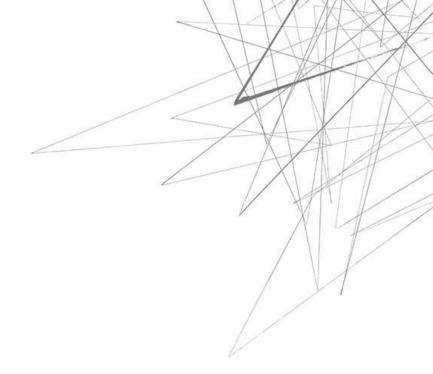
Sample Selection and Heckman Models

Applied Econometrics for Researchers, PhD Vera Rocha, CBS-SI, vr.si@cbs.dk
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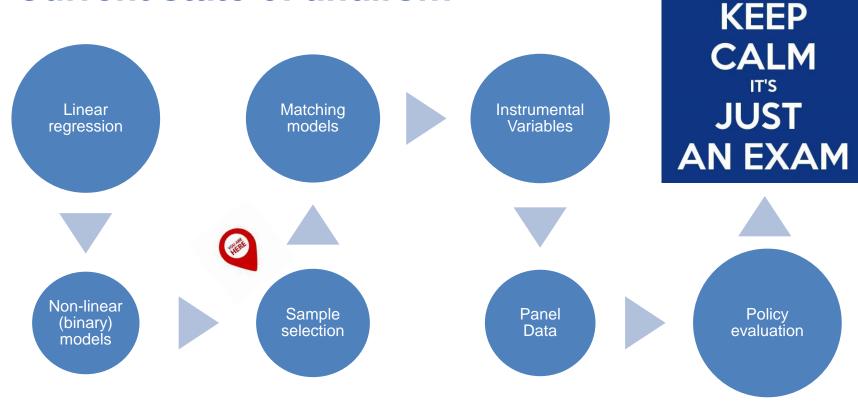








Current state of affairs...



Agenda for today

- 1. Forms of sample selection
- Implications of sample selection bias
- 3. Econometric models addressing sample selection (Heckman 2-step model)
- Sample selection in research (example)
- 5. Heckman models in Stata

Key readings:

- Certo, S. T., Busenbark, J. R., Woo, Y.,
 Semadeni, M. (2016), "Sample selection bias and Heckman models in strategic management research", Strategic Management Journal, 37, 2639-57.
- Cameron & Trivedi, Section 16.5 &
 Wooldridge, Sections 17.1-17.4.1.
- Naldi, L. Davidsson, P. (2014), "Entrepreneurial growth: The role of international knowledge acquisition as moderated by firm age", *Journal of Business Venturing*, 29, 687-703. (optional; only an applied example)



Assumptions of a Linear Regression Model

- 1. The regression model is linear in β and additive in u
- 2. The observations have been obtained as a random sample
- 3. No x variable is a linear function of (one or more) of the other x's (i.e., no exact multicollinearity)
- 4. u has a zero population mean and all x's are uncorrelated with u (zero correlation)
- 5. *u* has a constant variance (no heteroskedasticity)
- 6. *u* is normally distributed



OLS:
1-4: consistency
1-5: efficiency
6. Not needed
in "large
samples"



Assumptions of a Linear Regression Model

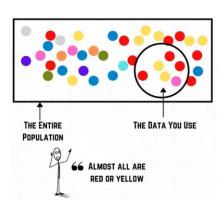
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Forms of Sample Selection (1/2)

I. Missing data

- Units could simply be "missing at random": we randomly miss observations
 on one or more of the variables that enter our model (testable!)
- Reduced sample, but <u>Assumption 2 is not violated</u>.

II. Deliberate selective sampling: e.g., stratified sampling

- Divide population into strata: exhaustive & non-overlapping groups
- Possible risk of over/undersampling of certain groups
- Still not problematic for Assumption 2



Forms of Sample Selection (2/2)

III. Non-random sampling

> A) <u>Exogenous</u> sample selection:

 Sample selection based on the independent variables; or, more generally, on factors that are independent of the error term of the equation we are considering

> B) Endogenous sample selection:

• Sample selection based on the **dependent variable**; or, more generally, on factors that are **related** to the error term of the equation we are considering

Examples of Non-Random Sampling (1/3)

A) Exogenous sample selection (sampling based on independent variables)

E.g.: Estimating a saving function

- $saving = \beta_0 + \beta_1 income + \beta_2 age + u$.
- A survey of adults with age > 45 → Nonrandom sample based on age
- But age is assumed to be exogenous (not correlated with the error term)
- The factor that determines the selection into the sample is <u>independent</u> of the error term
- OLS estimator is unbiased



Examples of Non-Random Sampling (2/3)

B1) Endogenous sample selection (sampling based on dependent variables)

E.g.: Determinants of income when income is top-coded at *k*

- $y = \beta_0 + \beta_1 x + u$.
- Actual y can only be observed if $y \le k$, where k is the top-coded value
- Often done to increase reporting rates
- Selection based on y causes a **selection bias** on $\widehat{\beta_1}$ if we use OLS
- Use Tobit (censored regression)* model instead, whenever k (cut-off point) is known/observed

^{*} Out of our scope in this course, but you can get a flavor of it here



Examples of Non-Random Sampling (3/3)

B2) Endogenous sample selection (sampling based on dependent variables)

E.g.: Effect of human capital (education & experience) on wages

- $y = \beta_0 + \beta_1 educ + \beta_2 exp + \cdots + u$
- y is only observed for individuals who are actually working; not all do!
- Potential non-random (i.e., selected) sample decision to work is likely related to unobserved factors that also influence wages (e.g., ability; reservation wages; education quality; network)
- Error term becomes correlated with one or several x's.

Incidental truncation problem: today's focus!

Non-Random Sampling: More Formally

Participation equation (e.g., labor market participation)

$$s = \begin{cases} 1 & if & s^* > 0 \\ 0 & if & s^* \le 0 \end{cases}$$

where
$$s^* = X_1' \beta_1 + \varepsilon_1$$

Outcome equation (e.g., wages)

$$y = \begin{cases} y^* & if & s^* > 0 \\ - & if & s^* \le 0 \end{cases}$$

where $y^* = X_2' \beta_2 + \varepsilon_2$

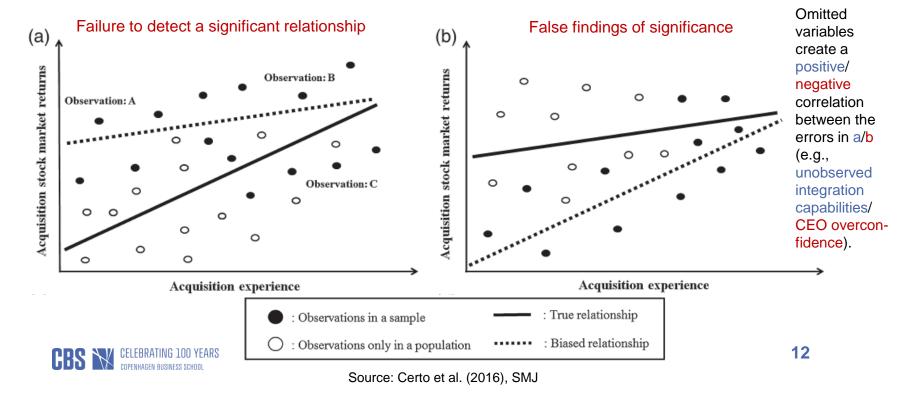
 ε_2 correlated with some variable(s) in X_2 (through the unobserved factors W) [when run on the observed subsample]

Likely correlated!
Unobserved factors
W may affect both
y and s

The cause of selection bias

Implications of Sample Selection Bias

Example: The role of acquisition experience (x) in stock market reactions to acquisition announcements (y). y only available for firms that actually complete acquisitions.



Heckman Two-Step Estimation Method

The two-step model (recall slide 11)

- The outcome equation: $y = X_2'\beta_2 + \varepsilon_2$, $\varepsilon_2 | X_2 \sim N(0, \sigma^2)$
- The selection equation: s=1 [$X_1'\beta_1+\varepsilon_1\geq 0$], where s=1 if we observe y, and zero otherwise; $\varepsilon_1|X_1\sim N(0,1)$

Assumptions

• Elements of X_1 and X_2 are always observed and are exogenous: $E(\varepsilon_1|X_1,X_2)=0, E(\varepsilon_2|X_1,X_2)=0$

• X_2 is a subset of X_1 : any x_{2j} is an element of X_1 , but some elements of X_1 are not in X_2 (exclusion restrictions)



Heckman Two-Step Model: Underlying Logic

The cause of bias

- $cov(\varepsilon_1, \varepsilon_2) \neq 0$ in the selected sample, so:
- $E(y|X_2, \varepsilon_2, s) = X_2'\beta_2 + \gamma_1 E(\varepsilon_2|\varepsilon_1)$ (instead of $X_2'\beta_2$ only)

The Heckman solution

- obtain a term to correct for the $E(\varepsilon_2|\varepsilon_1)$ term [the *first* step]
- use the term as an extra explanatory variable in the outcome equation [the second step]
- this (in principle!*) removes the part of the error that is correlated with X₂

How to estimate $E(\varepsilon_2|\varepsilon_1)$?

- If ε_1 and ε_2 are jointly normal, $E(\varepsilon_2|\varepsilon_1) = \rho \varepsilon_1$, where ρ is the correlation between ε_1 and ε_2 .
- We go from estimating $E(\varepsilon_2|\varepsilon_1)$ to estimating $E(\varepsilon_1|X_1,s=1)$ \rightarrow probit model in the first step

^{*}dependent on strong exclusion restrictions



Heckman Two-Step Model: Underlying Logic

...estimating $E(\varepsilon_1|X_1,s=1)$ with a probit model in the first step:

When
$$s=1$$
 (selection equation), $E(\varepsilon_1|X_1,s=1)=\phi(X_1'\beta_1)/\Phi(X_1'\beta_1)$

The Inverse Mills Ratio, $\lambda(X_1'\beta_1)$: a ratio beween the std normal pdf and std normal cdf, evaluated at $X_1'\beta_1$.

New Outcome Equation: $E(y|X_2, s = 1) = X_2'\beta_2 + \gamma_1\lambda(X_1'\beta_1)$

- If $\rho = 0$, no selection bias (The correlation between ε_1 and ε_2 does not cause a sample selection problem);
- If $\rho \neq 0$, a consistent estimate for β_1 using the *selected* sample can only be obtained if we include the IMR $-\lambda(X_1'\beta_1)$ as an additional regressor.



Heckman Model in Steps: Wrap-up

The first step

- Use the entire sample (all N observations) to estimate a <u>probit model</u> of s_i on X_{1i} $P(s=1|X_1)=\Phi(X_1'\beta_1)$
- Compute the Inverse Mills Ratio, $\widehat{\lambda}_i = \lambda(X_1'\widehat{\beta}_1)$ for each i. (Actually, we only need these for the observations with $s_i = 1$)

The second step

• Using the selected sample $(s_i = 1)$, run the regression $y = X_2' \beta_2 + \gamma_1 \lambda (X_1' \widehat{\beta_1}) + \varepsilon_2$ to obtain $\widehat{\beta_2}$, which is consistent and approximately normally distributed.

A test of selection bias: use the t statistic on $\hat{\lambda}$ (i.e., test significance of γ_1) as a test of H_0 : $\rho = 0$. (There is no sample selection problem)



Heckman 2-Step vs. ML Estimation

Note that: X_2 must be a subset of X_1

- If $X_1 = X_2$, $\hat{\lambda}$ can be highly correlated with $X_2 \to \text{high std errors for } \widehat{\beta_2}$ and identification hinges on functional form only: Not credible.
- Excluding one or more X_2 variables from X_1 helps identifying the $\widehat{\beta}_2$ parameters (but needs a theoretical argument!) **exclusion restrictions**
- Excluding x-variables from X_1 can lead to inconsistency (and would be hard to defend)

Heckman two-step estimation is a <u>second-best</u> alternative to ML

- Consistent but inefficient
- Introducing a measurement error problem



Sample Selection in Research: Example

Hypothesis 1. Acquisition of knowledge from international markets has a positive effect on a firm's <u>subsequent</u> entrepreneurial growth as reflected in an increase of

- a) sales generated in geographically new domestic markets,
- b) sales generated in geographically new international markets,
- c) sales from new products in domestic markets; and
- d) sales from new products in international markets.

Naldi & Davidsson (2014), JBV

• Sample of Swedish SMEs – initially 2,455 >>> 1,633 interviewed by phone, out of which 885 had international activity (eligible!) ←

218/885 suspended operations during the 6-y follow-up period

Final sample of 138 firms after all follow-ups

Sample attrition

Based on the DV!

Sample selection

(bias)



Using Heckman 2-step to address both issues

Sample selection bias

- Probit model for the likelihood of internationalization
- Exclusion restriction: whether or not the firm had a Swedish name

Sample attrition bias

- Probit model where the DV measures whether the firm participated in all survey rounds
- Exclusion restriction: firm's location in a metropolitan area

Respective IMRs obtained and used as a regressor in the 2nd step.

(How much do we trust these exclusion restrictions?)



Example

- Selection bias due to the fact that y is only observed for firms with international activity is significant, but does not seem to bias the coefficient of interest (acquisition of knowledge)
- Positive coefficient for Mills (selection bias) could suggest that unobserved factors that make firms more likely to internationalize also increase their growth through international sales. (though not a major concern here)



CEO age		Sales from n	new internation	nal markets	
CEO gender 3.18* 3.17* 3.02 2.10* (1.51) (1.50) (1.54) (0.96) CEO education -0.26 -0.31 -0.33 -0.50 (0.47) (0.45) (0.45) (0.45) (0.46) CEO prior leadership experience 0.017 0.058 0.050 0.060 (0.46) (0.48) (0.48) (0.48) (0.48) CEO prior experience-same industry -0.097 -0.091 -0.10 0.50 (0.45) (0.46) (0.47) (0.44) CEO prior experience-other industries 0.22 0.19 0.17 0.0070 (0.44) (0.45) (0.47) (0.40) Manufacturer 0.23 0.13 -0.034 -1.43 (0.51) (0.54) (0.85) (0.73) Service 0.70 0.54 0.52 -0.47 (0.53) (0.56) (0.56) (0.56) Retailer -12.7*** -12.8*** -11.6*** -12.2 (0.70) (0.69) (0.87) (0.70) Past performance -0.27 -0.27 -0.23 -0.65 (0.42) (0.41) (0.39) (0.29) Firm size -1.38 -1.37 -1.40 -0.96 (0.74) -0.75) -0.72) -0.96 Acquisition of knowledge 1.09*** 1.10*** 1.09*** 1.20** Firm age -0.0044 0.0015 0.0011 -0.00 (0.0087) (0.0088) (0.008 Mills (attrition) -0.31 (1.07) Mills (selection bias) -7.96**	·	Model 5a	Model 6a	Model 7a	Model 8a
CEO gender 3.18* 3.17* 3.02 2.10* (1.51) (1.50) (1.54) (0.96) CEO education -0.26 -0.31 -0.33 -0.50 (0.47) (0.45) (0.45) (0.45) (0.46) CEO prior leadership experience 0.017 0.058 0.050 0.060 (0.46) (0.48) (0.48) (0.48) (0.48) CEO prior experience-same industry -0.097 -0.091 -0.10 0.50 (0.45) (0.46) (0.47) (0.44) CEO prior experience-other industries 0.22 0.19 0.17 0.0077 (0.44) (0.45) (0.47) (0.40) Manufacturer 0.23 0.13 -0.034 -1.43 (0.51) (0.54) (0.85) (0.73) Service 0.70 0.54 0.52 -0.47 (0.53) (0.56) (0.56) (0.56) (0.61) Retailer -12.7*** -12.8*** -11.6*** -12.2 (0.70) (0.69) (0.87) (0.70) Past performance -0.27 -0.27 -0.23 -0.55 (0.42) (0.41) (0.39) (0.29) Firm size -1.38 -1.37 -1.40 -0.96 (0.74) - (0.75) - (0.72) - (0.85) Acquisition of knowledge 1.09*** 1.10*** 1.09*** 1.20** Firm age -0.0044 0.0015 0.0011 -0.00 Acquisition of knowledge * Firm age -0.0044 0.0015 0.0011 -0.00 Mills (attrition) -0.31 (1.07) Mills (selection bias) -7.96**	CEO age	-0.11**	-0.11**	-0.11**	-0.11**
(1.51) (1.50) (1.54) (0.96) CEO education			(0.037)		(0.036)
CEO education	CEO gender	3.18*	3.17*	3.02	2.10*
CEO prior leadership experience (0.47) (0.45) (0.45) (0.46) CEO prior experience-same industry -0.097 -0.091 -0.10 0.50 CEO prior experience-other industries 0.22 0.19 0.17 0.007 CEO prior experience-other industries 0.22 0.19 0.17 0.007 Manufacturer 0.23 0.13 -0.034 -1.43 Manufacturer 0.70 0.54 0.52 -0.47		(1.51)	(1.50)	(1.54)	(0.96)
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(0.46) (0.48) (0.48) (0.48) (0.48) (EO prior experience-same industry		(0.47)	(0.45)	(0.45)	(0.46)
CEO prior experience-same industry	CEO prior leadership experience	0.017	0.058	0.050	0.060
(0.45) (0.46) (0.47) (0.44) CEO prior experience-other industries 0.22 0.19 0.17 0.0070 (0.44) (0.45) (0.47) (0.40) Manufacturer 0.23 0.13 -0.034 -1.43 (0.51) (0.54) (0.85) (0.73) Service 0.70 0.54 0.52 -0.47 (0.53) (0.56) (0.56) (0.61) Retailer -12.7*** -12.8*** -11.6*** -12.2 (0.70) (0.69) (0.87) (0.70) Past performance -0.27 -0.27 -0.23 -0.65 (0.42) (0.41) (0.39) (0.29) Firm size -1.38 -1.37 -1.40 -0.96 (0.74) (0.75) (0.72) (0.86) Acquisition of knowledge 1.09*** 1.10*** 1.09*** 1.20** (0.29) Firm age -0.0044 0.0015 0.0011 -0.000 (0.008) (0.0087) (0.0074) (0.0072) Acquisition of knowledge * Firm age (0.0081) (0.0072) (0.0074) (0.0074) (0.0087) (0.0088) (0.0088) Mills (attrition) -0.31 (1.07) Mills (selection bias) 7.96**		(0.46)	(0.48)	(0.48)	(0.48)
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Manufacturer 0.23 0.13 -0.034 -1.43 (0.51) (0.54) (0.85) (0.73) (0.51) (0.54) (0.85) (0.73) (0.51) (0.54) (0.52) -0.47 (0.53) (0.56) (0.56) (0.56) (0.51) (0.57) (0.58) (0.56) (0		(0.45)	(0.46)	(0.47)	(0.44)
Manufacturer 0.23 0.13 -0.034 -1.43 (0.51) (0.54) (0.85) (0.73) Service 0.70 0.54 0.52 -0.47 (0.53) (0.56) (0.56) (0.61) Retailer -12.7*** -12.8*** -11.6*** -12.2 (0.70) (0.69) (0.87) (0.70) Past performance -0.27 -0.27 -0.23 -0.65 (0.42) (0.41) (0.39) (0.29) Firm size -1.38 -1.37 -1.40 -0.96 (0.74) (0.75) (0.72) (0.86) Acquisition of knowledge 1.09*** 1.10*** 1.09*** 1.20** (0.29) (0.26) (0.25) (0.26) Firm age -0.0044 0.0015 0.0011 -0.00 Acquisition of knowledge * Firm age -0.010 -0.01 -0.01 Mills (attrition) -0.31 (0.0087) (0.0088) (0.008 Mills (selection bias) 7.96**	CEO prior experience-other industries	0.22	0.19	0.17	0.0070
Service 0.70 0.54 (0.85) (0.73) Service 0.70 0.54 0.52 -0.47 (0.53) (0.56) (0.56) (0.56) (0.61) Retailer -12.7*** -12.8*** -11.6*** -12.2 (0.70) (0.69) (0.87) (0.70) Past performance -0.27 -0.27 -0.23 -0.65 (0.42) (0.41) (0.39) (0.29) Firm size -1.38 -1.37 -1.40 -0.96 (0.74) (0.75) (0.72) (0.86) Acquisition of knowledge 1.09*** 1.10*** 1.09*** 1.20** (0.29) (0.26) (0.25) (0.26) Firm age -0.0044 0.0015 0.0011 -0.00 (0.0081) (0.0072) (0.0074) (0.007 Acquisition of knowledge * Firm age (0.0081) (0.0072) (0.0088) (0.008 Mills (attrition) -0.31 (1.07) Mills (selection bias) 7.96**		(0.44)	(0.45)	(0.47)	(0.40)
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Retailer (0.53) (0.56) (0.56) (0.61) Retailer (-12.7*** -12.8*** -11.6*** -12.2 (0.70) (0.69) (0.87) (0.70) Past performance (0.42) (0.41) (0.39) (0.29) Firm size (0.74) (0.75) (0.72) (0.86) Acquisition of knowledge (0.29) (0.26) (0.25) (0.26) Firm age (0.09) (0.0072) (0.0074) (0.0072) Acquisition of knowledge * Firm age (0.0081) (0.0072) (0.0088) (0.0088) Mills (attrition) (0.0087) (0.0088) (0.0088) Mills (selection bias) 7.96**		(0.51)	(0.54)	(0.85)	(0.73)
Retailer -12.7*** -12.8*** -11.6*** -12.2 (0.70) (0.69) (0.87) (0.70) Past performance -0.27 -0.27 -0.23 -0.65 (0.42) (0.41) (0.39) (0.29) Firm size -1.38 -1.37 -1.40 -0.96 (0.74) (0.75) (0.72) (0.86) Acquisition of knowledge 1.09*** 1.10*** 1.09*** 1.20** (0.29) (0.26) (0.25) (0.26) Firm age -0.0044 0.0015 0.0011 -0.00 Acquisition of knowledge * Firm age -0.010 -0.01 -0.01 Mills (attrition) -0.31 (0.0087) (0.0088) (0.008 Mills (selection bias) 7.96**	Service	0.70	0.54	0.52	-0.47
Past performance (0.70) (0.69) (0.87) (0.70) Past performance -0.27 -0.27 -0.23 -0.65 (0.42) (0.41) (0.39) (0.29) Firm size -1.38 -1.37 -1.40 -0.96 (0.74) (0.75) (0.72) (0.86) Acquisition of knowledge 1.09*** 1.10*** 1.09*** 1.20** (0.29) (0.26) (0.25) (0.26) Firm age -0.0044 0.0015 0.0011 -0.00 (0.0081) (0.0072) (0.0074) (0.0074 Acquisition of knowledge * Firm age (0.0087) (0.0088) (0.008 Mills (attrition) -0.31 (1.07) Mills (selection bias) 7.96**		(0.53)	(0.56)	(0.56)	(0.61)
Past performance (0.70) (0.69) (0.87) (0.70) Past performance -0.27 -0.27 -0.23 -0.65 (0.42) (0.41) (0.39) (0.29) Firm size -1.38 -1.37 -1.40 -0.96 (0.74) (0.75) (0.72) (0.86) Acquisition of knowledge 1.09*** 1.10*** 1.09*** 1.20** (0.29) (0.26) (0.25) (0.26) Firm age -0.0044 0.0015 0.0011 -0.00 (0.0081) (0.0072) (0.0074) (0.0074 Acquisition of knowledge * Firm age (0.0087) (0.0088) (0.008 Mills (attrition) -0.31 (1.07) Mills (selection bias) 7.96**	Retailer	-12.7^{***}	-12.8***	-11.6***	-12.2^{***}
Firm size					(0.70)
Firm size	Past performance	-0.27	-0.27	-0.23	-0.65^{*}
(0.74) (0.75) (0.72) (0.86) Acquisition of knowledge 1.09*** 1.10*** 1.09*** 1.20** (0.29) (0.26) (0.25) (0.26) Firm age -0.0044 0.0015 0.0011 -0.00 (0.0081) (0.0072) (0.0074) (0.007 Acquisition of knowledge * Firm age (0.0087) (0.0088) (0.008 Mills (attrition) -0.31 (1.07) Mills (selection bias) 7.96**		(0.42)	(0.41)	(0.39)	(0.29)
Acquisition of knowledge 1.09*** 1.10*** 1.09*** 1.20**	Firm size	-1.38	-1.37	-1.40	-0.96
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		(0.74)		(0.72)	
Firm age -0.0044 0.0015 0.0011 -0.00 0.0072 0.0074 0.0074 0.0075 0.0011 0.0074 0.0075 0.0074 0.0074 0.0075 0.0074 0.0074 0.0075 0.0074 0.0074 0.0075 0.0087 0.0088 0.0088 0.0088 0.0089 0	Acquisition of knowledge	1.09***	1.10***	1.09***	1.20***
Firm age		(0.29)	(0.26)	(0.25)	(0.26)
Acquisition of knowledge * Firm age		-0.0044	0.0015	0.0011	-0.0080
(0.0087) (0.0088) (0.0088) Mills (attrition) -0.31 (1.07) Mills (selection bias) 7.96**		(0.0081)	(0.0072)	(0.0074)	(0.0078)
Mills (attrition) — 0.31 (1.07) Mills (selection bias) 7.96**	Acquisition of knowledge * Firm age		-0.010	-0.01	-0.01
(1.07) Mills (selection bias) 7.96** (2.30)			(0.0087)	(0.0088)	(0.0084)
Mills (selection bias) 7.96** (2.30)	Mills (attrition)			-0.31	
(2.30)				(1.07)	
(2.30)	Mills (selection bias)				7.96***
					(2.30)
Constant -4.04^* -3.87^* -3.47 -6.72	Constant	-4.04^{*}	-3.87^*	-3.47	-6.72^{***}
(1.90) (1.95) (2.59) (1.83)		(1.90)	(1.95)	(2.59)	(1.83)
LL -3.04 -3.03 -3.03 -2.97	LL	-3.04	-3.03	-3.03	-2.97

When can selection bias be really serious?

 Inverse Mills Ratio has a significant coefficient ⇔ the error terms of the selection and outcome equations are significantly correlated (i.e., ρ ≠ 0).

AND

- The independent variable(s) of interest must be a significant predictor in the selection equation
 - (test the significance of their coefficients in the first stage; also check the correlation between the *x* of interest and the *IMR*)

All in all, the significance of the IMR alone may not indicate (serious) sample selection bias. (Certo et al., 2016)



Heckman Models in STATA

dummy = 1 if wage observed (selection indicator) **Key STATA commands:** Heckman selection model (Two-Step) heckman wage edu exp, select married children edu exp) twostep possible exclusion restrictions outcome of interest (variables from X_1 not in X_2) (only observed for a selected sample: in = 1) Heckman selection model (Maximum Likelihood) heckman wage edu exp, select (in = married children edu exp)



Note that

<u>edu</u>and exp are

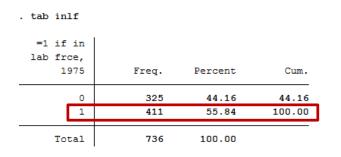
present in

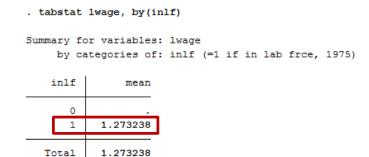
equations!

both

STATA example

Female wage offers in a sample of American women





Only 411 women in the labor force. Iwage is only observed for this subsample.

We are interested in the effect of **education** and **experience** on women's wage.

Any potential problem?



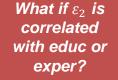
OLS in a selected sample

. reg lwage educ exper expersq

Source	SS	df	MS		er of ob		411
				F(3,	407)	=	30.53
Model	26.761859	3	8.92061966	Prob	> F	=	0.0000
Residual	118.924353	407	.292197428	R-sq	uared	=	0.1837
				- Adj	R-square	d =	0.1777
Total	145.686212	410	.355332225	Root	MSE	=	.54055
lwage	Coef.	Std. Err.	t	P> t	[95% (Conf.	Interval]
educ	.1056312	.0117748	8.97	0.000	.0824	841	.1287783
exper	.024	.0110258	2.18	0.030	.0023	254	.0456746
expersq	0004584	.0003257	-1.41	0.160	0010	986	.0001819
_cons	2742848	.166226	-1.65	0.100	6010	535	.052484

Only 411
women in the
labor force.
lwage is only
observed for
this subsample.

 $lwage = \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 exper^2 + \varepsilon_2$





Digging into the potential problem

educ	byte	%9.0g		years of	schooling		
Two-sample t test with equal variances							
Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]	
0	325 411	11.79692 12.6837	.1210353 .1120819	2.181995 2.272251	11.55881 12.46337	12.03504 12.90403	
combined	736	12.29212	.0838368	2.274435	12.12753	12.45671	
diff	8867752 .165744				-1.212164	5613864	
diff = mean(0) - mean(1) $t = -5.350$ Ho: diff = 0 degrees of freedom = 73							
	iff < 0 = 0.0000	Pr(Ha: diff != T > t) =	-		iff > 0) = 1.0000	

Any potential unobserved factors that can be correlated with both variables (and Iwage)?
What if we are observing a *positive selection* of women?



Women currently in the labor force are more educated and experienced than women outside of the labor force.

exper	byte	%9.0g		actual 1	labor mkt exper	
Two-sample	e t test wi	th equal var	riances			
Group	Obs	Mean	Std. Err.	Std. Dev	7. [95% Conf.	Interval]
0	325 411	7.461538 13.23844	.3837731 .3972984			
combined	736	10.6875	.2983866	8.095025	10.10171	11.27329
diff		-5.776904	.5622216		-6.880658	-4.67315
diff :	= mean(0) - = 0	mean(1)		degre	t ees of freedom	= -10.2751 = 734
	iff < 0) = 0.0000	Pr(Ha: diff !=			iff > 0) = 1.0000

Probit model of labor force participation

- . ** Probit model of labor force participation <=> Heckman 1st step
- . probit inlf educ exper expersq nwifeinc age kidslt6 kidsge6

```
Iteration 0: log likelihood = -505.12037
Iteration 1: log likelihood = -389.9303
Iteration 2: log likelihood = -389.31823
Iteration 3: log likelihood = -389.3179
Iteration 4: log likelihood = -389.3179
```

Probit regression Number of obs = 730LR chi2(7) = 231.60Prob > chi2 = 0.0000

Log likelihood = -389.3179

MUMBEL OF ODS	_	730	
LR chi2(7)	=	231.60	_
Prob > chi2	=	0.0000	Е
Pseudo R2	=	0.2293	tl
P> z [95%	Conf.	Interval]	

4 new variables included in this probit (1st step) model: reasoning?

inlf	Coef.	Std. Err.	Z	P> z	[95% Conf.	Interval]
educ	.1350915	.0258422	5.23	0.000	.0844417	.1857413
exper	.1308743	.0190863	6.86	0.000	.0934659	.1682828
expersq	0020464	.0006063	-3.38	0.001	0032348	0008581
nwifeinc	0115492	.0048753	-2.37	0.018	0211045	0019939
age	0534212	.0086097	-6.20	0.000	0702958	0365466
kidslt6	8738286	.1205009	-7.25	0.000	-1.110006	6376511
kidsge6	.036665	.0439668	0.83	0.404	0495083	.1228383
_cons	.1505762	.5179195	0.29	0.771	8645273	1.16568
	educ exper expersq nwifeinc age kidslt6 kidsge6	educ .1350915 exper .1308743 expersq0020464 nwifeinc0115492 age0534212 kidslt68738286 kidsge6 .036665	educ .1350915 .0258422 exper .1308743 .0190863 expersq0020464 .0006063 nwifeinc0115492 .0048753 age0534212 .0086097 kidslt68738286 .1205009 kidsge6 .036665 .0439668	educ .1350915 .0258422 5.23 exper .1308743 .0190863 6.86 expersq0020464 .0006063 -3.38 nwifeinc0115492 .0048753 -2.37 age0534212 .0086097 -6.20 kidslt68738286 .1205009 -7.25 kidsge6 .036665 .0439668 0.83	educ .1350915 .0258422 5.23 0.000 exper .1308743 .0190863 6.86 0.000 expersq0020464 .0006063 -3.38 0.001 nwifeinc0115492 .0048753 -2.37 0.018 age0534212 .0086097 -6.20 0.000 kidslt68738286 .1205009 -7.25 0.000 kidsge6 .036665 .0439668 0.83 0.404	educ .1350915 .0258422 5.23 0.000 .0844417 exper .1308743 .0190863 6.86 0.000 .0934659 expersq0020464 .0006063 -3.38 0.0010032348 nwifeinc0115492 .0048753 -2.37 0.0180211045 age0534212 .0086097 -6.20 0.0000702958 kidslt68738286 .1205009 -7.25 0.000 -1.110006 kidsge6 .036665 .0439668 0.83 0.4040495083

Use this probit model to estimate the IMR:

$$\frac{\phi(X_1'\beta_1)}{\Phi(X_1'\beta_1)}$$



OLS with mills as a regressor

. reg lwage educ exper expersq mills

Source	SS	df	MS	Number of obs	=	411
				F(4, 406)	=	22.86
Model	26.7758022	4	6.69395054	Prob > F	=	0.0000
Residual	118.91041	406	.292882783	R-squared	=	0.1838
				- Adj R-squared	=	0.1757
Total	145.686212	410	.355332225	Root MSE	=	.54119
'						
lwage	Coef.	Std. Err.	t	P> t [95% Co	onf.	<pre>Interval]</pre>

	lwage	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
	educ exper	.1068455 .0258102 0004956	.0130363 .0138088 .0003681	8.20 1.87 -1.35	0.000 0.062 0.179	.0812184 0013354 0012193	.1324727
	mills	.0241741	.1107941	0.22	0.827	1936276	.2419759
'	_cons	317652	.2592319	-1.23	0.221	8272563	.1919523

The t-test for the coefficient of *mills* suggests that selection bias is not a problem in the current data.



Heckman 2-step instead

- . * Automatically instead, using Heckman 2step
- . heckman lwage educ exper expersq, select(inlf = educ exper expersq nwifeinc age kidslt6 kidsge6) twostep

Heckman selection model -- two-step estimates (regression model with sample selection)

Number of obs	=	736
Censored obs	=	325
Uncensored obs	=	411

Wald chi2(3) 68.00 Prob > chi2 0.0000

Same results as an OLS including mills as a regressor.

	Coef.	Std. Err.	z	P> z	[95% Conf	. Interval]
lwage						
educ	.1068455	.0129602	8.24	0.000	.081444	.1322471
exper	.0258102	.013727	1.88	0.060	0010942	.0527145
expersq	0004956	.0003659	-1.35	0.176	0012129	.0002216
_cons	317652	.2577073	-1.23	0.218	8227489	.187445
inlf						
educ	.1350915	.0258422	5.23	0.000	.0844417	.1857413
exper	.1308743	.0190863	6.86	0.000	.0934659	.1682828
expersq	0020464	.0006063	-3.38	0.001	0032348	0008581
nwifeinc	0115492	.0048753	-2.37	0.018	0211045	0019939
age	0534212	.0086097	-6.20	0.000	0702958	0365466
kidslt6	8738286	.1205009	-7.25	0.000	-1.110006	6376511
kidsge6	.036665	.0439668	0.83	0.404	0495083	.1228383
_cons	.1505762	.5179195	0.29	0.771	8645273	1.16568
mills						
lambda	.0241741	.110135	0.22	0.826	1916865	.2400347
rho	0.04492				•	
sigma	.53814503					
229						



0.0449	rho
.5381450	sigma

Heckman Model by Maximum Likelihood

	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
lwage						
educ	.1063089	.0124263	8.56	0.000	.0819539	.1306639
exper	.0250121	.0125961	1.99	0.047	.0003242	.0497
expersq	0004792	.0003482	-1.38	0.169	0011617	.0002033
_cons	2985193	.2219991	-1.34	0.179	7336295	.136591
inlf						
educ	.1355116	.0259857	5.21	0.000	.0845806	.1864425
exper	.1309137	.0190936	6.86	0.000	.093491	.1683364
expersq	0020482	.0006068	-3.38	0.001	0032375	000859
nwifeinc	0116317	.0049004	-2.37	0.018	0212364	002027
age	0534266	.0086108	-6.20	0.000	0703035	0365497
kidslt6	8729929	.120636	-7.24	0.000	-1.109435	6365507
kidsge6	.0364668	.0439742	0.83	0.407	049721	.1226546
_cons	.1474565	.51828	0.28	0.776	8683536	1.163267
/athrho	.0251354	.1536882	0.16	0.870	276088	.3263589
/lnsigma	6199196	.0349263	-17.75	0.000	6883739	5514654
rho	.0251301	.1535912			2692806	.3152453
sigma	.5379877	.0187899	,		.5023924	.576105
lambda	.0135197	.0826562			1484835	.1755229

- Conclusions about
 educ and exper? An
 additional year of educ
 (exp) is associated with
 a 10.6% (2.5%*)
 increase in wage
 offered (note that wage is in logs).
- OLS would be consistent in this case, since selection bias is not a problem.

*on average, but at a sligthly decreasing rate

Can we trust our exclusion restrictions?

. reg lwage educ exper expersq nwifeinc age kidslt6 kidsge6

Source	SS	df	MS	Number of obs	=	411
				F(7, 403)	=	13.62
Model	27.8667467	7	3.98096382	Prob > F	=	0.0000
Residual	117.819465	403	.292355994	R-squared	=	0.1913
				Adj R-squared	=	0.1772
Total	145.686212	410	.355332225	Root MSE	=	.5407

lwage	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
educ	.1006009	.0125667	8.01	0.000	.0758964	.1253054
exper	.0230481	.0111876	2.06	0.040	.0010547	.0450414
expersq	0003973	.0003322	-1.20	0.232	0010504	.0002558
nwifeinc	.0042206	.0027117	1.56	0.120	0011103	.0095516
age	0035799	.0044829	-0.80	0.425	0123926	.0052329
kidslt6	0746493	.0730267	-1.02	0.307	2182103	.0689116
kidsge6	0141123	.0230222	-0.61	0.540	059371	.0311463
_cons	1127322	.2647116	-0.43	0.670	6331203	.4076559

test nwifeinc age kidslt6 kidsge6

```
(1) nwifeinc = 0
(2) age = 0
(3) kidslt6 = 0
(4) kidsge6 = 0
```

```
F(4, 403) = 0.94

Prob > F = 0.4379
```

By using these 4 variables as exclusion restrictions, we are assuming they are jointly insignificant in the main (wage) equation. They are indeed irrelevant for wages.



Yet, remember that selection bias may be serious in many other settings



"I'm pathetic, uninformed and don't give a damn.
Do you still want to continue with the poll?"



Wrap-up of today and next sessions

	Heckman models (11/09)	Matching Models (PSM) (11/16)	Instrumental Variables (11/23)
When	Y is missing in some cases (for a non-random reason)		
Problem	The missings in Y are driven by a "selection process"		
Stata commands	heckman, (twostep)		
Key tests	Significance of the IMR or of the <i>rho</i>		
Attention!	Need for valid exclusion restrictions; selection bias important when <i>IMR/rho</i> significant and <i>X</i> predicts selection (1st stage)		
First stage	Probit predicting selection into the sample (Y ≠ missing)		

