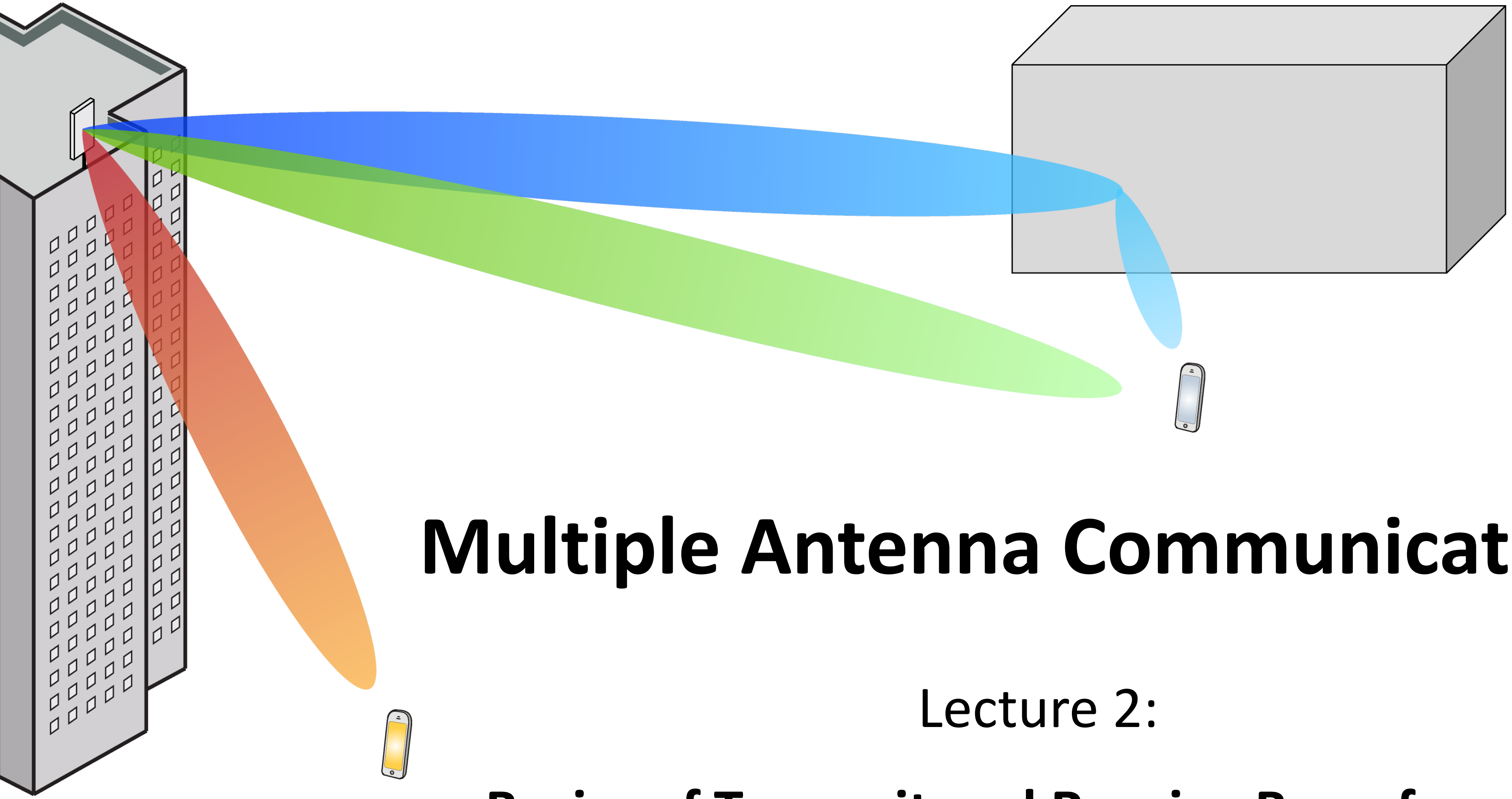


Multiple Antenna Communications

Lecture 2:

Basics of Transmit and Receive Beamforming

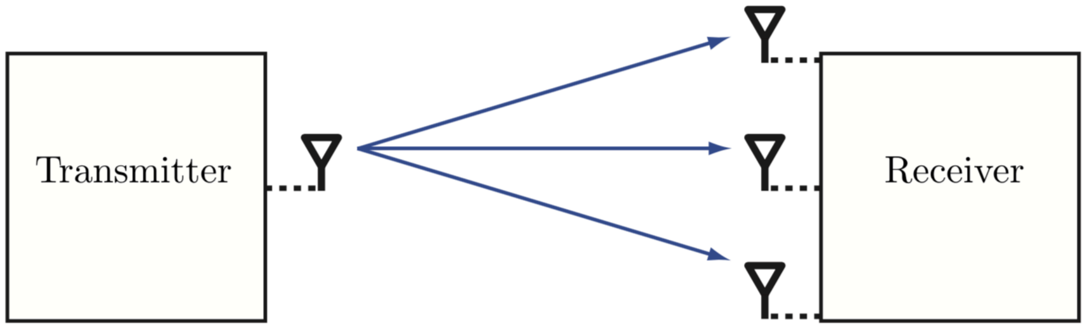
Emil Björnson

Outline

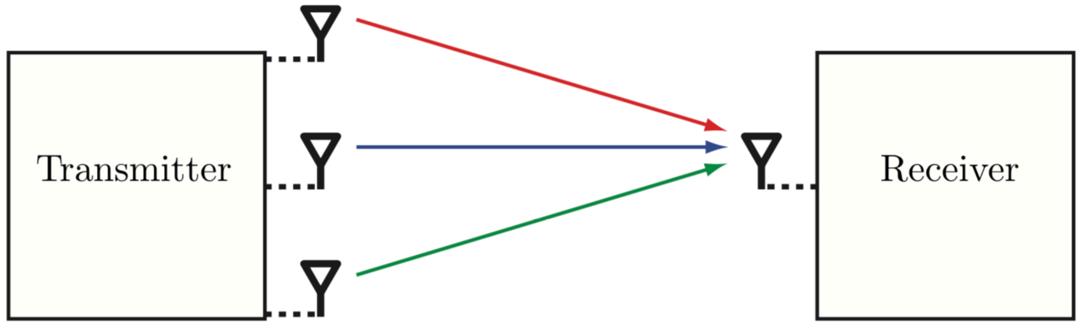
- Recap of single-antenna channel capacity
- Capacity of channels with multiple antennas at one side
- Channel models for line-of-sight channels
- Beamwidth with uniform linear arrays

Taxonomy: Point-to-point channels

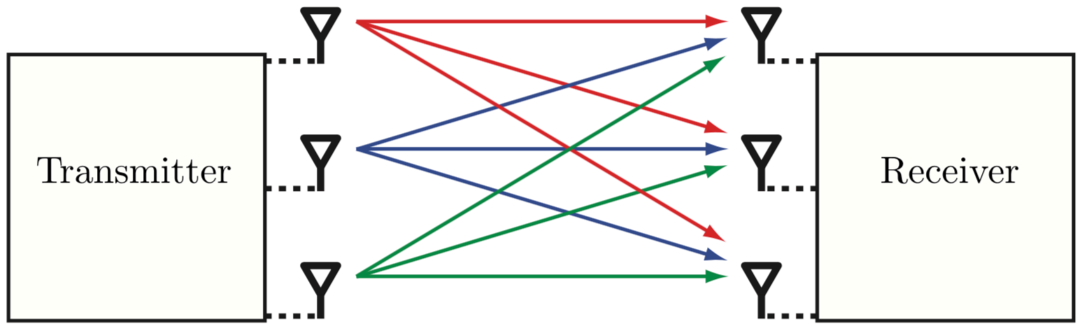
(a) Point-to-point SISO channel.



(b) Point-to-point SIMO channel.



(c) Point-to-point MISO channel.

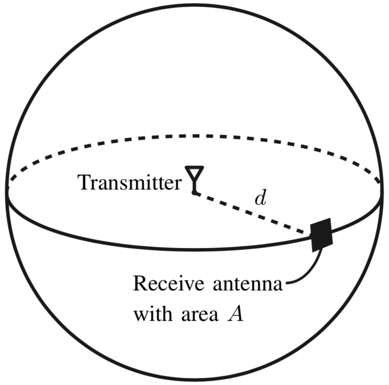


(d) Point-to-point MIMO channel.

Single-input single-output (SISO)
Single-input multiple-output (SIMO)

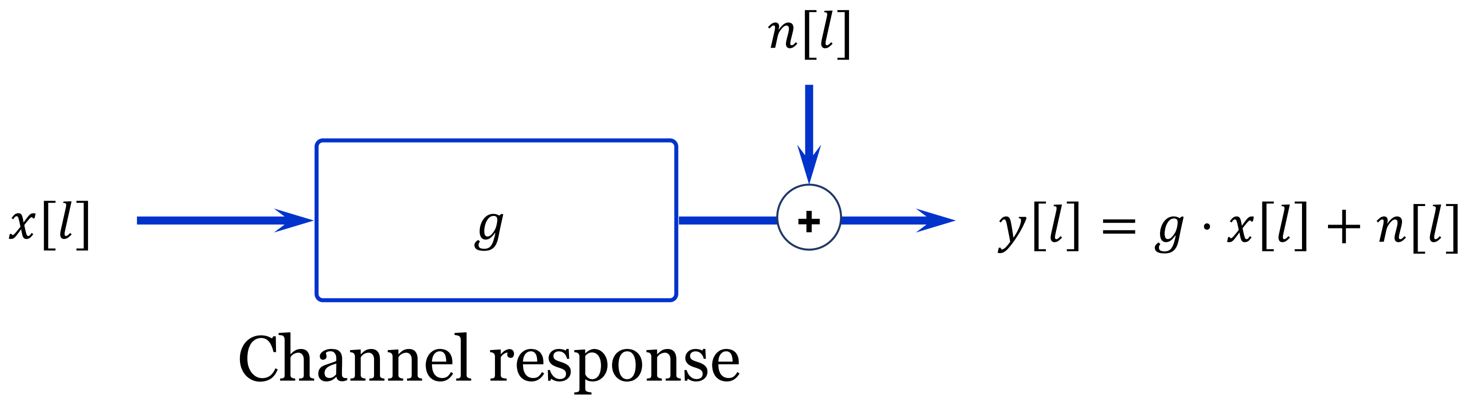
Multiple-input single-output (MISO)
Multiple-input multiple-output (MIMO)

Not all transmit power is received



- Wireless signals spread out as a sphere
  - Isotropic antenna: Fraction of received signal at distance d: $\frac{\lambda^2}{(4\pi d)^2}$
- Mobile communications: 70-130 dB is typically lost!
  - Channel gain $|g|^2$ is around -70 dB to -130 dB

Also known as *pathloss*



Capacity of memoryless SISO channel

- System model in symbol-sampled complex baseband:

$$y[l] = g \cdot x[l] + n[l]$$

  - $x[l] \sim CN(0, q)$, energy per symbol: $q = P/B$
  - $n[l] \sim CN(0, N_0)$

Memoryless channel: Drop dependence on l

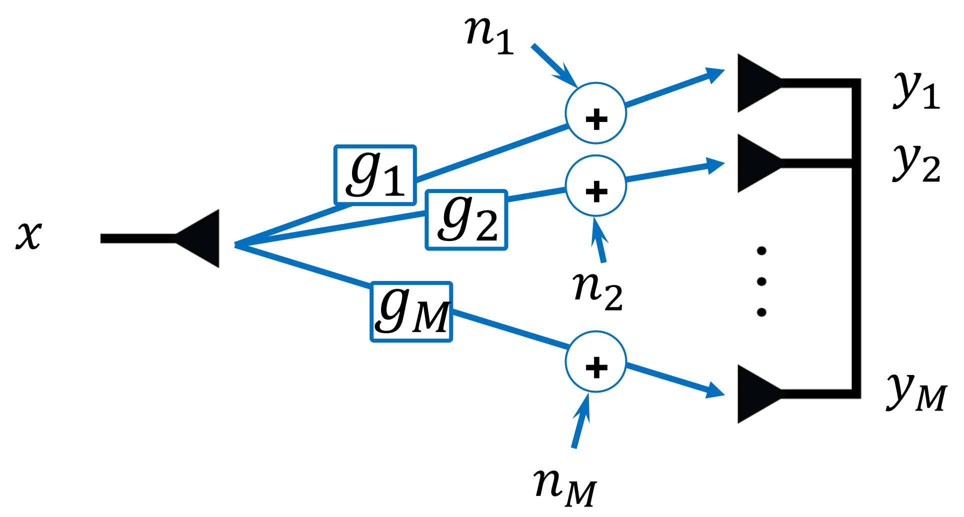
- Capacity:

$$\mathrm{C} = \log_2\left(1 + \frac{q|g|^2}{N_0}\right) \text{bits per symbol}$$

Received SNR

How can we increase the SNR?

Single-input multiple-output (SIMO) channel



Independent noise:
$n_m \sim CN(0, N_0)$

- Received vector signal:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} g_1 \\ \vdots \\ g_M \end{bmatrix} x + \begin{bmatrix} n_1 \\ \vdots \\ n_M \end{bmatrix}$$

  - Short form:

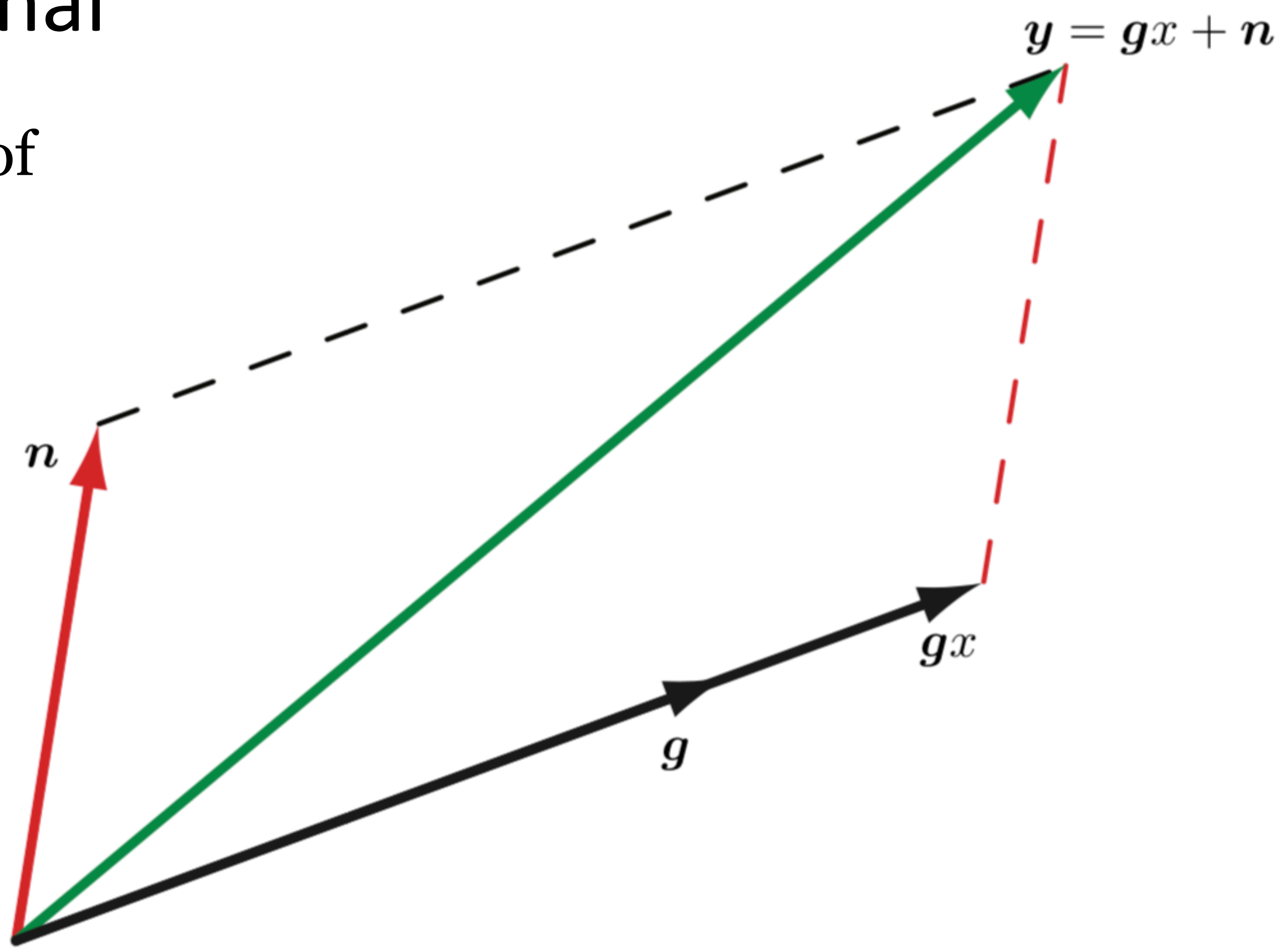
$$\boldsymbol{y} = \boldsymbol{g}x + \boldsymbol{n}$$

Geometry of received signal

- Received vector $\boldsymbol{y}$ is summation of
  - Desired signal $\boldsymbol{g}x$
  - Noise vector $\boldsymbol{n}$

- Want to turn $\boldsymbol{y}$ into an estimate of x

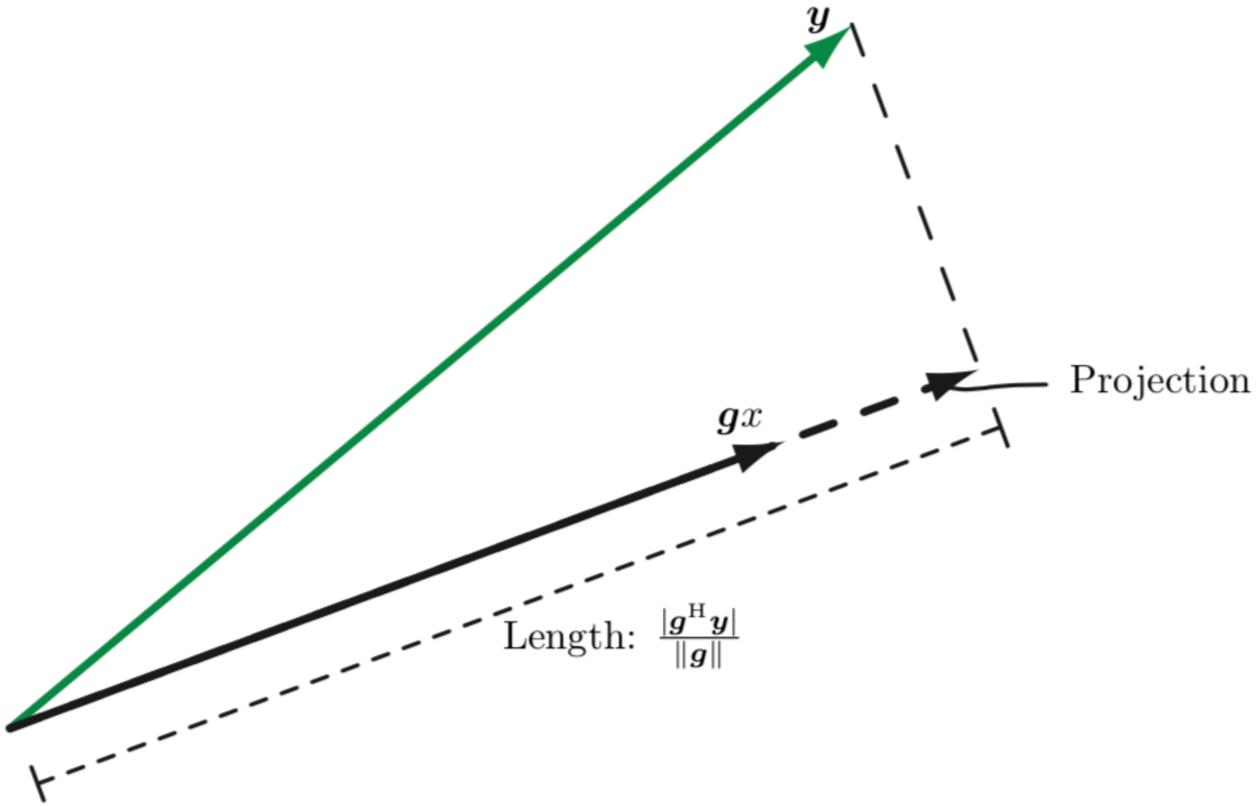
Only the part of $\boldsymbol{y}$ parallel to $\boldsymbol{g}$ contains the desired signal



Capacity of SIMO channel

- Maximum ratio combining (MRC):

$$\underbrace{\frac{\boldsymbol{g}^H}{\|\boldsymbol{g}\|}}_{\text{Combining vector}}\boldsymbol{y} = \underbrace{\frac{\boldsymbol{g}^H\boldsymbol{g}}{\|\boldsymbol{g}\|}}_{\text{Scalar } = \|\boldsymbol{g}\|}x + \frac{\boldsymbol{g}^H\boldsymbol{n}}{\|\boldsymbol{g}\|}$$

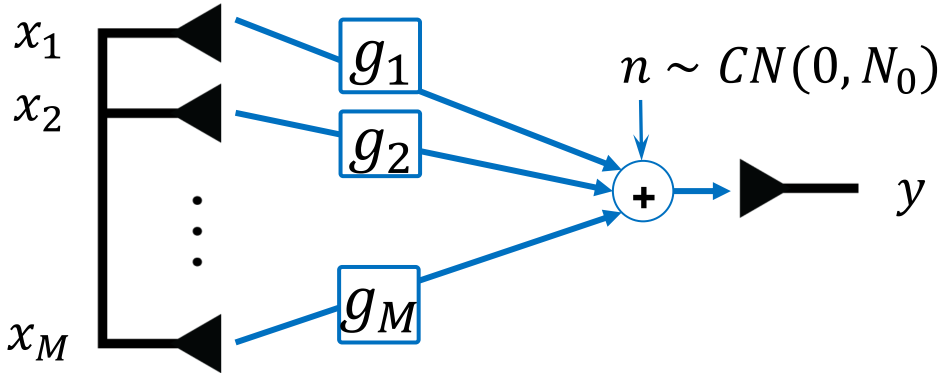


Like a SISO channel with $g = \|\boldsymbol{g}\|$

- Capacity:

$$C = \log_2\left(1 + \frac{q\|\boldsymbol{g}\|^2}{N_0}\right) \text{ bits per symbol}$$

Multiple-input single-output (MISO) channel



Inner product between $\boldsymbol{g}^*$ and $\boldsymbol{x}$

- Received vector signal:

$$y = [g_1 \quad \cdots \quad g_M] \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} + n = \boldsymbol{g}^T \boldsymbol{x} + n$$

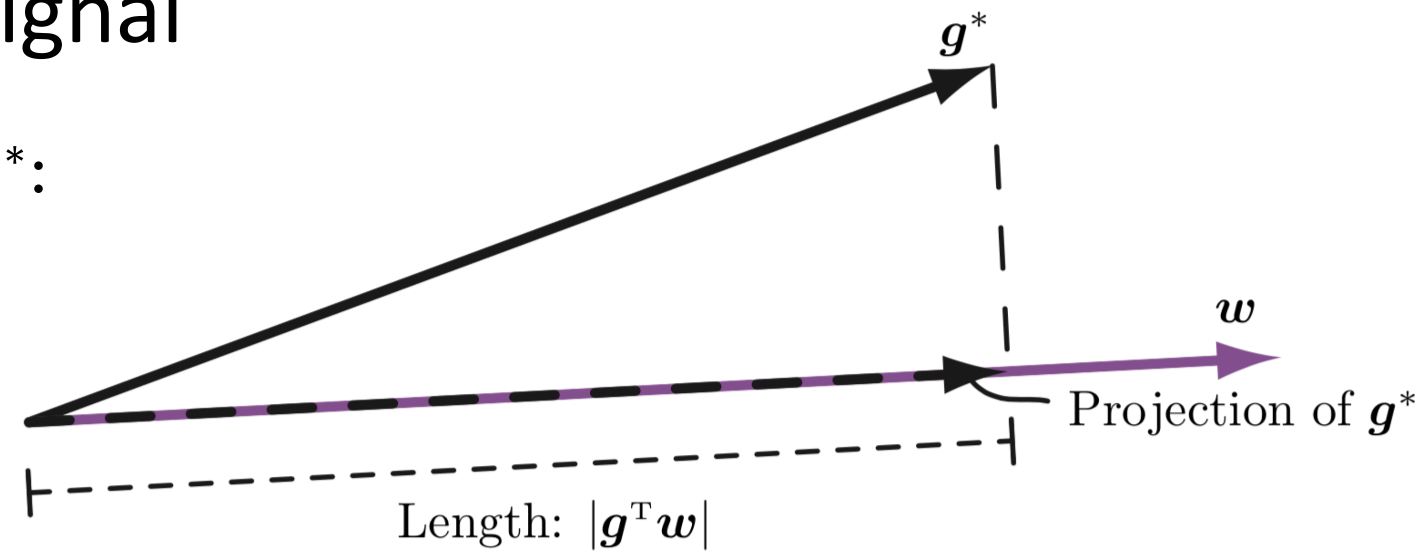
  - Precoding: $\boldsymbol{x} = \boldsymbol{w}\tilde{x}$

Information signal: $\tilde{x} \sim CN(0, q)$

Unit-norm precoding vector

Geometry of received signal

- Channel projects signal onto $\boldsymbol{g}^*$:



Maximum ratio transmission (MRT):

$$\boldsymbol{w} = \frac{\boldsymbol{g}^*}{\|\boldsymbol{g}\|}$$

$$y = \boldsymbol{g}^T \frac{\boldsymbol{g}^*}{\|\boldsymbol{g}\|} \tilde{x} + n = \|\boldsymbol{g}\| \tilde{x} + n$$

- Capacity:

$$\mathrm{C} = \log_2\left(1 + \frac{q\|\boldsymbol{g}\|^2}{N_0}\right) \text{ bits per symbol}$$

Receive and transmit beamforming

- Same capacity of SIMO and MISO channel

$$\mathrm{C} = \log_2\left(1 + \frac{q\|\boldsymbol{g}\|^2}{N_0}\right) \text{ bits per symbol}$$

  - Achieved by "beamforming" along channel vector $\boldsymbol{g}$
  - Beamforming gain of $\|\boldsymbol{g}\|^2$ (often proportional to M)
  - Antenna m contributes proportionally to $|g_m|^2$

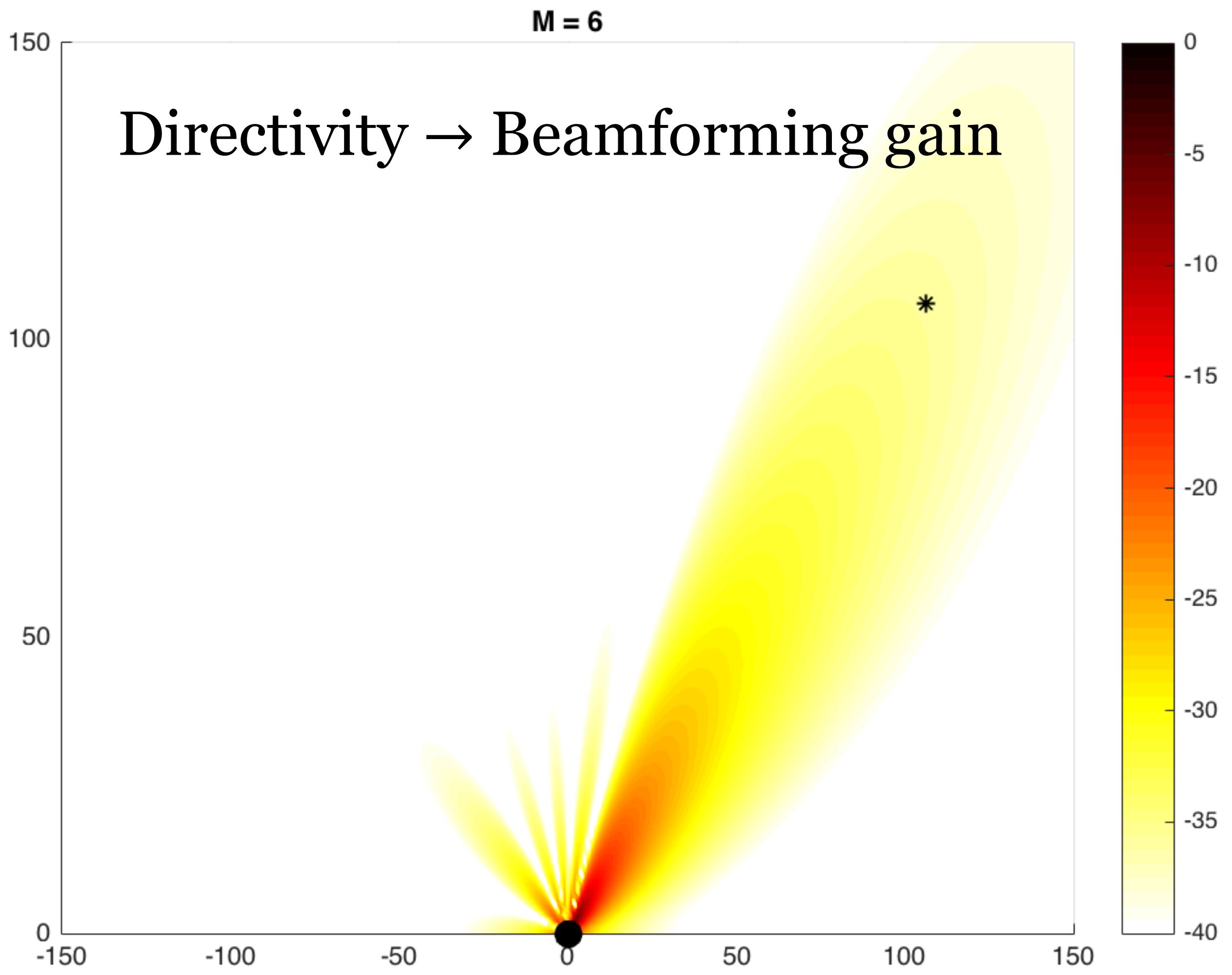
Several different names

Maximum ratio combining/transmission (MRC, MRT)
Conjugate beamforming, matched filtering

Transmit beamforming

- Other names:
  - Array gain
  - Power gain

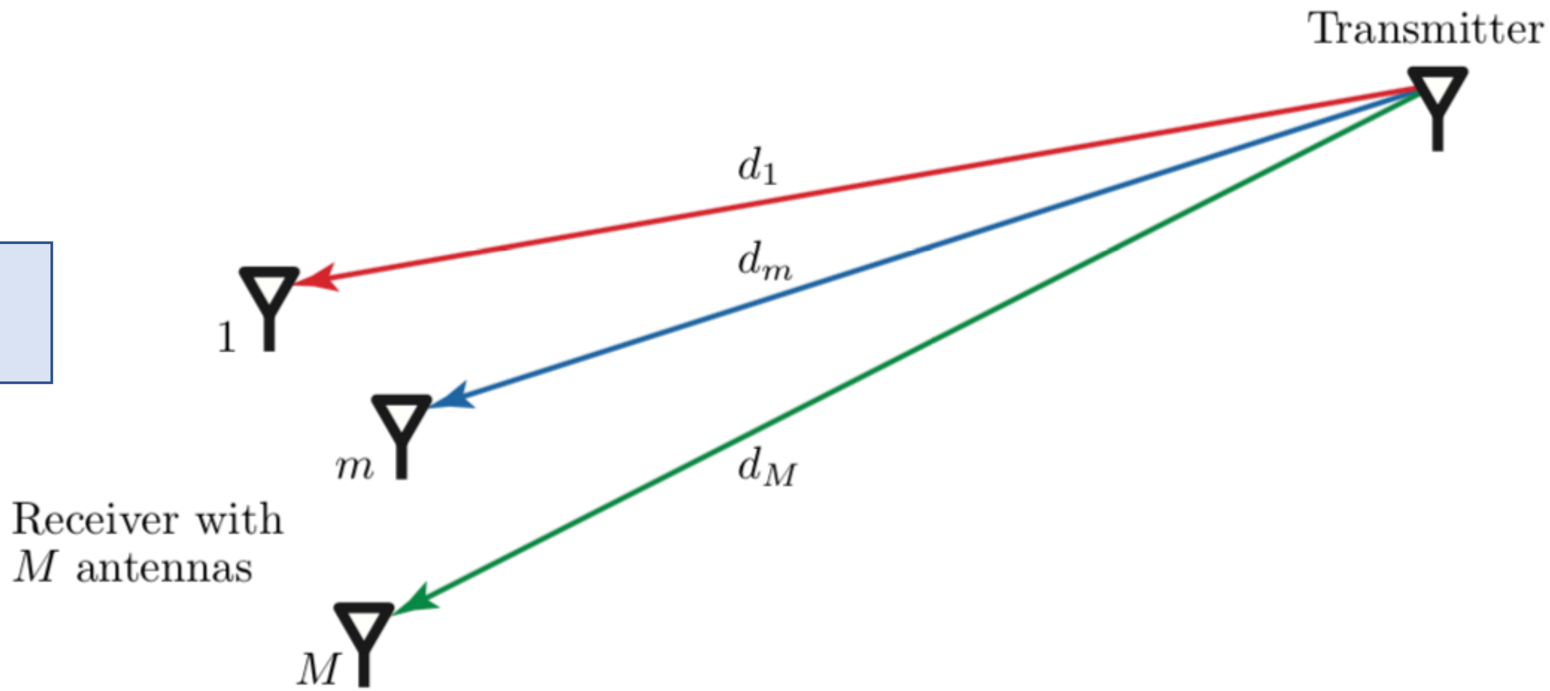
This is a valid interpretation only for line-of-sight channels



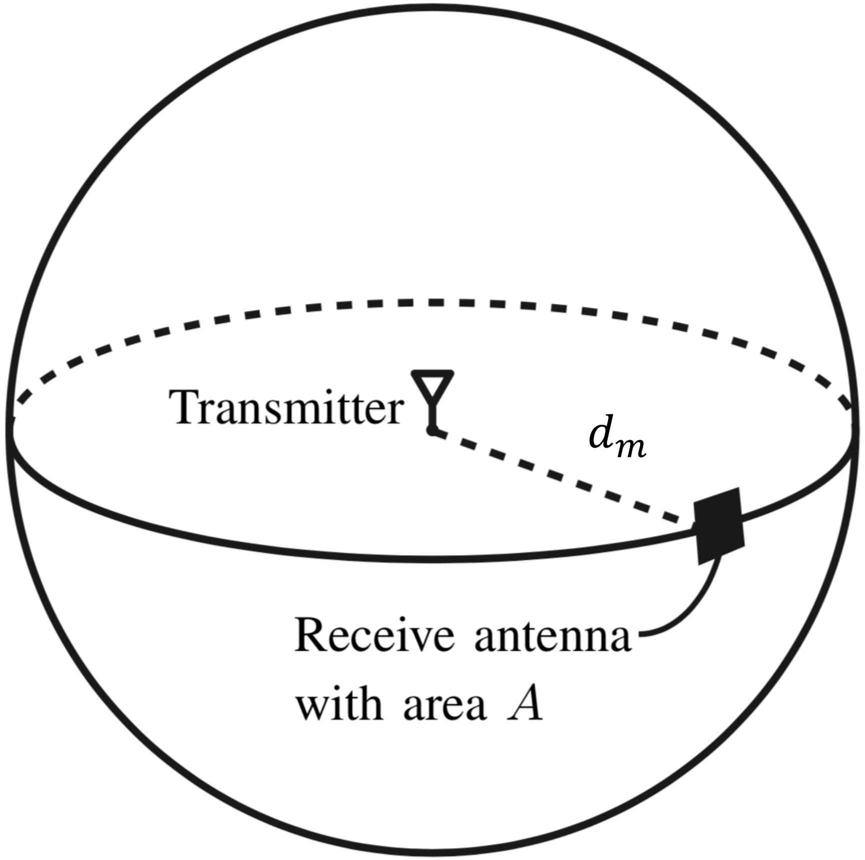
Free-space line-of-sight commmunication

- Direct path between transmitter and receiver
  - No other paths

We will derive a channel model



Complex-baseband model for antenna m



Propagation delay: $\tau_m = \frac{d_m}{c}$

Impulse response in passband: $h_{p,m}(t) = \alpha_m \delta(t + \eta - \tau_m)$

Amplitude loss: $\frac{\lambda}{4\pi d_m}$

Synchronization

$$\boxed{\text{Equivalent in complex baseband: } h_m(t) = \alpha_m e^{-j2\pi f_c t} \delta(t + \eta - \tau_m)}$$

Symbol-sampled system model:

$$y_m[l] = \sum_k x[k]\, \alpha_m e^{-j2\pi f_c\left(\frac{d_m}{c} - \eta\right)} \operatorname{sinc}\left(l - k + B\left(\eta - \frac{d_m}{c}\right)\right) + n[l]$$

$$= x[l]\alpha_m + n[l] \qquad \text{for } \eta = \frac{d_m}{c}$$

Same sampling time at all receive antennas

- Received at antenna m:

$$y_m[l] = \sum_k x[k]\,\alpha_m e^{-j2\pi f_c\left(\frac{d_m}{c}-\eta\right)} \mathrm{sinc}\left(l - k + B\left(\eta - \frac{d_m}{c}\right)\right) + n[l]$$

Narrowband assumption: $B\frac{d_m}{c}$ roughly the same for all antennas

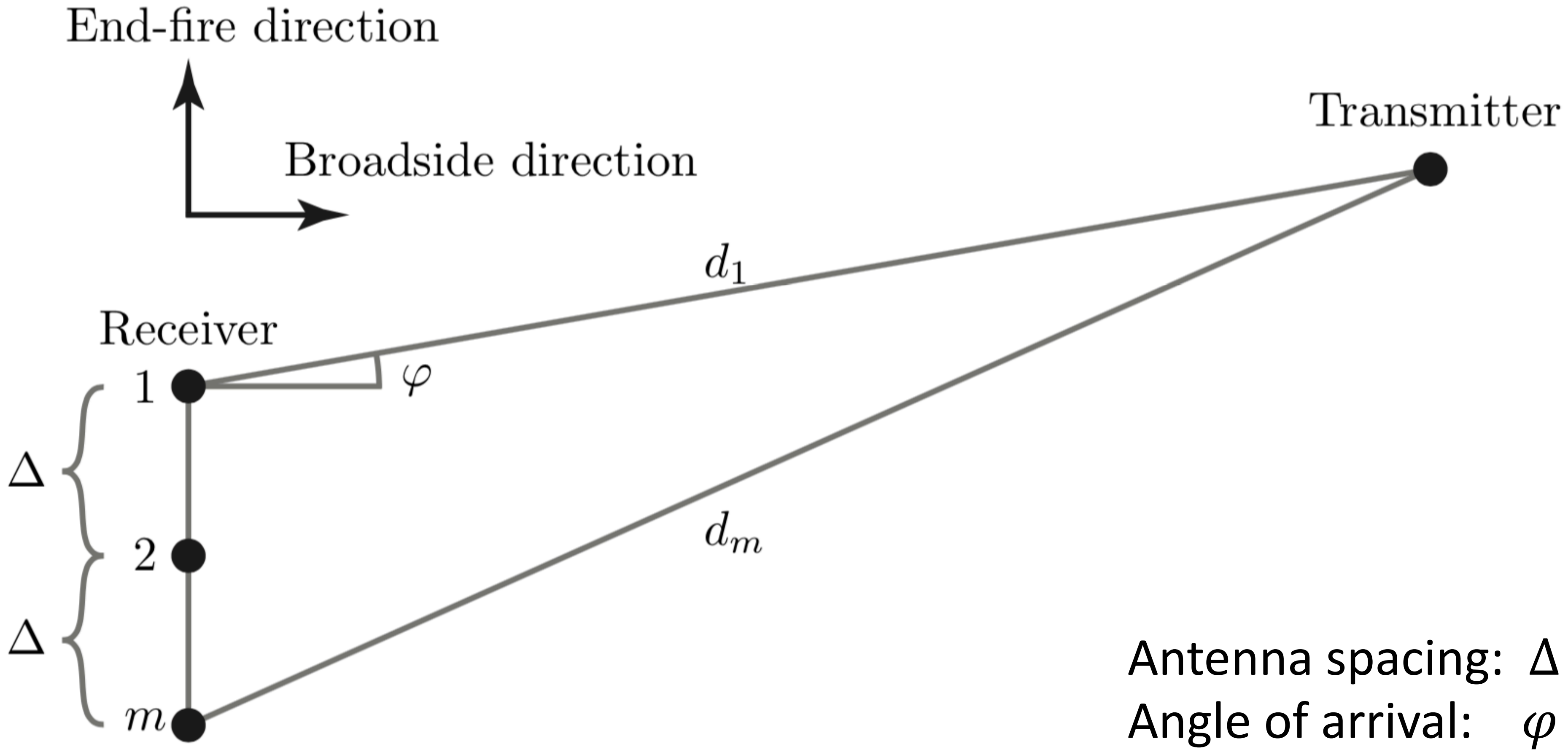
- Use antenna 1 as reference, $\eta = \frac{d_1}{c}$, and use $\lambda = \frac{c}{f_c}$:

$$y_m[l] = \sum_k x[k]\,\alpha_m e^{-j2\pi\left(\frac{d_m-d_1}{\lambda}\right)} \mathrm{sinc}\left(l - k + B\left(\frac{d_1 - d_m}{c}\right)\right) + n[l]$$

$$\approx x[l]\alpha_m e^{-j2\pi\left(\frac{d_m-d_1}{\lambda}\right)} + n[l]$$

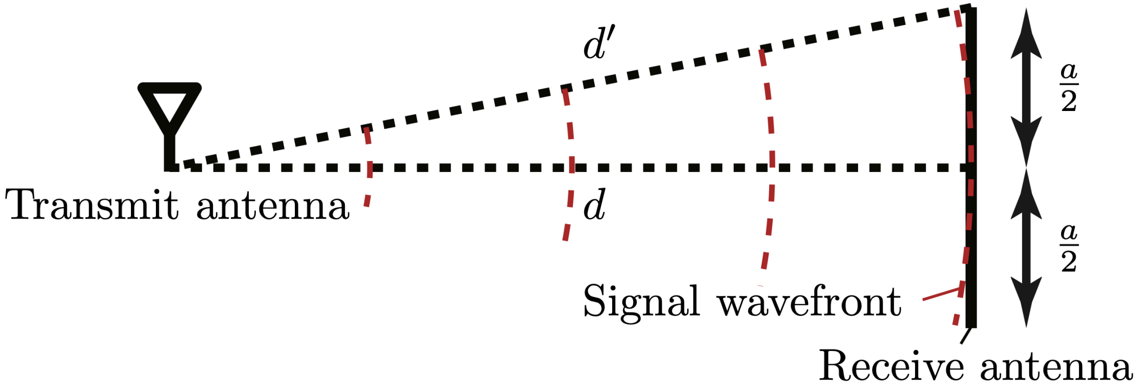
Channel model: $\boldsymbol{g} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_M e^{-j2\pi\left(\frac{d_M-d_1}{\lambda}\right)} \end{bmatrix}$

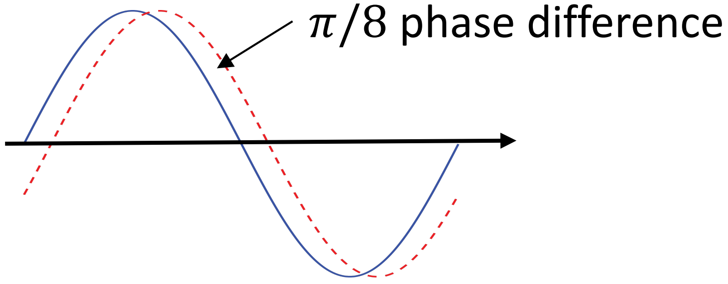
Uniform linear array (ULA)

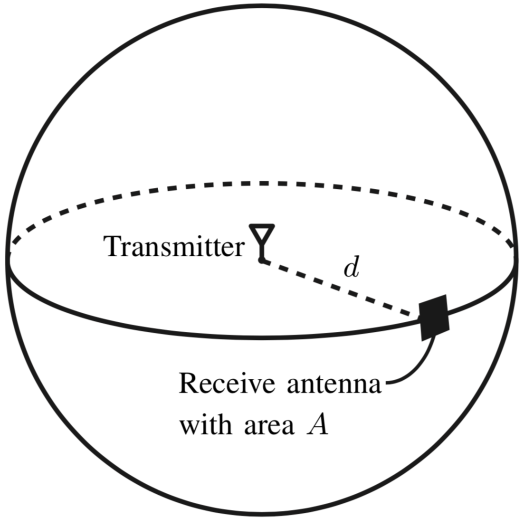


Antenna spacing: Δ
Angle of arrival: φ

Near-Field and Far-Field of an Antenna





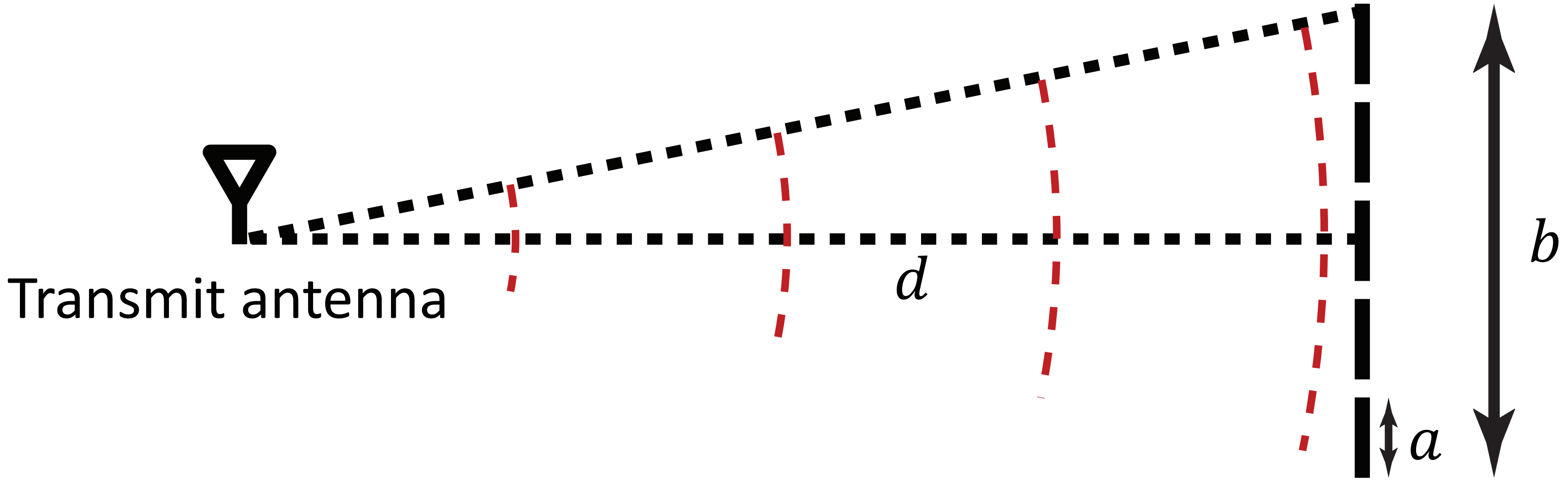


- Length difference: $d' - d = \sqrt{d^2 + \left(\frac{a}{2}\right)^2} - d \approx \frac{a^2}{8d}$
- At most $\pi/8$ phase diff: $\frac{\pi}{8} \geq \frac{2\pi}{\lambda}\frac{a^2}{8d} \rightarrow d \geq \frac{2a^2}{\lambda}$

Fraunhofer distance

Other electromagnetic near-field effects
Evanescent waves, Magnetic induction, …

Near-Field and Far-Field of an Array



Fraunhofer distance

Far-field of antenna: $d \geq 2a^2/\lambda$

Far-field of the array $d \geq 2b^2/\lambda$

Example: 3 GHz, $\lambda = 0.1$ m

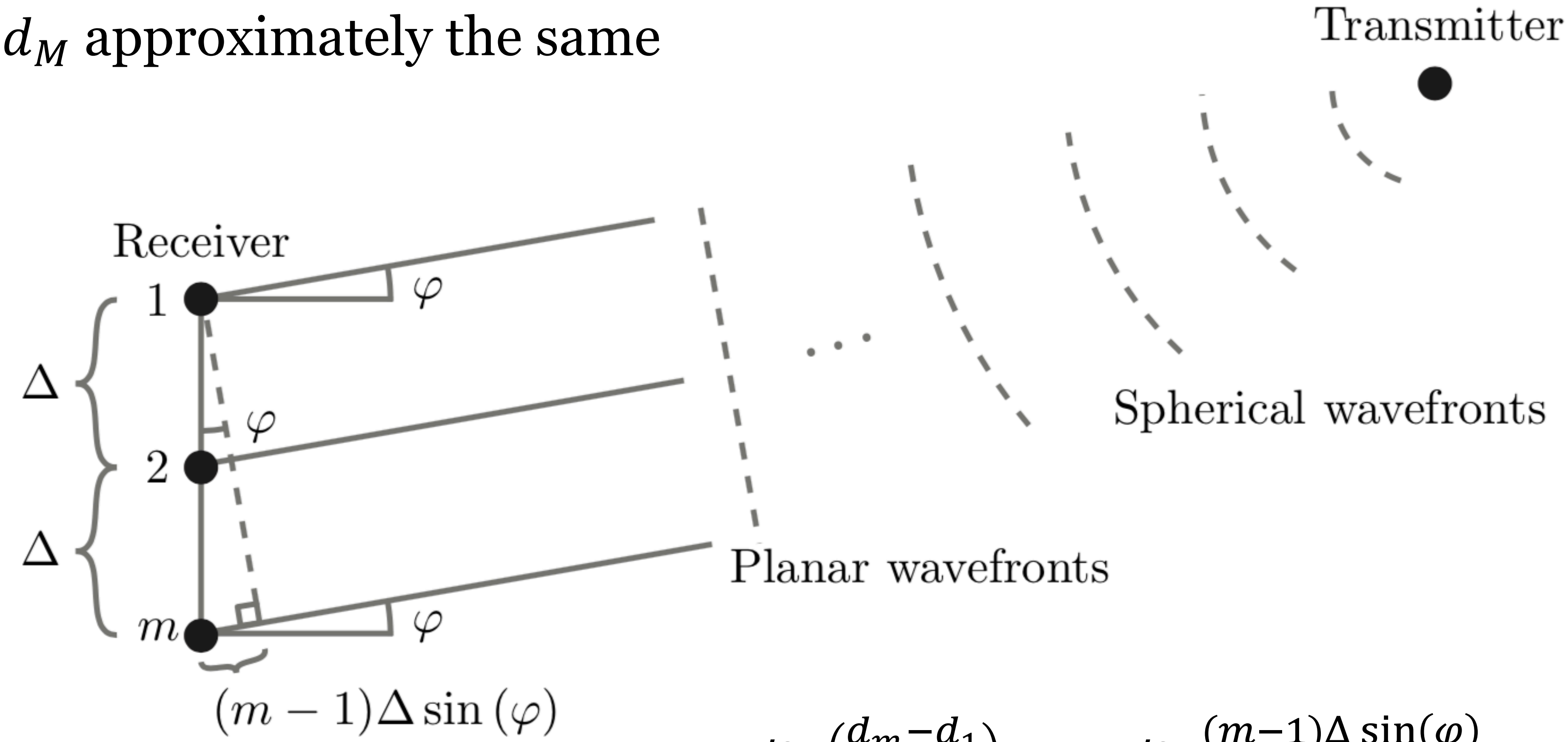
$b = 10\lambda = 1$ m: $\frac{2b^2}{\lambda} = 20$ m

We are typically in the far-field!

Far-field approximation with ULA

- Distances $d_1, \dots, d_M$ approximately the same

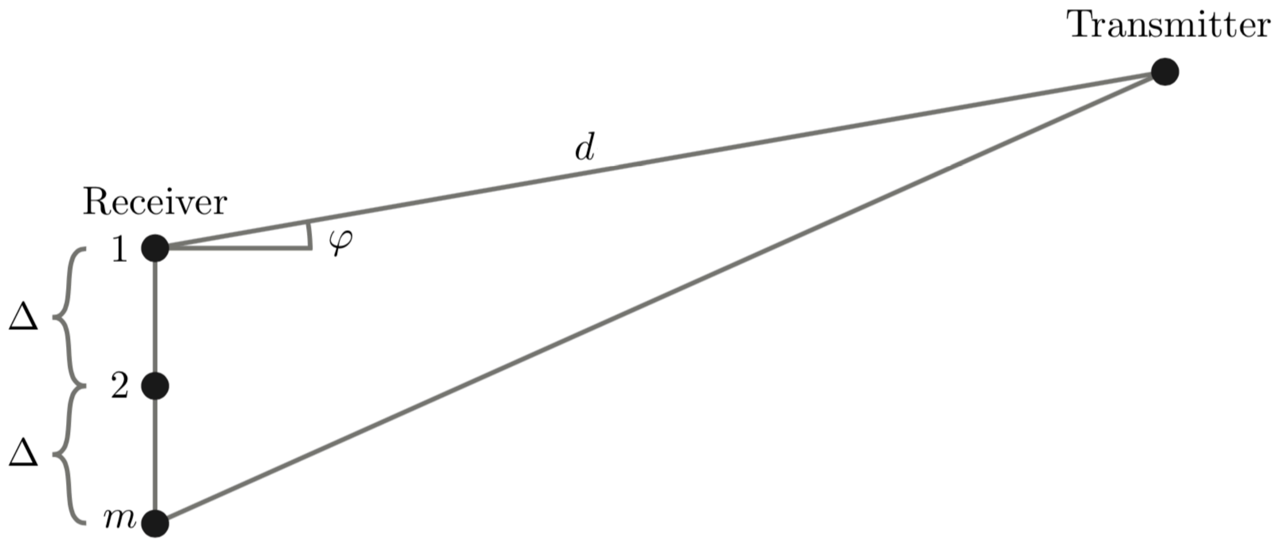
Approximately same channel gain:
$$\frac{\lambda^2}{(4\pi d_m)^2} \approx \frac{\lambda^2}{(4\pi d)^2}$$



$$e^{-j2\pi\left(\frac{d_m - d_1}{\lambda}\right)} = e^{-j2\pi\frac{(m-1)\Delta\sin(\varphi)}{\lambda}}$$

ULA channel model

- M antennas in receiver array
  - Angle of arrival φ
  - Wavelength λ
  - Distance Δ between antennas
  - Distance d to first antenna



- Channel vector:

$$\boldsymbol{g} = \frac{\lambda}{4\pi d}\begin{bmatrix} 1 \\ e^{-j2\pi\frac{\Delta \sin(\varphi)}{\lambda}} \\ \vdots \\ e^{-j2\pi\frac{(M-1)\Delta \sin(\varphi)}{\lambda}} \end{bmatrix}$$

Applies to both SIMO and MISO channels

Channel capacity with line-of-sight channel

- Recall:

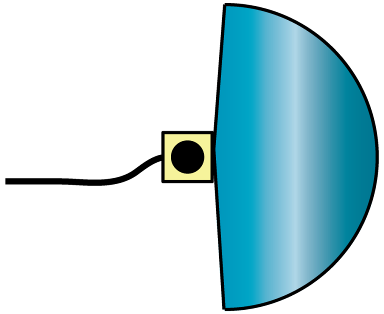
$$\mathrm{C} = \log_2\left(1 + \frac{q\|\boldsymbol{g}\|^2}{N_0}\right) \text{ bits per symbol}$$

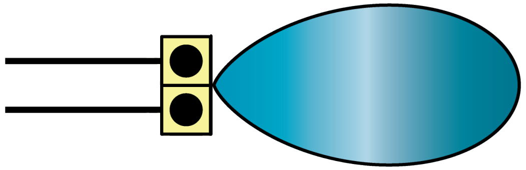
- We have $\|\boldsymbol{g}\|^2 = \beta M$ where $\beta = \frac{\lambda^2}{(4\pi d)^2}$:

$$\mathrm{C} = \log_2\left(1 + \frac{q\beta M}{N_0}\right)$$

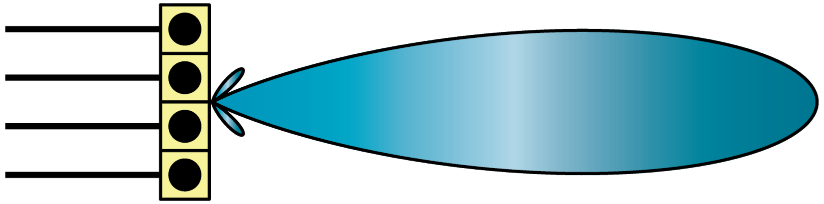
Beamforming gain: M

Illustrations of beamforming versus real beamforming





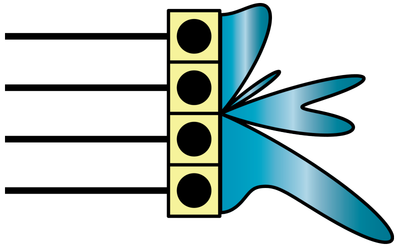
Same total power
2x maximum gain



Same total power
4x maximum gain

Question 1:
Is this an accurate illustration?

Question 2:
What does a ”beam” look like in non-line-of-sight channels?



Beamwidth

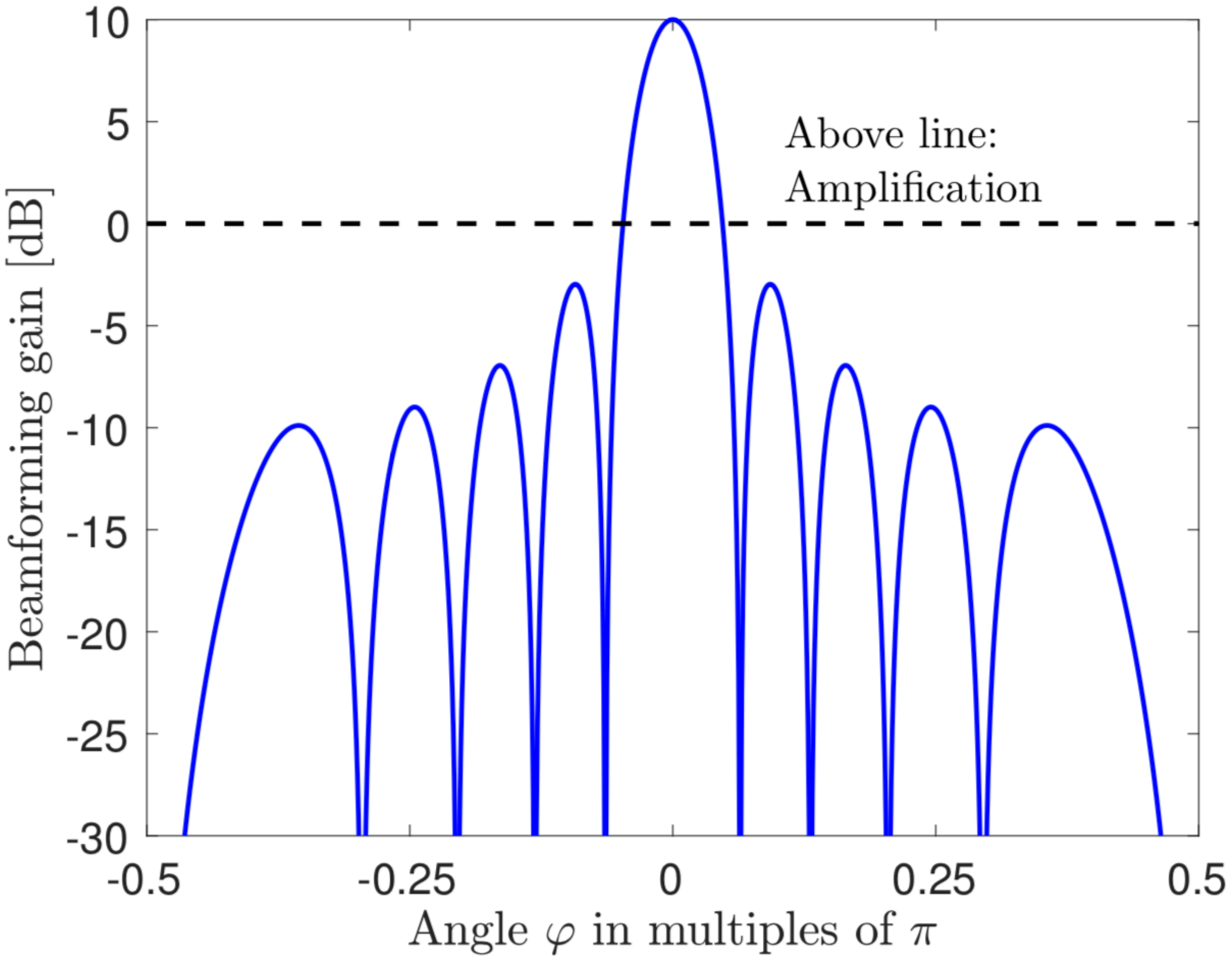
$$\frac{1}{M}\left\| \begin{bmatrix} 1 \\ e^{-j2\pi\frac{\Delta \sin(\varphi_{\text{beam}})}{\lambda}} \\ \vdots \\ e^{-j2\pi\frac{(M-1)\Delta \sin(\varphi_{\text{beam}})}{\lambda}} \end{bmatrix}^H \begin{bmatrix} 1 \\ e^{-j2\pi\frac{\Delta \sin(\varphi)}{\lambda}} \\ \vdots \\ e^{-j2\pi\frac{(M-1)\Delta \sin(\varphi)}{\lambda}} \end{bmatrix} \right\|^2$$

- Beamforming gain in more than one direction

- Example:
  $$M = 10, \Delta = \frac{\lambda}{2}, \varphi_{\text{beam}} = 0$$

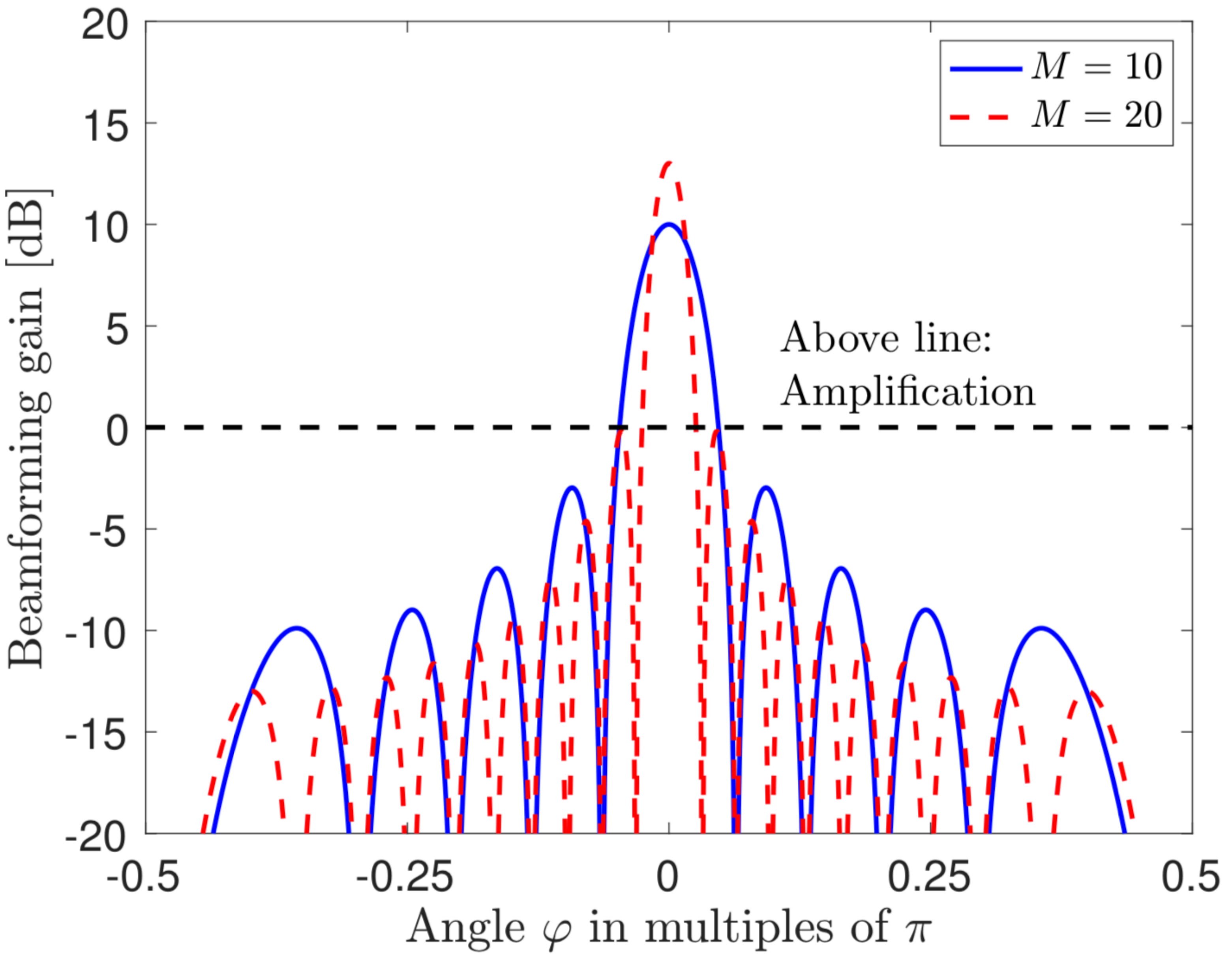
- First-null beamwidth:
  $$\approx \frac{4}{M}$$

More antennas:
Narrower beams



Two benefits of beamforming

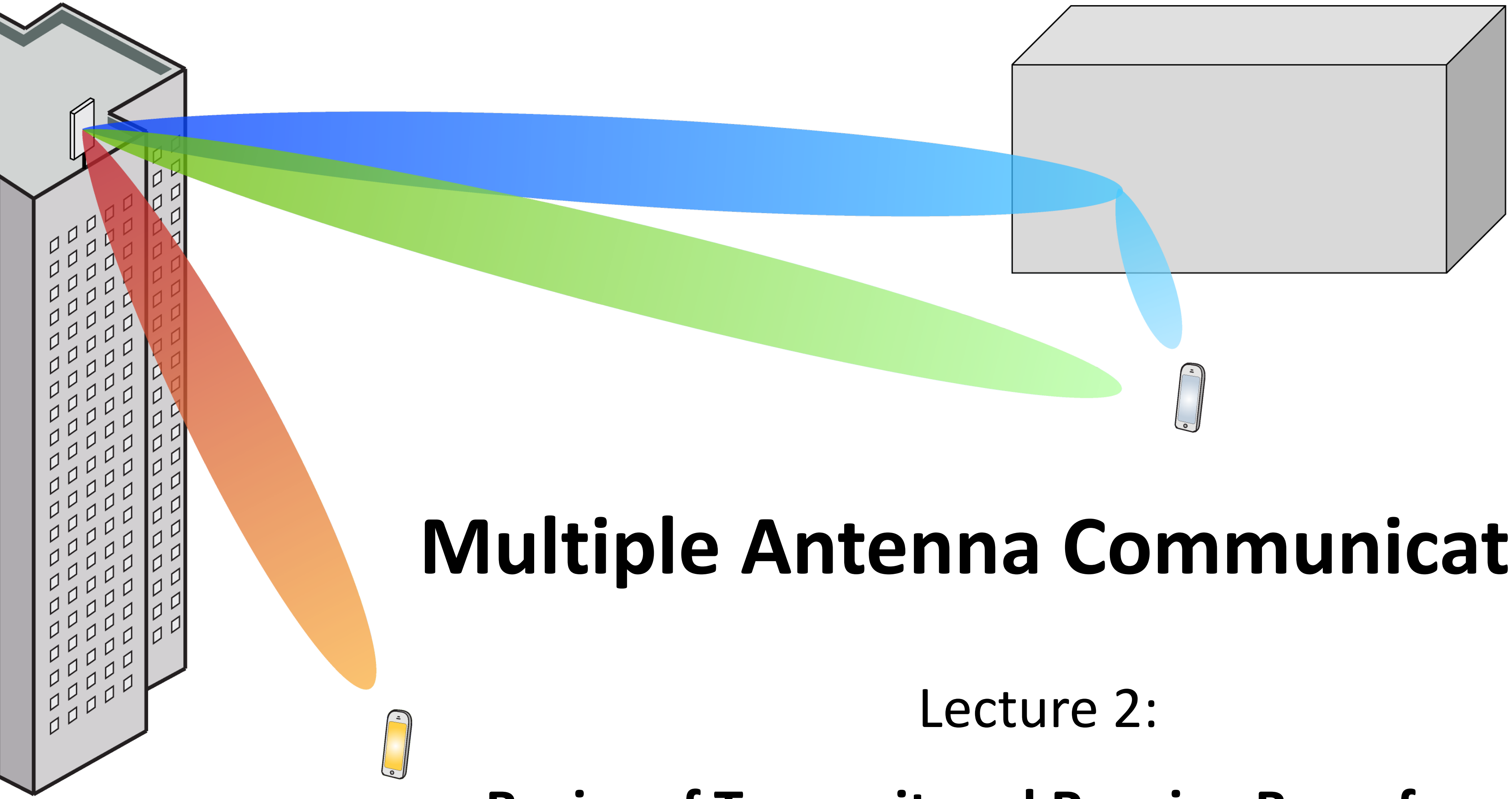
- Beamforming gain
  - Stronger signal at desired location
- Narrower beam
  - Less interference at undesired locations



Summary

- Same capacity of SIMO and MISO channels
  - Achieved by beamforming: maximum ratio combining/transmission

- Line-of-sight channels
  - All antennas contribute equally
  - Phase-shifts depend on angles
  - Beamforming gain of M
  - Beamwidth shrinks as $1/M$



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