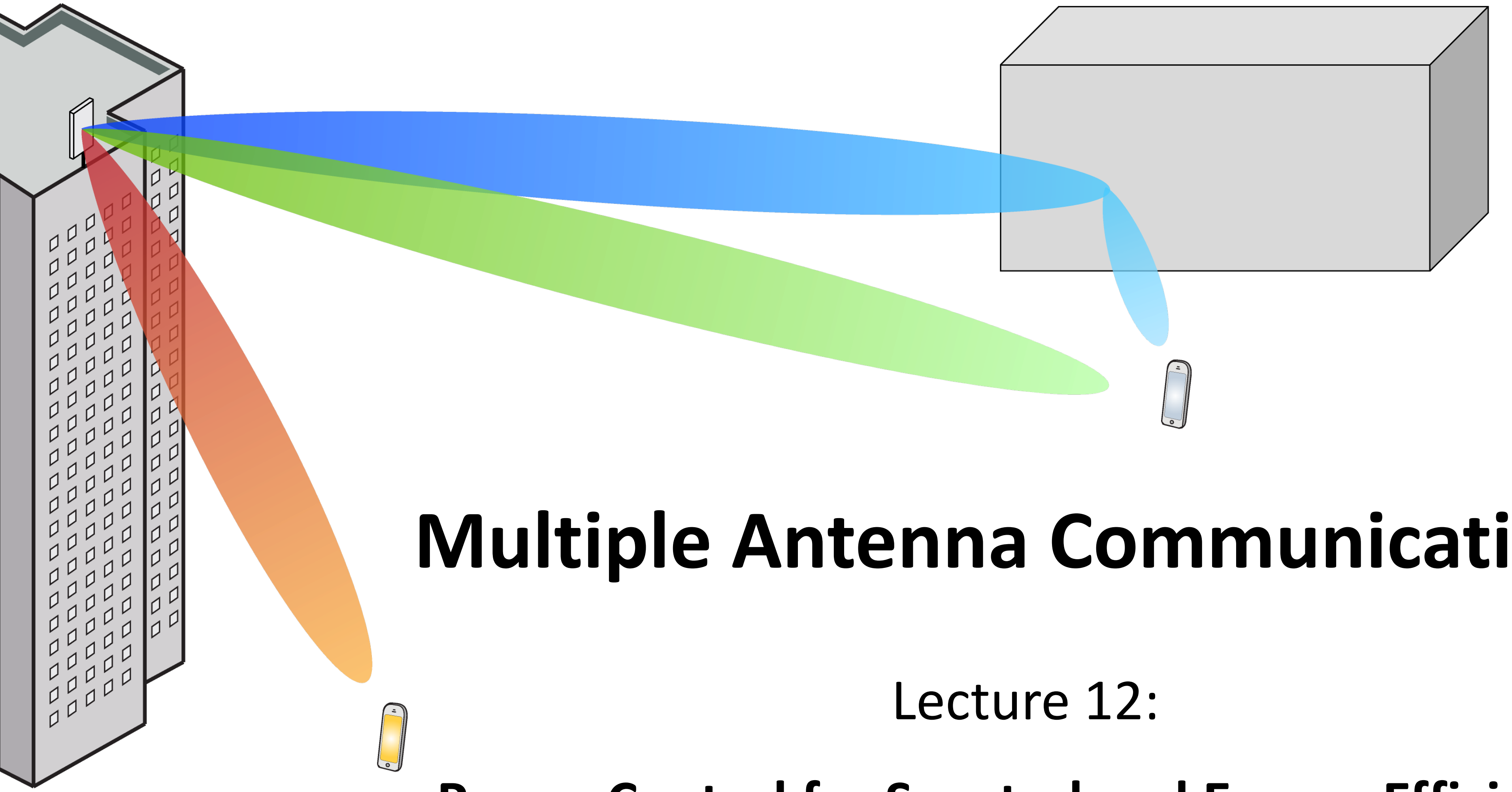


Multiple Antenna Communications

Lecture 12:

Power Control for Spectral and Energy Efficiency

Emil Björnson

Outline

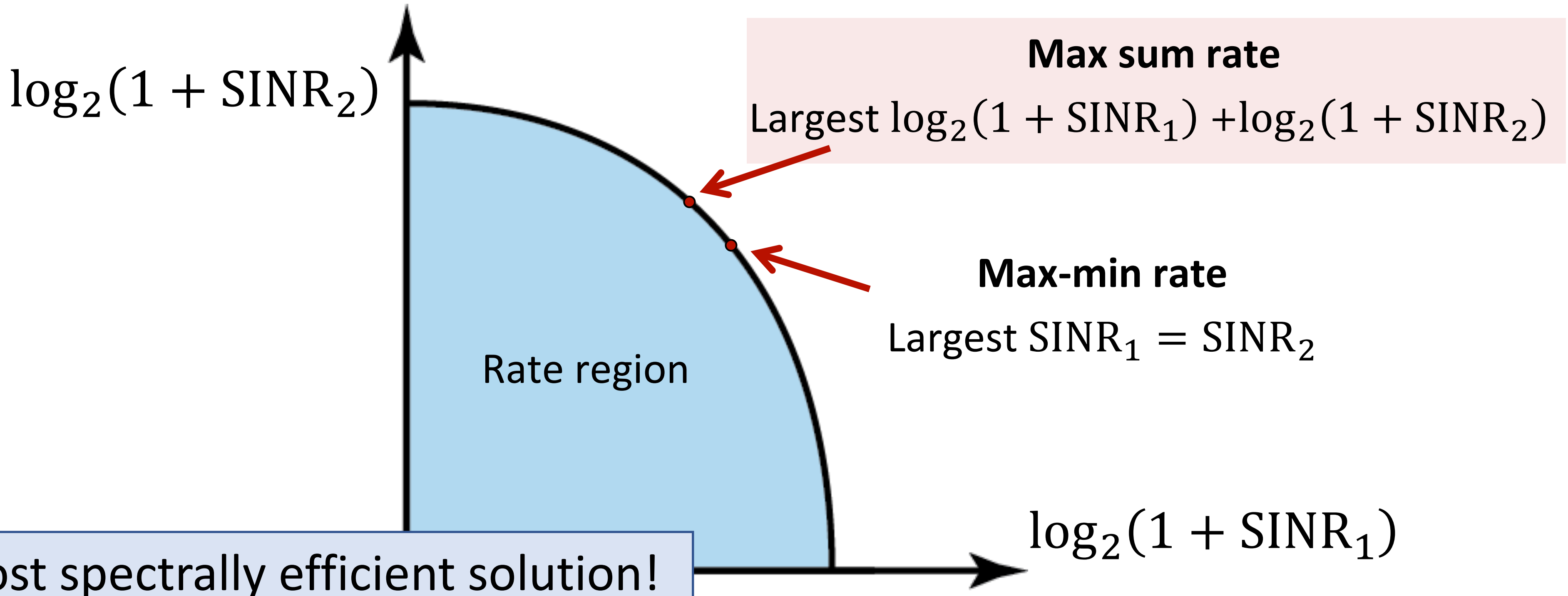
- Power control

- Spectral efficiency (sum rate) maximization
    - Uplink, downlink
    - Simulation example

- Energy efficiency maximization
    - Definition
    - Impact of beamforming and multipleixng

Power control

- Effective SINRs depend on $\eta_1, \dots, \eta_K$
  - Can be selected to achieve different operating points



Most spectrally efficient solution!

Downlink sum rate maximization

MR	ZF
$a_k = M\rho_{dl}\gamma_k$ $b_k = \rho_{dl}\beta_k$	$a_k = (M-K)\rho_{dl}\gamma_k$ $b_k = \rho_{dl}(\beta_k - \gamma_k)$

- Optimization problem:
  - Find $\eta_1, \ldots, \eta_K$ that maximizes

$$\sum_{k=1}^{K} \log_2\left(1 + \frac{a_k \eta_k}{1 + b_k \sum_{k'=1}^{K} \eta_{k'}}\right)$$

subject to $\sum_{k=1}^{K} \eta_k \leq 1,\ \eta_k \geq 0,\ k = 1, \ldots, K$

- Property 1: Sum rate is larger with $(c\eta_1, \ldots, c\eta_K)$ than with $(\eta_1, \ldots, \eta_K)$, for $c \geq 1$.
  - Consequence: Use maximum power, $\sum_{k=1}^{K} \eta_k = 1$

Maximize: $\sum_{k=1}^{K} \log_2\left(1 + \frac{a_k}{1+b_k}\eta_k\right)$ using “waterfilling”

Sum rate maximizing waterfilling power allocation

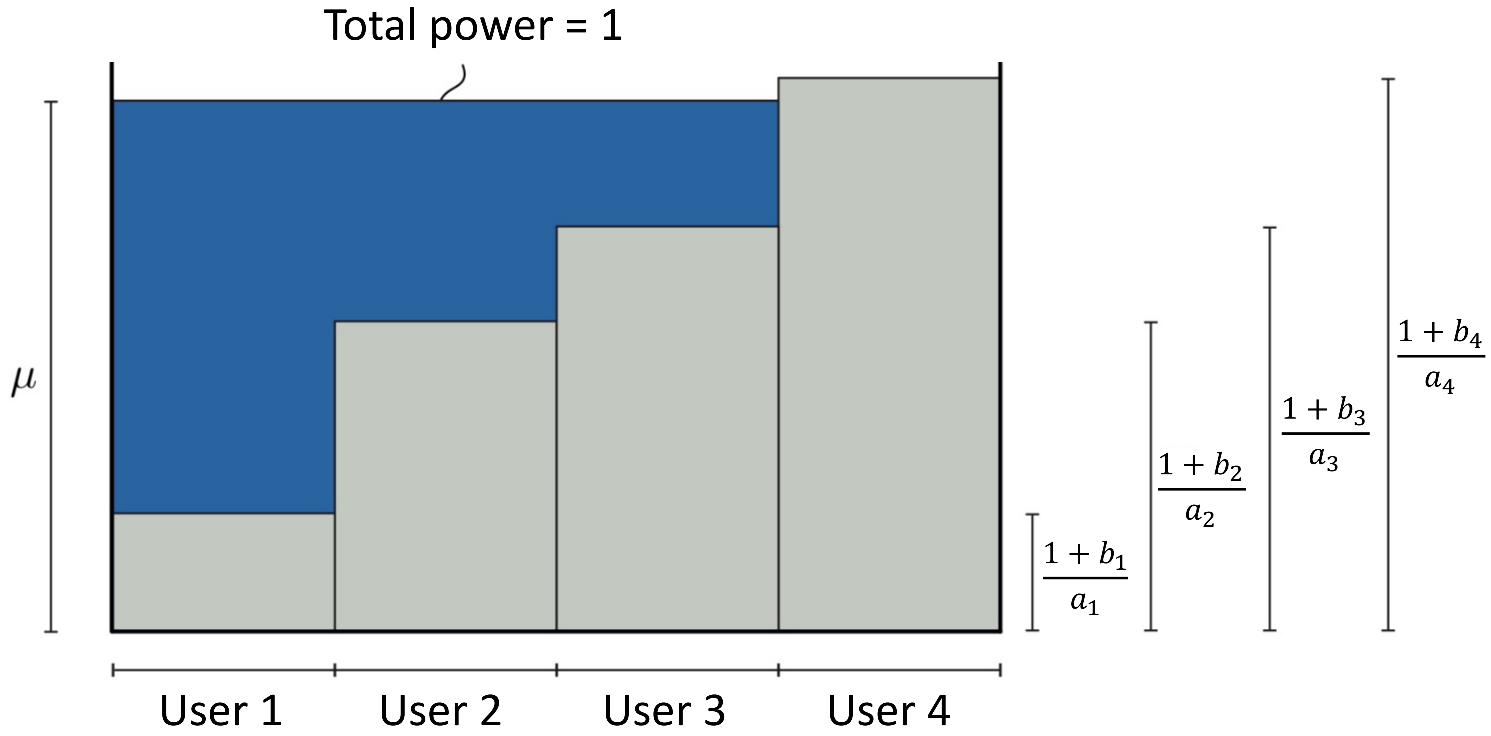
- After some optimization:

$$\eta_k = \max\left(\mu - \frac{1 + b_k}{a_k}, 0\right)$$

where μ is selected such that $\eta_1 + \cdots + \eta_K = 1$

Properties:

- Larger $a_k/(1 + b_k)$: More power
- Some users might get zero power



Users ordered based on channel quality

Uplink sum rate maximization

MR	ZF
$a_k = M\rho_{ul}\gamma_k$ $b_{k'} = \rho_{ul}\beta_{k'}$	$a_k = (M-K)\rho_{ul}\gamma_k$ $b_{k'} = \rho_{ul}(\beta_{k'} - \gamma_{k'})$

- Optimization problem:
  - Find $\eta_1, \ldots, \eta_K$ that maximizes

$$\sum_{k=1}^{K} \log_2\left(1 + \frac{a_k \eta_k}{1 + \sum_{k'=1}^{K} b_{k'}\eta_{k'}}\right)$$

subject to $0 \leq \eta_k \leq 1,\ k = 1, \ldots, K$

- Property 1: Denominator $(1 + \sum_{k'=1}^{K} b_{k'}\eta_{k'})$ is the same for every user
  - Change of variable:

$$s = \frac{1}{1 + \sum_{k'=1}^{K} b_{k'}\eta_{k'}}, \qquad x_k = \eta_k b_k s$$

Maximize

$$\sum_{k=1}^{K} \log_2\left(1 + \frac{a_k}{b_k} x_k\right)$$

Revised problem formulation

$$\underset{x_1,\ldots,x_K,s}{\text{maximize}} \quad \sum_{k=1}^{K} \log_2\left(1+\frac{a_k}{b_k}x_k\right)$$

$$\text{subject to} \;\; 0 \leq x_k \leq s b_k, \qquad k=1,\ldots,K$$

$$\sum_{k=1}^{K} x_k = 1-s$$

Convex optimization problem
Maximize a concave function
Linear constraints

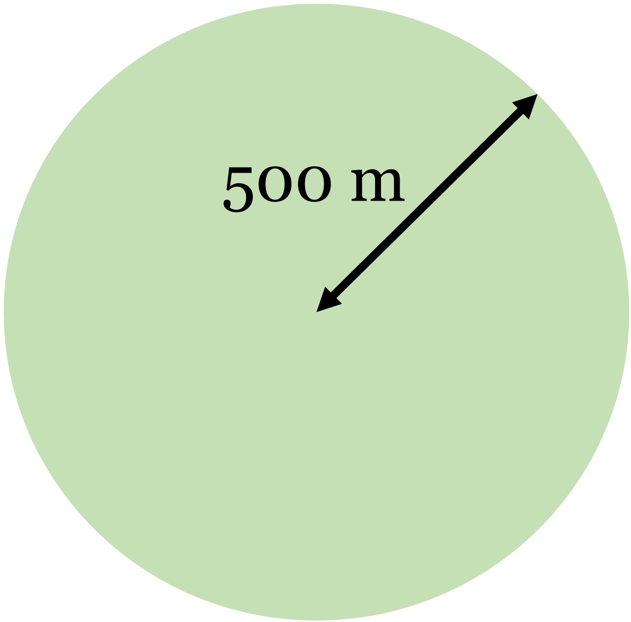
→

Solve with any convex optimization solver (e.g., CVX)

Simulation example: Urban deployment

- Single-cell setup
  - Circular cell with radius 500 m
  - Base station: $M = 100$ antennas
  - $K = 10$ uniformly users

- Important properties
  - Independent Rayleigh fading
  - No inter-cell interference
  - Carrier frequency: 2 GHz
  - Bandwidth 20 MHz



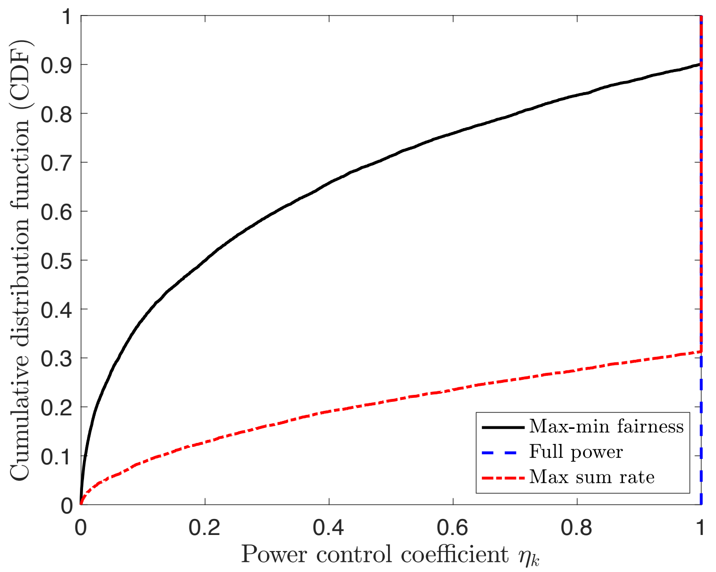
Same parameters as in last lecture

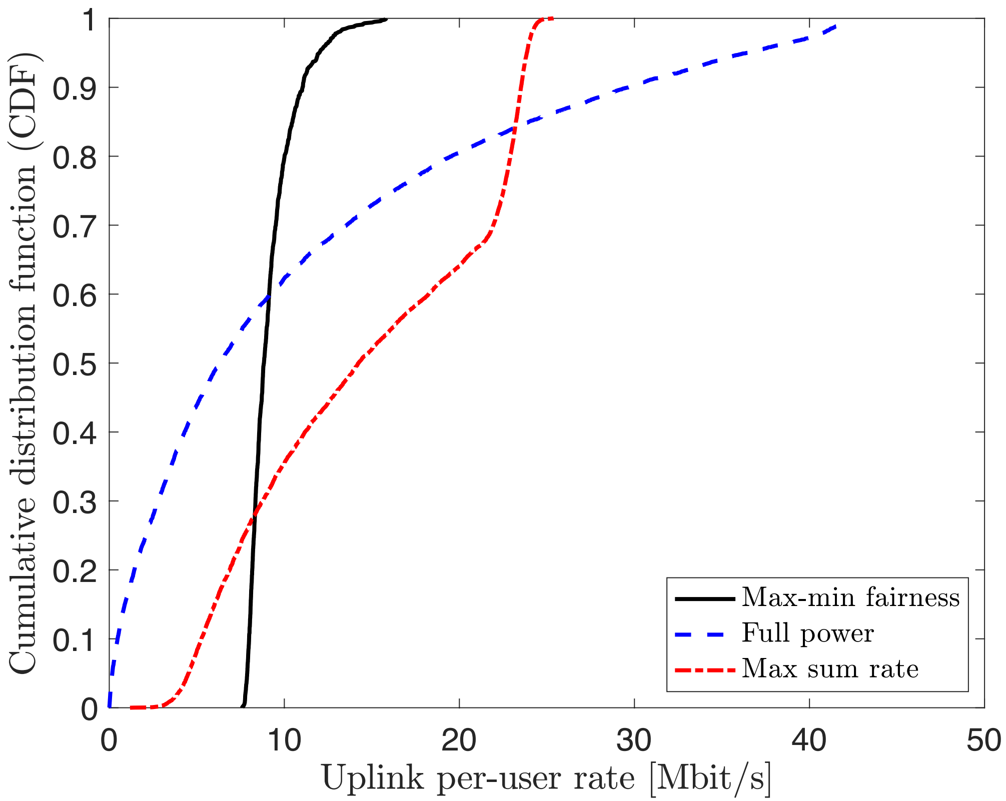
Uplink with power control

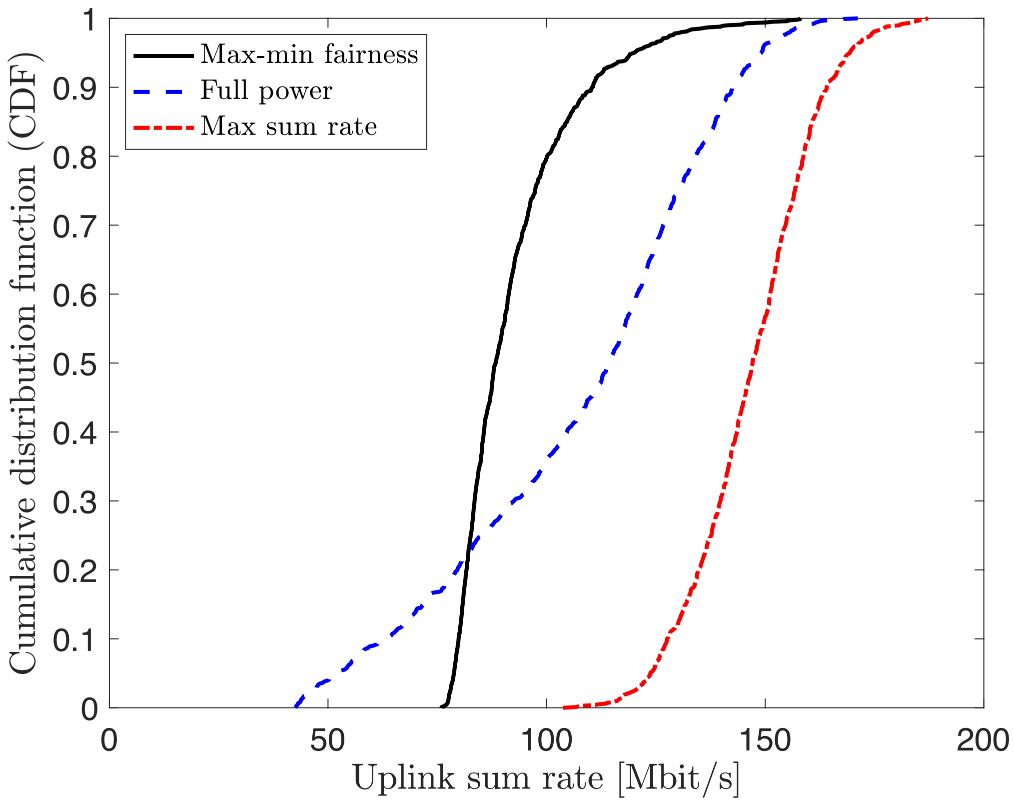
Sum rate maximization

Increase rates by sacrificing fairness

Focused on long-term fairness



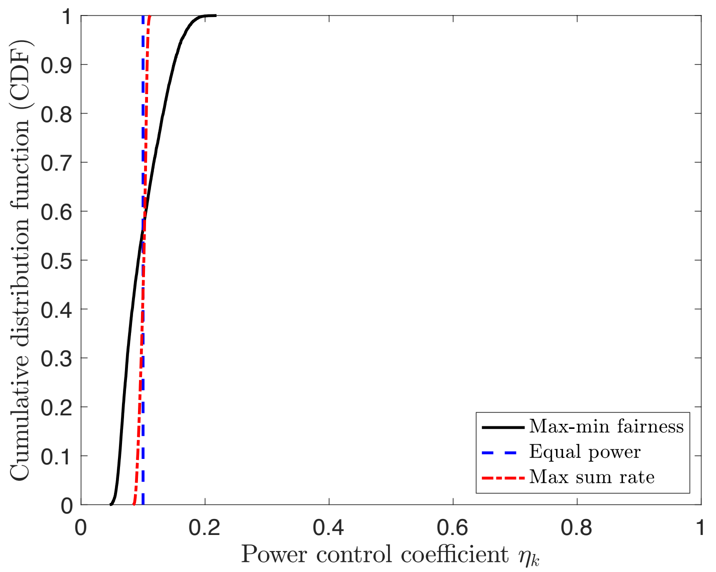


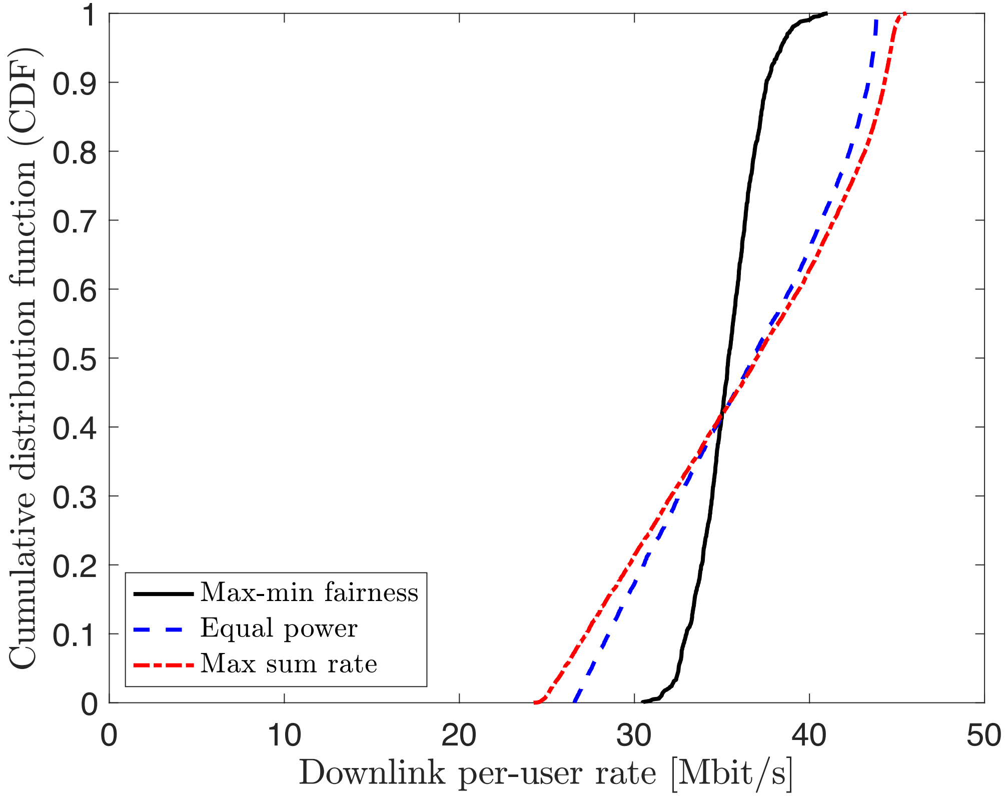


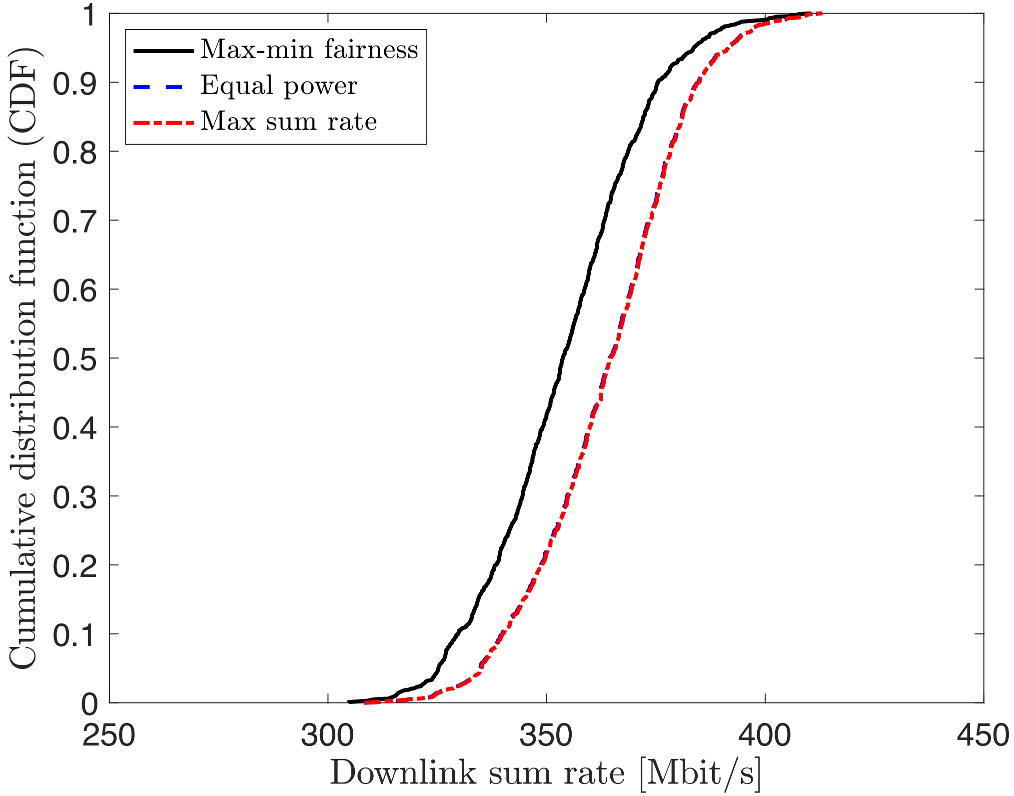
Downlink with power control

Sum rate maximization

Similar to equal power allocation







Power Control for Maximum Energy Efficiency

- Benefit-cost analysis:

Cost:
Energy consumption
(Watt=Joule/s)

→ **Communication system** →

Benefit:
Data rate
(bit/s)

- Benefit-cost ratio:

$$\text{Energy efficiency [bit/Joule]} = \frac{\text{Data rate [bit/s]}}{\text{Energy consumption [Joule/s]}}$$

Environmental concerns

Energy production is
mainly non-renewable

Economical concerns

Energy price: Joule/€
Other costs can also be included

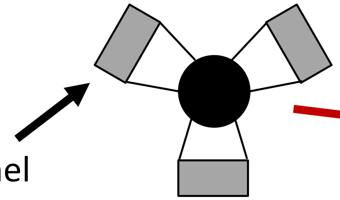
Example: Energy efficiency of 4G base station

Data throughput
Up to 28 Mbit/s

Energy consumption
1.35 kW

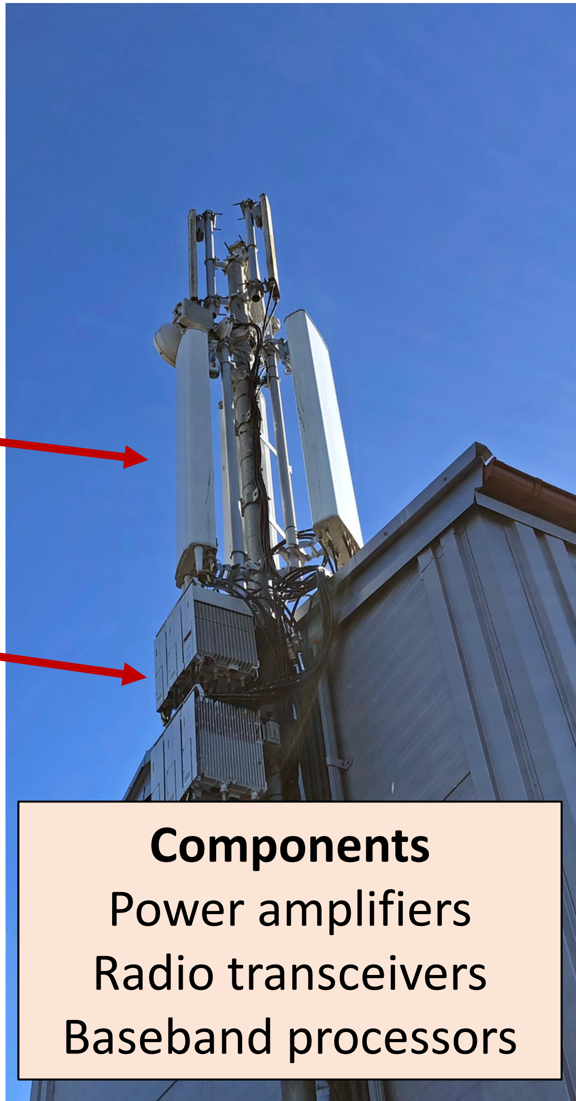
Energy efficiency
28 Mbit/s / 1.35 kW = 20 kbit/Joule

One site
Three base stations



Antenna panel

Power amplifiers

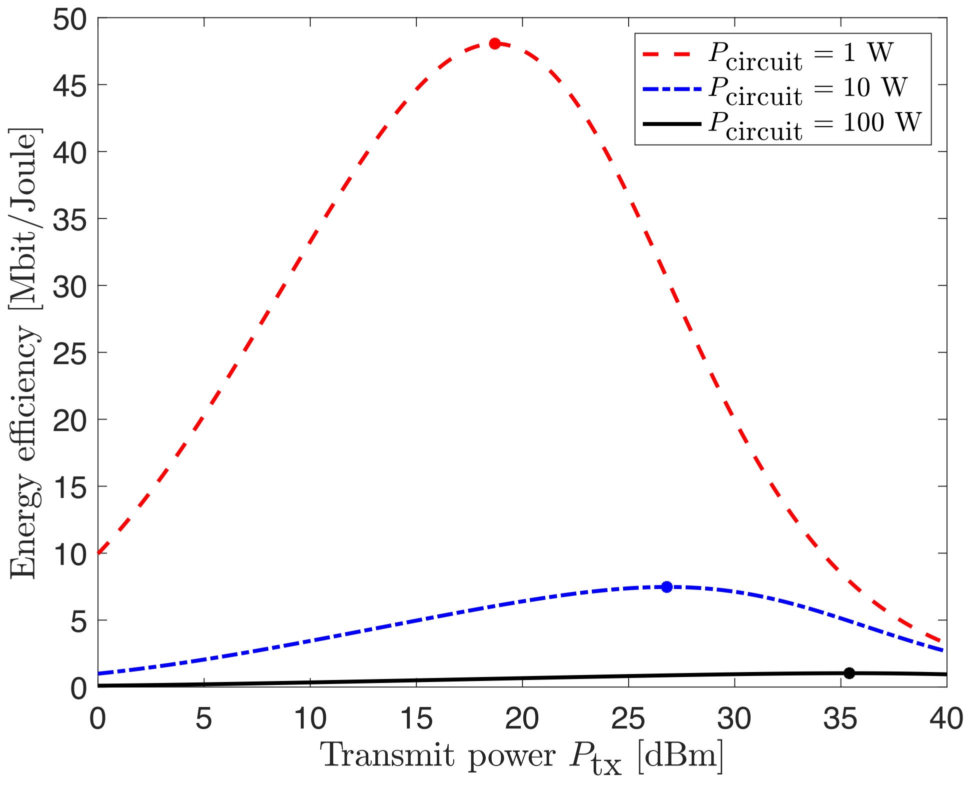


Components
Power amplifiers
Radio transceivers
Baseband processors

Reference:
Auer et al., “How much energy is needed to run a wireless network?,” IEEE Wireless Communication, 2011

Energy Efficient Power Control

- Consider point-to-point system:



Transmit power

Bandwidth

Pathloss

$$\text{Energy efficiency [bit/Joule]} = \frac{B \log_2\left(1 + \frac{P_{\text{tx}}\beta}{BN_0}\right)}{\frac{P_{\text{tx}}}{\mu} + P_{\text{circuit}}}$$

Noise power spectral density

Amplifier efficiency

Circuit power

Unimodal functions

Control transmit power to find optimum

Circuit power

Must be accurately modeled

$$B = 10 \text{ MHz}, \qquad \mu = 0.25, \qquad \frac{BN_0}{\beta} = 0 \text{ dBm}$$

Energy Efficiency and Beamforming

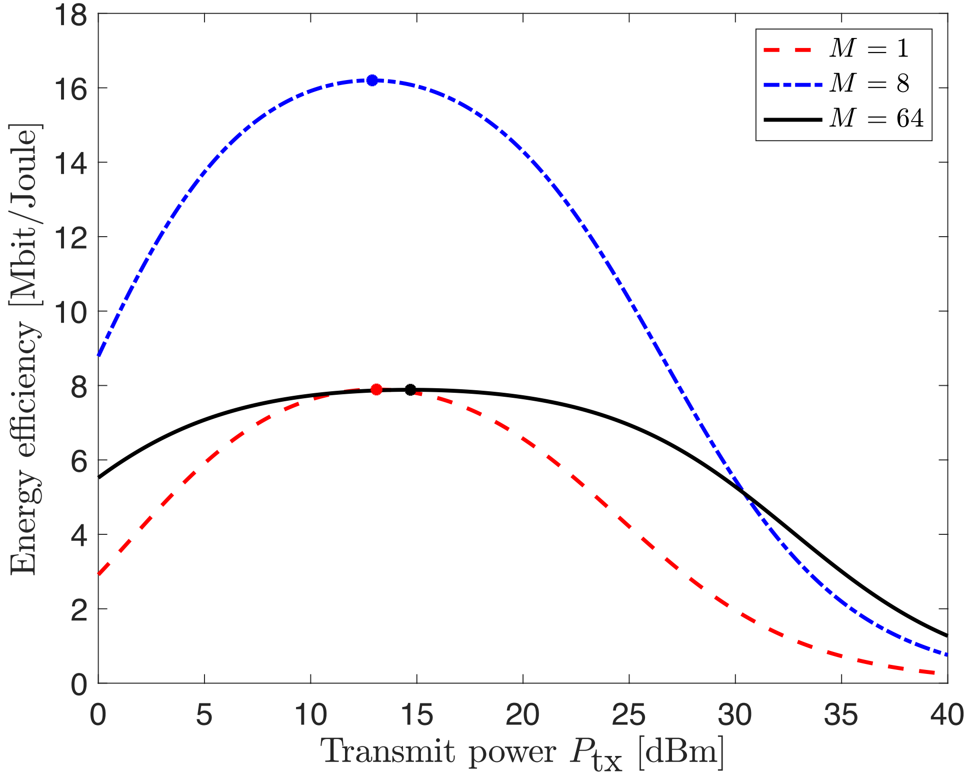
- Consider SIMO system:

SINR with MR

$$\text{Energy efficiency [bit/Joule]} = \frac{B \log_2\left(1 + \frac{M \rho_{ul} \gamma}{1 + \rho_{ul} \beta}\right)}{\frac{P_{\text{tx}}}{\mu} + P_{\text{circuit}} + M P_{\text{antenna}}}$$

Circuit power per antenna

$$\rho_{ul} = \frac{P_{tx}}{B N_0}$$



Beamforming can help reduce power

Received power proportional to MP_{tx}

$$B = 10 \text{ MHz}, \quad \mu = 0.25, \quad \frac{B N_0}{\beta} = 0 \text{ dBm}$$

$$P_{\text{circuit}} = 10 \text{ W}, \quad P_{\text{antenna}} = 0.1 \text{ W}$$

Energy Efficiency and Multiplexing

- Consider Massive MIMO system:

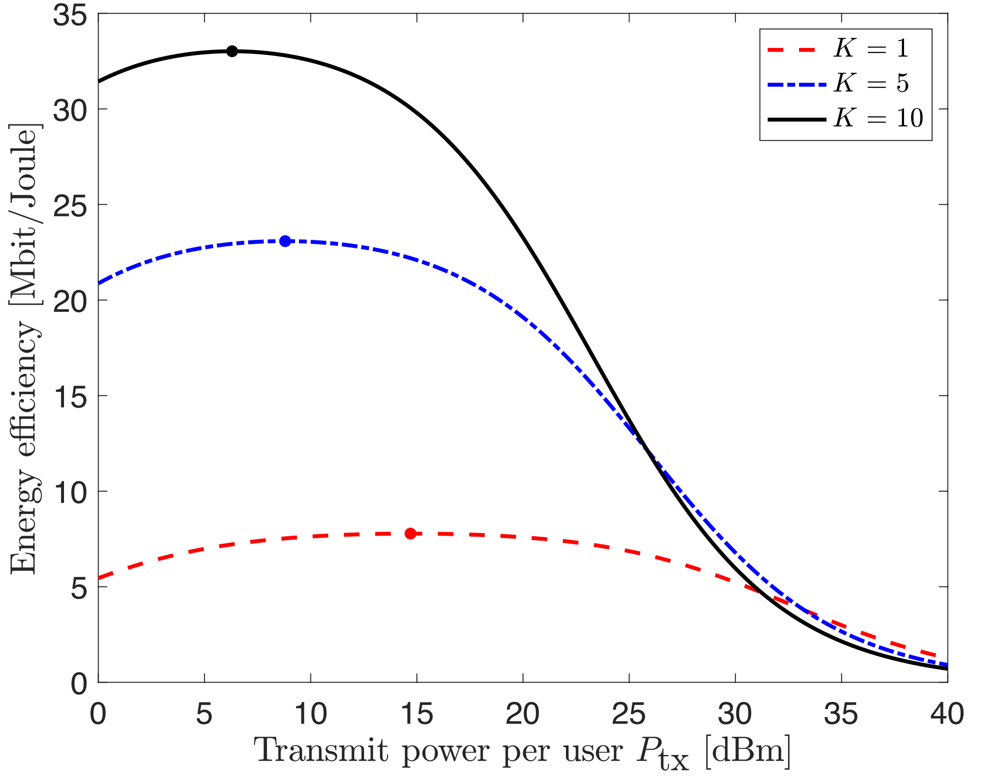
Uplink SINR with MR and K users

$$\text{Energy efficiency [bit/Joule]} = \frac{K\,B\log_2\left(1+\dfrac{M\rho_{ul}\gamma}{1+\rho_{ul}K\beta}\right)}{\dfrac{KP_{\text{tx}}}{\mu}+P_{\text{circuit}}+(M+K)P_{\text{antenna}}}$$

Total transmit power

Circuit power per antenna/users

$$\rho_{ul}=\frac{P_{tx}}{BN_0}$$



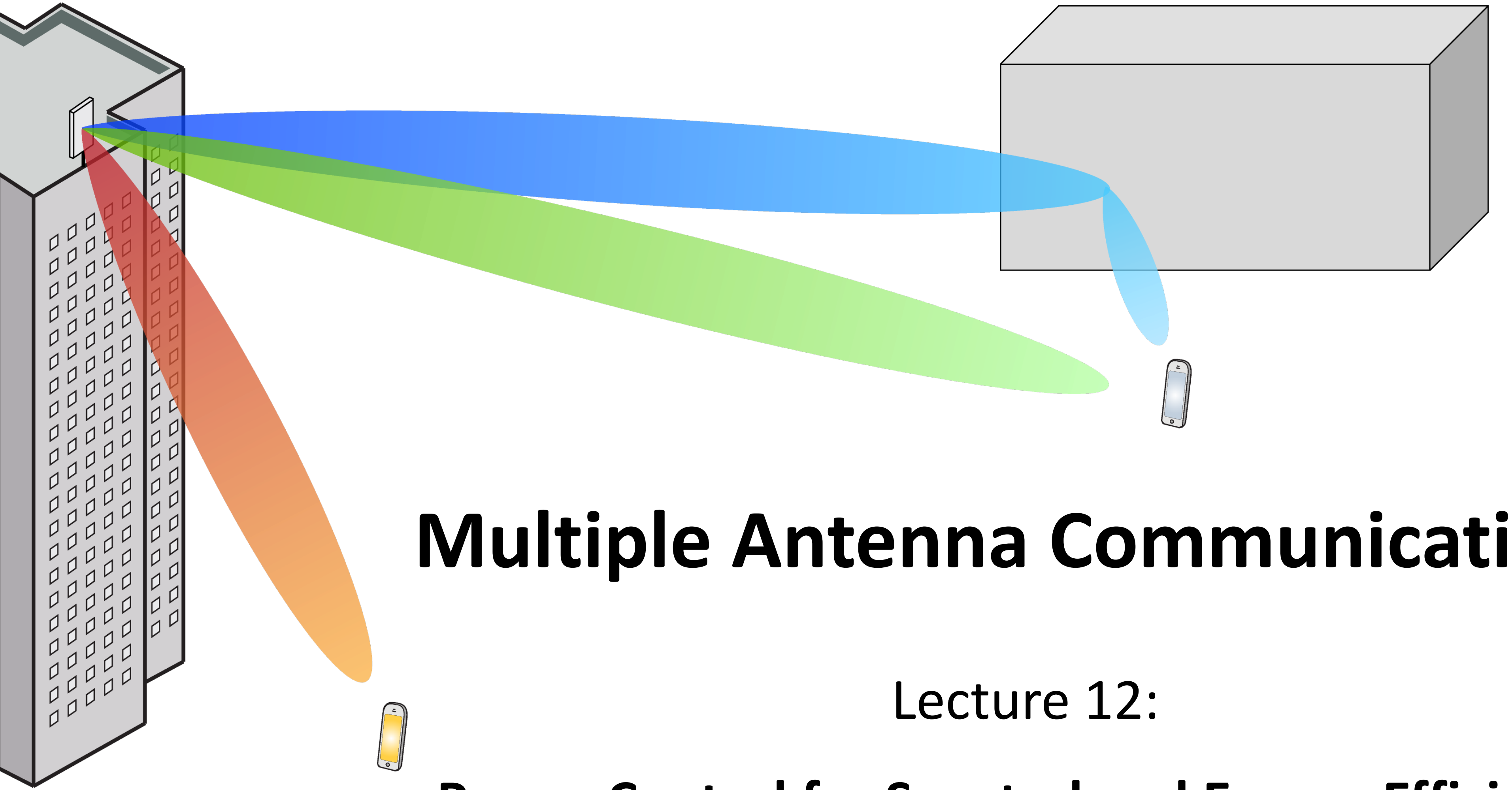
Multiplexing increases efficiency
Share power cost between users

$$B=10\text{ MHz},\quad \mu=0.25,\quad \frac{BN_0}{\beta}=0\text{ dBm}$$

$$P_{\text{circuit}}=10\text{ W},\quad P_{\text{antenna}}=0.1\text{ W},\quad M=64$$

Summary

- Power control used to increase efficiency
    - Spectral or energy efficiency
- Sum rate power control
    - Downlink: Waterfilling
    - Uplink: Convex optimization
- Energy efficiency
    - Rate divided by energy consumption
    - Accurate circuit power models needed
    - Beamforming and multiplexing useful



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