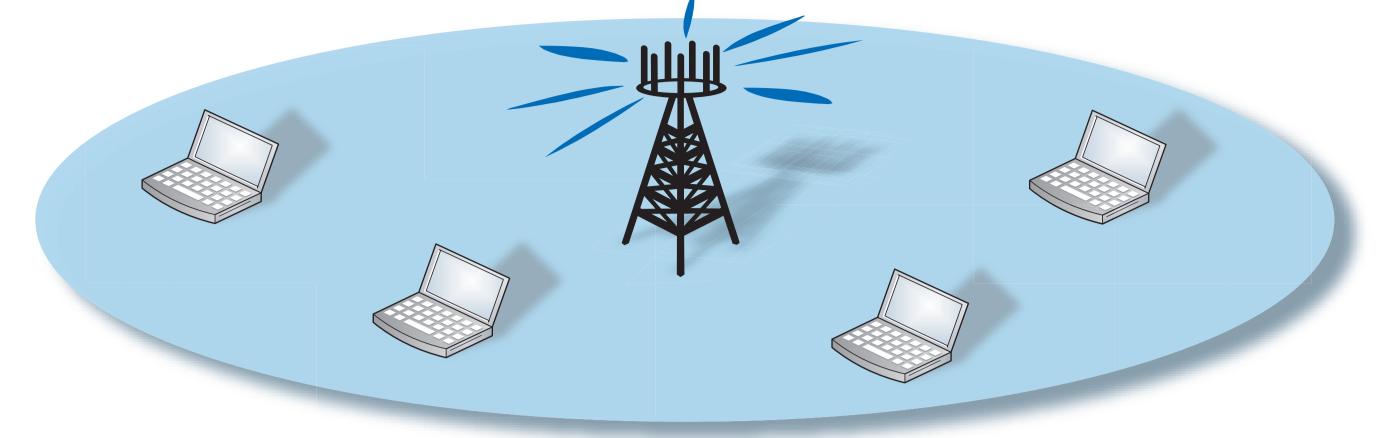


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# EXPLOITING LONG-TERM STATISTICS IN SPATIALLY CORRELATED MULTI-USER MIMO SYSTEMS WITH QUANTIZED CHANNEL NORM FEEDBACK

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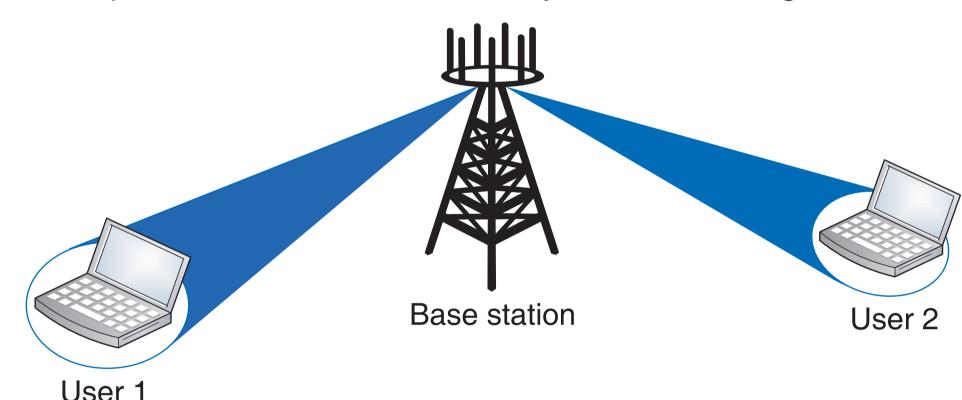


# **Spatial Correlation**

Consider the multi-antenna system

- ullet Elevated base station,  $n_T$  antennas.
- Multiple users,  $n_R$  antennas.

Some spatial directions are statistically favorable for a given user:



### **Excellent for simultaneous transmission to several users:**

- Spatial division multiple access (SDMA)
- Linear precoding and receive beamforming

# System Model

Urban environment with elevated base station:

- Spatially correlated transmitter.
- Independent receive antennas.

### **Channel model:**

Rayleigh fading multi-antenna channel to user k

$$\mathbf{H}_k = [\mathbf{h}_{k,1}, \dots, \mathbf{h}_{k,n_R}]^H \in \mathbb{C}^{n_R imes n_T}$$
 ,

with independent rows  $\mathbf{h}_{k,i} \in \mathcal{CN}(\mathbf{0},\mathbf{R}_k)$ .

Received signal when transmitting  $s_k$  to user k:

$$\mathbf{y}_k(t) = \mathbf{v}_k^H \mathbf{H}_k \bigg( \underbrace{\sqrt{p_k} \mathbf{w}_k s_k(t)}_{\text{signal}} + \sum_{j \neq k} \underbrace{\sqrt{p_j} \mathbf{w}_j s_j(t)}_{\text{interference}} \bigg) + \underbrace{\mathbf{n}_k(t)}_{\text{noise}, \sigma_k^2}$$

ullet Transmit beamformer  $\mathbf{w}_k \in \mathbb{C}^{n_T}$ , Receive beamformer  $\mathbf{v}_k \in \mathbb{C}^{n_R}$ .

### **Available channel state information:**

- Transmitter knows the statistics.
- ullet Receiver k knows both the channel realization  ${f H}_k$  and statistics.

# Performance and Beamforming

Performance measure: Total data rate of all users.
User rate increases with the signal-to-interference-and-noise ratio:

$$SINR_k = \frac{p_k \|\mathbf{v}_k^H \mathbf{H}_k \mathbf{w}_k\|^2}{\sum_{i \neq k} p_i \|\mathbf{v}_k^H \mathbf{H}_k \mathbf{w}_i\|^2 + \sigma_k^2}.$$

### Tricky performance optimization problem:

- Transmit beamformer  $\mathbf{w}_k$  affects all users (fairness).
- Transmitter only knows the channel statistics.
- Receiver is unaware of the other users' channels.

# Subspace Cancellation

In order to make robust SINR estimation possible:

ullet The effect of the receive beamformer  ${f v}_k$  predictable at transmitter.

Subspace partitioning of the covariance matrix  $\mathbf{R}_k$ :

$$\mathbf{R}_k = \left[\mathbf{u}_k^{(D)} \, \mathbf{U}_k^{(I)} \, \mathbf{U}_k^{(0)} \right] \left[ egin{matrix} \lambda_1 & & & \ & \ddots & \ & & \lambda_{n_T} \end{matrix} 
ight] \left[ \mathbf{u}_k^{(D)} \, \mathbf{U}_k^{(I)} \, \mathbf{U}_k^{(0)} 
ight]^H$$

- $ullet \mathbf{u}_k^{(D)}$  dominating eigenvector (largest eigenvalue).
- ullet  $\mathbf{U}_{\iota}^{(I)}$  eigenvector subspace of non-negligible eigenvalues.
- ullet  $\mathbf{U}_k^{(0)}$  eigenvector subspace with eigenvalues close to zero.

### **Proposed transmission strategy:**

- ullet Transmission along dominating eigenvector:  $\mathbf{w}_k = \mathbf{u}_k^{(D)}$ .
- ullet Interference in  $\mathbf{U}_k^{(I)}$  can be mitigated without loss of signal power.
- ullet Receiver cancels out the  $n_R-1$  strongest eigenvalues in  ${f U}_{k}^{(I)}$ .

# Dominating Eigenmode Receive beamforming Negligible Eigenmodes Non-negligible Interfering Eigenmodes

# SINR Estimation and Feedback

Effective channel with the proposed receive beamforming:

$$\widetilde{\mathbf{h}}_k^H = \mathbf{v}_k^H \mathbf{H}_k \in \mathcal{CN}(\mathbf{0}, \mathbf{Q}_k).$$

ullet Same eigenvectors in  ${f Q}_k$  as in  ${f R}_k$ , but the eigenvalues are

$$[\lambda_1,0,\ldots,0,\lambda_{n_R+1},\ldots,\lambda_{n_T}].$$

### **Important observations:**

- ullet The effective channel  $\widetilde{\mathbf{h}}_k$  is almost rank one (along  $\mathbf{u}_k^{(D)}$ ).
- ullet The distribution of  $oldsymbol{\mathbf{h}}_k$  is known at the transmitter.

Transmitter needs to estimate signal/interference powers  $\|\widetilde{\mathbf{h}}_k^H \mathbf{w}_i\|^2$ :

- Robust SINR estimation necessary for data rate adaptation.
- Estimation quality can be improved by feedback.
- The channel norm  $\|\widetilde{\mathbf{h}}_k\|^2$  measures the gain in all directions.

# Quantized Channel Norm Feedback

It have been shown before that

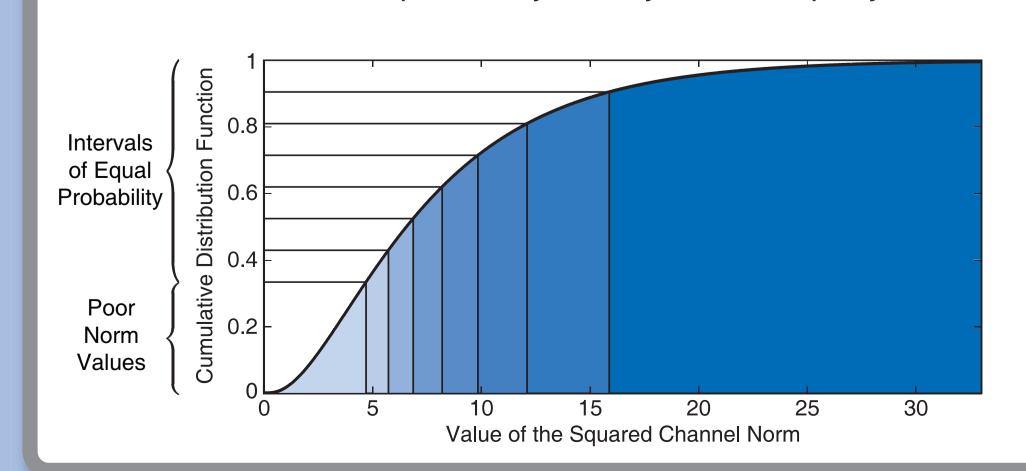
ullet Feedback of the exact norm  $\|\widetilde{\mathbf{h}}_k\|^2$  gives good SINR estimation.

### What happens when quantization is introduced?

- We have derived an MMSE estimator for signal/interference power.
- Exploits feedback of the quantized squared channel norm.
- Closed-form expressions are given for arbitrary quantization.

### How to quantize the channel norm optimally?

- Divide the probability density into intervals of equal probability!
  Should be based on the post-user-selection distribution.
- Proposed quantization (heuristic)
- One interval for really poor channel norms.
- The rest of the probability density divided equally.



## Performance Evaluation

### Base station

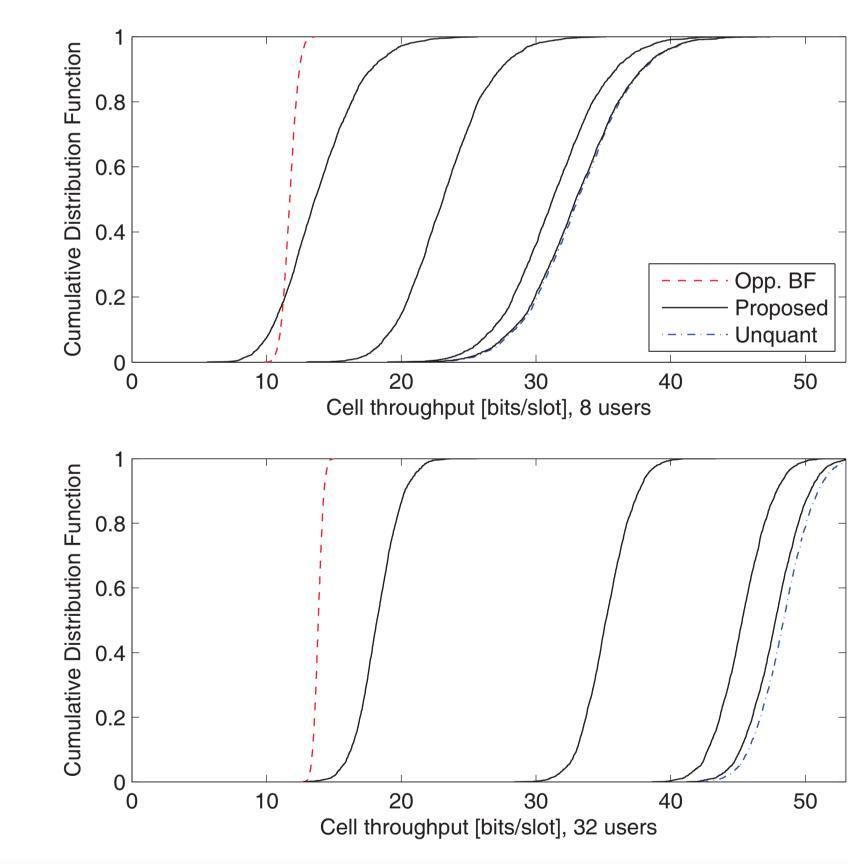
- 8 antennas in a uniform circular array (UCA).
- 15 degrees of angular spread.

### Mobile users

- 4 antennas at each user.
- Uniformly distributed users in the cell.

### **Transmission properties**

- Average signal-to-noise ratio (SNR) at cell boundary of 10 dB.
- Greedy resource allocation with proportional fairness.
- Outage probability of 5% (using SINR estimator with back-off).



### Observations

- 1 bit gives 50% of the performance gain, 3 bits give 90%.
- Opportunistic beamforming (exact feedback) is outperformed.

# Conclusions and Contributions

- Transmission takes place along the strongest eigenvector.
- Receiver cancels out interference in other eigen-subspaces.
- Feedback of channel norm makes reliable SINR estimation possible.
- Only a few bits are required to capture most of the feedback gain.