# A Multi-cell MMSE Precoder for Massive MIMO Systems and New Large System Analysis

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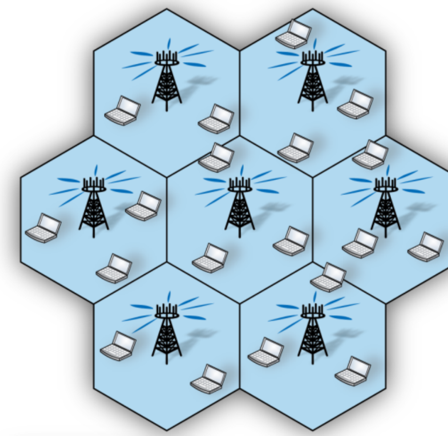
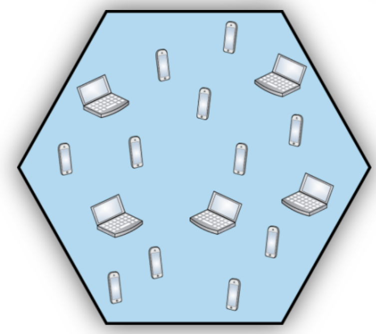
# Background & System Model

# Evolving Cellular Networks

- Cellular Network Architecture
  - Area divided into cells
  - One fixed base station (BS) serves all the users
  - Demand increases by 30-40% per year!



- Network Throughput [bit/s/km$^2$]
  - Consider a given area



- Simple Formula for Network Throughput:

$$\underbrace{\text{Throughput}}_{\text{bit/s/km}^2} = \underbrace{\text{Available spectrum}}_{\text{Hz}} \cdot \underbrace{\text{Cell density}}_{\text{Cell/km}^2} \cdot \underbrace{\text{Spectral efficiency}}_{\text{bit/s/Hz/Cell}}$$

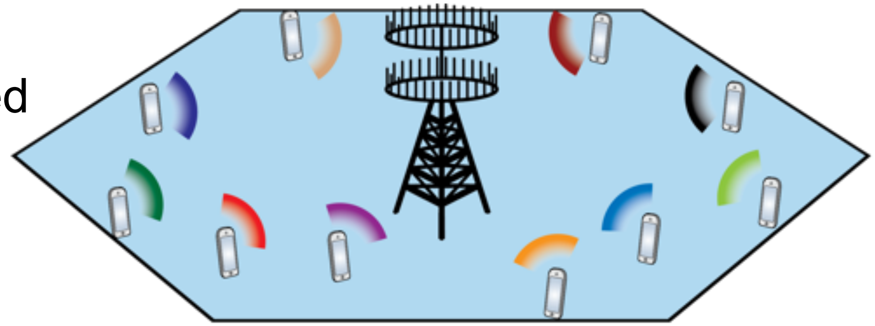
***Dominant factors in the past!***

***5G opportunity:***

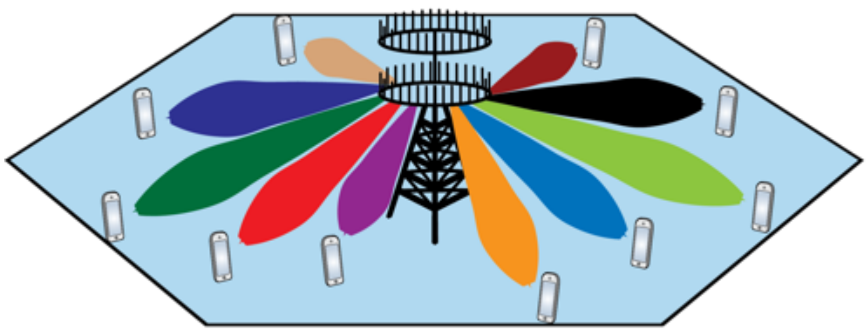
*Improve spectral efficiency*

# Aggressive Spatial Spectral Reuse

- Needed: Many active users per km$^2$
  - Interference Management is Key
  - "Small cells" are interference limited
- Massive MIMO
  - BSs with many antennas: $M$
  - Multiplexing of many users: $K$
  - Important: $M \gg K$
  - Uplink: Linear detection
  - Downlink: Linear precoding



Uplink

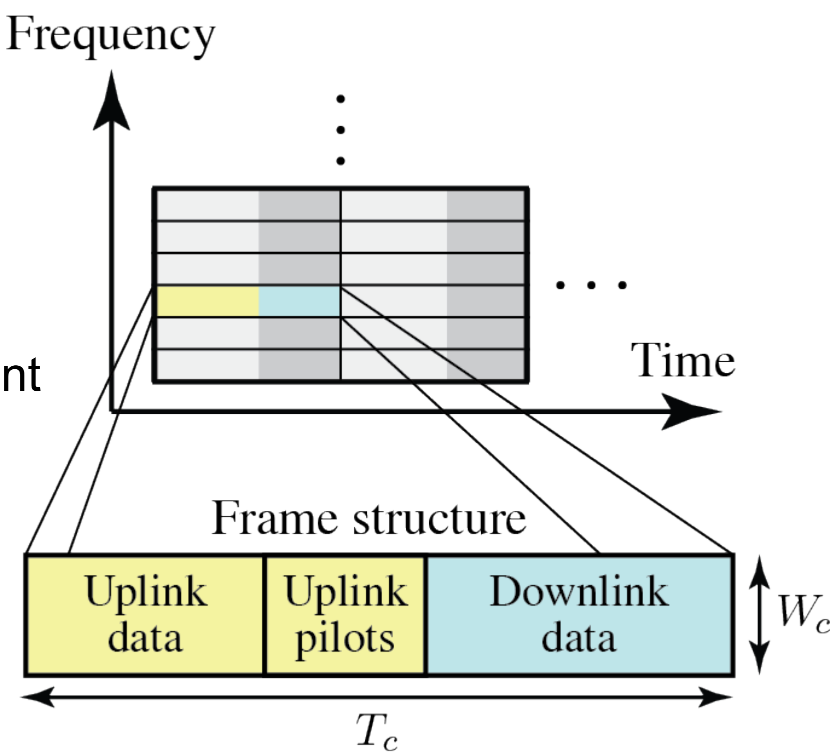


Downlink

***Array gain gives signal gain without causing more interference!***

# Massive MIMO Transmission Protocol

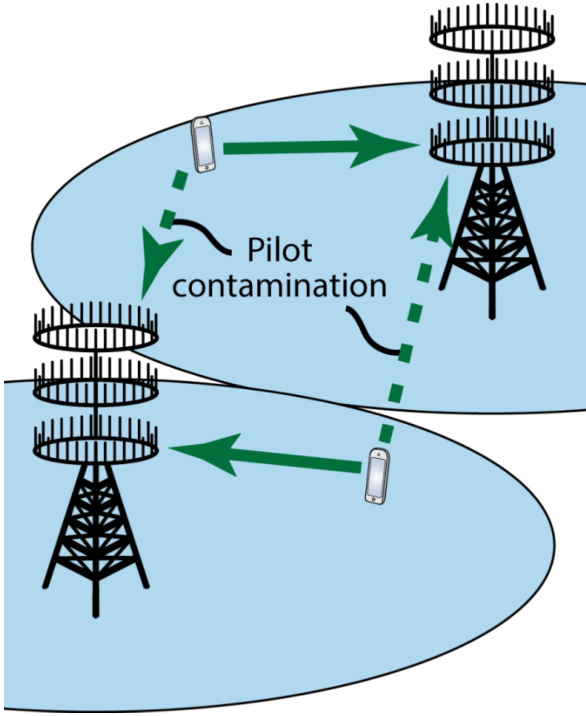
- Coherence Blocks
  - Fixed channel responses
  - Coherence time: $T_c$ s
  - Coherence bandwidth: $W_c$ Hz
  - Depends on mobility and environment
  - Block length: $S = T_c W_c$ symbols
  - Typically: $S \in [100{,}10000]$




- Time-Division Duplex (TDD)
  - Downlink and uplink on all frequencies
  - $B$ symbols/block for uplink pilots – for channel estimation
  - $S - B$ symbols/block for uplink and/or downlink payload data

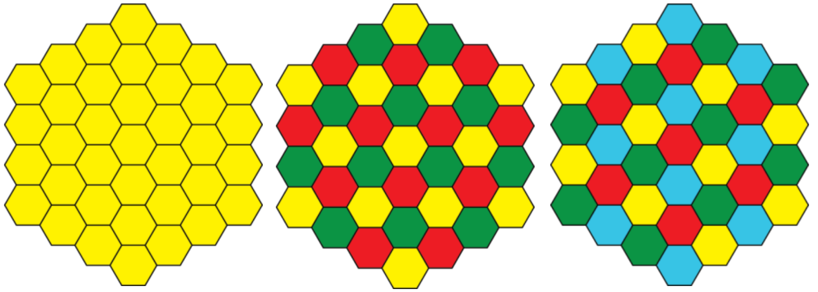
# Channel Acquisition in Massive MIMO

- Limited Number of Pilots: $B \leq S$
  - Must use same pilot sequence in multiple cells
  - Base stations cannot tell some users apart: *Essence of pilot contamination*




- Coordinated Pilot Allocation
  - Allocate pilots to users to reduce contamination
  - Scalability → No signaling between BSs

- Solution: Non-universal pilot reuse
  - Pilot reuse factor $\beta \geq 1$
  - Users per cell: $K = B/\beta$
  - Higher $\beta$→ Fewer users per cell, but reuse in fewer cells



***Reuse*** $\beta = 1$ ***Reuse*** $\beta = 3$ ***Reuse*** $\beta = 4$

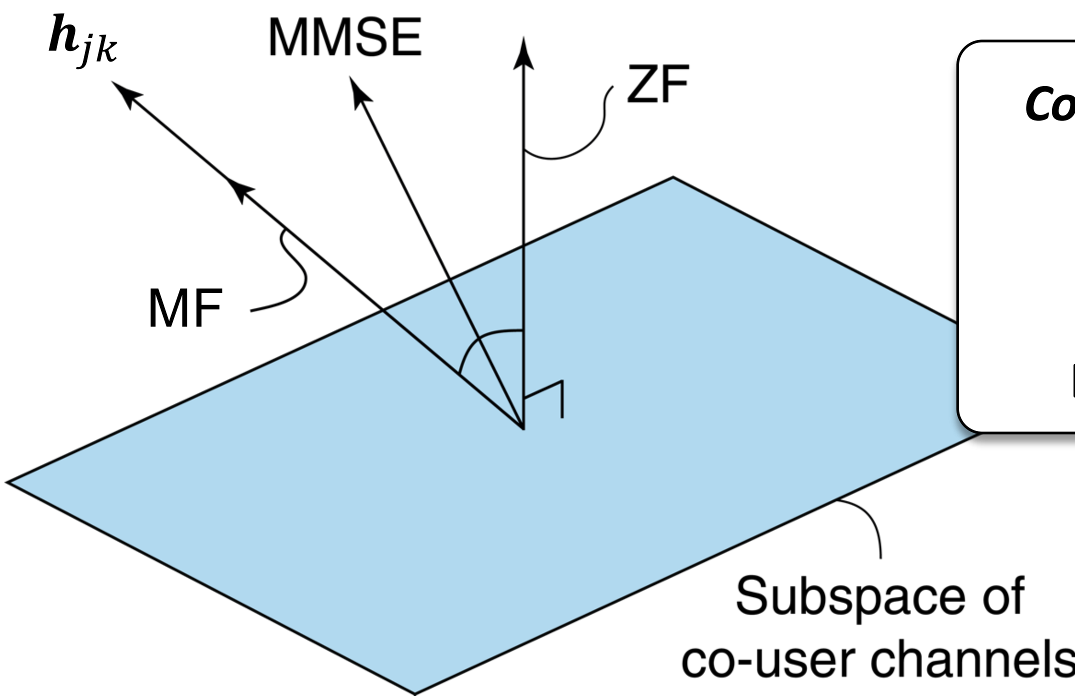
# Single-Cell Linear Detection

*Inactive* interference suppression

- MF: Maximize channel gain

*Active* interference suppression

- ZF: Minimize interference
- MMSE: Maximize SINR




***Commonly used also for:***

*Multi-cell cases*

*Downlink precoding*

But what is optimal?

# Multi-Cell MMSE Detection

- Optimal: Multi-Cell MMSE Detection

Estimated desired channel

$$\boldsymbol{g}_{jk} = \left( \sum_{l,m} p_{lm} \underbrace{\hat{\boldsymbol{h}}_{jlm} \hat{\boldsymbol{h}}_{jlm}^{H}}_{\text{All estimated channels}} + \boldsymbol{E}_j + \sigma^2 \boldsymbol{I} \right)^{-1} \hat{\boldsymbol{h}}_{jjk}$$

*Uplink powers*

*Sum of estimation error covariance matrices*

- Suppress both intra- and inter-cell interference


*H. Q. Ngo, M. Matthaiou, and E. G. Larsson, "Performance analysis of large scale MU-MIMO with optimal linear receivers," Swe-CTW, 2012.*

*K. F. Guo and G. Ascheid, "Performance analysis of multi-cell MMSE based receivers in MU-MIMO systems with very large antenna arrays," IEEE WCNC, 2013.*

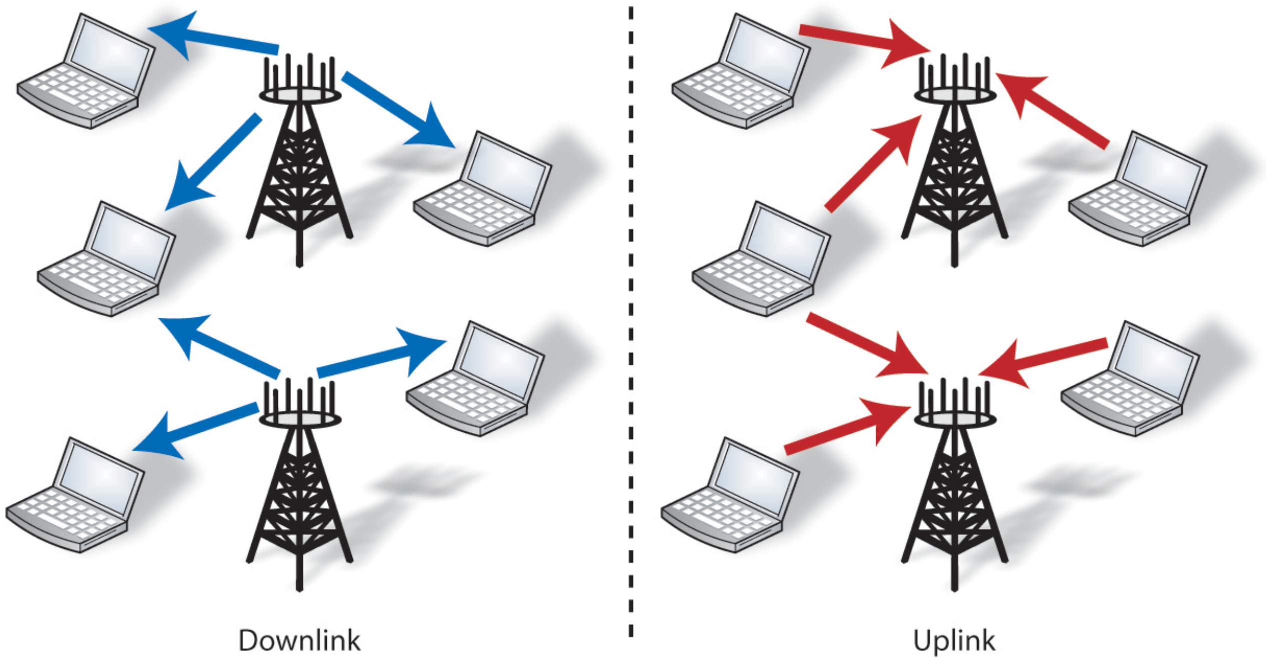
*Xueru Li, Emil Björnson, Erik G. Larsson, Shidong Zhou and Jing Wang, "A Multi-Cell MMSE Detector for Massive MIMO Systems and New Large System Analysis," IEEE GLOBECOM 2015.*

# Main Contributions:
# Multi-Cell Precoding

# Uplink-Downlink Duality

The same rates are achievable in the uplink and the downlink if

- Detection and precoding vectors are the same
- Same sum power, but optimized power allocation



# Proposal: Multi-Cell MMSE Precoding

- Multi-Cell MMSE Precoding

*Estimated desired channel*

$$\boldsymbol{w}_{jk} = \frac{1}{\lambda_{jk}} \boldsymbol{g}_{jk} = \frac{1}{\lambda_{jk}} \left( \sum_{l,m} p_{lm} \underbrace{\widehat{\boldsymbol{h}}_{jlm} \widehat{\boldsymbol{h}}_{jlm}^{H}}_{\text{All estimated channels}} + \boldsymbol{E}_j + \sigma^2 \boldsymbol{I} \right)^{-1} \widehat{\boldsymbol{h}}_{jjk}$$

*Uplink powers*

*Sum of estimation error covariance matrices*

***Features**:*

*Optimality property from uplink-downlink duality*

*Only requires “local” channels to BS $j$: Scalable!*

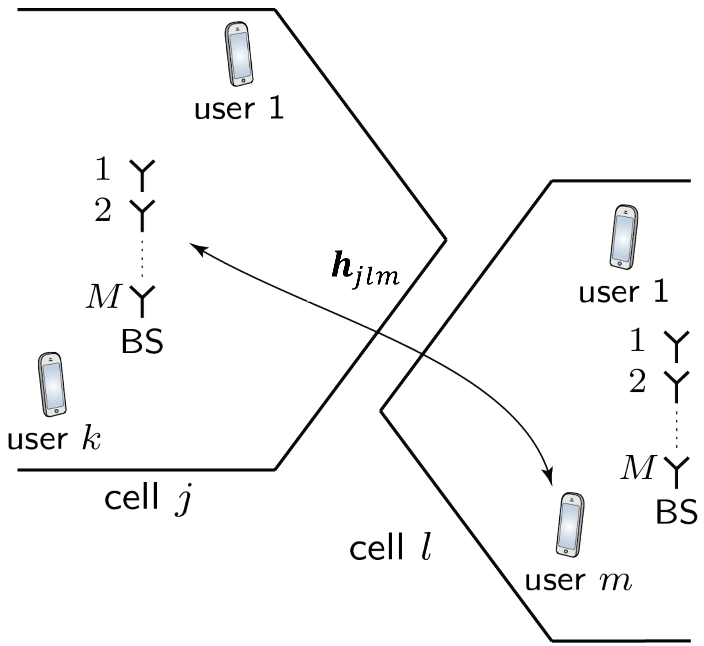
*Supports imperfect CSI with arbitrary pilot allocation*

# Property 1: Parallel Channel Estimates

- System Model
  - Channel from BS $j$ to user $m$ in cell $l$

$$\boldsymbol{h}_{jlm} \sim \mathcal{CN}\left(\mathbf{0}, d_{jlm}\mathbf{I}_M\right)$$

  - Uplink transmit power: $p_{lm}$
  - Pilot sequence: $\boldsymbol{v}_{i_{lm}} \in \{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_B\}$




- Uplink Pilot Transmission

$$\boldsymbol{Y}_j = \sum_l \sum_m \sqrt{p_{lm}}\, \boldsymbol{h}_{jlm} \boldsymbol{v}_{i_{lm}}^T + \boldsymbol{N}_j$$

  - Estimate of $\boldsymbol{h}_{jlm}$:

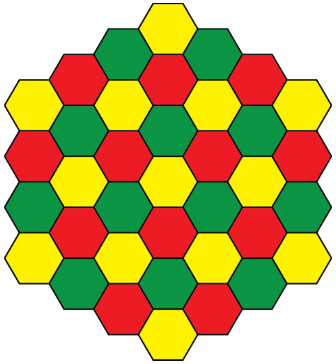
$$\widehat{\boldsymbol{h}}_{jlm} = \underbrace{\frac{\sqrt{p_{lm}}\, d_{jlm}}{\sum_{l'} \sum_k p_{l'k} d_{jl'k} \boldsymbol{v}_{i_{l'k}}^H \boldsymbol{v}_{i_{lm}} + \sigma^2}}_{\text{User-specific scalar}} \boldsymbol{Y}_j \boldsymbol{v}_{i_{lm}}^*$$

*Direction only depend on pilot sequence*

There are only $B$ channel directions to estimate!

# Property 2: Simplified Precoder Expression

**Reuse** $\beta = 3$



- Paradigm: Listen to all pilot sequences
  - $B$ estimated directions:

$$\widehat{\boldsymbol{H}}_{\mathcal{V},j} = \boldsymbol{Y}_j [\boldsymbol{v}_1^* \ \dots \ \boldsymbol{v}_B^*]$$

- Multi-Cell MMSE Precoding

$$\boldsymbol{w}_{jk} = \frac{1}{\lambda_{jk}} \left( \sum_{l,m} p_{lm} \widehat{\boldsymbol{h}}_{jlm} \widehat{\boldsymbol{h}}_{jlm}^H + \boldsymbol{E}_j + \sigma^2 \boldsymbol{I} \right)^{-1} \widehat{\boldsymbol{h}}_{jjk}$$

$$= \frac{1}{\lambda_{jk}} \left( \widehat{\boldsymbol{H}}_{\mathcal{V},j} \boldsymbol{\Lambda}_j \widehat{\boldsymbol{H}}_{\mathcal{V},j}^H + (\sigma^2 + \varphi_j) \boldsymbol{I} \right)^{-1} \widehat{\boldsymbol{h}}_{jjk}$$

*Diagonal matrix:*

*Uplink powers and scalars from estimates*

*Sum of estimation error variances*

Much easier to implement and analyze!

# Property 3: Achievable Downlink Performance

- Achievable Spectral Efficiency

$$R_{jk} = \left(1 - \frac{B}{S}\right) \log_2(1 + \eta_{jk})$$

with $$\eta_{jk} = \frac{\rho_{jk} \left| \mathbb{E}\{\boldsymbol{h}_{jjk}^H \boldsymbol{w}_{jk}\} \right|^2}{\sum_{l,m} \mathbb{E}\left\{ \left| \boldsymbol{h}_{ljk}^H \boldsymbol{w}_{lm} \right|^2 \right\} - \rho_{jk} \left| \mathbb{E}\{\boldsymbol{h}_{jjk}^H \boldsymbol{w}_{jk}\} \right|^2 + \sigma^2}$$

**Large-Scale Approximation:** $\eta_{jk} - \bar{\eta}_{jk} \to 0$ as $M, K \to \infty$ with a fixed ratio,

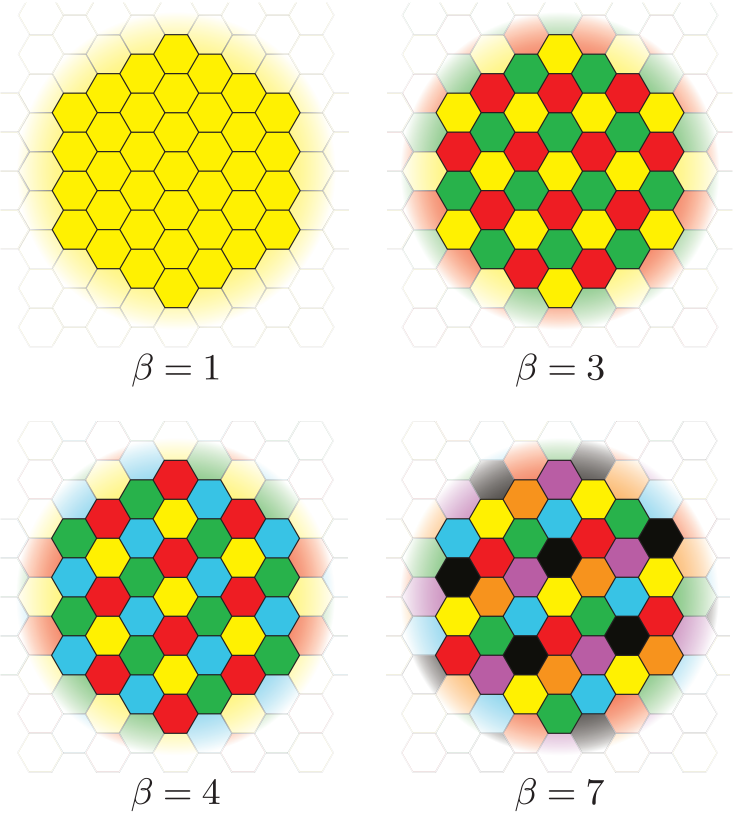
$$\bar{\eta}_{jk} = \frac{\rho_{jk} p_{jk} d_{jjk}^2 \frac{\delta_{jk}^2}{\vartheta_{jk}}}{\sum_{(l,m)\neq(j,k):\ \text{same pilot}} \rho_{lm} p_{jk} d_{ljk}^2 \frac{\delta_{lm}^2}{\vartheta_{lm}} + \sum_{(l,m):\ \text{diff pilot}} \frac{\rho_{lm} d_{ljk}}{M} \frac{\mu_{ljkm}}{\vartheta_{lm}} + \frac{\sigma^2}{M}}$$

*Computed using semi-closed form expressions from random matrix theory*

# Simulations

# Simulation Setup

- Hexagonal Network: 19 cells with wrap-around
  - Random user locations: $> 0.14 \cdot \text{radius}$
  - Shadowing: 2.2 dB standard deviation
  - Coherence block: $S = 500$
  - Pilot reuse patterns: $B = fK$




  - Uplink channel-inversion power control:

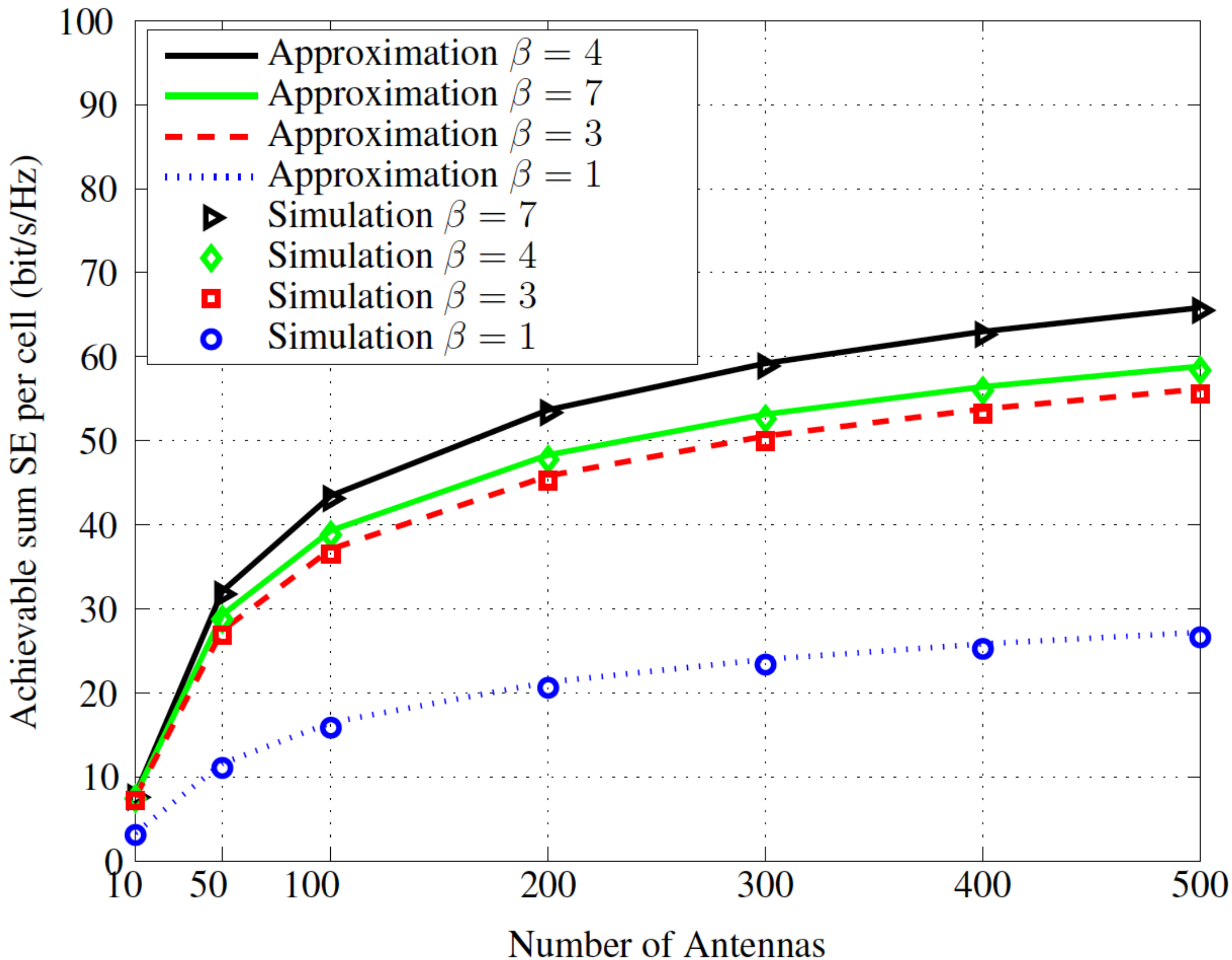
$$p_{jk} = \rho_{jk} = \frac{P}{d_{jjk}}$$

    Effective SNR $P/\sigma^2$, here: 0 dB

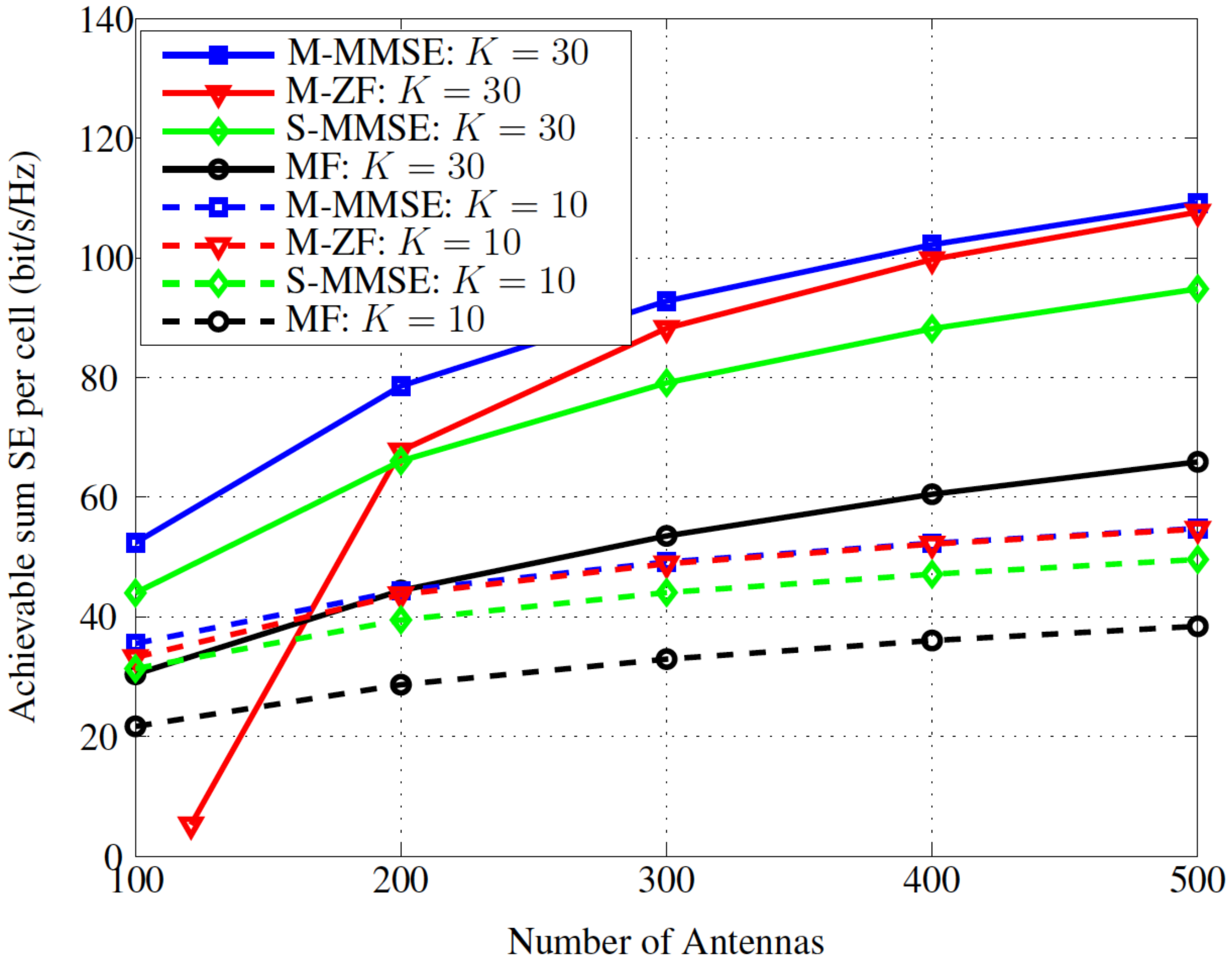
  - Downlink equal power control:

    Give – 3 dB at cell edge
    (ignoring shadowing)

# Reuse Factors and Large-Scale Approximations



# Comparison of Different Precoding Schemes




$\beta = 4$

# Summary

# Summary

- Proposal: Multi-Cell MMSE Precoding
  - Motivated by uplink-downlink duality
  - All channels = Only $B$ pilots to estimate channels from
  - Large-system performance analysis
  - Substantial gains over single-cell precoders

- Additional results
  - Large-scale approximation enables efficient power control optimization
  - X. Li, E. Björnson, E. G. Larsson, and J. Wang, "*Massive MIMO with multi-cell MMSE processing: exploiting all pilots for interference suppression*," IEEE Trans. Wireless Commun., Under review.

# Questions?

Emil Björnson

***Slides and papers available online:***

*http://www.commsys.isy.liu.se/en/staff/emibj29*

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