\sqrt{1-x^2}} \cdot (x)' =$

$y' = \frac{-2}{\sqrt{1-x^2}}$

б) $y = \left(\sin x + \frac{1}{x^2}\right) tgx =$

$= \left(\sin x + \frac{1}{x^2}\right)' tgx + \left(\sin x + \frac{1}{x^2}\right) tgx' =$

$= \left[\cos x \cdot (x)' + (x^{-2})'\right] tgx + \left(\sin x + \frac{1}{x^2}\right) \frac{1}{\cos^2 x} \cdot (x)' =$

$= \left[\cos x + (-2) \cdot x^{-2-1}\right] tgx + \left(\sin x + \frac{1}{x^2}\right) \frac{1}{\cos^2 x} =$

$= \left[\cos x - 2x^{-3}\right] tgx + \left(\sin x + \frac{1}{x^2}\right) \frac{1}{\cos^2 x} =$

$= \left(\cos x - \frac{1}{2x^3}\right) tgx + \left(\sin x + \frac{1}{x^2}\right) \frac{1}{\cos^2 x} =$

$= \cos x\, tgx - \frac{tgx}{2x^3} + \frac{\sin x}{\cos^2 x} + \frac{1}{x^2} \cdot \frac{1}{\cos^2 x}$

b) $y = \frac{\cos x}{e^{2x}+2}$ $y' = \frac{(\cos x)'(e^{2x}+2) + (\cos x)(e^{2x}+2)'}{(e^{2x}+2)^2} =$

$= \frac{(-\sin x)(e^{2x}+2) + (\cos x)(2e^{2x}+0)}{(e^{2x}+2)^2} =$

$= \frac{-\sin x e^{2x} - 2\sin x + \cos x 2e^{2x}}{(e^{2x}+2)^2}$

Зад. 4

$f(x) = \frac{x^3}{(2-x)^2}$, $x_0 = 6$

~~$f'(x) = \left(\frac{3x^2}{(2-x)^2}\right)'$~~ $\frac{36,6}{216}$

~~$f'(\pi)$~~

$f'(x) = \left(\frac{3x^3}{(2-x)^2}\right)' = \frac{3x^2}{(2-x)^2} = \frac{3.(6)^3}{(2-6)^2} =$

$= \frac{216}{(-4)^2} = \frac{216}{16}$

$f'(\pi) = \frac{3\pi^3}{(2-\pi)^2} = 3$

$f(\pi) = \frac{\pi^3}{(2-\pi)^2} = 1$

$f'(\pi)(x-\pi)f'(\pi) = 3(x-\pi)1$

Зад. 5

$f(x) = x^3+6x^2+9x$; $[0;3]$ $\boxed{D(-\infty;+\infty)}$

$f'(x) = 3x^2+12x+9$

$y' = 0$

$3x^2+12x+9=0 \mid :3$

$x^2+4x+9=0$

$\cancel{x_1}$ $(x-1)(x+9)=0$

$x_1=1$ $x_2=9$

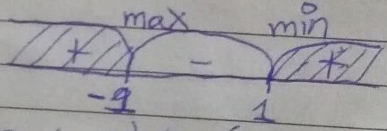
$y' > 0$

$3x^2+12x+9$

$x^2+4x+9=0$

$(x-1)(x+9)=0$

$x_1=1$ $x_2=-9$



$x \in (-\infty;-9) \cup (1;+\infty)$ $y\nearrow$ расте

$y' < 0$

$(x-1)(x+9) = 0$

$x_1=1$ $x_2=-9$

$x \in (-9;1)$ $y\swarrow$ намалява

$f(\overset{-9}{max}) = (-9)^3+6(-9)^2+9\cdot(-9) = -729+(-486)+$
$+(-81) = -729-486-81 = -1296$

$f(\overset{1}{min}) = 1+6+9 = 16$

$y(0) = 9$

y НМС $= 9$

$y(3) = 27+36+9 = 62$

y НГС $= 62$

Зад. 6

а) $\int \frac{\ln^2 x}{x} dx = \int \frac{\frac{1}{x}}{x} dx =$

б) $\int \frac{x^2+3}{x^2-1} dx = \int \frac{2x+3}{2x-1} dx =$

$=$

в) $\int \frac{dx}{x^2-5x+6} dx = \int \frac{dx}{(x+2)(x-3)} = \int$