

Multiple Antenna Communications

Background Lecture:
Complex Baseband Model

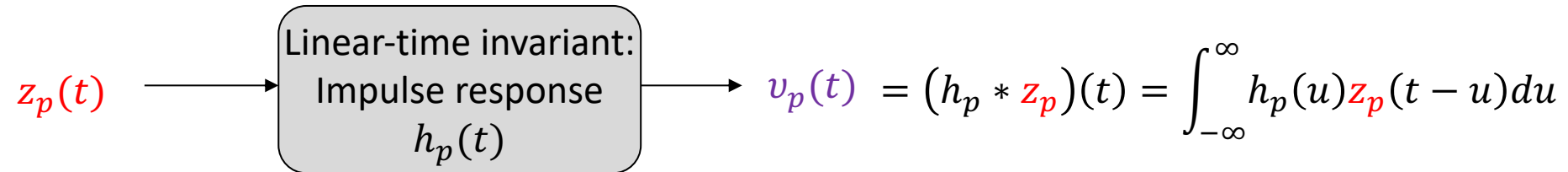
Emil Björnson

Outline

- Signals and systems
 - Fourier transform
- Complex baseband representation
 - Signals and communication channels
- Pulse amplitude modulation
 - Received signal
 - Narrowband channel

Linear time-invariant system

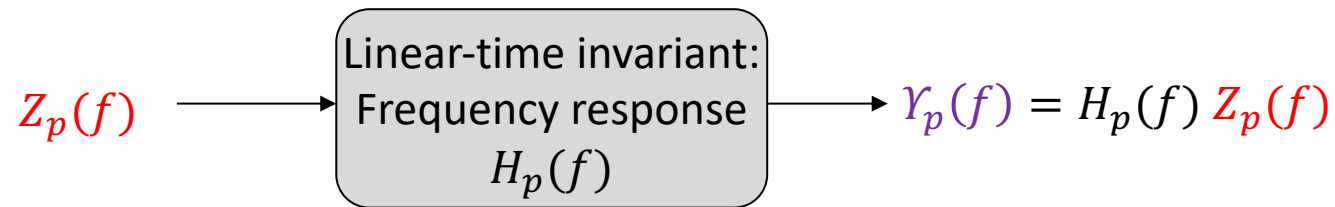
- System in time-domain:



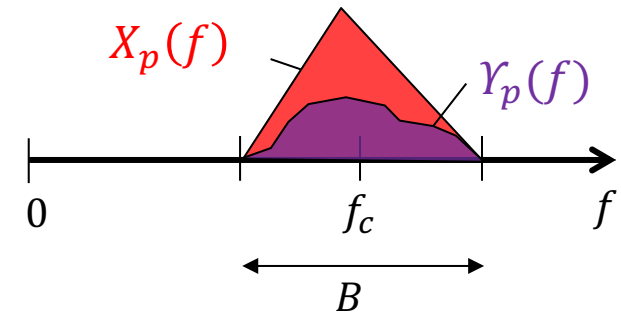
Impulse (Dirac function)

$$\delta(t): \int_{-\infty}^{\infty} x(t) \delta(t) dt = x(0)$$

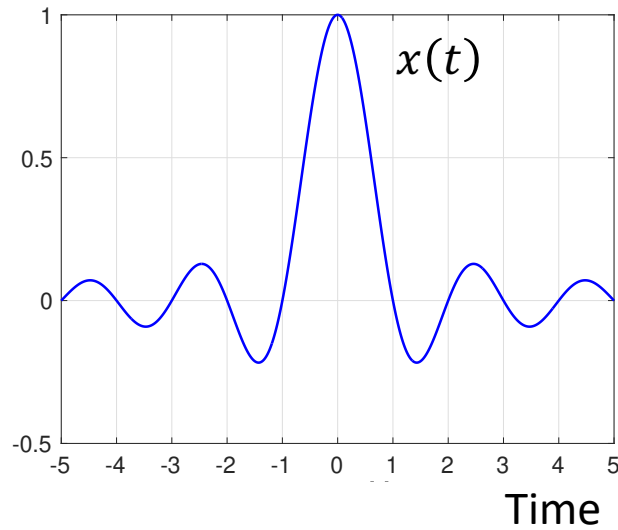
- System in frequency domain:



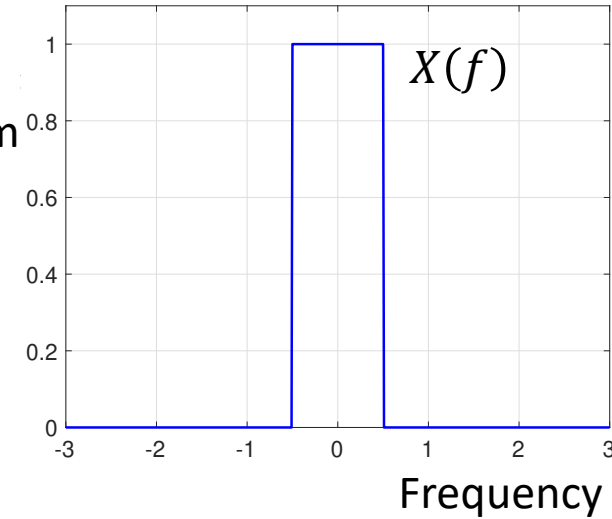
Passband signal:



Fourier transform



Fourier transform



- Definition: $X(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt$
- Exists if $\int_{-\infty}^{\infty} |x(t)| dt < \infty$

Example: Cosine

$$x(t) = \cos(2\pi f_c t)$$

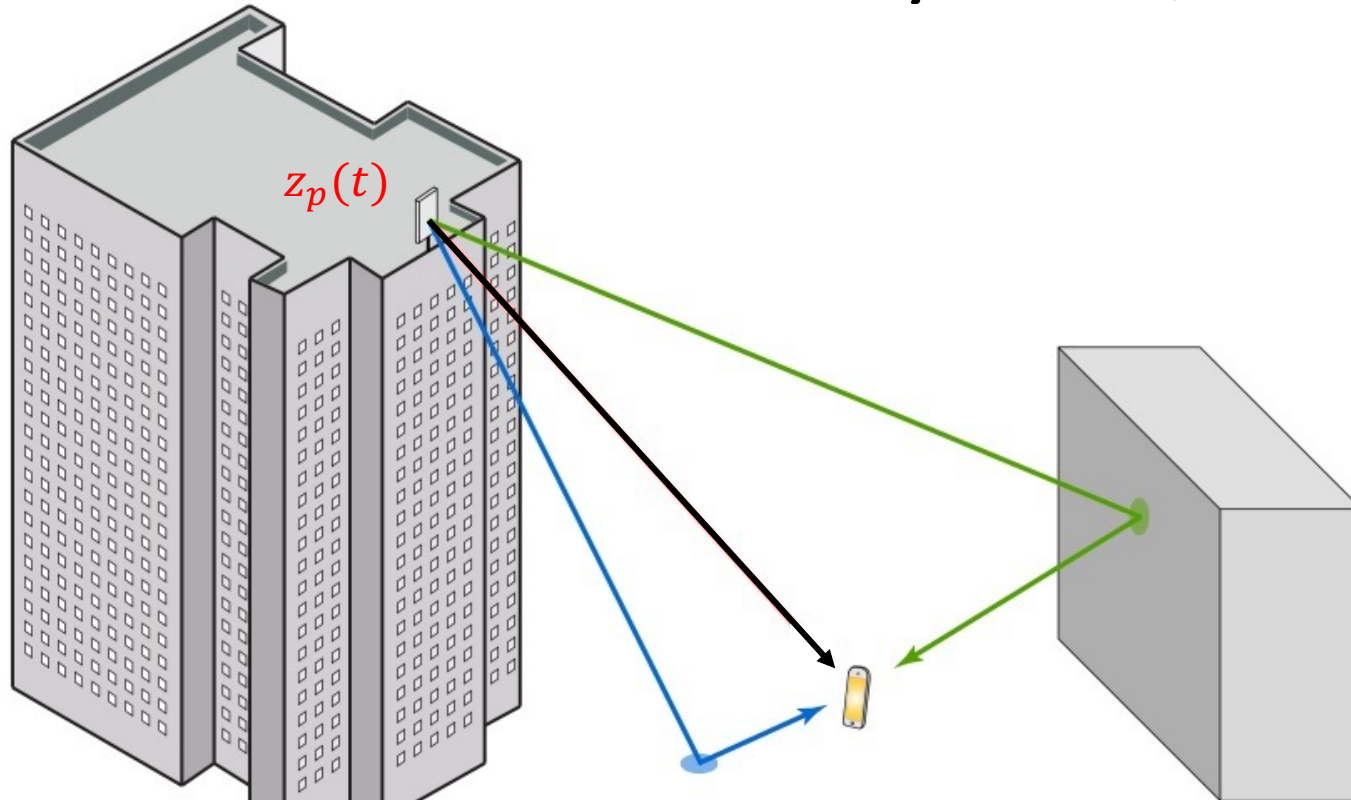
$$X(f) = \frac{1}{2} \delta(f - f_c) + \frac{1}{2} \delta(f + f_c)$$

Example: Sinc-function

$$x(t) = \text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$X(f) = 1 \text{ for } |f| \leq 1/2$$

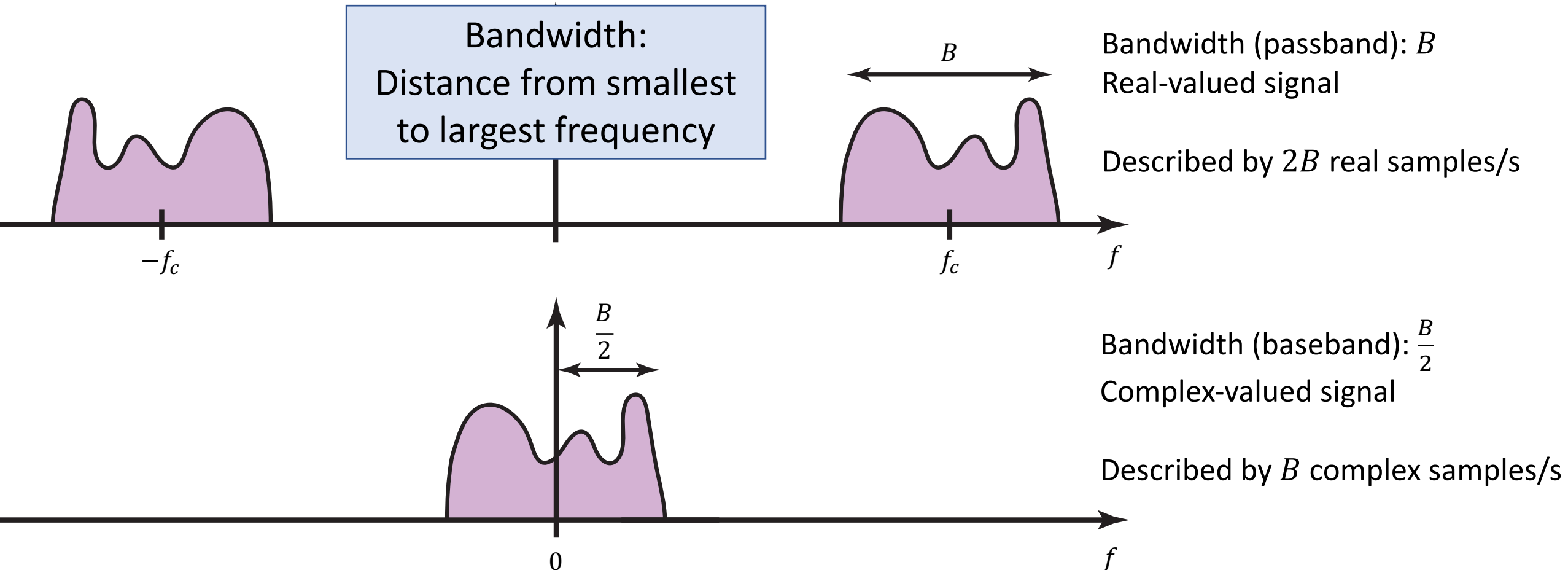
Wireless channels are systems/filters



$$v_p(t) = \alpha_1 z_p(t - \tau_1) + \alpha_2 z_p(t - \tau_2) + \alpha_3 z_p(t - \tau_3)$$

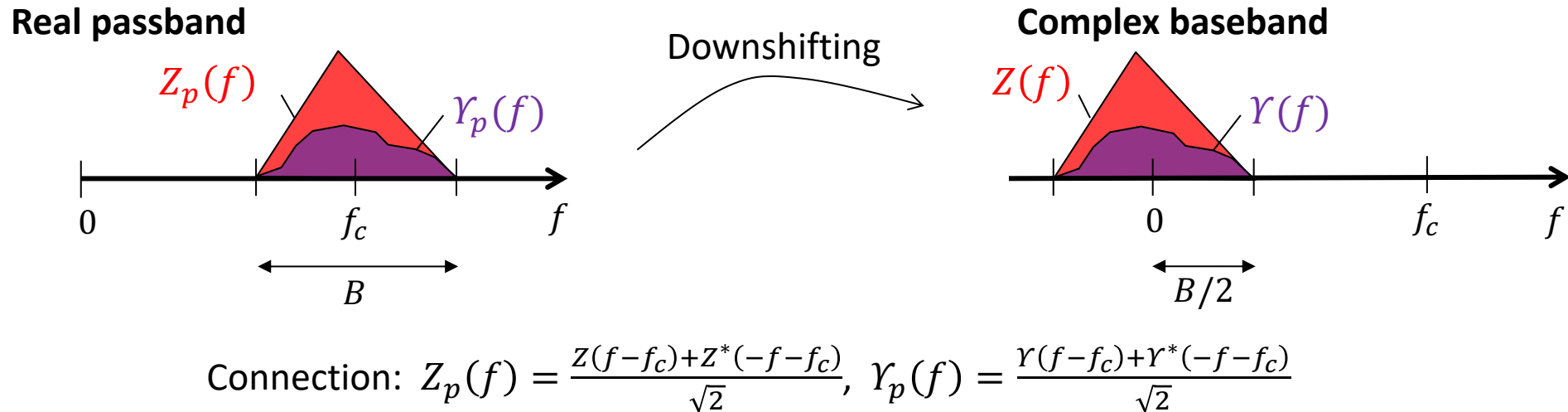
Impulse response: $h_p(t) = \alpha_1 \delta(t - \tau_1) + \alpha_2 \delta(t - \tau_2) + \alpha_3 \delta(t - \tau_3)$

Sampling theorem: Passband and baseband signals

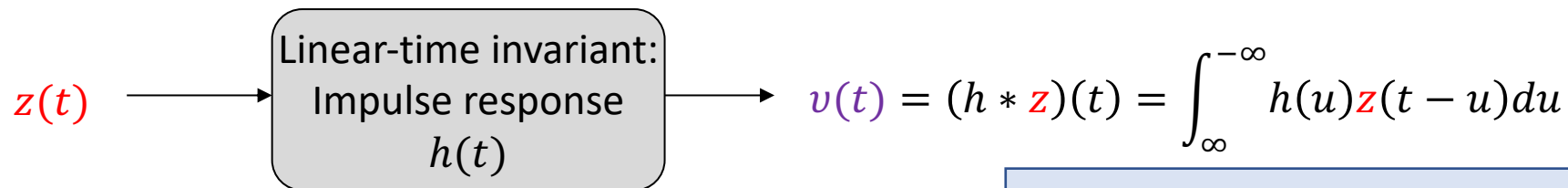


These samples are used to convey information

Communication theory is not developed for a passband



- Equivalent complex-baseband system:



Downshifted channel:

$$h(t) = h_p(t)e^{-j2\pi f_c t}, \quad H(f) = H_p(f + f_c)$$

How to transmit data?

- Transmit discrete sequence $x[k]$, $k = \text{integer}$
- Pulse amplitude modulation:

$$z(t) = \sum_k x[k]p\left(t - \frac{k}{B}\right)$$

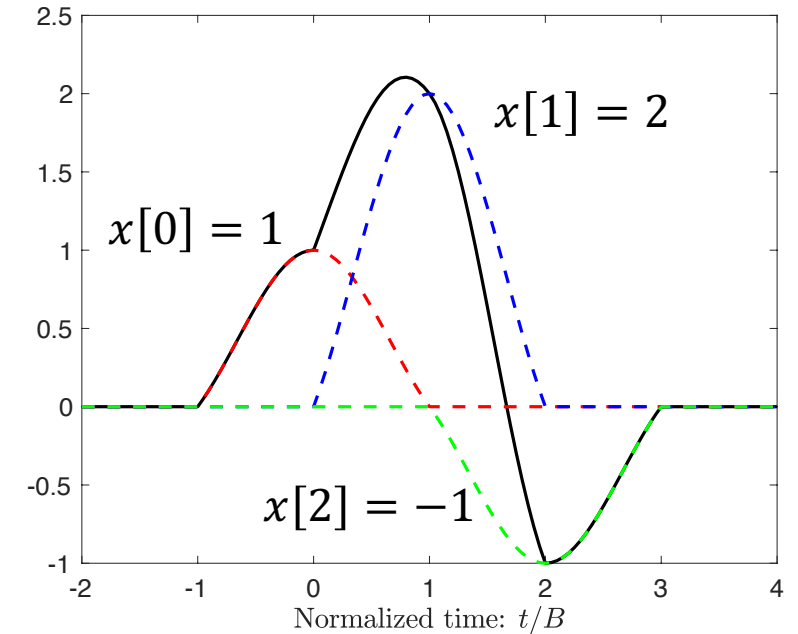
Use a pulse-form $p(t)$ satisfying the Nyquist criterion:

$$p\left(\frac{k}{B}\right) = 0 \text{ for integer } k \neq 0 \text{ and non-zero for } k = 0$$

- **Example:** $p(t) = \sqrt{B}\text{sinc}(Bt)$

Sampling of signal $z(t)$ at time $t = l/B$:

$$z\left(\frac{l}{B}\right) = \sum_k x[k]p\left(\frac{l-k}{B}\right) = x[l]$$



Reception with channel and noise

- Received signal (with **complex Gaussian noise**):

$$\mu(t) = (h * z)(t) + w(t)$$

- Lowpass filtering using $p(t) = \sqrt{B}\text{sinc}(Bt)$:

$$(p * \mu)(t) = \sum_k x[k] (p * h * p) \left(t - \frac{k}{B} \right) + (p * w)(t)$$

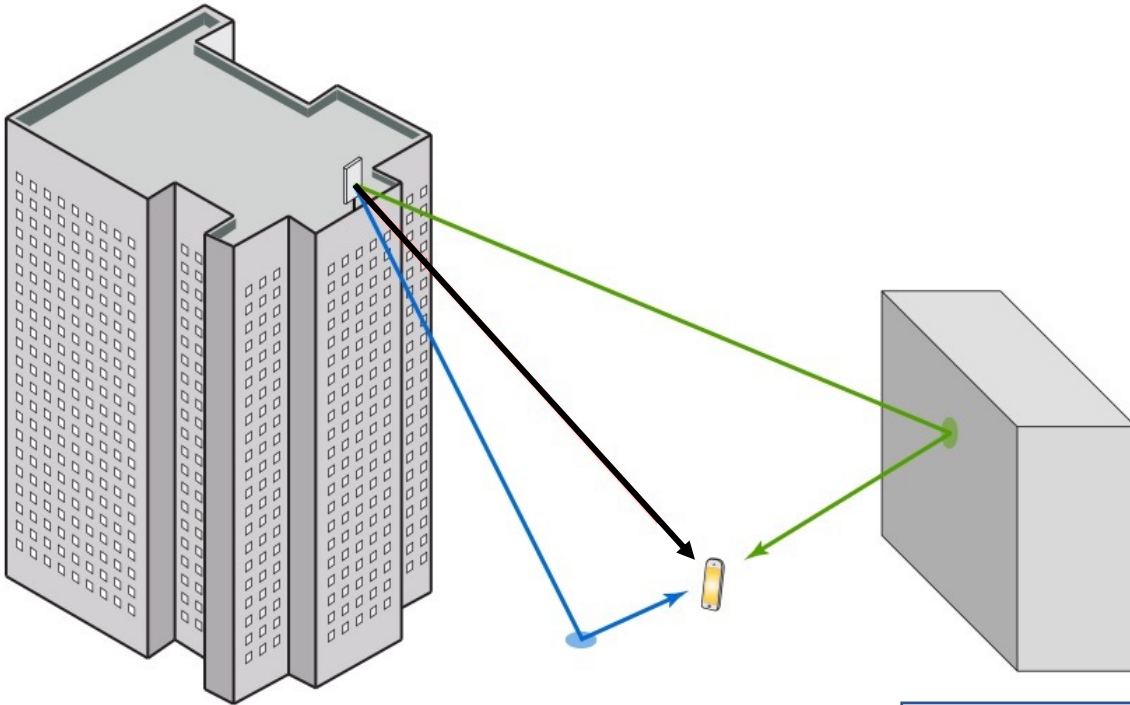
- Sample at time $t = l/B$:

$$y[l] = (p * \mu)(t) \Big|_{t=l/B} = \sum_k x[k] \underbrace{(p * h * p) \left(\frac{l-k}{B} \right)}_{\text{Effective pulse function}} + \underbrace{(p * w) \left(\frac{l}{B} \right)}_{n[l]}$$

Channel with memory?

$n[l]$
Complex Gaussian
noise $CN(0, N_0)$

Revisiting channel model



Original channel:

$$h_p(t) = \sum_{i=1}^L \alpha_i \delta(t + \eta - \tau_i)$$

Annotations:

- α_i : Amplitude loss
- η : Synchronization
- τ_i : Propagation delay

Equivalent in complex baseband:

$$h(t) = \sum_{i=1}^L \alpha_i e^{-j2\pi f_c t} \delta(t + \eta - \tau_i)$$

$$(p * h * p) \left(\frac{l-k}{B} \right) = \sum_{i=1}^L \alpha_i e^{-j2\pi f_c (\tau_i - \eta)} \text{sinc}((l-k) + B(\eta - \tau_i))$$

Narrowband assumption

- Small variations in delays (set $\eta \approx \tau_i$):

$$\text{sinc}((l - k) + B(\eta - \tau_i)) \approx \text{sinc}(l - k) = \begin{cases} 1, & l = k \\ 0, & l \neq k \end{cases}$$

- Symbol-sampled system model:

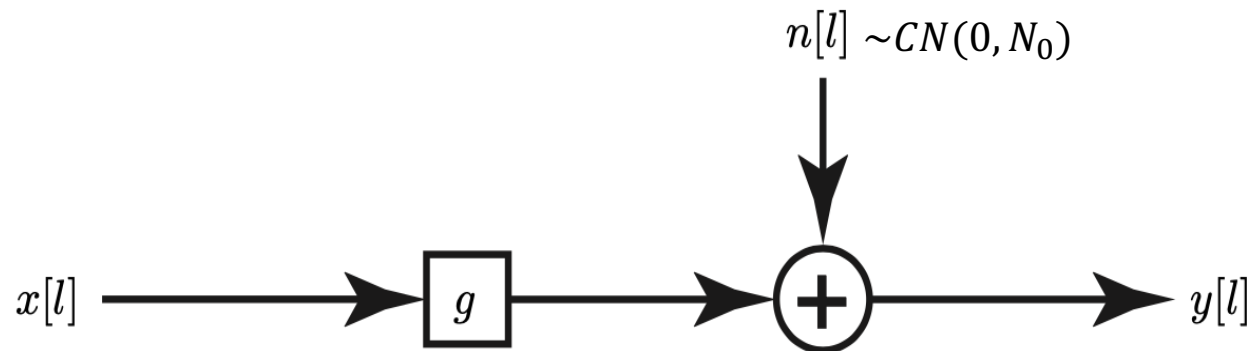
$$y[l] = \sum_k x[k] \underbrace{\sum_{i=1}^L \alpha_i e^{-j2\pi f_c(\tau_i - \eta)} \text{sinc}(l - k)}_g + n[l]$$

Discrete memoryless channel:

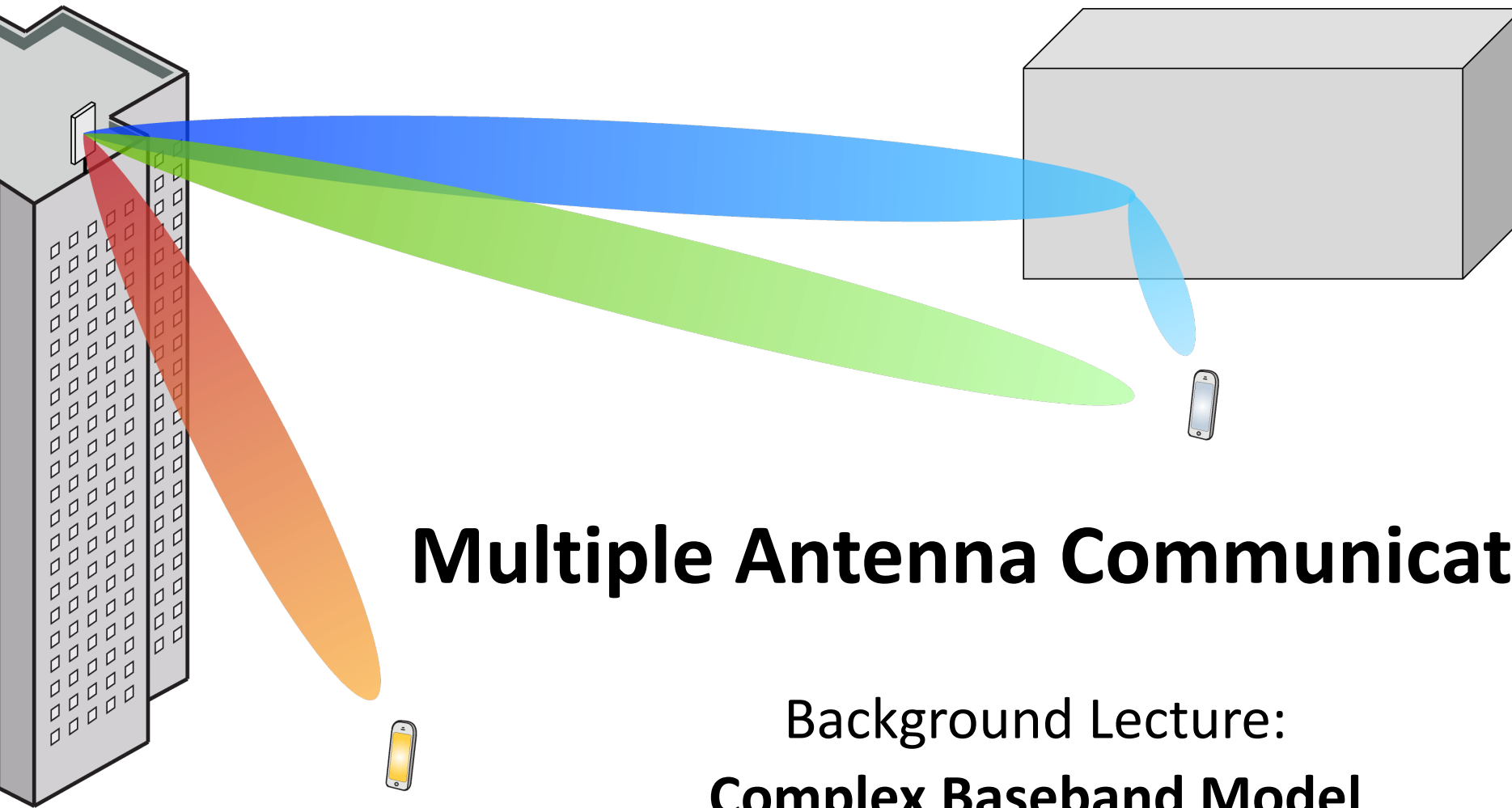
$$y[l] = g \cdot x[l] + n[l]$$

Summary

- Transmission in continuous time at frequency f_c
- Equivalent discrete description:



- B symbols per second (B = bandwidth)
- Memoryless channel: $y = g \cdot x + n$



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