Курсова работа 1 по Геометрия

За специалност „Информатика“

2 курс, задочно обучение

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КУРСОВА РАБОТА 1 по ГЕОМЕТРИЯ за специалност ИНФОРМАТИКА II курс задочно обучение

1. За кривата $c : \vec{r} = \left(\sqrt{2}\ln u, \frac{1}{u}, u\right)$ да се намери:

а) естествената параметризация;

б) дължината на дъгата, между равнините $\alpha\colon y = 1$ и $\beta\colon y = 2$;

в) уравнението на нормалната равнина за точка $P(u = 1) \in c$.

2. Да се пресметнат кривината и торзията на следните криви:

а) $c : \vec{r} = \left(u, \sqrt{u}, u^2\right)$ в произволна точка.

б) $c : \vec{r} = (\ln u, u^2, 2u)$ в точката $P(u = 1) \in c$.

3. Дадени са две C^0-непрекъснати криви в точката $P(-1,2)$ $\vec{f}(u) = (sinu - 1, 1 - cosu)$ и $\vec{g}(v) = (-v - 1, 2 + v^2), v \in [0,1]$. Да се изследва съставната крива за C^1-, C^2-, G^1-, G^2-непрекъснатост, както и за кривинна непрекъснатост в точката на съединяване.

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Зад. 1

$c: \vec{r} = \left(\sqrt{2}\ln u, \frac{1}{u}, u\right) \qquad u > 0$

$\dot{\vec{r}} = \left(\sqrt{2}\cdot\frac{1}{u}; -\frac{1}{u^2}; 1\right)$

$\dot{\vec{r}}^2 = \frac{2}{u^2} + \frac{1}{u^4} + 1 = \frac{2u^2+1+u^4}{u^4} =$

$= \frac{(u^2+1)^2}{u^4}$

$|\dot{\vec{r}}| = \sqrt{\dot{\vec{r}}^2} = \frac{u^2+1}{u^2}$

$S(u) = \int_{u_0=1}^{u} \frac{u^2+1}{u^2}\,du = \int_{u_0=1}^{u}\left(1+\frac{1}{u^2}\right)du =$

$= \int_{u_0=1}^{u} du + \int_{u_0=1}^{u}\frac{1}{u^2}\,du = u\Big|_1^u + \int_{u_0=1}^{u} u^{-2}\,du =$

$= u - 1 + \frac{u^{-1}}{-1}\Big|_1^u = u - 1 - \frac{1}{u}\Big|_1^u =$

$= u - 1 - \left(\frac{1}{u} - 1\right) = u - \cancel{1} - \frac{1}{u} + \cancel{1} = u - \frac{1}{u}$

$S = u - \frac{1}{u}, \quad Su = u^2 - 1$

$u^2 - Su - 1 = 0$

$u_{1,2} = \frac{S \pm \sqrt{S^2+4}}{2} \qquad u > 0$

$\Rightarrow u = \frac{S + \sqrt{S^2+4}}{2}$

=> Естествената параметризация на C има вида

$$C: \vec{r}\left(\sqrt{2}\ \ln\frac{s+\sqrt{s^2+4}}{2}\ ;\ \frac{2}{s+\sqrt{s^2+4}}\ ;\ \frac{s+\sqrt{s^2+4}}{2}\right)$$

б) Нека C пресича равнините α и β в точките A и B

$y = 1 \qquad \frac{1}{u} = 1 \qquad u = 1$

$y = 2 \qquad \frac{1}{u} = 2 \qquad \frac{1}{2} = u$

$$S(A,B) = \int_{\frac{1}{2}}^{1} \frac{u^2+1}{u^2}\,du = \int_{\frac{1}{2}}^{1}\left(1+\frac{1}{u^2}\right)du =$$

$$= \left(u - \frac{1}{u}\right)\Big|_{\frac{1}{2}}^{1} = 1 - 1 - \left(\frac{1}{2} - \frac{1}{\frac{1}{2}}\right) =$$

$$= -\frac{1}{2} + 2 = 1\frac{1}{2} = 1{,}5$$

в) $C: \vec{r}\left(\sqrt{2}\ln u\ ;\ \frac{1}{u}\ ;\ u\right)$

$\vec{r}'\left(\frac{\sqrt{2}}{u}\ ;\ -\frac{1}{u^2}\ ,\ 1\right)$

$$|\vec{r}'| = \sqrt{\left(\frac{\sqrt{2}}{u}\right)^2 + \left(-\frac{1}{u^2}\right)^2 + 1^2} = \sqrt{\frac{2}{u^2} + \frac{1}{u^4} + 1} =$$

$$= \sqrt{\frac{2u^2+1+u^4}{u^4}} = \sqrt{\frac{(u^2+1)^2}{u^2}} = \frac{u^2+1}{u^2}$$

Първият вектор от триедъра на Френе е единичният доп. вектор $\vec{t}$:

$$\vec{t} = \frac{\vec{r}'}{|\vec{r}'|} = \frac{1}{\frac{u^2+1}{u^2}}\left(\frac{\sqrt{2}}{u} ; -\frac{1}{u^2} ; 1\right) =$$

$$= \frac{u^2}{u^2+1}\left(\frac{\sqrt{2}}{u} ; -\frac{1}{u^2} ; 1\right) =$$

$$= \left(\frac{\sqrt{2}\,u}{u^2+1} ; -\frac{1}{u^2+1} ; \frac{u^2}{u^2+1}\right)$$

Единичният ~~б~~ бинормален вектор $\vec{b}$

$$\vec{b} = \frac{\vec{r}' \times \vec{r}''}{|\vec{r}' \times \vec{r}''|}$$

$$\vec{r}''\left(-\frac{\sqrt{2}}{u^2} ; \frac{2}{u^3} ; 0\right)$$

$$\vec{r}'\left(\frac{\sqrt{2}}{u} ; -\frac{1}{u^2} ; 1\right)$$

$$\vec{r}''\left(-\frac{\sqrt{2}}{u^2} ; \frac{2}{u^3} ; 0\right)$$

$$\vec{r}' \times \vec{r}''\left(-\frac{2}{u^3} ; -\frac{\sqrt{2}}{u^2} ; \frac{\sqrt{2}}{u^4}\right)$$

$$|\vec{r}' \times \vec{r}''| = \sqrt{\frac{4}{u^6} + \frac{2}{u^4} + \frac{2}{u^8}} = \sqrt{\frac{4u^2+2u^4+2}{u^8}} =$$

$$= \sqrt{\frac{2(u^4+2u^2+1)}{u^8}} = \sqrt{\frac{2(u^2+1)^2}{u^8}} =$$

$$= \frac{(u^2+1)}{u^4} \cdot \sqrt{2}$$

$$\vec{b} = \frac{\vec{r}' \times \vec{r}''}{|\vec{r}' \times \vec{r}''|} = \frac{u^4}{\sqrt{2}(u^2+1)}\left(-\frac{2}{u^3} ; -\frac{\sqrt{2}}{u^2} ; \frac{\sqrt{2}}{u^4}\right) =$$

$$= \left(-\frac{\sqrt{2}\,u}{u^2+1} ; -\frac{u^2}{u^2+1} ; \frac{1}{u^2+1}\right)$$

$$\vec{n} = \vec{b} \times \vec{t}$$

$$\vec{b}\left(-\frac{\sqrt{2}u}{u^2+1};\ -\frac{u^2}{u^2+1};\ \frac{1}{u^2+1}\right)$$

$$\vec{t}\left(\frac{\sqrt{2}u}{u^2+1};\ -\frac{1}{u^2+1};\ \frac{u^2}{u^2+1}\right)$$

$$\vec{b}\times\vec{t}\left(\frac{-u^4}{(u^2+1)^2}+\frac{1}{(u^2+1)^2};\ \frac{\sqrt{2}u}{(u^2+1)^2}+\frac{\sqrt{2}u^3}{(u^2+1)^2};\ \frac{\sqrt{2}u}{(u^2+1)^2}+\frac{\sqrt{2}u^3}{(u^2+1)^2}\right) =$$

$$=\left(\frac{(1-u^2)\cancel{(1+u^2)}}{(u^2+1)^{\cancel{2}}};\ \frac{\sqrt{2}u\cancel{(1+u^2)}}{(u^2+1)^{\cancel{2}}};\ \frac{\sqrt{2}u\cancel{(1+u^2)}}{(u^2+1)^{\cancel{2}}}\right)$$

$$\Rightarrow \vec{n}\left(\frac{1-u^2}{u^2+1};\ \frac{\sqrt{2}u}{u^2+1};\ \frac{\sqrt{2}u}{u^2+1}\right)$$

$$\vec{t}(1) = \left(\frac{\sqrt{2}}{2};\ -\frac{1}{2};\ \frac{1}{2}\right)$$

$$\vec{n}(1) = 0;\ \frac{\sqrt{2}}{2};\ \frac{\sqrt{2}}{2})$$

$$\vec{b}(1) = \left(-\frac{\sqrt{2}}{2};\ -\frac{1}{2};\ \frac{1}{2}\right)$$

$$\text{T. } P\left(\sqrt{2}\,.\,\ln 1;\ \frac{1}{1};\ 1\right)$$

$$\text{T. } P(\underset{x_0}{0};\ \underset{y_0}{1};\ \underset{z_0}{1})$$

$$\vec{t}\left(\frac{\sqrt{2}}{2}\to A;\ -\frac{1}{2}\to B;\ \frac{1}{2}\to C\right)$$

Използваме ур-ето

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0$$

$$\frac{\sqrt{2}}{2}(x-0) - \frac{1}{2}(y-1) + \frac{1}{2}(z-1) = 0 \quad |\cdot 2$$

$$\Rightarrow \sqrt{2}x - y + 1 + z - 1 = 0$$

$$\underline{\underline{\sqrt{2}x - y + z = 0}}$$

Зад. 2

a) $c: \vec{r} = (u, \sqrt{u}, u^2)$

$$\dot{\vec{r}} = \left(1, \frac{1}{2\sqrt{u}}, 2u\right) \qquad \ddot{\vec{r}}\left(0; \frac{1' \cdot 2\sqrt{u} - 1 \cdot (2\sqrt{u})'}{(2\sqrt{u})^2}; 2\right)$$

$$\ddot{\vec{r}} = \left(0; -2 \cdot \frac{1}{2\sqrt{u}}; 2\right)$$

$$\ddot{\vec{r}} = \left(0; -\frac{1}{4u\sqrt{u}}; 2\right)$$

$$\dddot{\vec{r}} = 0; \frac{0-(-1)\cdot 4 \cdot \frac{3}{2}u^{\frac{1}{2}}}{16u^3}; 0)$$

$$\dddot{\vec{r}} = \left(0; \frac{3\sqrt{u}}{8u^3}; 0\right)$$

$$|\dot{\vec{r}}| = \sqrt{1^2 + \left(\frac{1}{2\sqrt{u}}\right)^2 + (2u)^2} = \sqrt{1 + \frac{1}{4u} + 4u^2} =$$

$$= \sqrt{\frac{4u + 1 + 16u^3}{4u}}$$

$$\dot{\vec{r}} \times \ddot{\vec{r}} = \left(\frac{1}{2\sqrt{u}} \cdot 2 + \frac{\overset{1}{\cancel{2}}u}{\underset{2}{\cancel{4}}u\sqrt{u}}; 2u \cdot 0 - 2 \cdot 1; 1 \cdot \left(-\frac{1}{4u\sqrt{u}}\right) - 0\right)$$

$$\left.\begin{array}{l} \left(1; \frac{1}{2\sqrt{u}}; 2u\right) \\ \left(0; -\frac{1}{4u\sqrt{u}}; 2\right) \end{array}\right| = \left(\frac{3}{2\sqrt{u}}; -2; -\frac{1}{4u\sqrt{u}}\right)$$

$$|\vec{r}\,' \times \vec{r}\,''| = \sqrt{\left(\frac{3}{2\sqrt{u}}\right)^2 + (-2)^2 + \left(-\frac{1}{4u\sqrt{u}}\right)^2} =$$

$$= \sqrt{\frac{9}{4u} + 4 + \frac{1}{16u^3}} = \sqrt{\frac{36u^2 + 64u^3 + 1}{16u^3}}$$

$$\varkappa = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3} = \frac{\sqrt{\dfrac{36u^2 + 64u^3 + 1}{16u^3}}}{\left(\sqrt{\dfrac{4u + 1 + 16u^3}{4u}}\right)^3} =$$

$$= \frac{\dfrac{\sqrt{64u^3 + 36u^2 + 1}}{\cancel{4u\sqrt{u}}}}{\dfrac{(4u+1+16u^3)\sqrt{16u^3+4u+1}}{\cancel{4u}\,2\cancel{\sqrt{u}}}}$$

$$\varkappa = \frac{2\sqrt{64u^3 + 36u^2 + 1}}{(4u+1+16u^3)\sqrt{16u^3+4u+1}}$$

$$\tau = \frac{\vec{r}\,' \vec{r}\,'' \vec{r}\,'''}{|\vec{r}\,' \times \vec{r}\,''|^2}$$

$$\underbrace{\vec{r}\,' \vec{r}\,'' \vec{r}\,'''}_{\text{смесено произв.}} = (\vec{r}\,' \times \vec{r}\,'')\,\vec{r}\,''' =$$

$$= \left(\frac{3}{2\sqrt{u}}; -2; -\frac{1}{4u\sqrt{u}}\right)\left(0; \frac{3\sqrt{u}}{8u^3}; 0\right) =$$

$$= \frac{3}{2\sqrt{u}} \cdot 0 + (-2) \cdot \frac{3\sqrt{u}}{8u^3} + \left(-\frac{1}{4u\sqrt{u}} \cdot 0\right) =$$

$$= -\frac{3}{4}\,\frac{\sqrt{u}}{u^3}$$

$$\tau = \frac{-\frac{3}{4}\frac{\sqrt{u}}{u^3}}{\left(\sqrt{\frac{36u^2+64u^3+1}{16u^3}}\right)^2} =$$

$$= \frac{-\frac{3}{4}\frac{\sqrt{u}}{u^3}}{\frac{36u^2+64u^3+1}{16u^3}} = \frac{-3\sqrt{u}\cdot \cancel{16u^3}^{\,4}}{\cancel{4u^3}\cdot(36u^2+64u^3+1)}$$

$$\tau = -\frac{12\sqrt{u}}{36u^2+64u^3+1}$$

б) $C: \vec{r} = (\ln u;\ u^2;\ 2u)$ б т. $P(u=1)$

$\dot{\vec{r}} = \left(\frac{1}{u};\ 2u;\ 2\right)$

$\ddot{\vec{r}} = \left(-\frac{1}{u^2};\ 2;\ 0\right)$

$\dot{\vec{r}} \times \ddot{\vec{r}} = \left(2u\cdot 0 - 2\cdot 2;\ 2\cdot\left(-\frac{1}{u^2}\right) - 0\cdot\frac{1}{u};\ \frac{1}{u}\cdot 2 + \frac{2\cancel{u}}{u^{\cancel{2}}}\right) = \left(-4;\ -\frac{2}{u^2};\ \frac{4}{u}\right)$

$|\dot{\vec{r}} \times \ddot{\vec{r}}| = \sqrt{(-4)^2 + \left(-\frac{2}{u^2}\right)^2 + \left(\frac{4}{u}\right)^2} = \sqrt{16 + \frac{4}{u^4} + \frac{16}{u^2}} = \sqrt{\frac{16u^4+16u^2+4}{u^4}} = \sqrt{\frac{4(4u^4+4u^2+1)}{u^4}}$

$= \frac{2}{u^2}(2u^2+1)$

$$|\vec{r}\,'| = \sqrt{\left(\frac{1}{u}\right)^2 + (2u)^2 + 2^2} = \sqrt{\frac{1}{u^2} + 4u^2 + 4} =$$

$$= \sqrt{\frac{4u^4 + 4u^2 + 1}{u^2}} = \sqrt{\frac{(2u^2+1)^2}{u^2}} = \frac{2u^2+1}{u}$$

$$\varkappa = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3} = \frac{\frac{2}{u^2} \cdot (2u^2+1)}{\frac{(2u^2+1)^3}{u^3}} = \frac{2\cancel{(2u^2+1)} \cdot u^{\cancel{3}}}{\cancel{u^2}\, 2(u^2+1)^{\cancel{3}\,2}}$$

$$\varkappa(1) = \frac{2 \cdot 1}{(2 \cdot 1^2 + 1)^2} = \frac{2}{9} \qquad = \frac{2u}{(2u^2+1)^2}$$

$$\tau = \frac{\vec{r}\,' \vec{r}\,'' \vec{r}\,'''}{|\vec{r}\,' \times \vec{r}\,''|^2}$$

$$\vec{r}\,' \vec{r}\,'' \vec{r}\,''' = (\vec{r}\,' \times \vec{r}\,'')\, \vec{r}\,''' = \left(-4; -\frac{2}{u^2}; \frac{4}{u}\right)\left(\frac{2}{u^3}; 0; 0\right) =$$

$$= -4 \cdot \frac{2}{u^3} + \left(-\frac{2}{u^2}\right) \cdot 0 = \frac{4}{u} \cdot 0 =$$

$$= -\frac{8}{u^3}$$

$$\tau = \frac{-\frac{8}{u^3}}{\left(\frac{2}{u^2}(2u^2+1)\right)^2} = \frac{-8 \cdot u^4}{u^3 \cdot 4(2u^2+1)^2}$$

$$\tau = -\frac{2u}{(2u^2+1)^2}$$

$$\tau(1) = -\frac{2 \cdot 1}{(2 \cdot 1^2 + 1)^2} = -\frac{2}{9}$$

Зад. 3

$$\vec{f}(u) = (\sin u - 1,\ 1 - \cos u)$$

C^0 $\vec{g}(v) = (-v-1,\ 2+v^2)$ $v \in [0,1]$

Двете криви са непрекъснати. Има точка на съединяване и краят на ~~другата~~ едната ~~точка~~ съвпада с началото на другата $P(-1;2)$

$\sin u - 1 = -1$, $1 - \cos u = 2$ $u = \pi$

$\sin u = 0$ $\cos u = -1$

$-v-1=-1$ $2+v^2=2$

$v=0$ $v=0$

$\vec{f}(\pi) = (-1;2)$ $g(0) = (-1;2)$

$$\vec{f}(\pi) = \vec{g}(0)$$

$\Rightarrow$ двете криви се съединяват в т. $P(-1;2)$

Изследваме C^1 - непрекъснатост и G^1 - непрекъснатост след пресмятане на първите производни

$\vec{f}'(u) = (\cos u, \sin u)$ $\vec{f}'(\pi) = (-1;0)$

$\vec{g}'(v) = (-1;2v)$ $\vec{g}'(0) = (-1;0)$

$\vec{f}'(\pi) = \vec{g}'(0) \Rightarrow$ в точката на съединяване C^1 - непрекъснатост. G^1 - непрекъснатостта $\Rightarrow$ има и G^1 - непрекъснатост.

Пресмятаме вторите производни

$\vec{f}''(u) = (-\sin u; \cos u)$ $\quad f''(\pi) = (0; -1)$

$g''(v) = (0; 2)$ $\quad g''(0) = (0; 2)$

Тъй като $\vec{f}''(\pi) \neq \vec{g}''(0)$, то в точката на съединяване няма C^2-непрекъснатост. Трябва да проверим за наличие на G^2-непрекъснатост.

$\vec{f}''(\pi) - \vec{g}''(0) = \lambda \vec{f}'(\pi) \stackrel{\xi}{=} \lambda \vec{g}'(0)$

$\vec{f}''(\pi) - g''(0) = (0; -3)$ $\quad \vec{f}'(\pi) = \vec{g}'(0) = (-1; 0)$

Условието не е изпълнено, няма такова $\lambda \Rightarrow$ не съвз. G^2-непрекъснатост. $\varkappa$-непрекъснатост.

$$\varkappa \vec{f}(\pi) = \frac{|\vec{f}'(\pi) \times \vec{f}''(\pi)|}{|\vec{f}'(\pi)|^3}$$

$\vec{f}'(\pi) = (-1; 0)$ $\quad f'(\pi)(-1; 0; \underline{\underline{0}})$ Добавяме трети коорд.

$\vec{f}''(\pi) = 0; -1)$ $\quad \vec{f}''(\pi)(0; -1; \underline{\underline{0}})$

$\vec{f}'(\pi) \times \vec{f}''(\pi) = (0; 0; 1)$

$|\vec{f}'(\pi)| = \sqrt{1} = 1$ $\quad |\vec{f}'(\pi) \times \vec{f}''(\pi)| = 1$

$\Rightarrow \varkappa \vec{f}(\pi) = 1$

$\vec{g}'(0) = (-1;0)$ $\qquad$ $\vec{g}'(0) = (-1;0;0)$

$\vec{g}''(0) = (0;2)$ $\qquad$ $\vec{g}''(0) = (0;2;0)$

$\vec{g}'(0) \times \vec{g}''(0) = (0;0;-2)$

$|\vec{g}'(0) \times \vec{g}''(0)| = 2$

$|\vec{g}'(0)| = 1$

$$\varkappa_{\vec{g}}(0) = \frac{|\vec{g}'(0) \times \vec{g}''(0)|}{|\vec{g}'(0)|} = \frac{2}{1} = 2$$

$\varkappa_{\vec{f}}(\pi) \neq \varkappa_{\vec{g}}(0)$

$\Rightarrow$ няма $\varkappa$-непрекъснатост