Optimal Coordinated Beamforming in the Multicell Downlink with Transceiver Impairments

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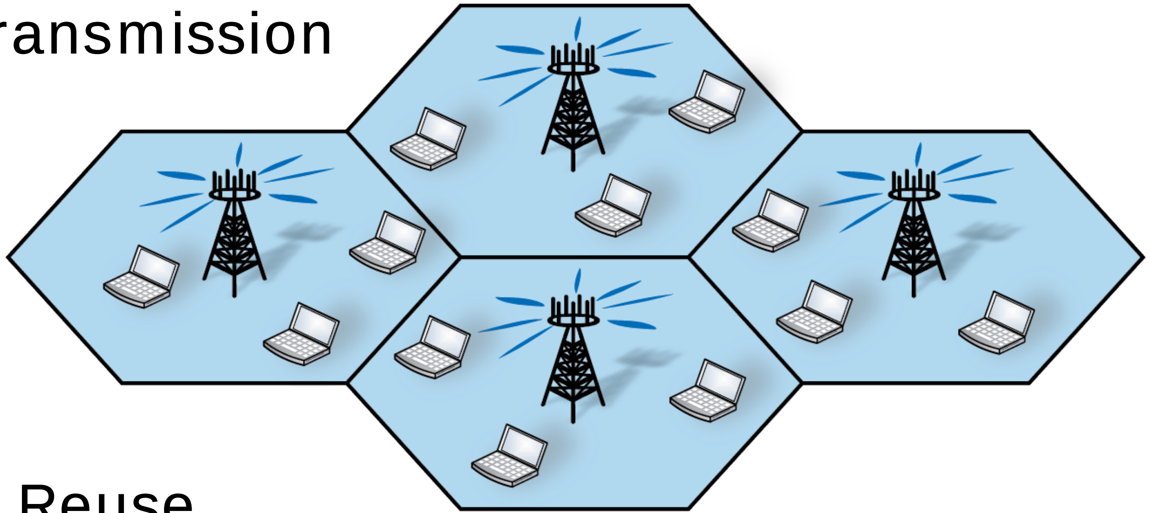
Introduction

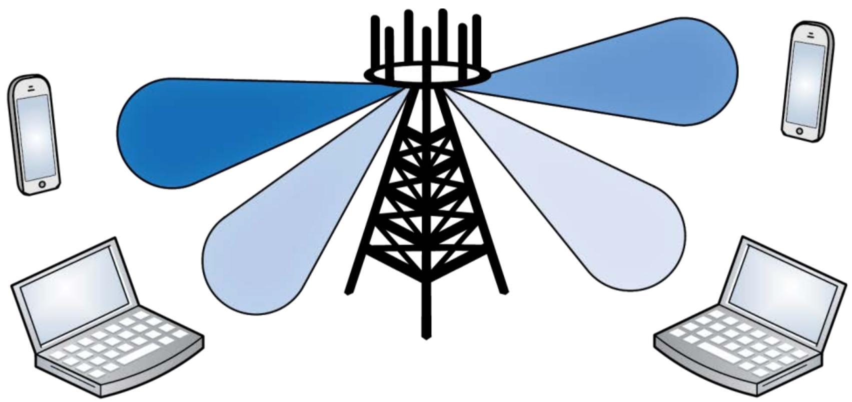
Coordinated Beamforming

- Downlink Multicell Transmission
  - N Base Stations
  - K Users per Cell



- Universal Frequency Reuse
  - Common Narrowband Frequency Resource
  - Limiting Factor: Inter-User Interference

- N_t-Antenna Base Stations
  - Beamforming: Spatially Directed Signals
  - Lower Interference



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Optimization of Beamforming

- Optimize System Utility
  - Many Possible Problem Formulations

- Two Main Categories of Optimization Problems

Focus in this Paper

- Convex Problems
  - Solvable in Practice (polynomial time)
  - Examples: Minimize power under rate constraints
    Maximize (weighted) worst-user rate

- Non-Convex Problems
  - Infeasible in Practice (exponential time)
  - Approximations Necessary
  - Examples: Weighted sum rate, Proportional fairness

Common Unrealistic Assumptions

- Unrealistic Assumptions Enable Analysis
  - Is Convexity Lost Otherwise?

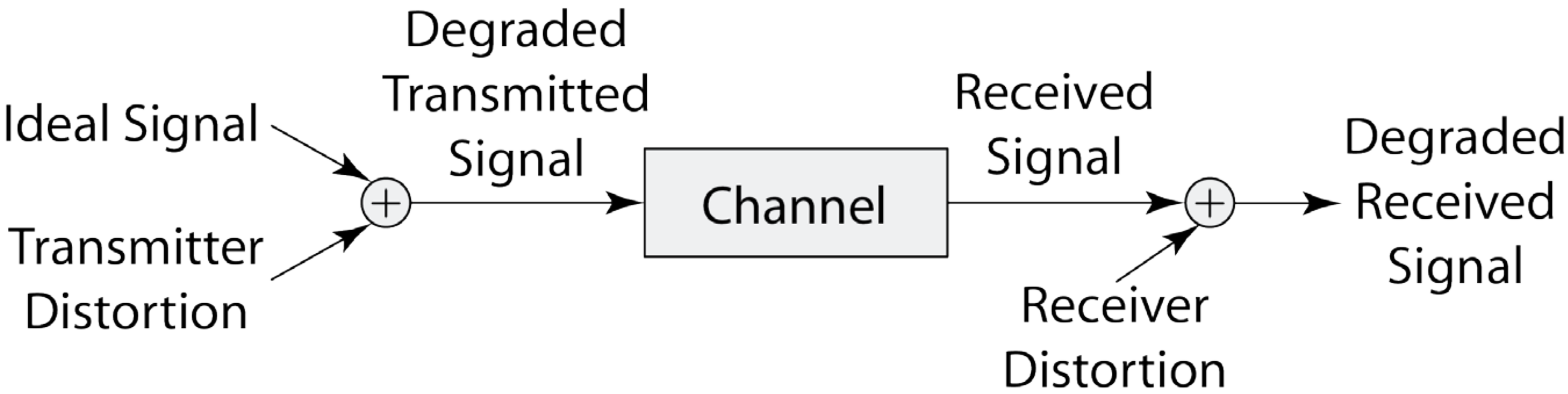
- Assumption: Perfect Channel Knowledge
  - Impractical: Estimation errors, feedback quantization, delays
  - Treated by Robust Optimization (convexity remains)
  - E. Björnson, G. Zheng, M. Bengtsson, B. Ottersten, “Robust Monotonic Optimization Framework for Multicell MISO Systems,” *IEEE Transactions on Signal Processing*, IEEE Trans. Signal Process., vol. 60, no. 5, 2012.

- Assumption: Centralized Optimization
  - Impractical: Limited backhaul, local computational resources
  - Handled by Primal/Dual Decomposition (convexity remains)
  - A. Tölli, H. Pennanen, and P. Komulainen, “Decentralized minimum power multi-cell beamforming with limited backhaul signaling,” *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, 2011.

Other Common Unrealistic Assumptions?

- Ideal Hardware is Commonly Assumed
  - Physical Transceivers Suffer From Impairments
  - Examples: Non-linear amplifiers, IQ imbalance, phase noise, carrier-frequency offset, quantization noise, etc.

- Degrading Impact on Transmission and Reception
  - Mismatch Between Ideal and Actual Signal
  - Distortion Power is Proportional to Signal Power



Transceiver Impairments

Transceiver Hardware Impairments

- Commonly Ignored in Beamforming Optimization
  - A Few Papers on Single-User Systems
  - Minor Impact on Single-User Low-Rate Transmission
  - Major Impact on
    1) High-rate transmission
    2) Inter-user interference
    3) Low-cost transceivers

- Exact Modeling
  - Separate distortion model of *each* component
  - Accurate but very hardware dependent

Focus in this Paper

- Simplified Modeling
  - Combined distortion model of *all* components
  - Accurate for residual distortion after calibration

Generalized System Model

- Parameters for User j in Cell i
  - Information Symbol: $x_{i,j} \sim \mathcal{CN}(0,1)$
  - Linear Beamforming: $\mathbf{w}_{i,j} \in \mathbb{C}^{N_t \times 1}$
  - Beamforming from Cell i: $\mathbf{W}_i = [\mathbf{w}_{i,1} \ldots \mathbf{w}_{i,K}] \in \mathbb{C}^{N_t \times K}$
  - Channel from Cell m: $\mathbf{h}_{m,i,j} \in \mathbb{C}^{N_t \times 1}$

- Received Signal at User j in Cell i

$$y_{i,j} = \sum_{m=1}^{N} \mathbf{h}_{m,i,j}^{H} \left(\sum_{k=1}^{K} \mathbf{w}_{m,k} x_{m,k} + \mathbf{z}_m^{(t)} \right) + z_{i,j}^{(r)}$$

Transmitter distortion

Receiver distortion

Characterization: Receiver Distortion

- Well-Modeled as Complex Gaussian: $z_{i,j}^{(r)} \sim \mathcal{CN}(0, \sigma_{i,j}^2)$
  - Aggregation of Many Impairments
  - Previously Verified by Measurements and Analysis

Received signal magnitude

Noise power

$$\sigma_{i,j}^2 = \sigma^2 + \nu^2 \left(\underbrace{\sqrt{\sum_{m=1}^{N} \|\mathbf{h}_{m,i,j}^H \mathbf{W}_m\|_F^2}}_{\text{Received signal magnitude}} \right)$$

Increasing convex function

- *Example*: $\nu(x) = \frac{\kappa_3}{100} x$
  - κ_3: Ratio of distortion to signal in percentage $(0 \leq \kappa_3 \leq 15)$
  - Smaller is Better

Characterization: Transmitter Distortion

- Also Well-Modeled as Gaussian: $\mathbf{z}_m^{(t)} \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_m)$
  - Linear with signal at low power
  - Faster than linear at high power

Error Vector Magnitude

$$\mathrm{EVM}_{m,n} = \frac{\eta\left(\|\mathbf{T}_n \mathbf{W}_m\|_F\right)}{\|\mathbf{T}_n \mathbf{W}_m\|_F}$$

$$\mathbf{C}_m = \begin{bmatrix} c_{m,1}^2 & & \\ & \ddots & \\ & & c_{m,N_t}^2 \end{bmatrix}, \quad c_{m,n} = \eta\left(\|\mathbf{T}_n \mathbf{W}_m\|_F\right)$$

Increasing convex function

Picks out transmit magnitude at *n*th antenna

- *Example*: $\eta(x) = \frac{\kappa_1}{100} x \left(1 + \left(\frac{x}{\kappa_2}\right)^4\right)$
  - κ_1: Base-level of distortion ($0 \leq \kappa_1 \leq 15$)
  - κ_2: Dynamic range of power amplifier (5^{th} order non-lin)

Optimization of Coordinated Beamforming

SINR ExpressionS

- Signal-to-interference-and-noise ratio of User j in Cell i:

$$\mathrm{SINR}_{i,j} = \frac{\overbrace{|\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}|^2}^{\text{Useful signal}}}{\underbrace{\sum_{l \neq j} |\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,l}|^2}_{\text{Intra-cell interference}} + \underbrace{\sum_{m \neq i} \|\mathbf{h}_{m,i,j}^H \mathbf{W}_m\|_F^2}_{\text{Inter-cell interference}} + \sum_{m,n} (\mathbf{h}_{m,i,j}^H \mathbf{T}_n \mathbf{h}_{m,i,j}) t_{m,n}^2 + r_{i,j}^2 + \sigma^2}$$

Transmitter distortion

Receiver distortion

- Extra variables:

$$\eta(\|\mathbf{T}_n \mathbf{W}_m\|_F) \leq t_{m,n} \quad \forall m,n$$

$$\nu\left(\sqrt{\sum_m \|\mathbf{h}_{m,i,j}^H \mathbf{W}_m\|_F^2}\right) \leq r_{i,j} \quad \forall i,j$$

  - Should be equality
  - If $t_{m,n}$, $r_{i,j}$ are seen as variables: Equality in optimal solution

Convexity is Retained

- Minimize Power under SINR Constraints: $\mathrm{SINR}_{i,j} \geq \gamma_{i,j}$
- Theorem: Solvable as Convex Optimization Problem

$$\underset{\beta, \mathbf{W}_i, t_{i,n}, r_{i,j} \, \forall i,j,n}{\text{minimize}} \quad \beta$$

$$\text{subject to} \quad t_{i,n} \geq 0, \; r_{i,j} \geq 0, \; \Im(\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}) = 0 \quad \forall i,j,n,$$

$$\mathrm{tr}(\mathbf{W}_i^H \mathbf{Q}_{i,k} \mathbf{W}_i) + \sum_n \mathrm{tr}(\delta \mathbf{Q}_{i,k} \mathbf{T}_n) t_{i,n}^2 \leq \beta q_{i,k} \quad \forall i,k,$$

$$\sqrt{\sum_m \|\mathbf{h}_{m,i,j}^H \mathbf{W}_m\|_F^2 + \sum_{m,n} (\mathbf{h}_{m,i,j}^H \mathbf{T}_n \mathbf{h}_{m,i,j}) t_{m,n}^2 + r_{i,j}^2 + \sigma^2} \leq \sqrt{1 + \frac{1}{\gamma_{i,j}}} \Re(\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}) \quad \forall i,j,$$

$$\eta(\|\mathbf{T}_n \mathbf{W}_m\|_F) \leq t_{m,n} \quad \forall m,n,$$

$$\nu\left(\sqrt{\sum_m \|\mathbf{h}_{m,i,j}^H \mathbf{W}_m\|_F^2}\right) \leq r_{i,j} \quad \forall i,j.$$

Main Point: Convexity is Retained Under Transceiver Impairments

Generalization of Optimization Problems

- (P1): Minimize Power under SINR/Rate Constraints
  - Convex Optimization Problem

- (P2): Maximize Worst-User Rate
  - Solved as Sequence of (P1)-Problems
  - (Quasi-)Convex Optimization Problem

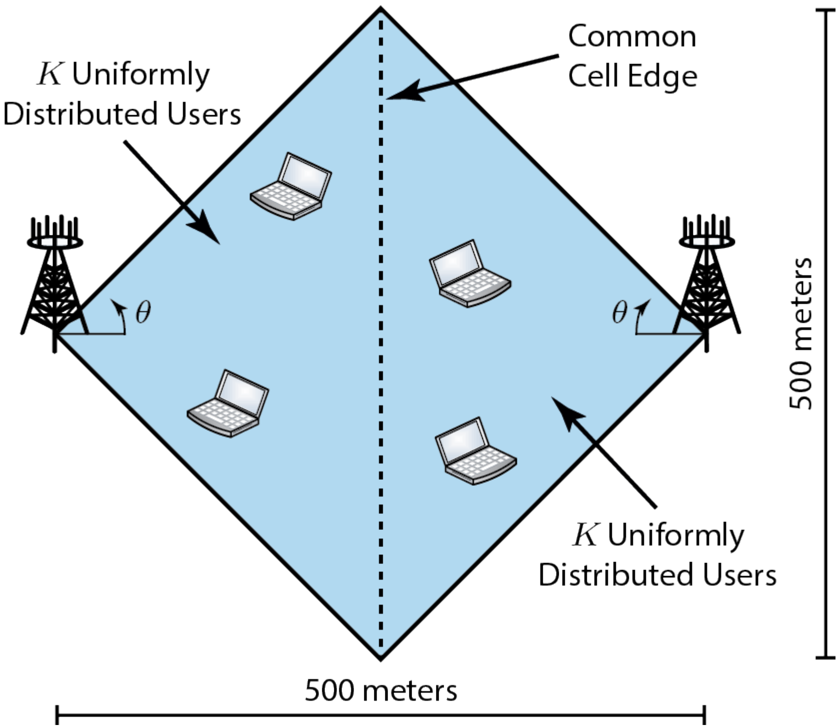
Numerical Examples

Simulation Scenario

- Maximize Worst-User Rate (Max-Min Fairness)

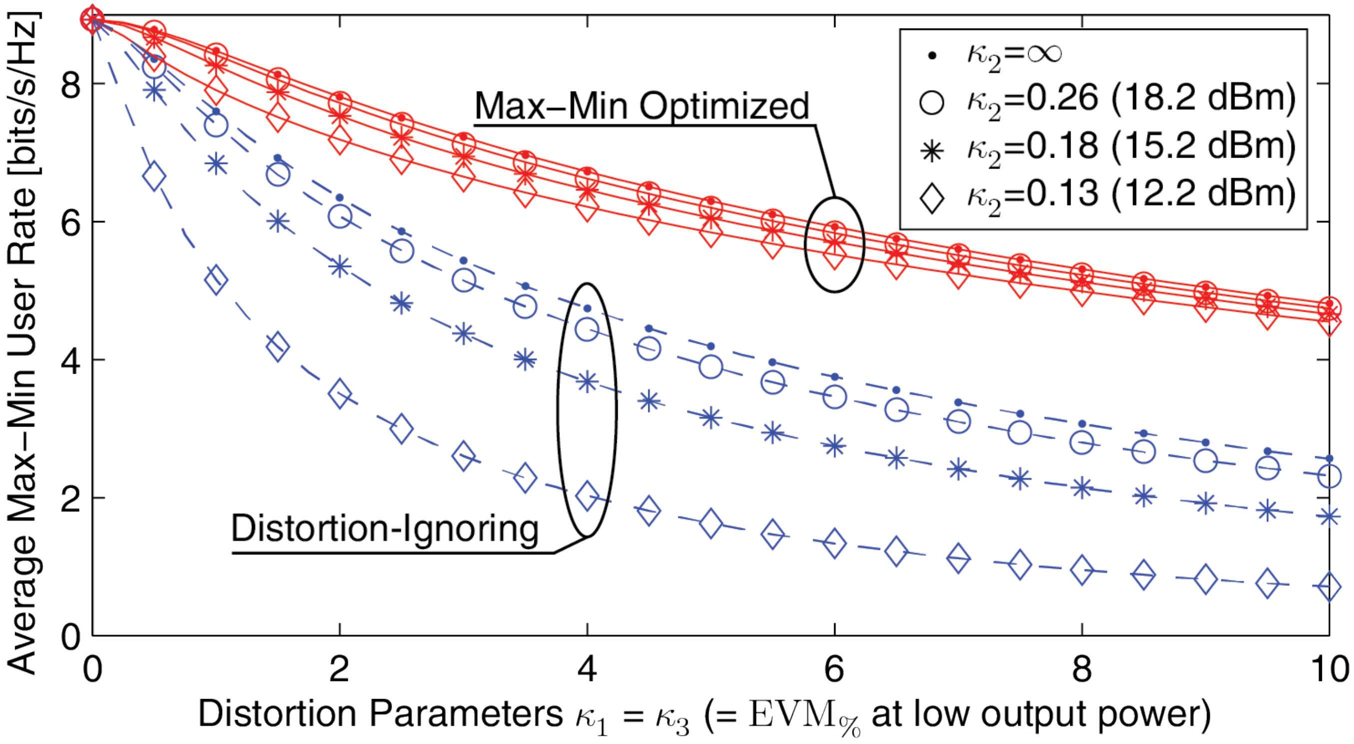
- Two Schemes:
  - Optimal Beamforming with Transceiver Impairments
  - Distortion-Ignoring Optimized Beamforming

- Simulation Scenario
  - 2 Base Stations
  - 3GPP LTE Case 1



Average Max-Min User Rate

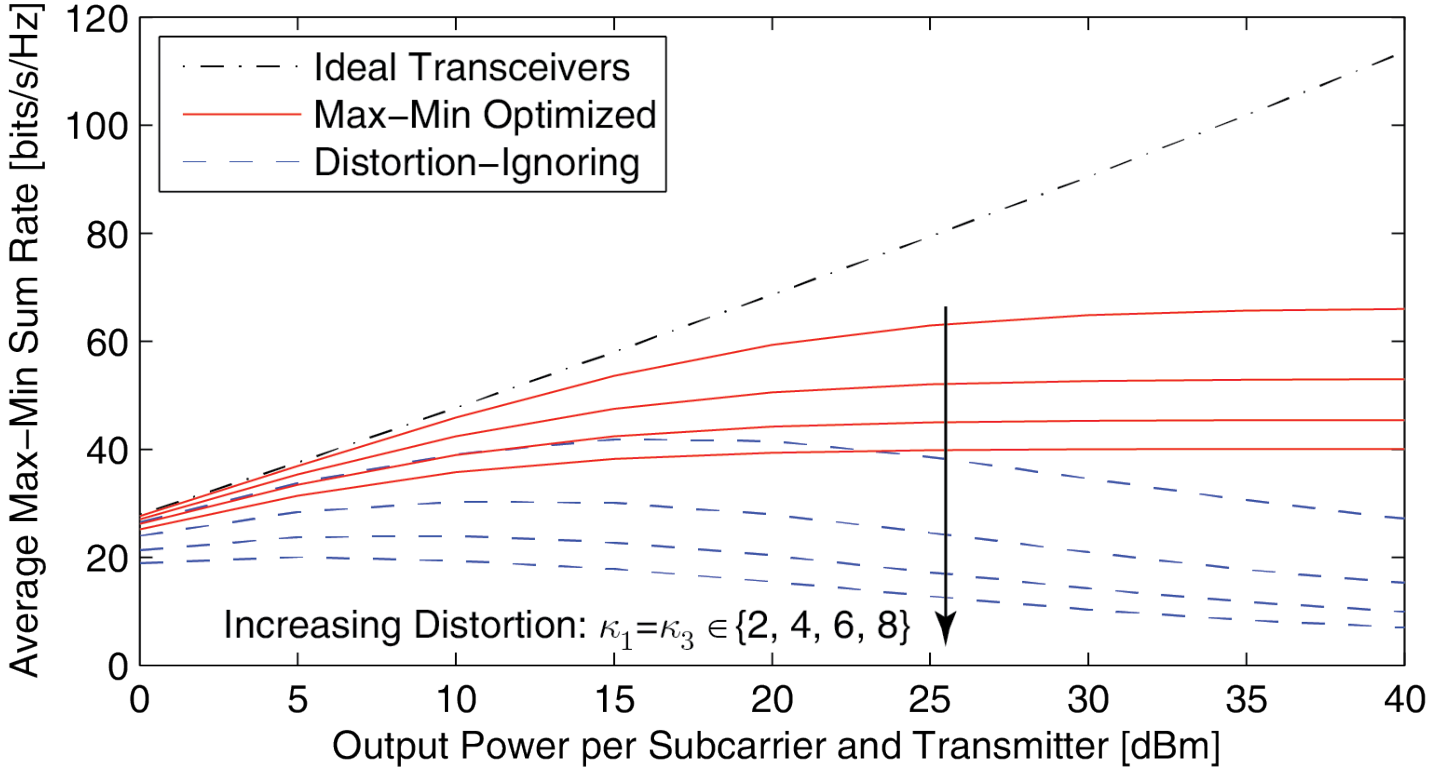
- Parameters
  - N_t = 4 antennas/BS, K = 2 users/cell
  - X-axis: $\kappa_1 = \kappa_3$ = EVM in % at transmitter/receiver



Conclusion: Smaller loss when optimized for impairments

Impact on Multiplexing Gain

- Parameters
  - N_t = 8 antennas/BS, K = 4 users/cell
  - X-axis: Transmit Power



Conclusion: Finite High-SNR Limit (No multiplexing gain)

Summary

Summary

- Transceiver Impairments
  - Physical Transceivers are Not Perfect
  - Small Impact in the Past
  - Major Impact in the Future: High spectral efficiency
Small inter-user interference

- Contributions
  - Tractable Mathematical Formulation
  - Minimize Power under SINR Constraints – Convex Problem
  - Maximize Worst-User Rate – Convex Problem

- Observations
  - Optimization Makes Degradations Much Smaller
  - Finite High-SNR Limit – No Multiplexing Gain

Additional Work

- Extension to General Multi-Cell Scenarios

    - E. Björnson, E. Jorswieck, "Optimal resource allocation in coordinated multi-cell systems," Foundations and Trends in Communications and Information Theory, to appear

- Analysis of Finite High-SNR Limit
    - Multiplexing is Very Useful – Although Multiplexing Gain is 0

    - E. Björnson, P. Zetterberg, M. Bengtsson, B. Ottersten, "Capacity Limits and Multiplexing Gains of MIMO Channels with Transceiver Impairments," IEEE Communications Letters, to appear

Thank You for Listening!

Questions?

All Papers Available:

http://flexible-radio.com/emil-bjornson

Backup Slides

Power Constraints

- Arbitrary Power Constraints in Cell i
  - Constraints:

$$\underbrace{\mathrm{tr}(\mathbf{W}_i^H \mathbf{Q}_{i,k} \mathbf{W}_i) + \sum_n \mathrm{tr}(\delta \mathbf{Q}_{i,k} \mathbf{T}_n) t_{i,n}^2}_{} \leq q_{i,k} \quad \forall i, k,$$

Positive semi-definite shape matrix

Positive limit

  - $0 \leq \delta \leq 1$ defines the extra power consumed by distortions
  - *Examples:* Per-antenna constraints
    Per-cell constraints
    Soft-shaping constraints