Post-User-Selection Quantization and Estimation of Correlated Frobenius and Spectral Channel Norms

Emil Björnson and Björn Ottersten

Royal Institute of Technology (KTH)

Stockholm, Sweden

Outline

- System Model
- Two Kinds of Channel Norms
- Feedback Quantization
- Estimation of SNR and Capacity
- Numerical Examples
- Summary

ROYAL INSTITUTE OF TECHNOLOGY

Contributions

- Two independent frameworks
- Entropy-maximizing Quantization
  - Based on pre- and post-user-selection statistics
- MMSE SNR/Capacity Estimation
  - Closed-form Expressions
  - Based on arbitrary quantization

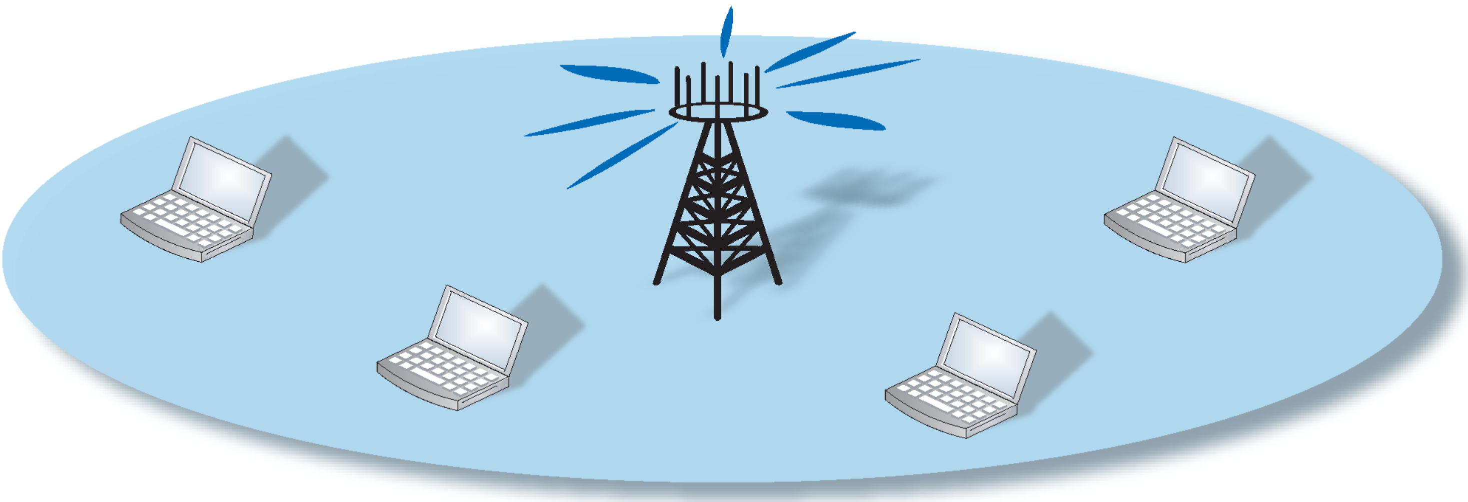
ROYAL INSTITUTE OF TECHNOLOGY

Known Channel Norm at Receiver → Feedback Quantization → Feedback-aided Norm Estimation at Transmitter

ROYAL INSTITUTE OF TECHNOLOGY

System Model

- Multi-user scenario
  - Scheduling (user selection)
  - Channel statistics are available
- Combination with norm/gain feedback
  - Improves scheduling and precoding



System Model (2)

- MIMO Communication:
  - n_T transmit antennas, n_R receive antennas
- Rayleigh fading, one-sided correlation:

$$\mathbf{y}_k(t) = \mathbf{H}_k \mathbf{x}(t) + \mathbf{n}_k,$$

  - $\mathbf{x}(t)$ transmitted signal, $E\{\|\mathbf{x}(t)\|_F^2\} = 1$
  - $\mathbf{y}_k(t)$ received signal, noise $\mathbf{n}_k \in \mathcal{CN}(\mathbf{0}, \mathbf{I}_{n_T})$
  - Distinct & Identical eigenvalues, or vice versa,

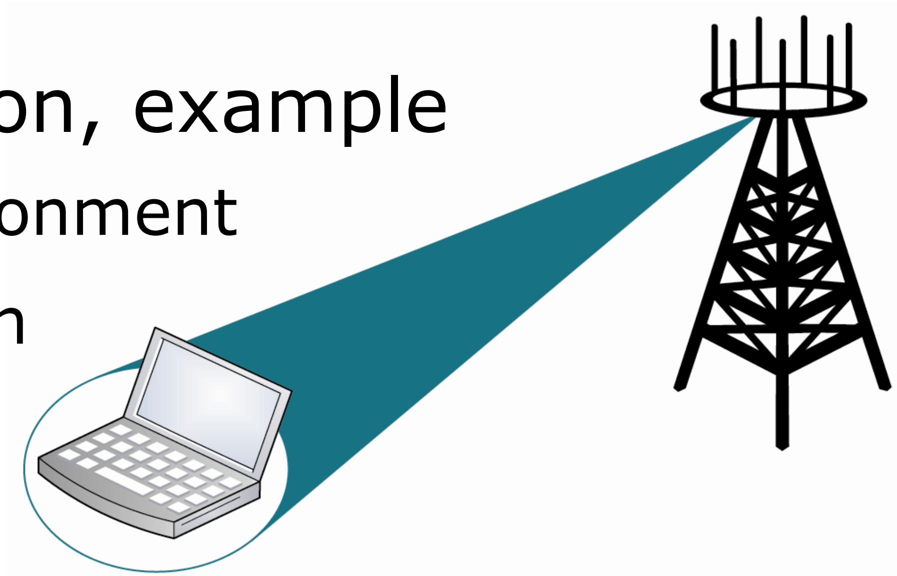
$$\mathbf{H}_k = \boxed{\mathbf{R}_{\mathrm{Rx},k}^{1/2}} \widetilde{\mathbf{H}}_k \boxed{\mathbf{R}_{\mathrm{Tx},k}^{1/2}}$$

Known channel statistics

ROYAL INSTITUTE OF TECHNOLOGY

System Model (3)

- One-sided correlation, example
  - Users in urban environment
  - Elevated base station



KTH VETENSKAP OCH KONST

ROYAL INSTITUTE OF TECHNOLOGY

- Why a limitation to such correlation?
  - Less complicated mathematical expressions
  - More general closed-form expressions exist
  - The principles hold for general correlation!

Two Kinds of Transmission

- Orthogonal Space-time Block Codes:
  - Perfect channel knowledge at receiver
  - Transmission in all spatial directions
  - Clever coding gives separable detection

ROYAL INSTITUTE OF TECHNOLOGY

- Maximum Ratio Transmission:
  - Perfect channel knowledge at receiver
  - Transmission in the strongest direction
  - Requires feedback of this direction
  - Gives better transmission performance

Two Kinds of Channel Norms

- Orthogonal Space-time Block Codes:

$$\mathrm{SNR}_k^{\mathrm{OSTBC}} = \frac{\|\mathbf{H}_k\|_F^2}{n_T} = \frac{1}{n_T}\sum_{i=1}^{n} \boxed{\sigma_i^2}$$

Squared singular values

$n = \min(n_T, n_R)$

  - Squared Frobenius norm

- Maximum Ratio Transmission:

$$\mathrm{SNR}_k^{\mathrm{MRT}} = \|\mathbf{H}_k\|_2^2 = \sigma_1^2$$

  - Squared Spectral norm
  - Antenna diversity provided by feedback gives *largest* singular value instead of *average*

ROYAL INSTITUTE OF TECHNOLOGY

Distribution of Channel Norms

- Cumulative distribution functions (CDFs)
  - Squared Frobenius norm

$$F_{\|\mathbf{H}\|_F^2}(\rho) = 1 - \frac{H_0(\rho)}{\prod_{i=1}^{n} \lambda_i^m} \sum_{k=1}^{n} \sum_{l=1}^{m} \frac{\Psi_{k,l,m}}{(-\frac{1}{\lambda_k})^{m-l+1}} e^{-\frac{\rho}{\lambda_k}} \sum_{j=0}^{m-l} \frac{(\frac{\rho}{\lambda_k})^j}{j!}$$

  - Squared Spectral norm

$$F_{\|\mathbf{H}\|_2^2}(\mu) = \frac{\det \begin{bmatrix} \mathbf{\Phi} \\ \mathbf{\Delta}(\mu) \end{bmatrix} (-1)^{m(n-s)} H_0(\mu)}{\det(\mathbf{V}) \prod_{i=1}^{s} (m-i)!}$$

  - Both can be expressed in closed form

ROYAL INSTITUTE OF TECHNOLOGY

Feedback Quantization

- Channel norms correspond to SNR
  - SNR feedback enables scheduling, precoding, and rate adaptation
  - Exact feedback requires infinitely many bits
- How to perform quantization?
  - Many different quality measures
  - Channel statistics known and should be used
- Entropy-maximizing quantization
  - Maximizes the information per feedback bit
  - Considered in this paper

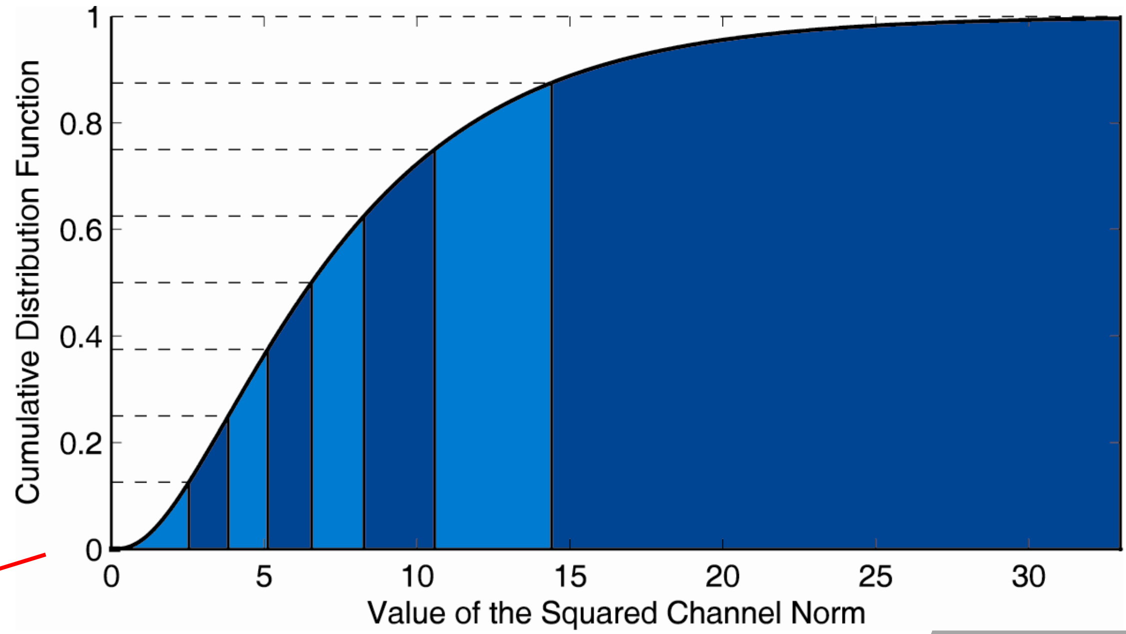
ROYAL INSTITUTE OF TECHNOLOGY

Feedback Quantization (2)

- Entropy of feedback bits maximized by:
  - Divide $[0,\infty)$ into 2^L intervals $[A_{i-1}, A_i)$ with $A_0 = 0, A_{2^L} = \infty$, and

$$A_i = F^{-1}\left(\frac{i}{2^L}\right), \quad i = 1, \ldots, 2^L - 1.$$

$F(\cdot)$ is the CDF (e.g., of a squared norm)

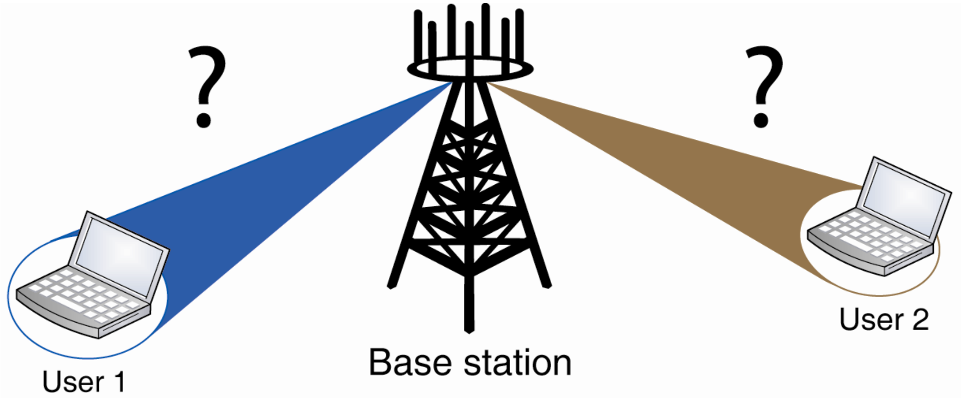


Example: 3 bits

ROYAL INSTITUTE OF TECHNOLOGY

Feedback Quantization (3)

- N users
  - Select M users
  - Spatial separability and user SNR are quality measures



ROYAL INSTITUTE OF TECHNOLOGY

- When will norm feedback be used?

1. User selection
2. Precoding and rate adaptation

  - #2 requires more accurate information
  - Better to optimize quantization for #2 than #1

Feedback Quantization (4)

- Post-user-selection statistics
  - Strong users more probable to be selected
  - Very difficult to describe this mathematically
- Proposed scheduler:
  - Select the M users with largest $F_k(\mathrm{SNR}_k)$
  - Good fairness, but ignores spatial separability
  - Post-user-selection CDF is $G_M(F_k(x))$,

where

Gives statistics for a selected user

$$\boxed{G_M(x)} = \sum_{i=0}^{M-1} \binom{N}{i} \frac{M-i}{M} x^{N-i} (1-x)^i$$

ROYAL INSTITUTE OF TECHNOLOGY

Estimation of SNR & Capacity

ROYAL INSTITUTE OF TECHNOLOGY

- Transmitter gets quantized norm feedback
  - Feedback says that norm belong to an interval
  - The value that best represents the squared norm depends on the statistics and application
- Estimation of functions of squared norm
  - We consider MMSE estimation given feedback
  - Two interesting functions: SNR and capacity
  - Closed-form expressions for these functions

Estimation of SNR & Capacity (2)

- MMSE Estimation of $g(\rho)$, with $\rho = \|\mathbf{H}_k\|_F^2$
  - Quantized feedback: $\rho \in [A, B)$

$$E\{g(\rho)|\mathcal{Q}_\rho\} = \frac{C}{\prod_{i=1}^{n} \lambda_i^m} \sum_{k=1}^{n} \sum_{l=1}^{m} \frac{\Psi_{k,l,m} G_{m-l,k}(A,B)}{(-1)^{m-l+1}(m-l)!},$$

  - SNR $g(\rho) = \rho$:

$$G_{N,k}(A,B) = \left[-\frac{(N+1)!}{(\frac{1}{\lambda_k})^{N+2}} e^{-\frac{\rho}{\lambda_k}} \sum_{i=0}^{N+1} \frac{(\frac{\rho}{\lambda_k})^i}{i!} \right]_A^B$$

  - Capacity $g(\rho) = \log_2(1+\rho)$:

    Similar closed-form expression as for SNR

Estimation of SNR & Capacity (3)

- MMSE Estimation of $g(\mu)$, with $\mu = \|\mathbf{H}_k\|_2^2$
  - Quantized feedback: $\mu \in [A, B)$

$$E\{g(\mu)|\mathcal{Q}_\mu\} = C \sum_{\alpha \in \mathcal{A}_n} \frac{\prod_{k=d+1}^{n} \lambda_{\alpha_k}^{t-k+1} \prod_{k=m-s+1}^{m} (k-1)!}{(-1)^{\mathrm{per}(\alpha)} \prod_{k=1}^{d} (-\lambda_{\alpha_k})^{d-k}}$$

$$\times \sum_{l=1}^{s} (-1)^l \sum_{\beta \in \mathcal{B}_{l,s}} \sum_{k=0}^{K_l(\beta)} \sum_{\tilde{k} \in \widetilde{\Omega}_k^{(l)}} \frac{1}{\tilde{k}_1! \cdots \tilde{k}_l!} \frac{\widetilde{G}_{k,\beta}(A,B)}{\prod_{i=1}^{l} \lambda_{\alpha_{(\beta_i+d)}}^{\tilde{k}_i}},$$

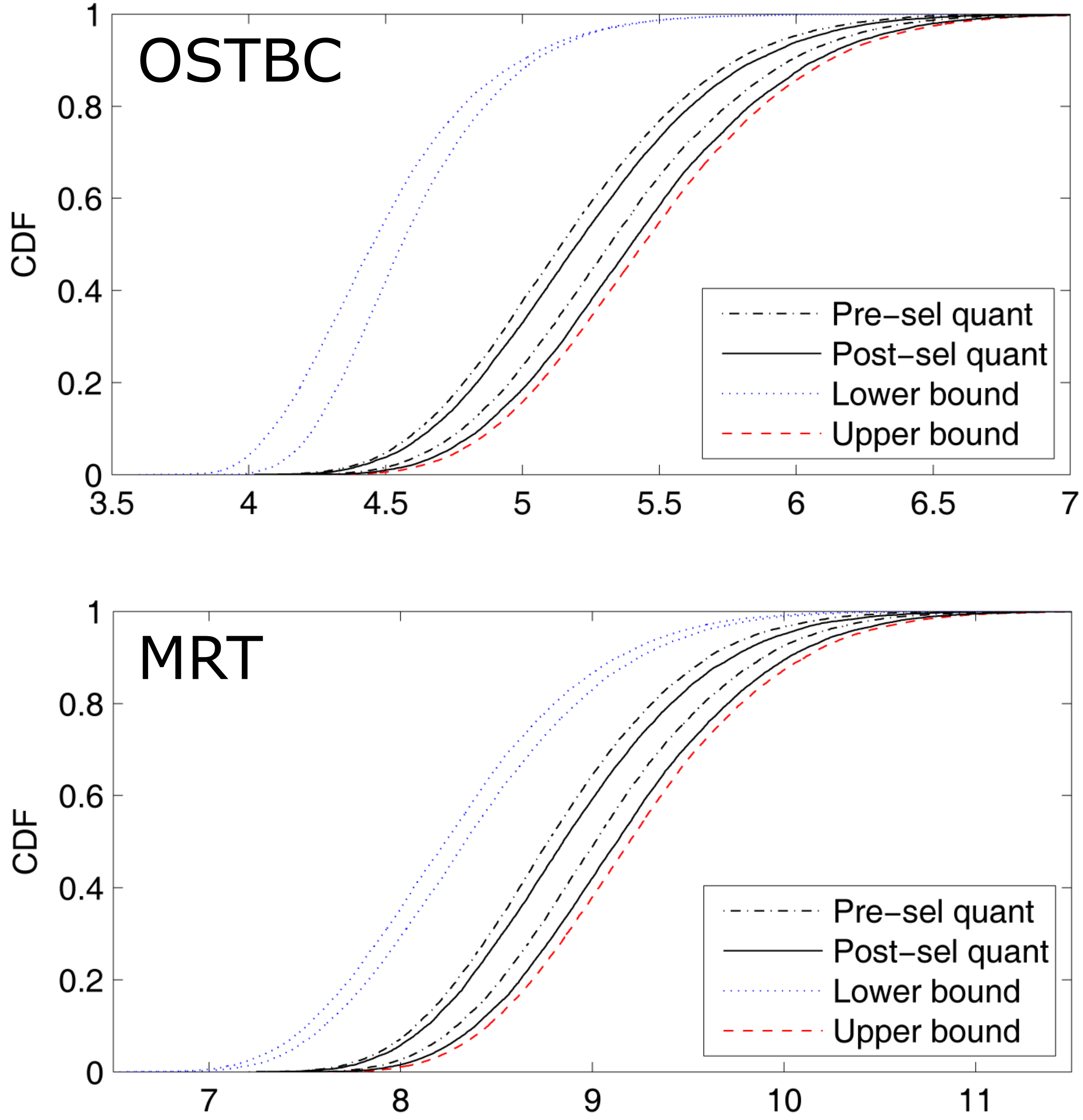
  - Closed-form expressions of $\widetilde{G}_{k,\beta}(A,B)$ for SNR $g(\mu) = \mu$ and capacity $g(\mu) = \log_2(1+\mu)$

Numerical Examples

- Downlink communication
  - Base station: 4 antennas, 15° angular spread
  - 8 Mobile users: 2 uncorrelated antennas
  - Circular cell, 10 dB at cell boundary
- 1 selected user (user k, highest $F_k(\mathrm{SNR}_k)$)
- OSTBC (coding rate ¾)
- MRT (perfect directional feedback)
- Quantized feedback of squared norm

ROYAL INSTITUTE OF TECHNOLOGY

- Entropy-maximizing *Pre*-user-selection Quantization
- Entropy-maximizing *Post*-user-selection Quantization
- Lower bound (no correlation, no estimation)
- Upper bound/capacity (exact feedback)
- 3 bits: 95% of capacity
- 5 bits: 99% of capacity



Summary

- OSTBCs: SNR = Squared Frobenius Norm
- MRT: SNR = Squared Spectral Norm
- Entropy-Maximizing Quantization Framework (pre- and post-user-selection)
- MMSE Estimation of the SNR and Capacity (given quantized norm feedback)
- 3 bits of norm feedback: 95% of capacity (with exact norm feedback)

ROYAL INSTITUTE OF TECHNOLOGY

Further Extensions

ROYAL INSTITUTE OF TECHNOLOGY

- Not limited to one-sided correlation
  - Similar expressions can be derived for general correlation (or Kronecker-structured)
- Proposed scheduler, just an example
  - This scheduler provides fairness and is analytical tractable, but somewhat idealized
  - When spatial separability is considered, the post-user-selection distribution can only be estimated numerically.
  - Important observation: Post-user-selection quantization can give a performance gain