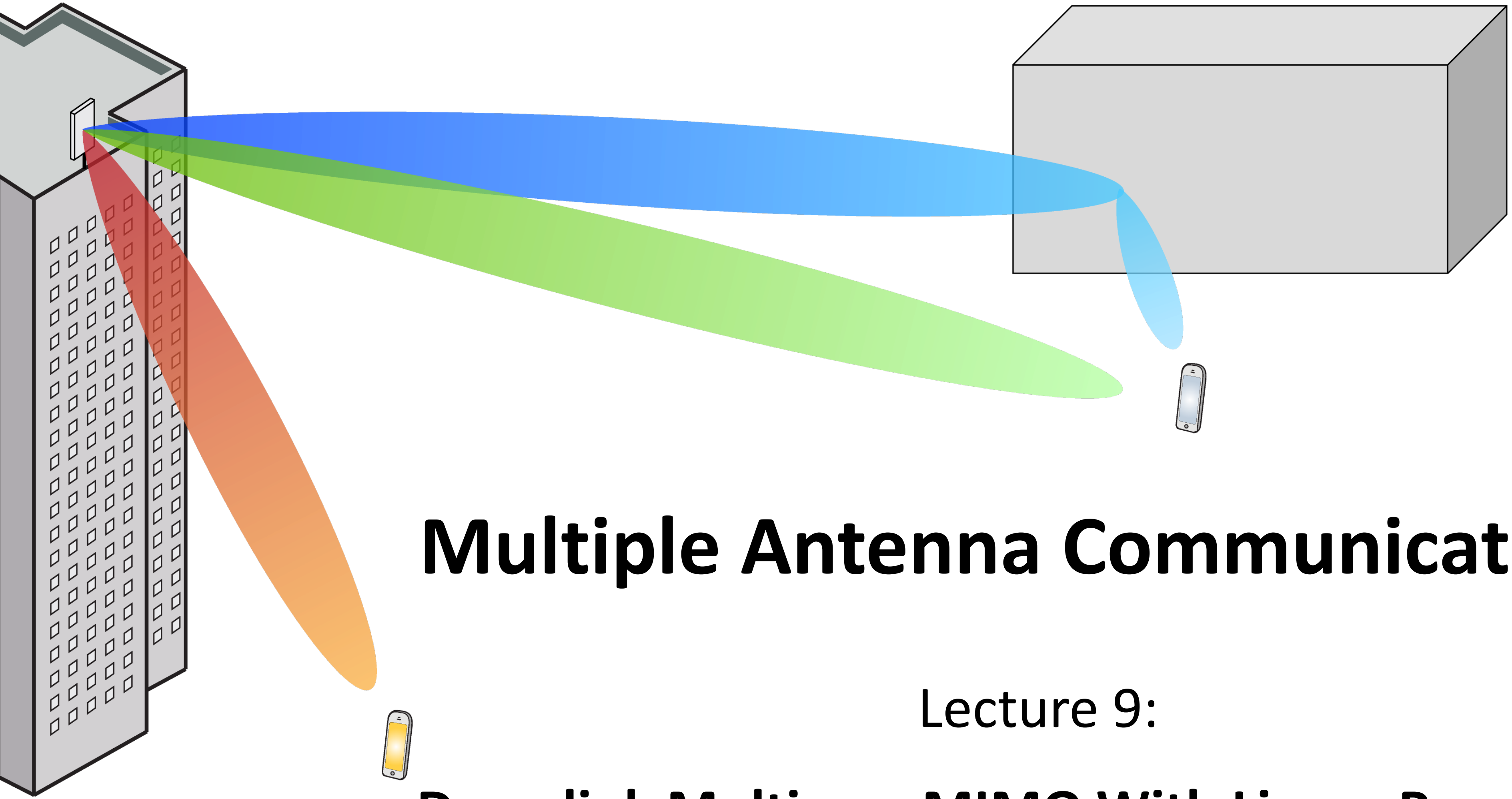


Multiple Antenna Communications

Lecture 9:

Downlink Multiuser MIMO With Linear Processing

Emil Björnson

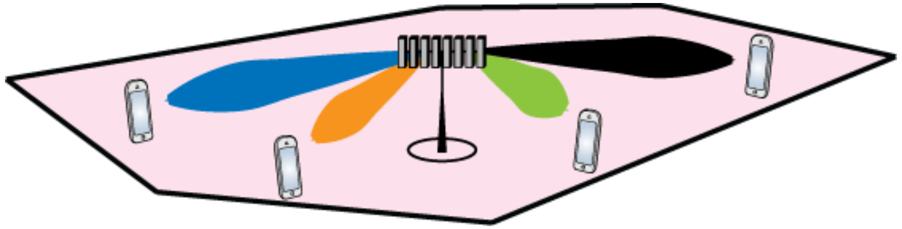
Outline

- Downlink communication
  - System model
  - Precoding

- Capacity lower bound
  - Any precoding
  - MR precoding

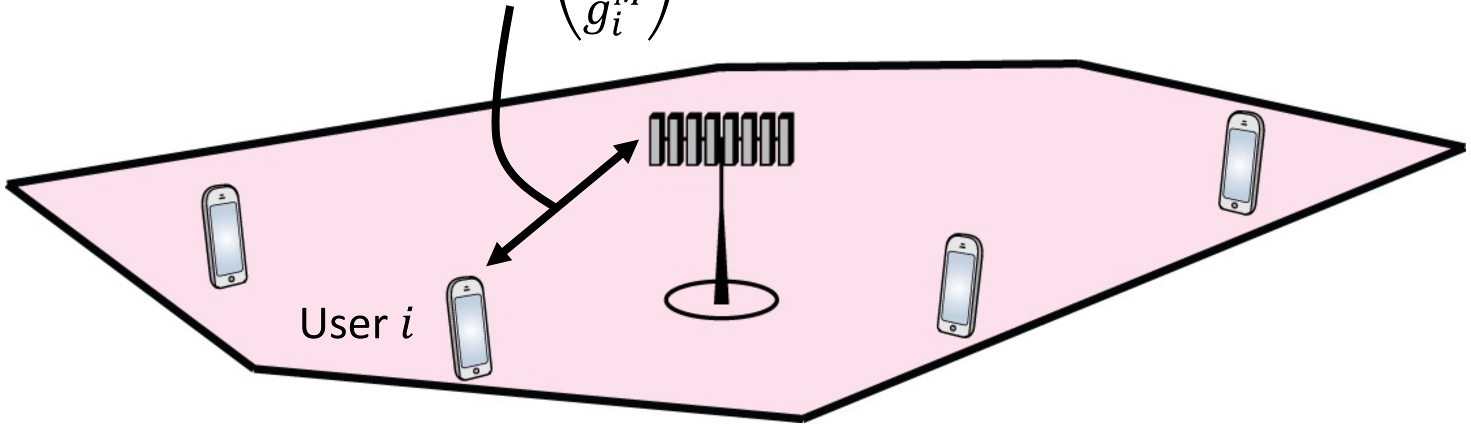
- Performance comparison:
  Uplink and downlink

Downlink communication



- Notation:

$$\boldsymbol{g}_i = \begin{pmatrix} g_i^1 \\ \vdots \\ g_i^M \end{pmatrix}$$



- Signal sent by base station

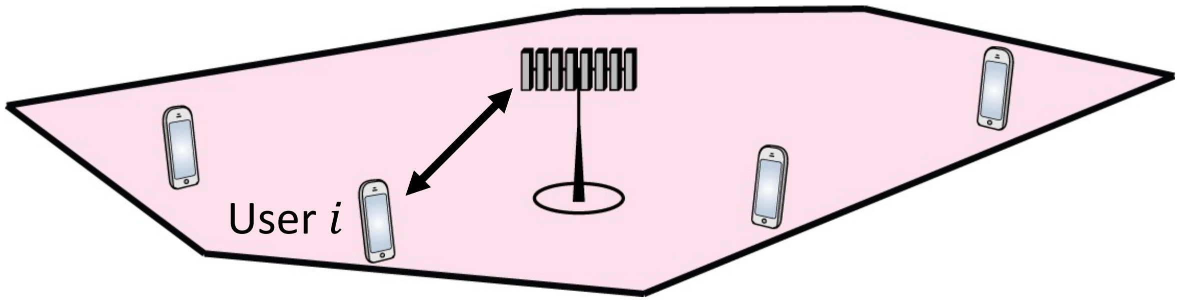
$$\sqrt{\rho_{dl}}\boldsymbol{x}$$

Transmit power

$M \times 1$ vector

Received signal at User i

- Transmitted signal: $\sqrt{\rho_{dl}}\boldsymbol{x}$
- Channel vector: $\boldsymbol{g}_i$
- Additive noise: $w_i \sim CN(0,1)$



Received signal:

$$y_i = \sqrt{\rho_{dl}}\boldsymbol{g}_i^T\boldsymbol{x} + w_i$$

Downlink Massive MIMO system model

- Received signal:

$$\boldsymbol{y} = \sqrt{\rho_{dl}}\boldsymbol{G}^T\boldsymbol{x} + \boldsymbol{w}$$

where

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_K \end{pmatrix} \quad \boldsymbol{G} = \begin{pmatrix} g_1^1 & \cdots & g_K^1 \\ \vdots & \ddots & \vdots \\ g_1^M & \cdots & g_K^M \end{pmatrix} \quad \boldsymbol{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_M \end{pmatrix} \quad \boldsymbol{w} = \begin{pmatrix} w_1 \\ \vdots \\ w_K \end{pmatrix}$$

  - Parameters are normalized: Maximum power is ρ_{dl}

$$\mathrm{E}\{\|\boldsymbol{x}\|^2\} \leq 1$$

  - Channel of user k: $g_k^1, \dots, g_k^M \sim CN(0, \beta_k)$
  - Normalized noise: $\boldsymbol{w} \sim CN(\boldsymbol{0}, \boldsymbol{I}_K)$

Large-scale fading coefficient

Linear precoding

If $\boldsymbol{b}_k$ is a preferred precoder, pick

$$\boldsymbol{a}_k = \frac{1}{\sqrt{E\{\|\boldsymbol{b}_k\|^2\}}}\boldsymbol{b}_k$$

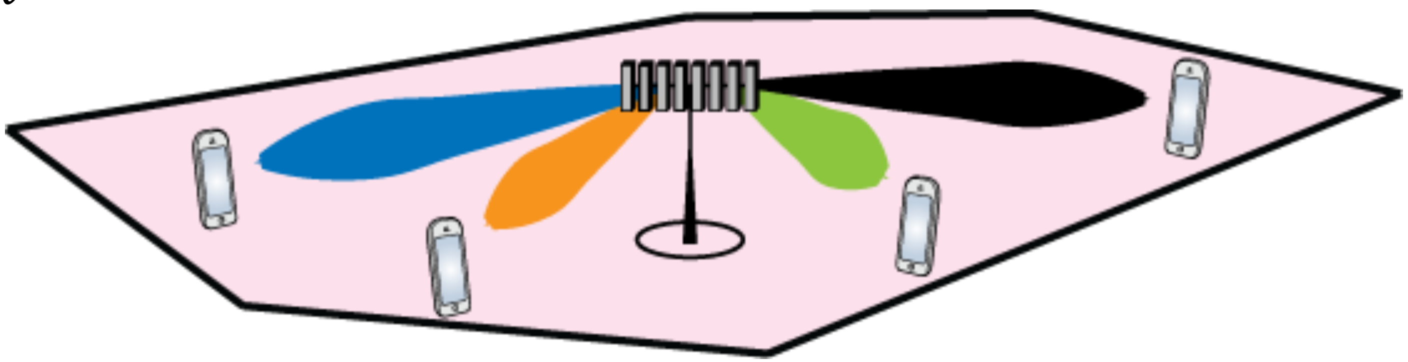
- Select transmitted signal as

$$\boldsymbol{x} = \sum_{k=1}^{K} \sqrt{\eta_k}\boldsymbol{a}_k q_k$$

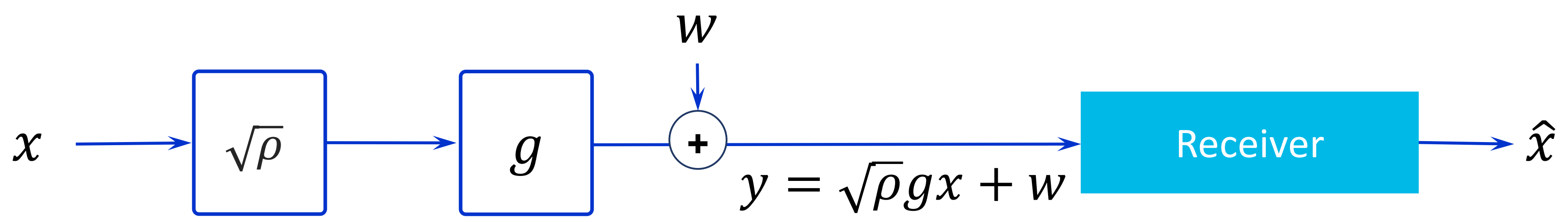
  - Message symbol to user k: q_k, $E\{|q_k|^2\} = 1$, zero mean
  - Precoding vector: $\boldsymbol{a}_k$, $E\{\|\boldsymbol{a}_k\|^2\} = 1$
  - Power control coefficient: $\eta_k \leq 1$

Total power constraint:

$$E\{\|\boldsymbol{x}\|^2\} = \sum_{k=1}^{K} \eta_k \leq 1$$



Capacity lower bound



- Desired signal x, transmit power ρ
- Deterministic channel coefficient g, known at receiver

Capacity lower bound:

$$C \geq \log_2\left(1 + \frac{\rho|g|^2}{Var\{w\}}\right)$$

Rewriting the received downlink signal

- Received signal:

$$y_i = \boldsymbol{g}_i^T \left(\sum_{k=1}^{K} \sqrt{\rho_{dl}\eta_k} \boldsymbol{a}_k q_k \right) + w_i = \underbrace{\sqrt{\rho_{dl}\eta_i} \boldsymbol{g}_i^T \boldsymbol{a}_i q_i}_{\text{Desired signal}} + \underbrace{\sum_{k=1, k\neq i}^{K} \sqrt{\rho_{dl}\eta_k} \boldsymbol{g}_i^T \boldsymbol{a}_k q_k + w_i}_{\text{Interference plus noise}}$$

Receiver does not know $\boldsymbol{g}_i^T \boldsymbol{a}_i$

But it knows that $\boldsymbol{g}_i^T \boldsymbol{a}_i \approx E\{\boldsymbol{g}_i^T \boldsymbol{a}_i\}$ if M is large

Add and subtract $E\{\boldsymbol{g}_i^T\boldsymbol{a}_i\}$

- Received signal:

$$y_i = \sqrt{\rho_{dl}\eta_i}\boldsymbol{g}_i^T\boldsymbol{a}_i q_i + \sum_{k=1,k\neq i}^{K}\sqrt{\rho_{dl}\eta_k}\boldsymbol{g}_i^T\boldsymbol{a}_k q_k + w_i$$

$$= \underbrace{\sqrt{\rho_{dl}\eta_i}}_{\sqrt{\rho}}\underbrace{E\{\boldsymbol{g}_i^T\boldsymbol{a}_i\}}_{g}\underbrace{q_i}_{q} + \underbrace{\sqrt{\rho_{dl}\eta_i}\left(\boldsymbol{g}_i^T\boldsymbol{a}_i - E\{\boldsymbol{g}_i^T\boldsymbol{a}_i\}\right)q_i + \sum_{k=1,k\neq i}^{K}\sqrt{\rho_{dl}\eta_k}\boldsymbol{g}_i^T\boldsymbol{a}_k q_k + w_i}_{w\text{: Interference plus noise}}$$

Almost like an AWGN channel!

Capacity lower bound:

$$C \geq \log_2\left(1 + \frac{\rho|g|^2}{Var\{w\}}\right)$$

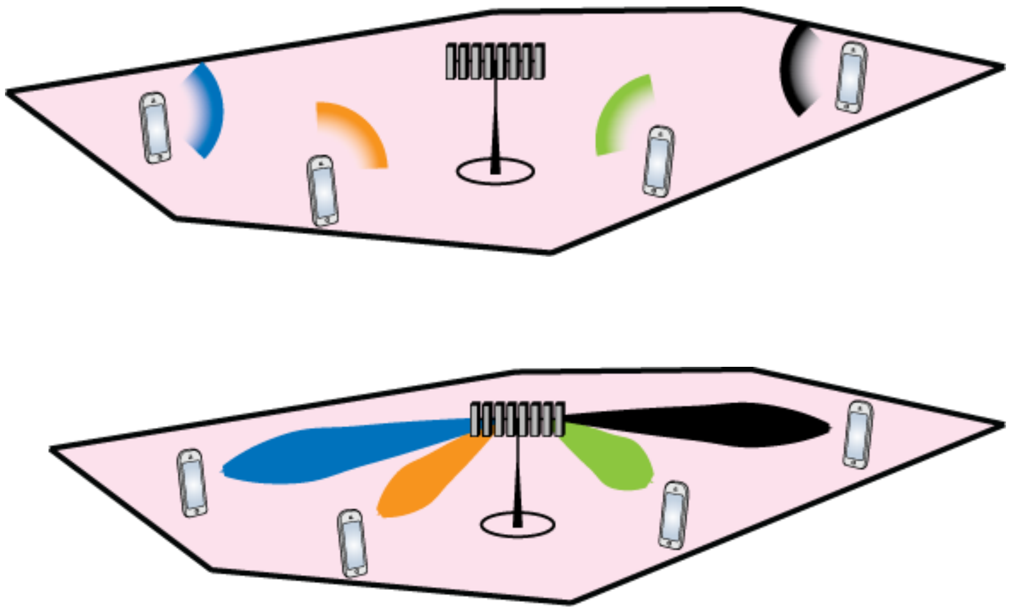
Capacity lower bound with any precoding

$$\log_2\left(1+\frac{\rho_{dl}\eta_i\left|E\{\boldsymbol{g}_i^T\boldsymbol{a}_i\}\right|^2}{\sum_{k=1}^{K}\rho_{dl}\eta_k E\left\{\left|\boldsymbol{g}_i^T\boldsymbol{a}_k\right|^2\right\}+1-\rho_{dl}\eta_i\left|E\{\boldsymbol{g}_i^T\boldsymbol{a}_i\}\right|^2}\right)$$

- Interpretation
  - Averaging over small-scale fading
  - Numerator: Proportional to $\left|E\{\boldsymbol{g}_i^T\boldsymbol{a}_i\}\right|^2$
  - Denominator: Sum of interference proportional to $E\left\{\left|\boldsymbol{g}_i^T\boldsymbol{a}_k\right|^2\right\}$ from all users plus noise variance

How to select precoding?

- Recall: Uplink processing
  - MMSE: $\boldsymbol{a}_i = \sqrt{\rho_{ul}\eta_i}\boldsymbol{B}_i^{-1}\widehat{\boldsymbol{g}}_i$
  - MR: $\boldsymbol{a}_i = \widehat{\boldsymbol{g}}_i$



Precoding principle

Transmit in the direction where you heard the users "most clearly"

- Downlink precoding schemes
  - MMSE: $\boldsymbol{a}_i = c_i\sqrt{\rho_{ul}\eta_i}\left(\boldsymbol{B}_i^{-1}\widehat{\boldsymbol{g}}_i\right)^*$ $\qquad c_i = \frac{1}{\sqrt{E\left\{\left\|\sqrt{\rho_{ul}\eta_i}\boldsymbol{B}_i^{-1}\widehat{\boldsymbol{g}}_i\right\|^2\right\}}}$
  - MR: $\boldsymbol{a}_i = c_i\widehat{\boldsymbol{g}}_i^*$ $\qquad c_i = \frac{1}{\sqrt{E\{\|\widehat{\boldsymbol{g}}_i\|^2\}}}$

Recall: Estimates of channels

- MMSE estimate of g_k^m from user k to antenna m
  - Estimate: $\hat{g}_k^m = E\{g_k^m | \boldsymbol{Y}'_p\} = \frac{\sqrt{\tau_p \rho_{ul}} \beta_k}{1+\tau_p \rho_{ul} \beta_k} [\boldsymbol{Y}'_p]_{mk} \sim CN(0, \gamma_k)$
  - Estimation error: $\tilde{g}_k^m = \hat{g}_k^m - g_k^m \sim CN(0, \beta_k - \gamma_k)$

    where

$$\gamma_k = \frac{\tau_p \rho_{ul} \beta_k^2}{1 + \tau_p \rho_{ul} \beta_k}$$

Downlink capacity lower bound with MR

$$C \geq \log_2\left(1 + \frac{M\rho_{dl}\eta_i\gamma_i}{\sum_{k=1}^{K}\rho_{dl}\eta_k\beta_i + 1}\right)$$

- Interpretation
  - Small-scale fading is not visible in this bound
  - **Numerator:**
    Coherent beamforming gain, grows with antennas M, power $\rho_{dl}\eta_i$ and estimation quality γ_i
  - **Denominator:**
    Sum of non-coherent interference from all users plus noise variance

Comparing uplink and downlink (with MR)

Uplink:

$$\log_2\left(1 + \frac{M\rho_{ul}\eta_i\gamma_i}{\sum_{k=1}^{K}\rho_{ul}\eta_k\beta_k + 1}\right)$$

Downlink:

$$\log_2\left(1 + \frac{M\rho_{dl}\eta_i\gamma_i}{\beta_i\sum_{k=1}^{K}\rho_{dl}\eta_k + 1}\right)$$

Similarities

- Same structure (beamforming gain M, powers $\rho_{ul/dl}\eta_i$)

Differences

- Uplink interference: From users ($\beta_1, \dots, \beta_K$)
- Downlink interference: From base station (β_i)

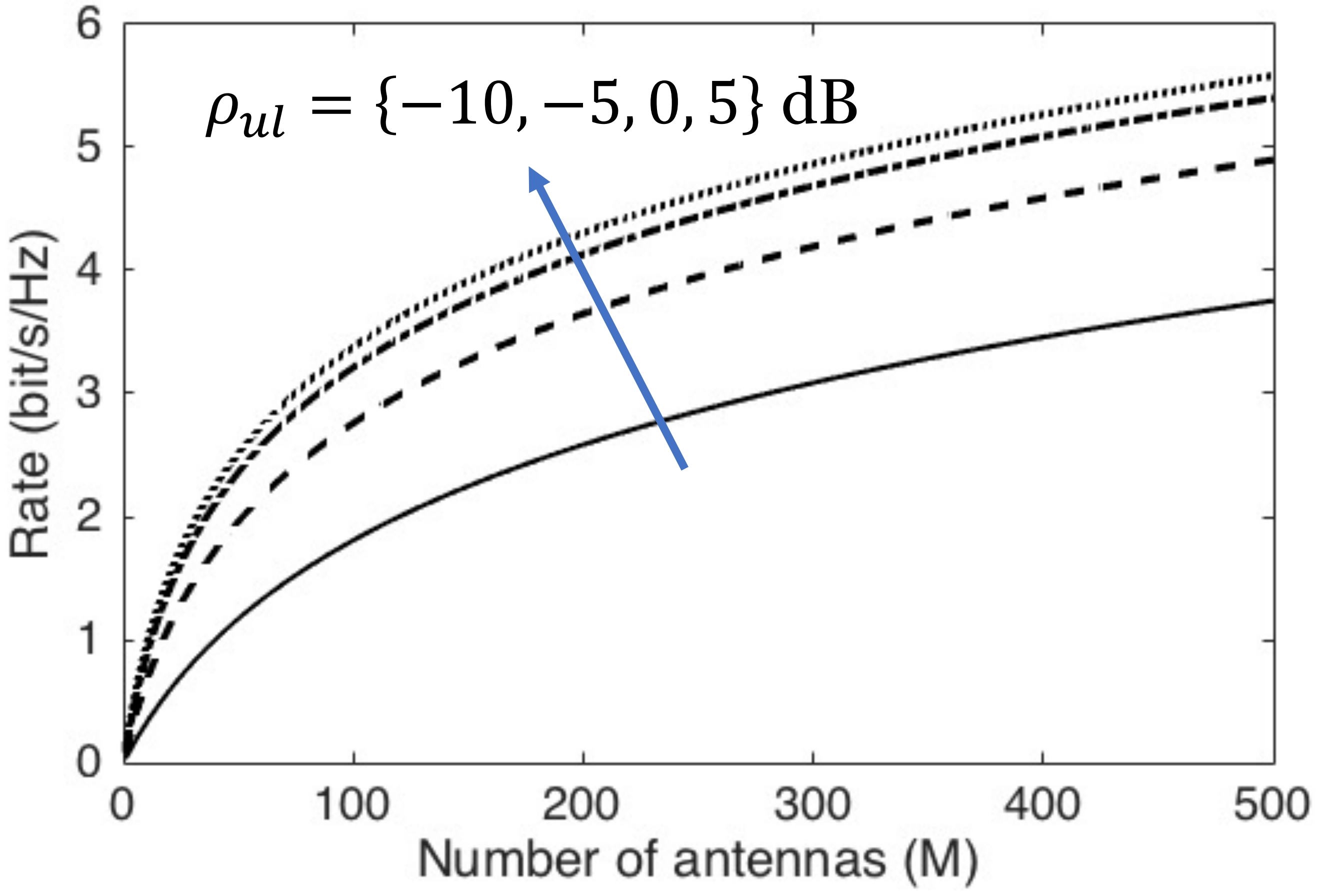
Example: Uplink rate, varying SNR

Assumptions:

$K = 10$
$\beta = 1$
$\tau_p = K$
$\eta_k = 1\ \forall k$

Same for DL if

$\rho_{dl} = K \cdot \rho_{ul}$
$\eta_k = 1/K$



Always better with more antennas

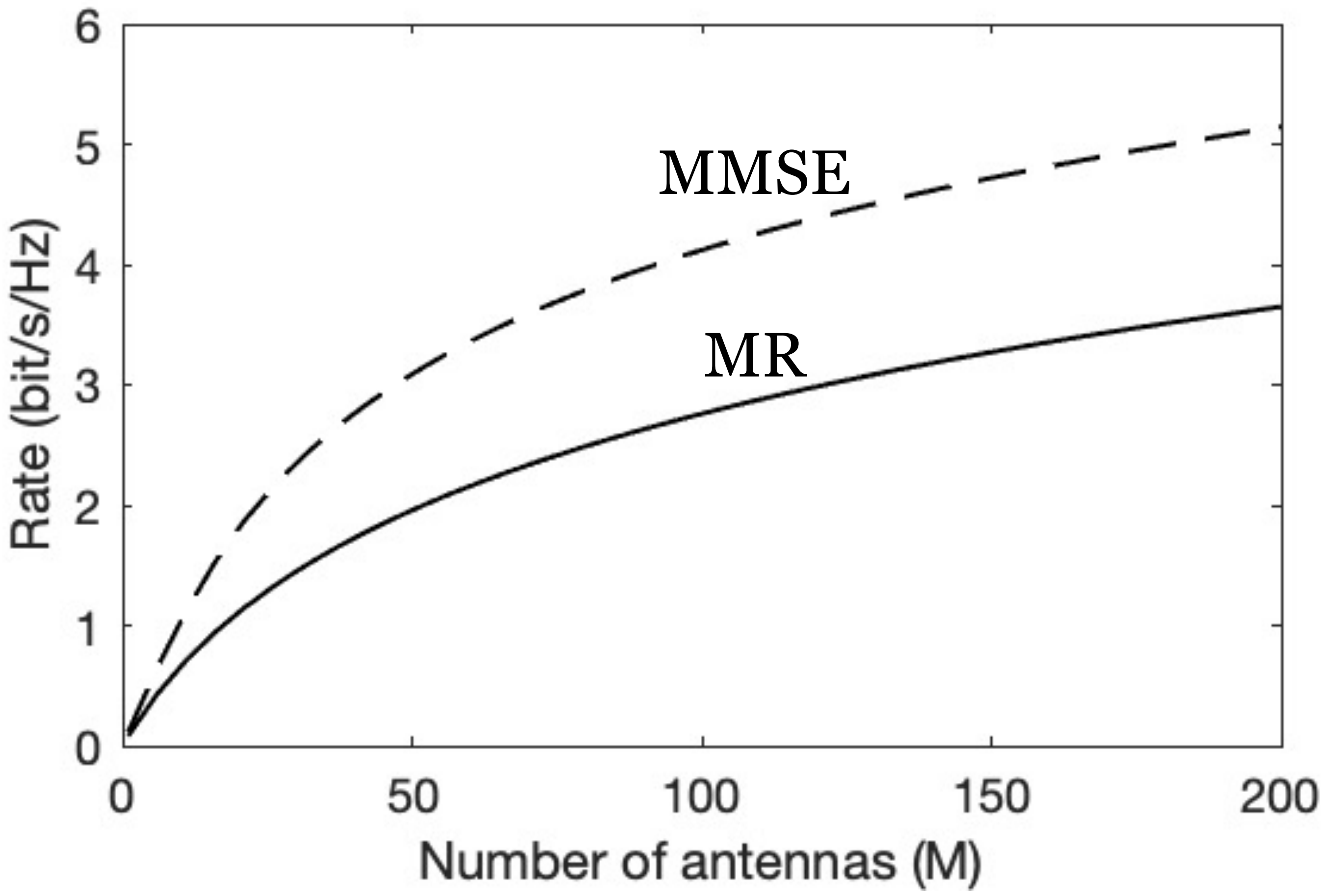
Example: Uplink rate, different schemes

Assumptions:

$K = 10$
$\beta = 1$
$\tau_p = K$
$\eta_k = 1\ \forall k$
$\rho_{ul} = -5\text{ dB}$

Similar for DL if

$\rho_{dl} = K \cdot \rho_{ul}$
$\eta_k = 1/K$



Same trend, but 40-60% higher rate with MMSE

Example: Rate when scaling number of users

Assumptions:

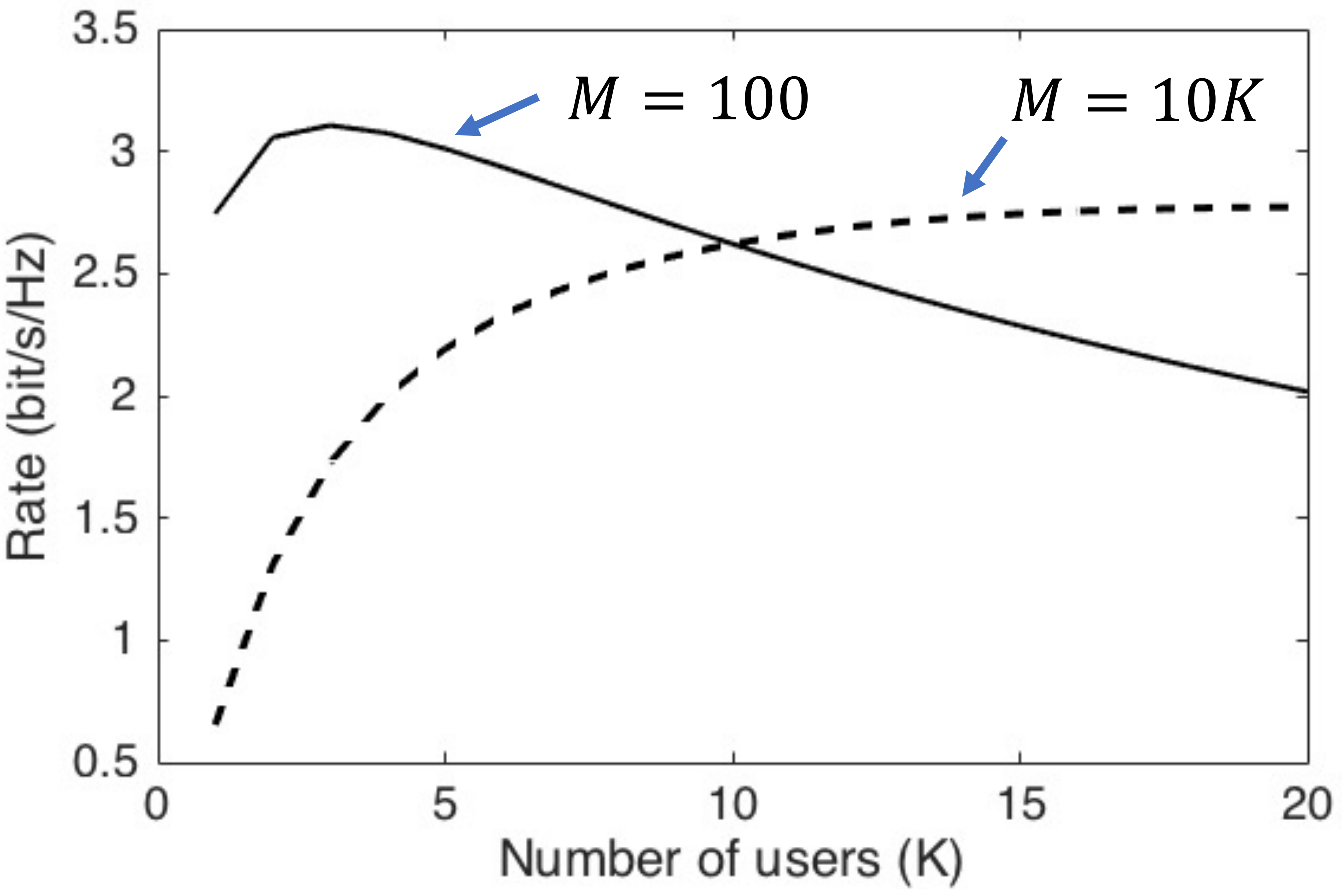
$\beta = 1$

$\tau_p = K$

$\eta_k = 1\ \forall k$

MR processing

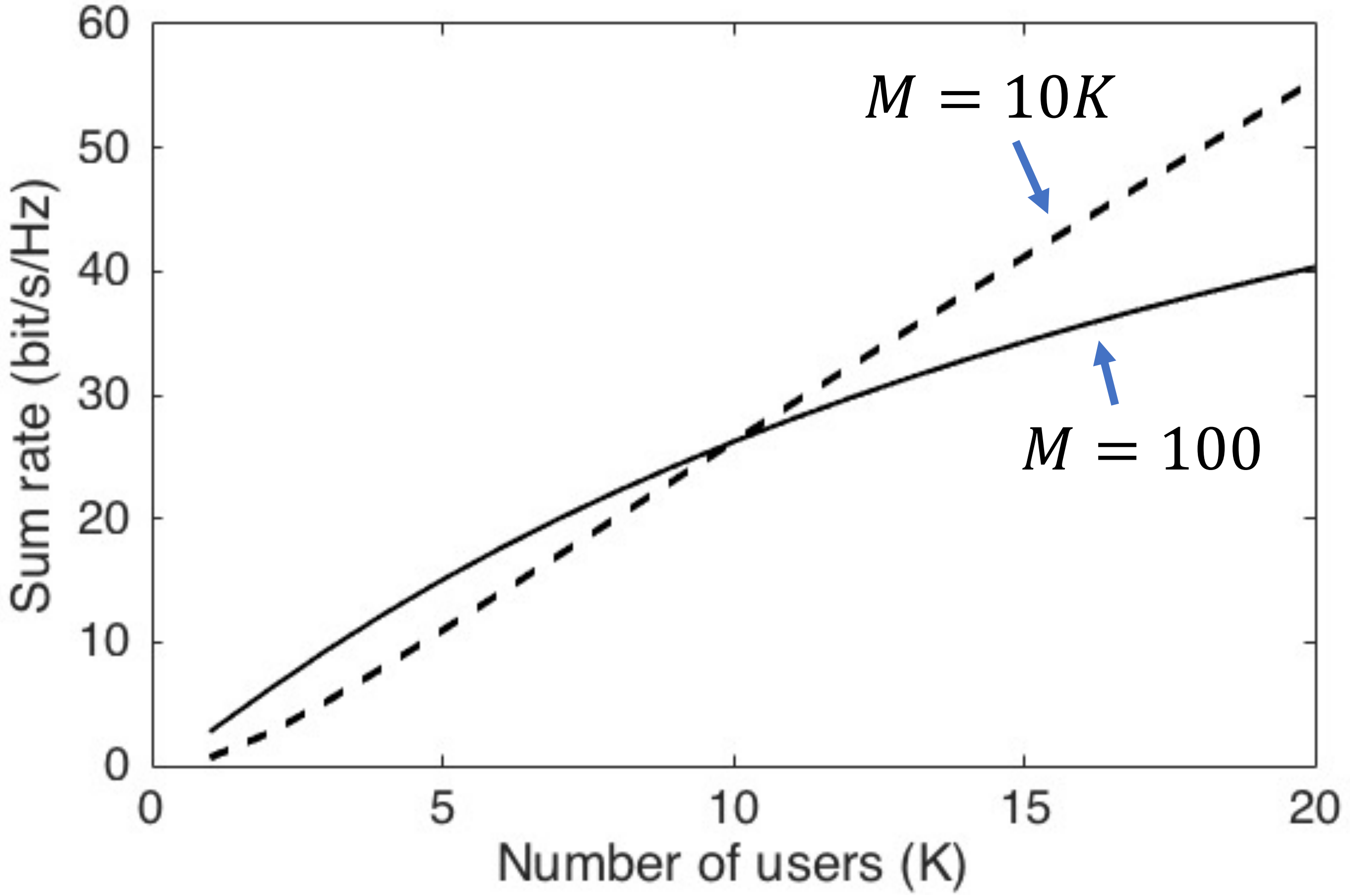
(γ_k grows with K)



Same behavior, but higher rates with MMSE

Example: Sum rate, scaling number of users

Assumptions:
$\beta = 1$
$\tau_p = K$
$\eta_k = 1 \; \forall k$
MR processing



Same behavior, but higher rates with MMSE

What are the benefits of MR processing?

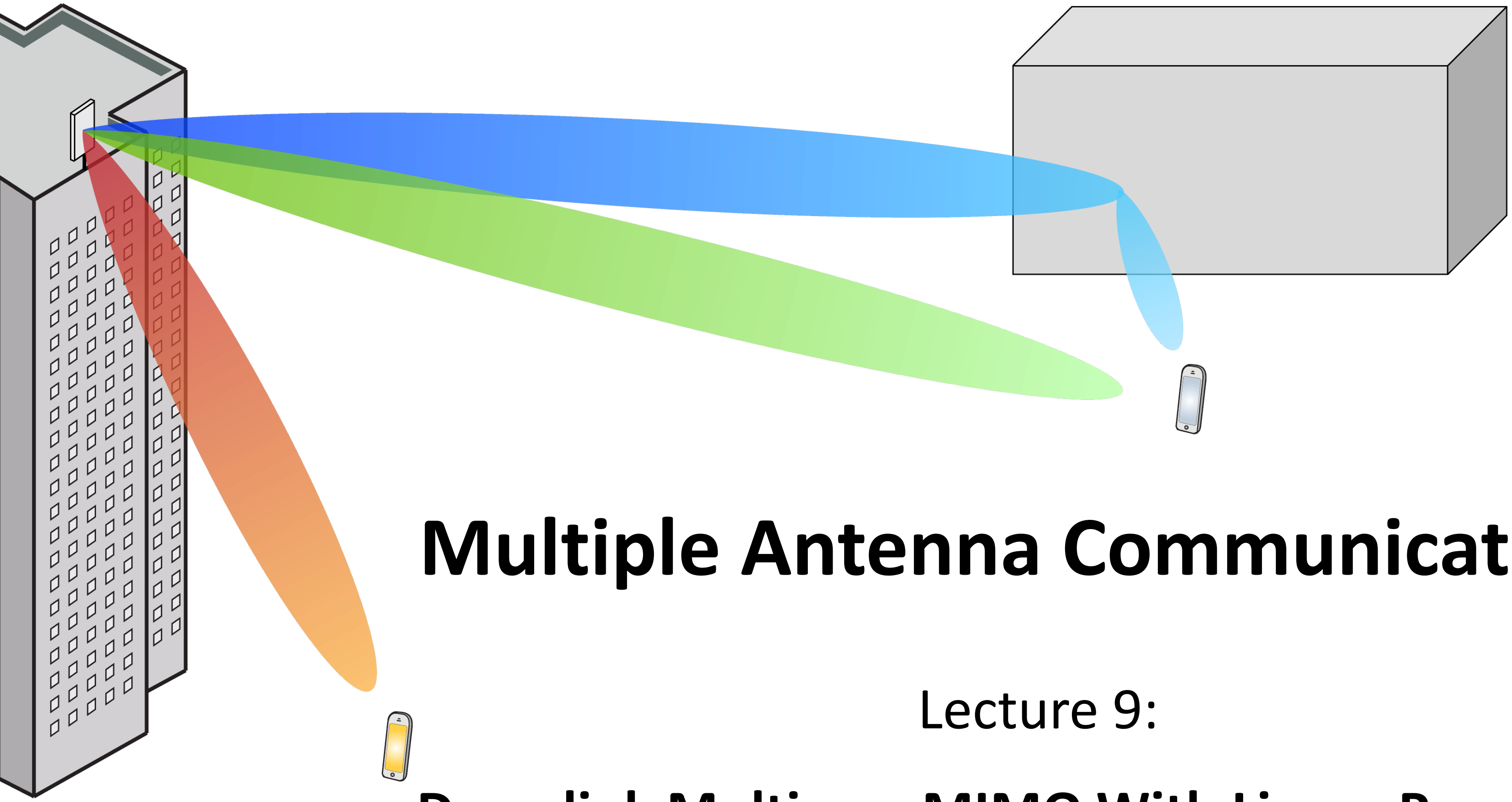
- Lower computational complexity
  - Substantial performance loss in theory
  - Practical loss is smaller since MR easier to implement

- Closed form bound on ergodic capacity
  - Typical shape of ergodic capacity bounds: $E\{\log_2(1 + \mathrm{SINR}_{\mathrm{random}})\}$
  - Treating channel as equal to its mean value: $\log_2(1 + \mathrm{SINR}_{\mathrm{constant}})$
  - Simple expression for $\mathrm{SINR}_{\mathrm{constant}}$ with MR

Summary

- Downlink communication
  - Rate expression for arbitrary precoding
  - Closed-form expression with MR precoding

- Insights
  - Uplink and downlink rates behave similarly
  - MMSE is substantially better than MR
  - Should increase the number of antennas when the number of users increase



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