

Multiple Antenna Communications

Lecture 5:

Introduction to Multiuser MIMO

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Outline

- Practical issues with point-to-point MIMO
- Introduction to multi-user MIMO
 - Uplink and downlink
 - Orthogonal access vs. spatial multiplexing
- Capacity region
 - Operating points

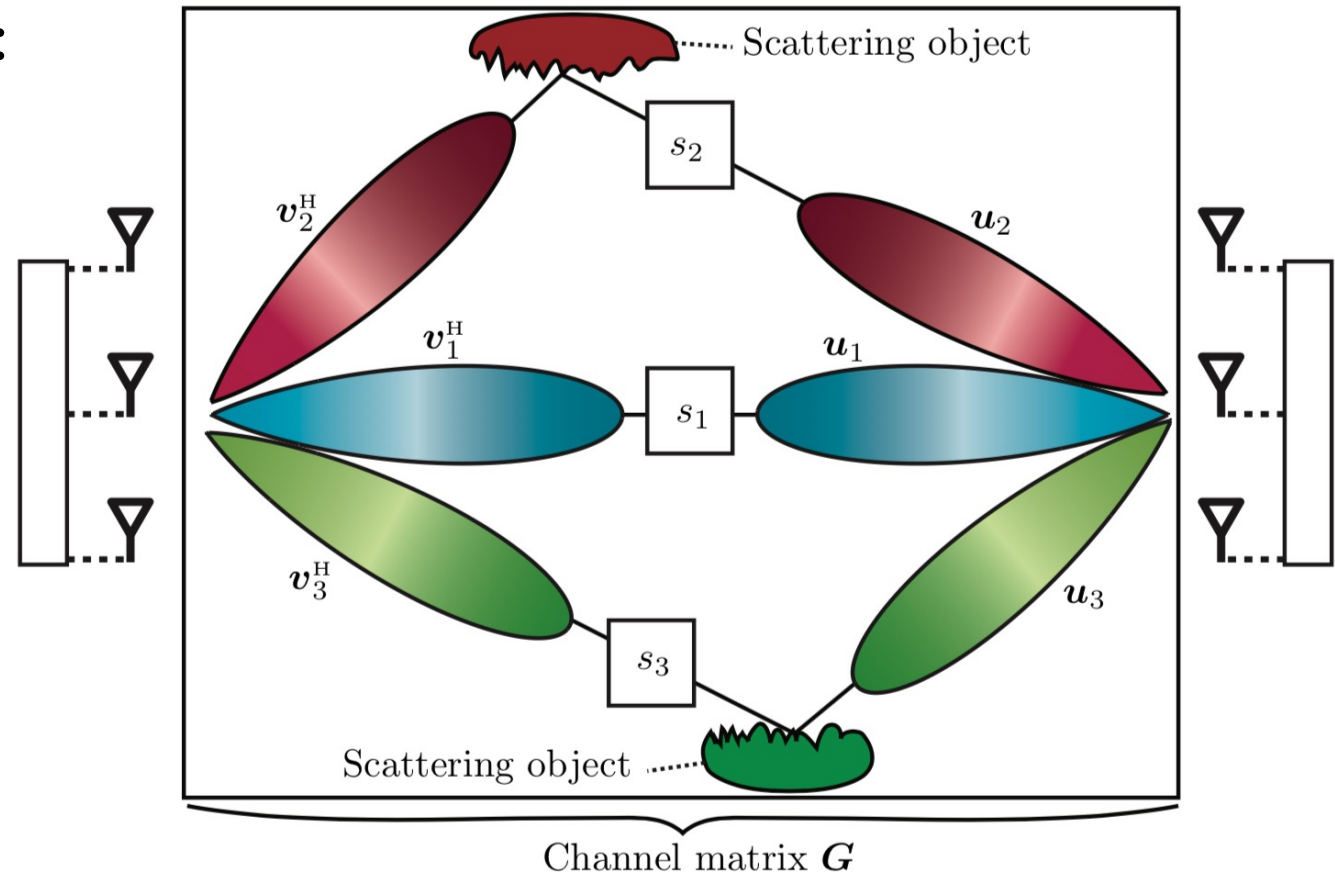
Recall: Point-to-Point MIMO Capacity

- Compute SVD of channel matrix:

$$\mathbf{G} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H = \sum_{k=1}^S s_k \mathbf{u}_k \mathbf{v}_k^H$$

- $\mathbf{\Sigma}$ “diagonal” with s_1, \dots, s_S
- $\mathbf{U} = [\mathbf{u}_1 \dots \mathbf{u}_M]$
- $\mathbf{V} = [\mathbf{v}_1 \dots \mathbf{v}_K]$

Decompose the channel into
 S parallel channels



Problems with point-to-point MIMO

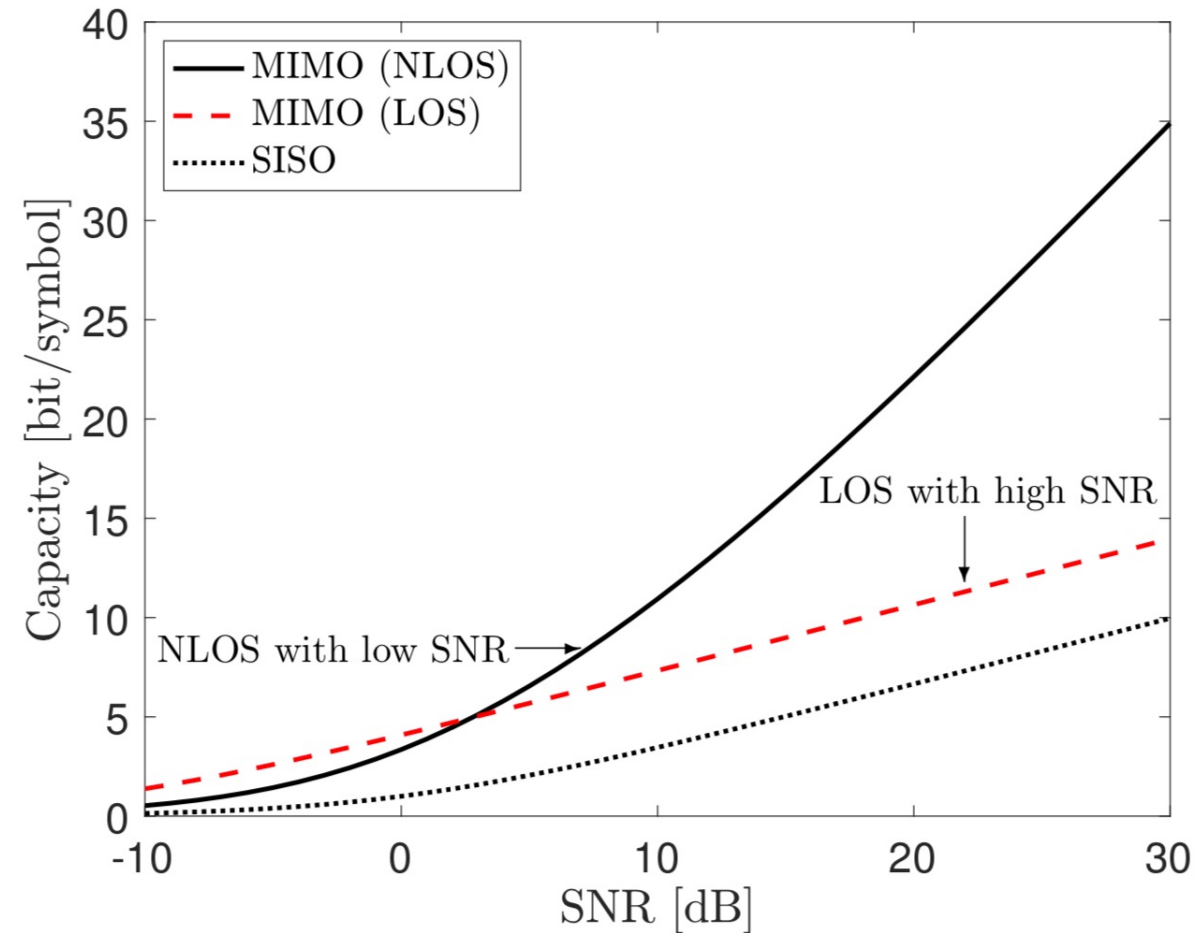
- Multiplexing gain: $S = \text{rank}(\mathbf{G})$
 - Line-of-sight: $S \approx 1$
 - Non-line-of-sight: $S = \min(M, K)$

Mainly beamforming gain:

- High SNR:
Likely to be in line-of-sight (LOS)
- Low SNR:
Likely to be non-line-of-sight (NLOS)

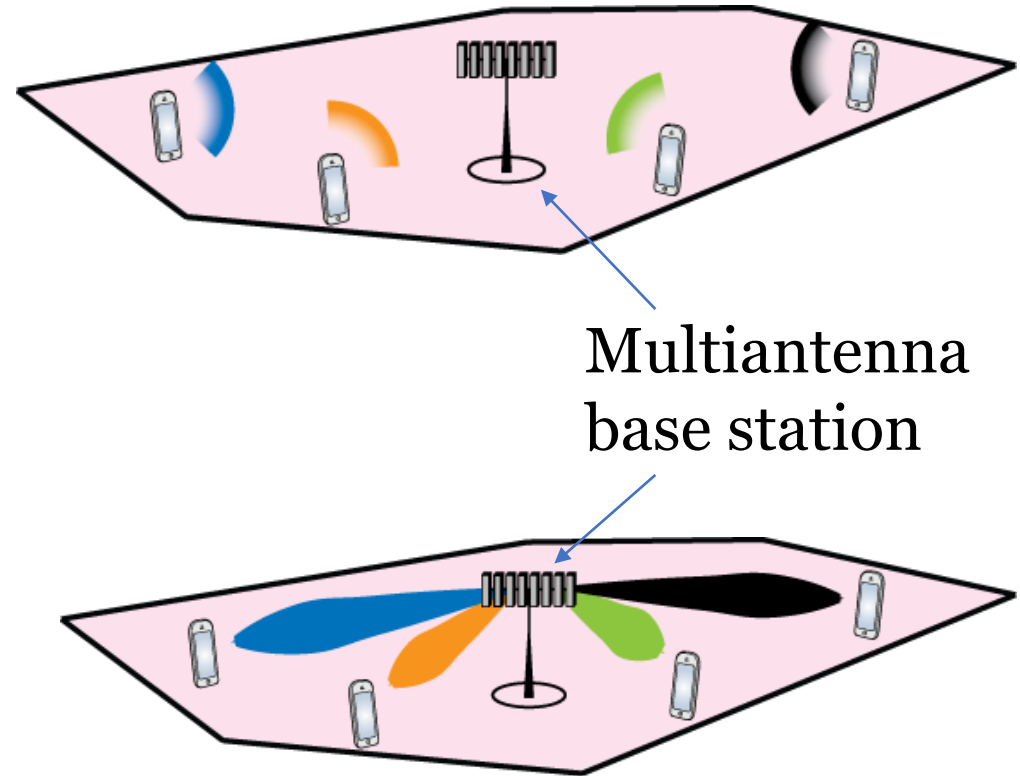
Not scalable:

User devices are small, cannot fit many antennas



Multiuser MIMO Communication

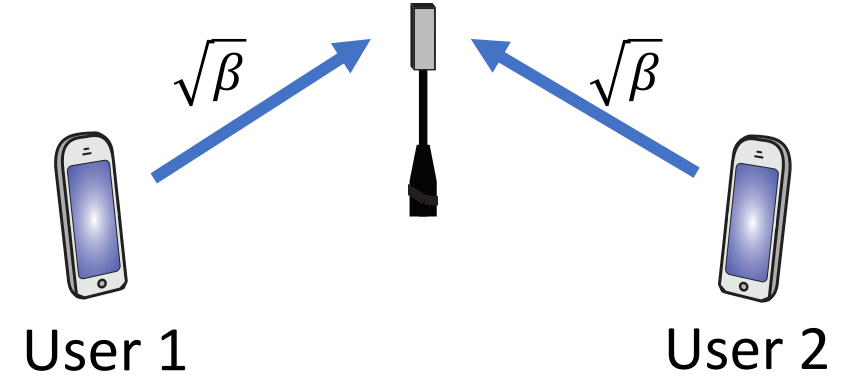
- Uplink
 - From users to base station
 - Multipoint-to-point MIMO
- Downlink
 - From base station to users
 - Point-to-multipoint MIMO



Orthogonal multiple access

- Two users want to communicate with base station
 - Power per user: P
 - Bandwidth: B
 - Noise power spectral density: N_0
- Divide bandwidth: αB to user 1, $(1 - \alpha)B$ to user 2

$$R_1 = \alpha B \log_2 \left(1 + \frac{P\beta}{\alpha B N_0} \right)$$
$$R_2 = (1 - \alpha) B \log_2 \left(1 + \frac{P\beta}{(1 - \alpha) B N_0} \right)$$



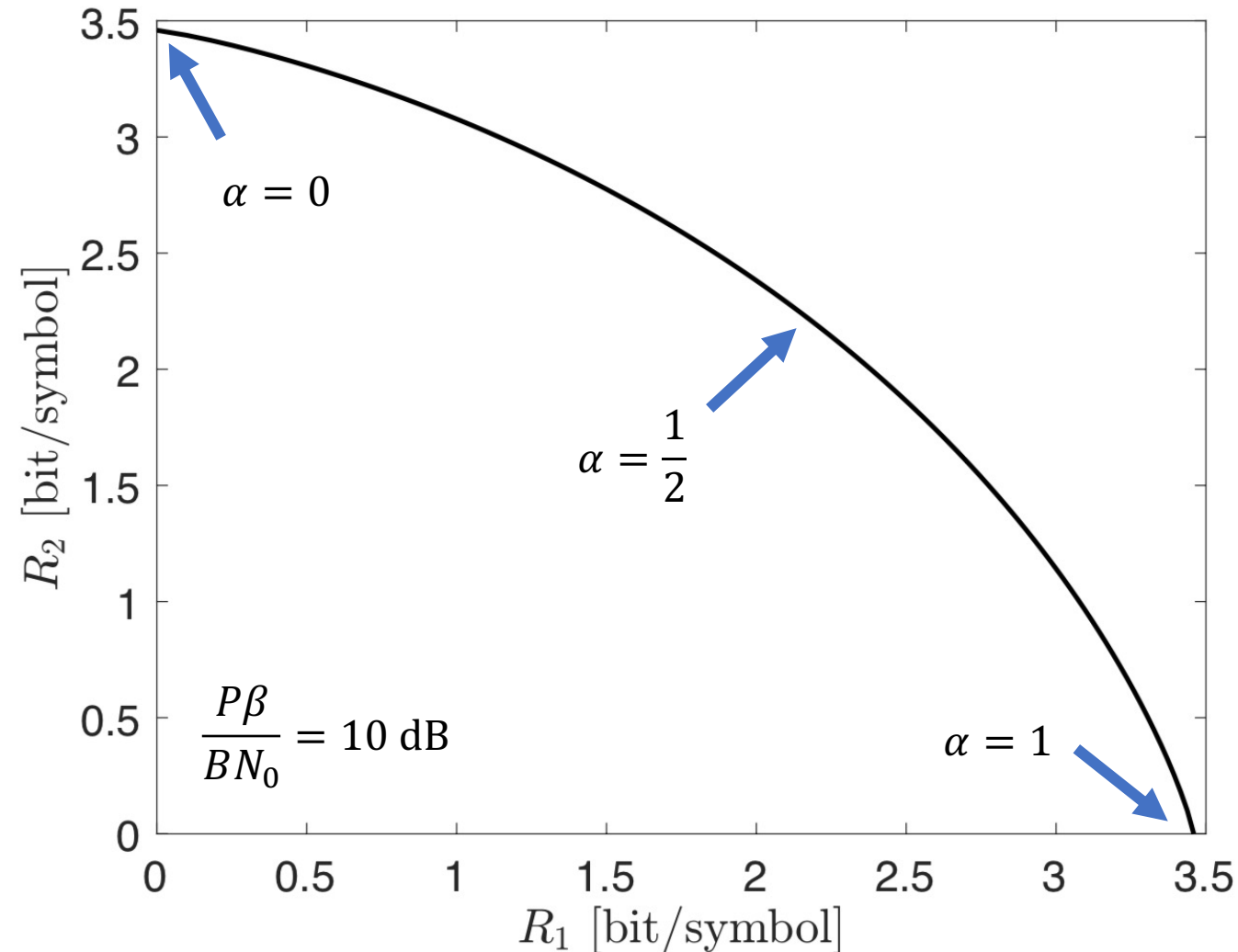
Orthogonal multiple access: Rate region

- Rates depend on α :

$$R_1 = \alpha B \log_2 \left(1 + \frac{P\beta}{\alpha B N_0} \right)$$

$$R_2 = (1 - \alpha) B \log_2 \left(1 + \frac{P\beta}{(1 - \alpha) B N_0} \right)$$

What is the preferred operating point?



Non-orthogonal multiple access

- Let both users transmit simultaneously:

$$y = x_1 + x_2 + n$$

From user 1
 $x_1 \sim \mathcal{CN}(0, P)$

Noise
(power BN_0)

Received signal

From user 2
 $x_2 \sim \mathcal{CN}(0, P)$

- Strategy:
 - Decode signal from user 1, treat interference as noise
 - Subtract x_1 : $y - x_1 = x_2 + n$. Decode signal from user 2:

$$R_1 = \log_2 \left(1 + \frac{P}{P + BN_0} \right)$$

$$R_2 = \log_2 \left(1 + \frac{P}{BN_0} \right)$$

Called: Successive interference cancelation

We can change the user order

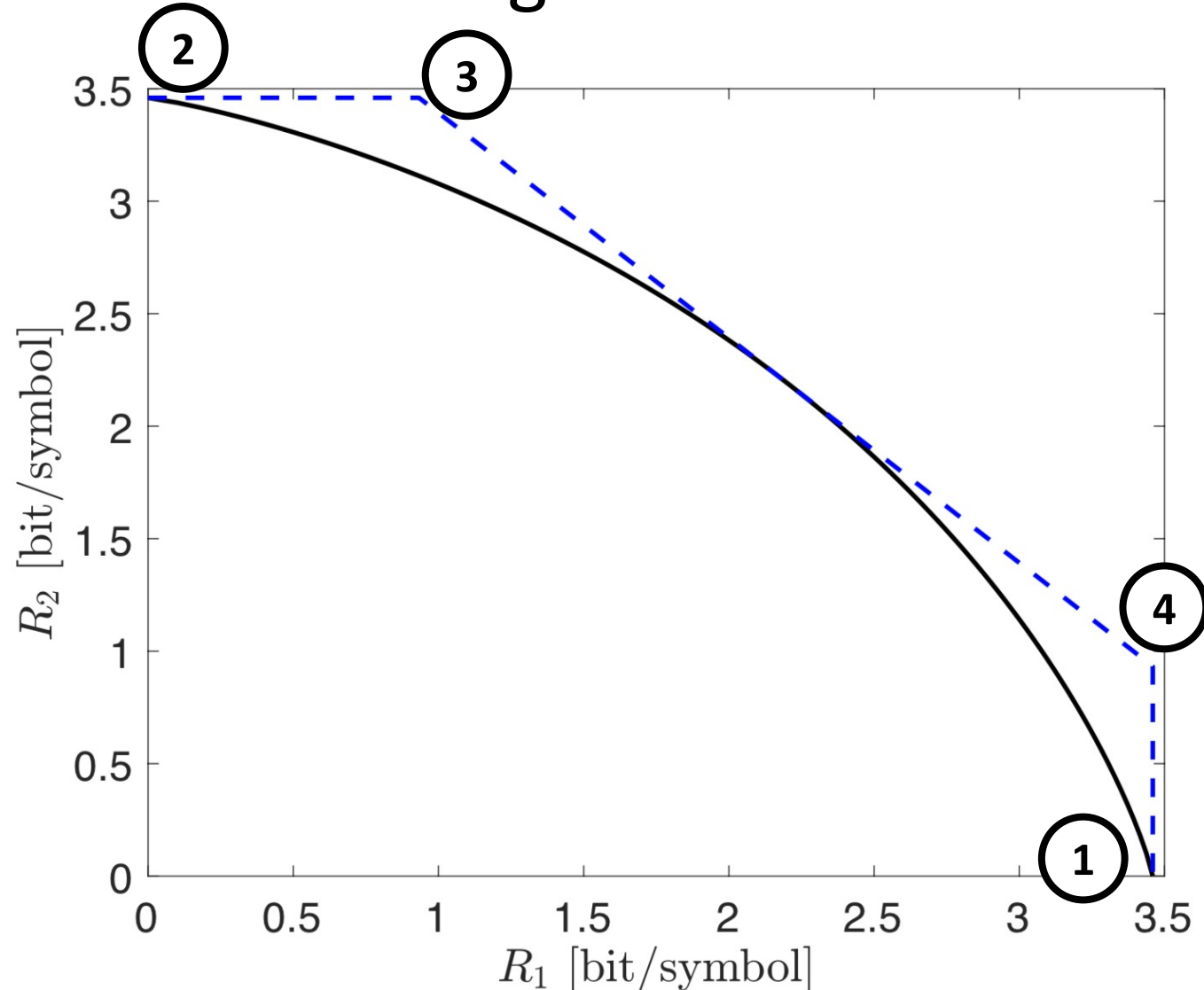
Non-orthogonal multiple access: Rate region

Four operating points (R_1, R_2) :

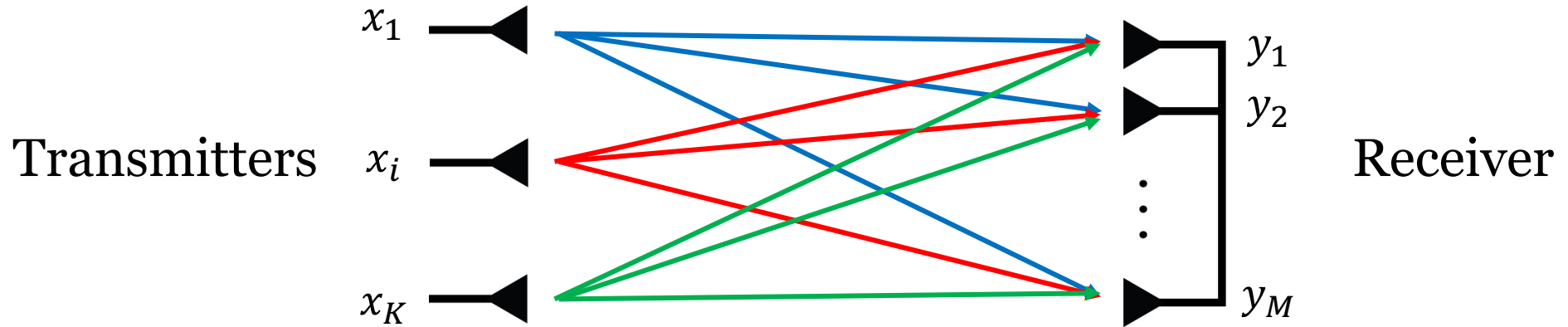
1. $\left(\log_2 \left(1 + \frac{P}{BN_0}\right), 0\right)$
2. $\left(0, \log_2 \left(1 + \frac{P}{BN_0}\right)\right)$
3. $\left(\log_2 \left(1 + \frac{P}{P+BN_0}\right), \log_2 \left(1 + \frac{P}{BN_0}\right)\right)$
4. $\left(\log_2 \left(1 + \frac{P}{BN_0}\right), \log_2 \left(1 + \frac{P}{P+BN_0}\right)\right)$

Time sharing:

We can achieve all points in between



Uplink in Multiuser MIMO



- Notation:
 - K single-antenna users, M base station antennas
 - Channel response g_i^j from user i to antenna j
 - Data signals x_1, \dots, x_K , received signals y_1, \dots, y_M

Uplink Multiuser MIMO: System model

- Received signal:

$$\mathbf{y} = \sqrt{\rho_{ul}} \mathbf{G} \mathbf{x} + \mathbf{w}$$

where

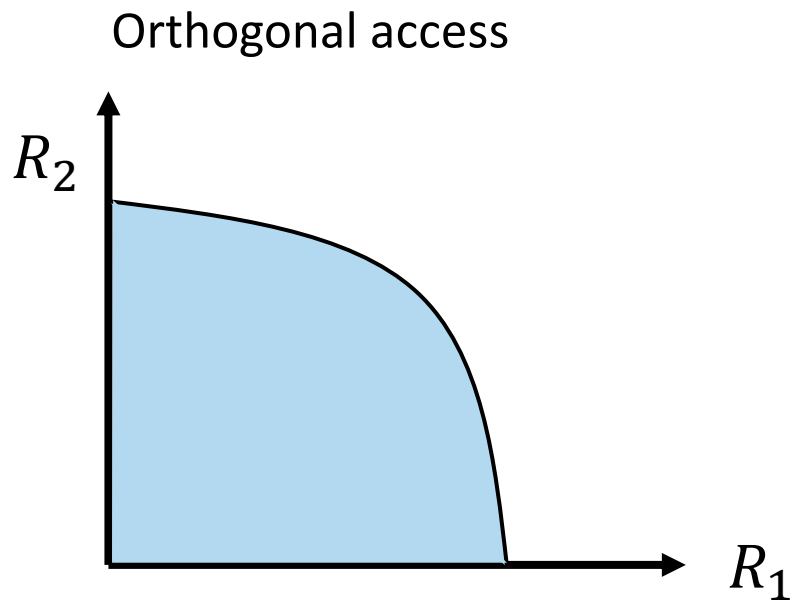
$$\mathbf{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_M \end{pmatrix} \quad \mathbf{G} = \begin{pmatrix} g_1^1 & \cdots & g_K^1 \\ \vdots & \ddots & \vdots \\ g_1^M & \cdots & g_K^M \end{pmatrix} \quad \mathbf{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_K \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_M \end{pmatrix}$$

- Parameters are normalized: SNR is ρ_{ul}
- Each users signal is power-limited as $\mathbb{E}\{|x_k|^2\} \leq 1$
- Normalized noise: $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$

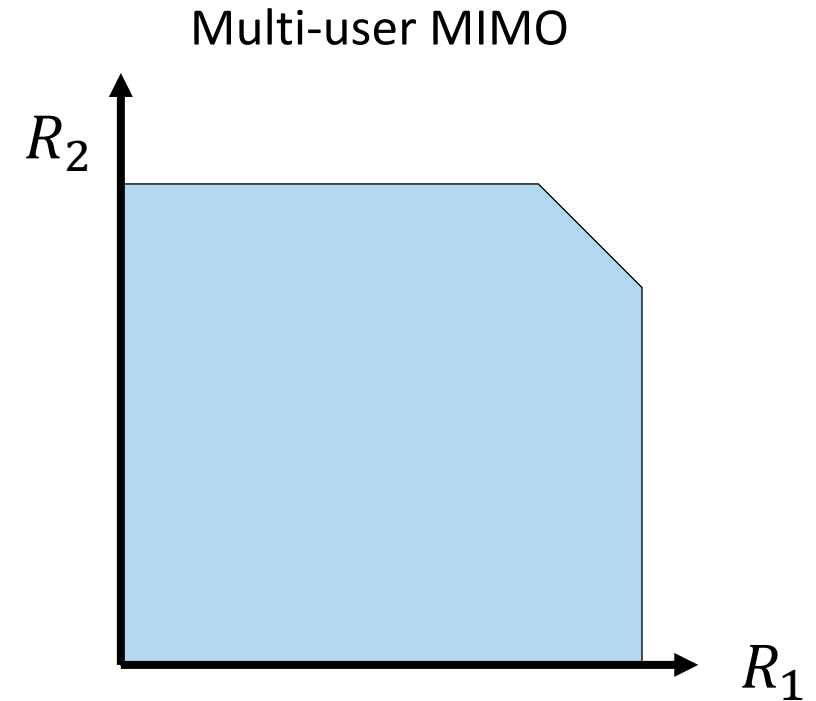
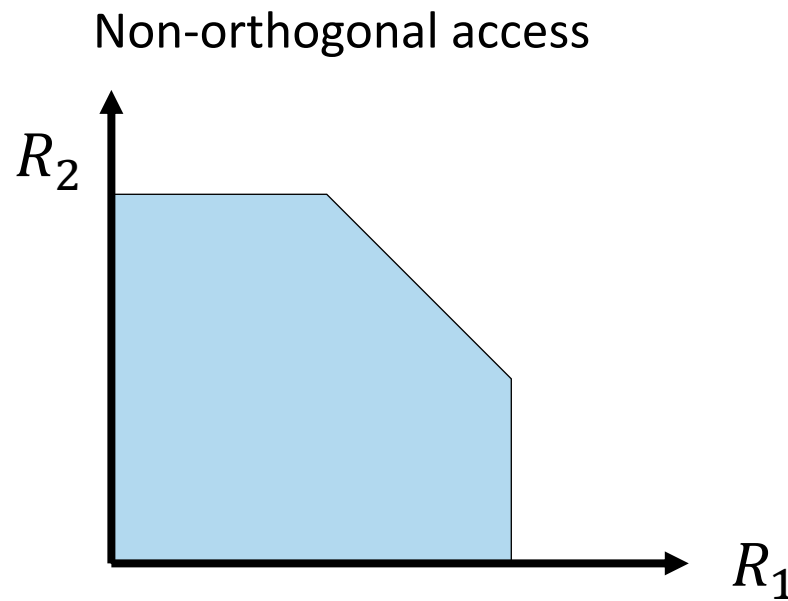
What is the difference from point-to-point MIMO?

- **Difference 1:** Users do not cooperate
 - x_1, \dots, x_K are independent signals
- **Difference 2:** Each user cares about its own performance
 - K user capacities instead of one capacity
- **Difference 3:** Each user has its own power budget
 - Power constraint: $E\{|x_k|^2\} \leq 1$
- **Difference 4:** The channel matrix \mathbf{G} is modeled differently
 - Each column can be modeled as a SIMO channel

Motivating example



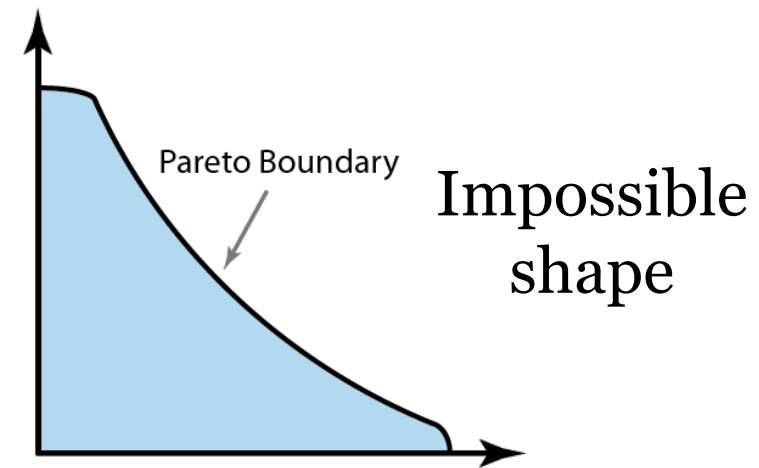
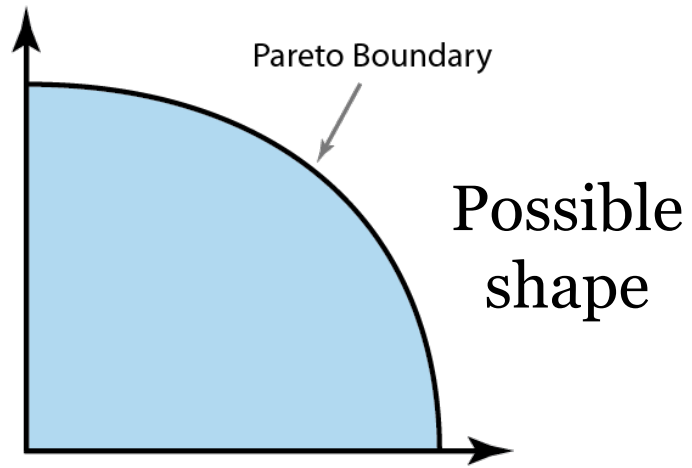
Capacity or rate region?
Capacity region is the largest possible rate region



Two benefits:
Beamforming gain for every user
Smaller interference loss

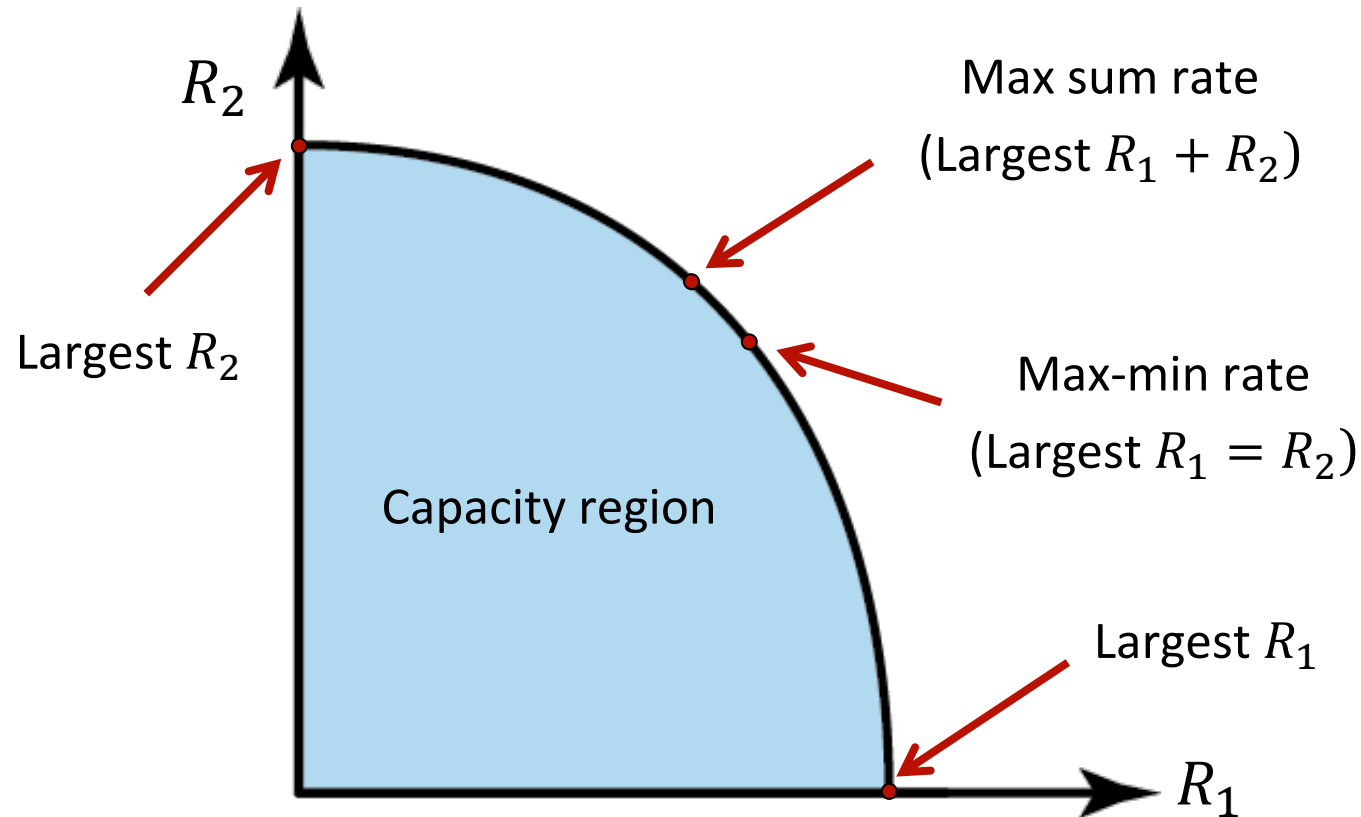
Shape of capacity region

- One can pick two points and use them fractions of the time
 - Similar to time-sharing
 - Hence: Line between any two points are in the region
 - Conclusion: Region must be a *convex set*



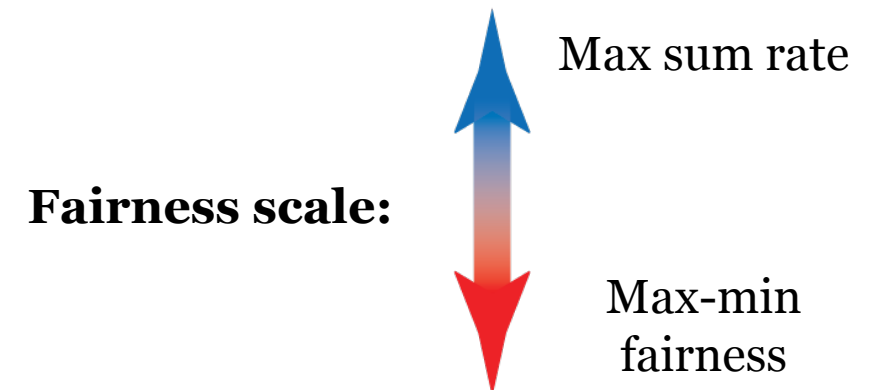
Points in the capacity region

- Combinations (R_1, R_2) of rates that can be simultaneously achieved



Many choices!

Only rule:
Always pick something
on the outer boundary
(Pareto boundary)!



Sum Capacity of Uplink Multiuser MIMO

- Recall: Received signal:

$$\mathbf{y} = \sqrt{\rho_{ul}} \mathbf{G} \mathbf{x} + \mathbf{w}$$

- Assume a deterministic \mathbf{G}

- Let all users transmit with full power: $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_K)$

Like a point-to-point MIMO channel

But with a “suboptimal” signal covariance matrix $\mathbf{Q} = \mathbf{I}_M$

- Sum rate: $R_1 + \dots + R_K = \log_2(\det(\mathbf{I}_M + \rho_{ul} \mathbf{G} \mathbf{G}^H))$

This is the sum capacity!

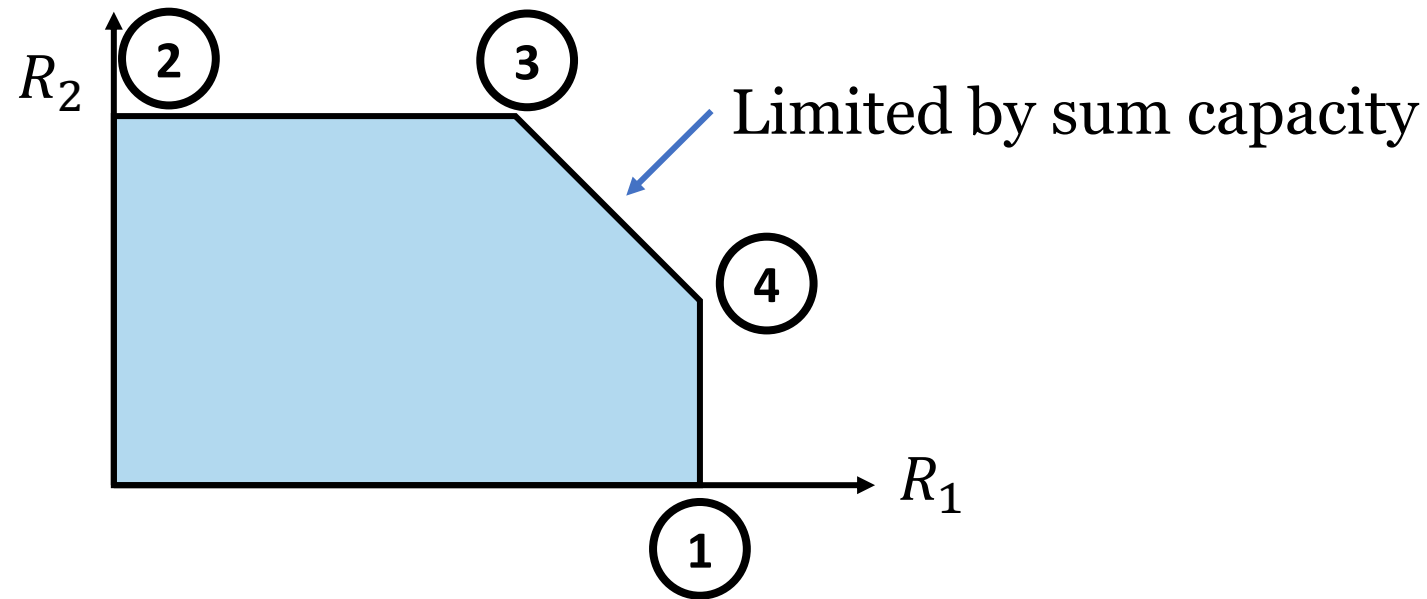
Achieved by successive interference cancellation
Decoding order determines who gets which share

Uplink Capacity Region with $K = 2$

- Region contains all (R_1, R_2) satisfying:

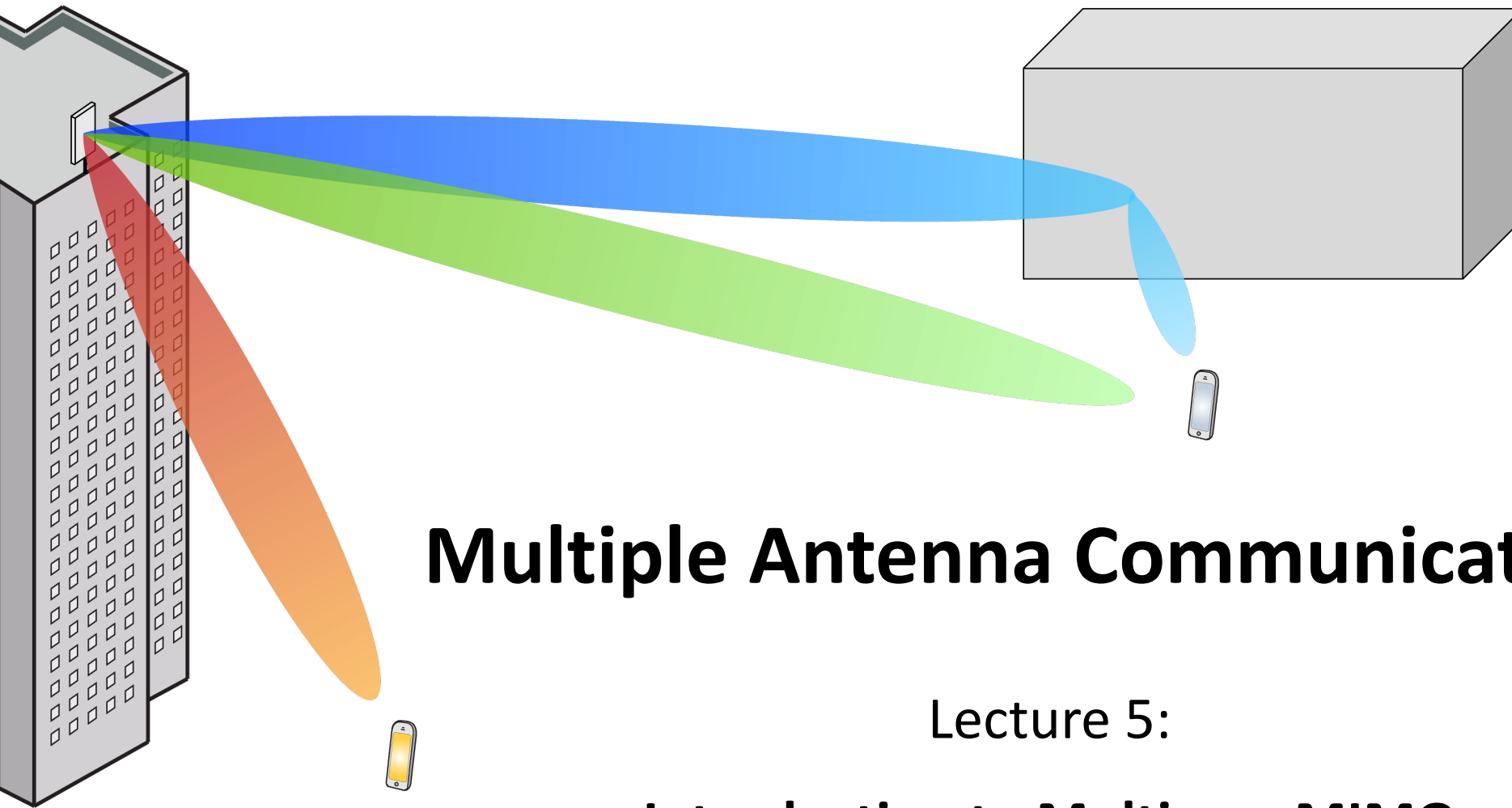
$$\begin{aligned} R_1 &\leq \log_2(1 + \rho_{ul} \|\mathbf{g}_1\|^2) \\ R_2 &\leq \log_2(1 + \rho_{ul} \|\mathbf{g}_2\|^2) \\ R_1 + R_2 &\leq \log_2(\det(\mathbf{I}_M + \rho_{ul} \mathbf{G} \mathbf{G}^H)) \end{aligned}$$

$$\mathbf{G} = [\mathbf{g}_1 \ \mathbf{g}_2]$$



Summary

- Point-to-point MIMO channels
 - Large multiplexing gains are hard to achieve in practice
- Multi-user MIMO channels
 - Similar system model
 - Key differences: Independent users, different power, different performance
 - Capacity and rate regions
 - Orthogonal and non-orthogonal access



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