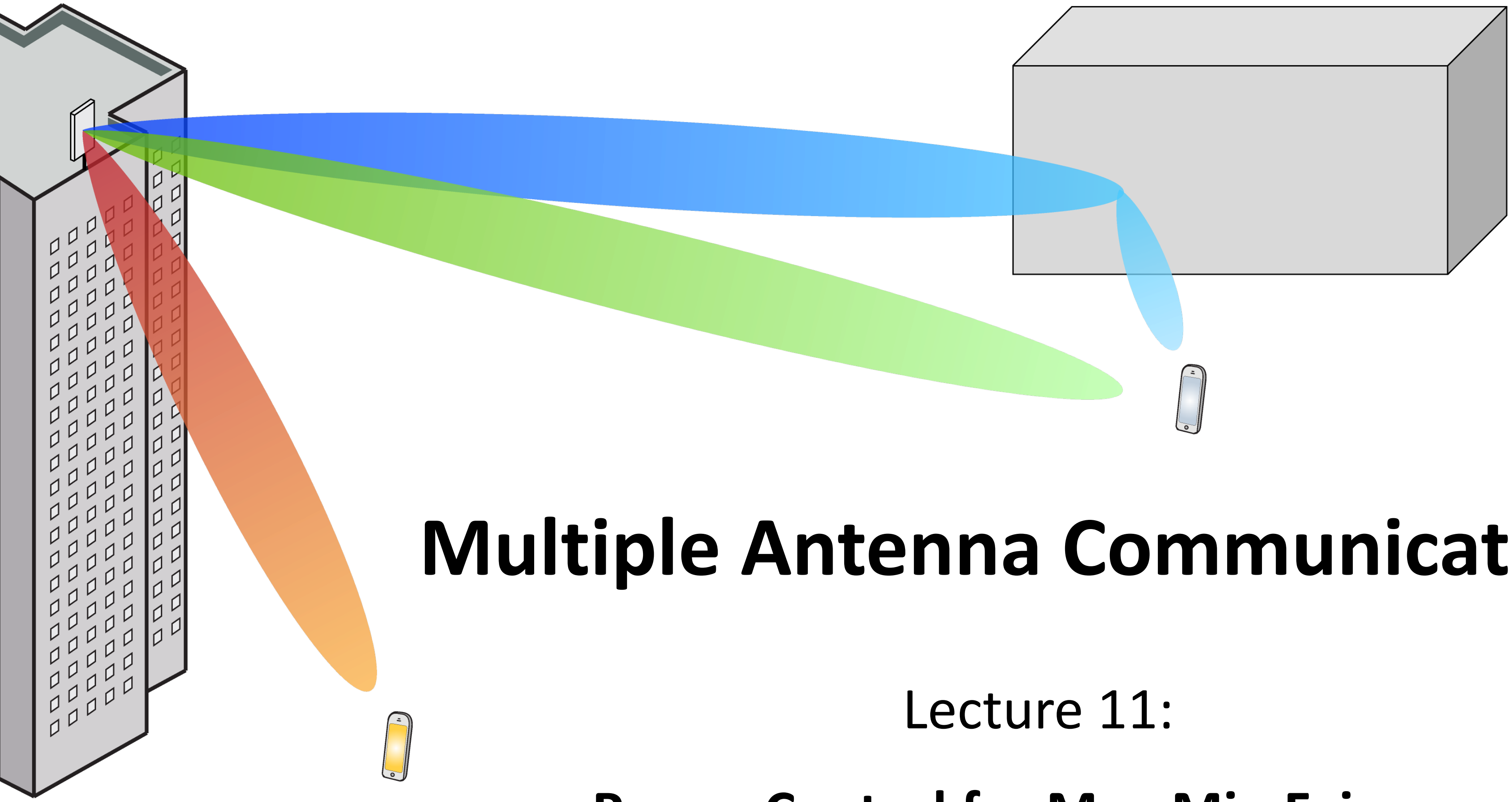


Multiple Antenna Communications

Lecture 11:

Power Control for Max-Min Fairness

Emil Björnson

Outline

- Power control
  - Purpose and operating points
  - Structure of the SINR
- Max-min fairness power control
  - Uplink
  - Downlink
- Simulation example

Single-cell effective SINRs

- Capacity lower bound for user k

$$\log_2(1 + \mathrm{SINR}_k)$$

where the "effective" SINR is

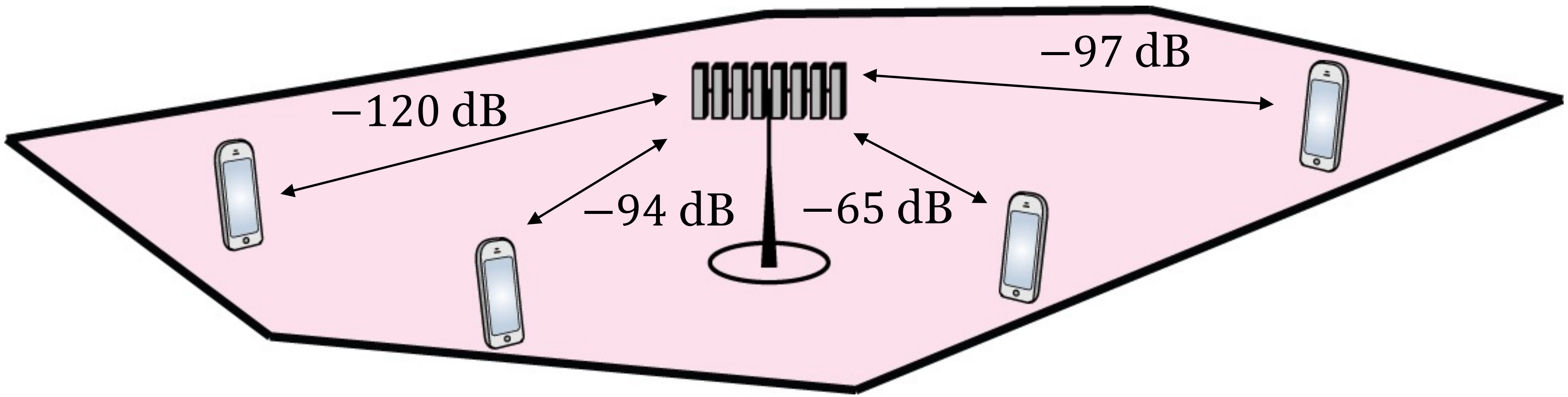
$$\mathrm{SINR}_k = \frac{a_k \eta_k}{1 + \sum_{k'=1}^{K} b_k^{k'} \eta_{k'}}$$

- With MR:

Uplink	Downlink
$a_k = M\rho_{ul}\gamma_k$ $b_k^{k'} = \rho_{ul}\beta_{k'}$	$a_k = M\rho_{dl}\gamma_k$ $b_k^{k'} = \rho_{dl}\beta_k$

Same structure with other methods than MR

Different channel conditions



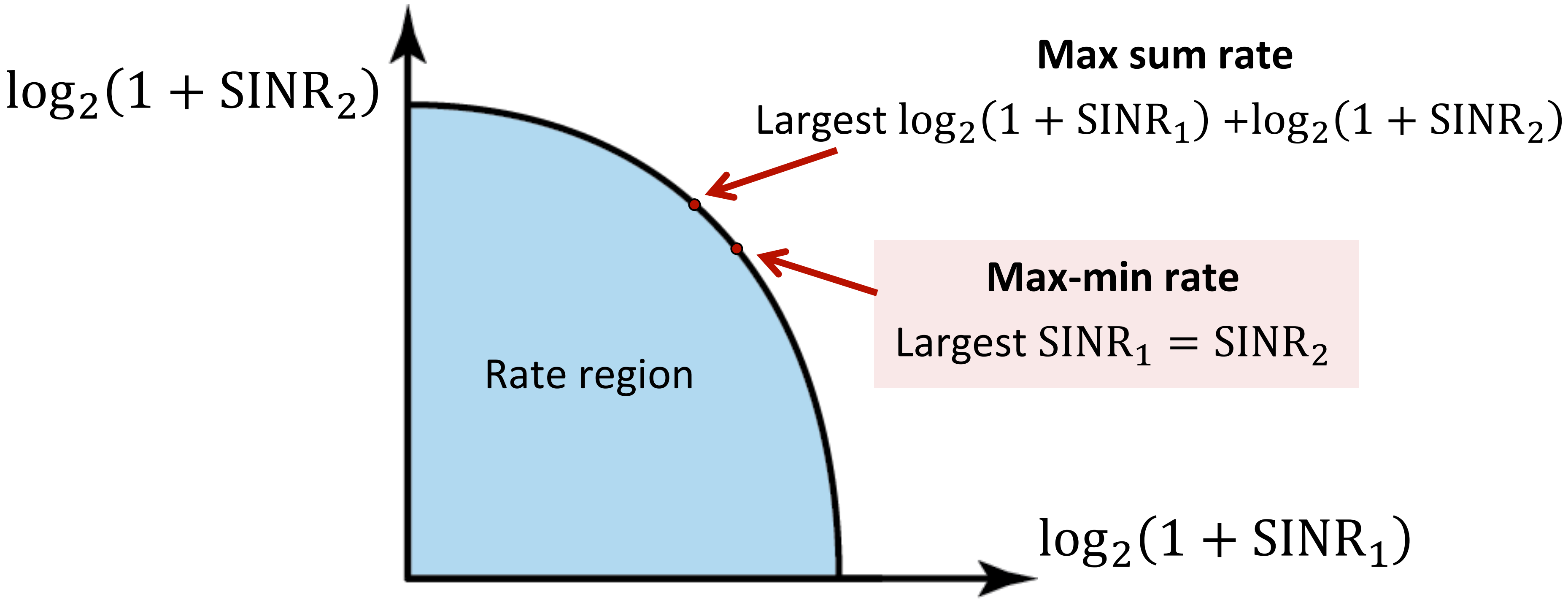
- Nature and hardware setup determine the values of

$$a_k = M\rho_{ul}\gamma_k, \qquad b_k^{k'} = \rho_{ul}\beta_{k'}$$

  - We can compensate by selecting $\eta_1, \ldots, \eta_K$

Power control

- Effective SINRs depend on $\eta_1, \dots, \eta_K$
  - Can be selected to achieve different operating points



Max-min fairness power control

- Maximize the minimum SINR (minimum rate)
  - Every user get the same performance
    (Prove by contradiction!)

- Power constraint
  - Uplink:

$$0 \leq \eta_k \leq 1, \qquad k = 1, \dots, K$$

  - Downlink:

$$\sum_{k=1}^{K} \eta_k \leq 1, \qquad \eta_k \geq 0, \qquad k = 1, \dots, K$$

Formulation as an optimization problem

$$
\begin{array}{ll}
\text{maximize} & \overline{\mathrm{SINR}} \\
\text{with respect to} & \eta_1, \ldots, \eta_K, \overline{\mathrm{SINR}} \\
\text{subject to} & \mathrm{SINR}_k \geq \overline{\mathrm{SINR}} \quad k = 1, \ldots, K \\
& \text{Power constraints}
\end{array}
$$

- Different solutions in uplink and downlink, since
  - SINR expressions are different
  - Power constraints are different

Uplink max-min fairness with MR

$$\mathrm{SINR}_k = \frac{M\rho_{ul}\gamma_k\eta_k}{1 + \sum_{k'=1}^{K} \rho_{ul}\beta_{k'}\eta_{k'}}$$

- Optimal power-control coefficients:

$$\eta_k = \frac{\min_{k'} \gamma_{k'}}{\gamma_k}$$

- Max-min SINR:

$$\overline{\mathrm{SINR}} = \frac{M\rho_{ul}}{\frac{1}{\min_{k'} \gamma_{k'}} + \rho_{ul} \sum_{k=1}^{K} \frac{\beta_k}{\gamma_k}}$$

Downlink max-min fairness with MR

$$\mathrm{SINR}_k = \frac{M\rho_{dl}\gamma_k\eta_k}{1 + \rho_{dl}\beta_k \sum_{k'=1}^{K} \eta_{k'}}$$

- Optimal power-control coefficients:

$$\eta_k = \frac{1 + \rho_{dl}\beta_k}{\rho_{dl}\gamma_k \left(\frac{1}{\rho_{dl}} \sum_{k'=1}^{K} \frac{1}{\gamma_{k'}} + \sum_{k'=1}^{K} \frac{\beta_{k'}}{\gamma_{k'}} \right)}$$

- Max-min SINR:

$$\overline{\mathrm{SINR}} = \frac{M\rho_{dl}}{\sum_{k=1}^{K} \frac{1}{\gamma_k} + \rho_{dl} \sum_{k=1}^{K} \frac{\beta_k}{\gamma_k}}$$

Insights from max-min power control

- Optimal power control

Uplink:

$$\eta_k = \frac{\min_{k'} \gamma_{k'}}{\gamma_k}$$

Downlink:

$$\eta_k = \frac{1 + \rho_{dl}\beta_k}{\rho_{dl}\gamma_k \left(\sum_{k'=1}^{K} \frac{1}{\gamma_{k'}} + \rho_{dl} \sum_{k'=1}^{K} \frac{\beta_{k'}}{\gamma_{k'}} \right)}$$

- Inversely proportional to $\gamma_k = \frac{\tau_p \rho_{ul} \beta_k^2}{1+\tau_p \rho_{ul} \beta_k}$ (approximately)
- Inversely proportional to β_k (approximately)
- Spend more power on the weakest users

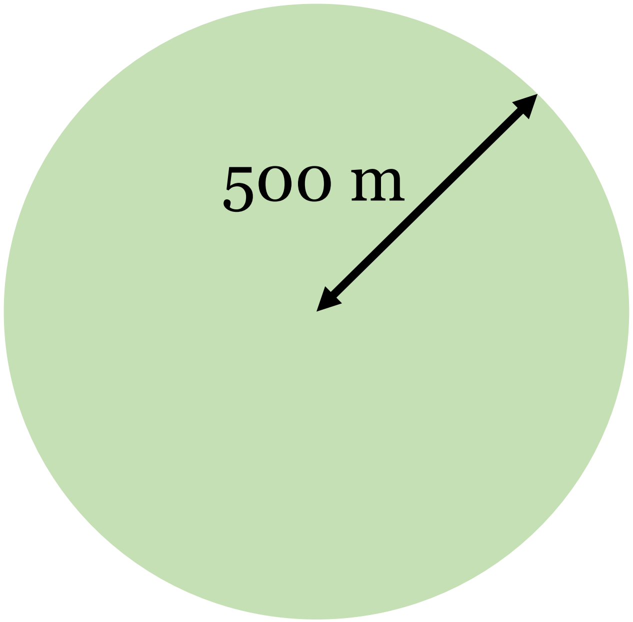
Who performs the optimization?

- Power-control coefficients depend on
  - Channel conditions: β_k, γ_k of all users
  - Maximum transmit power ρ_{ul}, ρ_{dl}
- Easiest to implement at the base station
  - Compute downlink coefficients and use them
  - Compute uplink coefficients and tell user k of η_k
- Update when users have moved or entered/exited

Simulation example: Urban deployment

- Single-cell setup
  - Circular cell with radius 500 m
  - Base station: $M = 100$ antennas
  - $K = 10$ uniformly users

- Important properties
  - Independent Rayleigh fading
  - No inter-cell interference
  - Carrier frequency: 2 GHz
  - Bandwidth 20 MHz



Some parameter values

Parameter	Value
Base station antenna gain	0 dBi
User terminal antenna gain	0 dBi
Base station receiver noise figure	7 dB
Terminal receiver noise figure	7 dB
Nominal noise temperature	300 K
Coherence time	1 ms
Coherence bandwidth	200 kHz

Parameter	Value
Base station antenna height	25 m
User terminal height	1.5 m
Path loss model (3GPP): $-25.3 - 37.6 \log_{10}\left(\frac{\text{distance}}{1\text{ m}}\right)$	
Maximum radiated power at base station	40 W
Maximum radiated power per user terminal	0.1 W

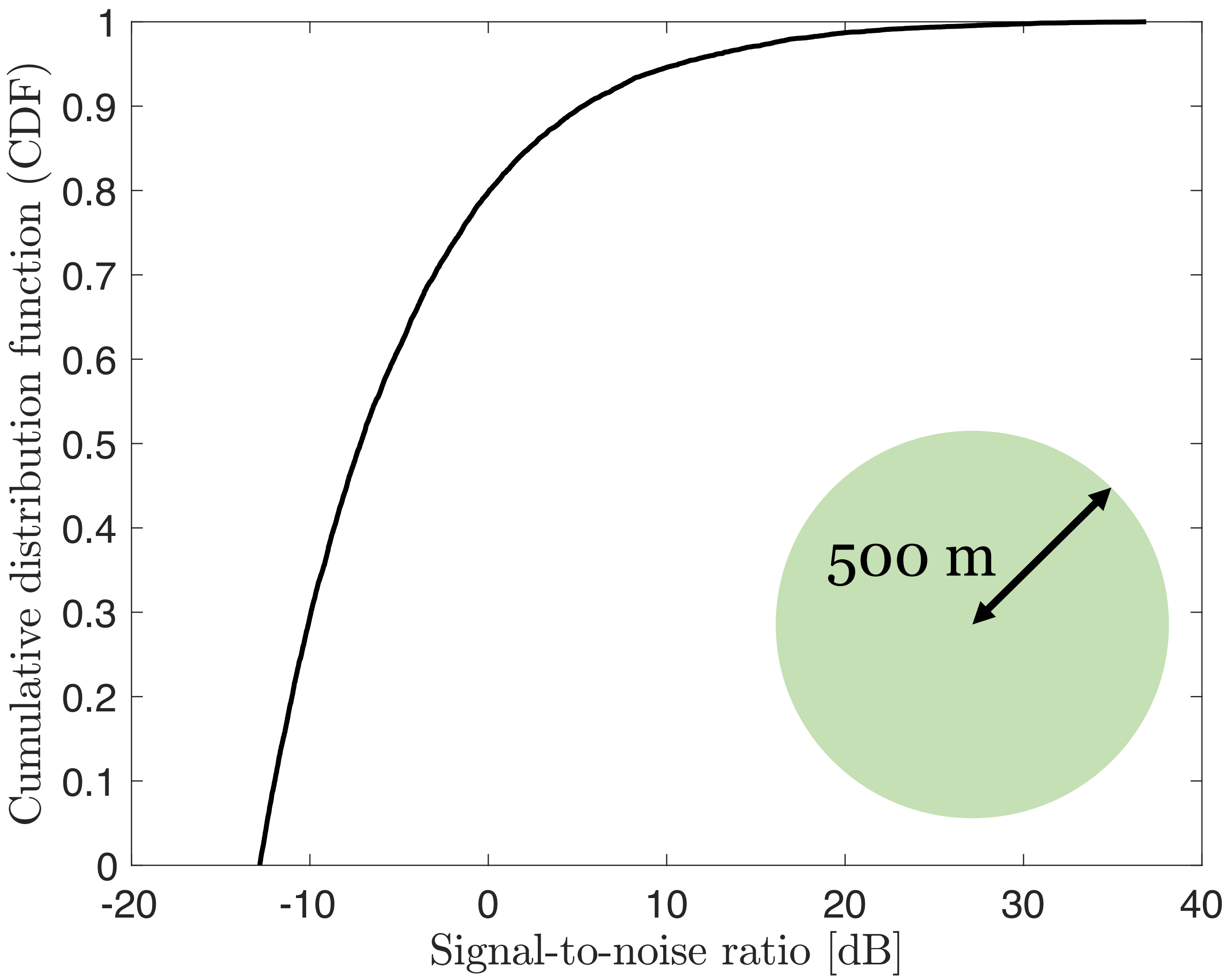
- 1 ms x 200 kHz = 200 samples per coherence interval
  - 10 for pilots, 63 for uplink (33%), 127 for downlink (67%)

Uplink signal-to-noise ratio (SNR)

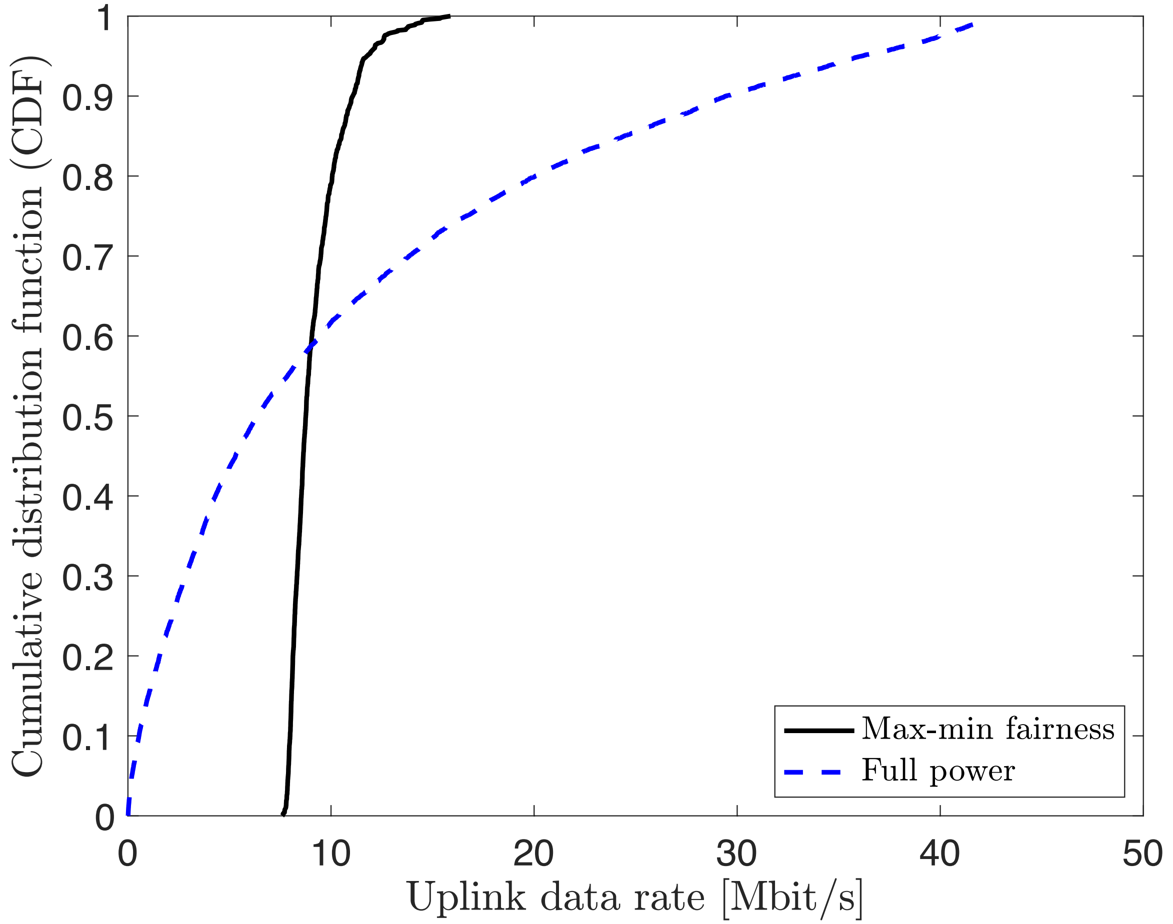
- Users at different locations
  - Cell edge
  - Cell center

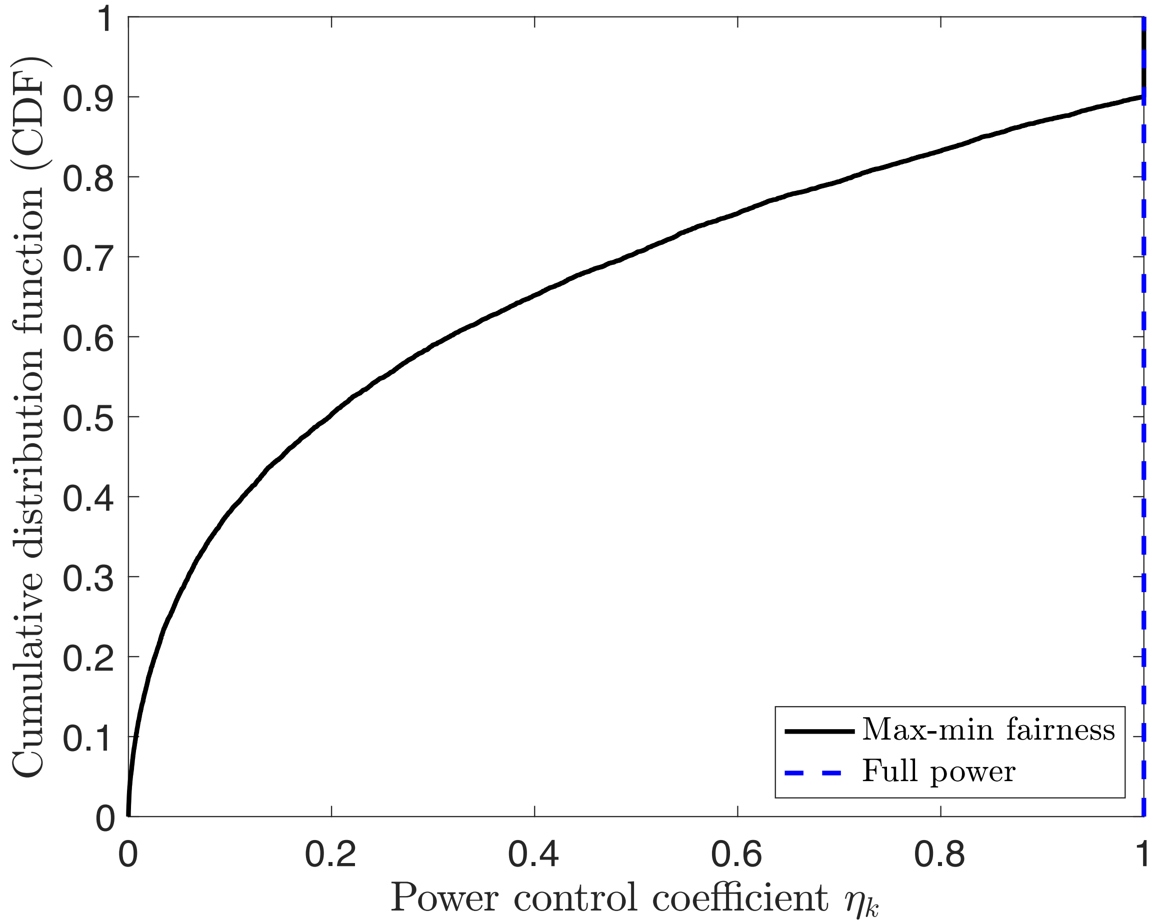
Simulation methodology

1) Drop K users in the cell
2) Compute β_k and γ_k
3) Compute power control η_k
4) Compute data rates

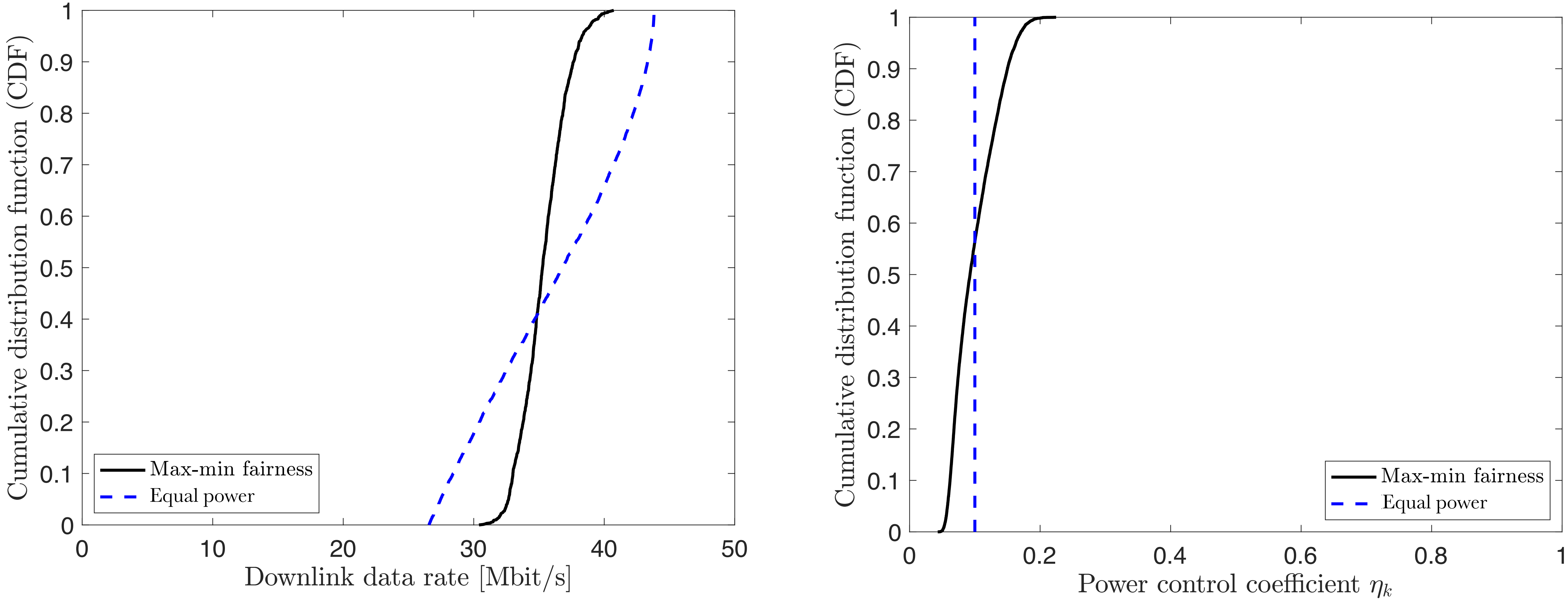


Case study: Uplink with power control



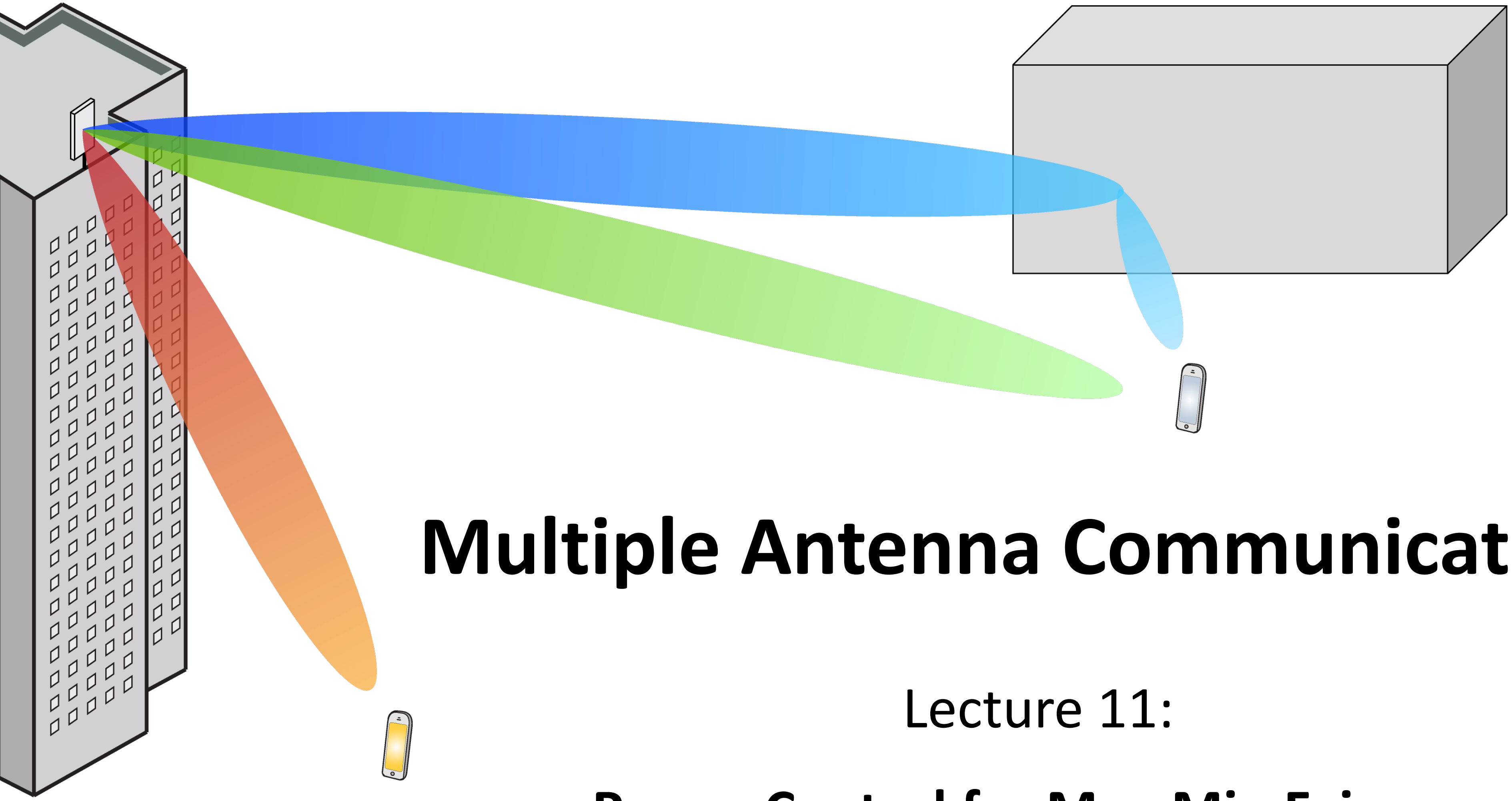


Case study: Downlink with power control



Summary

- User performance depends on transmit powers
  - Power control must be actively done
- Max-min fairness power control
  - Give everyone the same rate, maximize the common value
  - Power control coefficients can be computed in closed form
- Insight from simulations
  - Power control particularly important in uplink



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