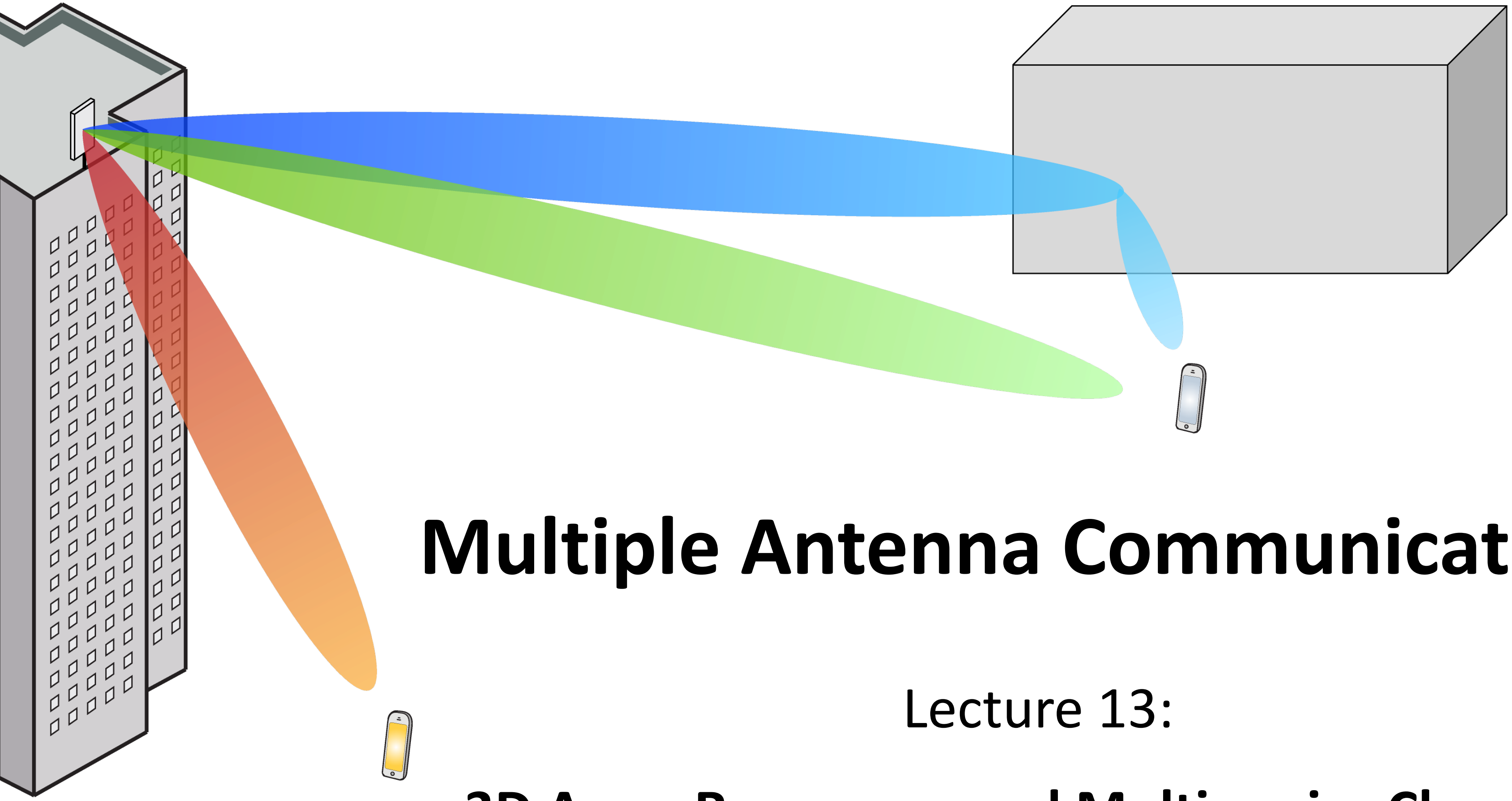


Multiple Antenna Communications

Lecture 13:

3D Array Responses and Multicarrier Channels

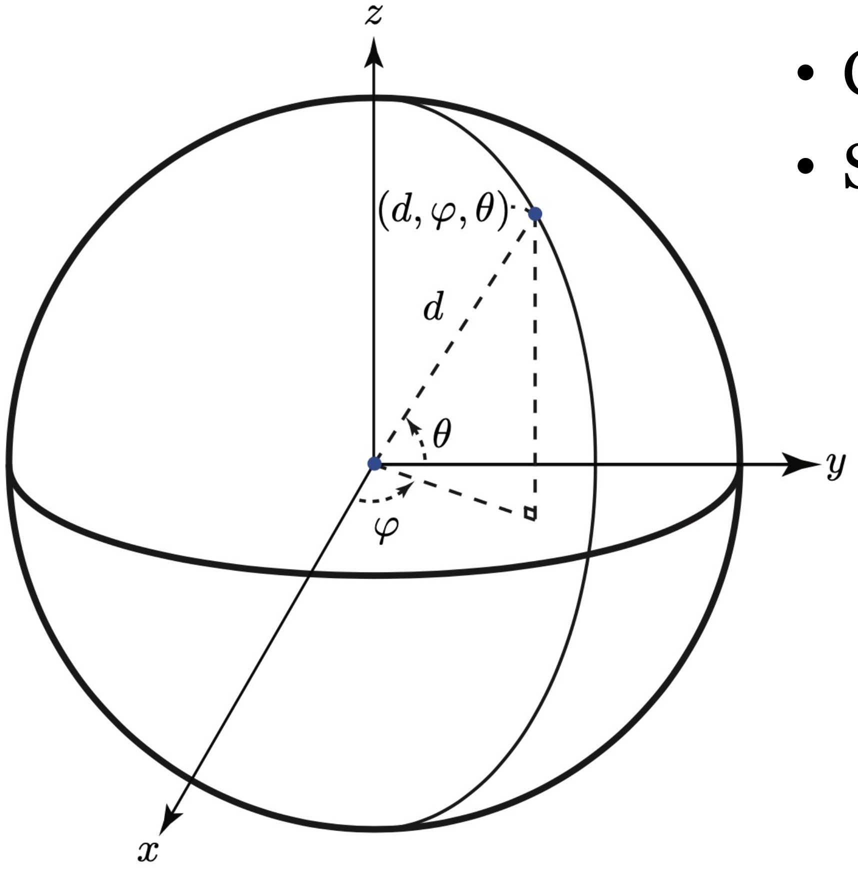
Emil Björnson

Outline

- Analysis of antenna arrays in three dimensions
  - Array response vectors with multiple users
  - Impact of antenna directivity

- Channel modeling
  - Scattering clusters
  - Multicarrier systems

Spherical coordinate system



Distance: $d \geq 0$

Azimuth angle: $-\pi \leq \varphi < \pi$

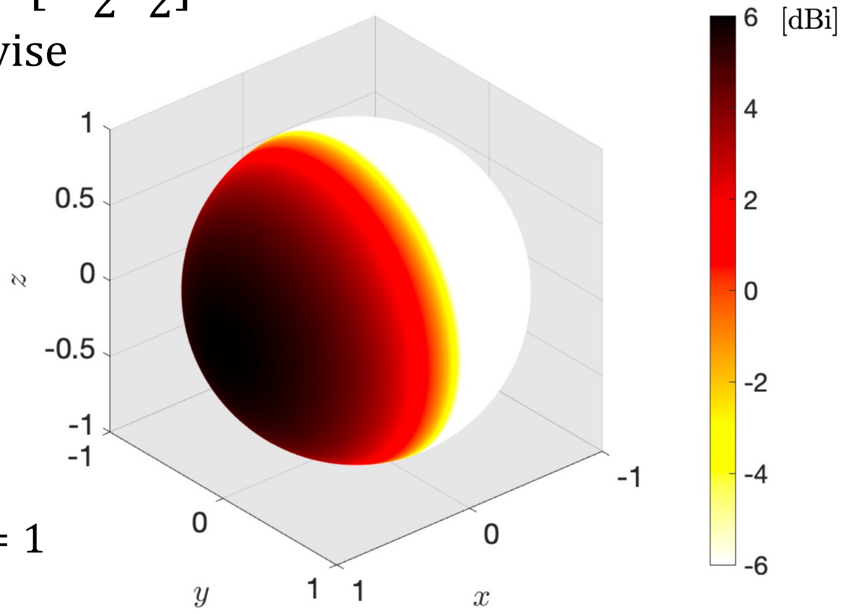
Elevation angle: $-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$

- Cartesian coordinates: (x, y, z)
- Spherical coordinates: (d, φ, θ)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = d \begin{bmatrix} \cos(\varphi)\cos(\theta) \\ \sin(\varphi)\cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

Example: Cosine antenna

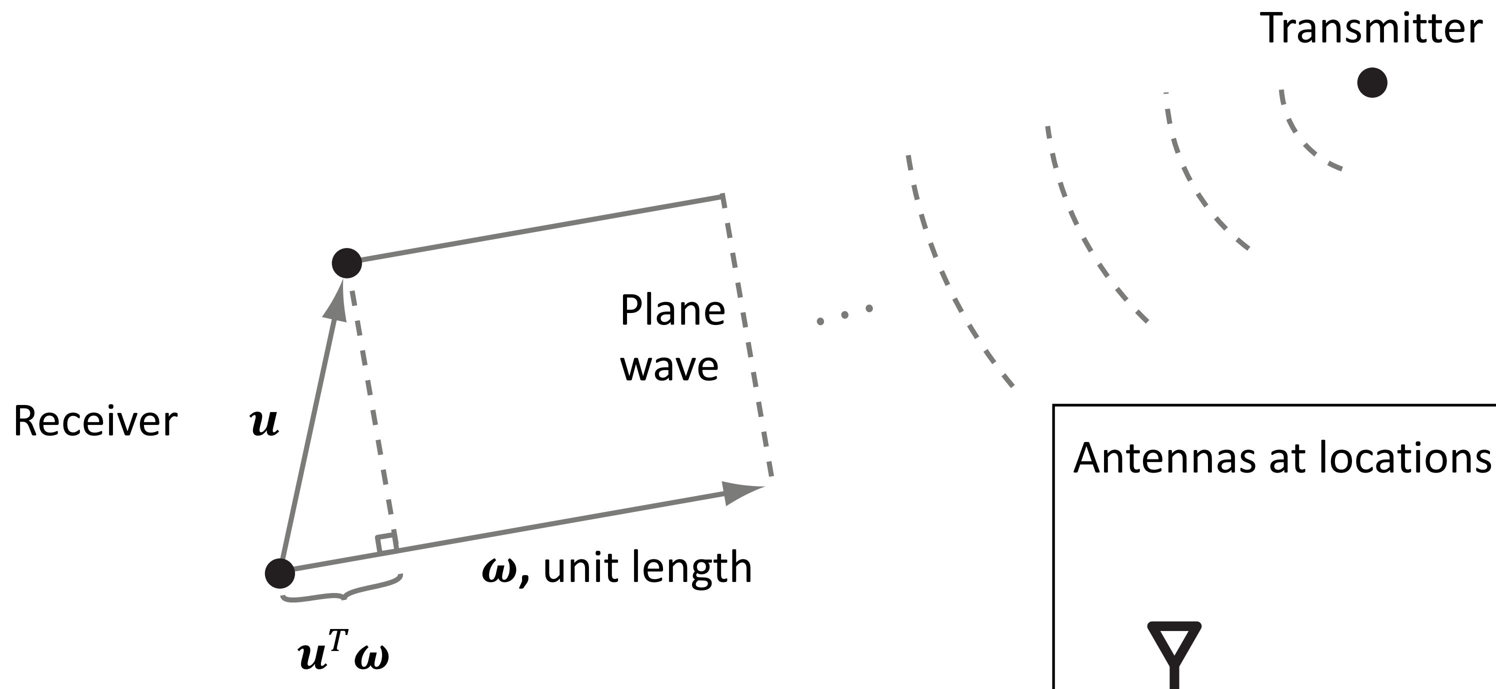
$$G(\varphi, \theta) = \begin{cases} 4\cos(\varphi)\cos(\theta), \varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ 0, \quad \text{otherwise} \end{cases}$$



Conservation of energy:

$$\frac{1}{4\pi} \int_{-\pi}^{\pi} \int_{-\pi/2}^{\pi/2} G(\varphi, \theta) \cos(\theta) d\theta d\varphi = 1$$

Array response in three dimensions



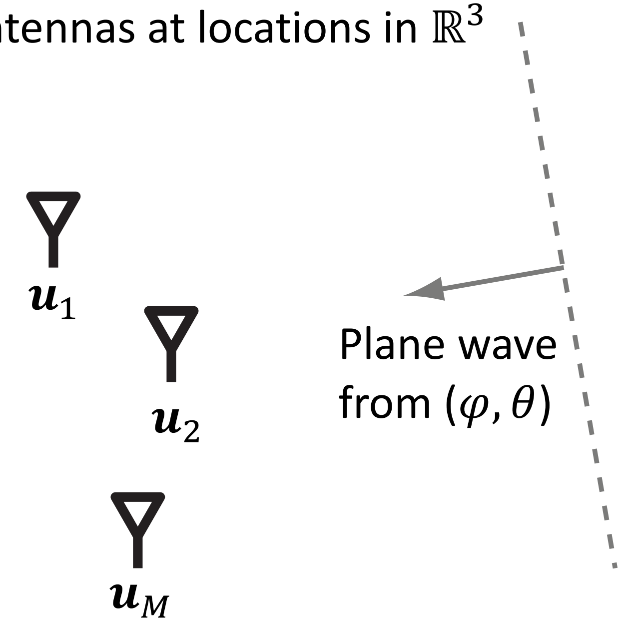
Unit length vector for direction (φ, θ):

$$\boldsymbol{\omega} = \begin{bmatrix} \cos(\varphi)\cos(\theta) \\ \sin(\varphi)\cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

Difference in distance: $\boldsymbol{u}^T\boldsymbol{\omega}$

Difference in phase: $\frac{2\pi}{\lambda}\boldsymbol{u}^T\boldsymbol{\omega}$

Antennas at locations in $\mathbb{R}^3$



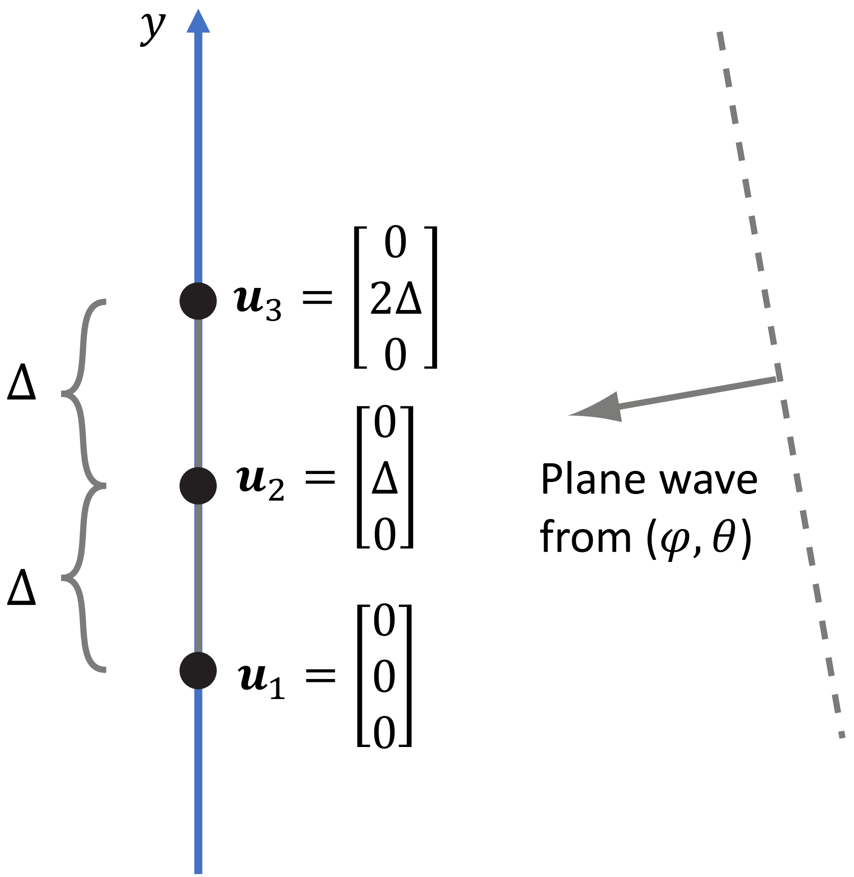
Array response:

$$\boldsymbol{a}(\varphi, \theta) = \begin{bmatrix} e^{j\frac{2\pi}{\lambda}\boldsymbol{u}_1^T\boldsymbol{\omega}} \\ e^{j\frac{2\pi}{\lambda}\boldsymbol{u}_2^T\boldsymbol{\omega}} \\ \vdots \\ e^{j\frac{2\pi}{\lambda}\boldsymbol{u}_M^T\boldsymbol{\omega}} \end{bmatrix}$$

Channel vector:

$$\boldsymbol{g} = \frac{\lambda}{4\pi d}\boldsymbol{a}(\varphi, \theta)$$

Uniform linear array (ULA) in three dimensions



$$\boldsymbol{u}_m^T \boldsymbol{\omega} = \begin{bmatrix} 0 \\ (m-1)\Delta \\ 0 \end{bmatrix}^T \begin{bmatrix} \cos(\varphi)\cos(\theta) \\ \sin(\varphi)\cos(\theta) \\ \sin(\theta) \end{bmatrix} = (m-1)\Delta \sin(\varphi)\cos(\theta)$$

Array response:

$$\boldsymbol{a}(\varphi, \theta) = \begin{bmatrix} e^{j\frac{2\pi}{\lambda}\boldsymbol{u}_1^T\boldsymbol{\omega}} \\ e^{j\frac{2\pi}{\lambda}\boldsymbol{u}_2^T\boldsymbol{\omega}} \\ \vdots \\ e^{j\frac{2\pi}{\lambda}\boldsymbol{u}_M^T\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} 1 \\ e^{j2\pi\frac{\Delta\sin(\varphi)\cos(\theta)}{\lambda}} \\ \vdots \\ e^{j2\pi\frac{(M-1)\Delta\sin(\varphi)\cos(\theta)}{\lambda}} \end{bmatrix}$$

If transmitter also in the xy-plane ($\theta = 0$): $\boldsymbol{a}(\varphi, \theta) = \begin{bmatrix} 1 \\ e^{j2\pi\frac{\Delta\sin(\varphi)}{\lambda}} \\ \vdots \\ e^{j2\pi\frac{(M-1)\Delta\sin(\varphi)}{\lambda}} \end{bmatrix}$

Array response with directive antennas

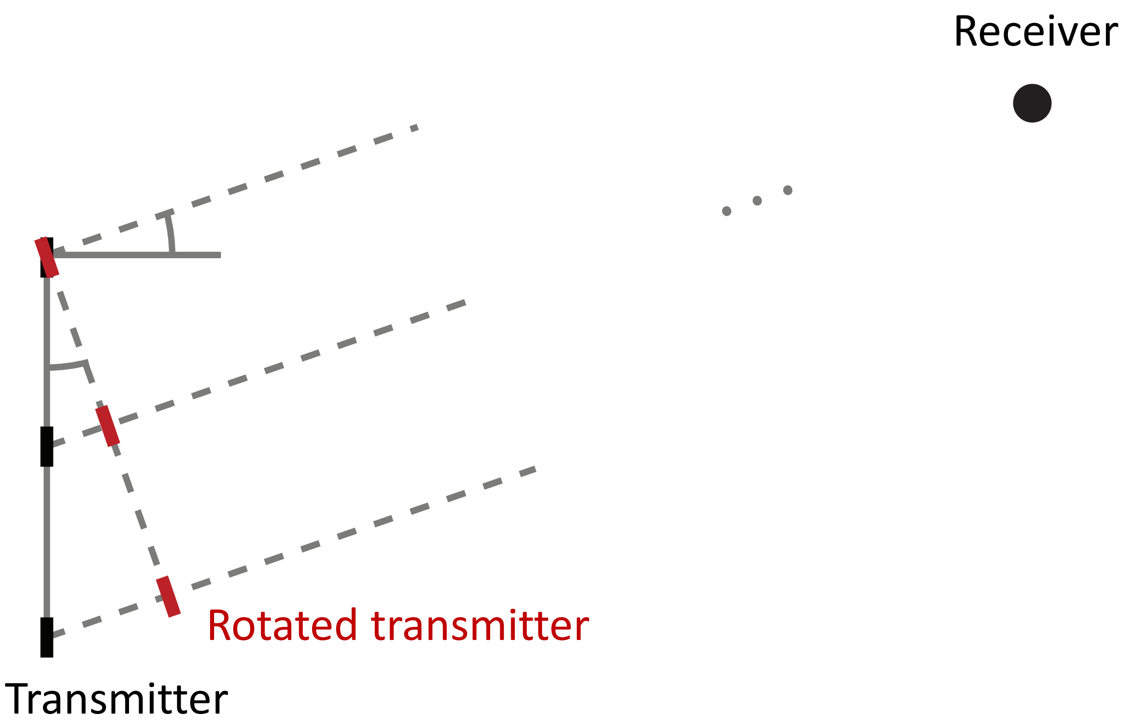
- Uniform linear array with M antennas
  - Directive receive antennas in array: Gain $G(\varphi,\theta)$
  - Isotropic transmit antenna at (d,φ,θ)

- Channel vector:

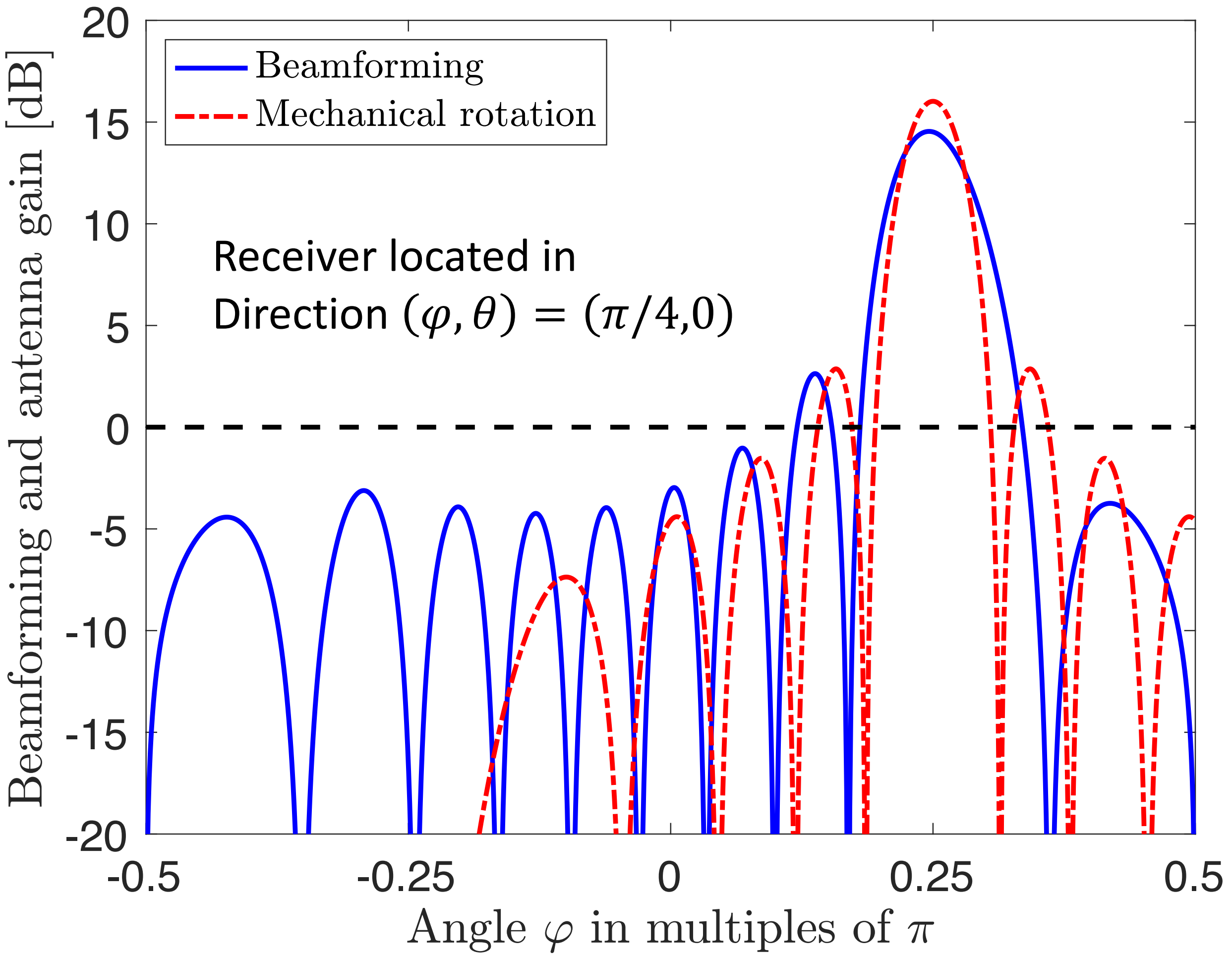
$$\boldsymbol{g} = \sqrt{G(\varphi,\theta)}\frac{\lambda}{4\pi d}\boldsymbol{a}(\varphi,\theta) = \sqrt{G(\varphi,\theta)}\frac{\lambda}{4\pi d}\begin{bmatrix} 1 \\ e^{j2\pi\frac{\Delta\sin(\varphi)\cos(\theta)}{\lambda}} \\ \vdots \\ e^{j2\pi\frac{(M-1)\Delta\sin(\varphi)\cos(\theta)}{\lambda}} \end{bmatrix}$$

$$\|\boldsymbol{g}\|^2 = \underbrace{G(\varphi,\theta)}_{\text{Amplification/attenuation}}\left(\frac{\lambda}{4\pi d}\right)^2\underbrace{\|\boldsymbol{a}(\varphi,\theta)\|^2}_{=M}$$

Electrical versus mechanical beamforming



$M = 10, \Delta = \frac{\lambda}{2}$



Cosine antennas

$$G(\varphi, \theta) = \begin{cases} 4\cos(\varphi)\cos(\theta), \varphi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ 0, \qquad \text{otherwise} \end{cases}$$

Effective isotropic radiated power (EIRP)

- Array response with same antenna gain for all antennas:

$$\sqrt{G(\varphi,\theta)}\boldsymbol{a}(\varphi,\theta)$$

  - Transmit power P

Definition: EIRP for beamforming in direction (φ,θ)

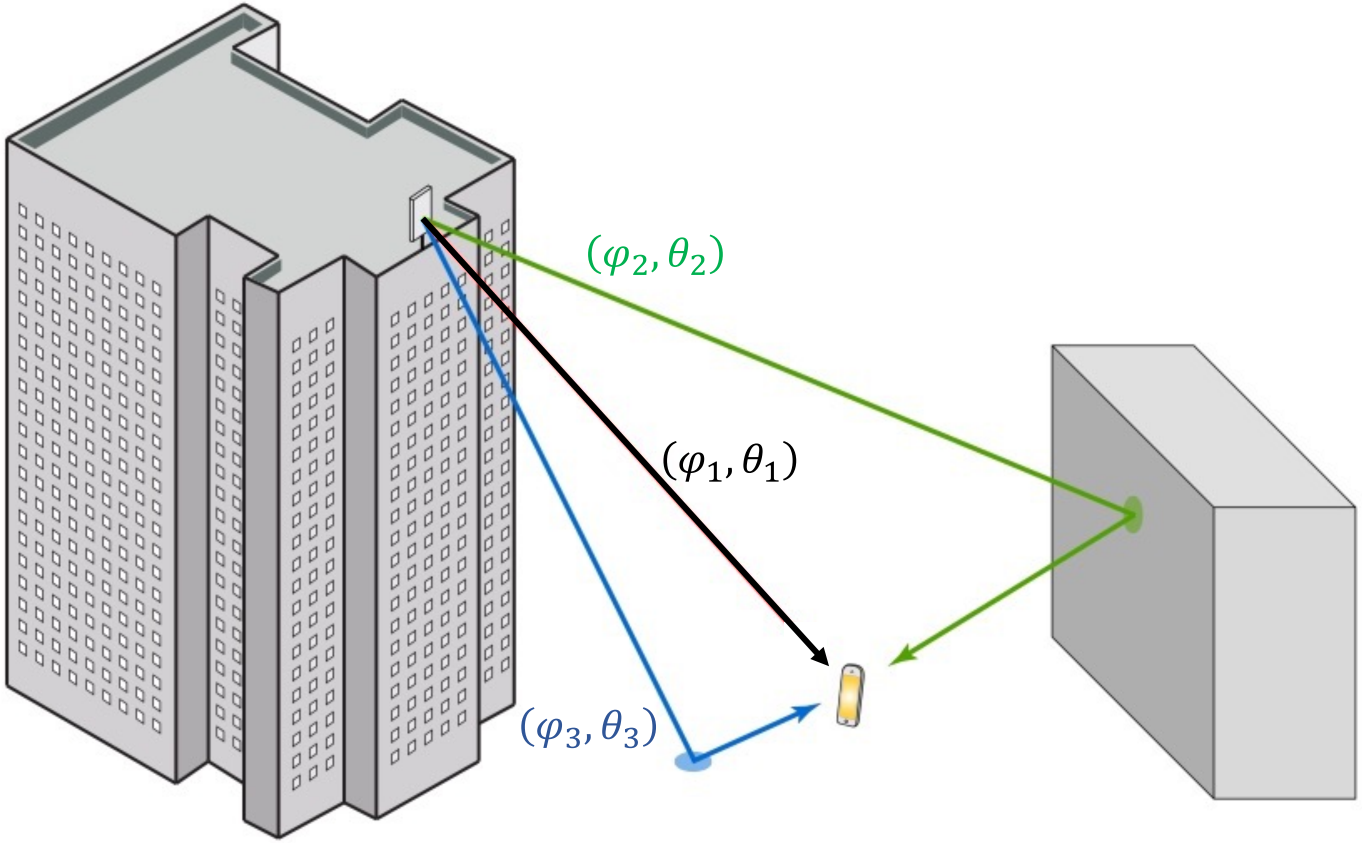
$$\text{EIRP}(\varphi,\theta) = P \cdot G(\varphi,\theta) \cdot \|\boldsymbol{a}(\varphi,\theta)\|^2$$

- Maximum EIRP:

$$\max_{\varphi,\theta} \text{EIRP}(\varphi,\theta) = P \cdot M \cdot \max_{\varphi,\theta} G(\varphi,\theta)$$

Spectrum licenses: Limits on transmit power and maximum EIRP

Sparse multipath propagation



Carrier frequency
Propagation delay

$$2\pi f_c \tau_i$$

$$\boldsymbol{g} = \sum_{i=1}^{L} \alpha_i e^{-j\psi_i} \boldsymbol{a}(\varphi_i, \theta_i)$$

Amplitude loss

Clustered scattering $(L = N_{\text{cl}} N_{\text{path}})$

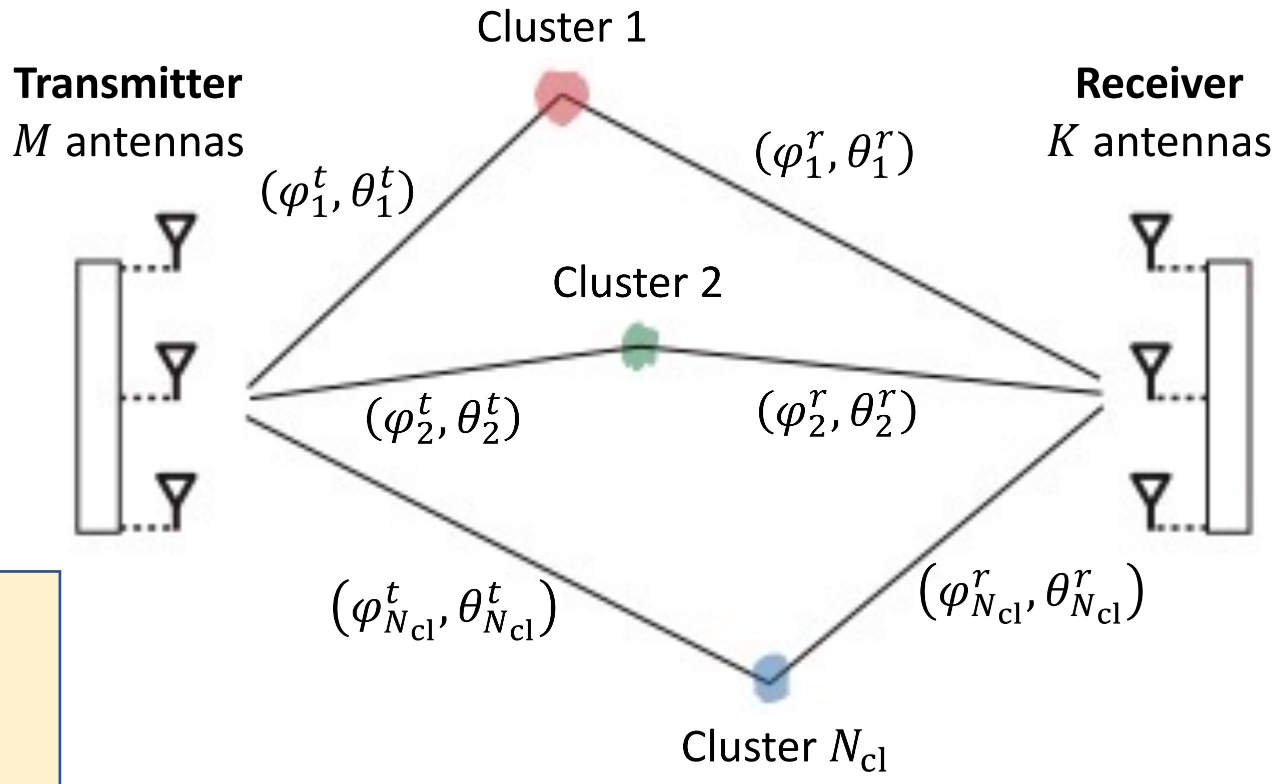
N_{cl} clusters with N_{path} paths

$$\boldsymbol{g} = \sum_{i=1}^{N_{\text{cl}}} \underbrace{\left(\sum_{n=1}^{N_{\text{path}}} \alpha_{n,i} e^{-j\psi_{n,i}} \right)}_{\sim\, CN(0,\beta_i) \text{ if } N_{\text{path}} \text{ is large}} \boldsymbol{a}(\varphi_i, \theta_i)$$

i.i.d. fading if clusters everywhere

Clustered MIMO channel

- Array responses
  - Transmitter: $\boldsymbol{a}_t(\varphi, \theta)$
  - Receiver : $\boldsymbol{a}_r(\varphi, \theta)$



Clustered scattering

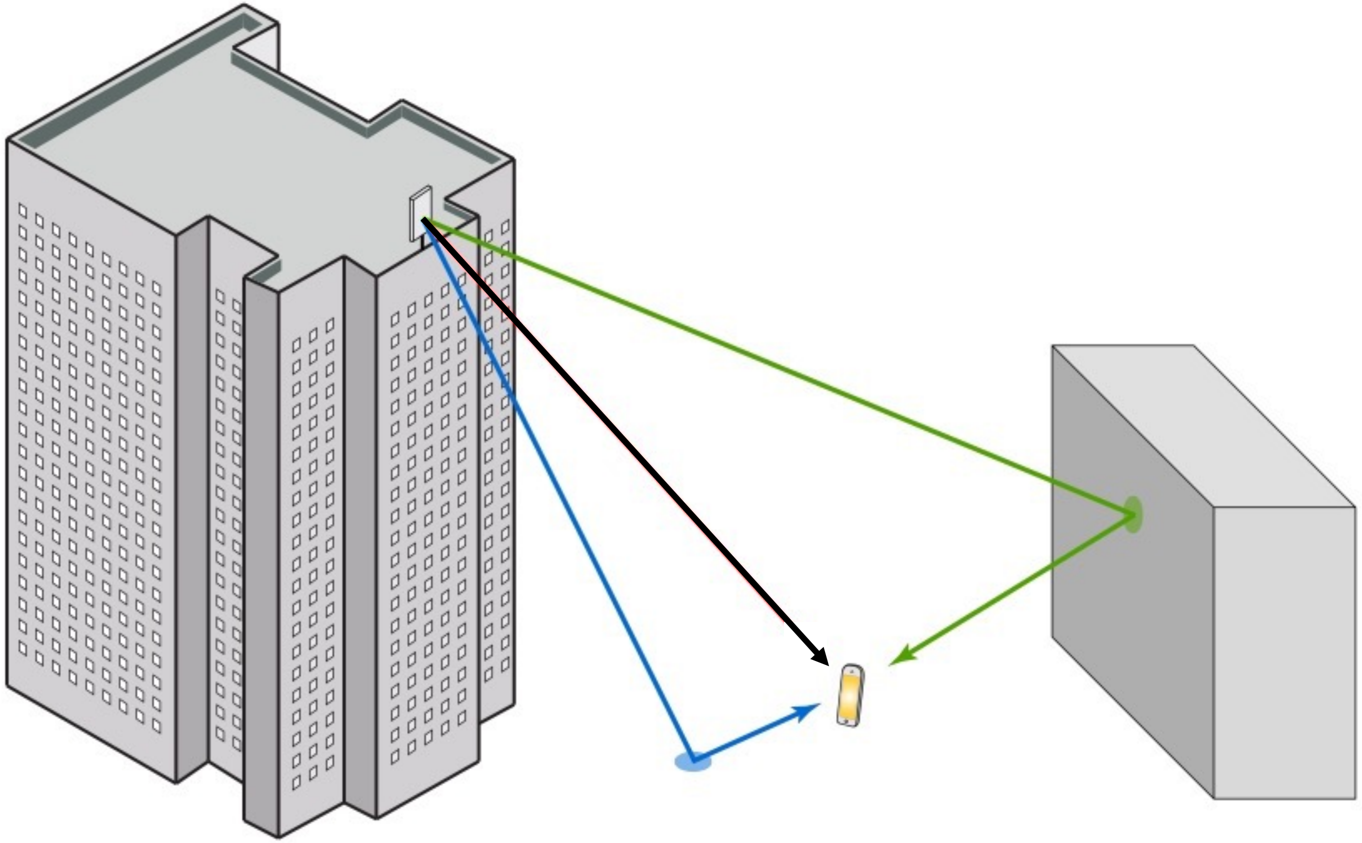
N_{cl} clusters with N_{path} paths

$$\boldsymbol{G} = \sum_{i=1}^{N_{\mathrm{cl}}} \underbrace{\left(\sum_{n=1}^{N_{\mathrm{path}}} \alpha_{n,i} e^{-j\psi_{n,i}} \right)}_{\sim\, CN(0,\beta_i) \text{ if } N_{\mathrm{path}} \text{ is large}} \boldsymbol{a}_r(\varphi_i^r, \theta_i^r) \boldsymbol{a}_t^T(\varphi_i^t, \theta_i^t)$$

Channel rank:

$\min(M, K, N_{cl})$

Channels with memory



Large bandwidth → Short symbol time

Paths arrive at different time

Discrete memoryless channel:

$$y[l] = g \cdot x[l] + n[l]$$

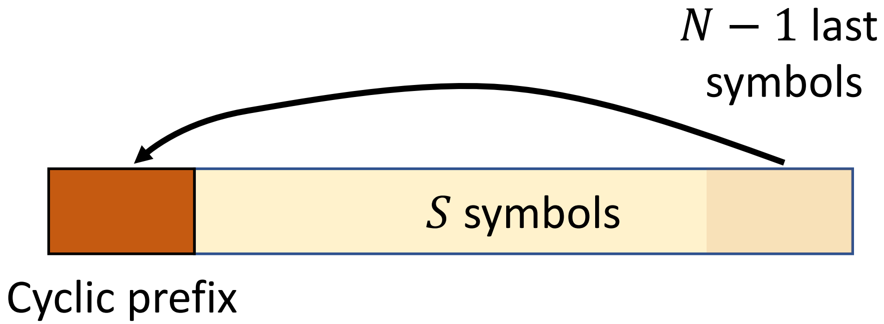
Discrete channel with memory:

$$y[l] = \sum_{n=0}^{N-1} x[l-n]g[n] + n[l]$$

Channel with N taps: $g[0], \dots, g[N-1]$

Orthogonal frequency-division multiplexing (OFDM)

- Method to manage intersymbol interference
  - Channel with N taps
  - Transmit block of $S \geq N$ symbols
  - Let $g[N], \dots, g[S-1] = 0$



Received signal in matrix form:

$$\begin{bmatrix} y[0] \\ y[1] \\ \vdots \\ y[S-1] \end{bmatrix} = \underbrace{\begin{bmatrix} g[0] & g[S-1] & \dots & g[1] \\ g[1] & g[0] & \ddots & \vdots \\ \vdots & \ddots & \ddots & g[S-1] \\ g[S-1] & \dots & g[1] & g[0] \end{bmatrix}}_{\mathbf{DFT}^H \cdot \mathrm{diag}(\bar{g}[0], \dots, \bar{g}[S-1]) \cdot \mathbf{DFT}} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[S-1] \end{bmatrix} + \begin{bmatrix} n[0] \\ n[1] \\ \vdots \\ n[S-1] \end{bmatrix}$$

Discrete Fourier transforms:

DFT is $S \times S$ matrix

$$[\mathbf{DFT}]_{k,m} = \frac{1}{\sqrt{S}} \mathrm{e}^{-j2\pi(k-1)(m-1)/S}$$

$$\bar{g}[i] = \sum_{l=0}^{N-1} g[l] e^{-j2\pi l i/S}$$

Point-to-point MIMO processing

Discrete memoryless channel

$$\bar{y}[i] = \bar{g}[i]\bar{x}[i] + \bar{n}[i]$$

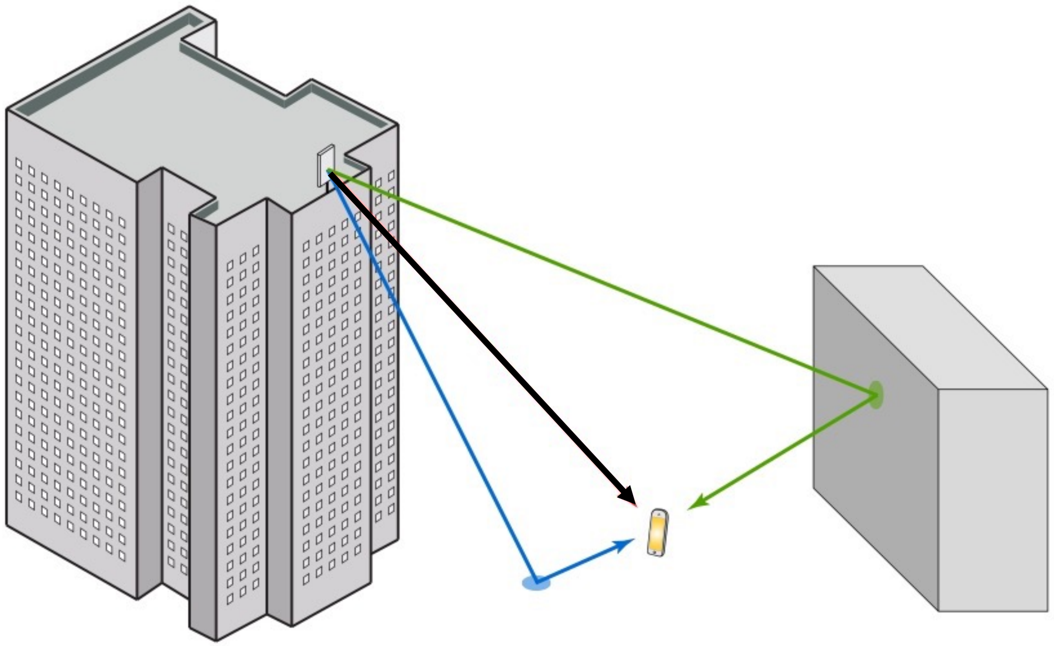
Subcarriers

$i = 0, \dots, S-1$

Multiple antenna OFDM

- Assume one cluster per tap:

$$\boldsymbol{g}[l] = \alpha_l e^{-j\psi_l}\boldsymbol{a}(\varphi_l, \theta_l)$$



Channel at subcarrier i:

$$\underbrace{\bar{\boldsymbol{g}}[i] = \sum_{l=0}^{N-1}\left(\alpha_l e^{-j\psi_l}\boldsymbol{a}(\varphi_l, \theta_l)\right)e^{-j2\pi l i/S}}_{\text{Different weighted sums at each subcarrier}}$$

Use different precoding at each subcarrier

If dominant LOS path

$$\bar{\boldsymbol{g}}[i] \approx \alpha_0 e^{-j\psi_0}\boldsymbol{a}(\varphi_0, \theta_0)$$

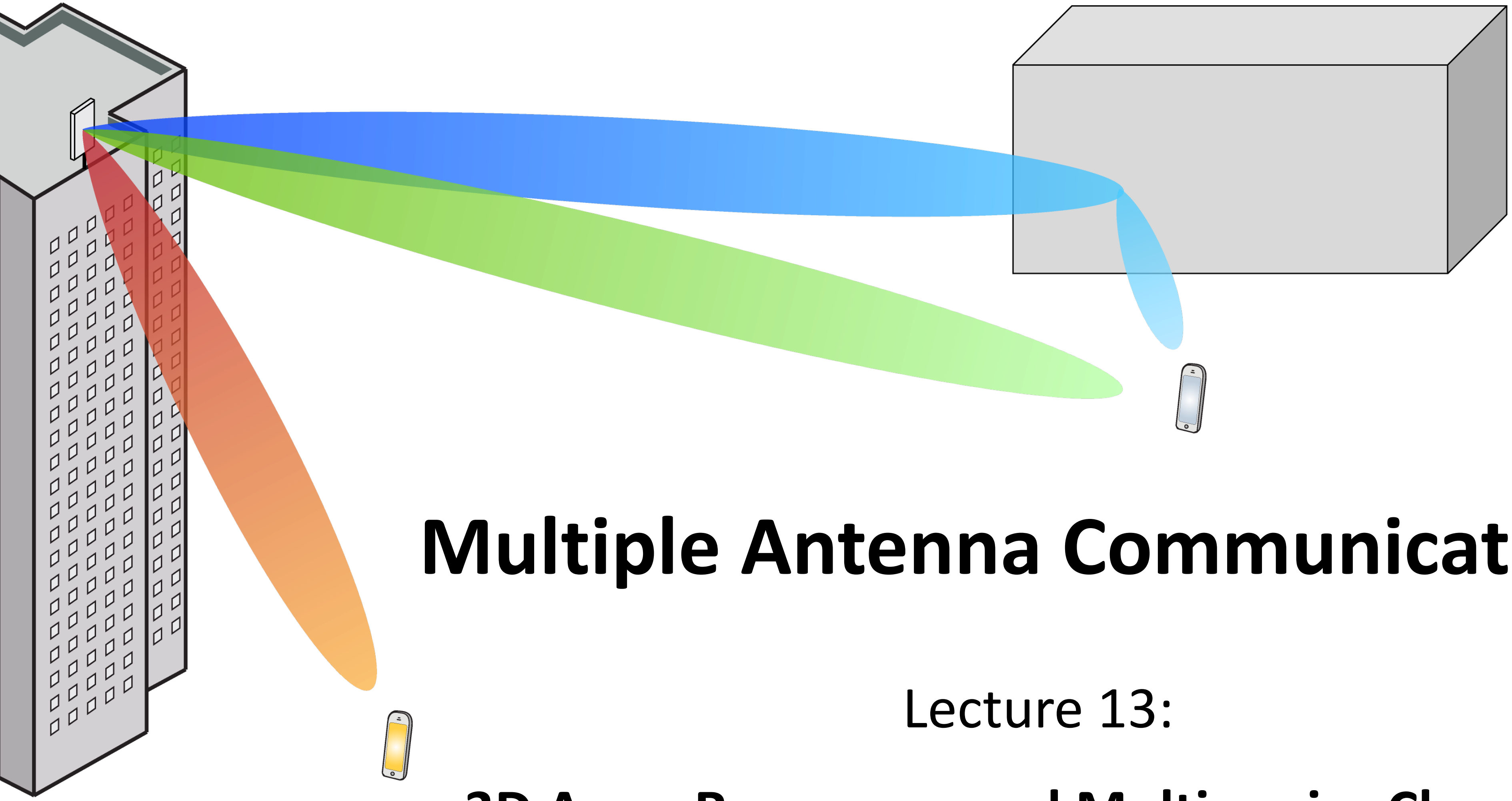
Same at all subcarriers!

Summary

- Array response
  - Depends on azimuth and elevation angle

- Important to model
  - Sparse multipath propagation
  - Impact of directive antennas

- Multicarrier OFDM systems
  - Resolve intersymbol interference
  - Different channels on the subcarriers



Multiple Antenna Communications

Lecture 13:

3D Array Responses and Multicarrier Channels

Emil Björnson