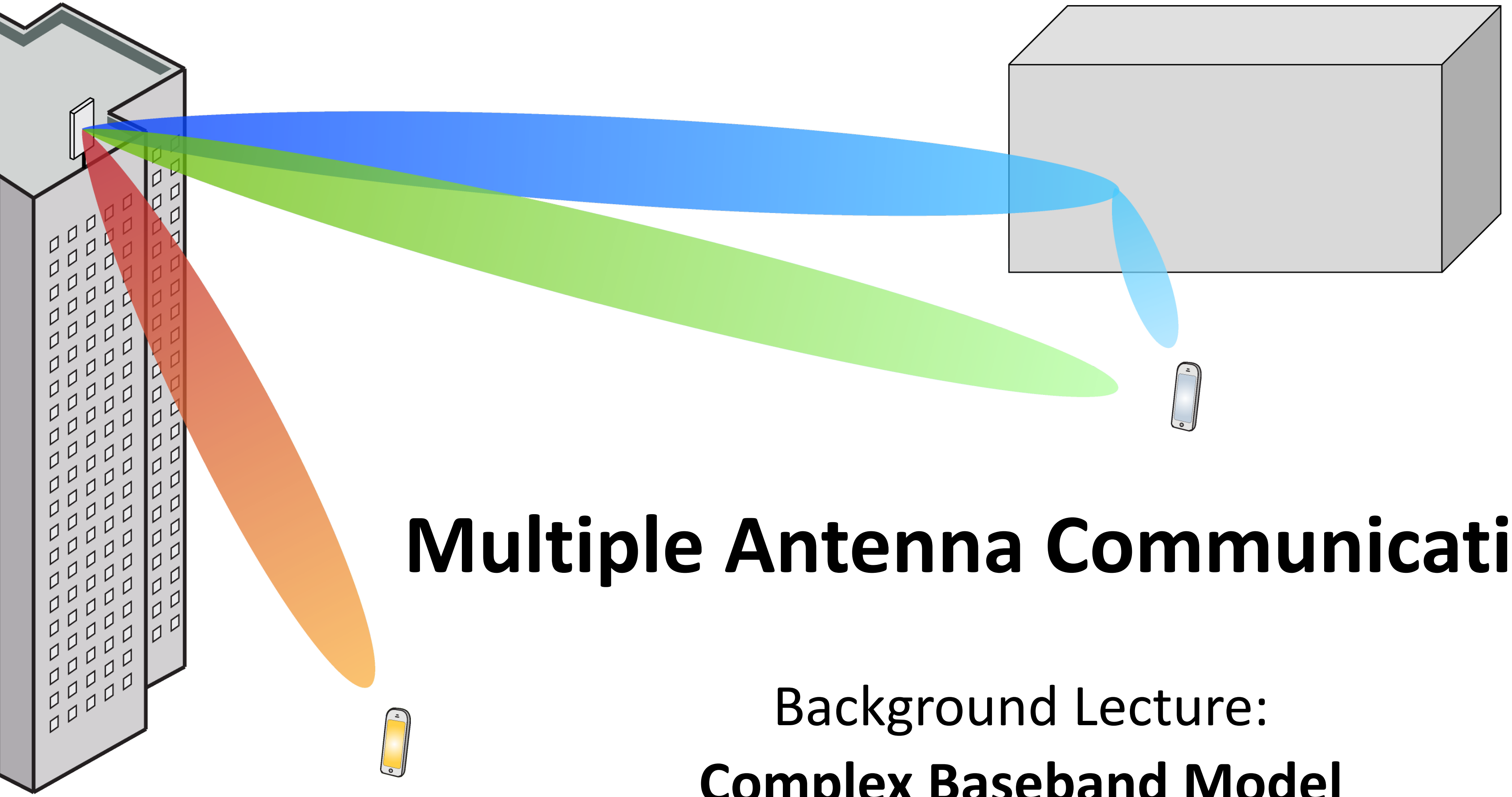


Multiple Antenna Communications

Background Lecture:

Complex Baseband Model

Emil Björnson

Outline

- Signals and systems
  - Fourier transform
- Complex baseband representation
  - Signals and communication channels
- Pulse amplitude modulation
  - Received signal
  - Narrowband channel

Linear time-invariant system

Impulse (Dirac function)

$$\delta(t):\ \int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$$

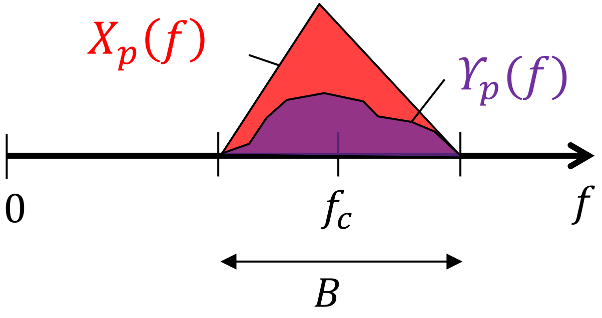
- System in time-domain:

$z_p(t)$ → Linear-time invariant: Impulse response $h_p(t)$ → $v_p(t) = (h_p * z_p)(t) = \int_{\infty}^{-\infty} h_p(u) z_p(t-u)du$

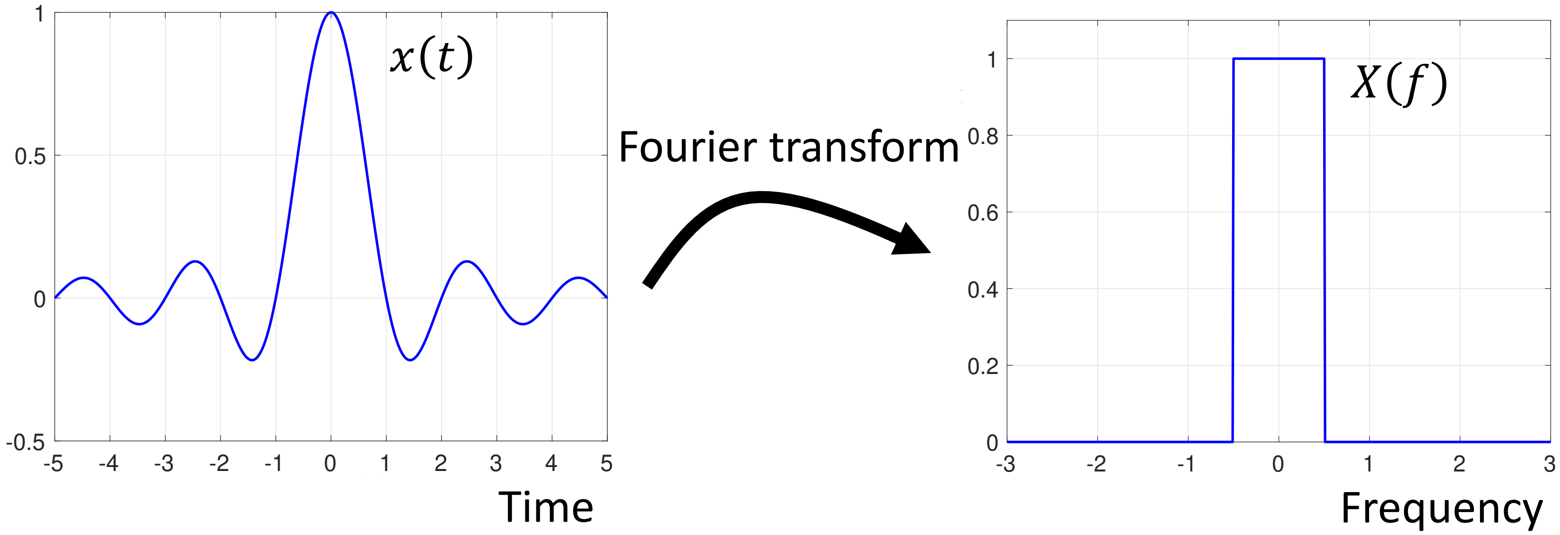
- System in frequency domain:

$Z_p(f)$ → Linear-time invariant: Frequency response $H_p(f)$ → $Y_p(f) = H_p(f)\, Z_p(f)$

Passband signal:



Fourier transform



- Definition: $X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$
  - Exists if $\int_{-\infty}^{\infty} |x(t)| dt < \infty$

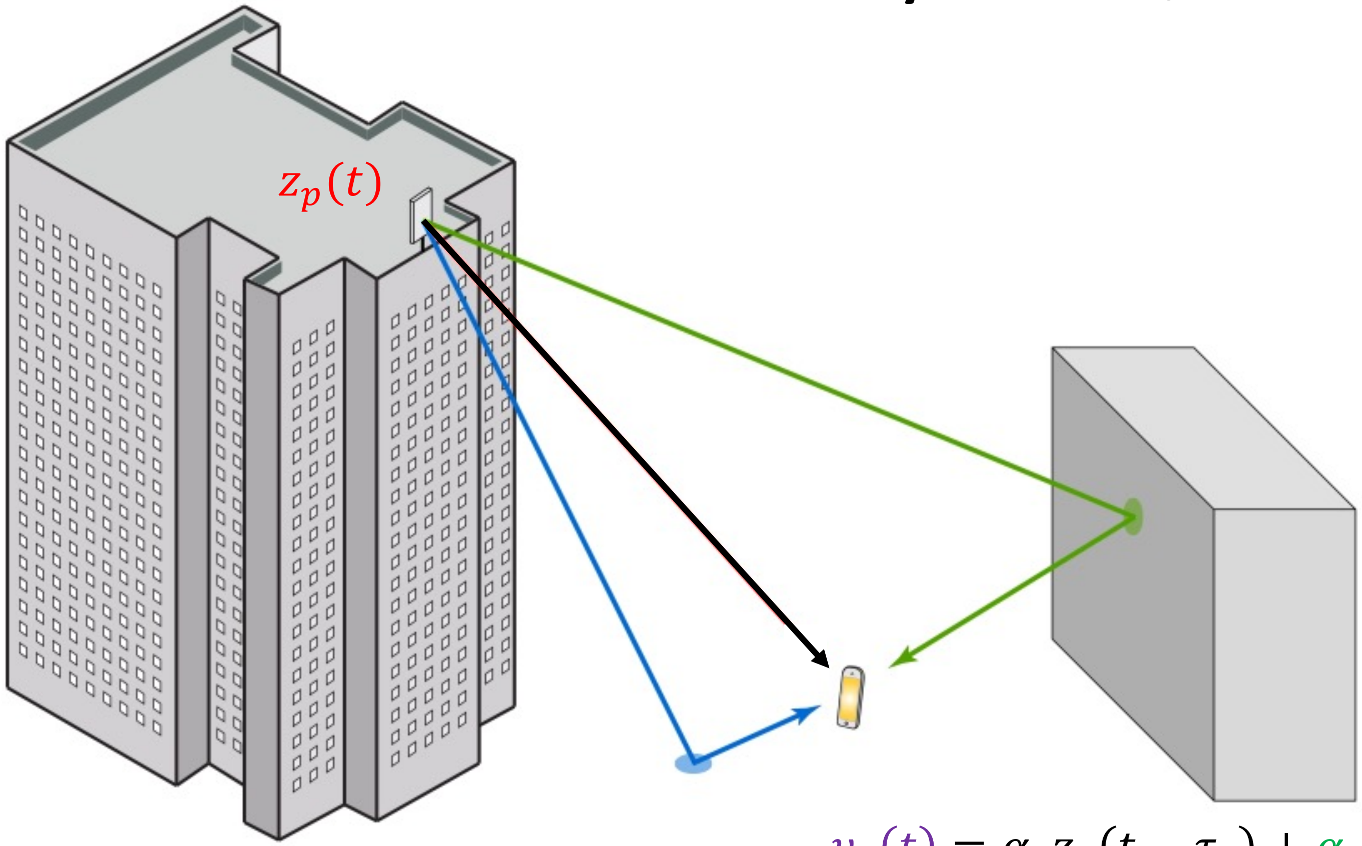
Example: Cosine

$$x(t) = \cos(2\pi f_c t)$$
$$X(f) = \frac{1}{2}\delta(f - f_c) + \frac{1}{2}\delta(f + f_c)$$

Example: Sinc-function

$$x(t) = \text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$
$$X(f) = 1 \text{ for } |t| \leq 1/2$$

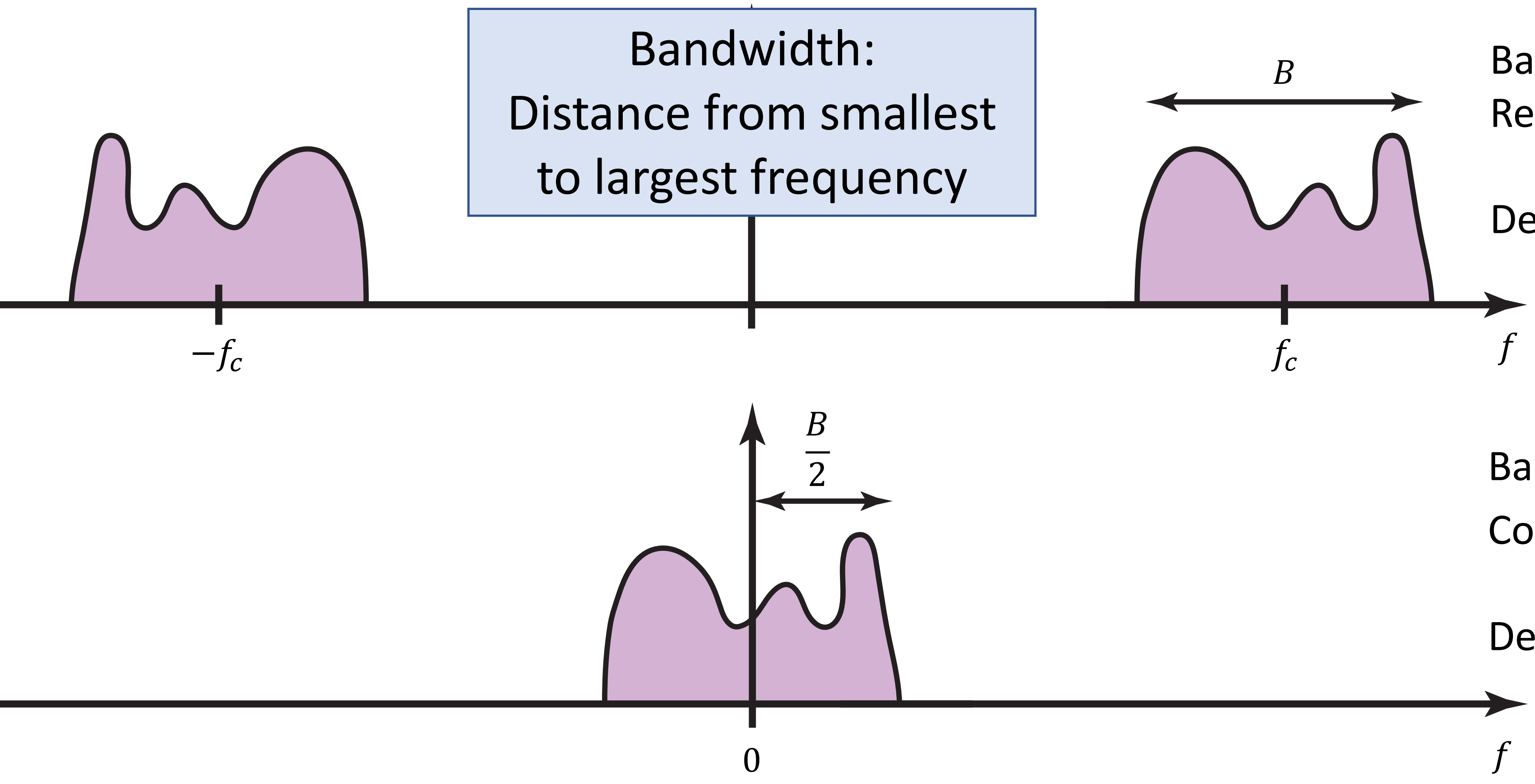
Wireless channels are systems/filters



$$v_p(t) = \alpha_1 z_p(t-\tau_1) + \alpha_2 z_p(t-\tau_2) + \alpha_3 z_p(t-\tau_3)$$

Impulse response: $h_p(t) = \alpha_1 \delta(t-\tau_1) + \alpha_2 \delta(t-\tau_2) + \alpha_3 \delta(t-\tau_3)$

Sampling theorem: Passband and baseband signals



Bandwidth (passband): B
Real-valued signal

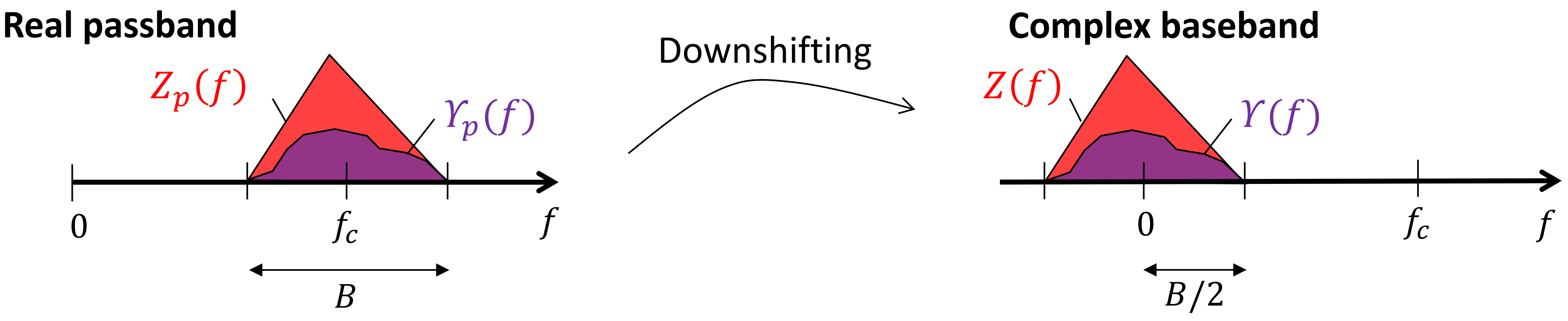
Described by $2B$ real samples/s

Bandwidth (baseband): $\frac{B}{2}$
Complex-valued signal

Described by B complex samples/s

These samples are used to convey information

Communication theory is not developed for a passband



Connection: $Z_p(f) = \frac{Z(f-f_c)+Z^*(-f-f_c)}{\sqrt{2}}$, $Y_p(f) = \frac{Y(f-f_c)+Y^*(-f-f_c)}{\sqrt{2}}$

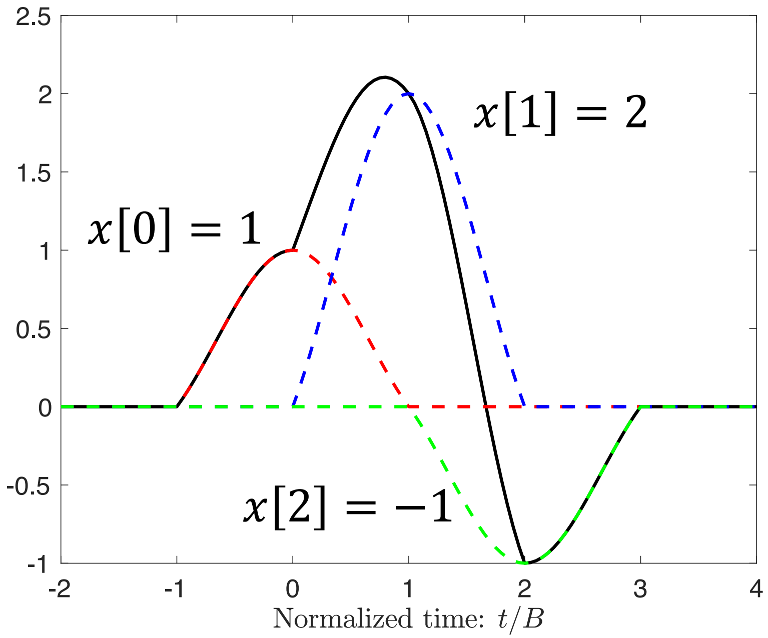
- Equivalent complex-baseband system:

$z(t)$ → Linear-time invariant: Impulse response $h(t)$ → $v(t) = (h * z)(t) = \int_{\infty}^{-\infty} h(u)z(t-u)du$

Downshifted channel:

$$h(t) = h_p(t)e^{-j2\pi f_c t}, \qquad H(f) = H_p(f + f_c)$$

How to transmit data?



- Transmit discrete sequence $x[k]$, $k =$ integer
- Pulse amplitude modulation:

$$z(t) = \sum_k x[k] p\left(t - \frac{k}{B}\right)$$

Use a pulse-form $p(t)$ satisfying the Nyquist criterion:

$p\left(\frac{k}{B}\right) = 0$ for integer $k \neq 0$ and non-zero for $k = 0$

- **Example:** $p(t) = \sqrt{B}\mathrm{sinc}(Bt)$

Sampling of signal $z(t)$ at time $t = l/B$:

$$z\left(\frac{l}{B}\right) = \sum_k x[k] p\left(\frac{l-k}{B}\right) = x[l]$$

Reception with channel and noise

- Received signal (with complex Gaussian noise):

$$\mu(t) = (h * z)(t) + w(t)$$

1. Lowpass filtering using $p(t) = \sqrt{B}\,\mathrm{sinc}(Bt)$:

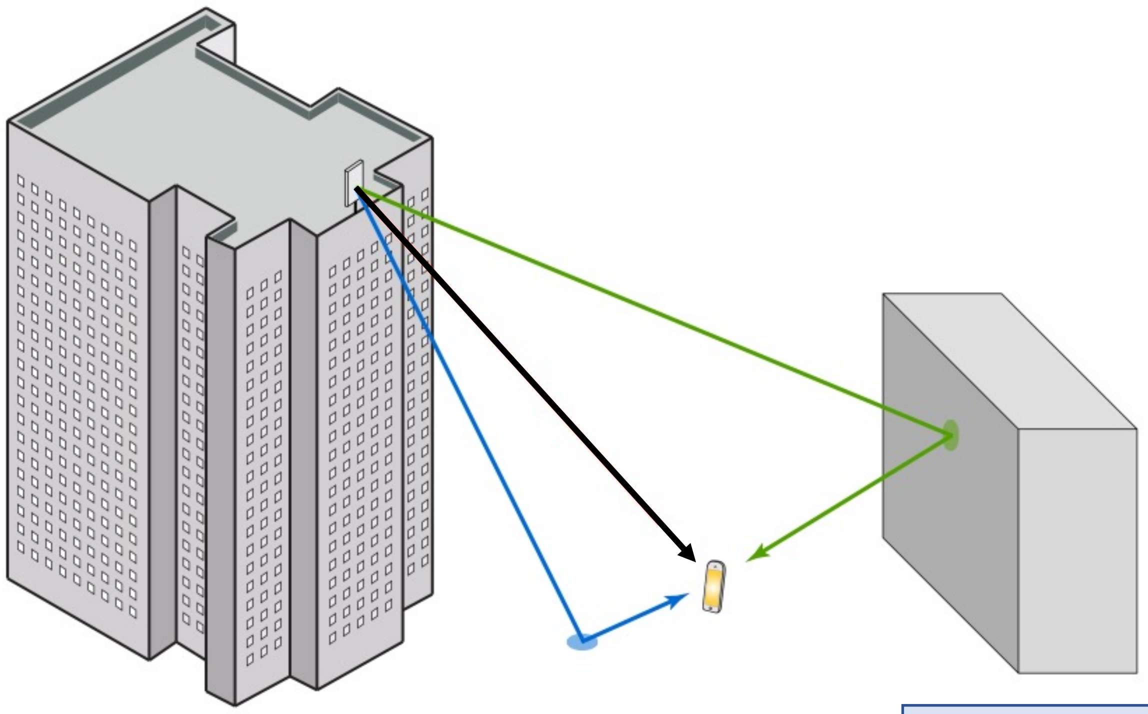
$$(p * \mu)(t) = \sum_k x[k]\,(p * h * p)\left(t - \frac{k}{B}\right) + (p * w)(t)$$

2. Sample at time $t = l/B$:

$$y[l] = (p * \mu)(t)\Big|_{t=l/B} = \sum_k x[k]\,\underbrace{(p * h * p)}_{\text{Effective pulse function}}\left(\frac{l-k}{B}\right) + \underbrace{(p * w)\left(\frac{l}{B}\right)}_{n[l]\ \text{Complex Gaussian noise}\ CN(0, N_0)}$$

Channel with memory?

Revisiting channel model



Original channel:

Propagation delay

$$h_p(t) = \sum_{i=1}^{L} \alpha_i \delta(t + \eta - \tau_i)$$

Amplitude loss

Synchronization

Equivalent in complex baseband:

$$h(t) = \sum_{i=1}^{L} \alpha_i e^{-j2\pi f_c t} \delta(t + \eta - \tau_i)$$

$$(p * h * p)\left(\frac{l-k}{B}\right) = \sum_{i=1}^{L} \alpha_i e^{-j2\pi f_c(\tau_i - \eta)} \operatorname{sinc}\big((l-k) + B(\eta - \tau_i)\big)$$

Narrowband assumption

- Small variations in delays (set $\eta \approx \tau_i$):

$$\operatorname{sinc}\big((l-k) + B(\eta - \tau_i)\big) \approx \operatorname{sinc}(l-k) = \begin{cases} 1, & l = k \\ 0, & l \neq k \end{cases}$$

- Symbol-sampled system model:

$$y[l] = \sum_k x[k] \underbrace{\sum_{i=1}^{L} \alpha_i e^{-j2\pi f_c(\tau_i - \eta)}}_{g} \operatorname{sinc}(l-k) + n[l]$$

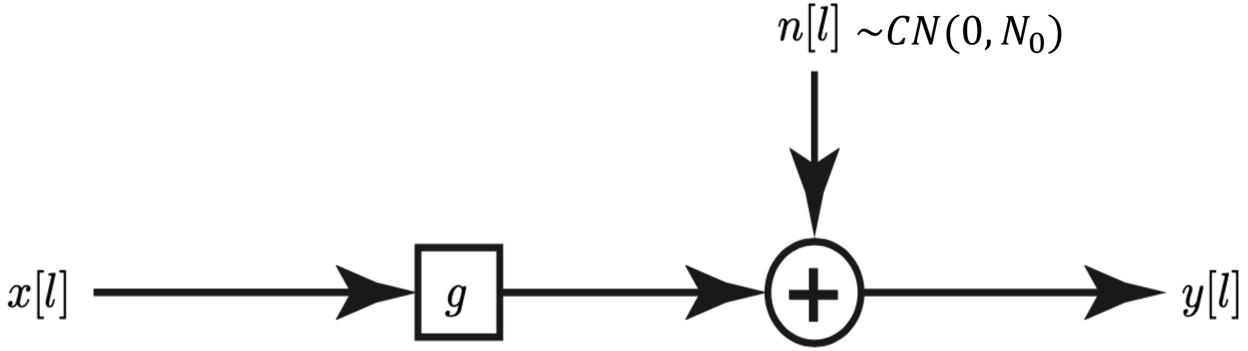
Discrete memoryless channel:

$$y[l] = g \cdot x[l] + n[l]$$

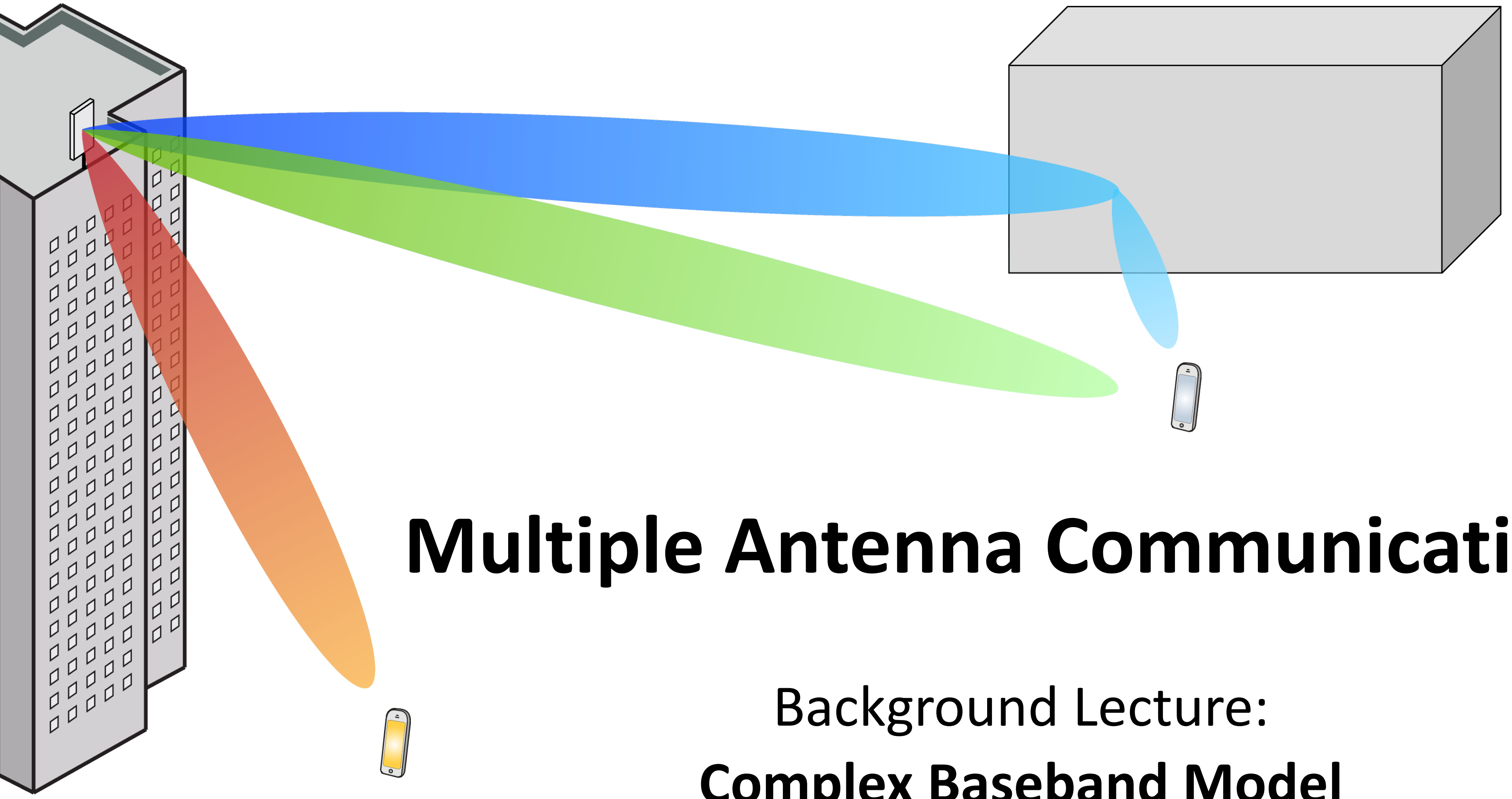
Summary

- Transmission in continuous time at frequency f_c

- Equivalent discrete description:



  - B symbols per second (B = bandwidth)
  - Memoryless channel: $y = g \cdot x + n$



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