Optimizing Multi-Cell Massive MIMO for Spectral Efficiency

How Many Users Should Be Scheduled?

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Typical Statements on Massive MIMO

- "*Massive MIMO improves spectral efficiency with orders of magnitude*"
  - This sounds promising but is vague!
  - Which gains can we expect in reality?

- "*Massive MIMO has an order of magnitude more antennas than users*"
  - This assumption reduces interference
  - But does it maximize any system performance metric?

- "*The pilot sequences are reused for channel estimation in every cell*"
  - This is an analytically tractable assumption
  - Are there no benefits of having more pilot sequences than that?

Partial Answers in This Paper

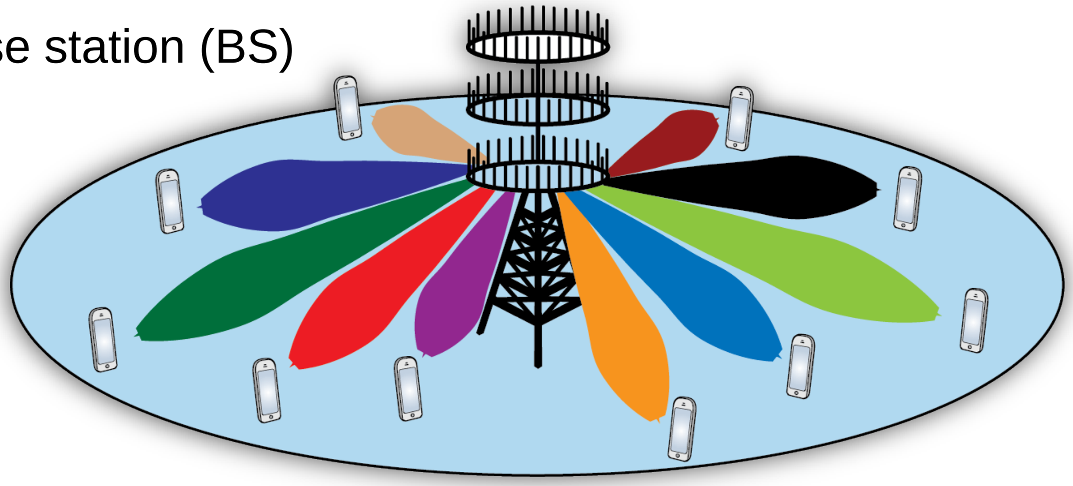
Goal: Optimize spectral efficiency for a given number of antennas

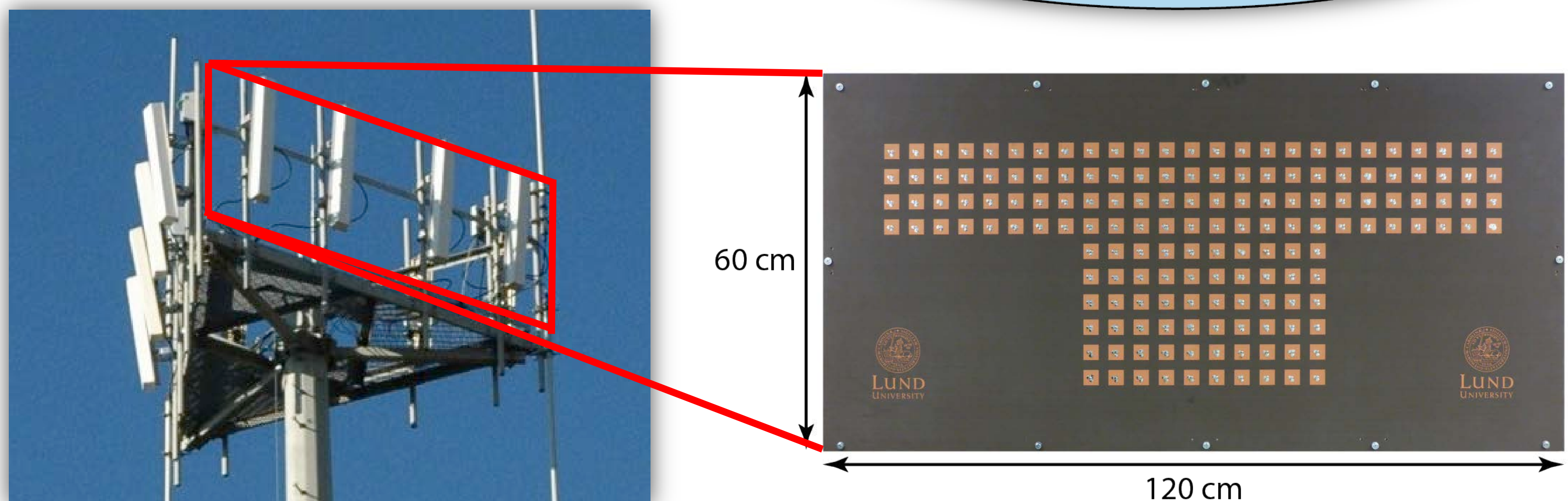
Variables: Number of users and pilot sequences

INTRODUCTION

What is Massive MIMO?

- A Grown Up Multi-User MIMO System
  - N active antennas at base station (BS)
  - K single-antenna users
  - Relation: $N \gg K$
  - Narrow beamforming
  - Less interference

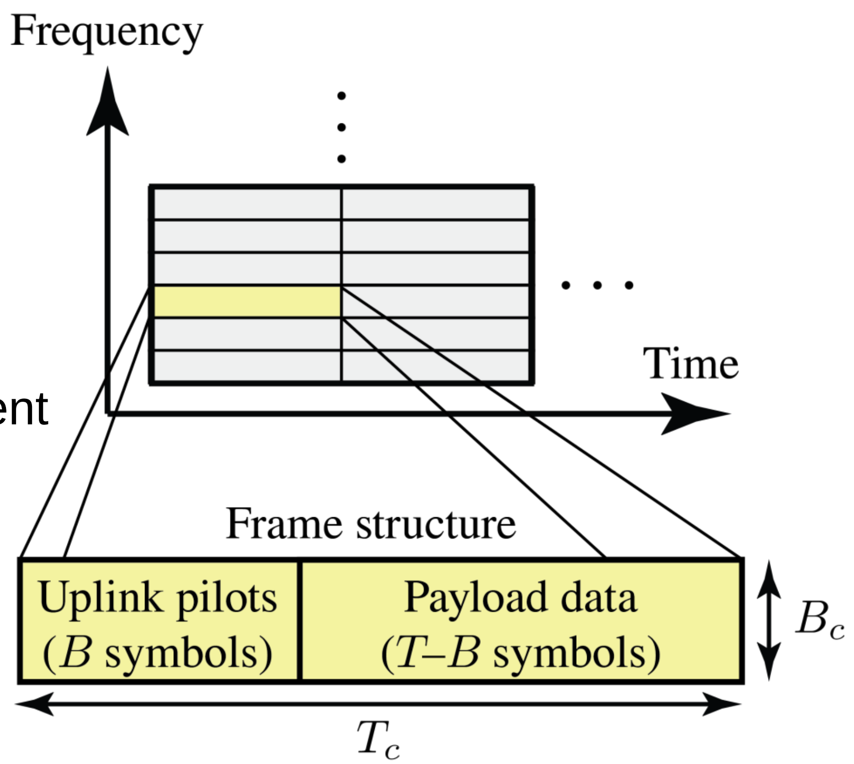




160 antenna elements, *LuMaMi testbed, Lund University*

Massive MIMO Transmission Protocol

- Coherence Blocks
  - Fixed channel responses
  - Coherence time: T_c s
  - Coherence bandwidth: B_c Hz
  - Depends on mobility and environment
  - Block length: $T = T_c B_c$ symbols
  - Typically: $T \in [100,10000]$

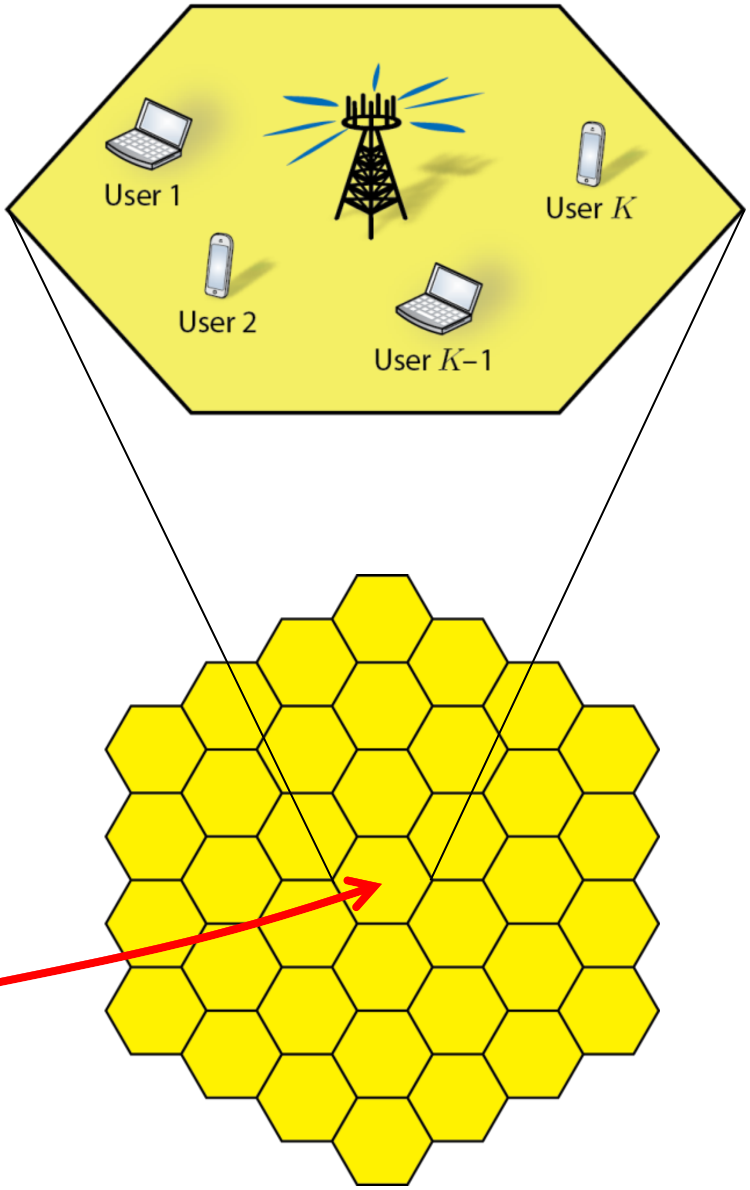


- Time-Division Duplex (TDD)
  - Downlink and uplink on all frequencies
  - B symbols/block for uplink pilots – for channel estimation
  - $T - B$ symbols/block for uplink and downlink payload data

This paper focus on uplink

Multi-Cell System

- Classic Hexagonal Cellular System
  - Infinitely large grid of cells
  - N antennas at each BS
  - K active users in each cell
  - Uniform user distribution in cells
  - Uncorrelated Rayleigh fading
  - Distance-dependent pathlosses



Every cell is "typical"

OPTIMIZING FOR SPECTRAL EFFICIENCY

Optimization of Spectral Efficiency

- Problem Formulation:

$$\underset{K,B}{\text{maximize}} \quad \text{spectral efficiency} \qquad [\text{bit/s/Hz/cell}]$$

for a given N *and* T

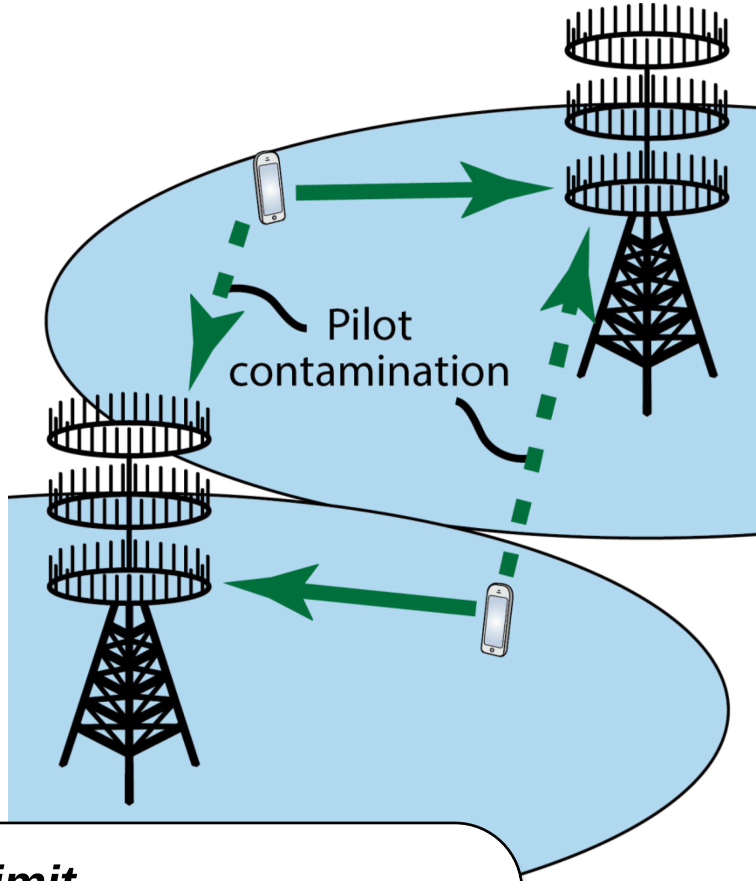
- Main Issue: Hard to Find Tractable Expressions
  - Interference depends on all users' positions!
  - Prior works: Explicit pathloss values *or* all pathloss are set equal
  - We want reliable quantitative results – independent of user locations

- Proposed Solution: Every user is "typical"
  - Same constant SNR: Power control inversely proportional to pathloss
  - Inter-cell interference: Code over variations in user locations in other cells

Impact of Pilot Length

- Limited Coherence Block Length T
  - Not more than T orthogonal pilots
  - Hence: $B \leq T$
  - Pilots must be reused across the cells

- Pilot Contamination
  - BS cannot tell difference between users
  - Interference cannot be suppressed by linear receive combining



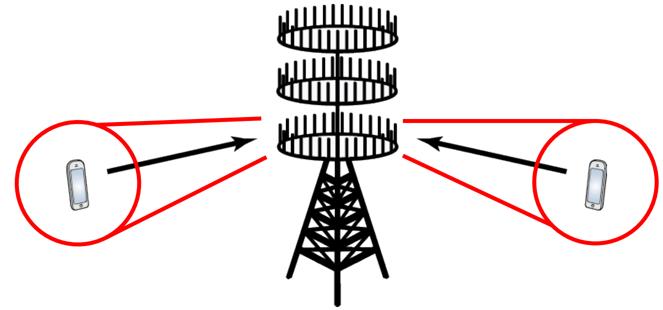
Upper Performance Limit

$$\mathrm{SINR} < \frac{1}{\sum \left(\frac{\text{Pathloss from contaminated interferer}}{\text{Pathloss to its base station}} \right)^2}$$

Can we control this limit?

Controlling Pilot Contamination

- Pilot Allocation
  - Control which users that use same pilots
  - Can be based on spatial correlation:
  - Drawback: Needs inter-cell coordination, scheduling makes fast variations

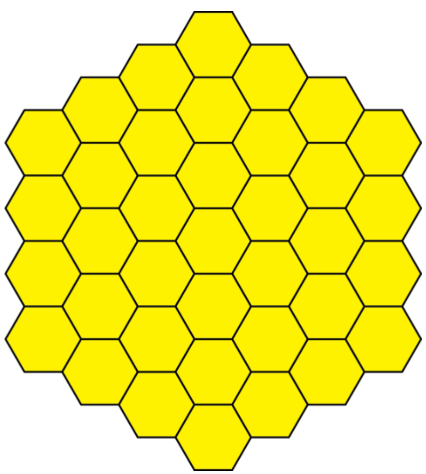


- Simple Pilot Allocation
  - More pilots than users: $B = \beta K$
  - Pilot reuse factor $\beta \geq 1$

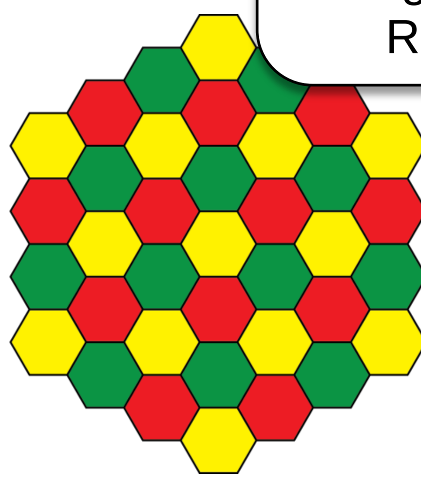
Benefit

Higher β → Interferers further away

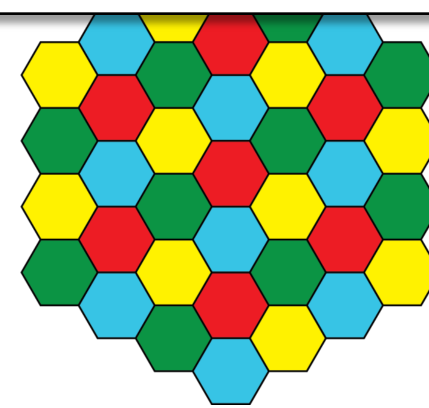
Change pilots randomly within cell →
Remove interference peaks



Reuse $\boldsymbol{\beta = 1}$



Reuse $\boldsymbol{\beta = 3}$



Reuse $\boldsymbol{\beta = 4}$

Analytic Contributions (1)

- New: Linear minimum mean-squared error (LMMSE) estimator
  - Arbitrary pilot allocation
  - Estimates effective power-controlled channels

- Limited Pilot Resolution
  - Each BS can estimate its channel to all users
  - B pilot sequences → Each BS can only see B channel directions
  - Hence: Channel estimates for users with same pilot are parallel!

Essence of pilot contamination

What if *β>1?*

Each BS can resolve channels to users in neighboring cells

Analytic Contributions (2)

- New: Closed-Form Achievable Spectral Efficiencies
  - Typical user – power control and averaging over inter-cell interference
  - Depend on N, K, β, and user distribution – not instantaneous locations
  - Scheme 1: Maximum ratio combining (MRC)
  - Scheme 2: Zero-forcing combining (ZFC)
  - Scheme 3: Pilot-based zero-forcing combining (P-ZFC)

$\beta = 1$: *Same as conventional ZFC*

$\beta > 1$: *Exploit unused pilot sequences to cancel inter-cell interference*

Asymptotic Limit $N \to \infty$***:*** *How many users to serve?*

Select $K = \frac{T}{2\beta}$ *users* → *Achieve spectral efficiency* $\frac{T}{4\beta}\log_2\left(1+\frac{1}{\mathrm{PC}(\beta)}\right)$

Pilot sequences of length $T/2$*:*
Spend half the frame on pilots!

Pilot contamination term:
Smaller if β *is larger!*

Spectral Efficiency Expressions

- Closed-Form Non-Asymptotic Expressions

$$\overline{\mathrm{SINR}}_{jk}^{\mathrm{MRC}} = \frac{B}{\left(\sum_{l\in\mathcal{B}} \mu_{jl}^{(1)} \frac{K}{N} + \frac{\sigma^2}{N\rho}\right)\left(\sum_{\ell\in\mathcal{B}}\sum_{m=1}^{K} \mu_{jl}^{(1)} \mathbf{v}_{i_{jk}}^{\mathrm{H}} \mathbf{v}_{i_{\ell m}} + \frac{\sigma^2}{\rho}\right) + \sum_{l\in\mathcal{B}}\sum_{m=1}^{K}\left(\mu_{jl}^{(2)} + \frac{\mu_{jl}^{(2)} - \left(\mu_{jl}^{(1)}\right)^2}{N}\right)\mathbf{v}_{i_{jk}}^{\mathrm{H}}\mathbf{v}_{i_{lm}} - B}$$

$$\overline{\mathrm{SINR}}_{jk}^{\mathrm{P\text{-}ZFC}} = \frac{B}{\sum_{l\in\mathcal{B}}\sum_{m=1}^{K}\left(\mu_{jl}^{(2)} + \frac{\mu_{jl}^{(2)} - \left(\mu_{jl}^{(1)}\right)^2}{N-B}\right)\mathbf{v}_{i_{jk}}^{\mathrm{H}}\mathbf{v}_{i_{lm}} + \left(\sum_{l\in\mathcal{B}}\sum_{m=1}^{K}\mu_{jl}^{(1)}\left(1 - \frac{B\mu_{jl}^{(1)}}{\sum_{\ell\in\mathcal{B}}\sum_{\tilde{m}=1}^{K}\mu_{j\ell}^{(1)}\mathbf{v}_{i_{lm}}^{\mathrm{H}}\mathbf{v}_{i_{\ell\tilde{m}}} + \frac{\sigma^2}{\rho}}\right) + \frac{\sigma^2}{\rho}\right)\left(\frac{\sum_{\ell\in\mathcal{B}}\sum_{m=1}^{K}\mu_{jl}^{(1)}\mathbf{v}_{i_{jk}}^{\mathrm{H}}\mathbf{v}_{i_{\ell m}} + \frac{\sigma^2}{\rho}}{N-B}\right) - B}$$

- Depends on:

1. N = Number of antennas
2. K = Number of users
3. Pilot sequence $\mathbf{v}_{i_{jk}}$ of user k in cell j
4. Propagation parameters:

$$\mu_{jl}^{(\gamma)} = \mathbb{E}_{\mathbf{z}_{lm}}\left\{\left(\frac{d_j(\mathbf{z}_{lm})}{d_l(\mathbf{z}_{lm})}\right)^{\gamma}\right\} \quad \text{for } \gamma = 1, 2.$$

Optimizing for Spectral Efficiency

NUMERICAL RESULTS

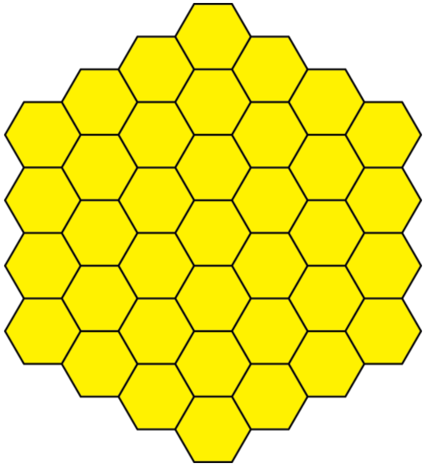
Optimization of Spectral Efficiency

- Problem Formulation:

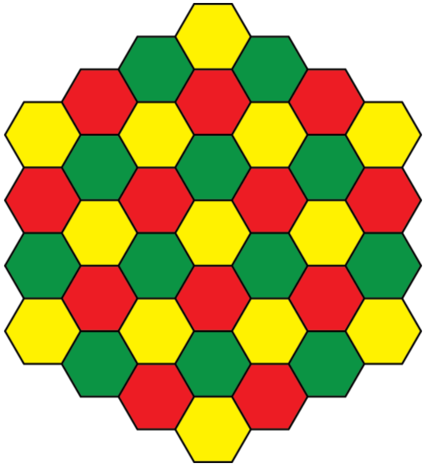
$$\underset{K,\beta}{\text{maximize}} \quad \text{spectral efficiency} \qquad [\text{bit/s/Hz/cell}]$$

for a given N *and* T

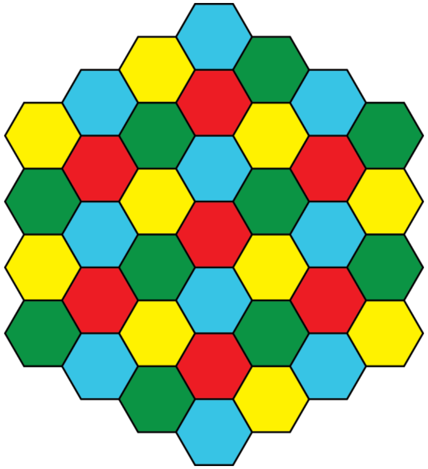
- Use new closed-form spectral efficiency expressions
- Compute average interference between different cells (a few minutes)
- Simply compute for different K and β and pick maximum (<1 minute)



Reuse $\boldsymbol{\beta = 1}$



Reuse $\boldsymbol{\beta = 3}$



Reuse $\boldsymbol{\beta = 4}$

Asymptotic Behavior

Assumptions

Uniform user distribution

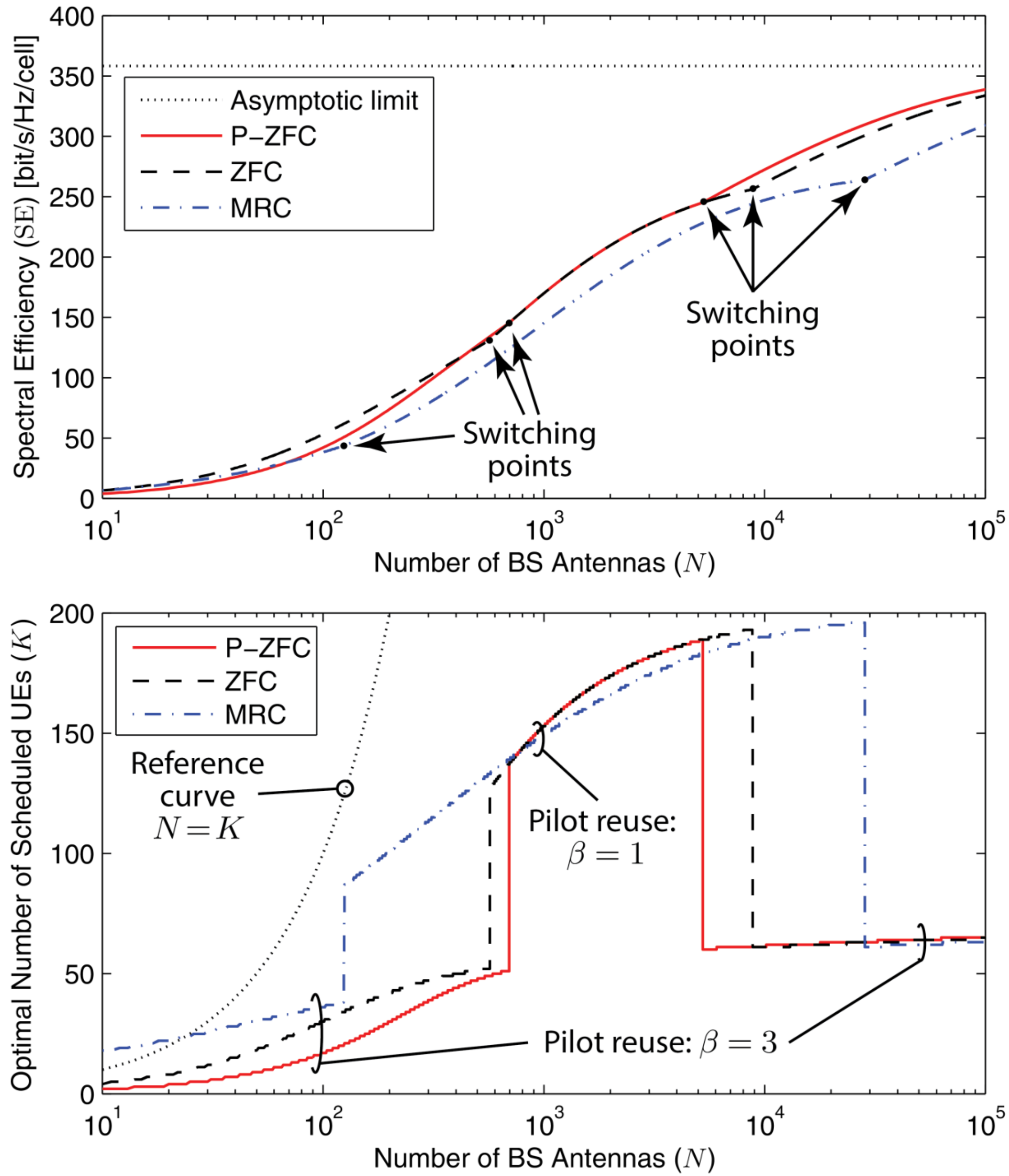
Pathloss exponent: 3.7

Coherence block: $T = 400$

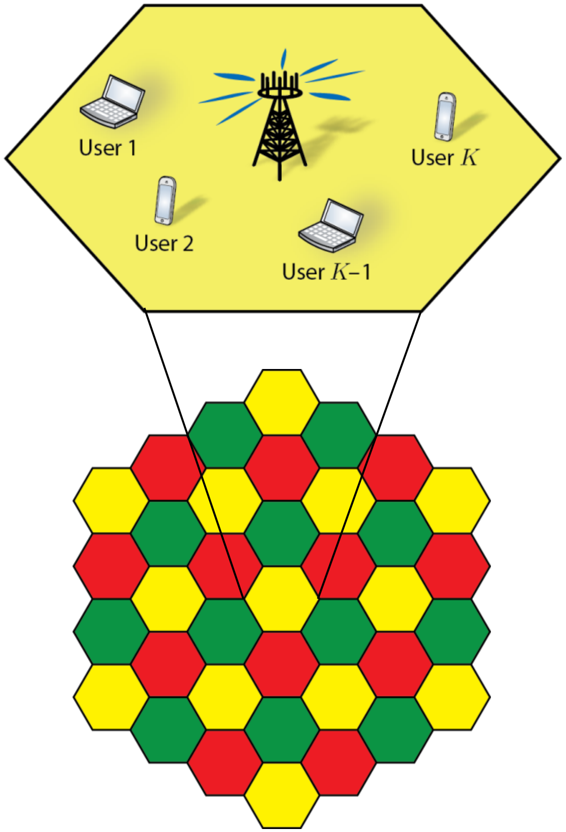
SNR 5 dB, Rayleigh fading

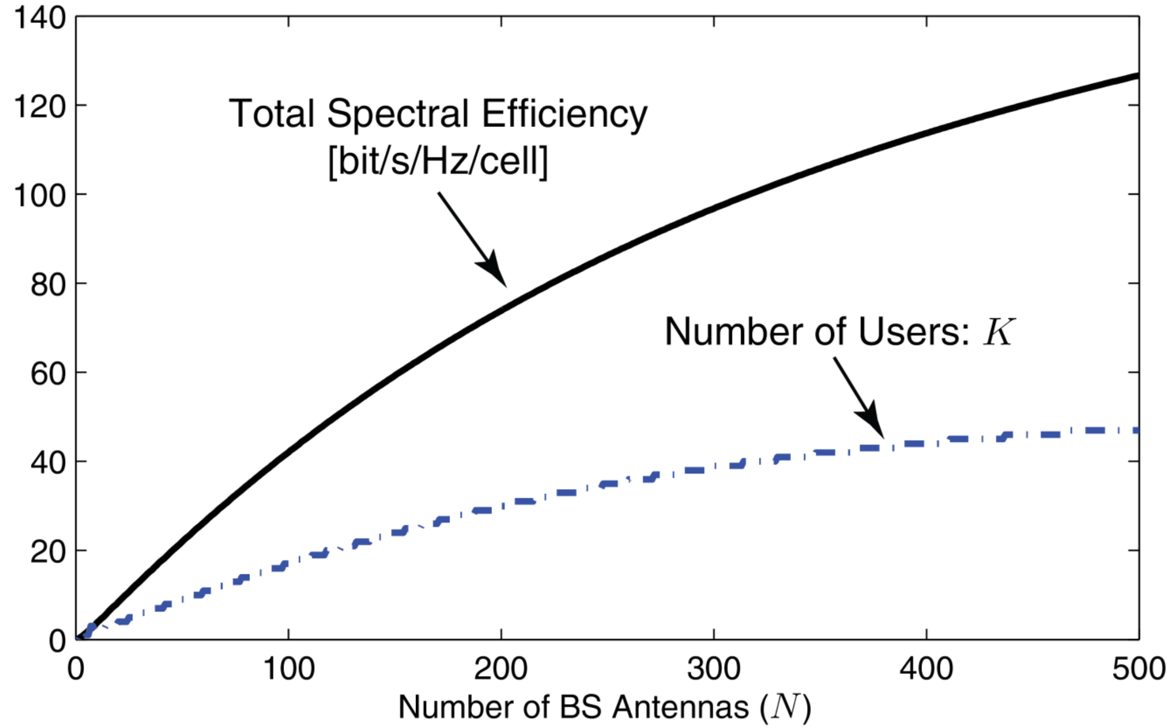
Observations

- *Asymptotic limits not obtained*
- *Reuse factor* $\beta = 3$ *is desired*
- *K is different for each scheme*
- *Small optimized performance difference between schemes*
- *Coordinated beamforming: Only useful at very large N*



Anticipated Spectral Efficiency





Further Assumptions

ZFC processing

Pilot reuse: $\beta = 3$

Observations

- *Baseline: 2.25 bit/s/Hz/cell (IMT-Advanced)*
- *Massive MIMO,* $N = 100$*: x20 gain* $(N/K \approx 6)$
- *Massive MIMO,* $N = 400$*: x50 gain* $(N/K \approx 9)$
- *Per scheduled user:* ≈ 2.5 *bit/s/Hz*

SUMMARY

Summary

- Quantitative Results
  - Massive MIMO can greatly increase spectral efficiency
  - >20x gain over IMT-Advanced is foreseen
  - High spectral efficiency per cell, not per user
  - MRC, ZFC, P-ZFC prefer different K and β
  - Fractional pilot reuse ($\beta = 3$) is often preferred

- Analytic Contributions
  - Channel estimator for arbitrary pilot allocation
  - Spectral efficiencies under power control and random user locations
  - No Monte-Carlo simulations needed: System-level results in a few minutes!
  - Asymptotic: Half coherence block spent on pilots

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QUESTIONS?

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