

Treatment Evaluation and Matching Methods

Applied Econometrics for Researchers, PhD
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Agenda

1. Treatment effects: what and why?
2. Key assumptions
3. Matching & Propensity Score Estimators
4. PSM in research and in Stata (examples)

Key readings:

- Caliendo, M., Kopeinig, S. (2008), "Some practical guidance for the implementation of propensity score matching", *Journal of Economic Surveys*, 22(1), 31-72.
- **Cameron & Trivedi**, Microeconometrics: Methods and Applications, **chapter 25**.
 - Alternatively, [Chapter 5 on Matching and Subclassification](#); Causal Inference, The Mixed Tape, by S. Cunningham
- Kaiser, U., Malchow-Møller, N. (2011), "Is self-employment really a bad experience? The effects of previous self-employment experience on subsequent wage-employment wages", *Journal of Business Venturing*, 26(5), 572-588. **(applied example)**

Treatment Evaluation

- **Objective:** To measure the **impact** of interventions (or choices) on outcomes of interest
- Examples (binary treatments)

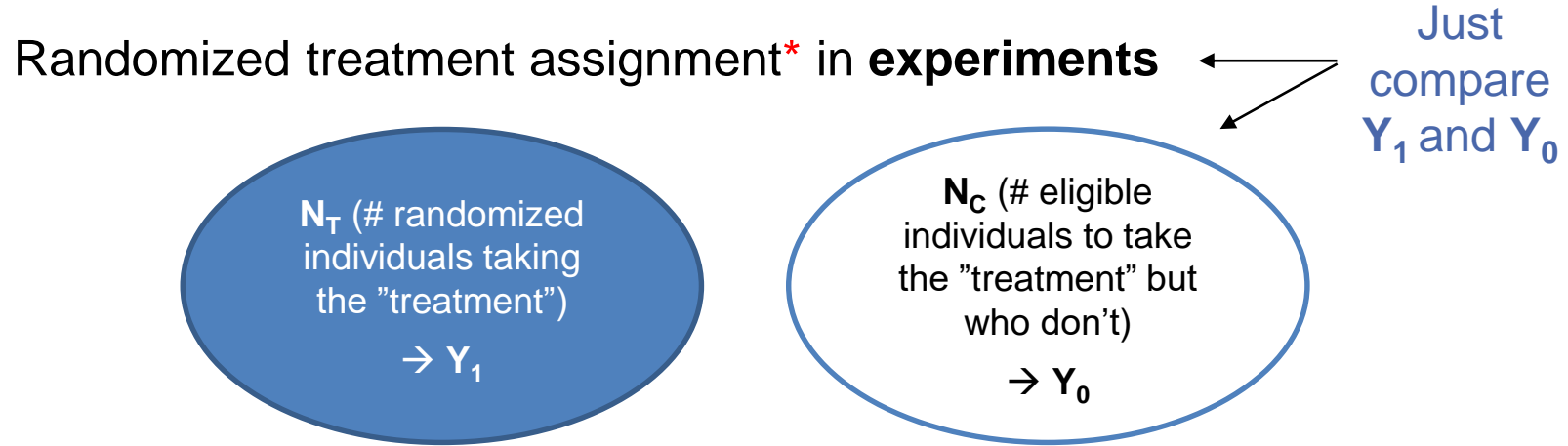
Origin in
medical trials;
increasingly
popular in
economic policy,
labor studies,
management &
strategy

Treatment	Outcome of interest
Medical treatment (e.g., new drug)	Health status (e.g., life expectancy)
Enrollment in labor training program	Productivity; Wages; Labor participation
Occupational choice (e.g., self-employment)	Lifetime earnings
Regulatory changes (e.g., tax or entry barriers reductions); subsidies	Individual/firm decisions

The Evaluation Problem

- We want to measure the **response to the treatment** relative to **some benchmark** (often no treatment)
- BUT no individual is simultaneously observed in both states
- We lack a **counterfactual**: how would the outcome of an average untreated individual change if s/he received the treatment?
- ALSO: Data often do not come from randomized experiments, but from (non-randomized) **observational studies**
 - Probable interferences in the causal connection between the treatment and the outcome

Treatment Effects Framework



*the assignment to the treatment ignores the possible impact of the treatment on the outcome (Y)

When using **observational data**, there is **no random assignment** to the treatment
 \Leftrightarrow Individuals might choose to be "treated" or other reasons might not make it random!

Key parameters of interest: ATE & ATET

$$\Delta = Y_1 - Y_0$$

Δ is not directly observable because no individual can be observed in both states

$$\mathbf{ATE} = E[\Delta] = E[Y_1 - Y_0]$$

$$\widehat{ATE} = \frac{1}{N} \sum_{i=1}^N [\Delta_i]$$

ATE is relevant when the treatment has universal applicability (not relevant for many policy studies; it includes the effect on persons for whom the treatment was never intended)

$$\mathbf{ATET} = E[\Delta \mid T = 1]$$

$$\widehat{ATET} = \frac{1}{N_T} \sum_{i=1}^{N_T} [\Delta_i \mid T = 1]$$

ATET is relevant to evaluate the effects on those for whom the treatment is actually intended (e.g., diabetes patients; unemployed individuals)

Identification

$$\begin{aligned}\text{ATET} &= E[Y_1 - Y_0 | T = 1] \\ &= E[Y_1 | T = 1] - E[Y_0 | T = 1]\end{aligned}$$

Not observed!

In experiments: We would use $E[Y_0 | T = 0]$.

In observational data: Not a good idea. What determines the treatment is also likely to determine Y .

So we need a method (e.g., matching) to generate a comparison group.

Can be further decomposed into:

$$= E[Y_1 | T = 1] - E[Y_0 | T = 0] + [E[Y_0 | T = 0] - E[Y_0 | T = 1]]$$

Observed

Unobserved difference in outcomes for nontreated, had they had the treatment (ideally = 0)



King ~~Prince~~ Charles

Male
Born in 1948
Raised in the UK
Married Twice
Lives in a castle
Wealthy and Famous



Ozzy Osbourne

Male
Born in 1948
Raised in the UK
Married Twice
Lives in a castle
Wealthy and Famous

Key Assumptions

Conditional Independence Assumption

(also referred to as *unconfoundedness*, *ignorability* or selection on observables assumption)

$$Y_0, Y_1 \perp T \mid X$$

Conditional on **X**, the outcomes are independent of the treatment assignment

⇔ **Random assignment to treatment**

If it holds, systematic differences in outcomes for persons with the same X can be attributed to T

(strong assumption, requires good data)

If **validated**, T is exogenous and matching methods are suitable. If **violated**, endogeneity is present – **later lectures!**

Note: If we are interested in ATET only, it is enough that: $Y_0 \perp T \mid X$

Key Assumptions

Overlap Assumption

(also referred to as *matching* or *common support* assumption)

$$0 < \Pr (T = 1 \mid \mathbf{X}) < 1$$

For each value of \mathbf{X} , there are **both treated and untreated cases** \Leftrightarrow for each treated individual there is another **comparable** (i.e., with similar \mathbf{X}) untreated individual

In other words: Persons with the same \mathbf{X} values have a positive probability of being both treated and non-treated; some randomness is needed that guarantees that persons with identical characteristics can be observed in both states.

If **violated**, we could have individuals with \mathbf{X} vectors who are all treated, and those with a different \mathbf{X} would all be untreated.

The concept of Propensity Score

When participation into the treatment is not random, but depends on a vector of variables \mathbf{X} (e.g., age, gender), the conditional probability of treatment participation is given by the **Propensity Score** $p(\mathbf{x})$:

$$\text{PS: } p(\mathbf{x}) = \Pr (T = 1 \mid \mathbf{X} = \mathbf{x})$$

Estimated
by
logit/probit

Balancing Condition: $T \perp \mathbf{x} \mid p(\mathbf{x})$

For individuals with the same propensity score, the assignment to the treatment is random and should look identical in terms of the \mathbf{X} vector

Testable!

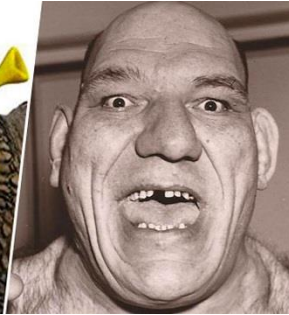
Matching methods: underlying logic

- If **assignment to T** directly depends on **(only) observed characteristics** of individuals (e.g., age, gender, socio-economic status), we can mimic an experimental setting by generating a **control group**, i.e. sample of **comparable individuals** (with comparable characteristics) who did not take the treatment. (selection on observables)
- But if there are **unobserved factors** that partly determine both T and Y (e.g., individual innate ability), matching methods are not enough to accurately measure ATET (selection on unobservables)
 - We will deal with this issue in later lectures
 - **For today: selection on observables**

Matching is persuasive and attractive if:

- We can control for a rich set of X variables
- There are many potential control units in the data

←
←
*Treated and
control subjects
as similar as
possible*



- ATET is the parameter of interest

Matching methods

Exact matching

- Practicable when the vector of covariates is discrete and the sample contains many observations at each distinct value of x
- Impractical when there are many (continuous) variables to match.
- Possibility: **cem** in Stata
- Data-hungry!

Propensity Score Matching

- Rather than matching on X , it **matches on a single metric**: the propensity score $p(x)$
- Control group = individuals whose $p(x)$ is *sufficiently close* to treated individuals
- Not so hungry in terms of data (as exact matching)
- **teffects psmatch** (or **psmatch2**) in Stata

When implementing matching, consider:

1) Whether to match *with* or *without* replacement

- **With:** a control individual can be used as a match multiple times (i.e., for multiple treated individuals)
- If with: higher matching quality on average (reduced bias), but higher variance
- **Without:** a control individual is matched to no more than one treated individual (more restrictive)
- If without: smaller comparison set; matches may not be so close in terms of $p(x)$ → increased bias (lower matching quality), but reduced variance
- TRADE-OFF BETWEEN **BIAS** AND **VARIANCE**!

When implementing matching, consider:

2) Number of control units to use in the comparison set

- **One single** (the closest) match – lower bias, increased variance
- **More than one match** ("oversampling") – lower variance, increased bias
 - Some may be poor matches. Possible solution: set a neighborhood for $p(x)$

3) Choice of matching method

- Depends on the data (how rich in terms of X variables and size of comparison group?)
- Depends also on choices made in 1) and 2)

Matching Algorithms (1/2)

Nearest Neighbor (NN)

- For each treated individual choose the match(es) where the **difference in $p(x)$ is smaller**
- Usually **with replacement** (so each untreated individual can be used multiple times as a match)
- **Oversampling** – possible to use more than one NN
- Note that **some matches may be poor** (even though they are the nearest, their PS may be far away)

→
alternative

Radius & Caliper Matching

- Establish a **neighborhood for the PS**; all matches falling within that **tolerance level (radius/caliper)** are used as matches
- **Bad (distant) matches are avoided**
- Possible to use one or more NN
- But if tolerance is too small, some units may not get matches

Matching Algorithms (2/2)

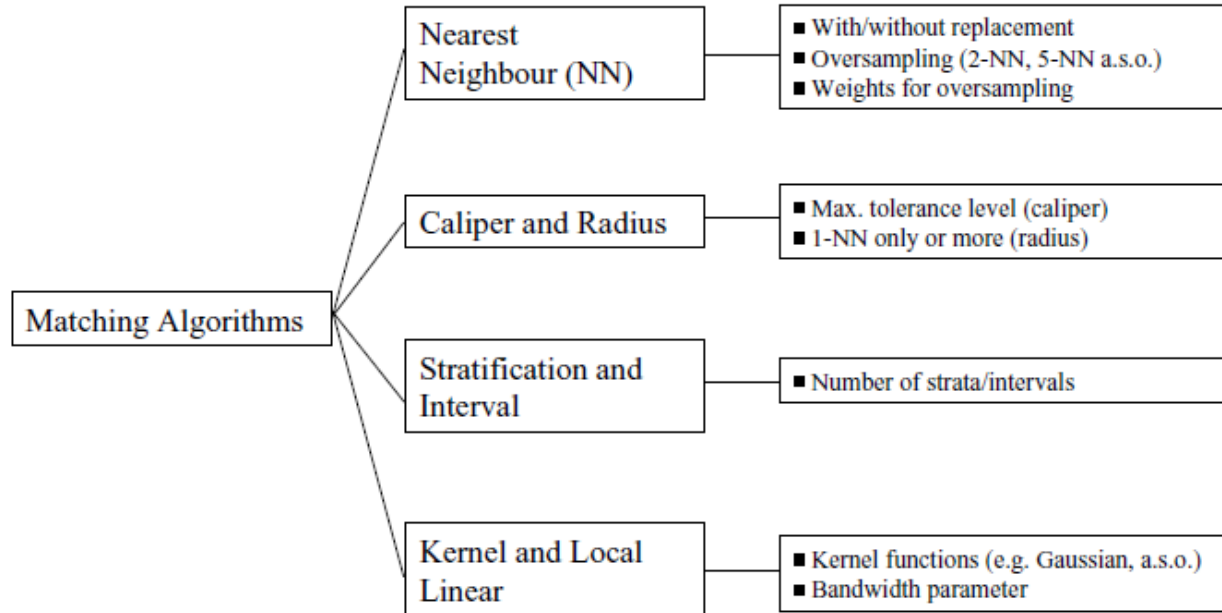
Kernel Matching

- **All treated** units are **matched with a weighted average** of **all control units**
- Weights are **inversely proportional to the Propensity Score distance** between treated and untreated units

Stratification or Interval Matching

- Divides the range of variation of the PS in **intervals** (default = 5)
- Within each interval, **T and C units have, on average, the same PS**
- ATET computed within each interval; global ATET = weighted average, depending on the distribution of treated units across the "blocks"
- How many intervals? **Balancing condition should be verified within each interval**

How to select a specific matching algorithm?



Source: Caliendo and Kopeinig (2008)

Trade-offs in terms of bias and efficiency

No winner for all situations! Depends on the data (*recall slides 13 & 16*).
In **smaller samples**, the choice might make a difference; in **larger samples**, all PSM methods should yield asymptotically similar results.

Decision	Bias	Variance
Nearest neighbour matching:		
<u>multiple neighbours/single neighbour</u>	(+)/(-)	(-)/(+)
<u>with caliper/without caliper</u>	(-)/(+)	(+)/(-)
Use of control individuals:		
<u>with replacement/without replacement</u>	(-)/(+)	(+)/(-)
Choosing method:		
NN matching/Radius matching	(-)/(+)	(+)/(-)
KM or LLM/NN methods	(+)/(-)	(-)/(+)

teffects
psmatch

Important to check after matching:

Overlap/Common Support Assumption

- $0 < \Pr(T = 1 | \mathbf{X}) < 1$
- **Visual analysis** of the region of overlap/common support – e.g.:
- Compare min and max of PS in T and C groups
- Compare **density distribution of the PS in both groups**
- If there is too much mass around 0 or 1, the overlap assumption may be violated \Leftrightarrow matching estimator not satisfactory

Balancing Condition/Matching Quality

- $T \perp \mathbf{x} | \mathbf{p}(\mathbf{x})$
- If matching is properly done, **T and C groups should have no significant differences in terms of X variables**
- E.g., **t-test** in covariate means;
tebalance after `teffects psmatch`
- If matching quality is not enough, **improve the estimation of the PS** (e.g., adding more variables, non-linear terms on some covariates, interaction terms)

Research example

Is self-employment really a bad experience? The effects of previous self-employment on subsequent wage-employment wages

Kaiser & Malchow-Møller (2011), JBV



The research setting

*Treatment
vs Control
groups*

Treatment: past self-employment (SE) experience (binary)

>>> then several different types of treatments

Outcome: subsequent earnings in wage employment (WE) – log (hourly wage) in 1996

Data: Danish men observed between 1990 and 1996; full-time wage-employed in both 1990 and 1996

PSM methods to estimate **ATET**

Ideal experiment: a fraction of individuals would be **randomly allocated to a (short) spell of SE** before being returned to WE, while the remaining individuals would be kept in WE during the whole period.

Observational data: having had SE experience is a **choice**; individuals with and without SE experience are likely to differ in a variety of characteristics

PSM to find "**clones**" (in terms of **observable characteristics**) for each individual with SE experience

Applying PSM to estimate the ATET

"...given a set of observable characteristics, x , – **which is not affected by treatment** – potential outcomes are independent of the assignment to treatment" \Leftrightarrow

conditional independence or unconfoundedness assumption
(recall slide 9)

"we cannot formally test if [this] assumption is satisfied. We do **formally test whether T and C observations no longer differ significantly wrt. observable characteristics after matching**" \Leftrightarrow **balancing condition**
(recall slides 11 & 21)

1-to-1 matching infeasible: why?

PSM instead: Probability of **treatment (spell in SE)** estimated with a **probit** model

Wide set of **X that affect both T assignment and Y** (e.g., tenure, age, sector, education, family background, regional conditions, employer characteristics, initial wage in 1990).

Several matching algorithms tested; **preference for nearest neighbor (single with replacement)** "since it reduces estimation bias at the cost of higher variance"

Applying PSM to estimate the (general) ATET

Before matching: former SE individuals **earn on average** (not controlling for X) **4% more** than consecutively employed individuals.

They also **differ in other observed characteristics**: e.g., shorter tenure, higher education levels, employed in smaller establishments.

Differences (in X) no longer significant **after matching**:
balancing property satisfied

Treatment groups	Control groups
	C_0
	WE throughout 1990–1996
T_1 : basic treatment: at least one spell of self-employment, no unemployment, no non-employment	<div>-0.0288^{***} 0.0063</div>
# obs.	534,456

ATET: a spell of SE between 1990 and 1996 goes along with a **reduction** in hourly wages in subsequent wage employment of **2.9%** (vs. OLS coefficient: 0.0013^{***})

Still possible to study more specific treatments

Treatment groups	Control groups	
	C ₀	C ₁
	WE throughout 1990–1996	As C ₀ but with job change
T ₁ : basic treatment: at least one spell of self-employment, no unemployment, no non-employment	−0.0288*** 0.0063	−0.0160*** 0.0064
T ₂ : as T ₁ but with WE-sector in 1996 = WE-sector in 1990		−0.0084 0.0079
T _{2a} : as T ₂ but with SE-sector = WE-sector in 1996		−0.0099 0.0098
T _{2b} : as T ₂ but with SE-sector = WE-sector in 1996		−0.0118 0.0133
T ₃ : as T ₁ but with WE-sector in 1996 = WE-sector in 1990		−0.0233** 0.0109
T _{3a} : as T ₃ but with SE-sector = WE-sector in 1996		0.0572*** 0.0219
T _{3b} : as T ₃ but with SE-sector ≠ WE-sector in 1996		−0.0405*** 0.0125
T _{4a} : as T ₁ but with employees in SE		0.0193 0.0122
T _{4b} : as T ₁ but with high income as SE		0.0629** 0.0296
# obs.	534,456	291,171

More refined control group: individuals who also experienced a job change in WE

Basic Treatment: wage reduction of 1.6%

Specific Treatment of SE in the same (different) sector of WE: wage increase of 5.7% (wage decrease of 4.1%) (OLS finds a 4% wage premium in both)

Propensity Score Matching in Stata

- For this course, give priority to **teffects psmatch**
- **Note:** **teffects psmatch** only allows matching with replacement and uses all "good neighbours" as matches
- An alternative could be the user-written command **psmatch2** (check syntax and description [here](#)). It allows several other matching algorithms (as discussed before).
- However, be aware that the standard errors obtained with **psmatch2** are not correctly estimated. **teffects psmatch** is therefore preferred, or should be used in combination with **psmatch2**. A comparison between the two can be found [here](#).

NOTE: For R, check chapter [5. Matching and subclassification](#), especially section 5.3.6. Nearest Neighbor Matching

teffects psmatch: Quick tour

```
teffects psmatch (y) (t x1 x2), atet
```

>> Estimates the **ATET of t on y by PSM**, using a logit model for t on $x1$ and $x2$. If probit is preferred, add "probit" option. If ATE is needed, drop the ATET option.

```
teffects psmatch (y) (t x1 x2), nn(3) gen(match)
```

>> At least 3 matches should be used per observation (default is 1)

>> **gen(match)** creates a new variable containing the ID of the nearest match(es)

```
predict ps0 ps1, ps
```

 >> predicts propensity score (probability of being in T vs C groups)

```
predict y0 y1, po
```

 >> predicts potential outcomes

```
teffects overlap
```

 >> produces graph to analyze overlap assumption

(...)

```
help teffects psmatch
```

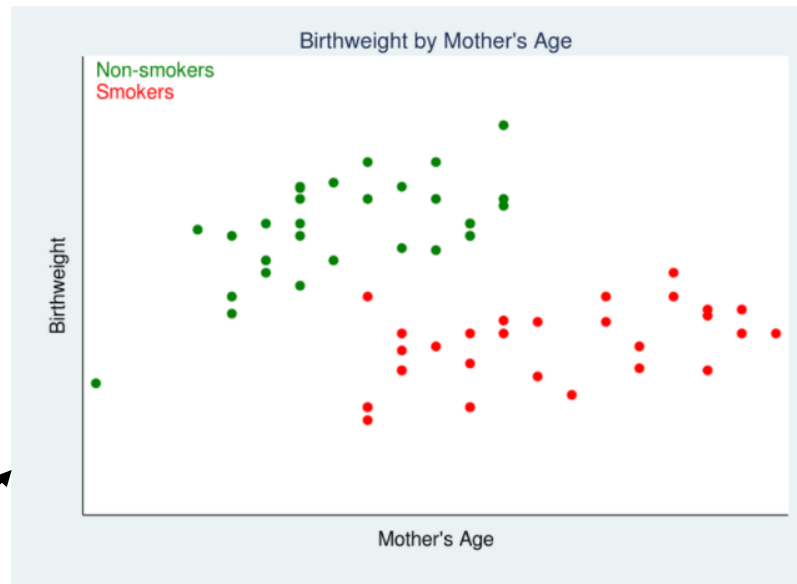
```
help teffects_postestimation
```

Stata Example

Effect of maternal smoking during pregnancy (**treatment**) on baby's weight at birth (**outcome**)

Why can't we estimate this effect by comparing the birth weights of babies of smoking vs. non-smoking mothers?

Mothers are **not randomly assigned to "smoking status"**; it is their choice. Other patterns?



(selected data points)

Before treatment evaluation: descriptives

- How many women are in the "treatment group" (i.e., smoked during pregnancy)?
- Observed difference in babies' birthweight of smoking vs. non-smoking mothers?
- Other observed differences between smoking vs. non-smoking mothers? e.g.:**
 - Marital status?
 - Education level?
 - First pregnancy?

1 if mother smoked	Freq.	Percent	Cum.
nonsmoker	3,778	81.39	81.39
smoker	864	18.61	100.00
Total	4,642	100.00	

```
. ttest bweight, by(mbsmoke)
```

Two-sample t test with equal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]	
nonsmoke	3,778	3412.912	9.284683	570.6871	3394.708	3431.115
smoker	864	3137.66	19.08197	560.8931	3100.207	3175.112
combined	4,642	3361.68	8.495534	578.8196	3345.025	3378.335
diff		275.2519	21.4528		233.1942	317.3096

diff = mean(nonsmoke) - mean(smoker) t = 12.8306
Ho: diff = 0 degrees of freedom = 4640

Ha: diff < 0
Pr(T < t) = 1.0000

Ha: diff != 0
Pr(|T| > |t|) = 0.0000

Ha: diff > 0
Pr(T > t) = 0.0000

*Critical to select which variables
predict the Propensity Score →*

What explains the prob(treatment)?

Logistic regression

Number of obs = 4,642

LR chi2(10) = 511.72

Prob > chi2 = 0.0000

Pseudo R2 = 0.1147

Log likelihood = -1974.8899


treatment →

Use pairwise correlations between X and the treatment variable & t-tests to identify observed differences between groups

	mbsmoke	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
	mmarried	-.962527	.1043015	-9.23	0.000	-1.166954	-.7580998
	mage	-.0238051	.0094441	-2.52	0.012	-.0423153	-.0052949
	medu	-.1084369	.0197706	-5.48	0.000	-.1471865	-.0696873
	foreign	-1.12312	.245545	-4.57	0.000	-1.60438	-.6418612
	alcohol	1.572497	.1849607	8.50	0.000	1.209981	1.935013
	deadkids	.3756484	.0909177	4.13	0.000	.1974529	.5538439
	monthslb	.005789	.0014805	3.91	0.000	.0028872	.0086907
	fedu	-.0576786	.012132	-4.75	0.000	-.0814569	-.0339004
	fbaby	-.2930148	.1048617	-2.79	0.005	-.49854	-.0874896
	frace	.5641113	.1138082	4.96	0.000	.3410514	.7871712
	_cons	1.143572	.2627225	4.35	0.000	.6286459	1.658499

Stata behind the scenes...

`predict ps0 ps1, ps`



	bweight	mbsmoke	ps0	ps1	match1	match2	match3
1	3459	nonsmoker	.890267	.109733	4043	.	.
4043	3629	smoker	.8904855	.1095145	2268	.	.

Individual #1, a non-smoker, is matched with only one smoker (individual #4043), who has the closest propensity score.

This individual is then matched to #2268, who has an even closer PS (0.109531).

	bweight	mbsmoke	ps0	ps1	match1	match2
11	3090	smoker	.9068304	.0931696	4170	1745
4170	3515	nonsmoker	.9068304	.0931696	11	.
1745	3572	nonsmoker	.9068304	.0931696	11	.

Individual #11 (treated) finds two good matches (untreated) with the same PS

Stata behind the scenes...

matches

```
predict y0 y1, po
predict te, te
```

	bweight	mbsmoke	match1	match2	ps0	ps1	y0	y1	te
3	3572	nonsmoker	3949	.	.9087363	.0912636	3572	2523	-1049
3949	2523	smoker	10	.	.9092206	.0907794	3880	2523	-1357

If "nonsmoker": y_0 = observed bweight and y_1 = bweight of the matched "smoker"

Treatment effect = $y_1 - y_0$

ATE = average of "te"

ATET = average of "te" for "smokers"

Variations: Minimum number of matches

```
teffects psmatch (bweight) (mbsmoke mmarried mage medu foreign  
alcohol deadkids monthslb fedu fbaby frace), nn(3)
```

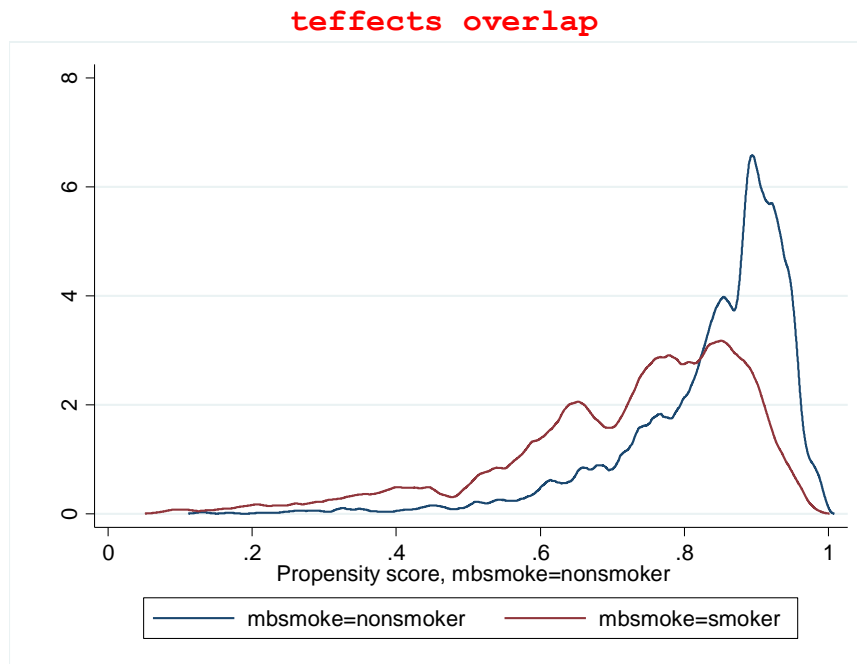
Treatment-effects estimation		Number of obs	=	4,642
Estimator	: propensity-score matching	Matches: requested	=	3
Outcome model	: matching	min	=	3
Treatment model	: logit	max	=	31

	bweight	AI Robust		z	P> z	[95% Conf. Interval]	
		Coef.	Std. Err.				
ATE							
	mbsmoke						
(smoker vs nonsmoker)		-216.7233	28.48581	-7.61	0.000	-272.5544	-160.8921

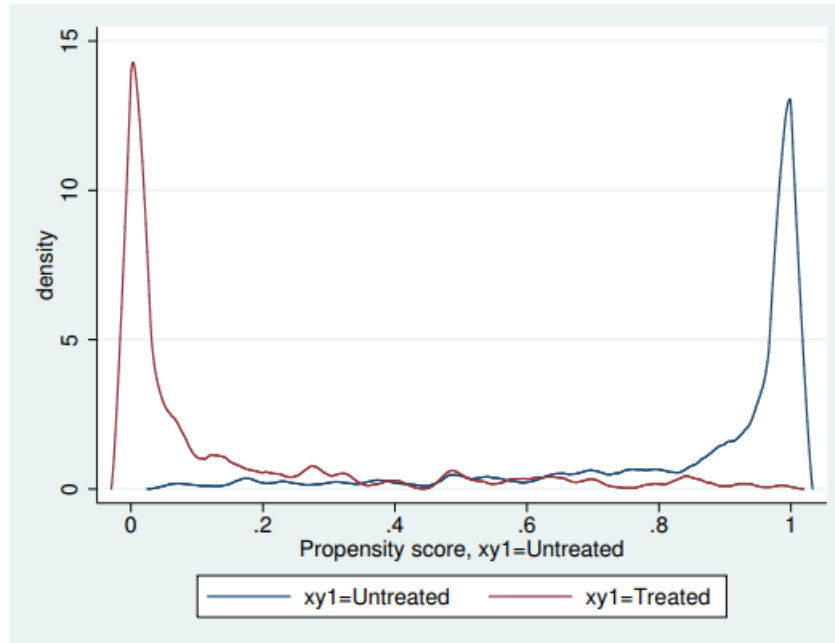
Remember the trade-off between bias and variance!

Overlap assumption

- Estimated density of the predicted probabilities
- Ideally not too much mass around 0 or 1
- The two estimated densities have most of their respective masses in regions in which they overlap each other
- **No evidence that the overlap assumption is violated.**



Violating Overlap Assumption: Example



Checking balancing condition (1/2)

- A covariate is said to be **balanced** when its distribution does not vary over treatment levels.
- A perfectly balanced covariate would have a **standardized difference of 0** and a **variance ratio of 1**.
- **Improved level of balance for all variables**, though for some it could be better (e.g., mother age/education).
- **To try to achieve better balance**, we could specify a richer model for the PS (e.g., with interactions between some covariates).

```
. tebalance summarize
```

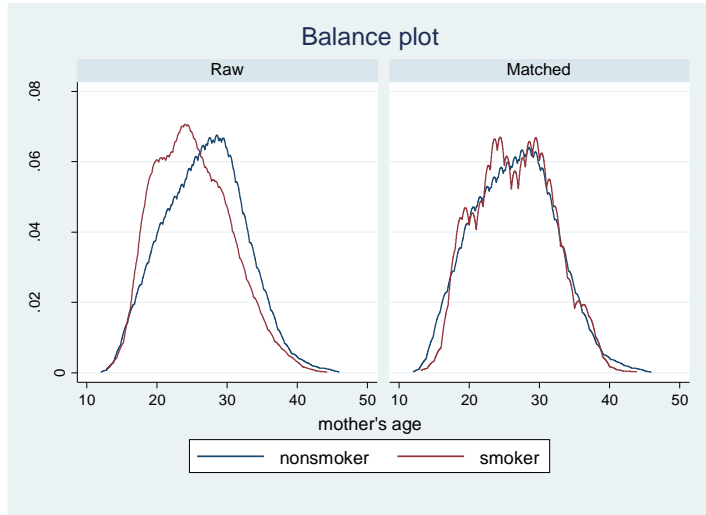
Covariate balance summary

	Raw	Matched
Number of obs =	4,642	9,284
Treated obs =	864	4,642
Control obs =	3,778	4,642

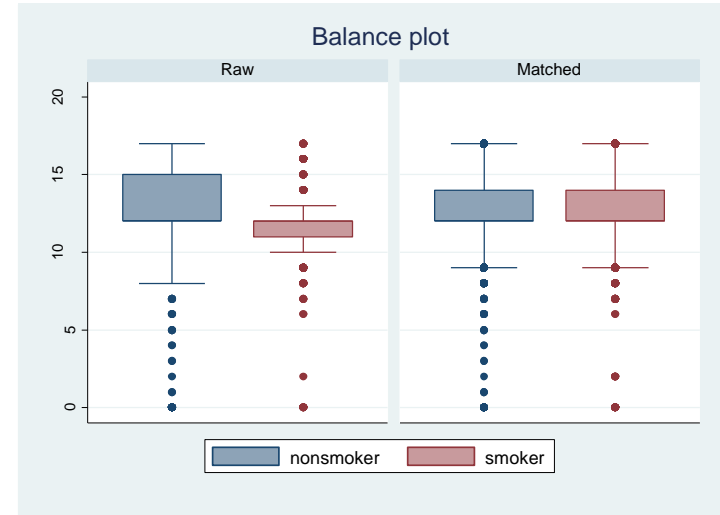
	Standardized differences		Variance ratio	
	Raw	Matched	Raw	Matched
mmarried	-.5953009	-.0131048	1.335944	1.011191
mage	-.300179	.0131895	.8818025	.8790964
medu	-.5474357	-.0151252	.7315846	.652641
foreign	-.1706164	-.0520006	.4416089	.7992838
alcohol	.3222725	.0573674	4.509207	1.358726
deadkids	.1613223	-.0669038	1.171182	.9225649
months1b	.1841973	-.0120765	1.373939	1.02189
fedu	-.5182535	-.0371648	1.385118	.8383112
fbaby	-.1663271	.0695698	.9430944	1.009759
frace	-.1755916	-.0418292	1.290599	1.06879

Checking balancing condition (2/2)

tebalance density mage



tebalance box medu



Wrap-up of today and next sessions

	Heckman models (11/09)	Matching Models (PSM) (11/16)	Instrumental Variables (11/23)
When	Y is missing in some cases (for a non-random reason)	X is a binary intervention/choice	...
Problem	The missings in Y are driven by a "selection process"	T & C groups are very different	...
Stata commands	<i>heckman, (twostep)</i>	<i>teffects psmatch, tebalance, teffects overlap</i>	...
Key tests	Significance of the IMR or of the <i>rho</i>	Balancing and overlapping conditions	...
Attention!	Need for valid exclusion restrictions; selection bias important when <i>IMR/rho</i> significant and X predicts selection (1 st stage)	T & C only matched on observable characteristics. If unobservables matter, PSM does not provide causal effects → IV	...
First stage	Probit predicting selection into the sample (Y ≠ missing)	Probit predicting probability of being treated (X)	...

Your roadmap when implementing PSM

1. **Model choice** (logit/probit) and **variable choice** (guided by theory and prior evidence; X should not be influenced by the treatment!)
Remember “Conditional Independence Assumption”
2. **Matching algorithm**: NN? Caliper? Number of NN? With or without replacement? (some Stata commands may limit your choices)
3. Check **overlap assumption** (visually + min/max of PS)
4. Check **balancing condition** (table of mean differences and variance ratio before and after matching; plots)
5. Satisfactory results? If not, **iterate and improve** PS model estimation (e.g. adding interaction terms, adding or removing variables) and follow the list again until you can “trust” your **ATET**

Remember:

“Matching is no ‘magic bullet’ that will solve the evaluation problem in any case. It should only be applied if the underlying identifying assumption* can be credibly invoked based on the informational richness of the data and a detailed understanding of the institutional set-up by which selection into treatment takes place.”

** selection on observables*

Caliendo & Kopeinig (2008)

Hopefully you disagree 😊



"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."