Beamforming and Quantized Feedback in Spatially Correlated Multi-User MIMO Systems

Emil Björnson
PhD Student in Telecommunications
Royal Institute of Technology (KTH)
Sweden

Biography

- Born 1983 in Malmö, Sweden.
- Master of Science in Engineering Mathematics, *Lund University, Sweden, Sept 2002 – Jan 2007.*
- Master Thesis: "*Beamforming Utilizing Channel Norm Feedback in Multiuser MIMO Systems*"
- PhD Student in Telecommunications at *Royal Institute of Technology (KTH)*, since Feb 2007.
- Two publications: SPAWC'07, ICASSP'08.

Outline

- Introduction to a Multi-user System
- How To Choose Performance Measure?
- Receive Beamforming with Subspace Cancellation
- Quantization of the Channel Norm
- Performance Evaluation
- Conclusions

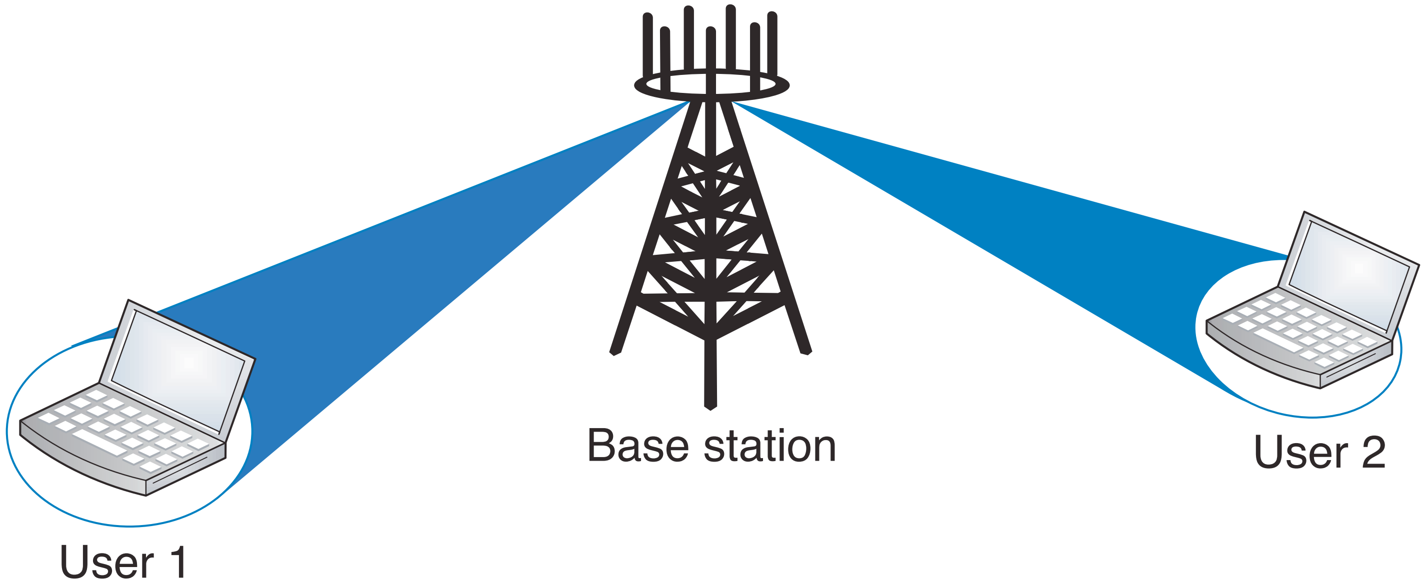
INTRODUCTION TO A MULTI-USER SYSTEM

Downlink multi-user system

- Downlink of multi-antenna system
  - Elevated base station, n_T antennas.
  - Multiple users, n_R antennas.

Spatial Correlation

- Some spatial directions are statistically more favorable for a given user:



- Excellent for simultaneous transmission to several users (beamforming, SDMA).

System Model

- Urban environment, elevated base station
  - Spatially correlated transmitter.
  - Independent receive antennas.

- Channel model:
  - Rayleigh fading multi-antenna channel to user k

$$\mathbf{H}_k = [\mathbf{h}_{k,1}, \ldots, \mathbf{h}_{k,n_R}]^H \in \mathbb{C}^{n_R \times n_T},$$

with independent rows $\mathbf{h}_{k,i} \in \mathcal{CN}(\mathbf{0}, \mathbf{R}_k)$.

Received signal

- Transmission of s_k to user k.

$$y_k(t) = \mathbf{v}_k^H \mathbf{H}_k \Bigg(\underbrace{\sqrt{p_k}\mathbf{w}_k s_k(t)}_{\text{signal}} + \sum_{j \neq k} \underbrace{\sqrt{p_j}\mathbf{w}_j s_j(t)}_{\text{interference}} \Bigg) + \underbrace{\mathbf{n}_k(t)}_{\text{noise},\sigma_k^2}$$

  - Transmit beamformer: $\mathbf{w}_k \in \mathbb{C}^{n_T}$
  - Receive beamformer: $\mathbf{v}_k \in \mathbb{C}^{n_R}$
- Transmitter knows statistics.
- Receiver k knows $\mathbf{H}_k$ and the statistics.

HOW TO CHOOSE PERFORMANCE MEASURE?

How to optimize performance?

- We need a performance measure:
  - Maximize sum of data rates of all users?
  - Maximize the minimal rate among all users?
- What about fairness?
- The measure depend on the application:
  - Balance between throughput and fairness.

Signal-to-interference/noise ratio

- Achievable rate depends on the SINR:

$$\mathrm{SINR}_k = \frac{p_k \|\mathbf{v}_k^H \mathbf{H}_k \mathbf{w}_k\|^2}{\sum_{i \neq k} p_i \|\mathbf{v}_k^H \mathbf{H}_k \mathbf{w}_i\|^2 + \sigma_k^2}.$$

- Tricky to maximize the SINR:
  - Transmit beamformer $\mathbf{w}_k$ affects all users.
  - Transmitter only knows the channel statistics.
  - Receiver is unaware of the other users' channels.

A suboptimal approach

- It's not enough to maximize the SINR
  - Robust estimation necessary at transmitter
- The effect of the receive beamformer should be predictable at transmitter.
- Next, a suboptimal approach is proposed
  - Clear framework for interference suppression.
  - Based on my paper from ICASSP'08.

RECEIVE BEAMFORMING WITH SUBSPACE CANCELLATION

Subspace Partitioning

- Partitioning of covariance matrix $\mathbf{R}_k$:

$$\mathbf{R}_k = [\mathbf{u}_k^{(D)}\, \mathbf{U}_k^{(I)}\, \mathbf{U}_k^{(0)}] \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_{n_T} \end{bmatrix} [\mathbf{u}_k^{(D)}\, \mathbf{U}_k^{(I)}\, \mathbf{U}_k^{(0)}]^H$$

  - $\mathbf{u}_k^{(D)}$ dominating eigenvector (largest eigenvalue).
  - $\mathbf{U}_k^{(I)}$ eigenvector subspace of non-negligible eigenvalues.
  - $\mathbf{U}_k^{(0)}$ eigenvector subspace with eigenvalues close to zero.

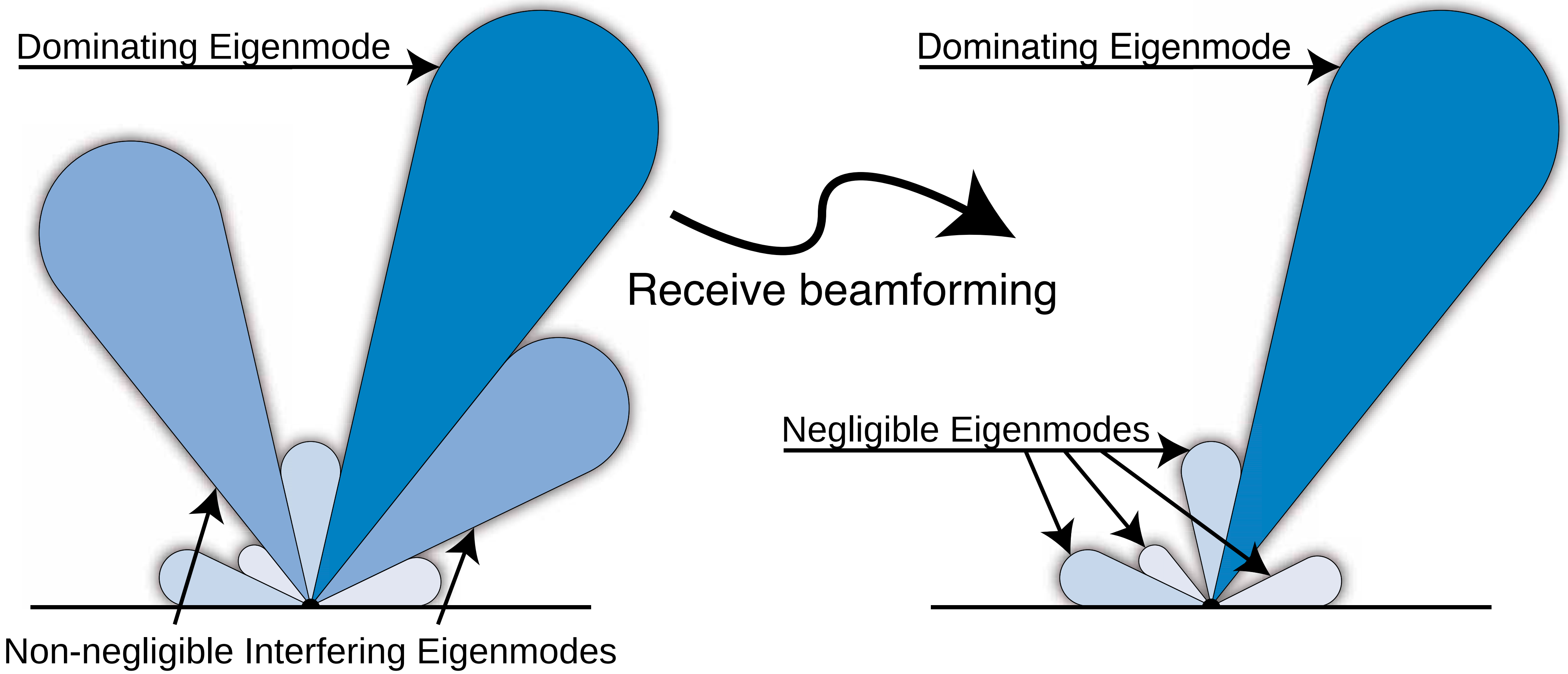
Proposed Transmission Strategy

- Signal power should be received along the dominating eigenvector:

$$\mathbf{w}_k = \mathbf{u}_k^{(D)}$$

- Interference in $\mathrm{U}_k^{(I)}$ can be mitigated without loss of signal power.
  - Receiver can cancel out the $n_R - 1$ strongest eigenvalues in $\mathrm{U}_k^{(I)}$.

Illustration of Receive Beamforming



Observe that the eigenvectors are unaffected.

How can it be done in practice?

- We cancel out transmissions within $\mathbf{U}_k^{(I)}$ by choosing
$$\mathbf{v}_k \in \text{null}\left((\mathbf{H}_k \mathbf{U}_k^{(I)})^H \right).$$
  - This will make
$$\mathbf{v}_k^H \mathbf{H}_k \mathbf{U}_k^{(I)} = 0.$$
- To make the nullspace one-dimensional:
  - Let $\mathbf{U}_k^{(I)}$ contain exactly $n_R - 1$ eigenvectors.

The Effective Channel

- With the proposed receive beamforming, the effective channel is

$$\widetilde{\mathbf{h}}_k^H = \mathbf{v}_k^H \mathbf{H}_k \in \mathcal{CN}(\mathbf{0},\mathbf{Q}_k).$$

  - Same eigenvectors in $\mathbf{Q}_k$ as in the original $\mathbf{R}_k$.
  - The eigenvalues of $\mathbf{Q}_k$ will be

$$[\lambda_1,0,\ldots,0,\lambda_{n_R+1},\ldots,\lambda_{n_T}].$$

  - The distribution is known at the transmitter!

Estimation of the SINR

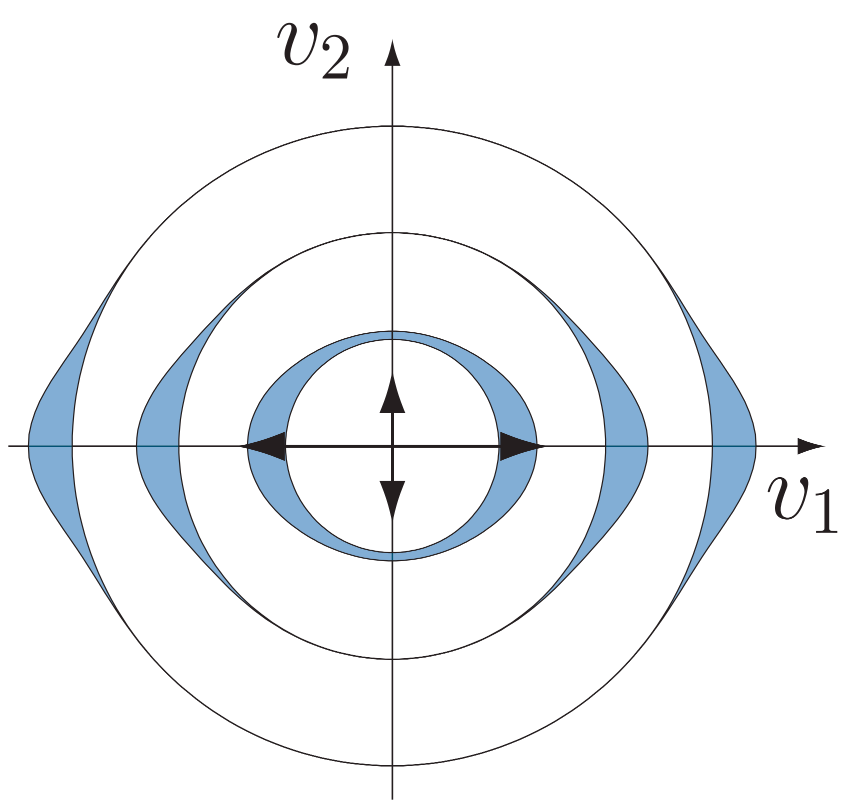
- With the proposed receive beamforming, the SINR becomes

$$\mathrm{SINR}_k = \frac{p_k \|\widetilde{\mathbf{h}}_k \mathbf{w}_k\|^2}{\sum_{i \neq k} p_i \|\widetilde{\mathbf{h}}_k \mathbf{w}_i\|^2 + \sigma_k^2}.$$

- Estimation of signal/interference power can be improved by feedback of $\|\widetilde{\mathbf{h}}_k\|^2$
  - For large n_R, the channel is almost rank one.
  - Statistics tell how $\|\widetilde{\mathbf{h}}_k\|^2$ is distributed spatially.

Illustration of norm and statistics

- Assume that v_1 has larger variance than v_2.
- Each circle represents a value of $\sqrt{v_1^2 + v_2^2}$.
- The statistics tell how the power is distributed between v_1 and v_2.
- The uncertainty reduces with increasing norm.



QUANTIZATION OF THE CHANNEL NORM

Feedback of the channel norm

- Feedback $\|\widetilde{\mathbf{h}}_k\|^2$ improves SINR estimation.
  - Formulas for MMSE estimation of signal/ interference powers in my ICASSP'08 paper.
- Example (power in one eigendirection):

$$E\{|v_l|^2|\rho\} = \frac{1}{g_\rho}\left[\frac{(A_\rho+\lambda_l)e^{-\frac{A_\rho}{\lambda_l}} - (B_\rho+\lambda_l)e^{-\frac{B_\rho}{\lambda_l}}}{\prod_{i\neq l}\left(1-\frac{\lambda_i}{\lambda_l}\right)} + \sum_{k\neq l}\frac{\lambda_l\left(e^{-\frac{A_\rho}{\lambda_l}} - e^{-\frac{B_\rho}{\lambda_l}}\right) - \lambda_k\left(e^{-\frac{A_\rho}{\lambda_k}} - e^{-\frac{B_\rho}{\lambda_k}}\right)}{\left(1-\frac{\lambda_k}{\lambda_l}\right)\prod_{i\neq k}\left(1-\frac{\lambda_i}{\lambda_k}\right)}\right]$$

How to quantize the norm?

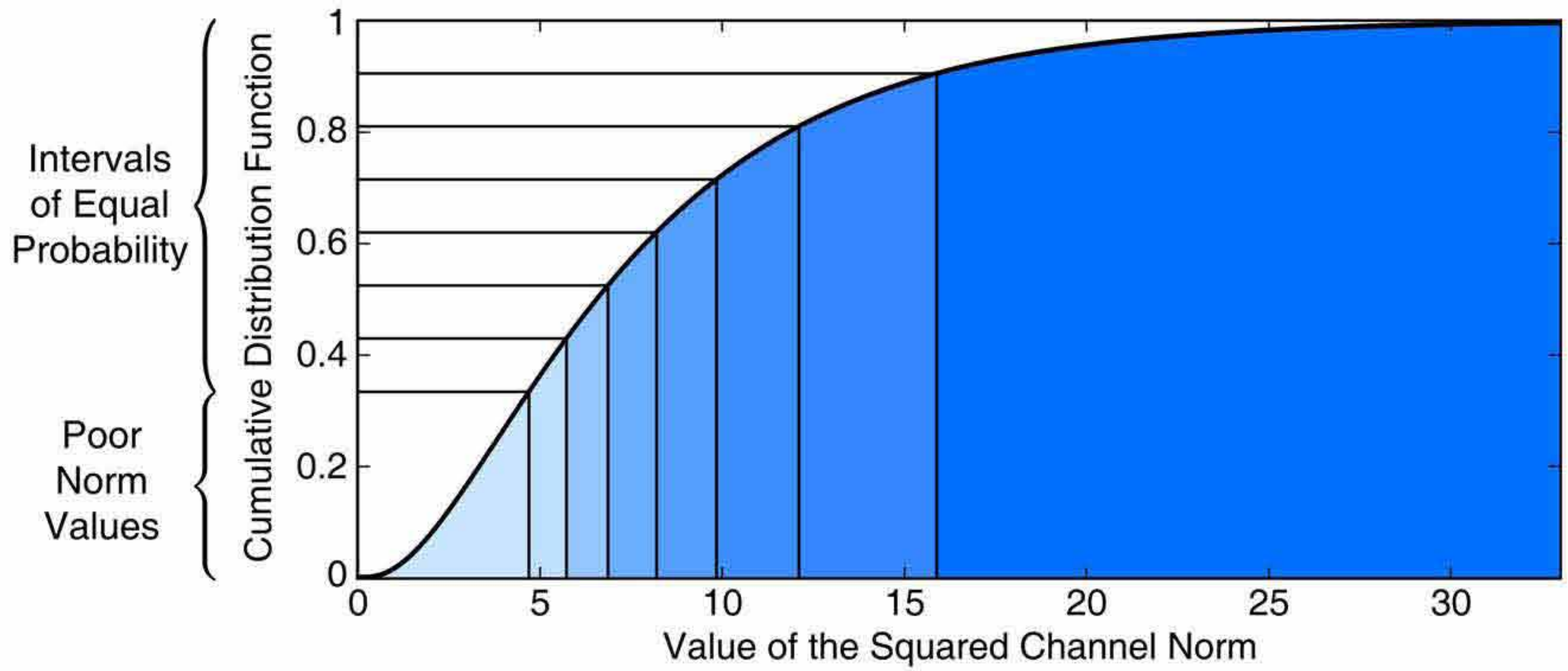
- Limited feedback capacity:
  - Only a few bits of feedback available per user.
- Maximum entropy quantization:
  - Divide probability density of the norm into interval of equal probability.
- When do we use the norm information?
  - SINR estimation after user selection.
  - Should use the post-user-selection distribution!

Post-User-Selection Distribution

- Users with strong values of $\|\widetilde{\mathbf{h}}_k\|^2$ are more probable to be selected.
  - The behavior depends on the scheduler.
  - Hard to derive post-user-selection probability since each user is unaware of other users.
- Possible to derive analytically for:
  - Select the M users (out of N) that have the largest CDF values.

Heuristic post-user-selection PDF

- Idea of the heuristic quantization:
  - One interval for really poor channel norms.
  - The rest of the probability mass divided equally.



PERFORMANCE EVALUATION

Simulation System

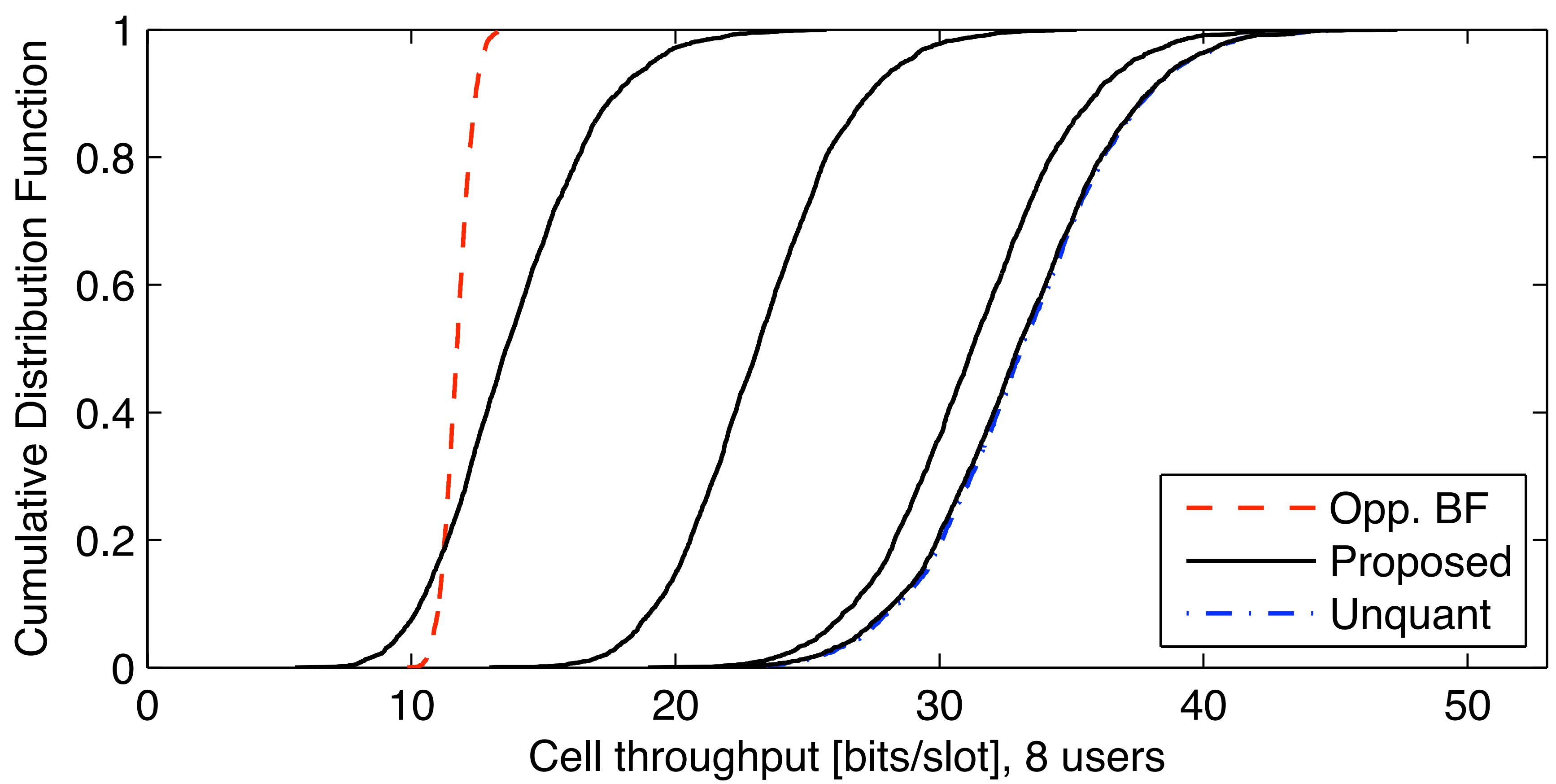
- Base Station
  - 8 antennas in a uniform circular array (UCA).
  - 15 degrees of angular spread.
- Mobile users
  - 4 antennas at each user.
  - Uniformly distributed in the cell.

Simulation Model

- Resource Allocation
  - Several users are selected in each time slot (greedily to maximize proportional fairness).
  - Transmit beamformers used to maximize the signal power, under the condition:
    - No interference allowed in other users dominating eigenmodes.
  - Equal power allocation, 10 dB at cell boundary.

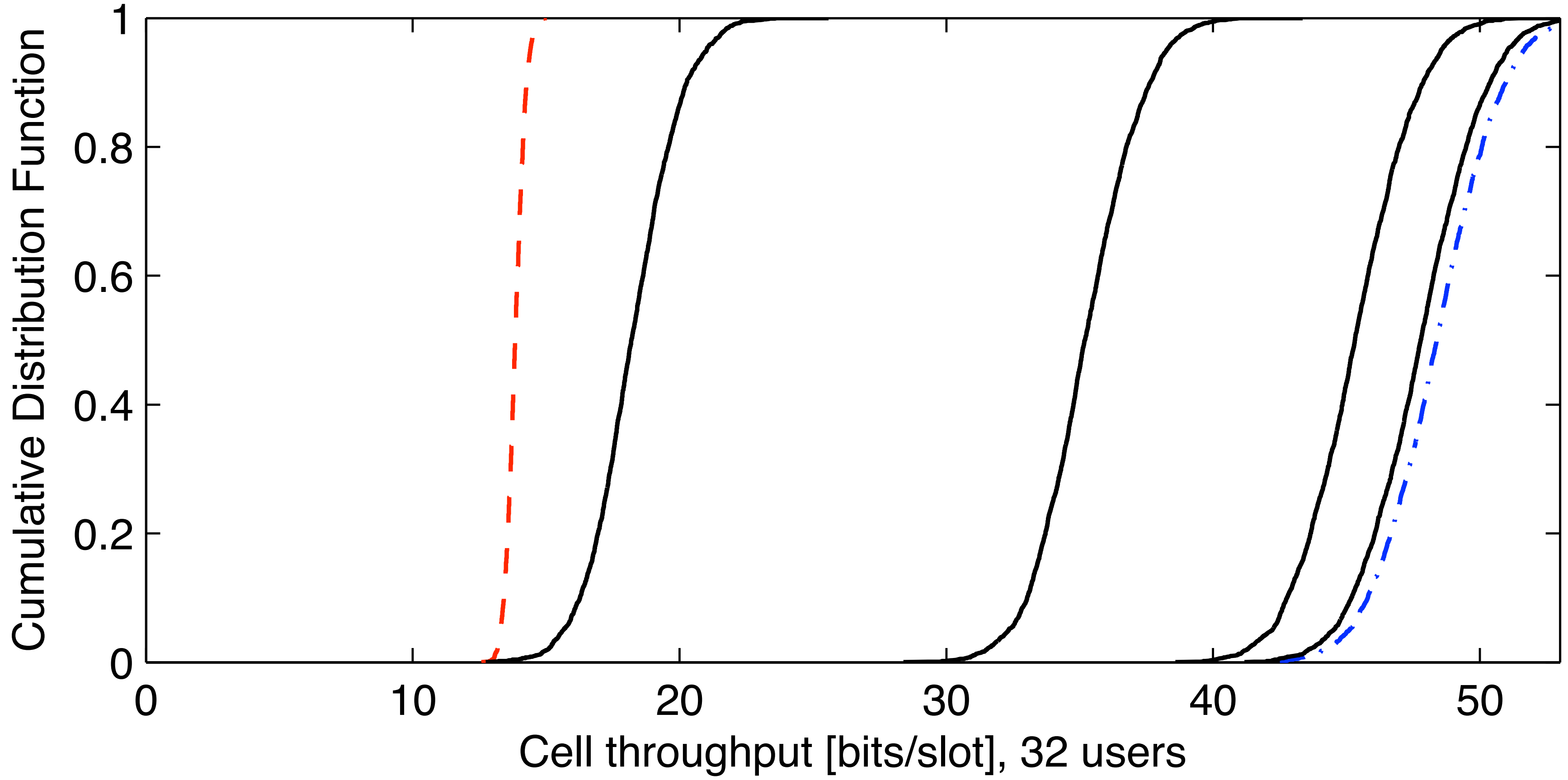
Simulation Results

- CDF of the cell throughput with 8 users:



Simulation Results

- CDF of the cell throughput with 32 users:



Observations

- 1 bit gives 50% of the performance gain, 3 bits gives 90%, 5 bits gives 99%.
- Multi-user opportunistic beamforming needs 3 bits just to choose a beamformer. It is clearly outperformed, even with exact feedback.

CONCLUSIONS

Conclusions

- When statistics are known at base station, transmission should take place along the strongest eigenvector.
- Receiver can cancel out interference in other eigenvector subspaces.
- Feedback of the channel norm makes reliable SINR estimation possible.
- Only a few bits are needed per user.

Some references

- References

  E. Björnson and B. Ottersten, "*Exploiting Long-Term Statistics in Spatially Correlated Multi-User MIMO Systems with Quantized Channel Norm Feedback.*"

  D. Hammarwall, "*Resource Allocation in Multi-Antenna Communication Systems with Limited Feedback,*" PhD Thesis.

- Even more details are given in an upcoming journal paper.