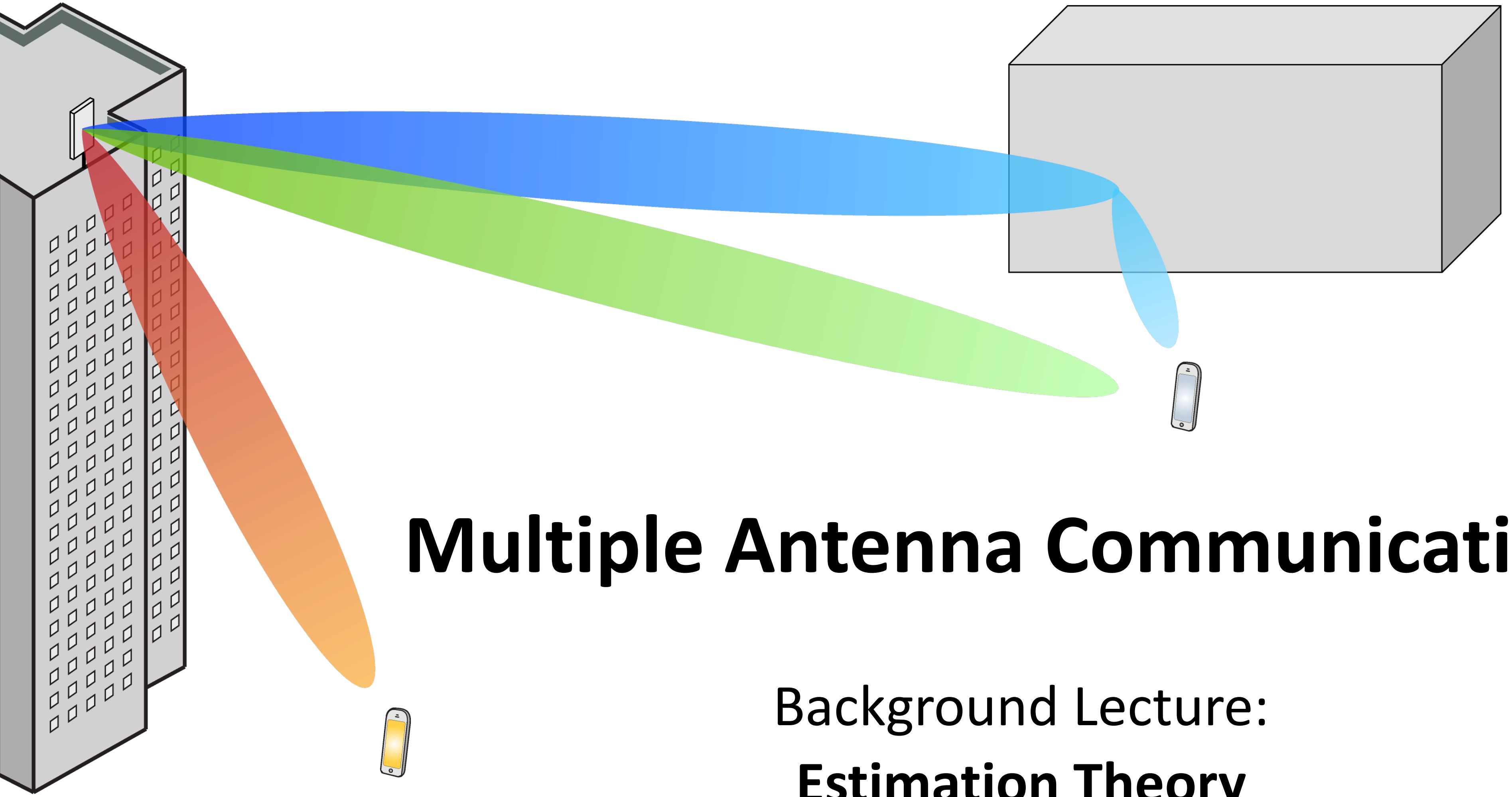


Multiple Antenna Communications

Background Lecture:

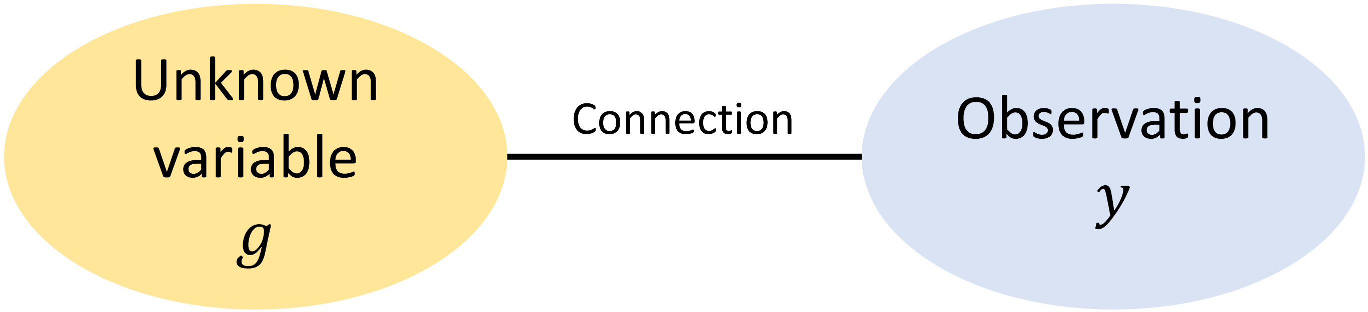
Estimation Theory

Emil Björnson

Outline

- Principle of estimating an unknown variable
- Minimum mean-squared error (MMSE) estimator
- Estimate Gaussian variable in Gaussian noise

Estimating an Unknown Variable



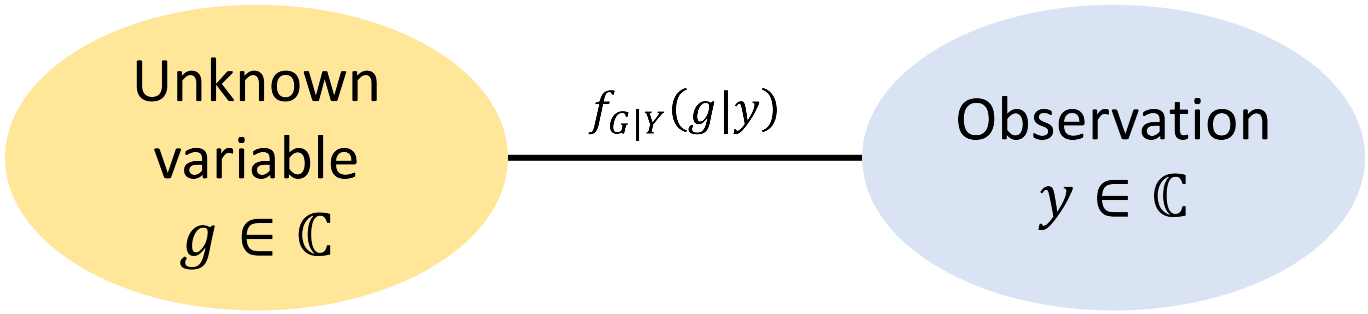
Two categories

1. Classical estimation: g is deterministic but unknown
2. Bayesian estimation: g is a realization of a random variable

Communication = transmission of long data packets

Easy to estimate deterministic variables
Challenging to estimate time-varying random variables

Principle of Bayesian estimation

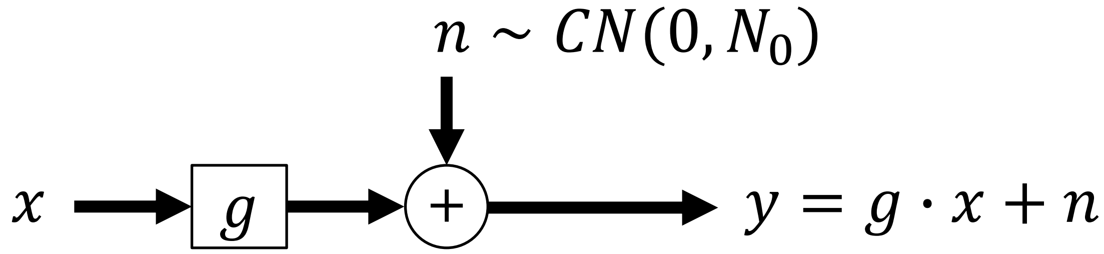


- Goal of estimation: Find a function $\hat{g}(y)$
    - Mean-squared error (MSE): $E\{|g - \hat{g}(y)|^2\}$

Minimum MSE (MMSE) estimator

$$\hat{g}(y) = E\{g|y\} = \int_{g \in \mathbb{C}} g f_{G|Y}(g|y) dg$$

Example: Estimation of a channel



- Prior: $g \sim CN(0, \beta)$

- Transmitted signal x: Choose a deterministic number

Called: pilot signal

- Observation y: Marginal distribution $y \sim CN(0, |x|^2\beta + N_0)$

  If g is known: $y - g \cdot x = n \sim CN(0, N_0)$

We need to determine $f_{G|Y}(g|y)$

Unknown variable
$g \in \mathbb{C}$

Connection

Observation
$y \in \mathbb{C}$

Finding the conditional PDF

The joint PDF of two random variables can be written as

$$f_{G,Y}(g,y) = f_{G|Y}(g|y)f_Y(y) = f_{Y|G}(y|g)f_G(g)$$

Bayes' theorem

$$f_{G|Y}(g|y) = \frac{f_{Y|G}(y|g)f_G(g)}{f_Y(y)}$$

In our example:

$$f_{G|Y}(g|y) = \frac{\frac{1}{\pi N_0}e^{-\frac{|y-xg|^2}{N_0}}\frac{1}{\pi\beta}e^{-\frac{|g|^2}{\beta}}}{\frac{1}{\pi(|x|^2\beta+N_0)}e^{-\frac{|y|^2}{|x|^2\beta+N_0}}} = \frac{1}{\pi\left(\frac{\beta N_0}{|x|^2\beta+N_0}\right)}e^{-\frac{\left|g-\frac{\beta x^*}{|x|^2\beta+N_0}y\right|^2}{\frac{\beta N_0}{|x|^2\beta+N_0}}}$$

$$g - \frac{\beta x^*}{|x|^2\beta+N_0}y \sim CN\left(0, \frac{\beta N_0}{|x|^2\beta+N_0}\right)$$

MMSE estimate of Gaussian variable in Gaussian noise

- Consider $y = g \cdot x + n$ where
  - x is a constant, $g \sim CN(0, \beta)$, $n \sim CN(0, N_0)$

MMSE estimator:

$$\hat{g} = E\{g|y\} = \frac{\beta x^*}{|x|^2\beta + N_0} y$$

Resulting MSE:

$$E\{|\hat{g} - g|^2\} = \frac{\beta N_0}{|x|^2\beta + N_0}$$

$$g - \frac{\beta x^*}{|x|^2\beta + N_0} y \sim CN\left(0, \frac{\beta N_0}{|x|^2\beta + N_0}\right)$$

Estimation error and its random distribution

- The estimation error is $\tilde{g} = \hat{g} - g$

Orthogonality principle:
$\hat{g}$ and $\tilde{g}$ are uncorrelated

- In our example, $g, \hat{g}, \tilde{g}$ are Gaussian distributed: $\hat{g}, \tilde{g}$ are independent

MSE

$$\tilde{g} \sim CN\left(0, \frac{\beta N_0}{|x|^2\beta + N_0}\right)$$

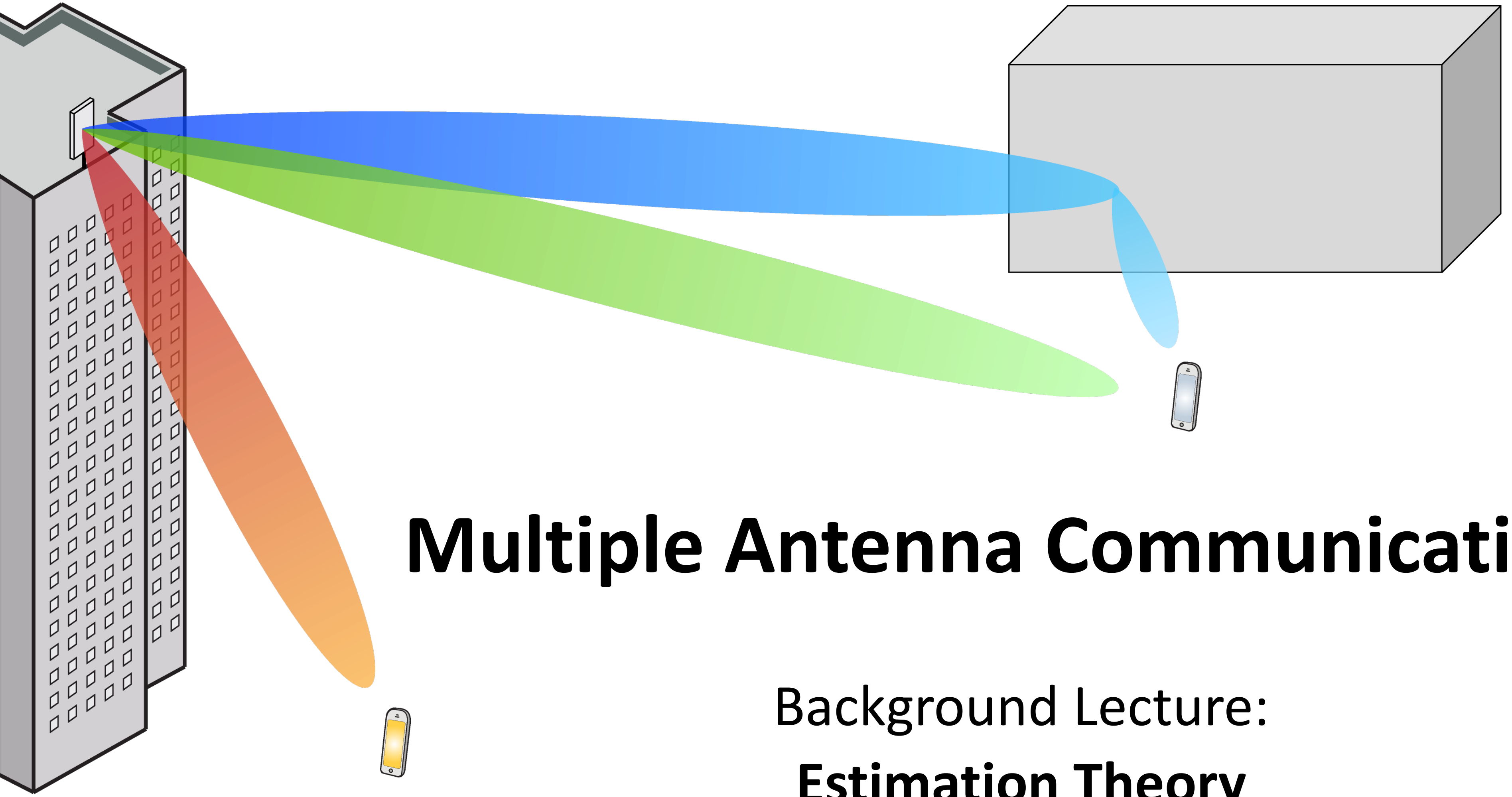
$$\hat{g} \sim CN\left(0, \beta - \frac{\beta N_0}{|x|^2\beta + N_0}\right) = CN\left(0, \frac{|x|^2\beta^2}{|x|^2\beta + N_0}\right)$$

Summary

- Estimate realizations of random variables
    - Based on observation and statistics

- MMSE estimator minimizes MSE

- Example of channel estimation
    - Transmit known signal
    - Scale the received signal correctly



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