IEEE Swedish Communication Technologies Workshop
19-21 October 2011, Stockholm

ROYAL INSTITUTE OF TECHNOLOGY

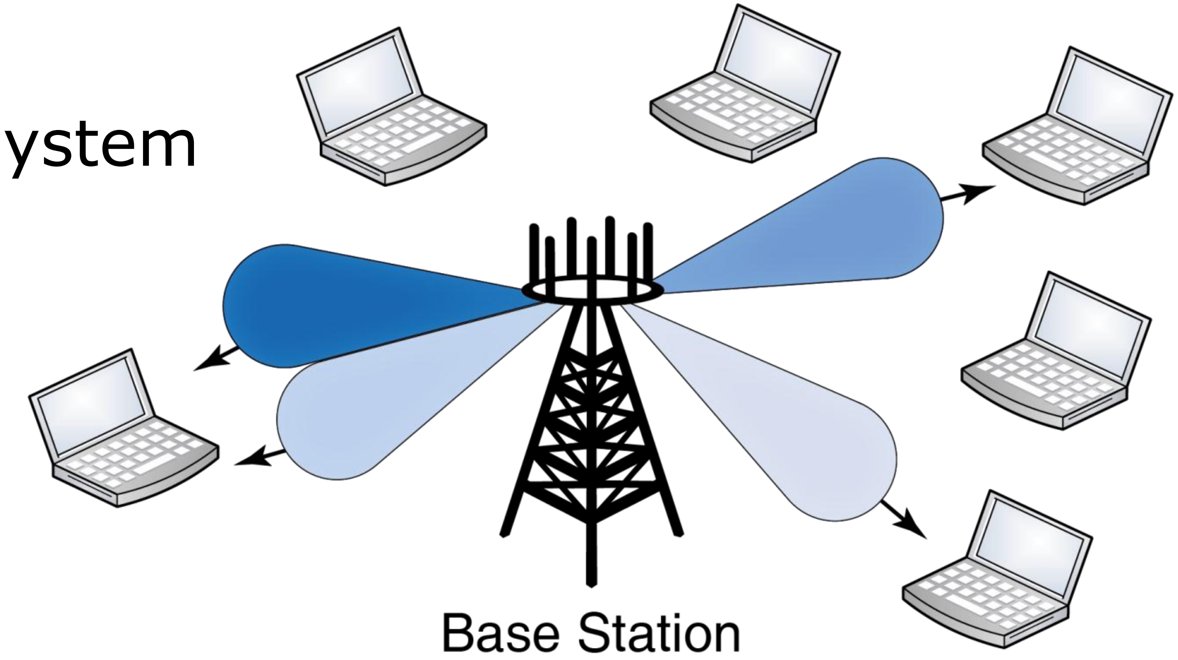
Receive Combining vs. Multistream Multiplexing in Multiuser MIMO Systems

Emil Björnson, Mats Bengtsson, Björn Ottersten

KTH Royal Institute of Technology
ACCESS Linnaeus Center
Signal Processing Lab
Stockholm, Sweden

Introduction

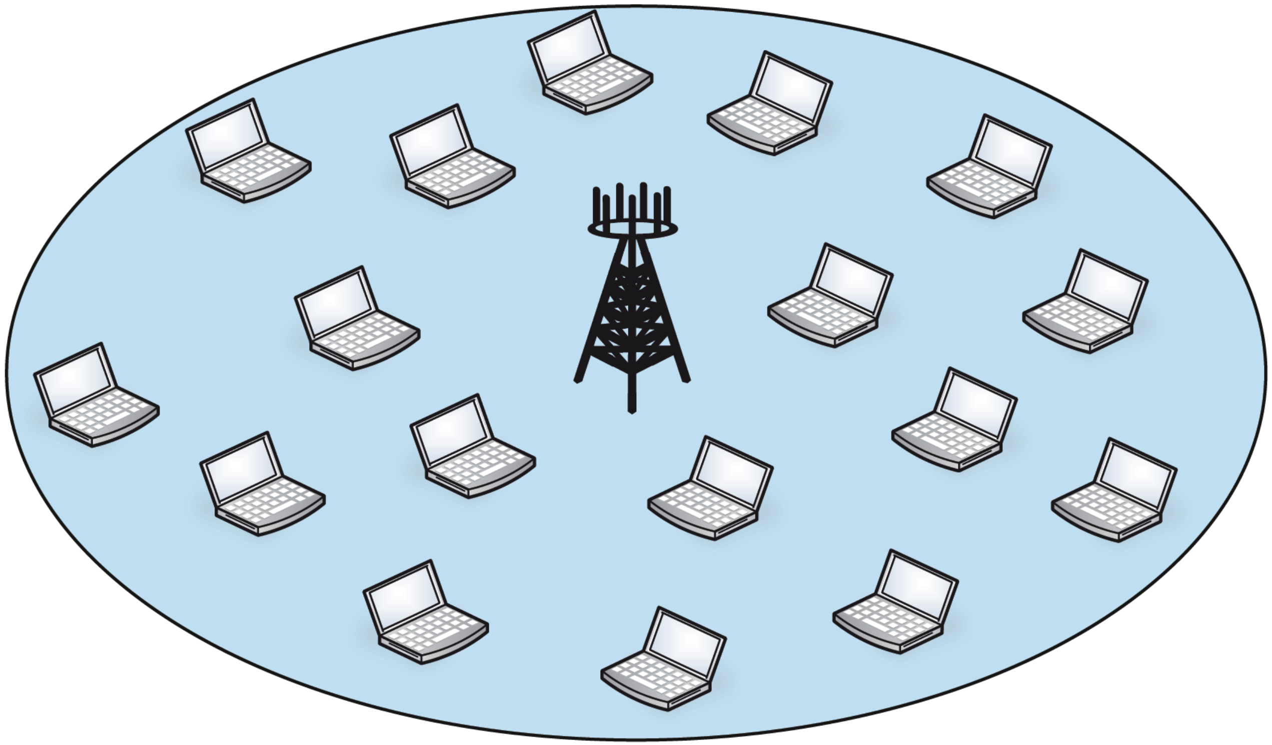
- Multiuser MIMO System
  - One Transmitting Base Station
  - Many Users
  - Multiple Antennas on all Devices



- Space Division Multiple Access (SDMA)
  - Downlink Transmission
  - Serve Multiple Users – Simultaneously
  - Beamforming: Spatially Directed Signals
  - Combining: Directed Reception (one or many streams)
  - Co-User Interference Control – Requires Channel Knowledge

Introduction (2)

- Basic Parameters
  - N Antennas at Base Station
  - K Users and M Antennas/User ($K \geq N \geq M$)
  - P = Total Transmit Power (Large)



ROYAL INSTITUTE OF TECHNOLOGY

Introduction (2)

- Basic Parameters
  - N Antennas at Base Station
  - K Users and M Antennas/User (K ≥ N ≥ M)
  - P = Total Transmit Power (Large)

- Number of Data Streams
  - Achievable Sum Rate Behaves as (large P)

$$\min(N, MK)\log_2(P) + O(1) = N\log_2(P) + O(1)$$ [bits/channel use]

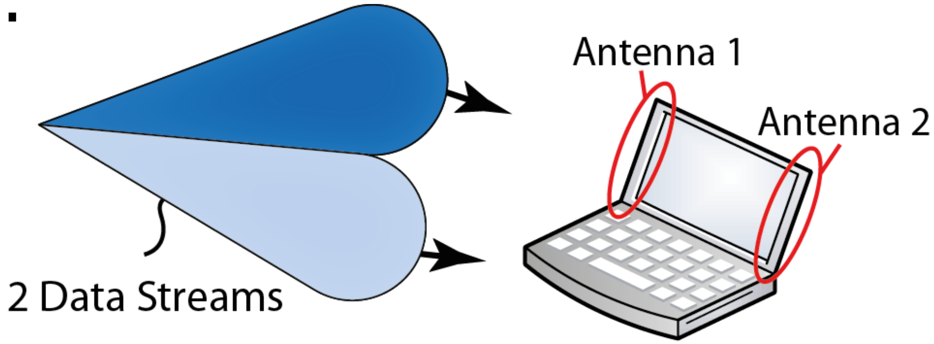
  - Maximal Multiplexing Gain: N
  - Depends on Transmit Antennas – Not on Receive Antennas
  - Achievable with N Data Streams and Linear Precoding

- Question: How to Allocate N Streams among Users?

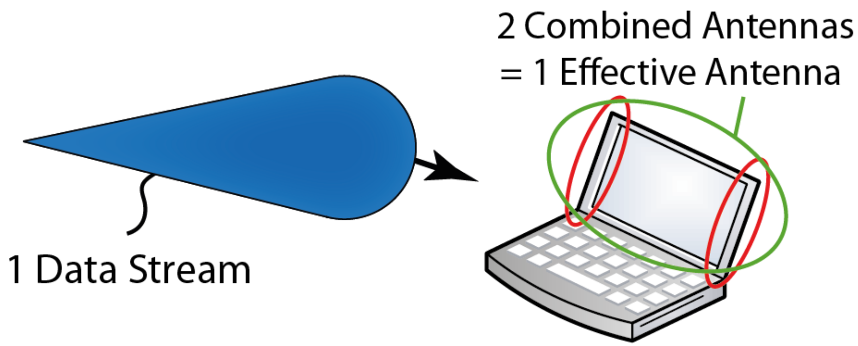
Introduction (3)

- *M* Antennas per User: How to Use them?

- Multistream Multiplexing:
  - Select up to *N*/*M* Users
  - *M* Data Streams per User
  - Manage Inter-Stream Interference at User-Side
  - Requires Rich Scattering



- Receive Combining:
  - Select up to *N* Users
  - One Data Stream per User
  - Form one Effective Antenna
  - Protect Against Bad Channels
  - Enables Interference Rejection



System Model and Linear Precoding

- Quasi-Static Flat-fading System:

$$\underbrace{\mathbf{y}_k}_{\substack{\text{Received Signal}\\\text{At User } k\\ M \times 1}} = \underbrace{\boxed{\mathbf{H}_k}}_{\substack{\text{Channel Matrix}\\ M \times N}} \sum_{l=1}^{K} \underbrace{\mathbf{V}_l}_{\substack{\text{Precoding}\\ N \times D}} \overbrace{\mathbf{d}_l}^{\substack{D \text{ streams/user}\\ D \times 1}} + \underbrace{\mathbf{n}_k}_{\substack{\text{White Gaussian Noise}\\ M \times 1}}$$

- Channel State Information (CSI)
  - Precoding Based on $\mathbf{H}_k$
  - Kronecker Model: $$\mathbf{H}_k = \underbrace{\mathbf{R}_{R,k}^{1/2}}_{\substack{\text{Receive-side}\\\text{correlation}}} \overbrace{\widetilde{\mathbf{H}}_k^{\text{i.i.d.}}}^{\mathcal{CN}(0,1)\text{-elements}} \underbrace{\mathbf{R}_{T,k}^{1/2}}_{\substack{\text{Transmit-side}\\\text{correlation}}}$$
  - Spatial Antenna Correlation:
    - Direction and quality correlated
    - Affects: Spread in singular values
    - High corr: Ill-conditioned matrix

Linear Precoding

Sum Rate Optimization:

Select Precoding

Satisfy Power Constraint

$$\underset{\{\mathbf{V}_k\}\forall k}{\text{maximize}} \quad \sum_{k=1}^{K} \log_2 \frac{\det\left(\mathbf{I} + \sum_{l=1}^{K} \mathbf{H}_k \mathbf{V}_l \mathbf{V}_l^H \mathbf{H}_k^H\right)}{\det\left(\mathbf{I} + \sum_{l \neq k} \mathbf{H}_k \mathbf{V}_l \mathbf{V}_l^H \mathbf{H}_k^H\right)},$$

$$\text{subject to} \quad \sum_{k=1}^{K} \mathrm{tr}\{\mathbf{V}_k \mathbf{V}_k^H\} \leq P.$$

- Block-Diagonalization (BD)
  - Select up to N/M Users: M Streams/User
  - Precoding to Remove Co-User Interference: $\mathbf{H}_k \mathbf{V}_l^{\text{BD}} = \mathbf{0} \quad l \neq k$

$$\begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 & \mathbf{V}_2 \end{bmatrix} = \begin{matrix} \boxed{\mathbf{H}_1 \mathbf{V}_1} & \\ & \boxed{\mathbf{H}_2 \mathbf{V}_2} \end{matrix}$$

  - Exact BD Requires Perfect CSI

Linear Precoding (2)

- Zero-Forcing with Combining (ZFC)
  - Select up to *N* Users, 1 Streams/User
  - User *k* Selects Unit-Norm Vector $\mathbf{r}_k$
    - Linear Combination of Antennas: Effective Channel $\mathbf{r}_k^H \mathbf{H}_k$
    - Preliminary choice: Good channel quality
    - Final choice: Balance channel quality and interference rejection
  - Zero-Forcing: Block-Diagonalization on Effective Channels

$$\begin{bmatrix} \mathbf{r}_1^H \mathbf{H}_1 \\ \mathbf{r}_2^H \mathbf{H}_2 \\ \mathbf{r}_3^H \mathbf{H}_3 \\ \mathbf{r}_4^H \mathbf{H}_4 \end{bmatrix} [\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{v}_4] = \begin{matrix} \boxed{\mathbf{r}_1^H \mathbf{H}_1 \mathbf{v}_1} & & & \\ & \boxed{\mathbf{r}_2^H \mathbf{H}_2 \mathbf{v}_2} & & \\ & & \boxed{\mathbf{r}_3^H \mathbf{H}_3 \mathbf{v}_3} & \\ & & & \boxed{\mathbf{r}_4^H \mathbf{H}_4 \mathbf{v}_4} \end{matrix}$$

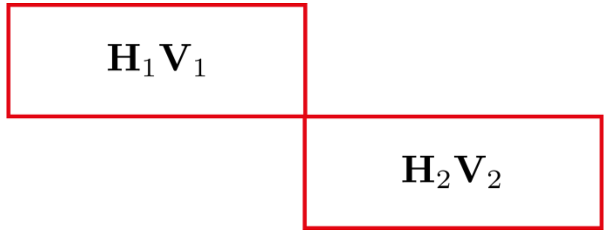
  - Exact ZFC Requires Perfect CSI (of effective channels)

Important: Not the only zero-forcing approach for M>1

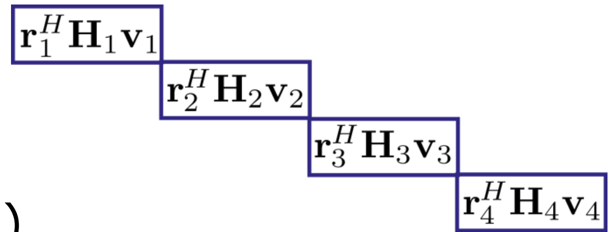
Problem Formulation

- Receive Combining vs. Multistream Multiplexing
  - Which Strategy Provides Highest Sum Rate?
    1. Block-Diagonalization (Multistream Multiplexing)
    2. Zero-Forcing with Combining (Receive Combining)

- What Can We Expect?
  - Block-Diagonalization (BD)
    - Less restrictive interference cancelation
    - Sensitive to ill-conditioned channels



  - Zero-Forcing with Combining (ZFC)
    - Quite robust to ill-conditioned channels
    - Enables interference rejection
      (update combining after precoding design)



Prior Work

- My intuition: Receive Combining is beneficial
  - Is My Intuition Wrong?
  - BD Much Better than ZFC in:

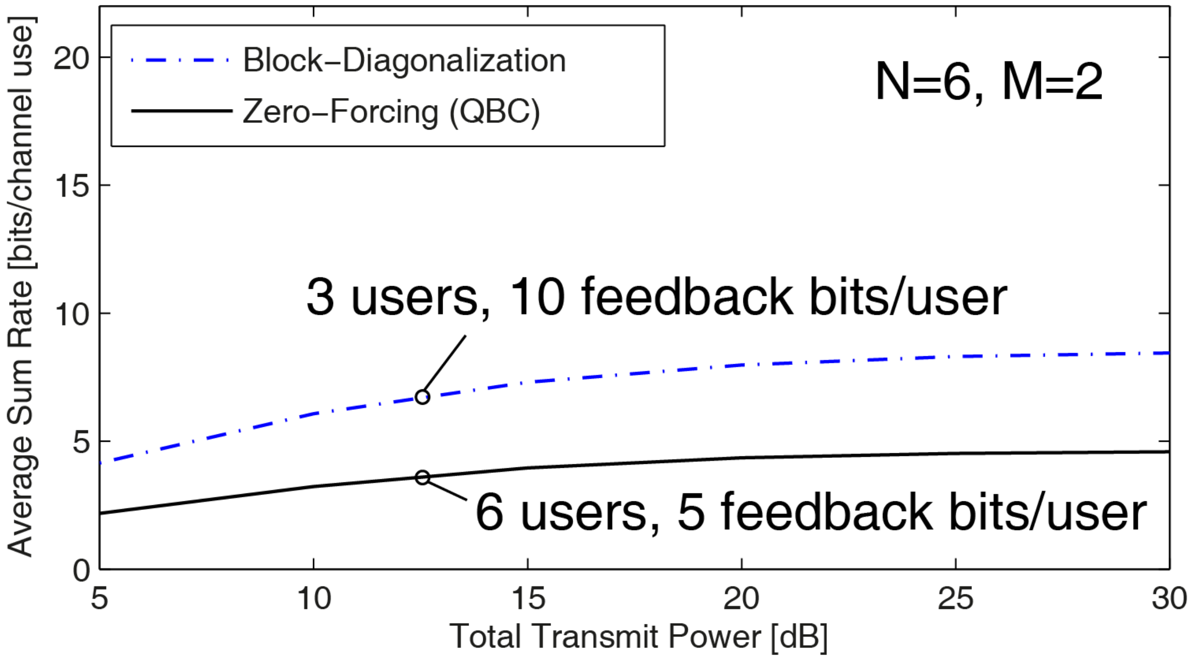
N. Ravindran and N. Jindal, "*Limited feedback-based block diagonalization for the MIMO broadcast channel*," IEEE J. Sel. Areas Commun., vol. 26, no. 8, pp. 1473–1482, 2008.

Homogenous users
i.i.d. Rayleigh fading

Random User Selection

Perfect CSI at users

Quantized CSI feedback
(Random Quantization)
(30 bits in total)



Analysis: Perfect CSI

Theorem 1 (*Spatial Antenna Correlation*): Assume

- Random User Selection (N/M users with BD, N with ZFC)
- Correlated Receive Antennas: $\mathbf{H}_k = \mathbf{R}_{R,k}^{1/2} \widetilde{\mathbf{H}}_k^{\text{i.i.d.}}$
- Same Eigenvalues: $\lambda_M \geq \ldots \geq \lambda_1 > 0$ (not necessary)

Average Asymptotic Difference (in Sum Rate):

$$\bar{\beta} = \lim_{P \to \infty} \mathbb{E}\{ f^{\text{sum rate, BD}}(P) - f^{\text{sum rate, ZFC}}(P) \}$$

$$\leq \underbrace{N \frac{\log_2(e)}{M} \sum_{i=1}^{M-1} \frac{M-i}{i}}_{\textit{Positive Constant}} + \underbrace{N \log_2 \left(\frac{ \left(\prod_{m=1}^{M} \lambda_m \right)^{1/M} }{\lambda_M} \right)}_{\textit{Increasingly Negative with Eigenvalue Spread}} .$$

- **Conclusion:** BD good at low correlation; ZFC good at high correlation — Confirmed with simulations

Analysis: Perfect CSI (2)

- User Selection
  - Find Strong Users – Easier with ZFC (largest singular value)
  - Find Near-Orthogonal Users – Could be Fewer that *N* streams
- Theorem 2:
  - Easier to Find Near-Orthogonal Users with ZFC
  - Natural: Only Consider one Direction per User

Impact of User Selection
(Greedy Selection Algorithm)

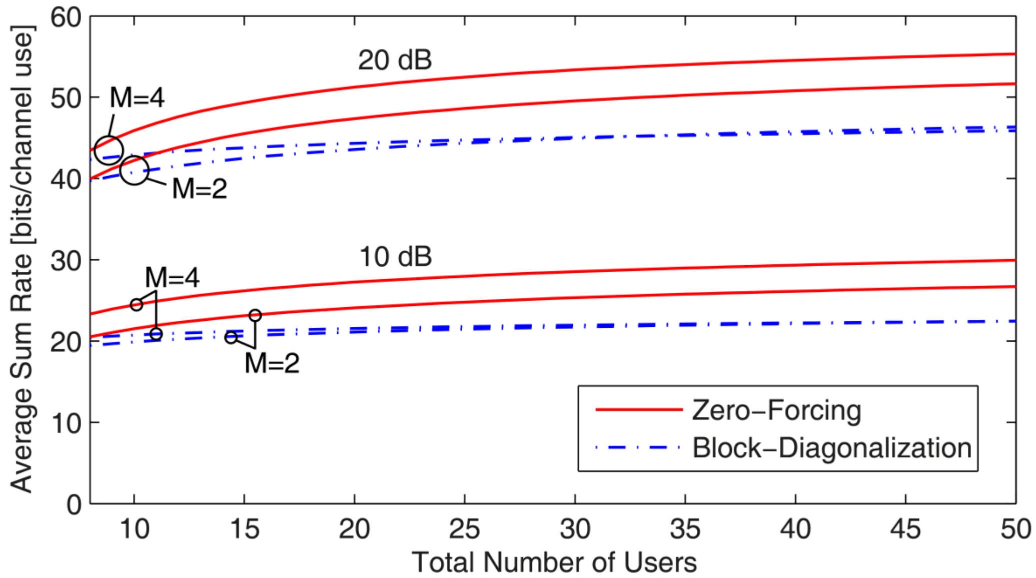
N=8 transmit antennas

M=2 or *M*=4 receive antennas

P = 10 or 20 dB transmit power

Uncorrelated channels

Observation: ZFC Superior to BD also at Uncorrelated Channels



Extension: Imperfect CSI

- Perfect CSI Cannot be Obtained
  - Estimation and Quantization Errors

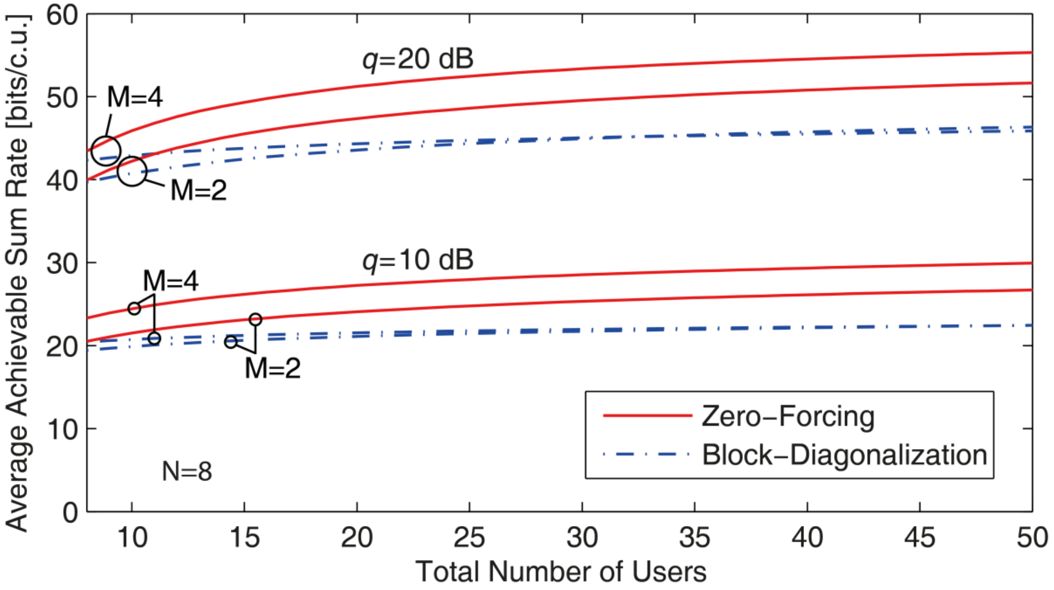
- How does Imperfect CSI affect our Problem?
  - Limited Uplink Resources for CSI Acquisition:
    - *Only obtain CSI for a fixed number of channel directions*
  - CSI from *M* times more users with ZFC than BD
    - *Further amplifies the larger user selection benefit for ZFC*
  - CSI Accuracy not an issue:
    - Estimation and Quantization Accuracy improves with Power
    - Bounded Loss for BD and ZF – No Loss in Multiplexing Gain

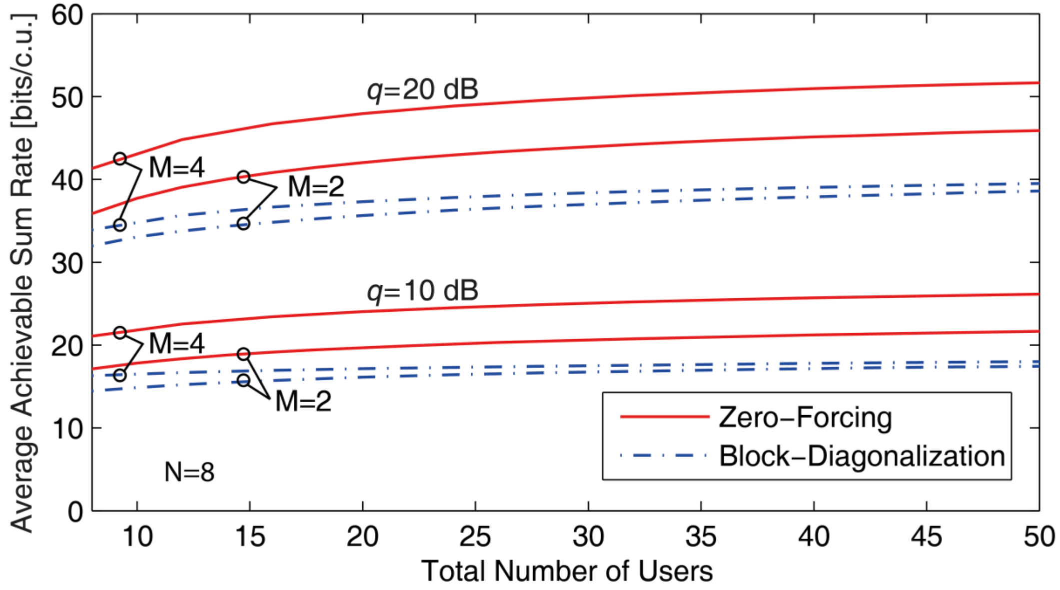
Shown analytically in the paper

Extension: Imperfect CSI (3)

Impact of User Selection
(Greedy Selection Algorithm)

Imperfect CSI Estimation: Same Simulation as with Perfect CSI
N=8 transmit antennas, M=2 or M=4 receive antennas
P = 10 or 20 dB transmit/training power

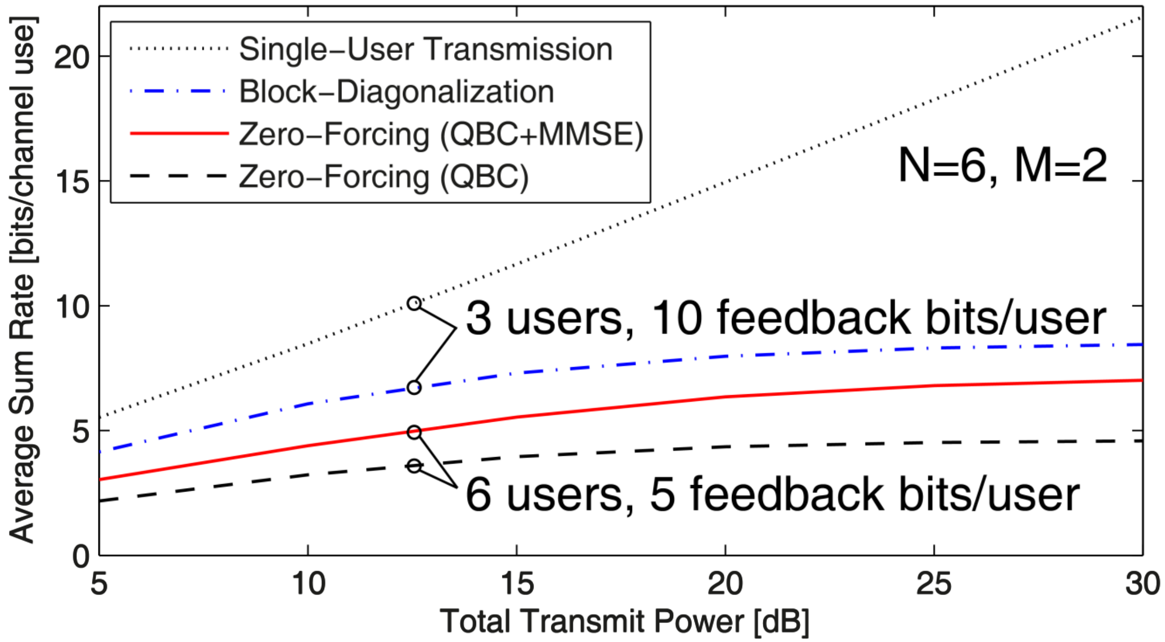




Observation: Performance Loss of Around 10%
ZFC Still Superior to BD - Even Larger Difference!

Back to Prior Work

- Our Results Shows Advantage of ZFC
  - Inconsistent with Prior Work? (Ravindran/Jindal, 2008)



  - Fixed Number of bits: Should Increase with Power
  - No User Selection: Select Fewer Users if poor CSI Accuracy
  - Did not Include: Single-user Transmission
  - Did not Update Combining in ZFC (after precoding)

Conclusion

1) Single-user transmission best in this scenario

2) To few bits to do SDMA – Both BD and ZFC are bad

Summary

- Multiantenna Downlink SDMA Transmission
  - Transmit antennas: Decides the Multiplexing Gain
  - Receive antennas: Combining or Multistream Multiplexing?
- How to Allocate *N* Data Streams?
  - Select Many Users: One Stream/User – Represented by ZFC
  - Select Few Users: Many Streams/User – Represented by BD
- Conclusions
  - With Sufficiently Good Channel Knowledge:

    Receive Combining Should be Used

    (Higher benefit from user selection, resilience to bad channels)
  - With Poor Channel Knowledge:

    Single-User Transmission is Best
- Implication: Simple User Hardware is Sufficient

Thank You for Listening!

Many More Mathematical Details in the Paper!

Questions?

Papers and Presentations Available:

http://www.ee.kth.se/~emilbjo