Discrete Response Models
Count Models

H.C. Kongsted
Copenhagen Business School, SÍ
hck.si@cbs.dk

Agenda

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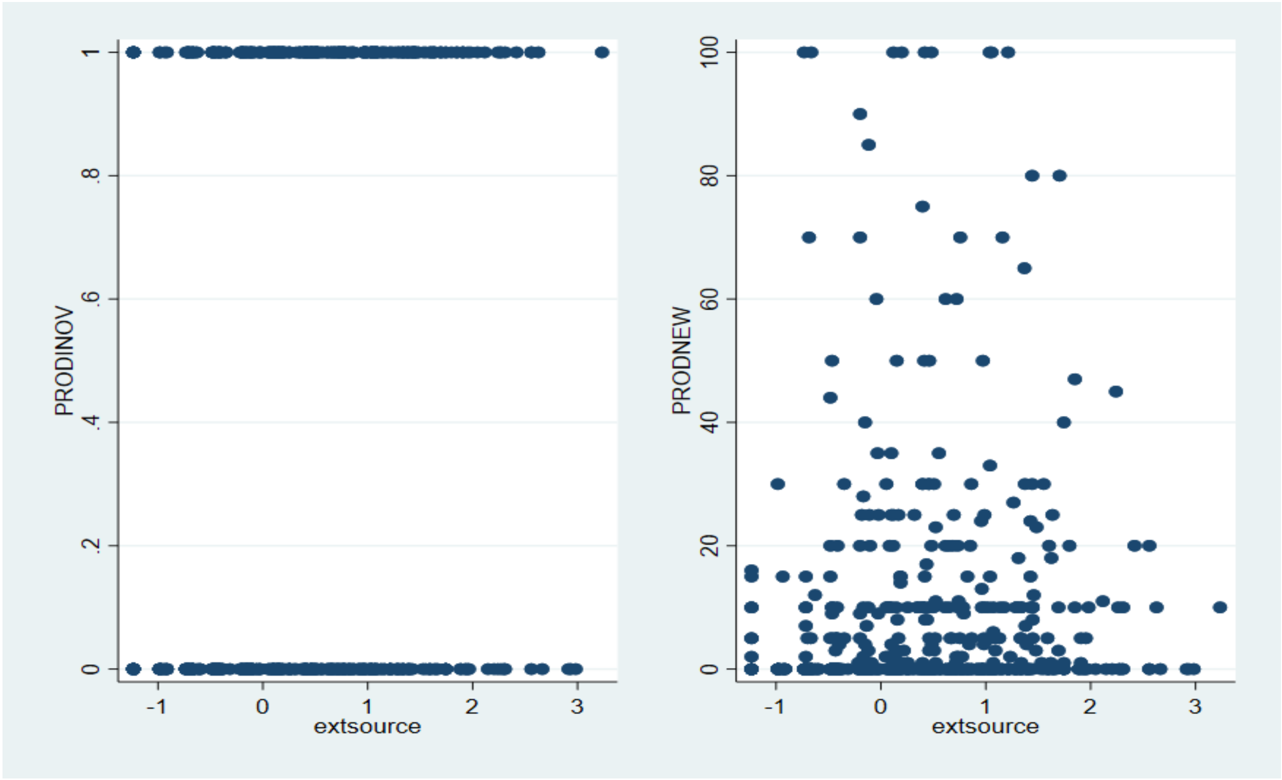
- Final extension to basic LRM: non-linear transformations of variables
- How to deal with **dependent variables** that have categorical or count outcomes?
- We will look at three (common) cases:

1. Binary response: 0/1 outcome
    - Logit (and probit) models
    - Marginal effects
    - Example: Coefficients and marginal effects
    - Comparing probit and logit models
2. Multinomial responses: More than two (unordered) outcomes
3. Count data: The possible outcomes are the natural counts 0,1,2,3,…

Logit (and probit) models

Binary response: The UK CIS example revisited



How to deal with the binary variable PRODINOV (0/1) as an outcome?

Dichotomous dependent variables

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- Running a linear regression by OLS is still a feasible option.
- In fact, the linear regression model provides a valid model for the *expected value* of Y given X, when the first 4 OLS assumptions are satisfied:

$$E(Y|X) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k$$

- The error term is heteroskedastic: We can fix that using robust s.e.
- Not always fully appropriate: the “expected” value for a particular unit could become negative, or larger than one.
- Readings:
  - Wooldridge, Introductory Econometrics, chapter 7.5, 17.1.
  - Hoetker SMJ 2006.

The logit model

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- A general binary response model:

$$P(y=1\,|\,\mathbf{x}) = G(\beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k) = G(\mathbf{x\beta}) = G(z)$$

- For the logit model, *G* is the logistic function:

$$G(z) = \exp(z)/[1+\exp(z)]$$

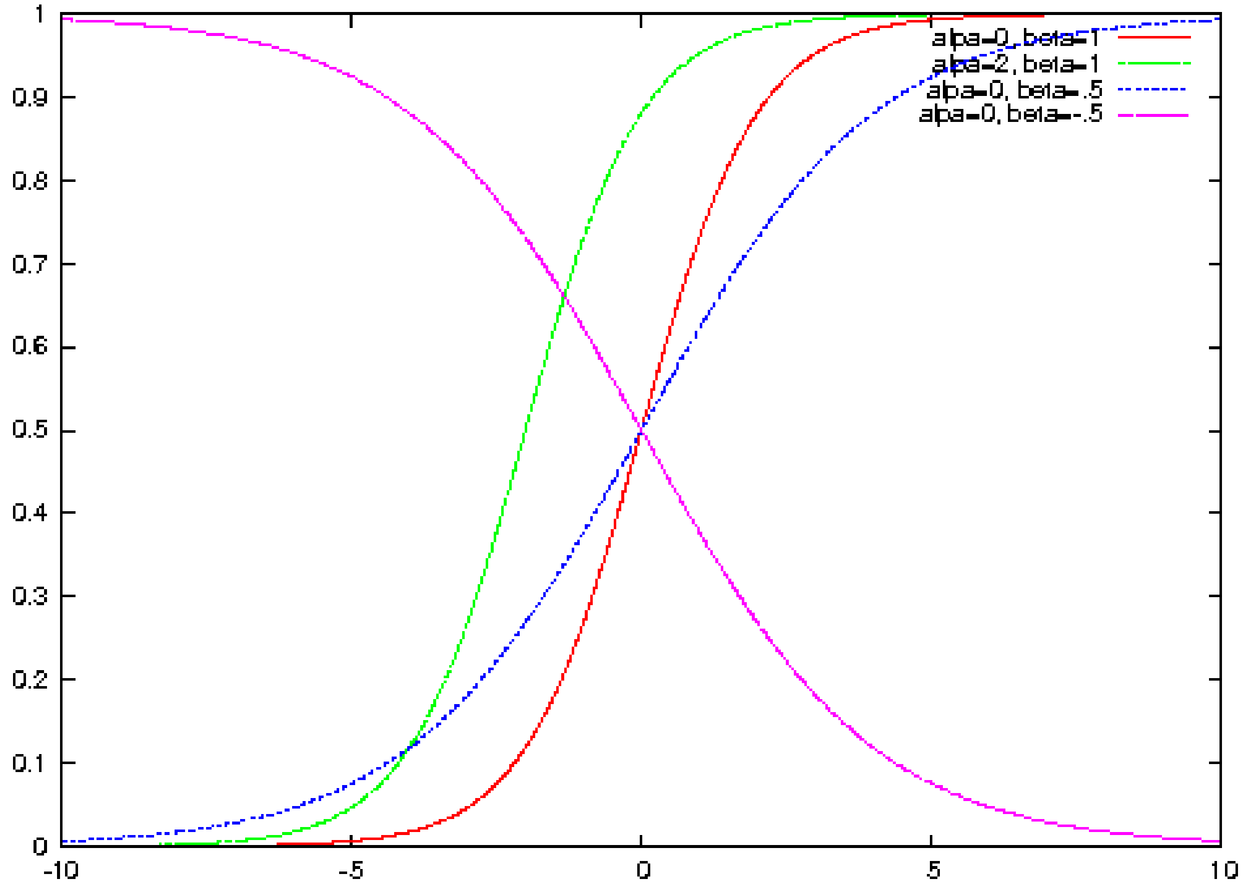
G is the cumulative distribution for a standard logistic random variable. *G* takes values strictly between 0 and 1: 0 < G(z) < 1.

- We use *G* as a model of the ***probability*** that *y* assumes the value 1 rather than 0. But this is also the expected value of Y:

$$E(y\,|\,\mathbf{x}) = 1\cdot P(y=1\,|\,\mathbf{x}) + 0\cdot P(y=0\,|\,\mathbf{x})$$
$$= P(y=1\,|\,\mathbf{x}) = G(\beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k)$$

- The model is estimated using the maximum likelihood method: maximizes **the log-likelihood function** $lnL(\beta)$ -> finds the estimates of $\beta_0, \beta_1, \ldots, \beta_k$ that make the actual outcome of y_i in the sample *i=1,2,…,n* **most likely**, given the values of the x_i's.

Logistic Function for $z = \alpha + \beta x$



The probit model

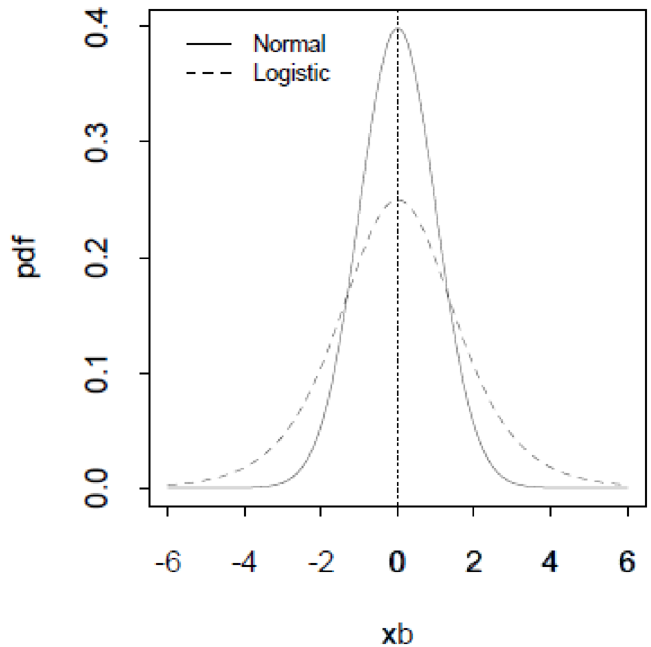
- Probit model: an alternative choice of probability distribution for a binary response model.
- Like logit, the focus is the *probability* that Y equals 1.
- In the probit model, *G* is the standard normal cumulative distribution function, which is expressed as an integral:

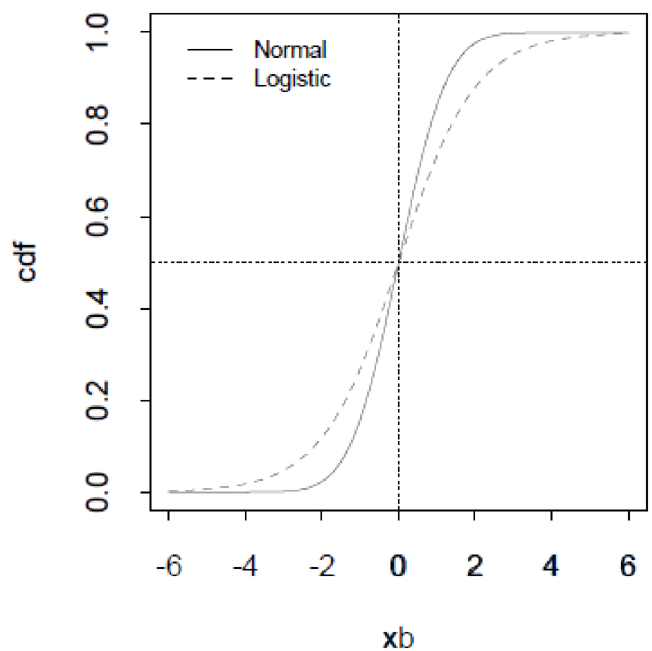
$$G(z) = \Phi(z) \equiv \int_{-\infty}^{z} \phi(v)dv$$

where $\varphi(z)$ is the standard normal density:

$$\phi(z) = (2\pi)^{-1/2} \exp(-z^2/2)$$

- Maximum likelihood is again used to estimate the β parameters.
- In many cases, the logit and probit results are "comparable." Let us focus on the logit case.





Both are symmetric; logistic has larger variance; tail probabilities differ.

Interpretation of coefficients

- Most often we are interested in the marginal effect of changing a given variable, x_j, on $E(Y|X)$, or equivalently, on $P(y=1|\, x)$.
- Because the logistic function is non-linear, the coefficients of the model (the β_j's) do not directly give the marginal effects.
- We can use the coefficients to tell us:
  - Sign: Whether there is a positive or negative effect of x_j, given that the relevant coefficient is statistically significant.
  - Significance: Whether there is an effect at all (is there a statistically significant effect? i.e., $\beta_j \neq 0$).

Example: logitexercise.do

- We estimate a logistic model, whereby product innovation (no=0/yes=1) is determined by whether or not a firm collaborates with a university and the size of the firm:

$$P(y=1|\mathbf{x}) = G(\beta_0 + \beta_1 x_1 + \beta_2 x_2)$$

- For comparison, we also run the simple linear regression.
- *y* is the indicator for innovating (prodinov)
- x_1 measures the size of the firm expressed as the log of number of employees (lempl00).
- x_2 is a binary variable expressing whether or not the firm is collaborating for innovation with a university (unic).

```
/* Note: define unic as: */
gen unic=0
replace unic=1 if punivl==1 | punivn==1 | punive==1 | punivu==1 | punivo==1
```

Logit *vs* linear regression

```
. * OLS for comparison
. reg prodinov lempl00 i.unic, r

Linear regression                               Number of obs     =        906
                                                F(2, 903)         =      54.87
                                                Prob > F          =     0.0000
                                                R-squared         =     0.1154
                                                Root MSE          =     .42903

------------------------------------------------------------------------------
             |               Robust
    prodinov | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
     lempl00 |   .0789866   .0107481     7.35   0.000     .0578924    .1000808
      1.unic |   .3990625   .0669843     5.96   0.000     .2675995    .5305255
       _cons |  -.0434095   .0415253    -1.05   0.296    -.1249069    .0380879
------------------------------------------------------------------------------


. logit prodinov lempl00 i.unic

Iteration 0:   log likelihood = -548.43996
Iteration 1:   log likelihood = -498.17582
Iteration 2:   log likelihood = -497.59473
Iteration 3:   log likelihood = -497.59414
Iteration 4:   log likelihood = -497.59414

Logistic regression                                     Number of obs =    906
                                                        LR chi2(2)    = 101.69
                                                        Prob > chi2   = 0.0000
Log likelihood = -497.59414                             Pseudo R2     = 0.0927

------------------------------------------------------------------------------
    prodinov | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
     lempl00 |   .4075079   .0568172     7.17   0.000     .2961482    .5188675
      1.unic |   1.798397   .3391331     5.30   0.000     1.133708    2.463086
       _cons |   -2.67987   .2538696   -10.56   0.000    -3.177446   -2.182295
------------------------------------------------------------------------------
```

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Logit: Marginal effects

- Logit coefficients do not directly measure marginal effects.
- The marginal effect corresponds to the effect of a one-unit increase (typically from the mean) of x_j on $P(y=1|\mathbf{x})$, holding other variables constant (typically at their means).
- Indeed, the marginal effect of x_j on the probability of obtaining y=1 (rather than y=0) depends on the value of the other variables in **x**.

Logit: Marginal effects

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- Consider the case where z = $\beta_0 + \beta_1 x_1 + \beta_2 x_2$

  That is:

  $$G(z) = \exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2)/[1 + \exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2)],$$

  Assume that x_1 is continuous: If we differentiate (check out your high school math!) partially with respect to x_1, we get:

  $$\frac{\partial G(z)}{\partial x_1} = \beta_1 \frac{\exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}{[1 + \exp(\beta_0 + \beta_1 x_1 + \beta_2 x_2)]^2}$$

- This is the formal expression for the marginal effect of x_1 in a logit model with two explanatory variables.
- From this expression we see that the marginal effect depends on the values of the x's, not only x_1, but also the value of x_2!!

Logit: Marginal effects

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- In other words, the marginal effect of x_1 will be different for different values of x_1 and x_2.
- We can examine the marginal effect at "representative" or "interesting values" of the independent variables.
- The "representative value" is normally taken to be the mean, so that the marginal effect of a given variable is examined for a one-unit increase **from the mean** with all other covariates **set to the their mean values**:
- "Conditional marginal effects" in Stata: `margins, dydx(*) atmeans`
- Or: we could calculate the marginal effect that applies to each firm with its specific values of the x's, and then average over firms:
- "Average marginal effect" in Stata: `margins, dydx(*)`

Same logit regression, but different ways to calculate marginal effects…

```
. logit prodinov lempl00 i.unic

------------------------------------------------------------------------------
    prodinov | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
     lempl00 |   .4075079   .0568172     7.17   0.000     .2961482    .5188675
      1.unic |   1.798397   .3391331     5.30   0.000     1.133708    2.463086
       _cons |   -2.67987   .2538696   -10.56   0.000    -3.177446   -2.182295
------------------------------------------------------------------------------

. margins, dydx(*)

. margins, dydx(*) atmeans
```

Use the Stata file logitexercise.do to calculate the marginal effects, and compare.

Logit: Marginal effects

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- So far: looked at the effect of a *continuous* variable on the (expectation of the) binary DV.
- However, if an independent variable, say x_2, is *binary*, the meaningfull effect is the effect of a discrete change from 0 to 1.
- The effect of changing x_2 from 0 to 1, holding other covariates constant, is given as:

$$G(\beta_0 + \beta_1 x_1 + \beta_2) - G(\beta_0 + \beta_1 x_1)$$

- Note: still varies with x_1.
- Stata will recognize this if you mark x_2 as a “factor” variable:
  `logit prodinov lempl00 i.unic`
- Output will say: `Note: dy/dx for factor levels is the discrete change from the base level.`

Measures of model fit

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- The "LR chi2(*k*)" test in the Stata output is similar to F-test of overall significance in linear regression: tells us whether all the estimates in the model combined are (in)significant.
- Under the null that there is in fact no effect of any of the explanatory variables, the test follows a chi-square distribution with *k* degrees of freedom.
- Alternatively, we can look at a (pseudo) R2. However, there are several ones available.
- All measures of fit in models for discrete data have problems (see the Hoetker 2006 SMJ article)
- So do not over-interpret these numbers!
- No simple measure of model fit equivalent to the R^2 exists for models for discrete data.

Logit vs. probit

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- There are usually no compelling theoretical grounds for preferring one over the other.
- If the outcomes in the sample are divided between a large majority and a small minority, results can differ. This is because the observations are then concentrated in a tail of the distribution where the logit and probit functions are somewhat different.
- The coefficient estimates for the logit model are approximately 1.6 times the size of those in the probit model.
- **But the marginal effects are typically strikingly similar (and close to those obtained in a linear regression).**
- Use the Stata file logitexercise.do to calculate the marginal effects for logit and probit, and compare.

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Multinomial logit

When to use a multinomial model?

- The dependent variable is categorical with more than two (mutually exclusive) outcomes
- The variable is nominal, rather than on an ordinal scale ⇔ the order of its categories plays no role in the multinomial model (except for interpretation)

  ⇨ … if outcomes are ordered (bad, OK, good), there are specialized alternatives for that (ordered probit, ordered logit)

- Examples:
  - Occupational choice (paid employment, self-employment, inactive, etc.)
  - Transport mode (bike, car, metro, etc.)

Readings:

- Cameron & Trivedi, Microeconometrics: Methods and Applications, section 15.4.
- Reichstein, T. and A. Salter (2006), Investigating the sources of process innovation among UK manufacturing firms, *Industrial and Corporate Change,* 15, 653–682. **(applied example)**
- Wooldridge, Econometric analysis of cross-section and panel data, section 15.9.1.

The multinomial logit model

- Binary response model: Two outcomes: *y = {0,1}*

$$P(y=1\,|\,\mathbf{x}) = G(\beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k) = G(x\boldsymbol{\beta})$$

- More than two outcome categories → we need more than one logit function (more than one set of betas)
  - In fact, we generally need *J* different logit functions when there are *J+1* possible outcomes in the dependent variable

- *J+1* outcomes: Label as *y=0, y=1, …, y=J*

$$P(y=j\,|\,\mathbf{x}) = \frac{\exp(x\beta_j)}{1+\sum_{h=1}^{J}\exp(x\beta_h)},\ j=1,2,\ldots,J.$$

- We model the ***probability*** that *y* takes the value *j,* rather than any of the *J* other outcomes, conditional on *x, with a* ***reference outcome*** *(e.g., y = 0)*

Example when J=2 (log odds ratios)

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- If *J=2*, there are three categories → Two logit functions are to be estimated.

- The logit function is conveniently summarized by:

$$
\left.
\begin{aligned}
g_1(\boldsymbol{x}) &= ln\frac{P(Y=1|\boldsymbol{x})}{P(Y=0|\boldsymbol{x})} = \boldsymbol{x}'\boldsymbol{\beta_1} \\
g_2(\boldsymbol{x}) &= ln\frac{P(Y=2|\boldsymbol{x})}{P(Y=0|\boldsymbol{x})} = \boldsymbol{x}'\boldsymbol{\beta_2}
\end{aligned}
\right\}
$$

*Note: The coefficients reflect the **change in the log of the relative risk between y=j and the baseline option** when x changes by 1 unit.*

- Note the **special interpretation of *beta*** in the logit specification: The ***log-odds ratios* are linear in *x*.**

Response probabilities

- Conditional probabilities in the case of three outcomes (0,1,2) are:

$$P(Y=0|\boldsymbol{x}) = \frac{1}{1+e^{g_1(\boldsymbol{x})}+e^{g_2(\boldsymbol{x})}}$$

$$P(Y=1|\boldsymbol{x}) = \frac{e^{g_1(\boldsymbol{x})}}{1+e^{g_1(\boldsymbol{x})}+e^{g_2(\boldsymbol{x})}}$$

Note: Response probabilities must sum to 1.

$$P(Y=2|\boldsymbol{x}) = \frac{e^{g_2(\boldsymbol{x})}}{1+e^{g_1(\boldsymbol{x})}+e^{g_2(\boldsymbol{x})}}$$

- Parameter estimates are found through a **maximum likelihood estimation** technique: Obtained by selecting the values of beta-parameters to make the actual sample most likely.

Multinomial logit in research: Types and sources of process innovation

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- Reichstein, T. & A. Salter (2006), Industrial and Corporate Change
- **Three values** of outcome variable: **No/Incremental/Radical** (they do not to impose any ordering!)
- “No innovation” as the reference category.
- Data from the UK CIS for 2,800+ firms.
- Characteristics of firms and their innovation strategies as explanatory variables.

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Reichstein/Salter example

- Two parameters for each explanatory variable
- E.g., the first describes how the share of sales from products new to the market influences the probability of being an **incremental innovator** <u>compared to</u> not being innovative.
- Corresponding coefficient for the outcome of the firm being a **radical innovator.**
- Wald test to see whether β_{i1} and β_{i2} are significantly different from each other.
- Post estimation (after **mlogit**) in Stata: **test [1]var=[2]var**

Table 3 Determinants of process innovation, results of multinomial logistic regression

Variables	Model 1		
	Incremental versus not innovative	Radical versus not innovative	Wald test
Share of sales from products			
New to the market	0.0176**	0.0407***	+
	(0.01)	(0.01)	
New to the firm	0.0140***	0.0115*	
	(0.00)	(0.01)	
Significantly improved	0.0111***	0.0196***	
	(0.00)	(0.01)	
Cost factor	0.9718***	1.2478***	+
	(0.08)	(0.11)	
Product factor	0.1467*	0.2774**	
	(0.09)	(0.12)	
Suppliers	0.5936***	0.4641***	
	(0.07)	(0.09)	
Customers	−0.1708**	−0.3203***	
	(0.07)	(0.10)	
Consultants	−0.2262***	−0.0744	
	(0.08)	(0.10)	
Universities	−0.0195	−0.0168	
	(0.09)	(0.11)	
Standards and regulations	−0.0277	−0.0329	
	(0.03)	(0.04)	
R & D	0.2633*	0.3905**	
	(0.15)	(0.19)	
Log (size)	0.1587***	0.2386***	
	(0.05)	(0.06)	
Investment expenditure/sales	0.0375	0.0111	
	(0.10)	(0.14)	
Training expenditure/sales	−0.0646	−0.2899	
	(0.38)	(0.83)	
Collaboration	0.6727***	1.2134***	+
	(0.16)	(0.19)	
Intercept	−2.8369***	−4.3807***	
	(0.32)	(0.45)	
Industry dummies		Yes	
Observations		2885	
Likelihood ratio		−1517.1	
Pseudo R^2		0.29	
Maximum variance inflation factor		2.51	

Limitations (1/2): IIA Assumption

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Independence from Irrelevant Alternatives (IIA) assumption

- Adding another alternative or removing one of the outcomes will not change the **relative probabilities** of the others
- Often considered very restrictive (e.g., probability of choosing between car & red bus will not change if a blue bus is also introduced)
- You may use the **mlogtest** post-estimation command in Stata to test the validity of this assumption
- Type *findit mlogtest* (part of the spost9_ado); For a Hausman test type ***mlogtest, hausman base***; for additional tests: ***mlogtest, iia***

If IIA assumption is violated, we may consider other potential models: nested logit or mixed logit *(out of our scope)*

Limitations (2/2): Marginal effects…

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…are complicated:

$$\frac{\partial P(y=j\,|\,\mathbf{x})}{\partial x_k} = P(y=j\,|\,\mathbf{x})\left(\beta_{jk} - \sum_{h=1}^{J}\beta_{hk}\exp(x\beta_h)/(1+\sum_{h=1}^{J}\exp(x\beta_h))\right), j = 1,2,\ldots,J.$$

- There is not necessarily a 1-to-1 correspondence between the beta's and the effect of changing any particular *x* variable, not even in terms of sign.
- Stata example: High school program choice ⇨ `hs_example.do`
- High school students' choice between general, vocational, and academic programs, modelled by their writing score and socio-economic status.

General vs Academic Vocational vs Academic

- https://stats.idre.ucla.edu/stata/dae/multinomiallogistic-regression/

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Count models

Count models

1. **Core model: Poisson regression**
    a. Purpose & assumptions
    b. Outcome of the regression and interpretation issues
    c. Applied examples (papers & exercises)
2. Alternative count models (Negative Binomial, Zero-Inflated Models)

Readings:

- Winkelmann, Econometric analysis of count data (2008), chapter 3.
- Cameron & Trivedi, Microeconometrics: Methods and Applications, chapter 20.
- Kaiser, U., Kongsted, H. C., Rønde, T. (2015), “Does the mobility of R&D labor increase innovation?”, *Journal of Economic Behavior & Organization*, 110, 91-105. **(applied example)**
- Wooldridge, Econometric analysis of cross-section and panel data, chapter 19.

When to use a count model?

- When the dependent variable assumes only counts (non-negative integer values): the number of occurrences of an event within a fixed period of time.
    - Number of patents applied for by a firm in a year
    - Number of emergency room episodes per day
    - Number of trades in a minute
    - Number of traffic incidents in a month
- There is no natural a priori **upper bound**.
- The **outcome is = 0 for at least some members** of the population.

Why not OLS?

Drawbacks of OLS similar to those for binary responses:

- The distribution of the dependent variable is **highly skewed.**
- In count data, $y \geq 0$, so E(y|x) should be non-negative for all x, but:
- with OLS we can have $\widehat{x\beta}<0$ **(negative values).**
- OLS will **predict non-integer values.**

A solution could be to use the log transformation: log(y) and run an OLS. But in count data, y=0 is an important part of the data (as we will see). → How to recover E(y|x) from log(y), or even log (y+1)?

Look for other functional forms that suit such data better.

Distributions of Count Data I: Poisson

Poisson probability distribution:

$$P(Y=y|\lambda)=\frac{e^{-\lambda}\lambda^{y}}{y!} \quad \text{for } y=0, 1, 2\ldots$$

- **λ** is the **mean** or **expected value** of a Poisson distribution: $\boldsymbol{E(Y)} =$ **λ**.

- **λ** is **also the variance** of a Poisson distribution: $\boldsymbol{Var(Y)} =$ **λ.**

- Poisson is a one parameter distribution (λ).

Examples of Poisson Distribution

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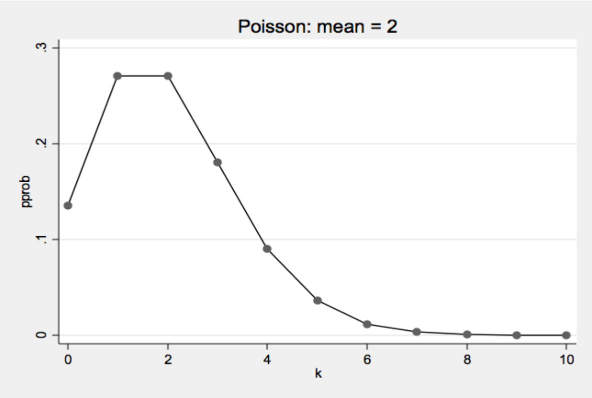

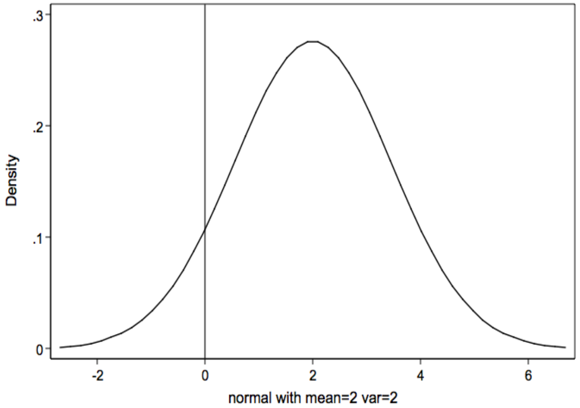

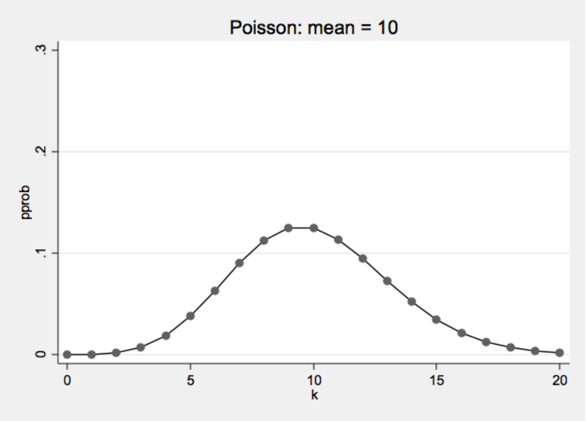

The Poisson distribution implies that the <u>mean of the variable is equal to its variance</u>

Distributions of Count Data II: Negative Binomial

The Negative Binomial takes into account **"over-dispersion"**: The variance often exceeds the mean.

- The Stata default specification is

$$Var(y) = E(y)[1 + \alpha E(y)]$$

- α is the over-dispersion parameter

- When α = 0 the negative binomial distribution is the same as a Poisson distribution. Stata provides a test that α = 0. If the data are overdispersed, then a Poisson model will be mis-specified.

- But consistency of a negative binomial regression relies on correctly specifying the variance equation.

- Poisson consistently estimates the expected value parameter **λ** irrespective of overdispersion. But important to use robust standard errors in that case.

Examples of Negative Binomial Distribution

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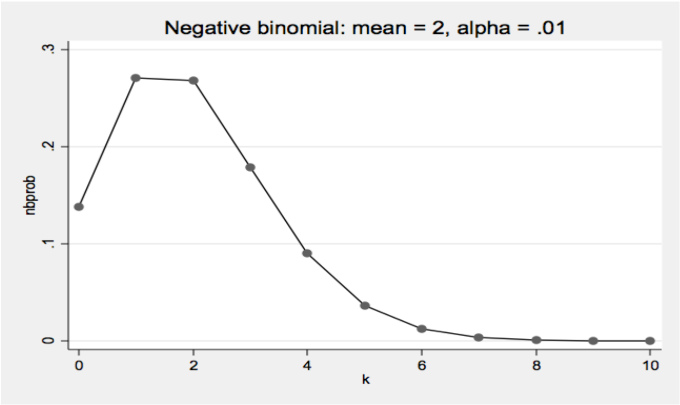

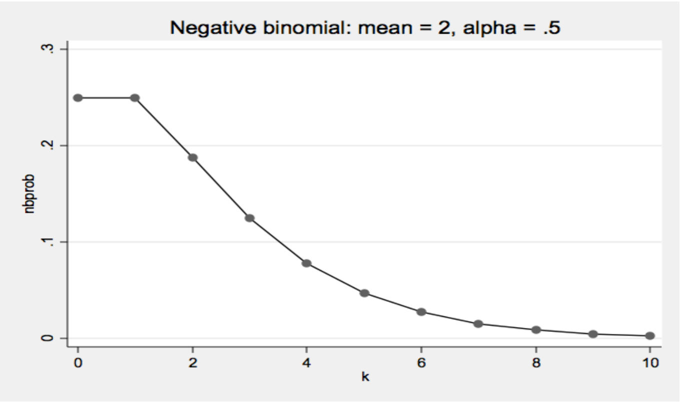

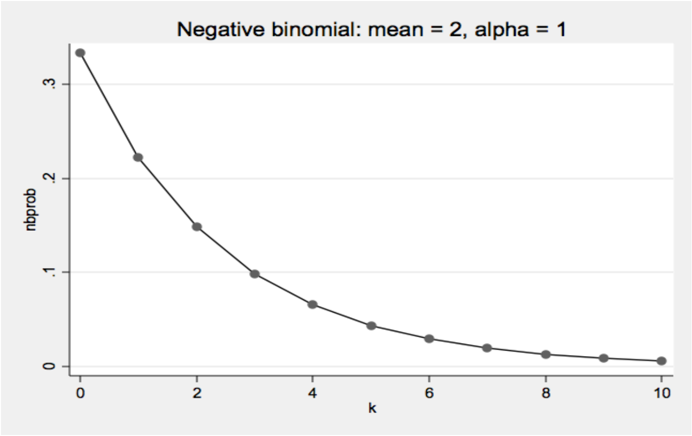

Count Data Regressions

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Model the **mean function:**

$$\lambda_i = E[Y_i \mid X_i] = \exp(X_i'\beta) = \exp(\beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k)$$

- Parameterizes the mean function as function of a set of covariates X. Count models are estimated using maximum likelihood.
- Interpretation: Log-linear function → $\log E[Y \mid X] = \beta_0 + \beta_1 x_1 + \ldots + \beta_k x_k$
- Partial effects $\dfrac{\partial E[Y \mid X]}{\partial x_j} = \exp(x\beta)\beta_j$

*Note that **marginal changes differ across individuals.** Report marginal changes at $\bar{x}$ or particular values of x.*

$$\text{Continuous variable: } \%\Delta E(y \mid X) \approx 100\beta_j \Delta x_j$$

$$\text{Dummy variable: } \%\Delta E(y \mid X) = 100(\exp(\beta_j) - 1)$$

*If the explanatory variables are in logs, the coefficients can be interpreted as **elasticities.***

Look up an example to see what that really means

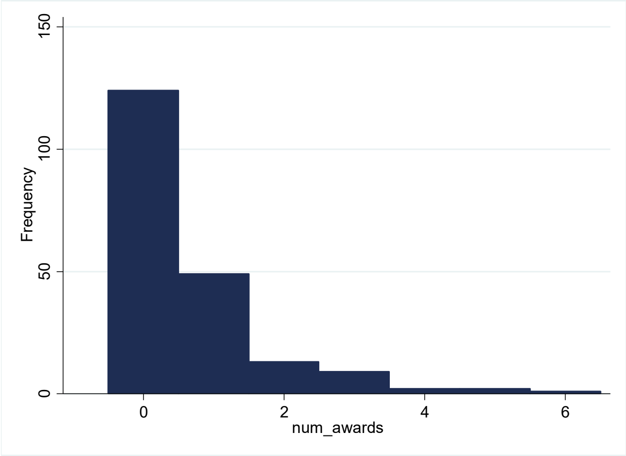
Stata Example 1: Poisson

DV: # Awards earned by students in a certain high school
Predictors: type of program (vocational, general, academic)
score in the final math exam

- Description of the data (Which distribution fits better?)
- Poisson regression (How to read the results?)
- Test the fit of the model (Does Poisson fit the data?)
- Marginal effects & Predicted counts
- Data generated to follow a Poisson distribution

Stata file `count.do`

Stata Example 1: Poisson



- The expected increase in log(# awards) for a 1-unit increase in math score is 0.07: Approx. 7 percent increase in expected number of awards.
- Compared to general program, the expected number of awards is exp(1.08) = 2.96 times the number expected for students in academic programs, or approx. 200% higher.

```
poisson num_awards i.prog math
```

```
Summary for variables: num_awards
     by categories of: prog (type of program)
```

prog	mean	sd	N
general	.2	.4045199	45
academic	1	1.278521	105
vocation	.24	.5174506	50
Total	.63	1.052921	200

```
Poisson regression                              Number of obs   =        200
                                                LR chi2(3)      =      98.22
                                                Prob > chi2     =     0.0000
Log likelihood = -182.75225                     Pseudo R2       =     0.2118
```

num_awards	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
prog						
academic	1.083859	.358253	3.03	0.002	.3816962	1.786022
vocation	.3698092	.4410703	0.84	0.402	-.4946727	1.234291
math	.0701524	.0105992	6.62	0.000	.0493783	.0909265
_cons	-5.247124	.6584531	-7.97	0.000	-6.537669	-3.95658

Goodness of Fit and Incidence Rate Ratios

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```
. poisson, irr

Poisson regression


Log likelihood = -182.75225
```

num_awards	IRR	Std. Err.	z
prog			
academic	2.956065	1.059019	3.03
vocation	1.447458	.6384309	0.84
math	1.072672	.0113695	6.62
_cons	.0052626	.0034652	-7.97

Incidence rate in academic programs is 3 times higher than in general programs, and increases 7% for every unit-increase in math scores.

Does the Poisson model fit our data?

```
estat gof

         Deviance goodness-of-fit =  189.4496
         Prob > chi2(196)         =    0.6182

         Pearson goodness-of-fit  =  212.1437
         Prob > chi2(196)         =    0.2040
```

In this case it does (no wonder, since the data are generated to fit the distribution).

If the test would be **significant**, it would indicate that the data did **not fit** the model well

(e.g., overdispersion?)

Marginal effects and predicted counts: example

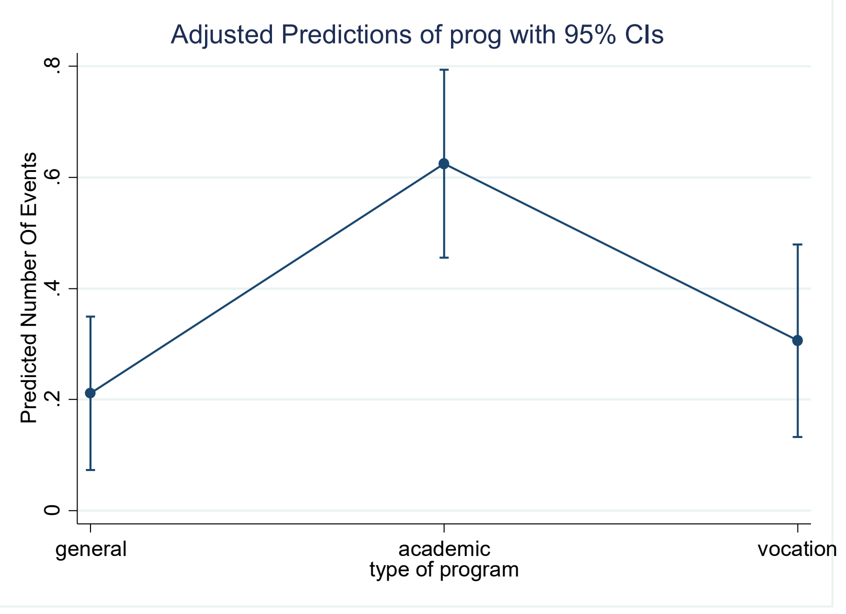
marginsplot

```
. margins prog, atmeans

Adjusted predictions                          Number of obs     =        200
Model VCE    : OIM

Expression   : Predicted number of events, predict()
at           : 1.prog          =        .225 (mean)
               2.prog          =        .525 (mean)
               3.prog          =         .25 (mean)
               math            =      52.645 (mean)

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        prog |
     general |    .211411   .0705011     3.00   0.003     .0732314    .3495906
    academic |   .6249446   .0862812     7.24   0.000     .4558366    .7940526
    vocation |   .3060086   .0883371     3.46   0.001     .1328711     .479146
------------------------------------------------------------------------------
```



Predicted # awards for general (academic) programs is 0.21 (0.62), holding math scores at its mean level.

Note that 0.6249/0.2114 = 2.96 → IRR we obtained before.

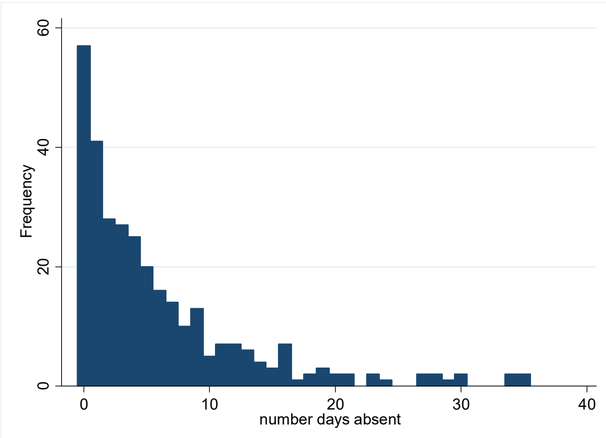
Stata Example 2: Negative Binomial

DV: Number of days absent during the school year (daysabs)

Predictors: Type of program and math scores, as before.

- Description of the data (Which distribution fits better?)
- Poisson & NegBin regression (How to read the results?)
- Test the fit of the model (Does Poisson fit the data?)
- Marginal effects & Predicted counts
- Data generated to follow a Negative Binomial distribution

Stata Example 2: Negative Binomial




```
. tabstat daysabs, by(prog) stats(mean v n)

Summary for variables: daysabs
     by categories of: prog

    prog |      mean  variance         N
---------+------------------------------
       1 |     10.65  67.25897        40
       2 |  6.934132  55.44744       167
       3 |  2.672897  13.93916       107
---------+------------------------------
   Total |  5.955414  49.51877       314
----------------------------------------
```

```
Negative binomial regression                    Number of obs =     314
                                                LR chi2(3)    =   61.69
Dispersion: mean                                Prob > chi2   = 0.0000
Log likelihood = -865.6289                      Pseudo R2     = 0.0344

------------------------------------------------------------------------------
     daysabs | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
        math |   -.005993   .0025072    -2.39   0.017     -.010907   -.001079
             |
        prog |
          2  |    -.44076    .182576    -2.41   0.016    -.7986025   -.0829175
          3  |  -1.278651   .2019811    -6.33   0.000    -1.674526    -.882775
             |
       _cons |   2.615265   .1963519    13.32   0.000     2.230423    3.000108
-------------+----------------------------------------------------------------
    /lnalpha |  -.0321895   .1027882                     -.2336506    .1692717
-------------+----------------------------------------------------------------
       alpha |   .9683231   .0995322                      .7916384    1.184442
------------------------------------------------------------------------------
```

Stata `nbreg` will estimate (as the default) the variance relationship:

$$Var(y) = E(y)[1 + \alpha E(y)]$$

Estimates are consistent if this is the correct model for the variance.

Stata Example 2: Alternative: Apply Poisson w/ robust standard errors

```
Poisson regression                                Number of obs =      314
                                                  Wald chi2(3)  =    69.35
                                                  Prob > chi2   =   0.0000
Log pseudolikelihood = -1328.6425                 Pseudo R2     =   0.1431

------------------------------------------------------------------------------
             |               Robust
     daysabs | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
        math |  -.0068084   .0023541    -2.89   0.004    -.0114223   -.0021944
             |
        prog |
          2  |  -.4398975   .1420843    -3.10   0.002    -.7183776   -.1614173
          3  |  -1.281364   .1822392    -7.03   0.000    -1.638546   -.9241819
             |
       _cons |   2.651974   .1473325    18.00   0.000     2.363207     2.94074
------------------------------------------------------------------------------
```

Technically: Poisson pseudo maximum likelihood w/ Huber-White
sandwich standard errors

Produces consistent estimates and ”correct” standard errors

In this case: Similar results.

Read more: Winkelmann (2008) chapter 3, pp. 63-126.

Too many zeros?

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When the data contain **“too many” zeros** (compared to Poisson/NegBin).

Two kinds of zeros generated by different processes? **True zeros** vs. **Excess zeros**

- In a **patenting example**, some firms active in R&D may get 0 because of **bad luck**, or they get 0 patents because **they were not active** active in R&D (”always zero”).

Zero-Inflated Models estimate the model in **two parts**:

- **Binary part**: 0 vs. 1

  (if 1, then ”always zero”; if 0, then Poisson or NegBin)

- **Count data part:** Poisson or NegBin distribution. **(zip vs zinb)**

Alternatively: restricting to subsample of ”active” firms based on independent variables (size, sector, pre-sample information).

Count models in applications: Kaiser et al. (2015)

Relationship between worker mobility and firms' inventive output (firm's patent applications per year)

	Poisson PSM		NegBin PSM	
	Coeff.	SE	Coeff.	SE
R&D worker shares				
Joiners from patenting firms	1.543***	0.400	1.608***	0.278
Joiners from non-patenting firms	0.506	0.385	0.362	0.336
Other joiners	1.238***	0.337	1.121***	0.274
Support	0.389	0.333	−0.109	0.203
Leavers to pat. firms	0.916**	0.464	0.668**	0.321
Leavers to non-pat. firms	−0.813	0.773	−0.486	0.424
Capital and R&D labor				
ln(total R&D workers)	0.384***	0.104	0.289***	0.059
ln(capital stock)	0.238***	0.068	0.138***	0.036
Lagged patent status and pre-sample variables				
Dummy patent $t-1$	2.026***	0.366	1.308***	0.138
Dummy patent $t-2$	1.080***	0.122	0.842***	0.107
ln(# pre-sample patents)	0.091	0.120	0.264***	0.087
Dummy pre-sample patent	−0.081	0.278	0.386	0.247
Number of observations and number of firms				
# of obs.	42,507		42,507	
# of firms	14,516		14,516	

Source: Kaiser et al. 2015, JEBO

What's next?

Friday at 9.00 on Zoom:

Workshop 3

Next lecture:

November 9 at 9.00:

Vera Rocha on Attrition and Selection Models