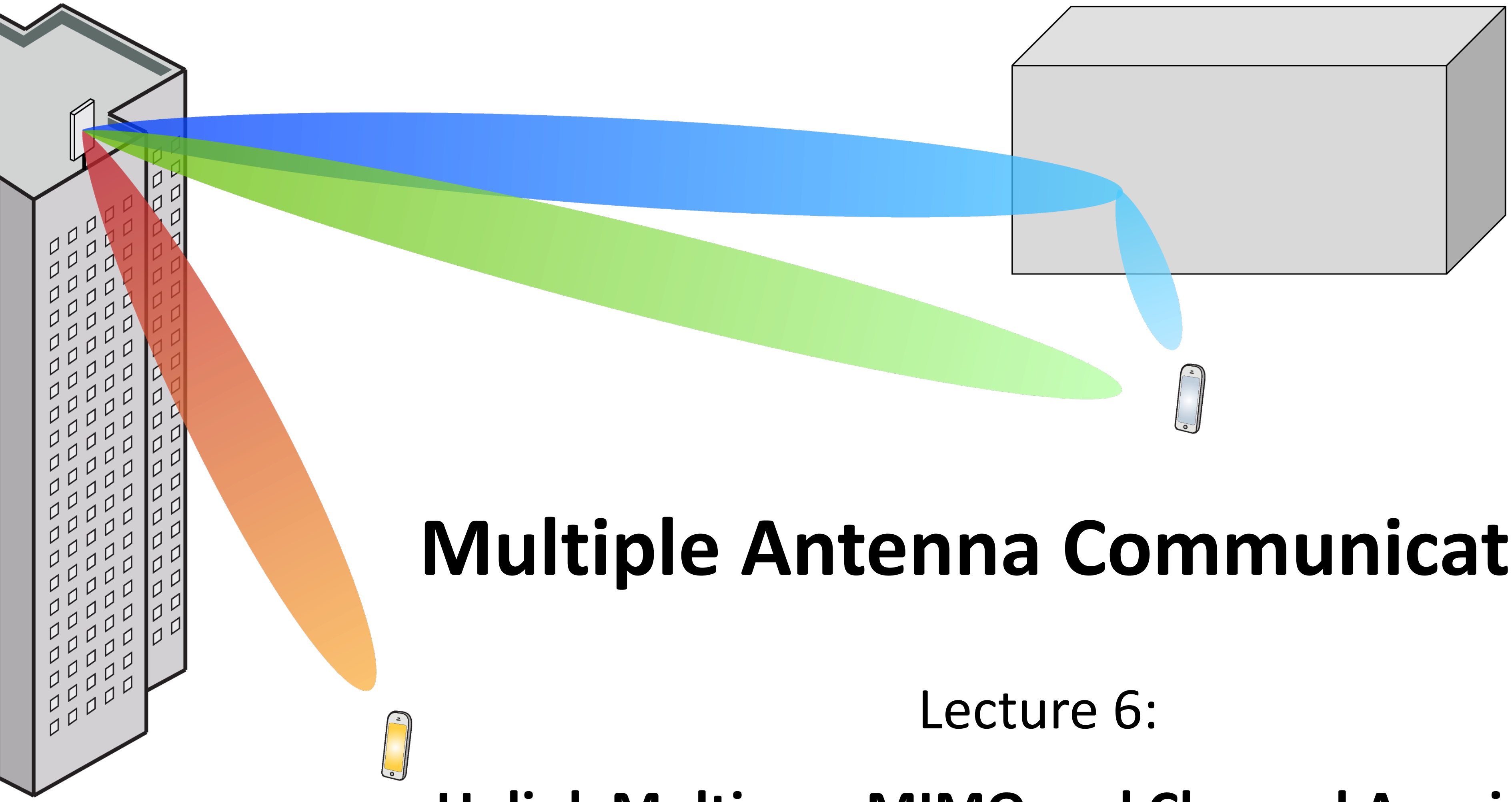


Multiple Antenna Communications

Lecture 6:

Uplink Multiuser MIMO and Channel Acquisition

Emil Björnson

Outline

- Channel coherence intervals
  - Coherence time and coherence bandwidth
- Massive MIMO
  - Motivation and basic properties
  - Duplexing modes
  - Uplink system model

Linear time-invariant channels

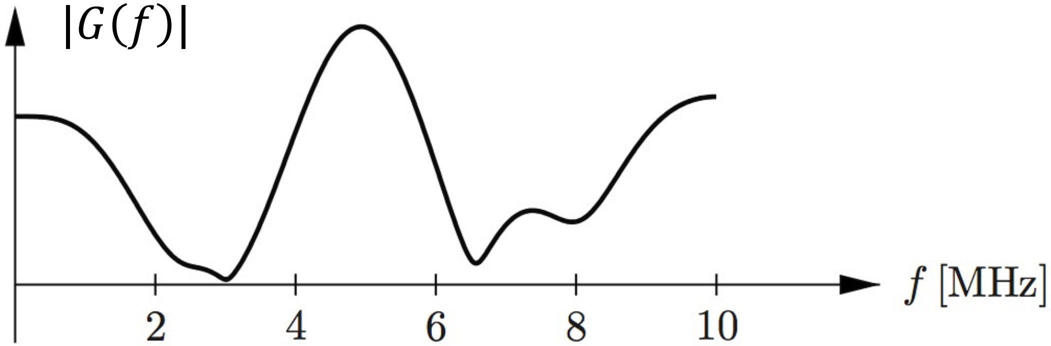
$$x(t) \longrightarrow \boxed{\text{Channel}} \longrightarrow y(\mathrm{t})$$

- Is the channel a linear time-invariant (LTI) filter?
- Linearity due to Maxwell's equations

- Coherence time T_c
  - Time that channel is *approximately* time-invariant
  - Simple model: $T_c = \lambda/(2v)$ or $T_c = \lambda/(4v)$ seconds

$\boxed{\text{Proportional to wavelength, inversely to speed } v}$

Channel dispersion

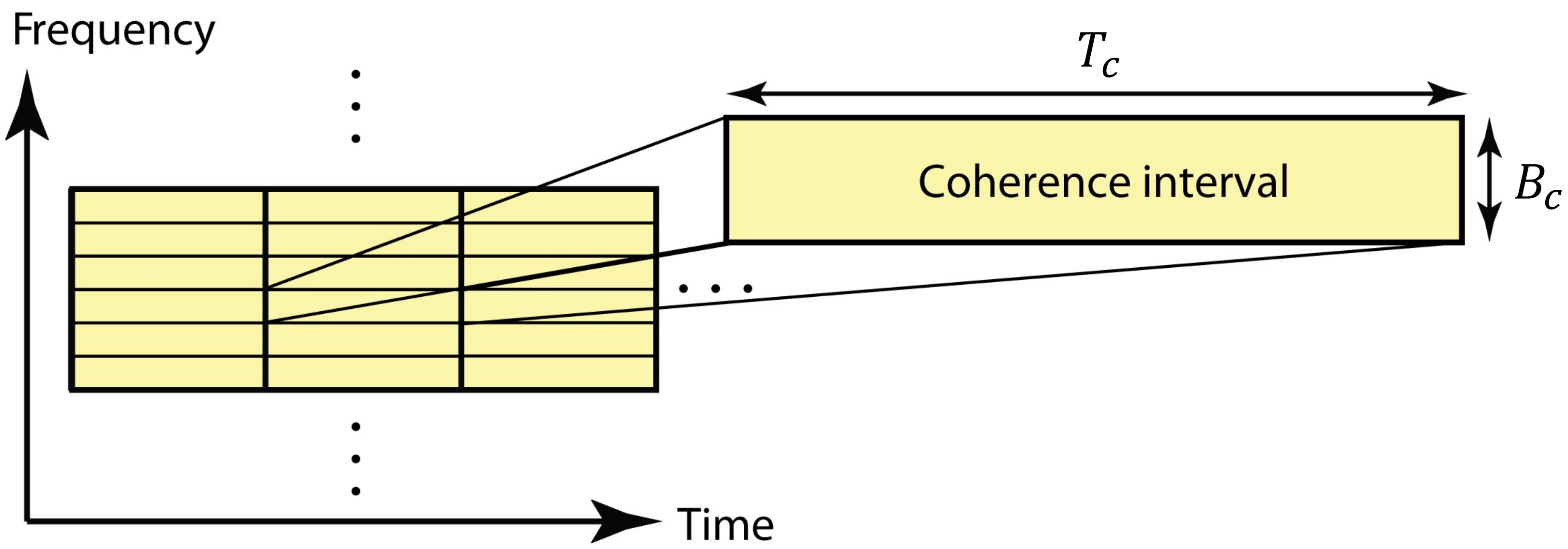


- Is the channel dispersive?

- Coherence bandwidth B_c
  - Bandwidth over which frequency response $G(f) \approx g$ is almost constant:
$$g(t) = g \cdot \delta(t)$$
  - Simple model: $B_c = c/|d_{\mathrm{max}} - d_{\mathrm{min}}|$ or $B_c = c/(2|d_{\mathrm{max}} - d_{\mathrm{min}}|)$ Hz

Inversely proportional to path length difference

Coherence interval (block fading)



- Divide bandwidth and time resources into coherence intervals
  - According to sampling theorem:
    $$\tau_c = B_c T_c \text{ complex samples}$$
  - Channel time-invariant and described by a scalar

This is an example of fast fading

How large is a coherence interval?

- **Example:** Support vehicular speed in suburban area:

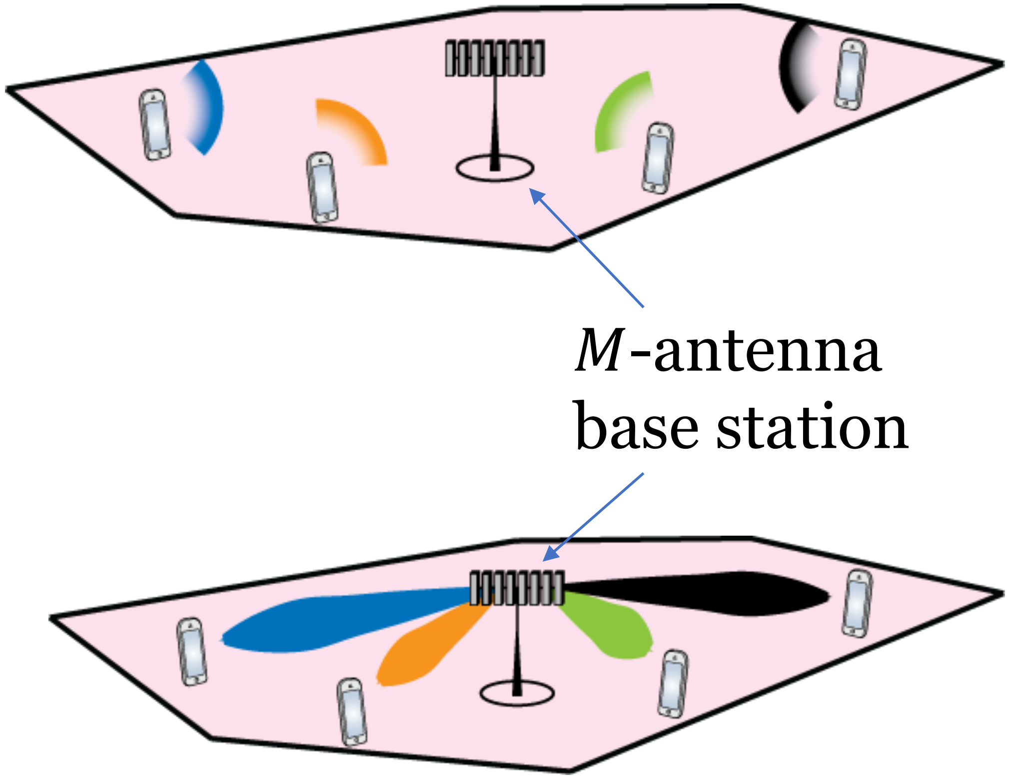
$$v = 30 \text{ m/s} = 108 \text{ km/h}$$
$$|d_{\max} - d_{\min}| = 1000 \text{ m}$$

- Typical carrier frequency: $f = 2$ GHz, $\lambda = 15$ cm
  - Coherence time: $T_c = \frac{\lambda}{2v} = \frac{0.15}{2 \cdot 30} = 2.5$ ms
  - Coherence bandwidth: $B_c = \frac{c}{|d_{\max} - d_{\min}|} = \frac{3 \cdot 10^8}{1000} = 300$ kHz

Coherence interval:
$\tau_c = 750$ complex samples

Recall: Multiuser MIMO Communication

- Uplink
  - From K users to base station
  - Multipoint-to-point MIMO
- Downlink
  - From base station to K users
  - Point-to-multipoint MIMO



Multiuser MIMO vs. Massive MIMO

- Conventional multiuser MIMO
  - $M \leq 8, \ K \leq 4$
  - Used in LTE and WiFi
  - Seldom reaches the $\min(M, K) = K$ capacity gain

- Massive MIMO
  - $M \approx 100, \ K \approx 10$ (or more)
  - More directive signals:
    Less randomness, larger beamforming gain, less interference

Motivation for Massive MIMO

- Recall: Sum Capacity with $K = 2$:

$$R_1 + R_2 = \log_2(\det(\boldsymbol{I}_M + \rho_{ul}\boldsymbol{G}\boldsymbol{G}^H)) = \log_2(\det(\boldsymbol{I}_2 + \rho_{ul}\boldsymbol{G}^H\boldsymbol{G}))$$

- If $\boldsymbol{G} = [\boldsymbol{g}_1\ \boldsymbol{g}_2]$: $\boldsymbol{G}^H\boldsymbol{G} = \begin{bmatrix} \|\boldsymbol{g}_1\|^2 & \boldsymbol{g}_1^H\boldsymbol{g}_2 \\ \boldsymbol{g}_2^H\boldsymbol{g}_1 & \|\boldsymbol{g}_2\|^2 \end{bmatrix}$

- Expanding sum capacity:

$$\log_2(\det(\boldsymbol{I}_2 + \rho_{ul}\boldsymbol{G}^H\boldsymbol{G})) = \log_2\left((1 + \rho_{ul}\|\boldsymbol{g}_1\|^2)(1 + \rho_{ul}\|\boldsymbol{g}_2\|^2) - \rho_{ul}^2\left|\boldsymbol{g}_1^H\boldsymbol{g}_2\right|^2\right)$$

$$\leq \log_2(1 + \rho_{ul}\|\boldsymbol{g}_1\|^2) + \log_2(1 + \rho_{ul}\|\boldsymbol{g}_2\|^2)$$

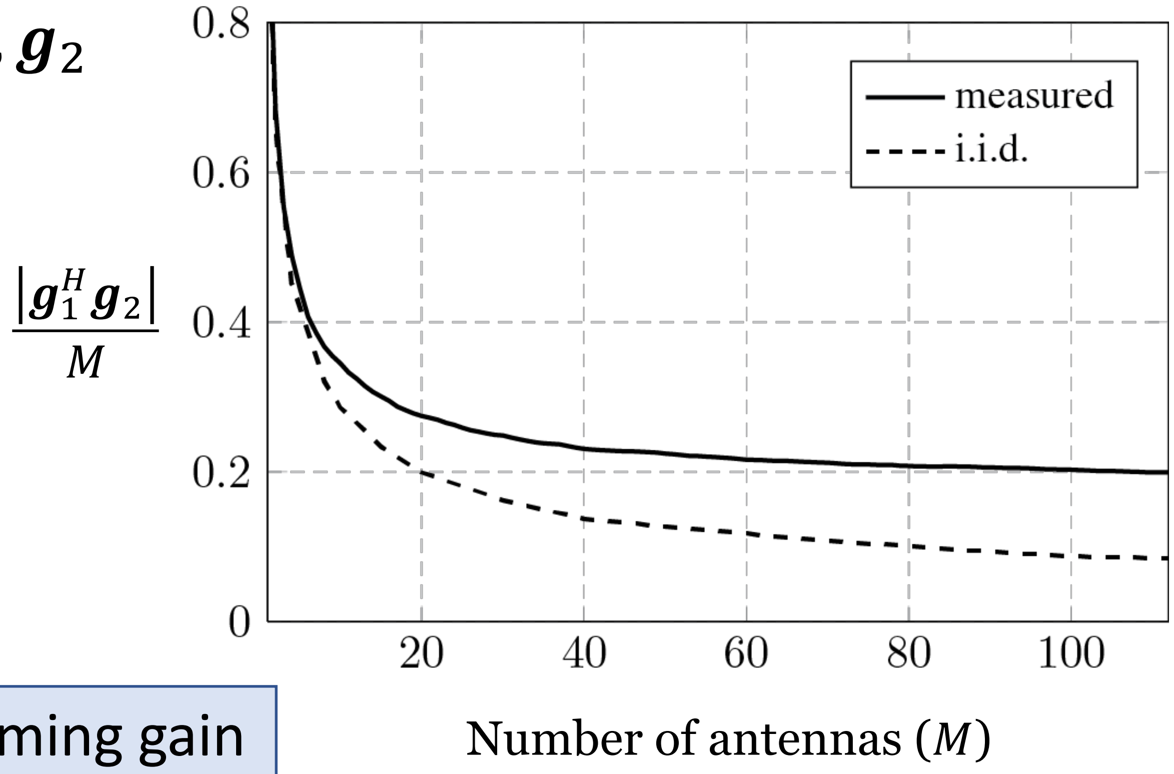
Equality if and only if $\boldsymbol{g}_1^H\boldsymbol{g}_2 = 0$

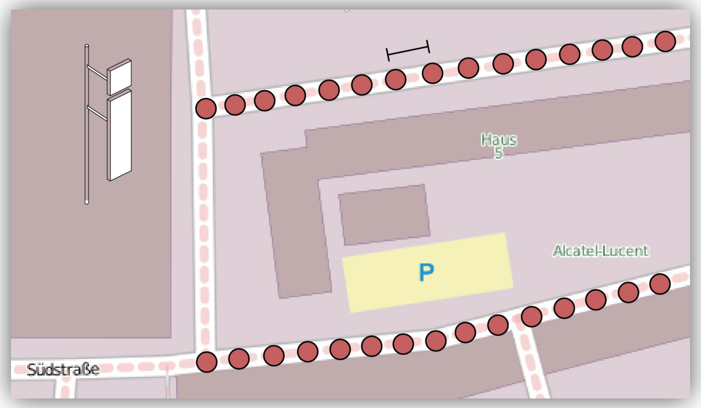
Motivation for Massive MIMO: Favorable propagation

- Consider two M-antenna channels $\boldsymbol{g}_1, \boldsymbol{g}_2$

Inner product $\frac{|\boldsymbol{g}_1^H \boldsymbol{g}_2|}{M}$ converges to zero as $M \to \infty$

Less interference





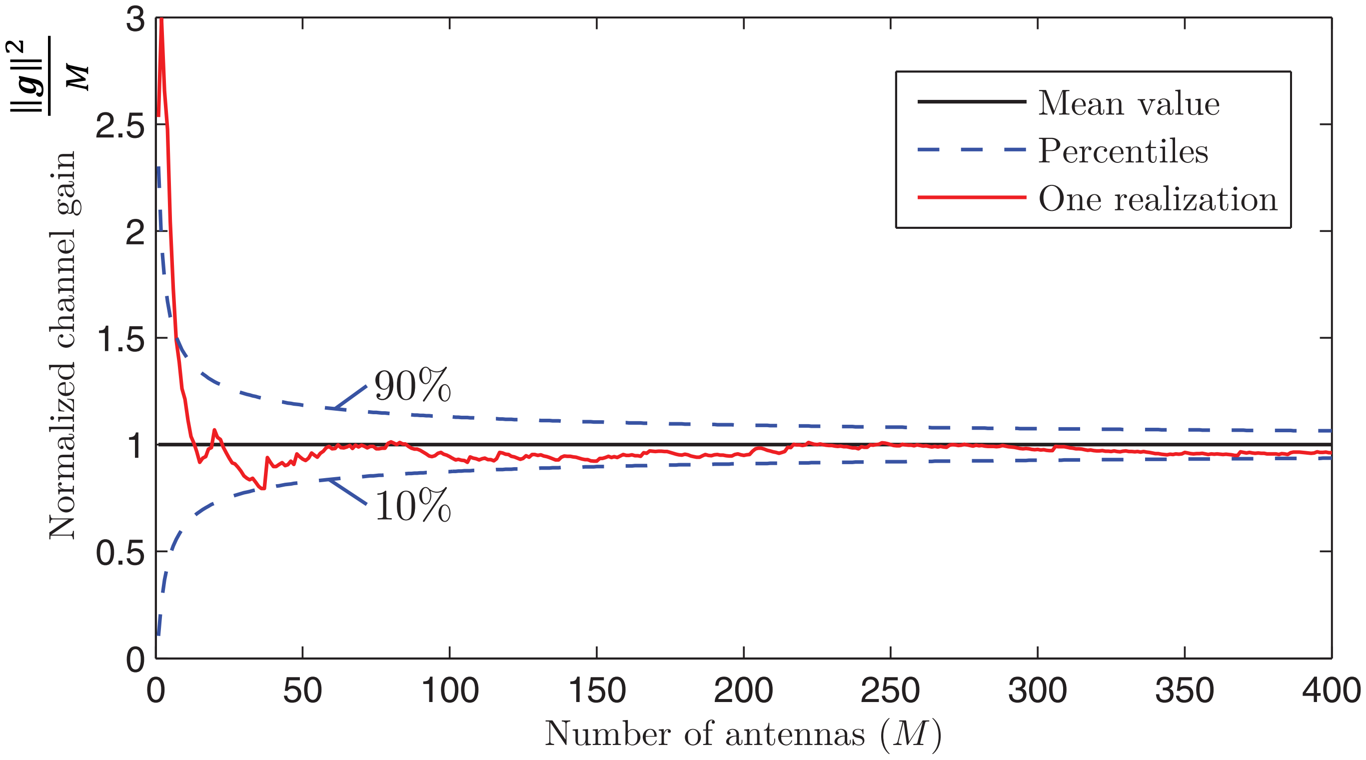
Related to beamforming gain and beamwidth

Reference: J. Hoydis, C. Hoek, T. Wild, and S. ten Brink, "Channel Measurements for Large Antenna Arrays," ISWCS 2012

Motivation for Massive MIMO: Channel hardening

- Consider an M-antenna channel $\boldsymbol{g} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_M)$

$$\frac{1}{M}\|\boldsymbol{g}\|^2 \text{ has } \begin{cases} \text{Mean: } 1 \\ \text{Variance: } 1/M \end{cases}$$



Consequence of spatial diversity:

$$\|\boldsymbol{g}\|^2 \approx \mathrm{E}\{\|\boldsymbol{g}\|^2\}$$

when M is large

Consequence of beamforming gain:

$$\|\boldsymbol{g}\|^2 \approx M$$

when M is large

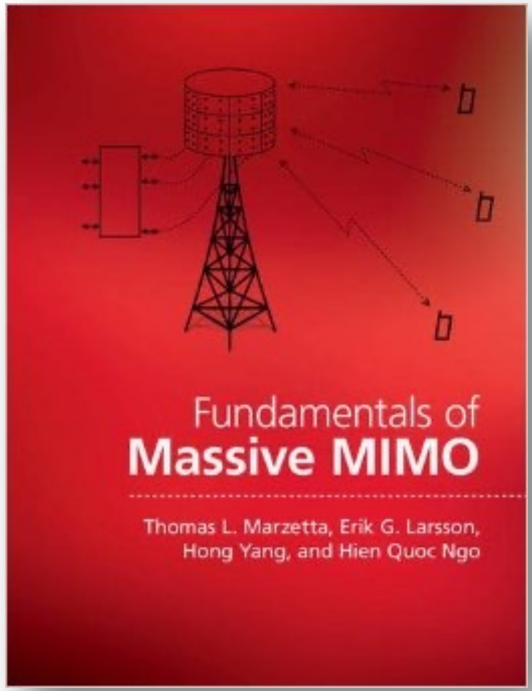
Background of Massive MIMO

- Proposed by
  Thomas L. Marzetta
  - Awarded *honorary doctor*
    at Linköping University, 2015

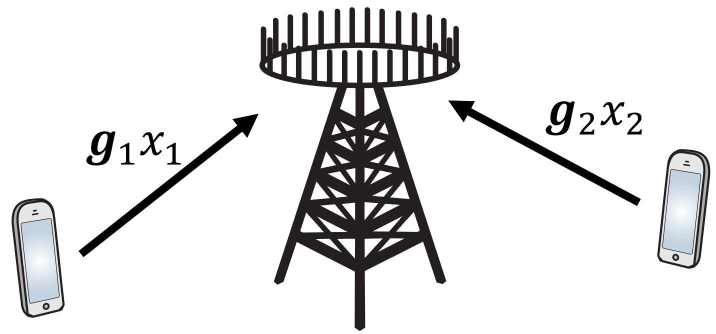
- First paper:

  "*Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas*," IEEE Trans. Wireless Communications, 2010.

Now a main component of 5G



Marzetta's asymptotic motivation

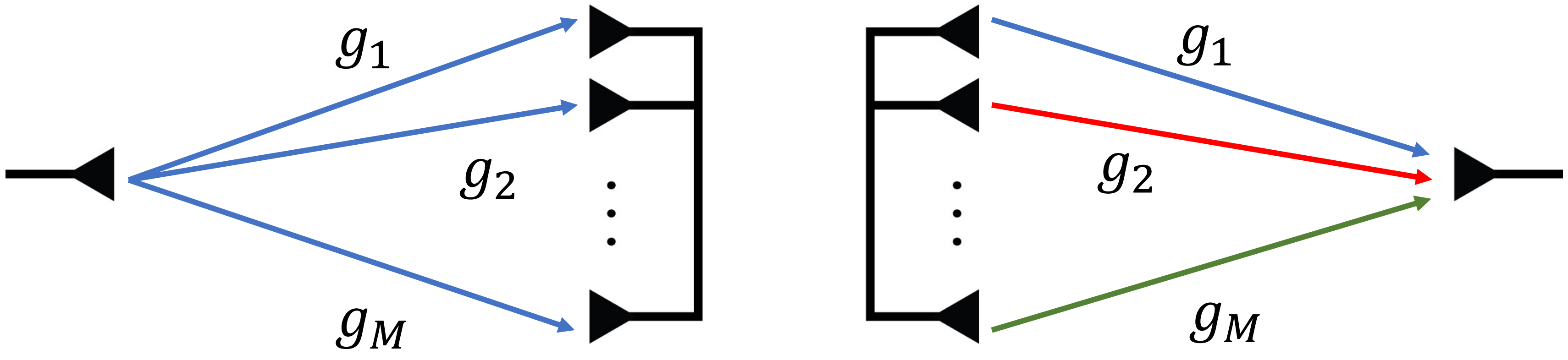


- Example: Uplink with i.i.d. Rayleigh fading
  - Two users, send signals x_k for $k = 1,2$
  - Channels: $\boldsymbol{g}_k = \left[g_k^1 \dots g_k^M\right]^T \sim CN(\boldsymbol{0}, \boldsymbol{I}_M)$
  - Noise: $\boldsymbol{w} \sim CN(\boldsymbol{0}, \boldsymbol{I}_M)$
  - Received: $\mathbf{y} = \boldsymbol{g}_1 x_1 + \boldsymbol{g}_2 x_2 + \boldsymbol{w}$
- Linear detector $\boldsymbol{a}_1 = \frac{1}{M}\boldsymbol{g}_1$ for User 1: $\tilde{y}_1 = \boldsymbol{a}_1^H \mathbf{y} = \boxed{\boldsymbol{a}_1^H \boldsymbol{g}_1} x_1 + \boxed{\boldsymbol{a}_1^H \boldsymbol{g}_2} x_2 + \boxed{\boldsymbol{a}_1^H \boldsymbol{w}}$
  - Signal remains: $\boldsymbol{a}_1^H \boldsymbol{g}_1 = \frac{1}{M}\|\boldsymbol{g}_1\|^2 \xrightarrow{M\to\infty} \mathrm{E}[|g_1^1|^2] = 1$
  - Interference vanishes: $\boldsymbol{a}_1^H \boldsymbol{g}_2 = \frac{1}{M}\boldsymbol{g}_1^H \boldsymbol{g}_2 \xrightarrow{M\to\infty} \mathrm{E}[g_1^{1*} g_2^1] = 0$
  - Noise vanishes: $\boldsymbol{a}_1^H \boldsymbol{w} = \frac{1}{M}\boldsymbol{g}_1^H \boldsymbol{w} \xrightarrow{M\to\infty} \mathrm{E}[g_1^{1*} w_1] = 0$

Asymptotically noise/interference-free communication: $\tilde{y}_1 \xrightarrow{M\to\infty} x_1$

Estimating the channel responses

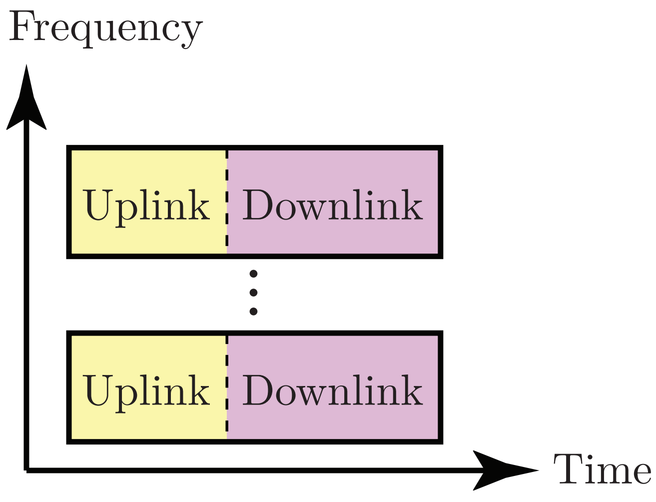
- Channels: K users with M-length channels
  - Estimate MK coefficients in each coherence interval
- Basic principle: Send known *pilot* signal



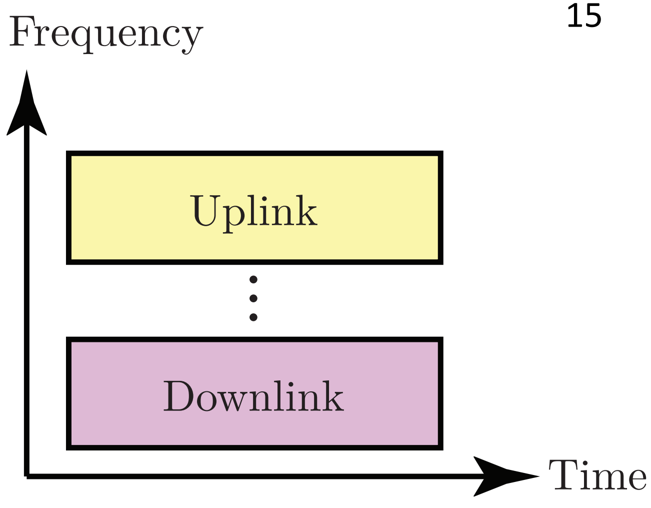
One pilot: Estimate all coefficients

M pilots to estimate all coefficients

Duplexing



TDD operation



FDD operation

- Time-division duplex (TDD): Separate uplink and downlink in time
  - K pilots are needed
- Frequency-division duplex (FDD): Separate uplink and downlink in frequency
  - M pilots are needed

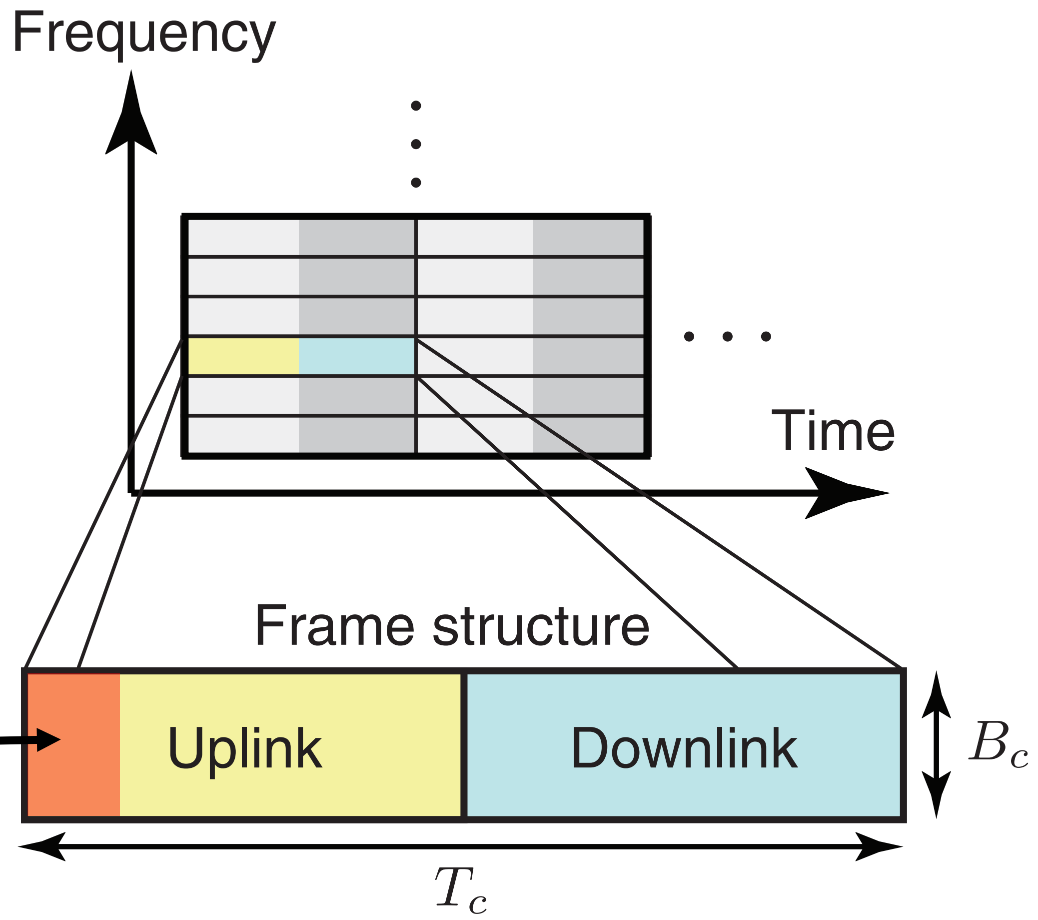
- Example: $M = 100, K = 10$, $\tau_c = 200$

TDD operation is key!

Operating a TDD Massive MIMO System

- Frames matched to coherence intervals
  - Fixed single-tap channel responses
  - Coherence time: T_c s
  - Coherence bandwidth: B_c Hz
  - Interval contains $\tau_c = T_c B_c$ symbols

Uplink pilots:
Enable channel estimation



Uplink Massive MIMO system model

- Received signal:

$$\boldsymbol{y} = \sqrt{\rho_{ul}}\boldsymbol{G}\boldsymbol{x} + \boldsymbol{w}$$

where

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_M \end{pmatrix} \quad \boldsymbol{G} = \begin{pmatrix} g_1^1 & \cdots & g_K^1 \\ \vdots & \ddots & \vdots \\ g_1^M & \cdots & g_K^M \end{pmatrix} \quad \boldsymbol{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_K \end{pmatrix} \quad \boldsymbol{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_M \end{pmatrix}$$

- Parameters are normalized: Maximum power is ρ_{ul}
  $x_1, \dots, x_K$ has power ≤ 1
- Channel of user k: $g_k^1, \dots, g_k^M \sim CN(0, \beta_k)$
- Normalized noise: $\boldsymbol{w} \sim CN(\boldsymbol{0}, \boldsymbol{I}_M)$

Large-scale fading coefficient

Modeling of power and large-scale fading coefficients

- Maximum SNR of user k is $\rho_{ul}\beta_k$

- How to model ρ_{ul}?

$$\rho_{ul} = \frac{\text{Uplink radiated power} \cdot \text{Antenna gains}}{BN_0}$$

$N_0 = 10^{-17}$

- How to model β_k?
  - Example: 3GPP-type model at distance d_k:

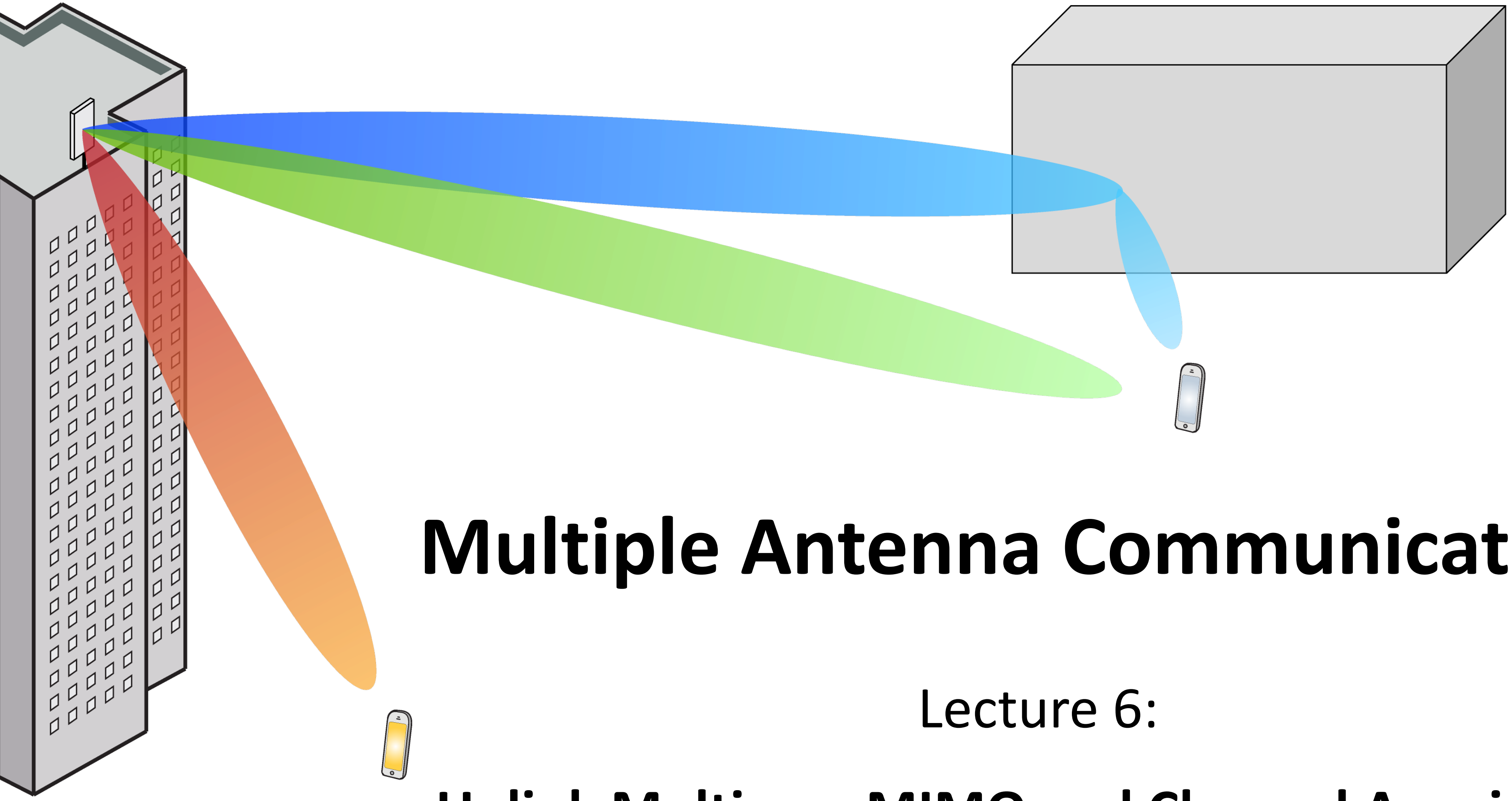
$$\beta_k = 10^{-1.53}\left(\frac{d_k}{1\text{ m}}\right)^{-3.76} \qquad \text{for } d_k \geq 35\text{ m}$$

Typical values:
$B = 10$ MHz
Radiated power: 100 mW
Antenna gains = 0 dBi

Typical values:
$d_k = 35$ m: $\beta_k = -73$ dB
$d_k = 1$ km: $\beta_k = -128$ dB

Summary

- Massive MIMO is multi-user MIMO with many antennas and users
  - No strict definition exists
- TDD operation
  - Divide time-frequency resources into frames
  - Match frame size to coherence intervals
  - Send uplink pilots for channel estimation
  - Switch between uplink/downlink data in same coherence interval



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