PIMRC 2013 Tutorial

Optimal Resource Allocation in Coordinated Multi-Cell Systems

Emil Björnson[1] and Eduard Jorswieck[2]

[1] Signal Processing Lab., KTH Royal Institute of Technology, Sweden and
Alcatel-Lucent Chair on Flexible Radio, Supélec, France

[2] Dresden University of Technology,
Department of Electrical Engineering and Information Technology, Germany

8 September 2013

Biography: Emil Björnson

- 1983: Born in Malmö, Sweden
- 2007: Master of Science in Engineering Mathematics, Lund University, Sweden
- 2011: PhD in Telecommunications, KTH, Stockholm, Sweden
  Advisors: Björn Ottersten, Mats Bengtsson
- 2012: Recipient of International Postdoc Grant from Sweden. Work with Prof. Mérouane Debbah at Supélec on “Optimization of Green Small-Cell Networks”

Biography: Eduard Jorswieck

- 1975: Born in Berlin, Germany
- 2000: Dipl.-Ing. in Electrical Engineering and Computer Science, TU Berlin, Germany
- 2004: PhD in Electrical Engineering, TU Berlin, Germany
  Advisor: Holger Boche
- 2006: Post-Doc Fellowship and Assistant Professorship at KTH Stockholm, Sweden
- 2008: Full Professor and Head of Chair of Communications Theory at TU Dresden, Germany

Book Reference

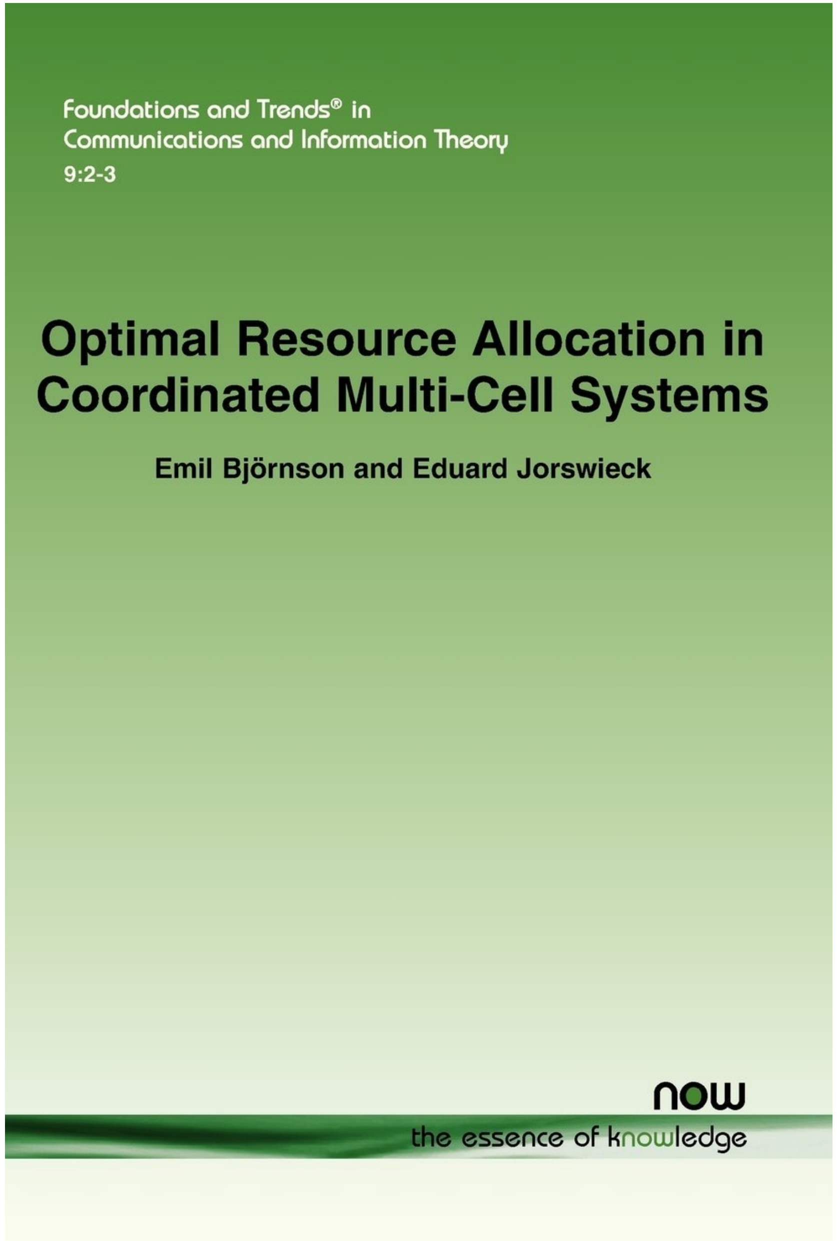
- Tutorial is Based on a Recent Book:

Optimal Resource Allocation in Coordinated Multi-Cell Systems

Research book by E. Björnson and E. Jorswieck

Foundations and Trends in Communications and Information Theory, Vol. 9, No. 2-3, pp. 113-381, 2013

- 270 pages
- E-book for free (from our homepages)
- Printed book: Special price $35, use link: https://ecommerce.nowpublishers.com/shop/add_to_cart?id=1595
- Matlab code is available online



Check out: http://flexible-radio.com/emil-bjornson

General Outline

- Book Consists of 4 Chapters

- Introduction
  - Problem formulation and general system model

- Optimal Single-Objective Resource Allocation
  - Which problems are practically solvable?

- Structure of Optimal Resource Allocation
  - How does to optimal solution look like?

Covered by
Emil Björnson
in first 90 mins

- Extensions and Generalizations
  - Applications to 9 current research topics
  - E.g., channel uncertainty, distributed optimization, hardware impairments, cognitive radio, etc.

Covered by
Eduard Jorswieck
in second 90 mins

Outline: Part 1

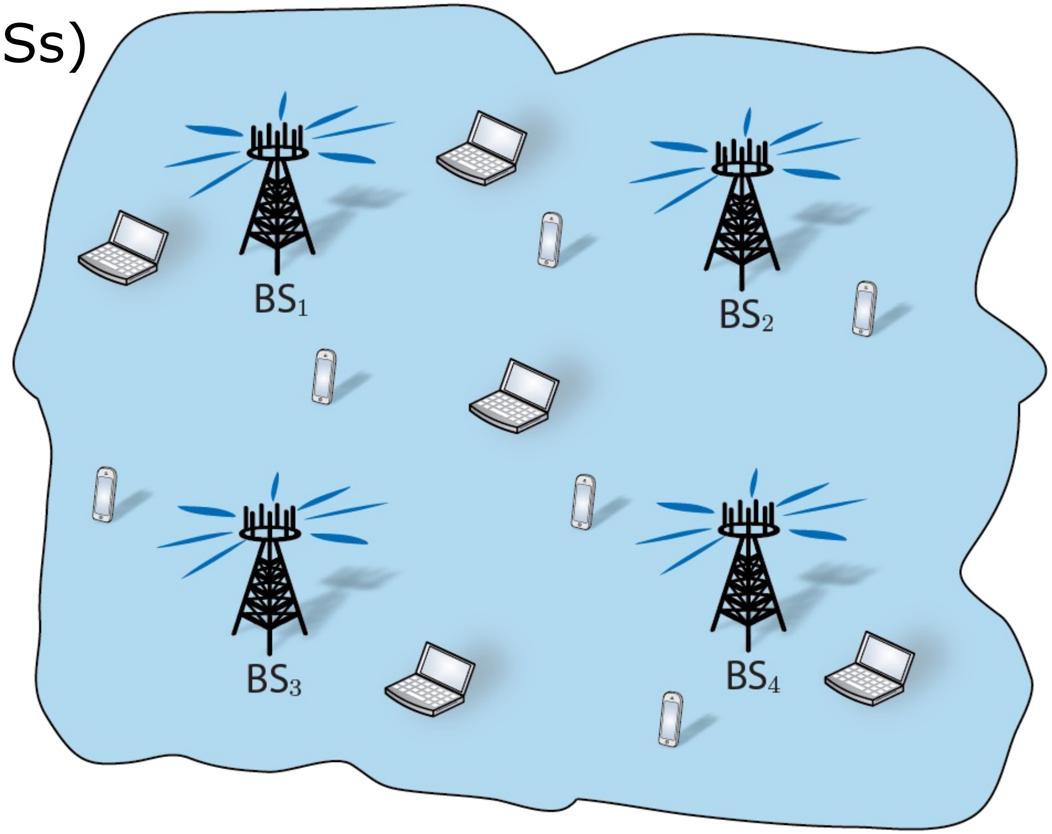
- Introduction
  - Multi-cell structure, system model, performance measure
- Problem Formulation
  - Resource allocation: Multi-objective optimization problem
- Subjective Resource Allocation
  - Utility functions, different computational complexity
- Structure of Optimal Beamforming
  - Beamforming parametrization and its applications

Section

Introduction

Introduction

- Problem Formulation (vaguely):
  - Transfer information wirelessly to users
  - Divide radio resources among users (time, frequency, space)
- Downlink Coordinated Multi-Cell System
  - Many transmitting base stations (BSs)
  - Many receiving users
- Sharing a Frequency Band
  - All signals reach everyone!
- Limiting Factor
  - Inter-user interference

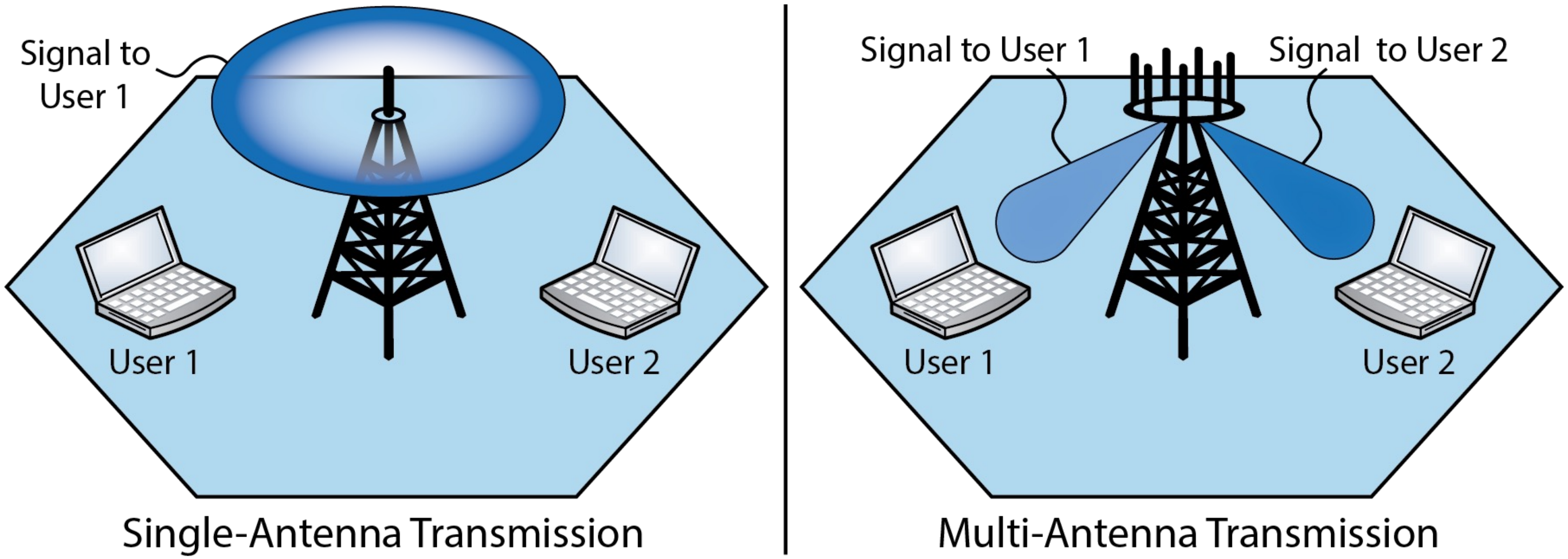


Introduction: Multi-Antenna Transmission

- Traditional Ways to Manage Interference
  - Avoid and suppress in time and frequency domain
  - Results in orthogonal access techniques: TDMA, OFDMA, etc.

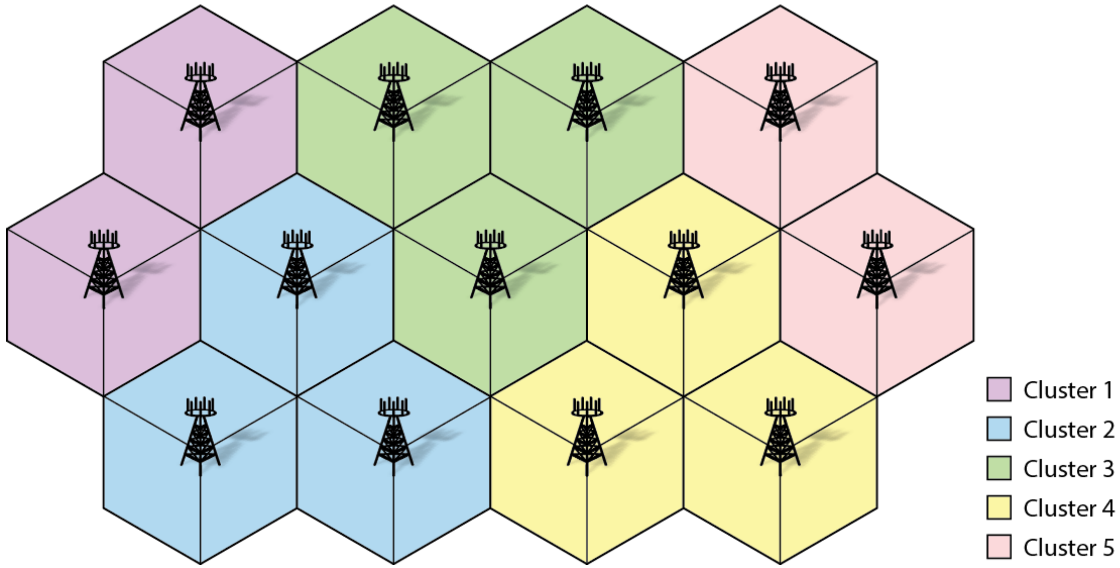
- Multi-Antenna Transmission
  - Beamforming: Spatially directed signals
  - Adaptive control of interference
  - Serve multiple users: Space-division multiple access (SDMA)

Main difference from classical resource allocation!



Introduction: From Single-Cell to Multi-Cell

- Naïve Multi-Cell Extension
  - Divide BS into disjoint clusters
  - SDMA within each cluster
  - Avoid inter-cluster interference
  - Fractional frequency-reuse



- Coordinated Multi-Cell Transmission
  - SDMA in multi-cell: All BSs collaborate
  - Frequency-reuse one: Interference managed by beamforming
  - Many names: *co-processing*, *coordinated multi-point (CoMP)*, *network MIMO*, *multi-cell processing*

- Almost as One Super-Cell
  - But: Different data knowledge, channel knowledge, power constraints!

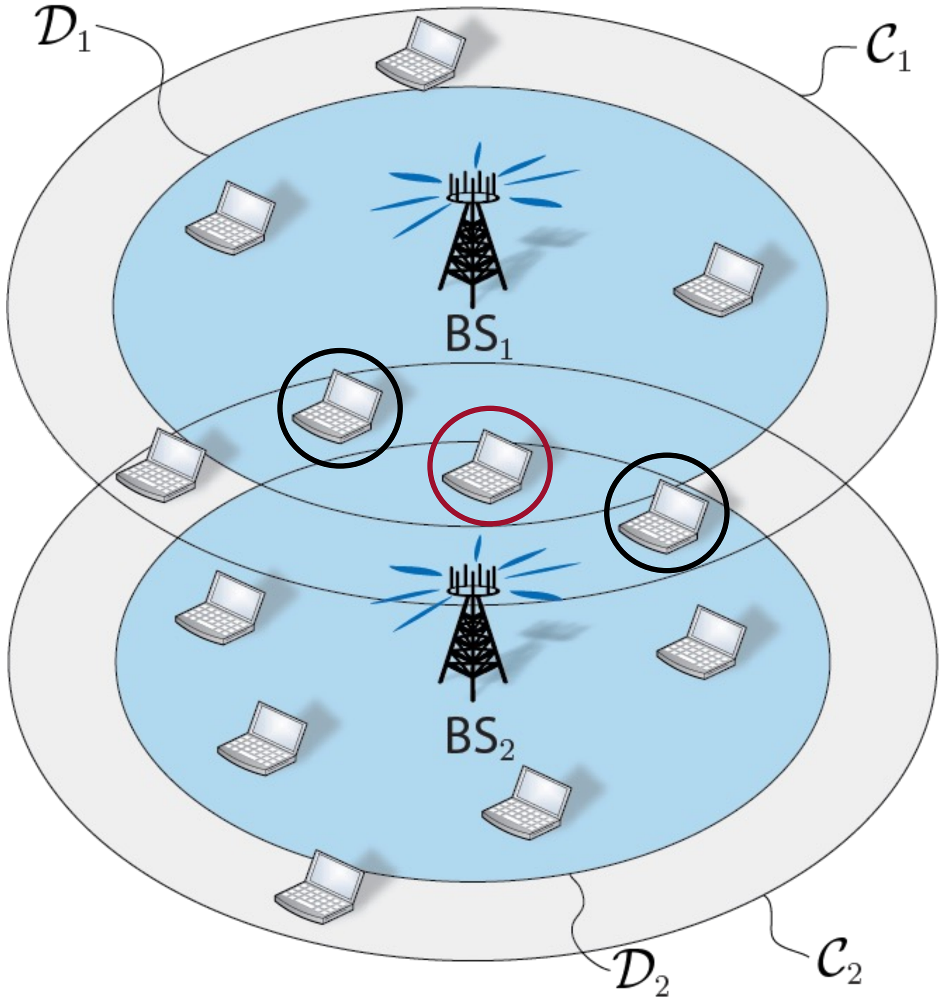
Basic Multi-Cell Coordination Structure

- General Multi-Cell Coordination
  - Adjacent base stations coordinate interference
  - Some users served by multiple base stations

Dynamic Cooperation Clusters

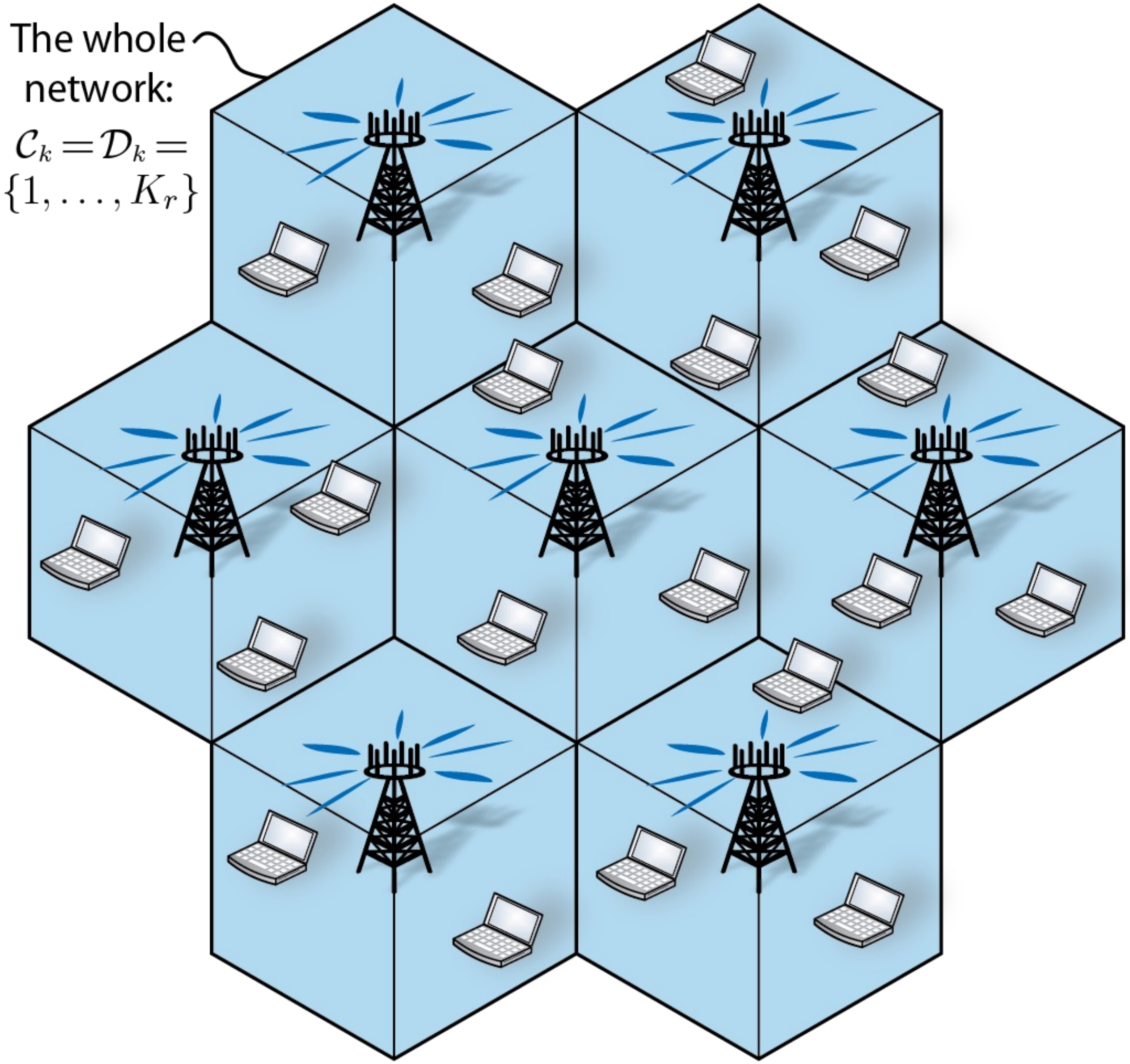
- Inner Circle $\mathcal{D}_k$: Serve users with data
- Outer Circle $\mathcal{C}_k$: Suppress interference
- Outside Circles:

  Negligible impact
  Impractical to acquire information
  Difficult to coordinate decisions



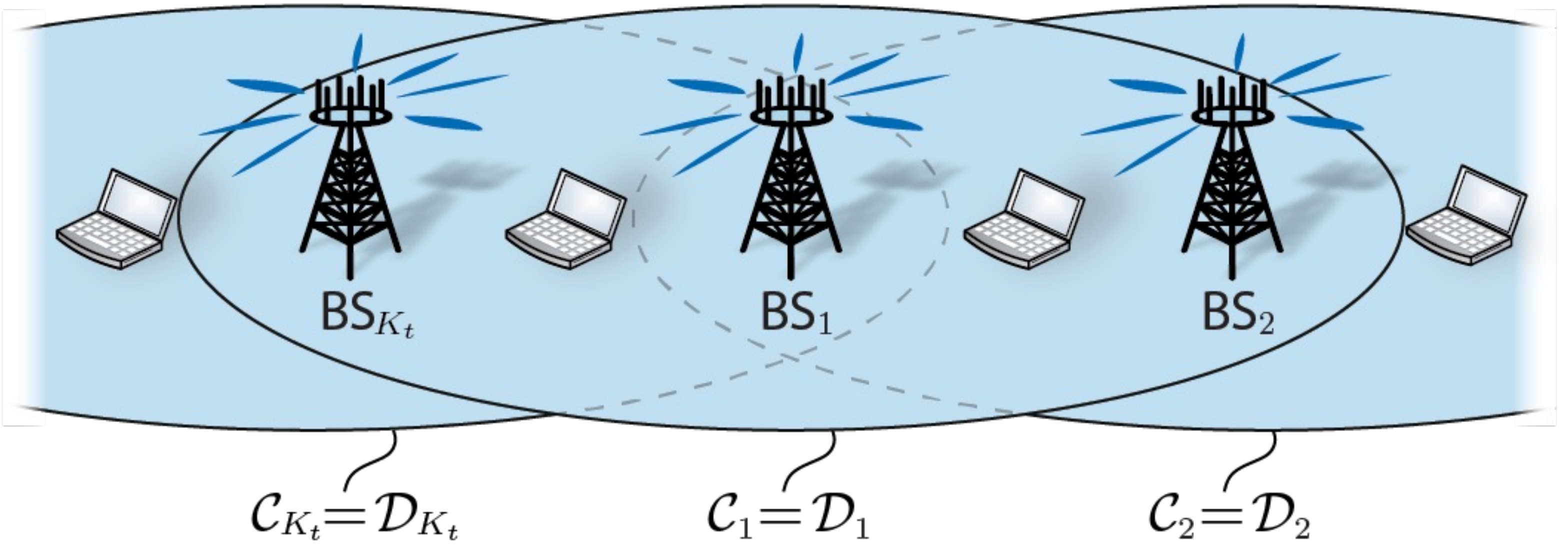
- E. Björnson, N. Jaldén, M. Bengtsson, B. Ottersten, "Optimality Properties, Distributed Strategies, and Measurement-Based Evaluation of Coordinated Multicell OFDMA Transmission," IEEE Trans. on Signal Processing, 2011.

Example: Ideal Joint Transmission



- All Base Stations Serve All Users Jointly = One Super Cell

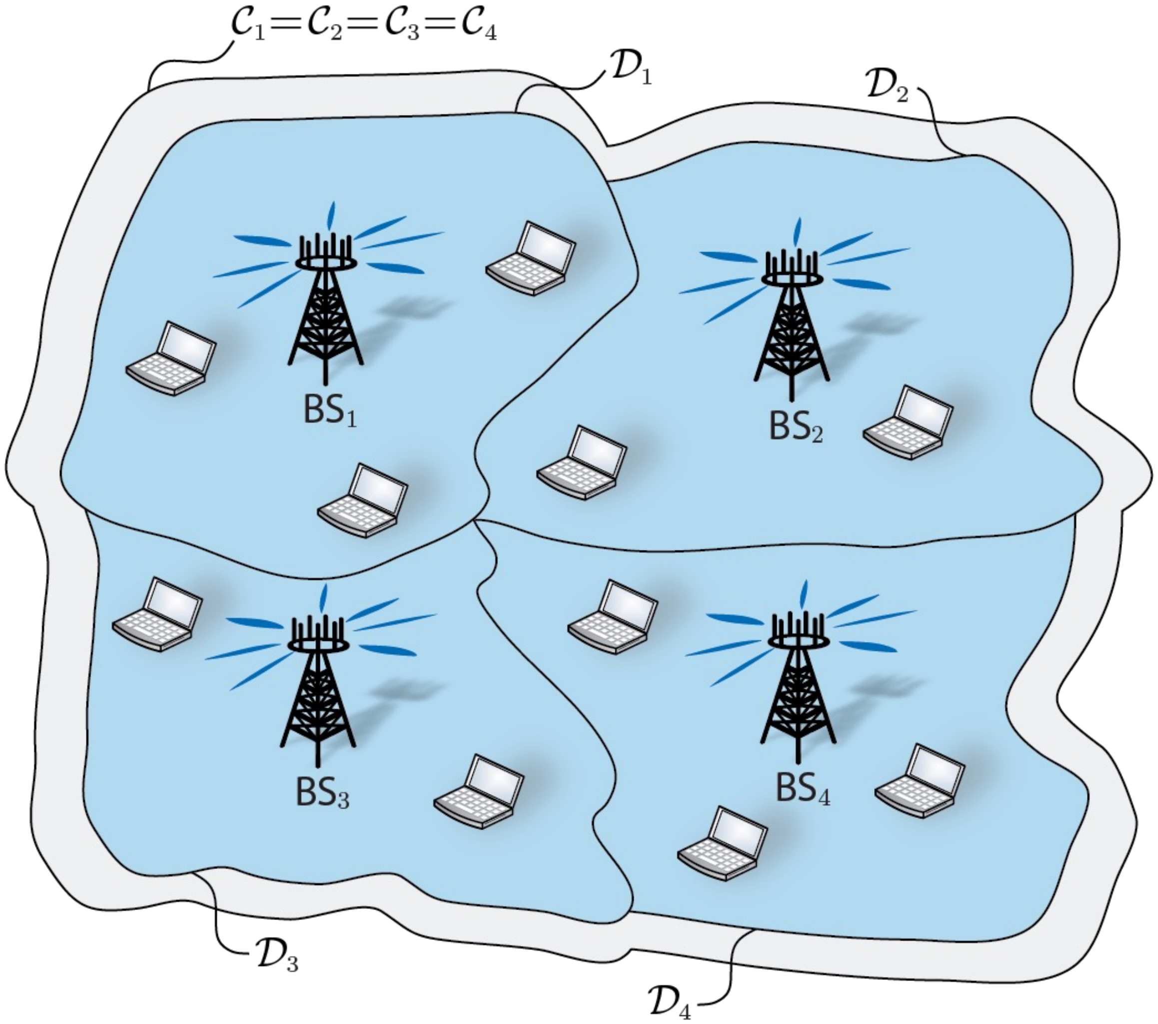
Example: Wyner Model



- Abstraction: User receives signals from own and neighboring base stations

- One or Two Dimensional Versions
- Joint Transmission or Coordination between Cells

Example: Coordinated Beamforming

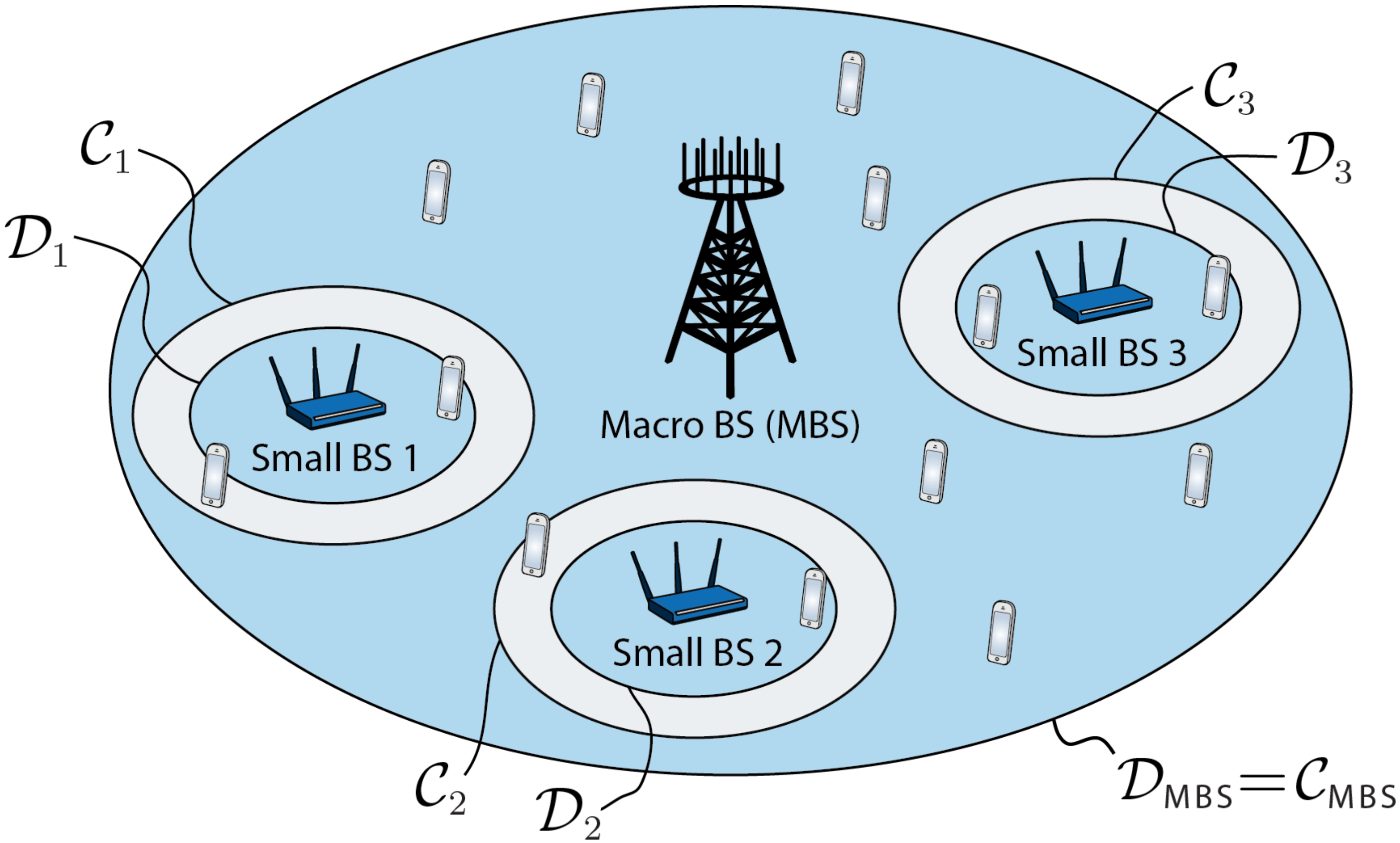


Special Case

Interference channel

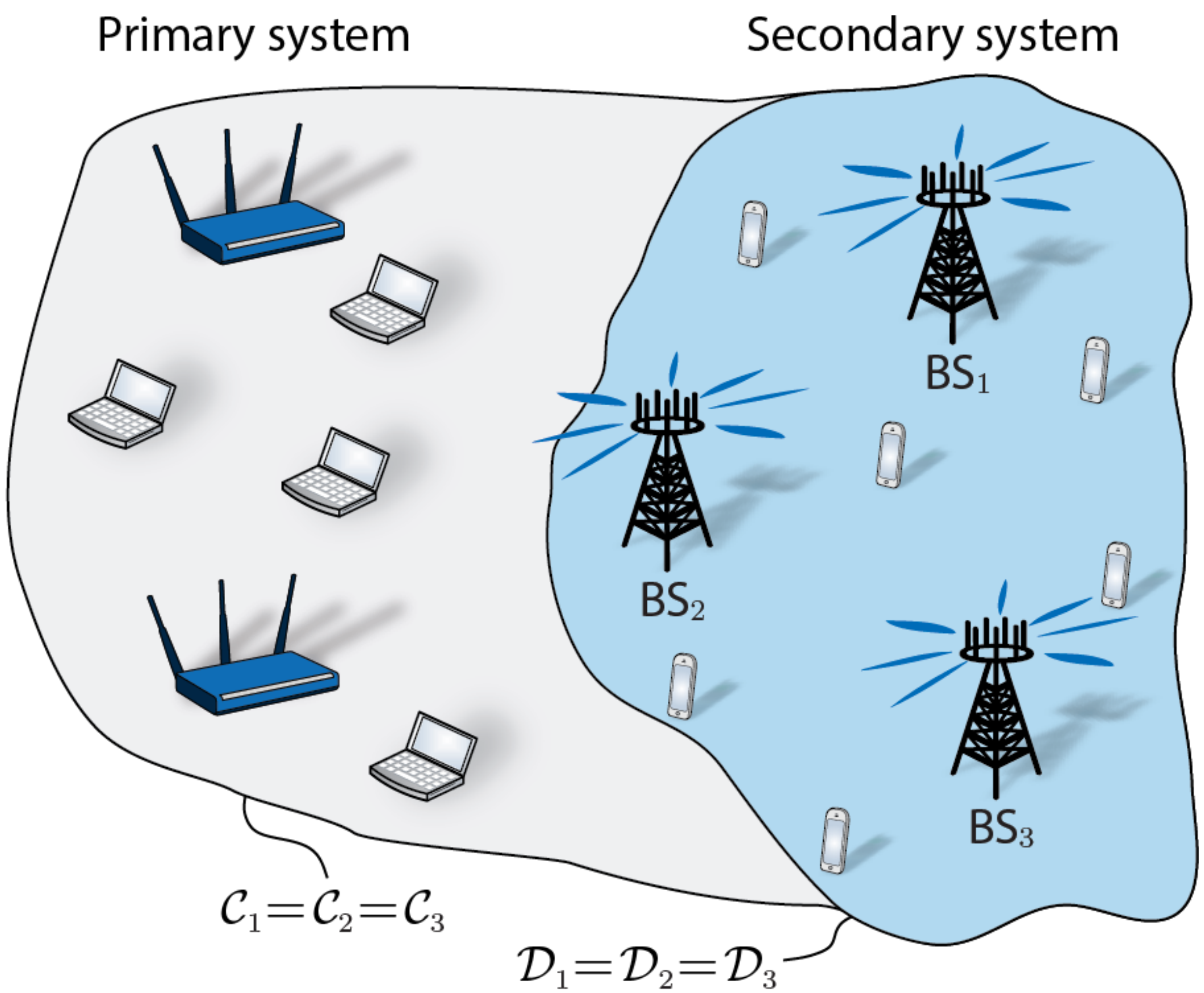
- One Base Station Serves Each User
- Interference Coordination Across Cells

Example: Soft-Cell Coordination



- Heterogeneous Deployment
  - Conventional macro BS overlaid by short-distance small BSs
  - Interference coordination and joint transmission between layers

Example: Cognitive Radio



Other Examples

Spectrum sharing between operators

Physical layer security

- Secondary System Borrows Spectrum of Primary System
  - Underlay: Interference limits for primary users

Resource Allocation: First Definition

- Problem Formulation (imprecise):
  - Select beamforming to maximize "system utility"
  - Means: Allocate power to users and in spatial dimensions
  - Satisfy: Physical, regulatory & economic constraints

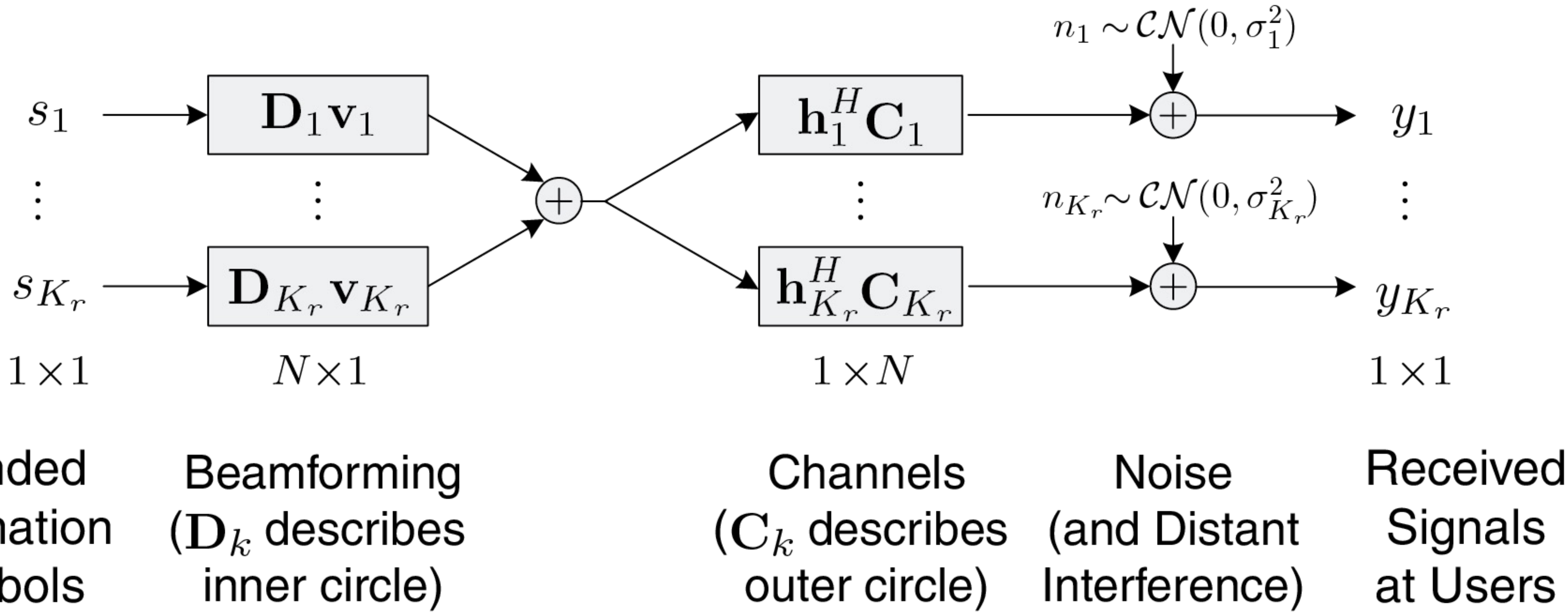
- Some Assumptions:
  - Linear transmission and reception
  - Perfect synchronization (whenever needed)
  - Flat-fading channels (e.g., using OFDM)

  - Perfect channel knowledge
  - Ideal transceiver hardware
  - Centralized optimization

  Will be relaxed in Part 2

Multi-Cell System Model

- K_r Users: Channel vector $\mathbf{h}_k = [\mathbf{h}_{1k}^T \ldots \mathbf{h}_{K_t k}^T]^T$ to User k from all BSs
- N_j Antennas at jth BS (dimension of $\mathbf{h}_{jk}$)
- $N = \sum_j N_j$ Antennas in Total (dimension of $\mathbf{h}_k$)



One System Model for Any Multi-Cell Scenario!

Multi-Cell System Model: Dynamic Cooperation Clusters

- How are $\mathbf{D}_k$ and $\mathbf{C}_k$ Defined?
  - Consider User k:

$$\mathbf{D}_k = \begin{bmatrix} \mathbf{D}_{1k} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{D}_{K_t k} \end{bmatrix} \quad \text{where } \mathbf{D}_{jk} = \begin{cases} \mathbf{I}_{N_j}, & \text{if } k \in \mathcal{D}_j, \\ \mathbf{0}_{N_j}, & \text{otherwise,} \end{cases}$$

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{C}_{1k} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{C}_{K_t k} \end{bmatrix} \quad \text{where } \mathbf{C}_{jk} = \begin{cases} \mathbf{I}_{N_j}, & \text{if } k \in \mathcal{C}_j, \\ \mathbf{0}_{N_j}, & \text{otherwise.} \end{cases}$$

- Interpretation:
  - Block-diagonal matrices
  - $\mathbf{D}_k$ has identity matrices for BSs that send *data*
  - $\mathbf{C}_k$ has identity matrices for BSs that can/should *coordinate* interference

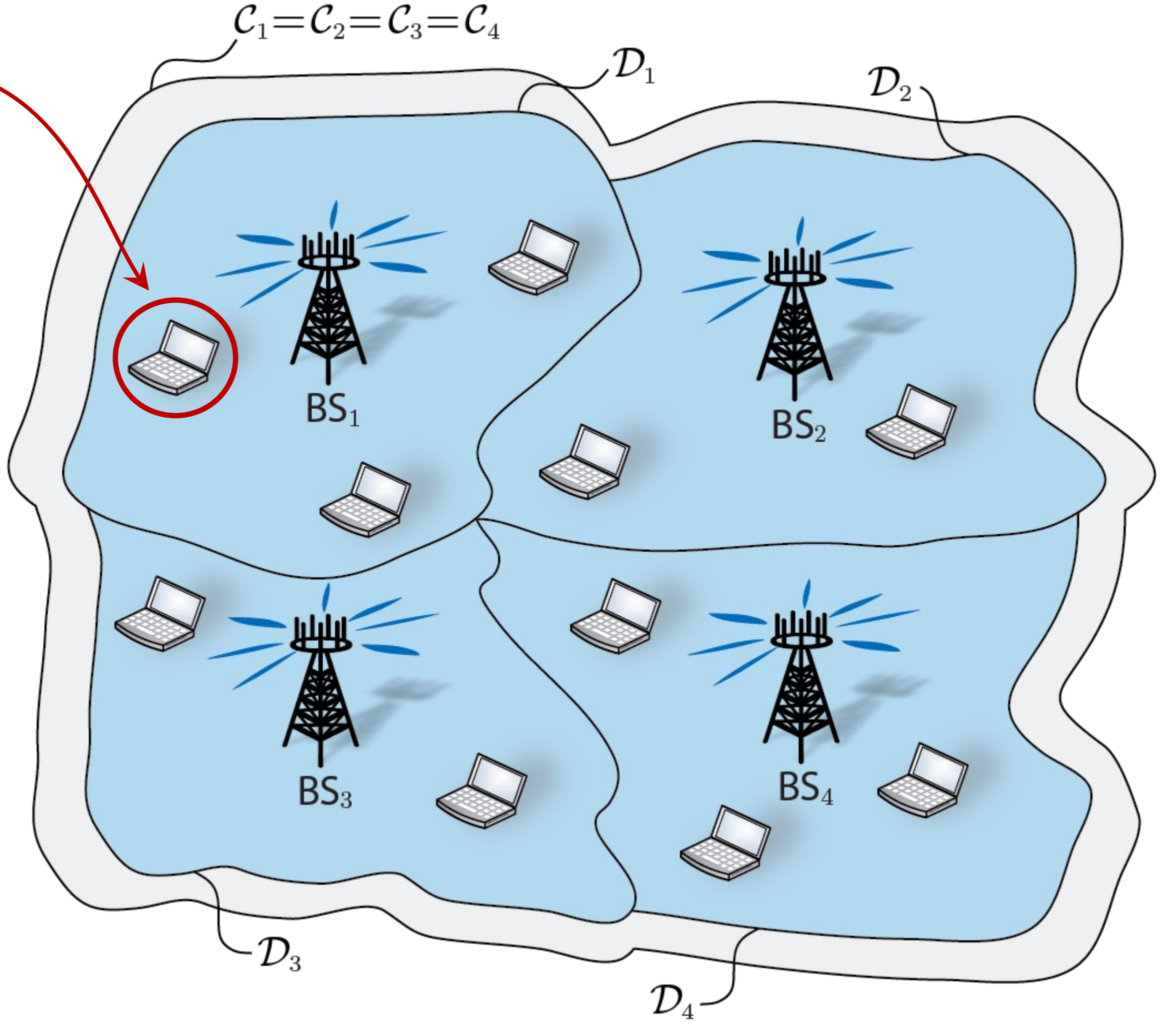
Multi-Cell System Model: Dynamic Cooperation Clusters (2)

- Example: Coordinated Beamforming
  - This is User k
  - Beamforming: $\mathbf{D}_k \mathbf{v}_k$
    Data only from BS_1:

$$\mathbf{D}_k = \begin{bmatrix} \mathbf{I}_{N_1} & & & \\ & \mathbf{0}_{N_2} & & \\ & & \mathbf{0}_{N_3} & \\ & & & \mathbf{0}_{N_4} \end{bmatrix}$$

  - Effective channel: $\mathbf{C}_k \mathbf{h}_k$
    Interference from all BSs:

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{I}_{N_1} & & & \\ & \mathbf{I}_{N_2} & & \\ & & \mathbf{I}_{N_3} & \\ & & & \mathbf{I}_{N_4} \end{bmatrix}$$



Multi-Cell System Model: Power Constraints

- Need for Power Constraints
  - Limit radiated power according to regulations
  - Protect dynamic range of amplifiers
  - Manage cost of energy expenditure
  - Control interference to certain users

All at the same time

- L General Power Constraints:

$$\sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l, \quad l = 1, \ldots, L$$

Weighting matrix
(Positive semi-definite)

Limit
(Positive scalar)

Multi-Cell System Model: Power Constraints (2)

- Recall: $$\sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l$$

- Example 1, Total Power Constraint: $L = 1$: $\mathbf{Q}_{1k} = \mathbf{I}_N$

$$q_1 = \text{Maximal total power}$$

- Example 2, Per-Antenna Constraints:

$$L = N\text{:}\ \ \mathbf{Q}_{1k} = \mathrm{diag}(1, 0, \ldots, 0),\ \ \ldots,\ \ \mathbf{Q}_{Nk} = \mathrm{diag}(0, \ldots, 0, 1)$$
$$q_l = \text{Maximal power at } l\text{th antenna}$$

- Example 3, Control Interference to User i

$$\mathbf{Q}_{1k} = \mathbf{h}_i \mathbf{h}_i^H \text{ for all } k \neq i, \quad \mathbf{Q}_{1i} = \mathbf{0}_N$$
$$q_1 = \text{Maximal interference power caused to User } i$$

Introduction: How to Measure User Performance?

- Mean Square Error (MSE)
  - Difference: transmitted and received signal
  - Easy to Analyze
  - Far from User Perspective?
- Bit/Symbol Error Probability (BEP/SEP)
  - Probability of error (for given data rate)
  - Intuitive interpretation
  - Complicated & ignores channel coding
- Information Rate
  - Bits per “channel use”
  - Mutual information: perfect and long coding
  - Anyway closest to reality?

All improves with SINR:

$$\frac{\text{Signal}}{\text{Interference + Noise}}$$

Introduction: Generic Measure User Performance

- Generic Model
  - Any function of signal-to-interference-and-noise ratio (SINR):

$$g_k(\mathrm{SINR}_k) = g_k\left(\frac{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2}{\sigma_k^2 + \sum_{i \neq k} |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2}\right) \quad \text{for User } k$$

  - Increasing and continuous function
  - For simplicity: $g_k(0) = 0$

- Examples:
  - Information rate: $g_k(\mathrm{SINR}_k) = \log_2(1 + \mathrm{SINR}_k)$
  - MSE: $g_k(\mathrm{SINR}_k) = \frac{\mathrm{SINR}_k}{1+\mathrm{SINR}_k}$

- Complicated Function
  - Depends on all beamforming vectors $\mathbf{v}_1, \ldots, \mathbf{v}_{K_r}$

Section: Introduction

Questions?

Section

Problem Formulation

Problem Formulation

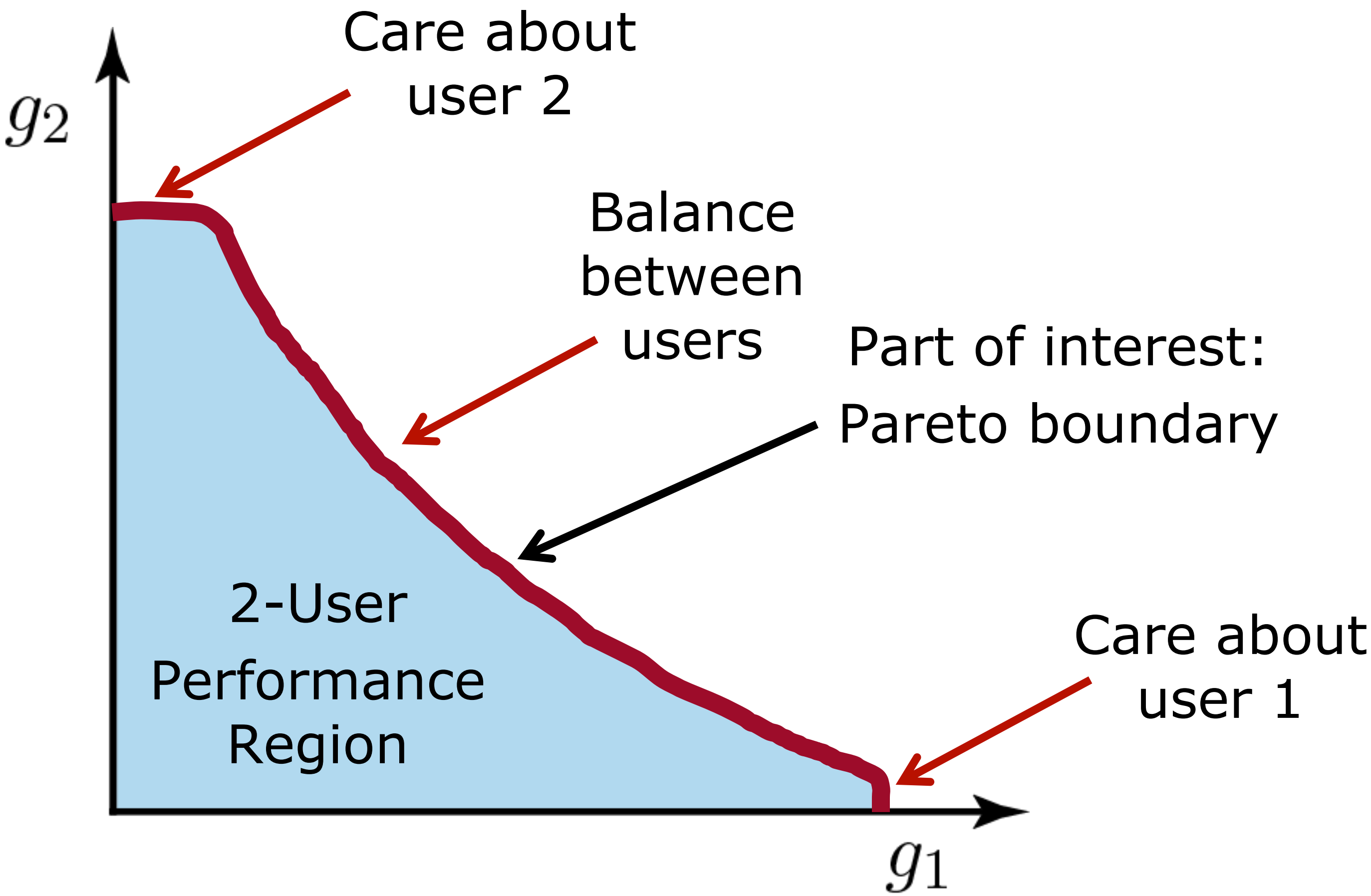
- General Formulation of Resource Allocation:

$$\begin{aligned} \underset{\mathbf{v}_1,\ldots,\mathbf{v}_{K_r}}{\text{maximize}} \quad & \{g_1(\text{SINR}_1),\ldots,g_{K_r}(\text{SINR}_{K_r})\} \\ \text{subject to} \quad & \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \quad \forall l. \end{aligned}$$

- Multi-Objective Optimization Problem
  - Generally impossible to maximize for all users!
  - Must divide power and cause inter-user interference

Performance Region

- Definition: Achievable Performance Region $\mathcal{R}$
  - Contains all feasible combinations $\{g_1(\mathrm{SINR}_1), \ldots, g_{K_r}(\mathrm{SINR}_{K_r})\}$
  - Feasible = Achieved by some $\mathbf{v}_1, \ldots, \mathbf{v}_{K_r}$ under power constraints



Pareto Boundary

Cannot improve for any user without degrading for other users

Other Names

Rate Region
Capacity Region
MSE Region, etc.

Performance Region (2)

- Definitions of Pareto Boundary
  - Strong Pareto point: Improve for any user → Degrade for other user
  - Weak Pareto point: Cannot simultaneously improve for *all* users

- Weak Definition is More Convenient
  - Boundary is compact and simply-connected

Optimality Condition 1

Sending one stream per user is sufficient (assumed earlier)

Optimality Condition 2

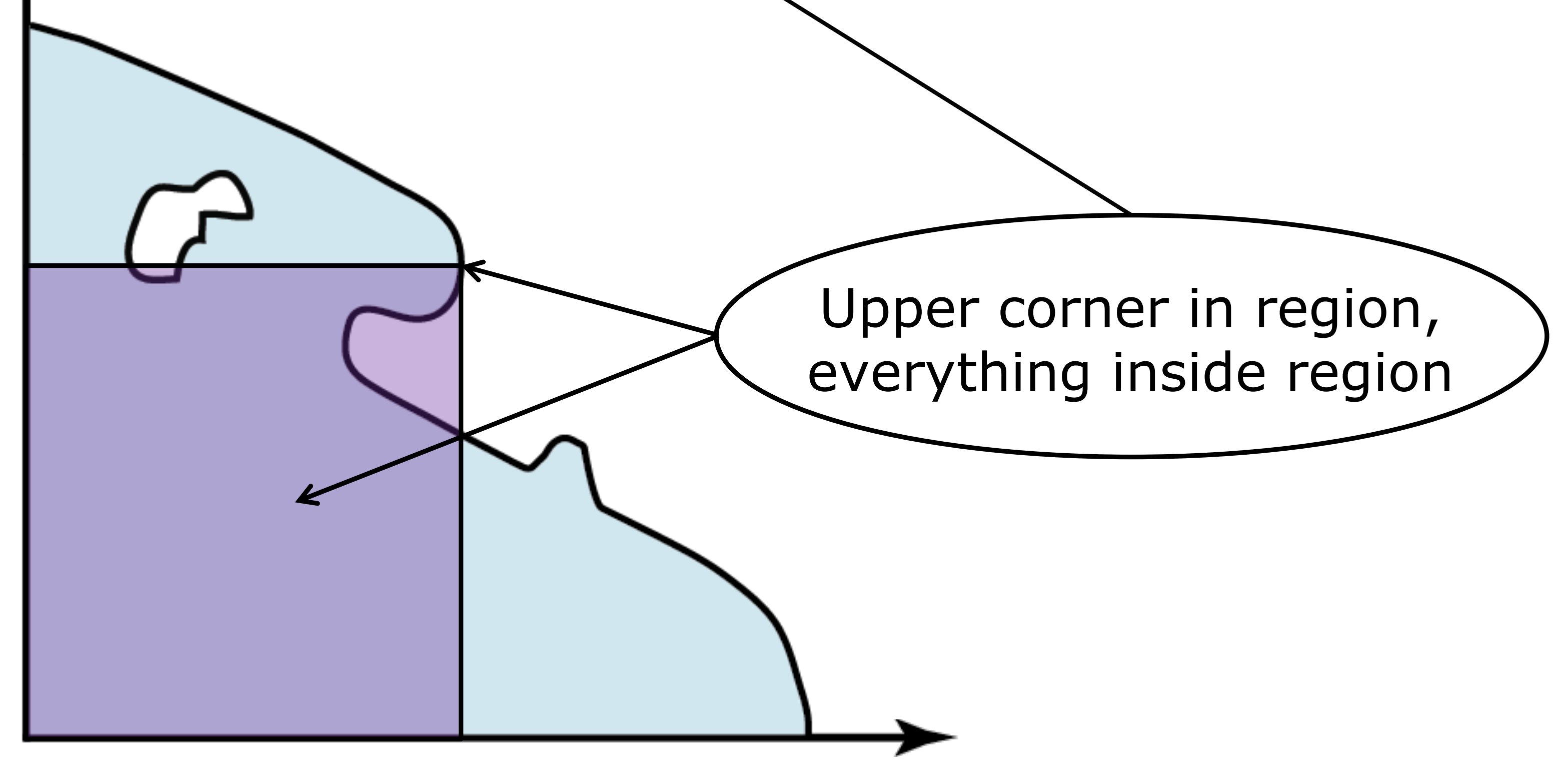
At least one power constraint is active (=holds with equality)

- X. Shang, B. Chen, H. V. Poor, "Multiuser MISO Interference Channels With Single-User Detection: Optimality of Beamforming and the Achievable Rate Region," IEEE Trans. on Information Theory, 2011.
- R. Mochaourab and E. Jorswieck, "Optimal Beamforming in Interference Networks with Perfect Local Channel Information," IEEE Trans. on Signal Processing, 2011.

Performance Region (3)

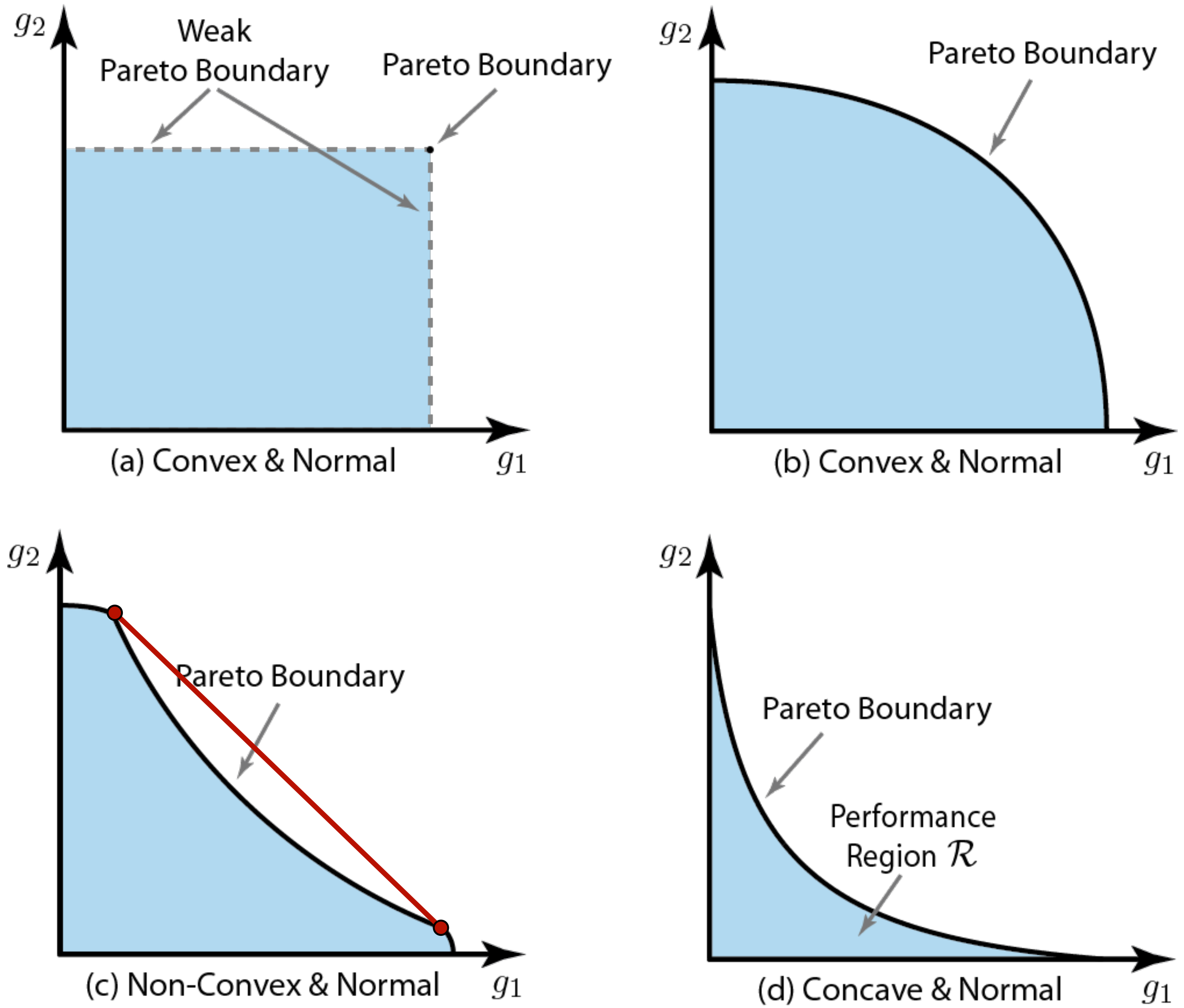
- Can the region have any shape?

- No! Can prove that:
  - Compact set
  - Normal set



Performance Region (4)

- Some Possible Shapes



User-Coupling

Weak: Convex
Strong: Concave

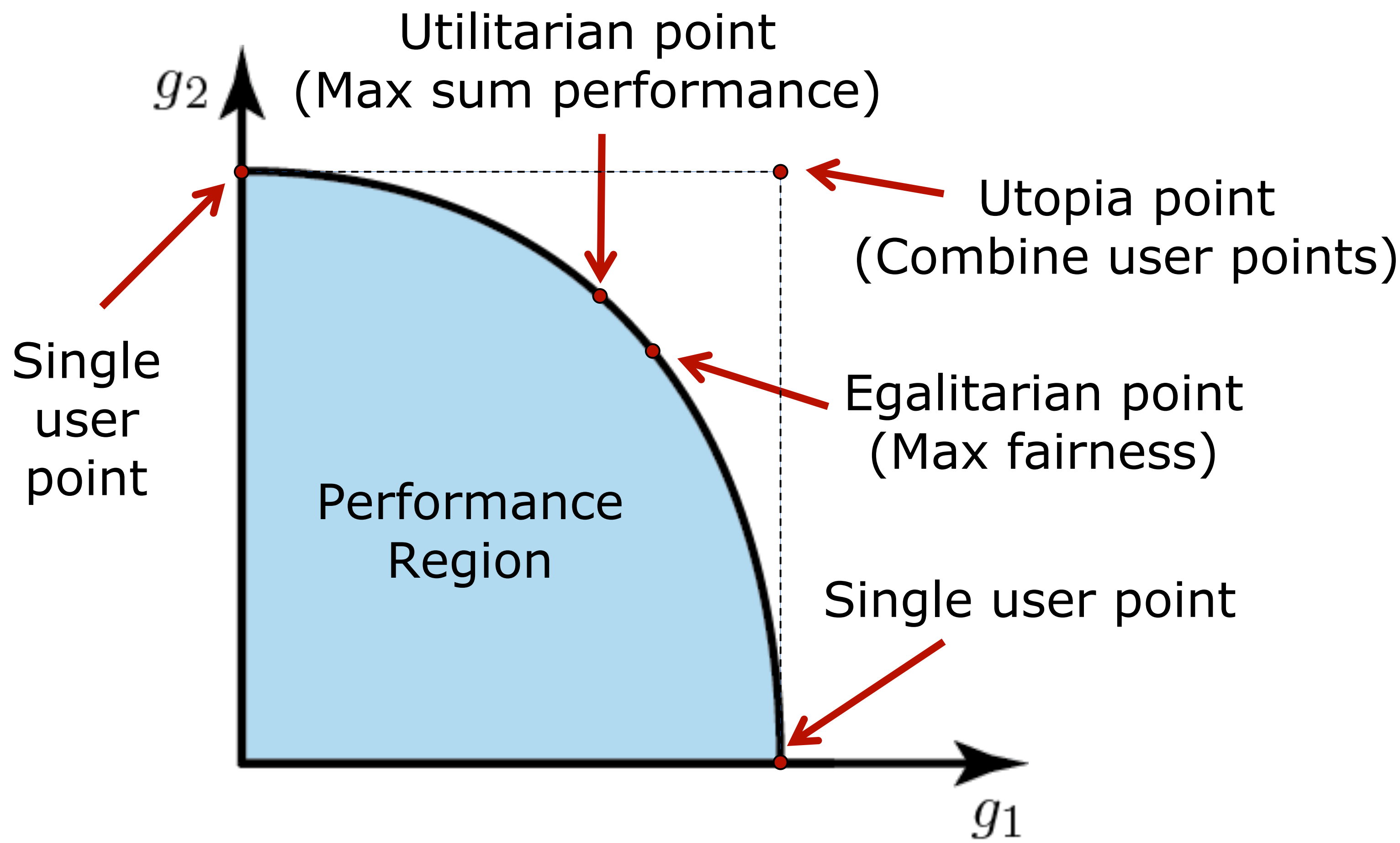
Scheduling

Time-sharing for strongly coupled users

Select multiple points
Hard: Unknown region

Performance Region (5)

- Which Pareto Optimal Point to Choose?
  - Tradeoff: Aggregate Performance vs. Fairness



No Objective Answer

Utopia point outside of region

Only subjective answers exist!

Section: Problem Formulation

Questions?

Section

Subjective Resource Allocation

Subjective Approach

- System Designer Selects Utility Function $f : \mathcal{R} \rightarrow \mathbb{R}$
  - Describes subjective preference
  - Increasing and continuous function

Put different weights to move between extremes

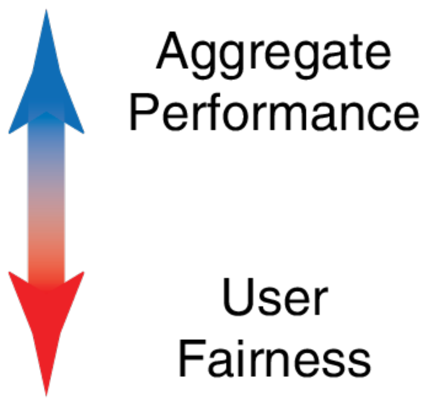
- Examples:

Sum performance: $f(\mathbf{g}) = \sum_k g_k$

Proportional fairness: $f(\mathbf{g}) = \prod_k g_k$

Harmonic mean: $f(\mathbf{g}) = K_r \left(\sum_k g_k^{-1}\right)^{-1}$

Max-min fairness: $f(\mathbf{g}) = \min_k g_k$



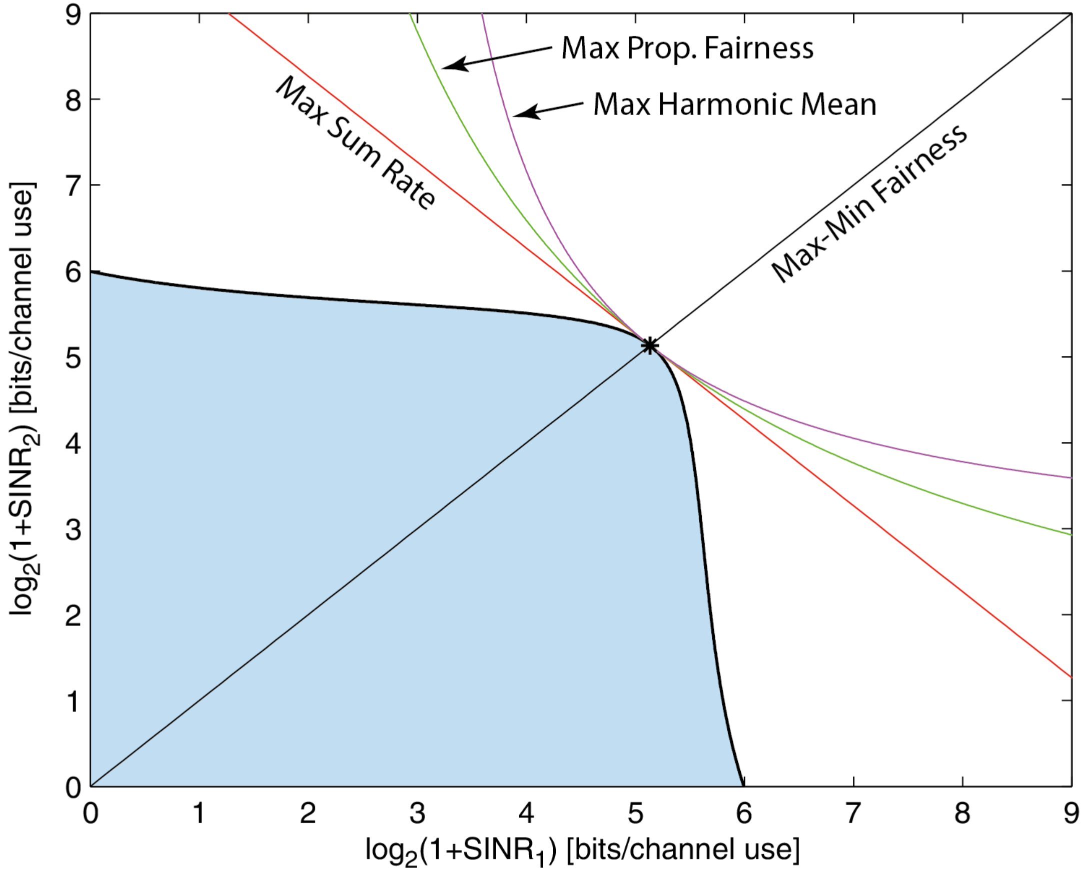
Known as *A Priori* Approach

Select utility function before optimization

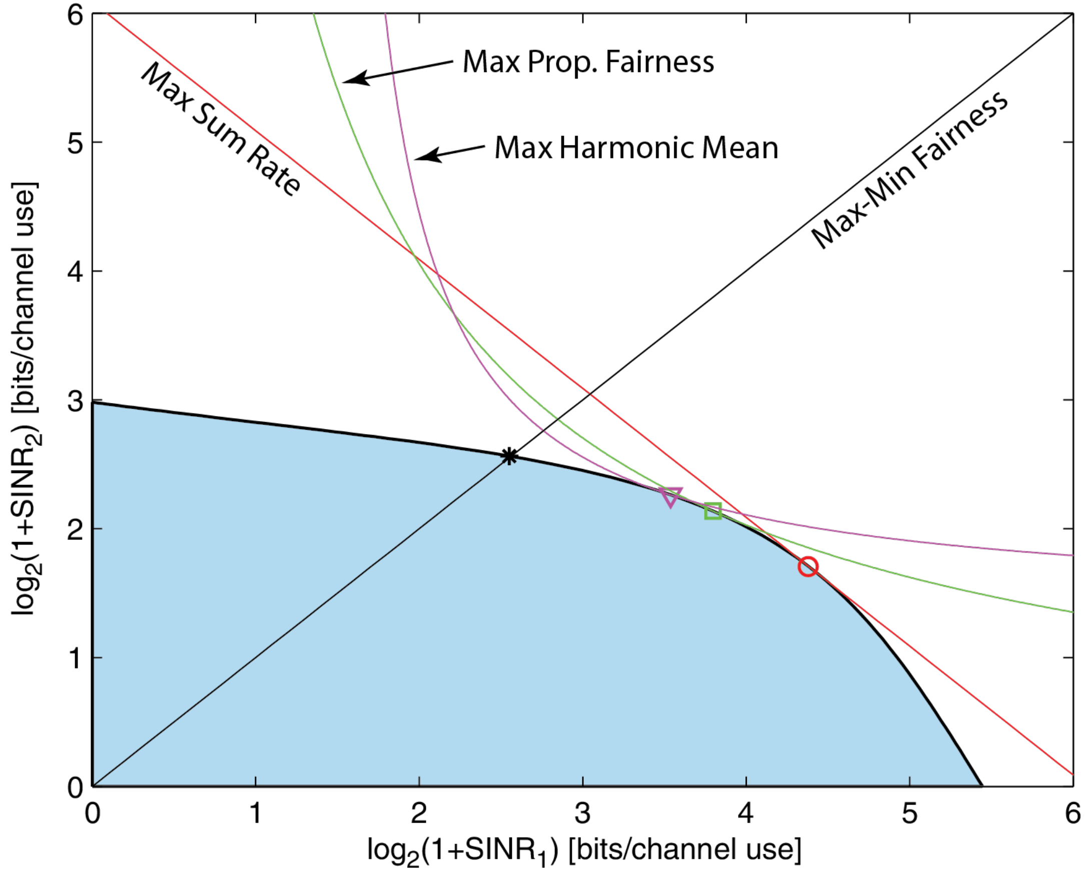
Subjective Approach (2)

- Utilities Functions Has Different Shapes
  - Curve: $f(\mathbf{g}) = \text{constant}$
  - Optimal constant: Curve intersects optimum

$$f(\mathbf{g}) = \sum_k g_k$$
$$f(\mathbf{g}) = \prod_k g_k$$
$$f(\mathbf{g}) = K_r \left(\sum_k g_k^{-1}\right)^{-1}$$
$$f(\mathbf{g}) = \min_k g_k$$



Symmetric region: Same point



Asymmetric region: Different points

Subjective Approach (3)

- Utility Function gives Single-Objective Optimization Problem:

$$\underset{\mathbf{v}_1,\ldots,\mathbf{v}_{K_r}}{\text{maximize}} \quad f(\mathbf{g}) \qquad \text{subject to} \quad \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \quad \forall l.$$

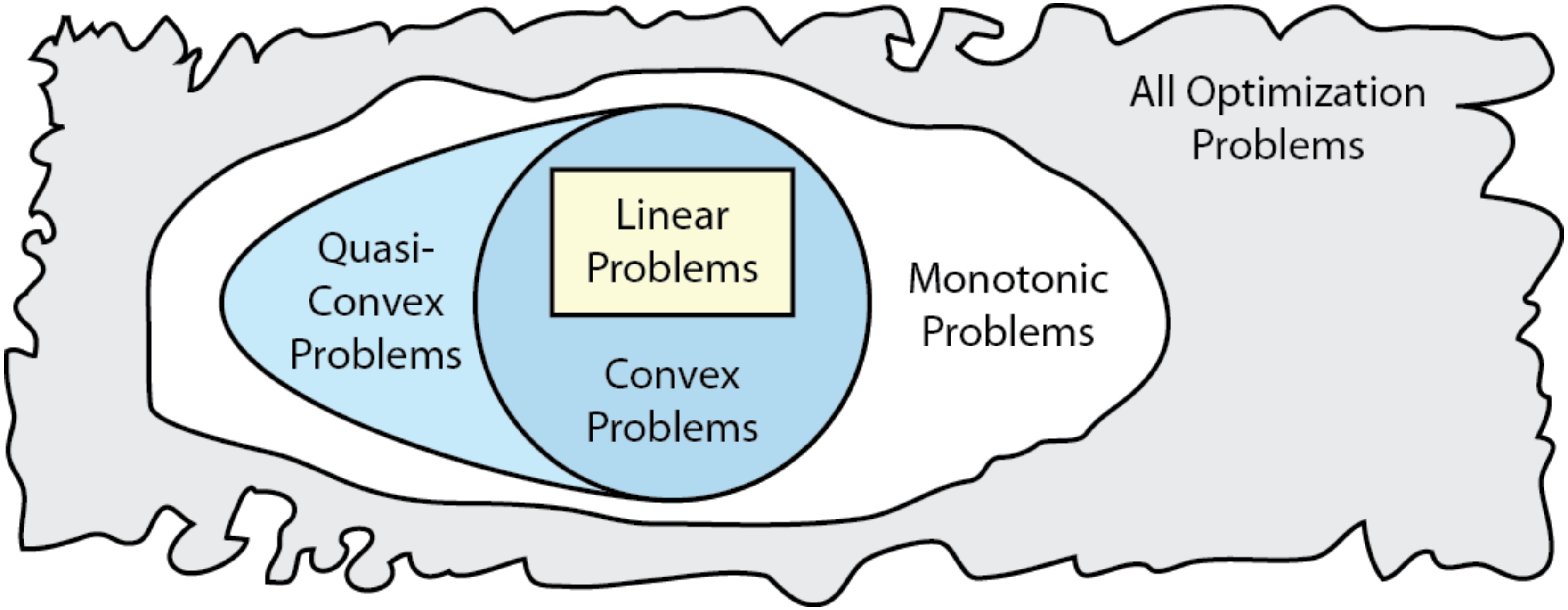
- This is the Starting Point of Many Researchers
  - Although Selection of f is Inherently Subjective
Affects the Solvability

Pragmatic Approach

Try to Select Utility Function to Enable Efficient Optimization

Complexity of Single-Objective Optimization Problems

- Classes of Optimization Problems
  - Different scaling with number of parameters and constraints



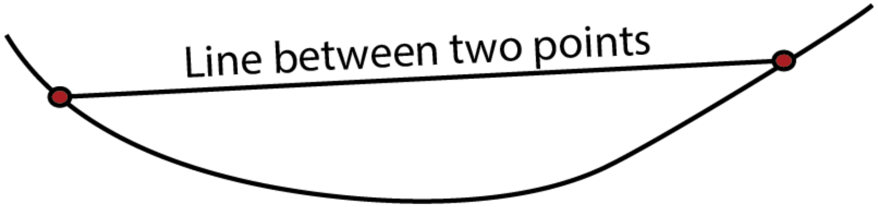
- Main Classes
  - Convex: Polynomial time solution ← Practically solvable
  - Monotonic: Exponential time solution ← Approximations needed
  - Arbitrary: More than exponential time ← Hard to even approximate

Complexity of Resource Allocation Problems

- What is a Convex Problem?
  - Recall definitions:

Convex Function

For any two points on the graph of the function, the line between the points is above the graph



Examples: $x^2,\ e^x,\ -\log_2(x)$

Convex Problem

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & f_0(\mathbf{x}) \\ \text{subject to} \quad & f_m(\mathbf{x}) \leq 0 \quad m = 1, \ldots, M \end{aligned}$$

Convex if objective f_0 and constraints $f_1, \ldots, f_M$ are convex functions

Complexity of Resource Allocation Problems (2)

- When is Resource Allocation a Convex Problem?
  - Original problem:

$$\underset{\mathbf{v}_1,\ldots,\mathbf{v}_{K_r}}{\text{maximize}} \quad f(\mathbf{g}) \qquad \text{subject to} \quad \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \quad \forall l.$$

  - Rewritten problem (replace SINR_k with variable γ_k):

$$\underset{\mathbf{v}_k,\gamma_k \forall k}{\text{minimize}} \quad -f\left(g_1(\gamma_1),\ldots,g_{K_r}(\gamma_{K_r})\right)$$

$$\text{subject to} \quad |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2 \geq \gamma_k \Big(\sigma_k^2 + \sum_{i \neq k} |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2 \Big) \quad \forall k,$$

$$\sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \quad \forall l.$$

Can be selected to be convex

SINR constraints: Main complication!

Convex power constraints

Classification of Resource Allocation Problems

- Classification of Three Important Problems
  - The “Easy” problem
  - Weighted max-min fairness
  - Weighted sum performance

- We will see: These have Different Complexities
  - Difficulty: Too many spatial degrees of freedom
  - Convex problem only if search space is particularly limited
  - Monotonic problem in general

Complexity Example 1: The "Easy" Problem

- Given Any Point $(\tilde{g}_1, \dots, \tilde{g}_{K_r})$ or SINRs $(\gamma_1, \dots, \gamma_{K_r})$
  - Find beamforming $\mathbf{v}_1, \dots, \mathbf{v}_{K_r}$ that attains this point
  - Fixed SINRs make the constraints convex:

Second order cone: Convex

$$|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2 \geq \gamma_k \left(\sigma_k^2 + \sum_{i \neq k} |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2 \right) \longrightarrow \left\| \begin{matrix} \mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_1 \mathbf{v}_1 \\ \vdots \\ \mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_{K_r} \mathbf{v}_{K_r} \\ \sigma_k \end{matrix} \right\| \leq \sqrt{\frac{1+\gamma_k}{\gamma_k}} \Re(\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k)$$

  - Global solution in polynomial time – use CVX, Yalmip

Total Power Constraints
- M. Bengtsson, B. Ottersten, "Optimal Downlink Beamforming Using Semidefinite Optimization," Proc. Allerton, 1999.
- A. Wiesel, Y. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," IEEE Trans. on Signal Processing, 2006.

Per-Antenna Constraints
- W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," IEEE Trans. on Signal Processing, 2007.

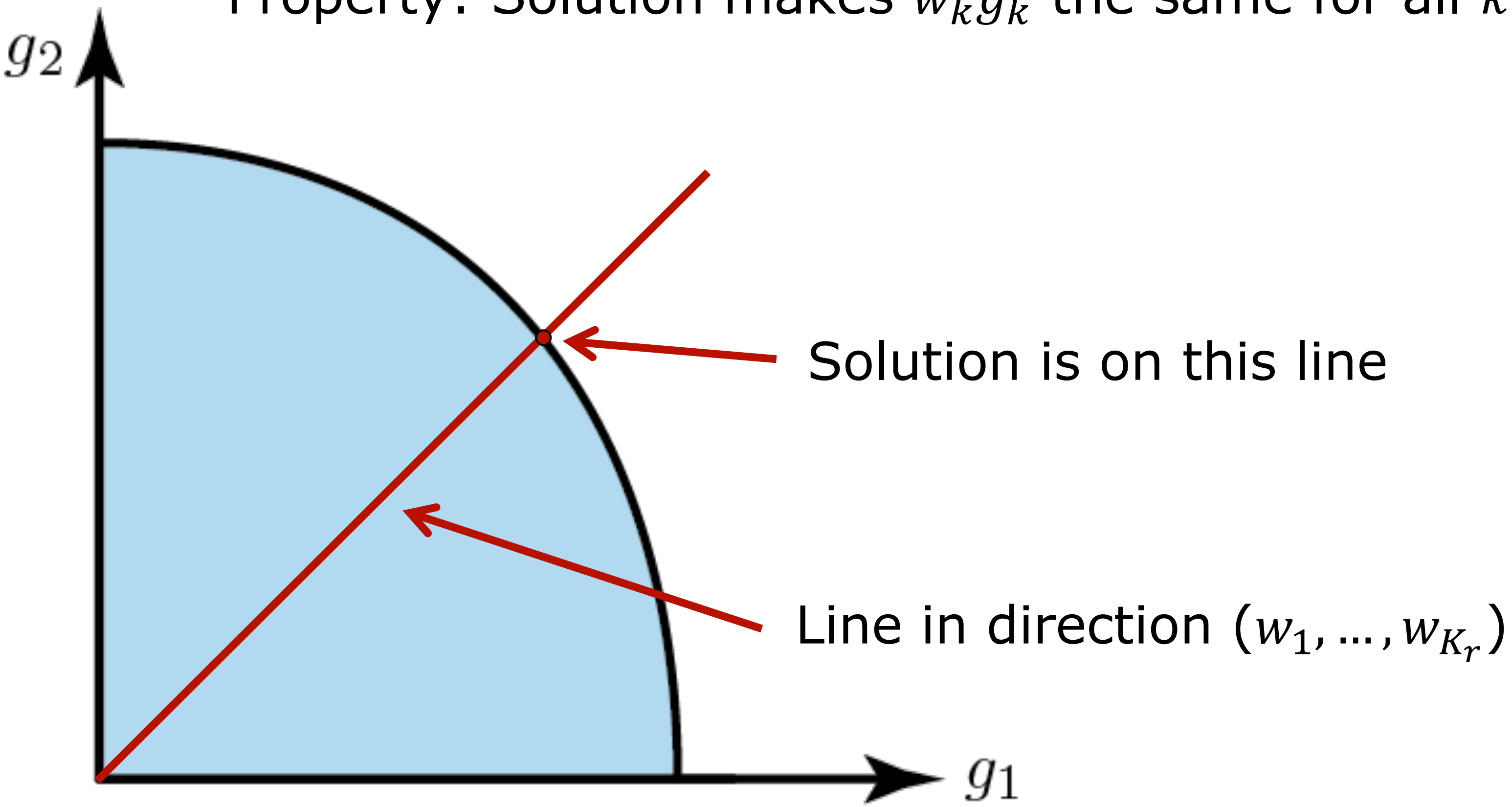
General Constraints
- E. Björnson, G. Zheng, M. Bengtsson, B. Ottersten, "Robust Monotonic Optimization Framework for Multicell MISO Systems," IEEE Trans. on Signal Processing, 2012.

Complexity Example 2: Max-Min Fairness

- How to Classify Weighted Max-Min Fairness?

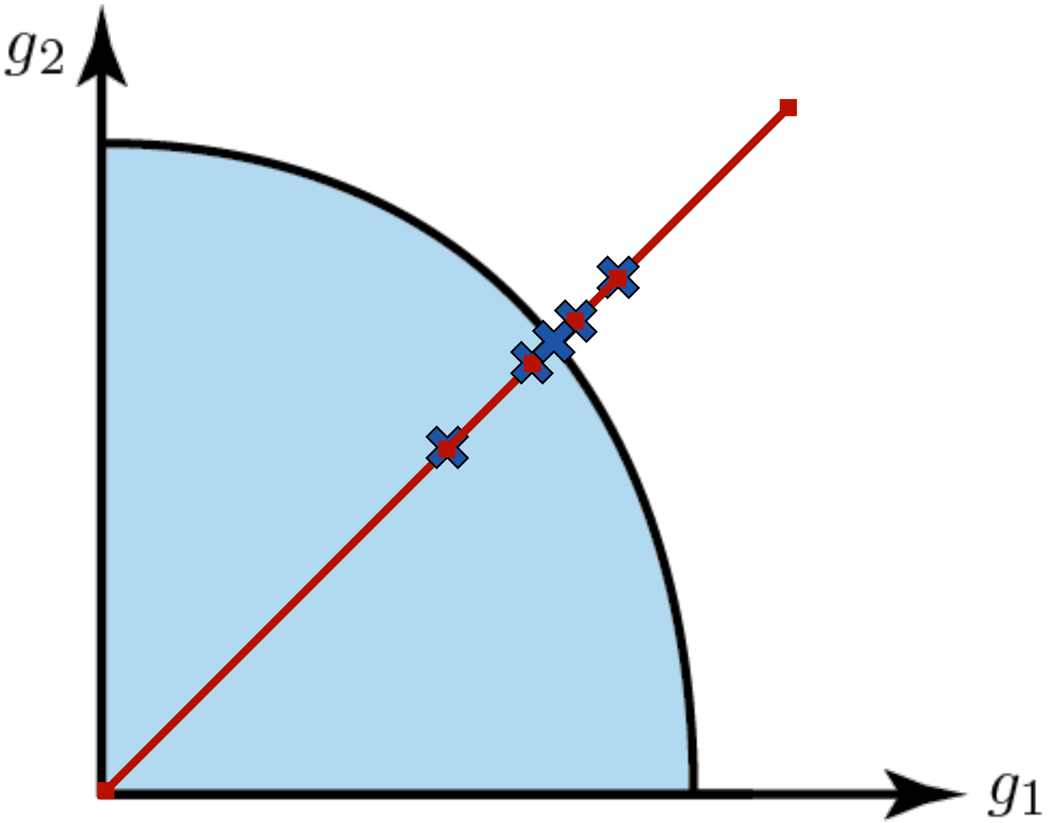
$$\underset{\mathbf{v}_1,\ldots,\mathbf{v}_{K_r}}{\text{maximize}} \quad f(\mathbf{g}) = \min_k w_k g_k \quad \text{subject to} \quad \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \quad \forall l.$$

  - Property: Solution makes $w_k g_k$ the same for all k



Complexity Example 2: Max-Min Fairness (2)

- Simple Line-Search: Bisection
  - Iteratively Solving Convex Problems (i.e., quasi-convex)



1. Find start interval
2. Solve the "easy" problem at midpoint
3. If feasible:
   Remove lower half
   Else: Remove upper half
4. Iterate

Subproblem: Convex optimization
Line-search: Linear convergence
One dimension (independ. #users)

Complexity Example 2: Max-Min Fairness (3)

- Classification of Weighted Max-Min Fairness:
  - **Quasi-convex problem** (belongs to convex class)
  - Polynomial complexity in #users, #antennas, #constraints
  - Might be feasible complexity in practice

Early work

- T.-L. Tung and K. Yao, "Optimal downlink power-control design methodology for a mobile radio DS-CDMA system," in IEEE Workshop SIPS, 2002.

Main references

- M. Mohseni, R. Zhang, and J. Cioffi, "Optimized transmission for fading multiple-access and broadcast channels with multiple antennas," IEEE Journal on Sel. Areas in Communications, 2006.
- A. Wiesel, Y. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," IEEE Trans. on Signal Processing, 2006.

Channel uncertainty

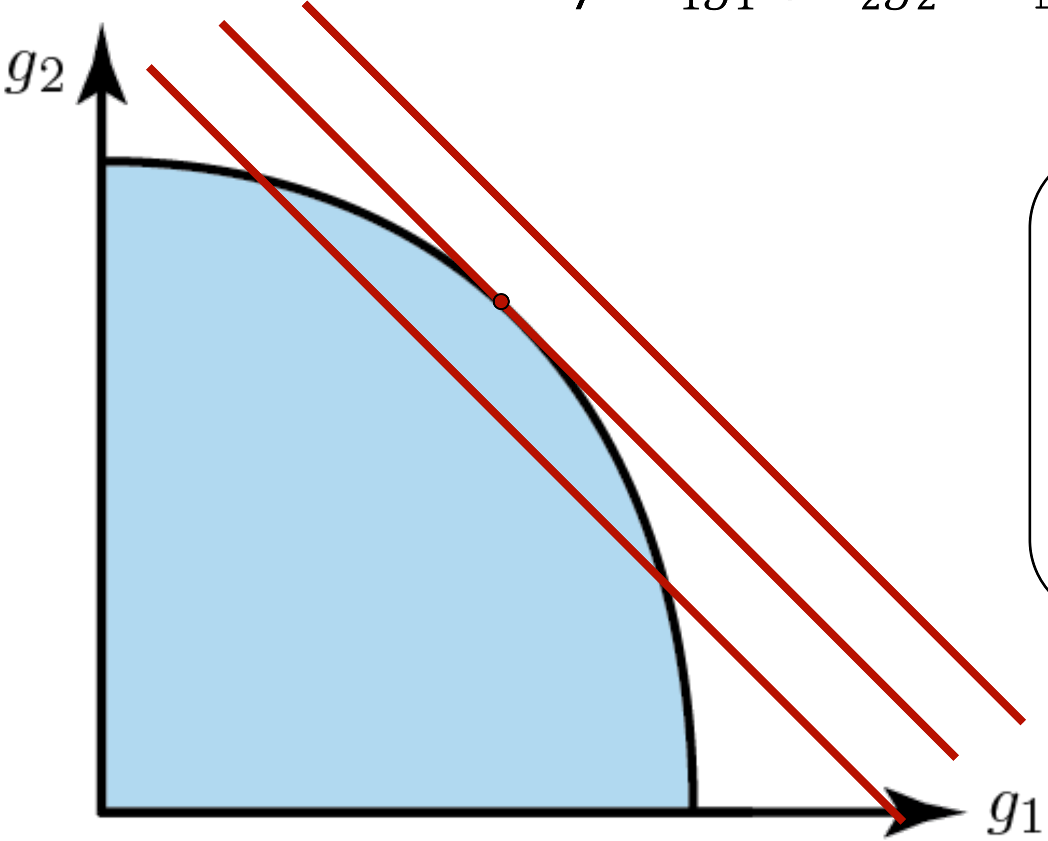
- E. Björnson, G. Zheng, M. Bengtsson, B. Ottersten, "Robust Monotonic Optimization Framework for Multicell MISO Systems," IEEE Trans. on Signal Processing, 2012.

Complexity Example 3: Weighted Sum Performance

- How to Classify Weighted Sum Performance?

$$\underset{\mathbf{v}_1,\ldots,\mathbf{v}_{K_r}}{\text{maximize}} \quad f(\mathbf{g}) = \sum_{k=1}^{K_r} w_k g_k \quad \text{subject to} \quad \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \quad \forall l.$$

  - Geometrically: $w_1 g_1 + w_2 g_2 =$ `opt-value` is a line



Opt-value is unknown!

- Distance from origin is unknown
- Line → Hyperplane (dim: #user – 1)
- Harder than max-min fairness
- Non-convex problem

Complexity Example 3: Weighted Sum Performance (2)

- Classification of Weighted Sum Performance:
  - Non-convex problem
  - Power constraints: Convex
  - Utility: Monotonic increasing/decreasing in beamforming vectors
  - Therefore: **Monotonic problem**

- Can There Be a Magic Algorithm?
  - No, provably NP-hard (Non-deterministic Polynomial-time hard)
  - Exponential complexity but in which parameters? (#users, #antennas, #constraints)

- Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE Journal of Sel. Topics in Signal Processing*, 2008.
- Y.-F. Liu, Y.-H. Dai, and Z.-Q. Luo, "Coordinated beamforming for MISO interference channel: Complexity analysis and efficient algorithms," IEEE Trans. on Signal Processing, 2011.

Complexity Example 3: Weighted Sum Performance (3)

- Are Monotonic Problems Impossible to Solve?
  - No, not for small problems!
- Monotonic Optimization Algorithms
  - Improve Lower/upper bounds on optimum: $f_{\min} \leq f_{\text{opt}} \leq f_{\max}$
  - Continue until $f_{\max} - f_{\min} < \varepsilon$
  - Subproblem: Essentially weighted max-min fairness problem

---

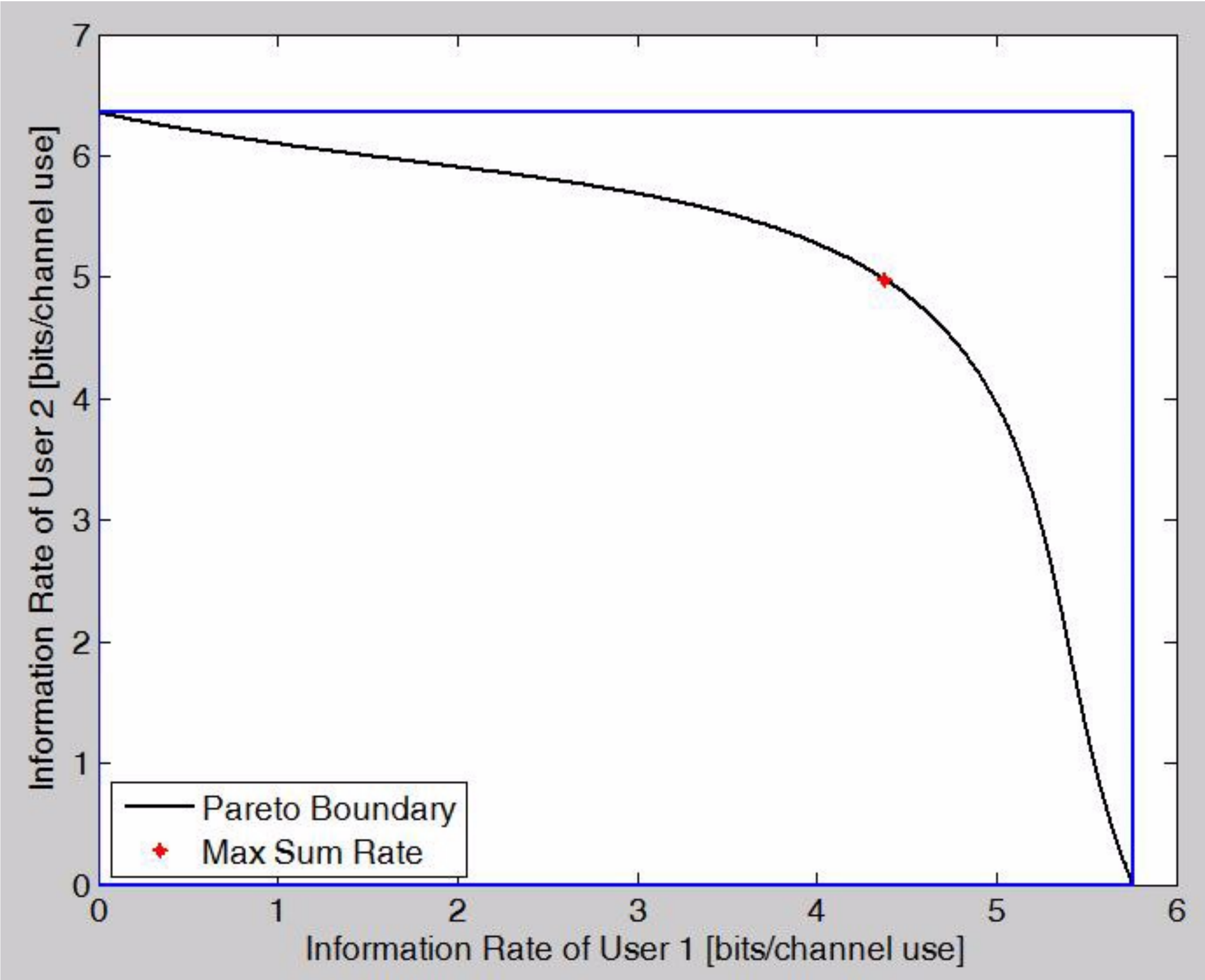
Monotonic optimization
- H. Tuy, "Monotonic optimization: Problems and solution approaches," SIAM Journal of Optimization, 2000.

Early works
- L. Qian, Y. Zhang, and J. Huang, "MAPEL: Achieving global optimality for a non-convex wireless power control problem," IEEE Trans. on Wireless Commun., 2009.
- E. Jorswieck, E. Larsson, "Monotonic Optimization Framework for the MISO Interference Channel," IEEE Trans. on Communications, 2010.

Polyblock algorithm
- W. Utschick and J. Brehmer, "Monotonic optimization framework for coordinated beamforming in multicell networks," IEEE Trans. on Signal Processing, 2012.

BRB algorithm
- E. Björnson, G. Zheng, M. Bengtsson, B. Ottersten, "Robust Monotonic Optimization Framework for Multicell MISO Systems," IEEE Trans. on Signal Processing, 2012.

Complexity Example 3: Weighted Sum Performance (4)

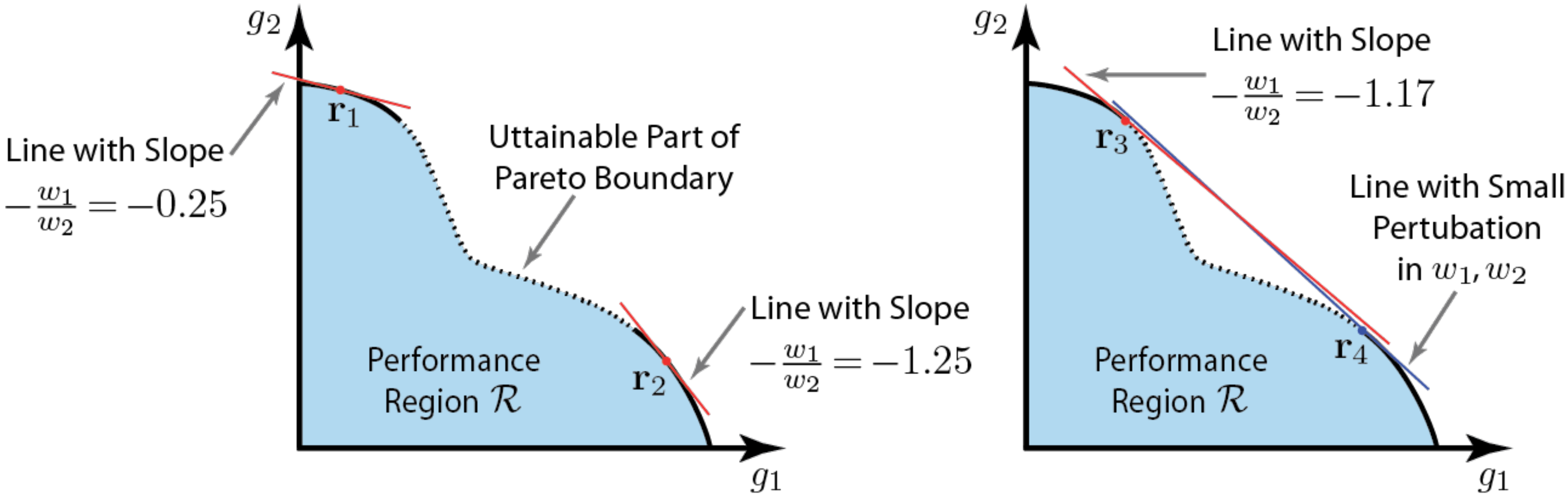


Branch-Reduce-Bound (BRB) Algorithm

- Global convergence
- Accuracy ε>0 in finitely many iterations
- Exponential complexity only in #users (K_r)
- Polynomial complexity in other parameters (#antennas, #constraints)

Complexity Example 3: Weighted Sum Performance (5)

- Maximizing Sum Performance has High Complexity
- Other Shortcomings
  - Not all Pareto Points are Attainable
  - Weights have no Clear Interpretation
  - Not Robust to Perturbations



Summary: Complexity of Resource Allocation Problems

	General	Zero Forcing	Single Antenna
Sum Performance	NP-hard	Convex	NP-hard
Max-Min Fairness	Quasi-Convex	Quasi-Convex	Quasi-Convex
"Easy" Problem	Convex	Convex	Linear
Proportional Fairness	NP-hard	Convex	Convex
Harmonic Mean	NP-hard	Convex	Convex

- Recall: The SINR constraints are complicating factor

$$\underbrace{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2}_{\text{Signal}} \geq \underbrace{\gamma_k}_{\text{SINR}} \left(\sigma_k^2 + \underbrace{\sum_{i \neq k} |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2}_{\text{Interference}} \right)$$

Three conditions that simplify:

1. Fixed SINRs γ_k ("easy" problem)
2. Allow no interference (zero-forcing)
3. Multiplication → Addition (change of variable, single antenna BSs)

Summary: Complexity of Resource Allocation Problems (2)

- Recall: All Utility Functions are Subjective
  - Pragmatic approach: Select to enable efficient optimization

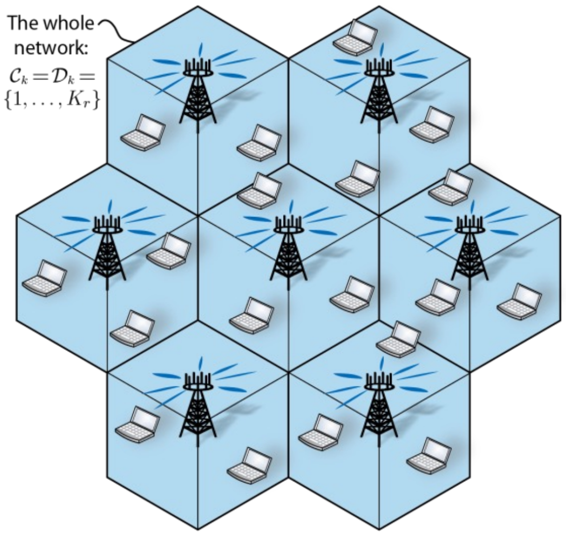
- Good Choice: Any Problem with Polynomial complexity
  - Example: Weighted max-min fairness
  - Use weights to adapt to other system needs

	General	Zero Forcing	Single Antenna
Sum Performance		Convex	
Max-Min Fairness	Quasi-Convex	Quasi-Convex	Quasi-Convex
“Easy” Problem	Convex	Convex	Linear
Proportional Fairness		Convex	Convex
Harmonic Mean		Convex	Convex

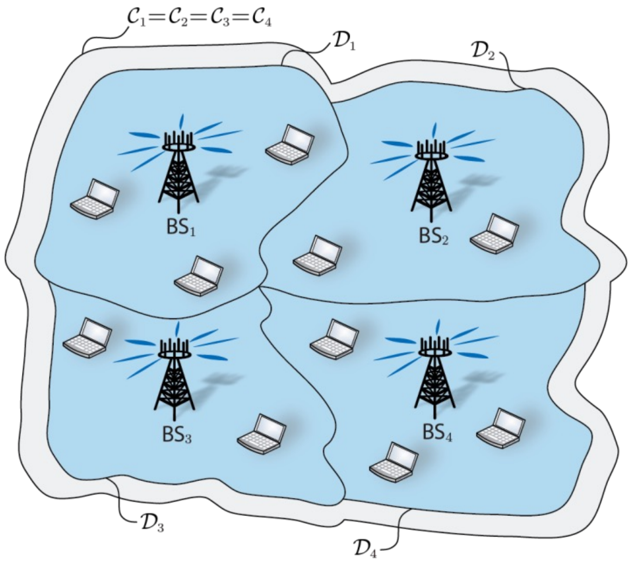
- Bad Choice: Weighted Sum Performance
  - Generally NP-hard: Exponential complexity (in #users)
  - Should be avoided – Sometimes needed (virtual queuing techniques)

Summary: Complexity of Resource Allocation Problems (3)

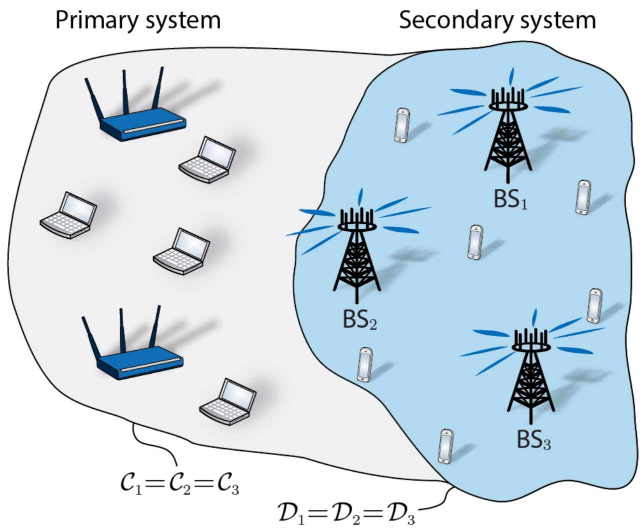
- Complexity Analysis for Any Dynamic Cooperation Clusters
  - Same optimization algorithms!
  - Extra characteristics can sometime simplify
  - Multi-antenna transmission: More complex, higher performance



Ideal Joint Transmission



Coordinated Beamforming



Underlay Cognitive Radio

Section: Subjective Resource Allocation

Questions?

Section

Structure of Optimal Beamforming

Parametrization of Optimal Beamforming

- $K_r N$ Complex Optimization Variables: Beamforming vectors $\mathbf{v}_1, \ldots, \mathbf{v}_{K_r}$
  - Can be reduced to $K_r + L - 2$ positive parameters

- Any Resource Allocation Problem Solved by

$$\mathbf{v}_k^{(\text{Optimal})} = \sqrt{p_k} \Big(\underbrace{\sum_{l=1}^{L} \frac{\mu_l}{q_l} \mathbf{Q}_{lk}}_{\text{Rotation by Constraints}} + \underbrace{\sum_{i=1}^{K_r} \frac{\lambda_i}{\sigma_i^2} \mathbf{D}_k^H \mathbf{C}_i^H \mathbf{h}_i \mathbf{h}_i^H \mathbf{C}_i \mathbf{D}_k}_{\text{Rotation by Inter-User Channels}} \Big)^{-1} \underbrace{\mathbf{D}_k^H \mathbf{C}_k^H \mathbf{h}_k}_{\text{Channel}}$$

$$\underbrace{\begin{bmatrix} p_1 & \ldots & p_{K_r} \end{bmatrix}}_{\text{Power Allocation}} = \begin{bmatrix} \gamma_1 \sigma_1^2 & \ldots & \gamma_{K_r} \sigma_{K_r}^2 \end{bmatrix} \mathbf{M}^{\dagger}, \quad [\mathbf{M}]_{ik} = \begin{cases} |\mathbf{h}_i^H \mathbf{C}_i \mathbf{D}_i \mathbf{v}_i|^2, & i = k, \\ -\gamma_k |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2, & i \neq k, \end{cases}$$

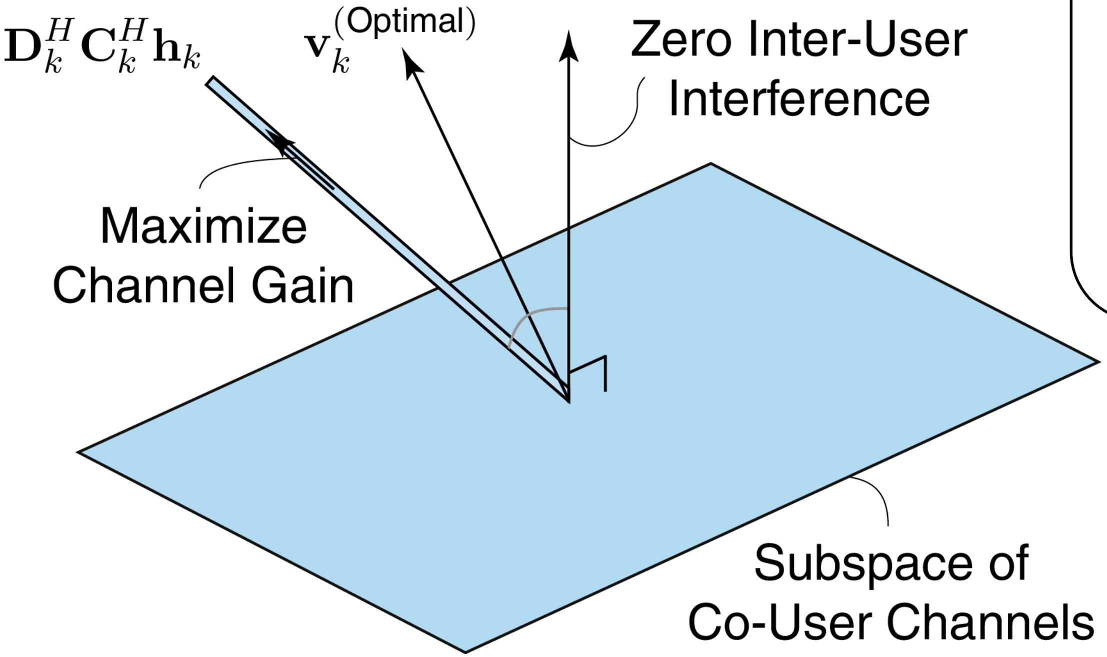
$$\gamma_k = \frac{\lambda_k}{\sigma_k^2} \mathbf{h}_k^H \mathbf{D}_k \Big(\sum_{l=1}^{L} \frac{\mu_l}{q_l} \mathbf{Q}_{lk} + \sum_{i \neq k} \frac{\lambda_i}{\sigma_i^2} \mathbf{D}_k^H \mathbf{C}_i^H \mathbf{h}_i \mathbf{h}_i^H \mathbf{C}_i \mathbf{D}_k \Big)^{-1} \mathbf{D}_k^H \mathbf{h}_k$$

  - Priority of User k: λ_k
  - Impact of Constraint l: μ_l

$\Big\}$ Lagrange multipliers of "Easy" problem

Parametrization of Optimal Beamforming (2)

- Geometric Interpretation:



Tradeoff

- Maximize signal vs. minimize interference
- Hard to find optimal tradeoff

- Special Case: $K_r = 2$
  - Beamforming: Linear combination of channel and zero-forcing direction

---

Early work
- E. A. Jorswieck, E. G. Larsson, D. Danev, "Complete characterization of the Pareto boundary for the MISO interference channel," IEEE Trans. on Signal Processing, 2008.

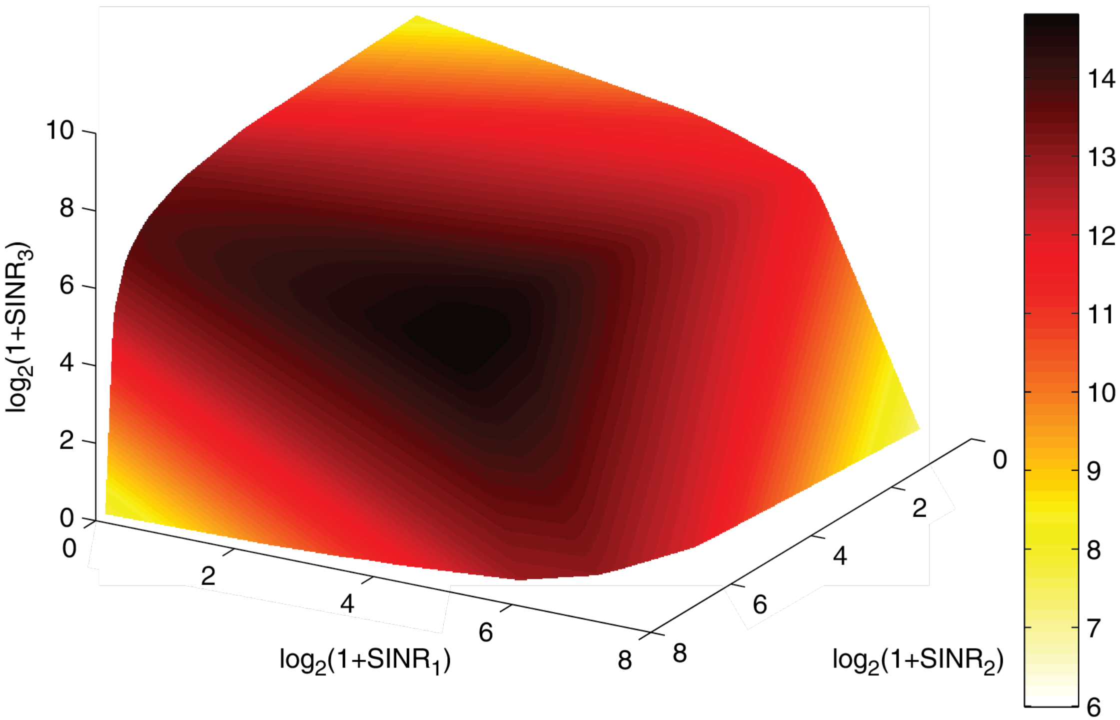
State-of-the-art
- E. Björnson, M. Bengtsson, B. Ottersten, "Pareto Characterization of the Multicell MIMO Performance Region With Simple Receivers," IEEE Trans. on Signal Processing, 2012.

Application 1: Generate Performance Region

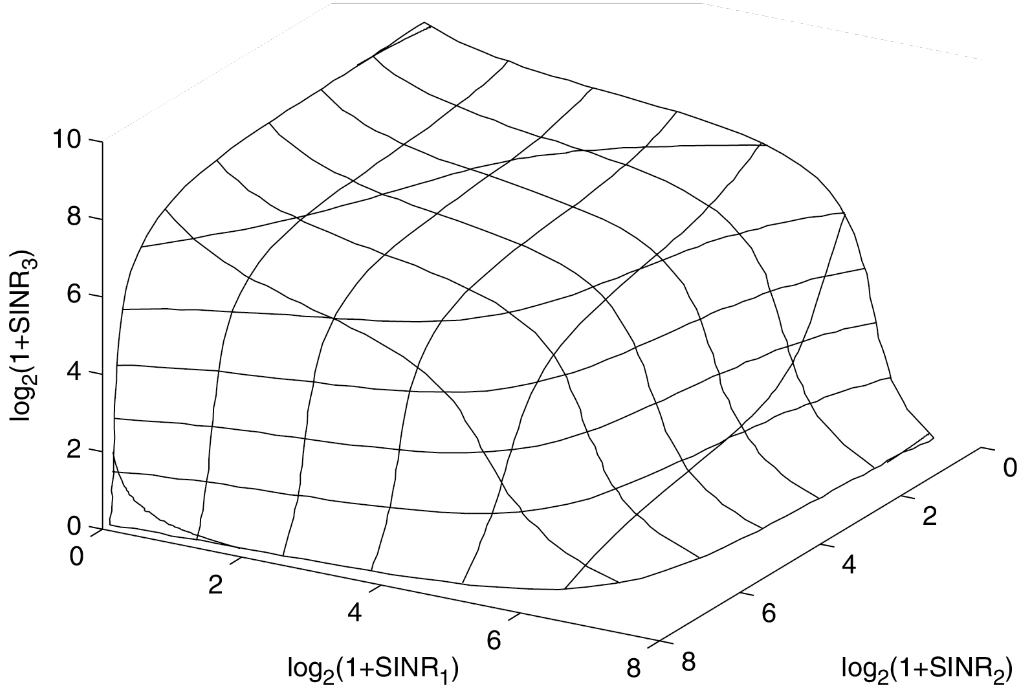
- Performance Region is Generally Unknown
  - Compact and normal
  - Perhaps non-convex

***A Posteriori* Approach**

Look at region at select operating point



Approach 1:
Vary parameters in parametrization



Approach 2:
Maximize sequence of utilities $f()$

Application 2: Heuristic Beamforming

- Parametrization: Foundation for Low-Complexity Beamforming
  - Select parameters heuristically

- One Approach – Many Names
  - Set all parameters to same value:

$$\mathbf{v}_k^{(\text{Heuristic})} = \sqrt{p_k} \left(\sum_{l=1}^{L} \frac{1}{L q_l} \mathbf{Q}_{lk} + \sum_{i=1}^{K_r} \frac{1}{K_r \sigma_i^2} \mathbf{D}_k^H \mathbf{C}_i^H \mathbf{h}_i \mathbf{h}_i^H \mathbf{C}_i \mathbf{D}_k \right)^{-1} \mathbf{D}_k^H \mathbf{C}_k^H \mathbf{h}_k$$

  - Ideal joint transmission: $\mathbf{v}_k^{(\text{Heuristic})} = \sqrt{p_k} \left(\frac{1}{q_1} \mathbf{I}_N + \sum_{i=1}^{K_r} \frac{1}{K_r \sigma_i^2} \mathbf{h}_i \mathbf{h}_i^H \right)^{-1} \mathbf{h}_k$

  - Proposed many times (since 1995):

    Transmit Wiener/MMSE filter, Signal-to-leakage beamforming, Virtual-uplink MVDR beamforming, Regularized zero-forcing, Virtual SINR beamforming, etc.

Application 3: Behavior at low/high SNRs

- Recall: Parametrization
  - Assume total power constraint

$$\mathbf{v}_k^{(\text{Optimal})} = \sqrt{p_k}\left(\frac{1}{q_1}\mathbf{I}_N + \sum_{i=1}^{K_r}\frac{\lambda_i}{\sigma_i^2}\mathbf{D}_k^H\mathbf{C}_i^H\mathbf{h}_i\mathbf{h}_i^H\mathbf{C}_i\mathbf{D}_k\right)^{-1}\mathbf{D}_k^H\mathbf{C}_k^H\mathbf{h}_k$$

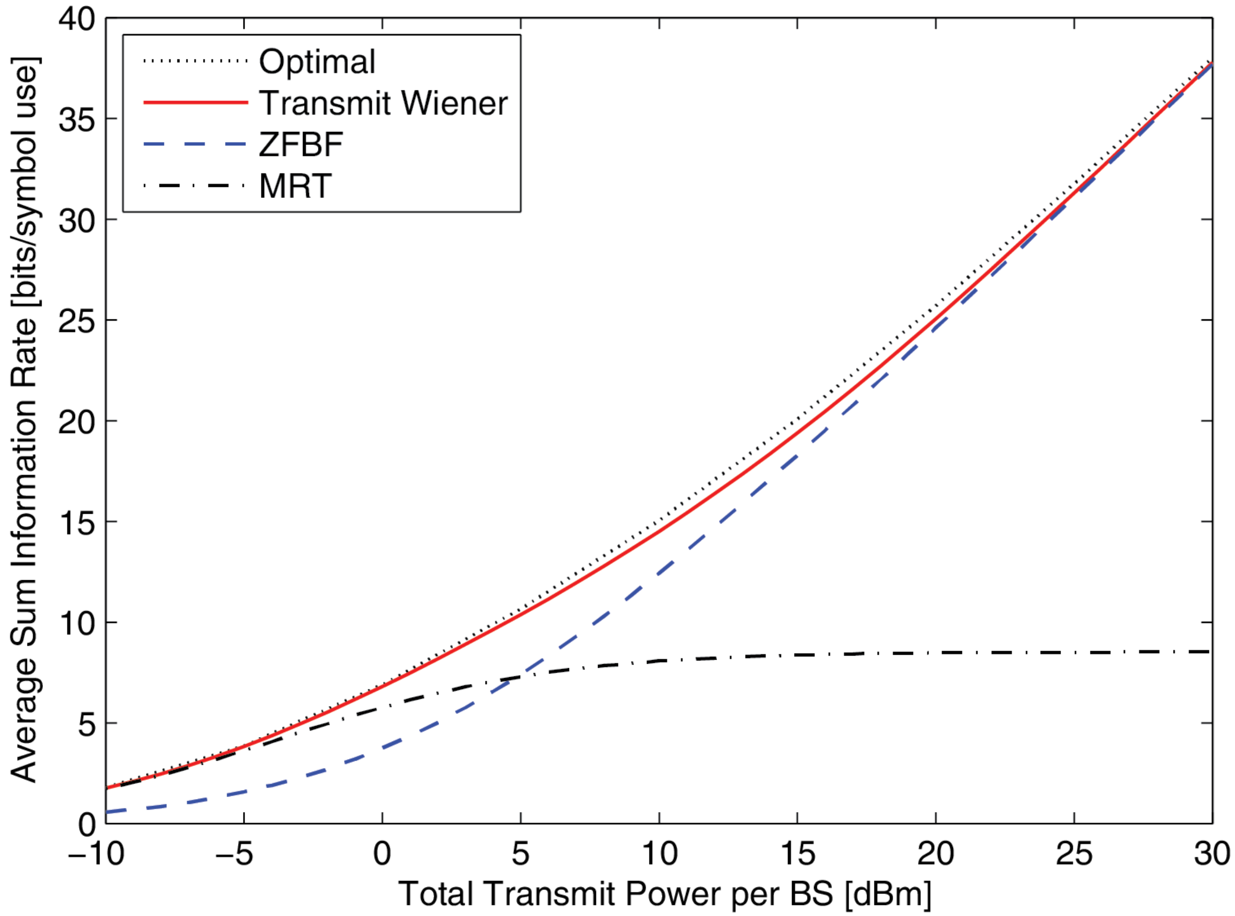
- Low SNR: $\sigma_i^2 \to \infty$
  - Inverse → Identity matrix: Beamforming in channel direction
  - Name: Maximum ratio transmission (MRT)

- High SNR: $q_1 \to \infty$
  - Inverse → Project orthogonal to co-users
  - Name: Zero-forcing beamforming (ZFBF)

Application 3: Behavior at low/high SNRs (2)

- Example: 4-User Interference Channel
  - Maximize sum information rate, 4 antennas/transmitter

- Four Strategies:
  - Optimal beamforming (BRB algorithm)
  - Transmit Wiener filter
  - ZFBF, MRT



Observations

- MRT good at low SNR
- ZFBF good at high SNR
- Wiener filter always good

Section: Structure of Optimal Beamforming

Questions?

Summary: Part 1

Summary

- Multi-Cell Multi-Antenna Resource Allocation
  - Divide power between users and spatial directions
  - Solve a multi-objective optimization problem
  - Pareto boundary: Set of efficient solutions
- Subjective Utility Function
  - Selection has fundamental impact on solvability
  - Multi-antenna transmission: More possibilities – higher complexity
  - Pragmatic approach: Select to enable efficient optimization
  - Polynomial complexity: Weighted max-min fairness etc.
  - Not solvable in practice: Weighted sum performance etc.
- Structure of Optimal Beamforming
  - Simple parametrization – balance signal and interference
  - Foundation for low-complexity beamforming
  - Easy to generate Pareto boundary

Coffee Break

- Thank you for listening!
  - Questions?

- After the Break, Part 2 Application:
  - Robustness to channel uncertainty
  - Distributed resource allocation
  - Transceiver hardware impairments
  - Multi-cast transmission
  - Multi-carrier systems
  - Multi-antenna users
  - Design of cooperation clusters
  - Cognitive radio systems
  - Physical layer security