PIMRC 2013 Tutorial
Optimal Resource Allocation in Coordinated Multi-Cell Systems

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Introduction: Part II

- The multi-cell system model analyzed in Part I was based on three simplifying assumptions: **perfect CSI**, **unlimited backhaul capacity**, and **ideal transceiver hardware**.
- These assumptions are generalized in Part II to more **realistic conditions**. We show that many of the previous results are readily generalizable.
- Furthermore, we describe how the system model can be extended to also incorporate **multi-cast transmission**, **multi-carrier systems**, and **multi-antenna receivers**.
- Finally, we discuss some recent work on the design of **dynamic cooperation clusters** and show that our framework is applicable in **cognitive radio systems** and for **physical layer security**.
- There are many **open problems** in each topic and in the combination of multiple topics.

Outline

Communications Theory

1. Robustness to Channel Uncertainty
2. Distributed Resource Allocation
3. Transceiver Hardware Impairments
4. Multi-Cast Transmission
5. Multi-Carrier Systems
6. Multi-Antenna Users
7. Design of Dynamic Cooperation Clusters
8. Cognitive Radio Systems
9. Physical Layer Security

Channel Uncertainty Model

- The **channel uncertainty** originates from a variety of sources; for example, imperfect channel estimation, feedback quantization, inadequate channel reciprocity, and delays in CSI acquisition on fading channels.
- The **additive error model** is given by

$$\mathbf{h}_k = \widehat{\mathbf{h}}_k + \tilde{\boldsymbol{\epsilon}}_k \quad \forall k \tag{1}$$

  where $\widehat{\mathbf{h}}_k = [\widehat{\mathbf{h}}_{1k}^T \, \ldots \, \widehat{\mathbf{h}}_{K_t k}^T]^T \in \mathbb{C}^{N \times 1}$ is the CSI available at the base stations and $\tilde{\boldsymbol{\epsilon}}_k \in \mathbb{C}^{N \times 1}$ is the combined error vector.
- The system cannot account for any error; the **stochastic error vector** $\tilde{\boldsymbol{\epsilon}}_k$ could potentially cancel out the nominal vector as $\tilde{\boldsymbol{\epsilon}}_k = -\widehat{\mathbf{h}}_k$ or be very large (unbounded for Rayleigh fading channels).
- Therefore, we consider a subset of error vectors, **the uncertainty set**, that has high probability of containing the error

Ellipsoidal Uncertainty Set

- We define jointly for all base stations to a certain user, the ellipsoidal uncertainty set

$$\mathcal{U}_k(\widehat{\mathbf{h}}_k, \mathbf{B}_k) = \left\{ \mathbf{h}_k : \mathbf{h}_k = \widehat{\mathbf{h}}_k + \mathbf{B}_k \boldsymbol{\epsilon}_k, \ \|\boldsymbol{\epsilon}_k\|_2 \leq 1 \right\} \quad \forall k. \qquad (2)$$

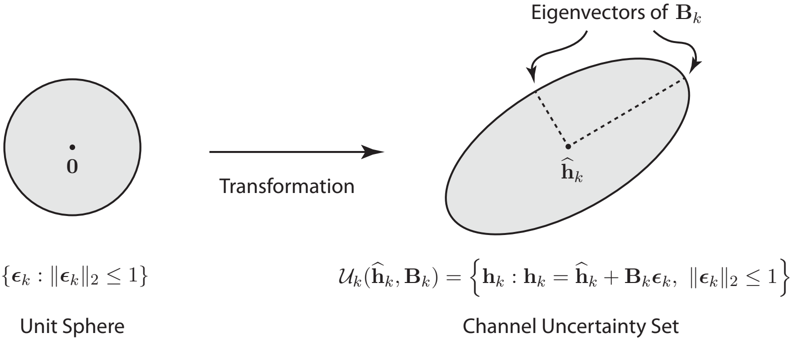


Illustration of the **ellipsoidal uncertainty set** $\mathcal{U}_k(\widehat{\mathbf{h}}_k, \mathbf{B}_k)$ in (2) and the unit sphere $\{\boldsymbol{\epsilon}_k : \|\boldsymbol{\epsilon}_k\|_2 \leq 1\}$ that it is created from.

Example: CSI Estimation Uncertainty

- Modeling the channels by $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_k)$, and performing **channel estimation** results in estimation errors $\tilde{\boldsymbol{\epsilon}}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{E}_k)$.
- The **error covariance matrix** $\mathbf{E}_k$ depends on $\mathbf{R}_k$ and on the type of channel estimation. It is block-diagonal as $\mathbf{E}_k = \mathrm{diag}(\mathbf{E}_{1k}, \ldots, \mathbf{E}_{K_t k})$ if the channel from each base station to each user is estimated separately.
- The estimation error $\tilde{\boldsymbol{\epsilon}}_k$ belongs with probability $\tilde{p}_k$ to the ellipsoidal set $\{\tilde{\boldsymbol{\epsilon}}_k : 2\tilde{\boldsymbol{\epsilon}}_k^H \mathbf{E}_k^{-1} \tilde{\boldsymbol{\epsilon}}_k \leq \chi^2_{\tilde{p}_k}(2N)\}$, where $\chi^2_{\tilde{p}_k}(n)$ denotes the $\tilde{p}_k$-percentile of the chi-squared distribution with n degrees of freedom.
- Alternatively, the estimation error $\tilde{\boldsymbol{\epsilon}}_{jk}$ of the channel component $\mathbf{h}_{jk}$ belongs with probability $\tilde{p}_{jk}$ to the ellipsoidal set $\{\tilde{\boldsymbol{\epsilon}}_{jk} : 2\tilde{\boldsymbol{\epsilon}}_{jk}^H \mathbf{E}_{jk}^{-1} \tilde{\boldsymbol{\epsilon}}_{jk} \leq \chi^2_{\tilde{p}_{jk}}(2N_j)\}$.

Worst-Case Robust Optimization Problem

- We formulate the **feasibility problem with SINR requirements** $\mathrm{SINR}_k \geq \gamma_k$ as

$$\begin{aligned} &\text{find} && \mathbf{S}_1 \ldots, \mathbf{S}_{K_r} && (3)\\ &\text{s. t.} && \frac{\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H \mathbf{C}_k^H \mathbf{h}_k}{\sigma_k^2 + \mathbf{h}_k^H \mathbf{C}_k (\sum_{i \neq k} \mathbf{D}_i \mathbf{S}_i \mathbf{D}_i^H) \mathbf{C}_k^H \mathbf{h}_k} \geq \gamma_k \quad \forall \mathbf{h}_k \in \mathcal{U}_k \ \forall k, && (4)\\ & && \textstyle\sum_{k=1}^{K_r} \mathrm{tr}(\mathbf{Q}_{lk} \mathbf{S}_k) \leq q_l \quad \forall l && (5) \end{aligned}$$

  where the beamforming vectors $\mathbf{v}_k \mathbf{v}_k^H$ have been replaced by signal correlation matrices $\mathbf{S}_k$.
- This is a generalization of the "easy" problem.
- Is **single-stream beamforming** (rank one $\mathbf{S}_k$) still optimal?

S-Procedure

Lemma (S-Procedure)

Let $\theta_m(\boldsymbol{\epsilon}) = \boldsymbol{\epsilon}^H \mathbf{Z}_m \boldsymbol{\epsilon} + \mathbf{z}_m^H \boldsymbol{\epsilon} + \boldsymbol{\epsilon}^H \mathbf{z}_m + \tilde{z}_m$, $m = 1, 2$, *be two quadratic functions in* $\boldsymbol{\epsilon}$ *and let* $\mathbf{Z}_m$ *be Hermitian. Suppose it exists* $\hat{\boldsymbol{\epsilon}}$ *such that* $\theta_1(\hat{\boldsymbol{\epsilon}}) > 0$, *then the implication*

$$\theta_1(\boldsymbol{\epsilon}) \geq 0 \Rightarrow \theta_2(\boldsymbol{\epsilon}) \geq 0 \tag{6}$$

holds true if and only if it exists $\lambda \geq 0$ *such that*

$$\begin{bmatrix} \mathbf{Z}_2 & \mathbf{z}_2 \\ \mathbf{z}_2^H & \tilde{z}_2 \end{bmatrix} - \lambda \begin{bmatrix} \mathbf{Z}_1 & \mathbf{z}_1 \\ \mathbf{z}_1^H & \tilde{z}_1 \end{bmatrix} \succeq \mathbf{0}. \tag{7}$$

Feasibility of Robust SINR Constraints

Communications Theory

Theorem

Let $\mathbf{S}_1, \dots, \mathbf{S}_{K_r}$ *be a transmit strategy and let* $\gamma_k \geq 0$ *be given, then the robust SINR constraint*

$$\frac{\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H \mathbf{C}_k^H \mathbf{h}_k}{\sigma_k^2 + \mathbf{h}_k^H \mathbf{C}_k (\sum_{i \neq k} \mathbf{D}_i \mathbf{S}_i \mathbf{D}_i^H) \mathbf{C}_k^H \mathbf{h}_k} \geq \gamma_k \quad \forall \mathbf{h}_k \in \mathcal{U}_k(\widehat{\mathbf{h}}_k, \mathbf{B}_k) \tag{8}$$

is fulfilled if and only if it exists $\lambda_k \geq 0$ *such that*

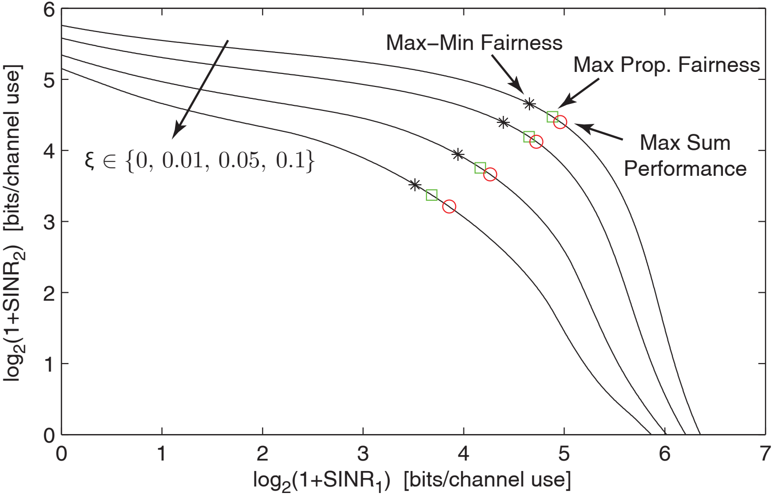
$$\begin{bmatrix} \mathbf{B}_k^H \mathbf{A}_k \mathbf{B}_k & \mathbf{B}_k^H \mathbf{A}_k \widehat{\mathbf{h}}_k \\ \widehat{\mathbf{h}}_k^H \mathbf{A}_k \mathbf{B}_k & \widehat{\mathbf{h}}_k^H \mathbf{A}_k \widehat{\mathbf{h}}_k - \sigma_k^2 \end{bmatrix} + \begin{bmatrix} \lambda_k \mathbf{I}_N & \mathbf{0}_{N \times 1} \\ \mathbf{0}_{1 \times N} & -\lambda_k \end{bmatrix} \succeq \mathbf{0}_{N+1} \tag{9}$$

where $\mathbf{A}_k = \frac{1}{\gamma_k} \mathbf{C}_k \mathbf{D}_k \mathbf{S}_k \mathbf{D}_k^H \mathbf{C}_k^H - \sum_{i \neq k} \mathbf{C}_k \mathbf{D}_i \mathbf{S}_i \mathbf{D}_i^H \mathbf{C}_k^H$.

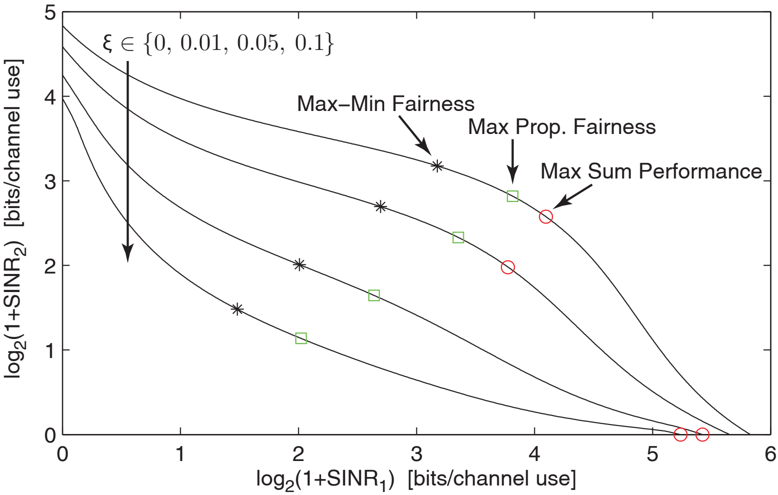
Observations on Robust SINR Opt.

- This theorem converts the infinite number of SINR constraints into just **one (linear) semi-definite constraint per user**—at the cost of adding K_r extra variables $\{\lambda_k\}_{k=1}^{K_r}$ that indirectly represents the worst channel conditions in the uncertainty set.
- Channel uncertainty naturally increases the **computational complexity**, but Theorem 2 shows that the robust problem is also convex. The complexity is thus still polynomial in the number of antennas N, users K_r, and power constraints L.
- The transmit strategy $\mathbf{S}_1^*, \ldots, \mathbf{S}_{K_r}^*$ that solves might in general have $\mathrm{rank}(\mathbf{S}_k^*) > 1$ for some users. As **single-stream beamforming** is always sufficient under perfect CSI, we can however expect it to also work well when the uncertainty is small.

Illustrations: Uncertainty I



Illustrations: Uncertainty II



Further Remarks on Channel Uncertainty

- We can solve any **robust multi-cell resource allocation** problem as a sequence of the **convex problems** described above.
- We can expect a **single-stream beamforming** solution.
- Worst-case robustness under separate uncertainty

Theorem

For a MISO interference channel with per-transmitter constraints of q_j, all Pareto optimal points of the robust performance region are achieved by beamforming vectors $\mathbf{v}_{j_k k}(\lambda_k)$ for $\lambda_k \in [0,1]^{K_r-1}$ for $k = 1, \dots, K_r$:

$$\begin{aligned} \mathbf{v}_{j_k k}(\lambda_k) \quad = \quad & \underset{\mathbf{v}_{j_k k}}{\arg\max}\; \Re(\hat{\mathbf{h}}_{j_k k}^H \mathbf{C}_{j_k k} \mathbf{v}_{j_k k}) - \|\mathbf{B}_{j_k k}^H \mathbf{C}_{j_k k} \mathbf{v}_{j_k k}\| \\ & \text{subject to} \quad \Im\left(|\hat{\mathbf{h}}_{j_k k}^H \mathbf{C}_{j_k k} \mathbf{v}_{j_k k} \right) = 0, \|\mathbf{v}_{j_k k}\| \leq q_{j_k}, \\ & |\hat{\mathbf{h}}_{j_k i}^H \mathbf{C}_{j_k i} \mathbf{v}_{j_k k}| + \|\mathbf{B}_{j_k i} \mathbf{C}_{j_k i} \mathbf{v}_{j_k k}\|_2 \leq \sqrt{\lambda_{ki} \Gamma_{ki}} \quad \forall i \neq k, \end{aligned}$$

for fixed values of Γ_{ki}. The elements in λ_k are denoted λ_{ki} for all $i \neq k$.

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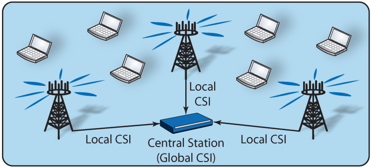
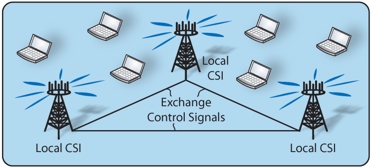
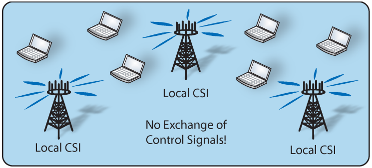
Distributed Implementation



(a) Centralized Optimization and Resource Allocation



(b) Distributed Allocation with Control Signaling (Subsection 4.2.1)



(c) Truly Distributed Allocation (Subsection 4.2.2)

The very essence of resource allocation problems is the **coupling between the users**, in terms of inter-user interference and power constraints.

Different implementations of resource allocation in multi-cell systems: (a) Global CSI is gathered at a central station that allocates resources; (b) Base stations perform distributed resource allocation by iteratively exchanging control variables (but not CSI); and (c) Base stations perform distributed resource allocation without exchanging any information (but the problem formulation and local CSI is available).

Example Decomposition

As a first step, the **feasibility problem with SINR requirements** (with $\mathrm{SINR}_k \geq \gamma_k \, \forall k$) is rewritten as

$$\begin{aligned} \text{find} \;& \mathbf{v}_k, \Theta_{ik}, \widetilde{\Theta}_{ik}, q_{lk} \, \forall k, i, l, \quad i \neq k \\ \text{subject to} \;& \frac{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2}{\sigma_k^2 + \sum_{i \neq k} \widetilde{\Theta}_{ik}^2} \geq \gamma_k \quad \forall k, \\ & |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2 \leq \Theta_{ik}^2, \quad \Theta_{ik} \leq \widetilde{\Theta}_{ik} \quad \forall k, i, \; i \neq k, \\ & \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_{lk}, \quad \sum_{i=1}^{K_r} q_{li} \leq q_l \quad \forall l, k \end{aligned}$$

by adding non-negative **auxiliary variables** $\Theta_{ik}, \widetilde{\Theta}_{ik}, q_{lk}$. The squared variable Θ_{ik}^2 is the actual interference generated at User k by signals intended for User i, while $\widetilde{\Theta}_{ik}^2$ is its believed value in the beamforming optimization for User k.

- **Dual decomposition approach.**
- Lagrange multipliers are denoted y_{ik} and z_l for the interference and power consistency constraints, respectively, decompose the problem into K_r

$$\begin{aligned} \underset{\mathbf{v}_k,\{\Theta_{ki}\}_{\forall i},\{\widetilde{\Theta}_{ik}\}_{\forall i},\{q_{lk}\}_{\forall l}}{\text{minimize}} \quad & \sum_{i \neq k} \left(y_{ki}\Theta_{ki} - y_{ik}\widetilde{\Theta}_{ik} \right) + \sum_{l=1}^{L} z_l q_{lk} \\ \text{subject to} \quad & \frac{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2}{\sigma_k^2 + \sum_{i \neq k} \widetilde{\Theta}_{ik}^2} \geq \gamma_k \quad \forall k, \\ & \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_{lk}, \quad q_{lk} \leq q_l \quad \forall l, \\ & |\mathbf{h}_i^H \mathbf{C}_i \mathbf{D}_k \mathbf{v}_k|^2 \leq \Theta_{ki}^2 \quad i \neq k \end{aligned}$$

- with a **master dual problem**

$$\underset{\{y_{ik}\}_{\forall k,i},\{z_l\}_{\forall l}}{\text{maximize}} \sum_{k=1}^{K_r} \sum_{i \neq k} y_{ik} \left(\Theta_{ik}^* - \widetilde{\Theta}_{ik}^* \right) + \sum_{l=1}^{L} z_l \left(\sum_{k=1}^{K_r} q_{lk}^* - q_l \right),$$

where $\Theta_{ik}^*, \widetilde{\Theta}_{ik}^*, q_{lk}^*, \mathbf{v}_k^*$ are the subproblem solutions.

- This decomposition enables an iterative procedure where the subproblems are solved for constant values on the Lagrange multipliers $y_{ki}, y_{ik} \ \forall i \neq k$ and $z_l \ \forall l$.
- This is called **dual decomposition** because the Lagrange multipliers are viewed as constants in the subproblems, and not the coupling variables themselves.
- The **master problem** has a more complicated structure and is typically solved by subgradient methods.
- This implies that the Lagrange multipliers in iteration $n+1$ are achieved as

$$y_{ik}^{(n+1)} = \left[y_{ik}^{(n)} - \xi(\widetilde{\Theta}_{ik}^{(n)} - \Theta_{ik}^{(n)}) \right]_{+} \quad (10)$$

$$z_l^{(n+1)} = \left[z_l^{(n)} - \xi\big(q_l - \sum_{k=1}^{K_r} q_{lk}^{(n)}\big) \right]_{+} \quad (11)$$

where $\xi > 0$ is the step size.

Decomposition Remarks

- Since the approach described above provides a distributed solution to resource allocation with fixed SINR requirements, it can also be used as a **subproblem in algorithms that successively increase the SINR requirements** for the purpose of obtaining a Pareto optimal point.
- Therefore, it can be **generalized** to find certain **operating points** on the Pareto boundary as, e.g., the Max-min fairness point.
- To summarize, the dual decomposition approach should be seen as proof-of-concept: convex and quasi-convex resource allocation problems can be implemented in a distributed fashion by **exchanging control variables** rather than CSI.

Decomposition Example Illustration

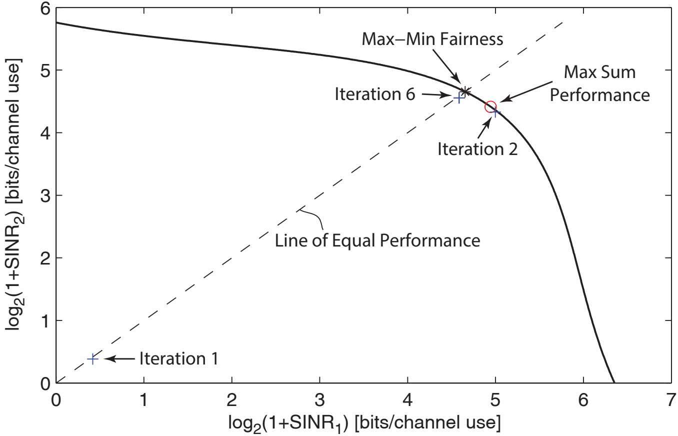


Illustration of the **convergence of the distributed resource allocation**. Max-min fairness is the system utility function and 98% of the optimal performance is achieved after 6 iterations.

Hardware Distortion Model

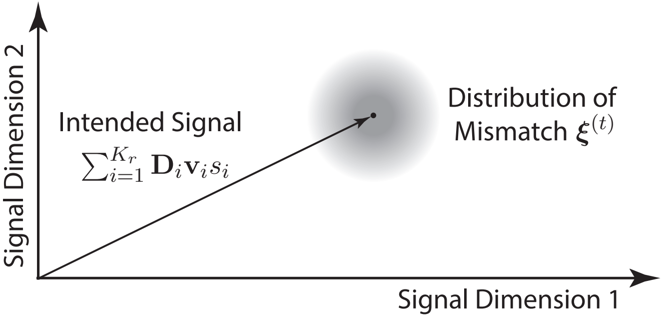
- **Practical transceiver hardware** suffer from impairments: phase-noise, I/Q imbalance, nonlinearities, etc.
- These impairments distort the transmitted and received signals.
- Residual distortion noise **remain after calibration algorithms**.
- A **generalization** of the multi-cell system model is given by

$$y_k = \mathbf{h}_k^H \mathbf{C}_k \left(\sum_{i=1}^{K_r} \mathbf{D}_i \mathbf{v}_i s_i + \boldsymbol{\xi}^{(t)} \right) + n_k + \xi_k^{(r)} \tag{12}$$

  where $\mathbf{v}_i s_i$ is the transmitted signal to User i.
- The new terms $\boldsymbol{\xi}^{(t)} \in \mathbb{C}^N$ and $\xi_k^{(r)} \in \mathbb{C} \ \forall k$ are the additive **transmitter-distortion** and **receiver-distortion**, respectively.
- These are modeled as **zero-mean complex Gaussian** and are statistically independent of the data signals, but the covariance matrices depend on the power of the transmitted and received signals, respectively.

Additive Impairment Mismatch

- The transmitter-distortion $\boldsymbol{\xi}^{(t)}$ describes the **mismatch between the aggregate data signal** $\sum_{i=1}^{K_r} \mathbf{D}_i \mathbf{v}_i s_i$ designed by the resource allocation and what is **actually transmitted by the RF hardware**. The mismatch is due to transceiver hardware impairments in the transmitter.



Additive Mismatch Model

- We assume that the elements of $\boldsymbol{\xi}^{(t)}$ are **uncorrelated**, which can be confirmed by measurements on decoupled antenna branches, i.e., $\boldsymbol{\xi}^{(t)} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Xi})$ where $\boldsymbol{\Xi} \in \mathbb{C}^{N \times N}$ is a diagonal covariance matrix.
- The **signal power** allocated to the nth transmit antenna can be computed as $\|\mathbf{T}_n \mathbf{V}_{\text{tot}}\|_F^2$, where $\mathbf{T}_n \in \mathbb{C}^{N \times N}$ is zero except at the nth diagonal element and $\mathbf{V}_{\text{tot}} = [\mathbf{D}_1 \mathbf{v}_1 \ \ldots \ \mathbf{D}_{K_r} \mathbf{v}_{K_r}]$.
- The **distortion** at the n-th antenna increases with $\|\mathbf{T}_n \mathbf{V}_{\text{tot}}\|_F^2$, thus

$$\boldsymbol{\Xi} = \begin{bmatrix} \big(\eta_1(\|\mathbf{T}_1 \mathbf{V}_{\text{tot}}\|_F)\big)^2 & & \\ & \ddots & \\ & & \big(\eta_N(\|\mathbf{T}_N \mathbf{V}_{\text{tot}}\|_F)\big)^2 \end{bmatrix} \tag{13}$$

where $\eta_n(\cdot)$ is a continuous and monotonically increasing function.

Error Vector Magnitude (EVM)

- The **error vector magnitude** (EVM) is the ratio between the average distortion magnitude and the average transmit magnitude, defined as

$$\mathrm{EVM}_n^{(t)} = \sqrt{\frac{\mathbb{E}\left\{\left|[\boldsymbol{\xi}^{(t)}]_n\right|^2\right\}}{\mathbb{E}\left\{\left|\left[\sum_{k=1}^{K_r}\mathbf{D}_k\mathbf{v}_k s_k\right]_n\right|^2\right\}}} = \frac{\eta_n\left(\|\mathbf{T}_n\mathbf{V}_{\mathsf{tot}}\|_F\right)}{\|\mathbf{T}_n\mathbf{V}_{\mathsf{tot}}\|_F} \qquad (14)$$

  and is often reported as a percentage.
- The EVM requirements in **3GPP Long Term Evolution** (LTE) are 8-17.5 % at the transmitter, depending on the anticipated spectral efficiency.

Power Constraints with Distortion

- Due to the statistical independence of all terms, the **power constraints** are generalized as

$$\sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k + \mathrm{tr}(\mathbf{Q}_{l\xi} \mathbf{\Xi}) \leq q_l \qquad l = 1, \ldots, L. \tag{15}$$

- The **weighting matrix** $\mathbf{Q}_{l\xi} \in \mathbb{C}^{N \times N}$ is positive semi-definite and should typically have the same structure as $\mathbf{Q}_{lk}$, but we might have $\mathrm{tr}(\mathbf{Q}_{l\xi}) < \mathrm{tr}(\mathbf{Q}_{lk})$.
- An **alternative model** is to keep the original power constraints (i.e., set $\mathbf{Q}_{l\xi} = \mathbf{0}_N$) and simply reduce each limit q_l to account for the distortions.

- The **receiver-distortion** $\xi_k^{(r)}$ of User k models the mismatch between ideal and practical reception:

$$\xi_k^{(r)} \sim \mathcal{CN}(0, \sigma_{k,\xi}^2) \quad \text{where} \quad \sigma_{k,\xi} = \nu_k \left(\|\mathbf{h}_k^H \mathbf{C}_k \mathbf{V}_{\text{tot}}\|_F \right) \qquad (16)$$

  and $\nu_k(\cdot)$ is a continuous and monotonically increasing function.
- The main error sources are **phase noise** and **I/Q-imbalance**, thus we can expect $\nu_k(\cdot)$ to behave as $\nu_k(x) = \mathrm{EVM}_k^{(r)} x$, where $\mathrm{EVM}_k^{(r)}$ is a constant EVM-term.

- The generalized system model in (12) results in generalized power constraints (15) and a **generalized SINR** expression for User k:

$$\mathrm{SINR}_k = \frac{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2}{\sigma_k^2 + \sigma_{k,\xi}^2 + \sum_{i \neq k} |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2 + \mathbf{h}_k^H \mathbf{C}_k \mathbf{\Xi} \mathbf{C}_k^H \mathbf{h}_k}. \quad (17)$$

- Otherwise the **multi-objective** and **single-objective resource allocation** problems are the **same**.
- The **feasibility problem** is generalized (where $\{\gamma_k\}_{k=1}^{K_r}$ fixed)

$$\begin{aligned} \text{find} \;\; & \mathbf{v}_1 \ldots, \mathbf{v}_{K_r} \qquad (18) \\ \text{subject to} \;\; & |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2 \geq \gamma_k \Big(\sigma_k^2 + \sigma_{k,\xi}^2 \\ & \quad + \sum_{i \neq k} |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2 + \mathbf{h}_k^H \mathbf{C}_k \mathbf{\Xi} \mathbf{C}_k^H \mathbf{h}_k \Big) \quad \forall k, \\ & \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k + \mathrm{tr}(\mathbf{Q}_{l\xi} \mathbf{\Xi}) \leq q_l \quad \forall l. \end{aligned}$$

Convex Reformulation under Impairments

Corollary

The feasibility problem in (18) *can be reformulated as*

$$\text{find } \mathbf{v}_k, t_n, \rho_k \quad \forall k, n \tag{19}$$

$$\text{subject to } \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k + \sum_{n=1}^{N} \operatorname{tr}(\mathbf{Q}_{l\xi} \mathbf{T}_n) t_n^2 \leq q_l \quad \forall l, \tag{20}$$

$$\sqrt{\sigma_k^2 + \rho_k + \sum_{i=1}^{K_r} |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2 + \sum_{n=1}^{N} t_n^2 \mathbf{h}_k^H \mathbf{C}_k \mathbf{T}_n \mathbf{C}_k^H \mathbf{h}_k} \leq \sqrt{\frac{1+\gamma_k}{\gamma_k}} \Re(\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k) \quad \forall k, \tag{21}$$

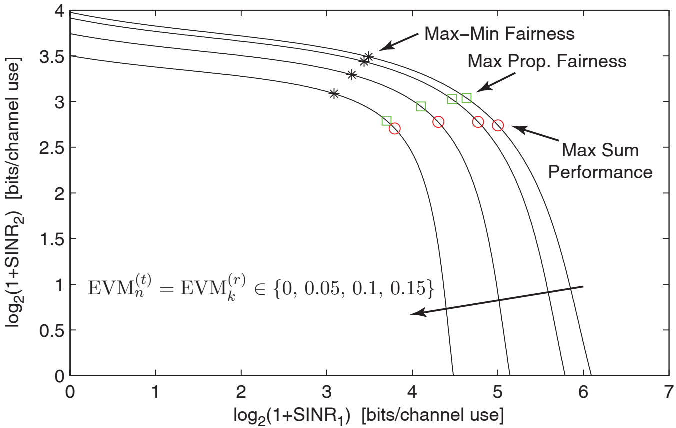
$$\Im(\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k) = 0 \quad \forall k, \tag{22}$$

$$\eta_n(\|\mathbf{T}_n \mathbf{V}_{tot}\|_F) \leq t_n \quad \forall n, \tag{23}$$

$$\nu_k\left(\|\mathbf{h}_k^H \mathbf{C}_k \mathbf{V}_{tot}\|_F\right) \leq \rho_k \quad \forall k \tag{24}$$

and is **jointly convex** *in the beamforming vectors and in the auxiliary optimization variables* $\{t_n\}_{n=1}^{N}$, $\{\rho_k\}_{k=1}^{K_r}$ **if** $\eta_n(\cdot)$ *and* $\nu_k(\cdot)$ *are* **monotonically increasing convex** *functions.*

Rate Region under Hardware Impairments



Performance regions for two different channel realizations under global joint transmission with **different levels of (linear) transceiver hardware impairments**; the EVM at the transmitters and receivers is 0, 5, 10, or 15 percent.

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Problem Statement: Multicast

- The maximization of the minimum achievable SNR leads to

$$\max_{\mathbf{v}:\|\mathbf{v}\|\leq q} \min_{k\in\mathcal{K}_j} \frac{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}|^2}{\sigma_k^2}. \tag{25}$$

- Alternatively, the power minimization under SNR requirement γ

$$\min_{\mathbf{v}} \ \|\mathbf{v}\|^2 \quad \text{subject to} \quad \frac{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}|^2}{\sigma_k^2} \geq \gamma \quad \forall k \in \mathcal{K}_j. \tag{26}$$

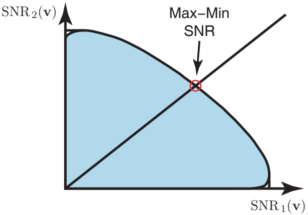
Corollary

For a fixed total transmit power q, the multi-cast beamforming vector which solves (25) *is given by*

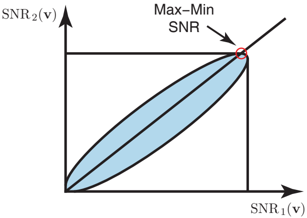
$$\mathbf{v}_k^*(\lambda) = q\, \boldsymbol{v}_{\max}\left(\sum_{k\in\mathcal{K}_j} \lambda_k \mathbf{D}_k^H \mathbf{C}_k^H \mathbf{h}_k \mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k\right) \tag{27}$$

for some set of $|\mathcal{K}_j|$ parameters that satisfies $\lambda_k \geq 0$ and $\sum_{k\in\mathcal{K}_j} \lambda_k = 1$.

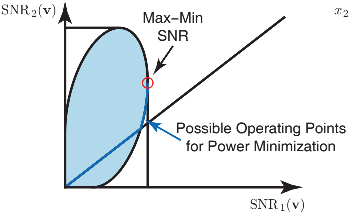
Multicast Illustration



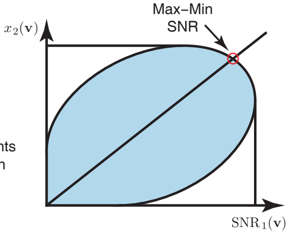
(a) Almost Orthogonal Channels



(b) Almost Equal Channels



(c) Unequal Path Losses



(d) Equal Path Losses

1. Robustness to Channel Uncertainty
2. Distributed Resource Allocation
3. Transceiver Hardware Impairments
4. Multi-Cast Transmission
5. Multi-Carrier Systems
6. Multi-Antenna Users
7. Design of Dynamic Cooperation Clusters
8. Cognitive Radio Systems
9. Physical Layer Security

Multi-Carrier System Model

- The received signal at User k on the cth subcarrier is

$$y_{kc} = \mathbf{h}_{kc}^H \mathbf{C}_k \sum_{i=1}^{K_r} \mathbf{D}_i \mathbf{v}_{ic} s_{ic} + n_{kc} \qquad c = 1, \ldots, K_c. \tag{28}$$

- Assuming that all signal and noise variables are independent, the **SINR at User k on the cth subcarrier** is

$$\mathrm{SINR}_{kc}(\mathbf{v}_{1c}, \ldots, \mathbf{v}_{K_r c}) = \frac{|\mathbf{h}_{kc}^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_{kc}|^2}{\sigma_{kc}^2 + \sum_{i \neq k} |\mathbf{h}_{kc}^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_{ic}|^2} \tag{29}$$

with $\mathbf{v}_{kc}$ is the beamforming vector and the **power constraints** are

$$\sum_{k=1}^{K_r} \sum_{c=1}^{K_c} \mathbf{v}_{kc}^H \mathbf{Q}_{lkc} \mathbf{v}_{kc} \leq q_l \qquad l = 1, \ldots, L \tag{30}$$

where $\mathbf{Q}_{lkc} \succeq \mathbf{0}_N$ might model subcarrier-specific characteristics.

Multi-Carrier Resource Allocation

- The **multi-objective resource allocation** problem under multi-carrier transmission can be formulated as

$$\max_{\mathbf{v}_{kc} \succeq \mathbf{0}_N \, \forall k,c} \quad \{g_1, \ldots, g_{K_r}\} \tag{31}$$

$$\text{subject to} \quad g_k = g_k\left(\{\mathrm{SINR}_{kc}(\mathbf{v}_{1c}, \ldots, \mathbf{v}_{K_r c})\}_{c=1}^{K_c}\right) \quad \forall k, \tag{32}$$

$$\sum_{k=1}^{K_r} \sum_{c=1}^{K_c} \mathbf{v}_{kc}^H \mathbf{Q}_{lkc} \mathbf{v}_{kc} \leq q_l \quad \forall l. \tag{33}$$

- The overwhelming multi-carrier complexity can be handled by dividing the subcarriers into subsets of manageable size and solve these separately (cf. *physical resource blocks* in 3GPP LTE).

Multi-Carrier Parametrization

Corollary

Every feasible point $\mathbf{g} \in \mathcal{R}$ *is achieved by beamforming vectors* $\mathbf{v}_{kc} = \sqrt{p_{kc}}\bar{\mathbf{v}}_{kc}$ *for all* k, c*, where*

$$\bar{\mathbf{v}}_{kc} = \frac{\mathbf{\Psi}_{kc}^{\dagger}\mathbf{D}_k^H\mathbf{h}_{kc}}{\|\mathbf{\Psi}_{kc}^{\dagger}\mathbf{D}_k^H\mathbf{h}_{kc}\|}, \tag{34}$$

$$\begin{bmatrix} p_{1c} & \dots & p_{K_r c} \end{bmatrix} = \begin{bmatrix} \gamma_{1c}\sigma_{1c}^2 & \dots & \gamma_{K_r c}\sigma_{K_r c}^2 \end{bmatrix} \mathbf{M}_c^{\dagger}, \tag{35}$$

$$\mathbf{\Psi}_{kc} = \left(\sum_{l=1}^{L} \frac{\mu_l}{q_l}\mathbf{Q}_{lkc} + \sum_{i=1}^{K_r} \frac{\lambda_{ic}}{\sigma_{ic}^2}\mathbf{D}_k^H\mathbf{C}_i^H\mathbf{h}_{ic}\mathbf{h}_{ic}^H\mathbf{C}_i\mathbf{D}_k \right), \tag{36}$$

$$\gamma_{kc} = \frac{\lambda_{kc}}{\sigma_{kc}^2}\mathbf{h}_{kc}^H\mathbf{D}_k\left(\mathbf{\Psi}_{kc} - \frac{\lambda_{kc}}{\sigma_{kc}^2}\mathbf{D}_k^H\mathbf{C}_k^H\mathbf{h}_{kc}\mathbf{h}_{kc}^H\mathbf{C}_k\mathbf{D}_k\right)^{\dagger}\mathbf{D}_k^H\mathbf{h}_k, \tag{37}$$

$$[\mathbf{M}_c]_{ik} = \begin{cases} |\mathbf{h}_{ic}^H\mathbf{C}_i\mathbf{D}_i\bar{\mathbf{v}}_{ic}|^2, & i = k, \\ -\gamma_{kc}|\mathbf{h}_{kc}^H\mathbf{C}_k\mathbf{D}_i\bar{\mathbf{v}}_{ic}|^2, & i \neq k, \end{cases} \tag{38}$$

for some non-negative parameters $\{\lambda_{kc}\}_{k=1,c=1}^{K_r,K_c}$ *and* $\{\mu_l\}_{l=1}^{L}$.

System and Channel Model

- Resource allocation is harder to optimize with multi-antenna users.
- Consider a **two-user MIMO IC** denoted by $\mathrm{TX}_i \mapsto \mathrm{RX}_i, i = 1, 2$, where each transmitter TX_i and each receiver RX_i are equipped with $N_T \geq 2$ and $N_R \geq 2$ antennas and a single data stream is transmitted to each user.
- In this IC, the **received signal** at RX_i is modelled as

$$y_i = \mathbf{g}_i^H \left(\mathbf{H}_{ii} \mathbf{v}_i s_i + \mathbf{H}_{ki} \mathbf{v}_k s_k + \mathbf{n}_i \right) \qquad i,\ k \in \{1, 2\}, k \neq i$$

  where $s_i \sim \mathcal{CN}(0, 1)$ is the transmitted symbol of TX_i by the transmit beamforming vector $\mathbf{v}_i \in \mathbb{C}^{N_T \times 1}$.
- The matrices $\mathbf{H}_{ii}, \mathbf{H}_{ki} \in \mathbb{C}^{N_R \times N_T}$ are **channel matrices** of direct link $\mathrm{TX}_i \mapsto \mathrm{RX}_i$ and cross-talk link $\mathrm{TX}_k \mapsto \mathrm{RX}_i$, respectively.
- Each transmitter has a **power constraint** set to 1 and define the set of feasible transmit beamforming vectors as $\mathcal{V} \triangleq \left\{ \mathbf{v} \in \mathbb{C}^{N_T \times 1} : \|\mathbf{v}\|^2 \leq 1 \right\}$.

Achievable Rates

- The **achievable rate** of the link $\mathrm{TX}_i \mapsto \mathrm{RX}_i$ is given by

$$R_i(\mathbf{v}_1, \mathbf{v}_2, \mathbf{g}_i) = \log_2\left(1 + \mathrm{SINR}_i(\mathbf{v}_1, \mathbf{v}_2, \mathbf{g}_i)\right) \tag{39}$$

  where $\mathrm{SINR}_i(\mathbf{v}_1, \mathbf{v}_2, \mathbf{g}_i) = \frac{|\mathbf{g}_i^H \mathbf{H}_{ii} \mathbf{v}_i|^2}{\sigma_i^2 + |\mathbf{g}_i^H \mathbf{H}_{ki} \mathbf{v}_k|^2}$.
- With **linear MMSE** receive filter

$$\mathrm{SINR}_i(\mathbf{v}_1, \mathbf{v}_2) = \mathbf{v}_i^H \underbrace{\mathbf{H}_{ii}^H \left(\sigma_i^2 \mathbf{I} + \mathbf{H}_{ki} \mathbf{v}_k {\mathbf{v}_k}^H {\mathbf{H}_{ki}}^H\right)^{-1} \mathbf{H}_{ii}}_{\triangleq \mathbf{A}_i(\mathbf{v}_k)} \mathbf{v}_i.$$

Proposition

For the two-user single-beam MIMO IC, the SINR can be reformulated as

$$\mathrm{SINR}_i(\mathbf{v}_1, \mathbf{v}_2) = \sin^2(\theta_{H,i}) \cdot \frac{\|\mathbf{H}_{ii}\mathbf{v}_i\|^2}{\sigma_i^2} + \cos^2(\theta_{H,i}) \cdot \frac{\|\mathbf{H}_{ii}\mathbf{v}_i\|^2}{\sigma_i^2 + \|\mathbf{H}_{ki}\mathbf{v}_k\|^2}$$

where $\cos(\theta_{H,i}) = \left| \overrightarrow{\mathbf{H}_{ii}\mathbf{v}_i}^H \cdot \overrightarrow{\mathbf{H}_{ki}\mathbf{v}_k} \right|$ and $\theta_{H,i} \in [0, \pi/2]$.

Pareto Boundary Points

Communications Theory

Proposition

For the two-user single-stream MIMO IC, all the operating points on the strict Pareto boundary can be achieved only when both the transmitters spend the full power, i.e., $\|\mathbf{v}_1\|^2 = \|\mathbf{v}_2\|^2 = 1$.

- A **single-user point** can be easily achieved when only TX_i works and simultaneously operates "egoistically" to maximize its own rate.
- The **maximum achievable rate** $\overline{R_i}$ of the link $\mathrm{TX}_i \mapsto \mathrm{RX}_i$ and its associated **"egoistic" strategy** $\mathbf{v}_i^{\mathrm{Ego}}$ are

$$\overline{R_i} = \log_2\left(1 + \frac{\lambda_1(\mathbf{H}_{ii}^H \mathbf{H}_{ii})}{\sigma_i^2}\right), \quad \mathbf{v}_i^{\mathrm{Ego}} = \mathbf{u}_1(\mathbf{H}_{ii}^H \mathbf{H}_{ii}) \quad \forall i. \qquad (40)$$

Ending Points of Strict PB

- Each **ending point of the strict Pareto boundary** can be achieved when one transmitter employs an **"altruistic"** strategy to create no interference to the other receiver and simultaneously to maximize its own rate and the other transmitter operates **"egoistically"**.

Proposition

$E1(\overline{R}_1, \underline{R}_2)$ can be achieved by $(\mathbf{v}_1^{\text{Ego}}, \mathbf{v}_2^{\text{Alt}})$, where $\overline{R}_1$ and $\mathbf{v}_1^{\text{Ego}}$ are as above and

$$\begin{aligned} \underline{R}_2 &= \log_2\left(1 + \mathbf{v}_2^{\text{Alt},H} \mathbf{A}_2(\mathbf{v}_1^{\text{Ego}}) \mathbf{v}_2^{\text{Alt}}\right), \\ \mathbf{v}_2^{\text{Alt}} &= \overrightarrow{\Pi^{\perp}_{\mathbf{H}_{21}^H \mathbf{H}_{11} \mathbf{v}_1^{\text{Ego}}} \mathbf{u}_1\left(\mathbf{B}_1, \Pi^{\perp}_{\mathbf{H}_{21}^H \mathbf{H}_{11} \mathbf{v}_1^{\text{Ego}}}\right)}, \end{aligned} \tag{41}$$

with $\mathbf{B}_1 \triangleq \Pi^{\perp}_{\mathbf{H}_{21}^H \mathbf{H}_{11} \mathbf{v}_1^{\text{Ego}}} \mathbf{A}_2(\mathbf{v}_1^{\text{Ego}}) \Pi^{\perp}_{\mathbf{H}_{21}^H \mathbf{H}_{11} \mathbf{v}_1^{\text{Ego}}}$.

- An arbitrary point $(R_1^\star, R_2^\star)$ on the **non-strict Pareto boundary** can be computed as

$$R_i^\star = \gamma \cdot \underline{R}_i \text{ and } R_k^\star = \overline{R}_k, \ \forall i, \ k \in \{1,2\}, \ k \neq i$$

where $i = 1$ and $i = 2$ correspond to the horizontal part and the vertical part, respectively.

- The **ZF transmit strategies**

$$\mathbf{v}_1^{\text{ZF}} = \mathbf{u}_\ell(\mathbf{H}_{22}^H \mathbf{H}_{12}, \mathbf{H}_{21}^H \mathbf{H}_{11}), \forall \ell \in \{1, 2, ..., N_T\},$$

$$\mathbf{v}_2^{\text{ZF}} = \sum_{\ell=1}^{N_T-1} c_\ell \mathbf{u}_\ell(\Pi^\perp_{\mathbf{H}_{21}^H \mathbf{H}_{11} \mathbf{v}_1^{\text{ZF}}}), \tag{42}$$

where $\{c_\ell\}_{\ell=1}^{N_T-1}$ are complex-valued numbers with $\sum_{\ell=1}^{N_T-1} |c_\ell|^2 = 1$.

- **ZF achievable rates** are

$$R_i^{\text{ZF}}(\mathbf{v}_i^{\text{ZF}}, \mathbf{v}_2^{\text{ZF}}) = \log_2 \left(1 + \frac{\left\| \mathbf{H}_{ii} \mathbf{v}_i^{\text{ZF}} \right\|^2}{\sigma_i^2} \right) \quad \forall i. \tag{43}$$

Strict Pareto Boundary

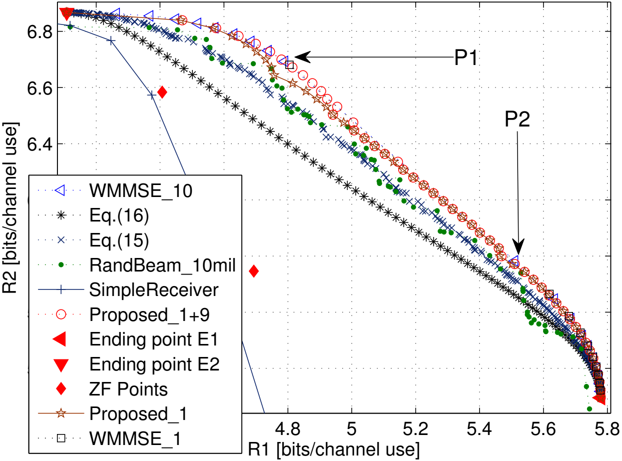
Communications Theory

- We propose the following **optimization problem**

$$(\mathrm{P0}) \begin{cases} \underset{\mathbf{v}_1, \mathbf{v}_2 \in \mathcal{W}_{FP}}{\text{maximize}} & \mathrm{SINR}_1(\mathbf{v}_1, \mathbf{v}_2) \\ \text{subject to} & \mathrm{SINR}_2(\mathbf{v}_1, \mathbf{v}_2) = \mathrm{SINR}_2^\star. \end{cases}$$

  where $\mathrm{SINR}_2^\star \in (2^{\underline{R}_2} - 1, 2^{\overline{R}_2} - 1)$ is an **SINR constraint**, and $\mathbf{v}_1, \mathbf{v}_2$ should be in $\mathcal{V}_{FP} \triangleq \left\{ \mathbf{v} \in \mathbb{C}^{N_T \times 1} : \|\mathbf{v}\|^2 = 1 \right\}$.
- Then, $(R_1^\star, R_2^\star) = (\log_2(1 + \mathrm{SINR}_1(\mathbf{v}_1^\star, \mathbf{v}_2^\star)), \log_2(1 + \mathrm{SINR}_2^\star(\mathbf{v}_1^\star, \mathbf{v}_2^\star)))$ is achieved by the optimal solution $(\mathbf{v}_1^\star, \mathbf{v}_2^\star)$ to $(\mathrm{P0})$.
- **Direct joint optimization** of $\mathbf{v}_1$ and $\mathbf{v}_2$ is analytically **intractable**.
- To solve $(\mathrm{P0})$, an **alternating optimization** algorithm is applied to optimize $\mathbf{v}_1$ and $\mathbf{v}_2$ alternatively.

Rate Region Boundary



Achievable boundary with SNR=10dB and $N_T = 3, N_R = 2$ for a random Gaussian channel data.

Multi-User MIMO IC Extension

Achievable rate of the link $\mathrm{TX}_k \mapsto \mathrm{RX}_k$

$R_k(\{\mathbf{v}_k\}_{\mathcal{K}}) = \log_2\left(1 + \mathrm{SINR}_k(\{\mathbf{v}_k\}_{\mathcal{K}})\right)\ \forall k \in \mathcal{K} = \{1, ..., K\}$, where

$$\mathrm{SINR}_k(\{\mathbf{v}_i\}_{\mathcal{K}}) = \mathbf{v}_k^H \underbrace{\mathbf{H}_{kk}^H \left(\sum_{i \neq k} \mathbf{H}_{ik}\mathbf{v}_i{\mathbf{v}_i}^H{\mathbf{H}_{ik}}^H + \sigma_k^2\mathbf{I}\right)^{-1} \mathbf{H}_{kk}}_{\mathbf{A}_k(\mathbf{v}_{-k})} \mathbf{v}_k \quad (44)$$

is the SINR expression of the kth user. $\mathbf{v}_{-k}$ denotes $\{\mathbf{v}_i\}_{\mathcal{K}\backslash\{k\}}$.

- Fix all but one SINR values

$$(\mathrm{Q0}) \begin{cases} \underset{\{\mathbf{v}_i\}_{\mathcal{K}}}{\text{maximize}} & \mathrm{SINR}_1(\{\mathbf{v}_i\}_{\mathcal{K}}) \\ \text{subject to} & \mathrm{SINR}_k(\{\mathbf{v}_i\}_{\mathcal{K}}) = \mathrm{SINR}_k^\star, \quad \forall k \in \mathcal{K}\backslash\{1\}. \\ & \mathbf{v}_i^H\mathbf{v}_i \le 1, \quad \forall i \in \mathcal{K}. \end{cases}$$

$(\mathrm{Q0})$ is a non-convex problem w.r.t. K beamformers, i.e., $\{\mathbf{v}_i\}_{\mathcal{K}}$.

Algorithm 3 The Proposed Alternating Optimization Algorithm for K-user Case

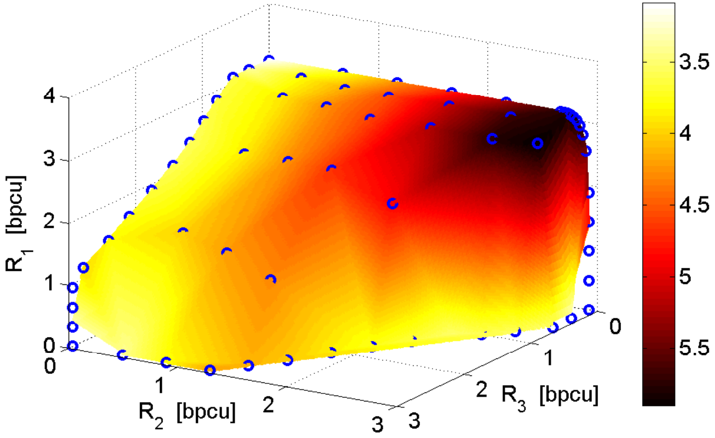
Input: Rate requirements $\{R_k^\star\}_{\mathcal{K}\setminus\{1\}}$ where $R_k^\star = \log_2(1+\mathrm{SINR}_k^\star)$ and $\mathrm{SINR}_k^\star \in \left(0, \frac{1}{\sigma_k^2}\lambda_1(\boldsymbol{H}_{kk}\boldsymbol{H}_{kk}^H)\right]$.

Output: A convergent operating point $(R_1^{(\ell)}, R_2^\star, ..., R_K^\star)$ with $\{\boldsymbol{w}_i^{(\ell)}\}_{\mathcal{K}}$.

begin

- Initialization: Set a feasible $\boldsymbol{w}_{-1}^{(0)}$, $\ell = 0$.
- **while** *some convergence criterion is not satisfied* **do**
  - $\ell{+}{+}$.
  - **for** $k = 1 \to K$ **do**
    - Given $\boldsymbol{w}_{-k}^{(\ell-1)}$, obtain an optimal $\boldsymbol{W}_k$ to (Qk′);
    - Extract a tight/approximate $\boldsymbol{w}_k$ from $\boldsymbol{W}_k$ to (Qk).
    - **if** $K \geq 5$ and $\mathrm{SINR}_1\left(\{\boldsymbol{w}_i^{(\ell)}\}_{i\leq k}, \{\boldsymbol{w}_i^{(\ell-1)}\}_{i>k}\right) < \mathrm{SINR}_1\left(\{\boldsymbol{w}_i^{(\ell)}\}_{i<k}, \{\boldsymbol{w}_i^{(\ell-1)}\}_{i\geq k}\right)^8$ **then**
      - $\boldsymbol{w}_k^{(\ell)} = \boldsymbol{w}_k^{(\ell-1)}$;
  - Compute $R_1^{(\ell)} = \log_2\left(1 + \mathrm{SINR}_1\left(\{\boldsymbol{w}_k^{(\ell)}\}_{\mathcal{K}}\right)\right)$.

3D Achievable Rate Region



Achievable rate region for three-user MIMO IC with SNR=0dB and $N_T = 3, N_R = 2$. The color bar shows the sum rate.

Dynamic Cooperation Cluster Model

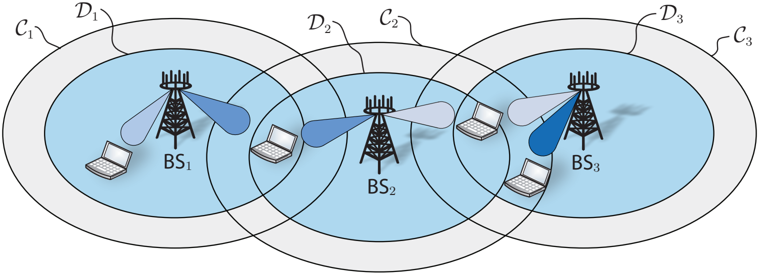


Illustration of the overlapping nature of **dynamic cooperation clusters**. BS_j forms different cooperation constellations when serving different users. $\mathcal{D}_j$ are the users that it serves with **data** and $\mathcal{C}_j$ are the users considered in the spatial interference **coordination**.

Resource Allocation and DCC

- While this tutorial provides a thorough framework for resource allocation for given DCCs, the **practical design of DCCs** is a relatively new and unexplored research area.
- The sets $\{\mathcal{D}_j\}_{j=1}^{K_r}$ of users served by each base station are selected under the following conditions:
  - Each active user should have a **master base station** (MBS) that guarantees its data services.
  - The **backhaul infrastructure** should support the joint transmission to a user, i.e., joint transmission is used only when the increase in throughput outweighs the increased demands on the backhaul.
  - **Proximity** is not measured geographically but by the average channel gain, taking possible differences in transmit power between base stations into account.
  - The base stations and many objects in the propagation environment are **static**.

RA and DCC - user sets

- The sets $\{\mathcal{C}_j\}_{j=1}^{K_r}$ of users that are considered in the beamforming at each base station are selected under the following conditions:
  - The channels between the base station and all the users that it includes in its **interference coordination** need to be **estimated**.
  - All neighboring base stations in TDD can **estimate** their own individual channel components by listening to the **same** uplink training signal from a user.
  - All neighboring base stations in FDD can **overhear** the feedback intended for their neighbors and thus obtain channels from a user to multiple base stations.
  - Both CSI estimation errors and **tolerance to CSI errors** increase with distance.

Simple Clustering Algorithm

- A **simple clustering algorithm** would be to include User k in $\mathcal{D}_j$ if the average channel gain $\mathbb{E}\{\|\mathbf{h}_{jk}\|_2^2\}$ (over some suitable time-window) is above a certain threshold value. The fulfillment of this condition is rechecked at the same **time-scale** as the estimate of $\mathbb{E}\{\|\mathbf{h}_{jk}\|_2^2\}$ is updated. User k appoints base station

$$m = \arg\max_{\{j:\, k \in \mathcal{C}_j\}} \mathbb{E}\{\|\mathbf{h}_{jk}\|_2^2\} \tag{45}$$

  as its MBS, which is the one with the strongest channel.

- The user then computes the ratio between the average channel gain of the MBS and the gain of the **second strongest base station**,

$$\frac{\mathbb{E}\{\|\mathbf{h}_{mk}\|_2^2\}}{\max_{\{j:k\in\mathcal{C}_j\}\setminus\{m\}} \mathbb{E}\{\|\mathbf{h}_{jk}\|_2^2\}}. \tag{46}$$

  Joint transmission occurs if this is above a threshold.

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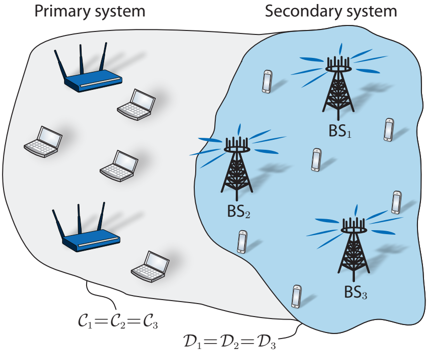
Cognitive Radio Introduction

- In a **cognitive radio** scenario, the systems are capable of detecting their environment and reconfiguring their operations accordingly.
- Consider a network composed of licensed **primary users**. Additional users, called **secondary users**, having cognitive radio capabilities, can be supported without degrading the QoS of the primary users.
- CR systems can be categorized into three models:
  - An **interweave** cognitive radio is an intelligent wireless communication system that periodically monitors the radio spectrum, intelligently detects occupancy in the different parts of the spectrum, and then opportunistically communicates over spectrum holes with minimal interference to the active primary users.
  - The **underlay** paradigm mandates that concurrent non-cognitive and cognitive transmissions may occur only if the interference generated by the cognitive devices at the non-cognitive receivers is below some acceptable threshold.
  - Cognitive radio systems that have cooperation with the primary system as key feature are denoted as **overlay** cognitive radio systems.

Underlay Cognitive Radio Model



- We focus on the underlay cognitive radio system and begin with null-shaping for all $k \in \mathcal{K}_{\text{primary}}$

$$\mathbf{v}_k^H \mathbf{Q}_{kl} \mathbf{v}_k = 0 \quad (47)$$

or general interference temperature constraints (ITC)

$$\mathbf{v}_k^H \mathbf{Q}_{kl} \mathbf{v}_k \leq q_l. \quad (48)$$

Null Shaping Constraints

- Collect all **null-shaping constraints** for transmission to User k in a matrix

$$\mathbf{Z}_k = \left[\mathbf{Q}_{k1} \, \ldots \, \mathbf{Q}_{kK_{\text{primary}}}\right]. \tag{49}$$

- To satisfy the null-shaping constraints, we can define a new **effective channel** of User k as

$$\tilde{\mathbf{h}}_k = \mathbf{\Pi}^{\perp}_{\mathbf{Z}_k} \mathbf{h}_k \tag{50}$$

  by projecting the original channel vector $\mathbf{h}_k$ onto the **null-space** of the null-shaping matrix $\mathbf{Z}_k$.
- Based on the effective channels $\tilde{\mathbf{h}}_k$ in (50), the **complete framework** developed in this tutorial can be applied.

Interference Temperature Constraints

- Let us consider a simple two-link example: primary link one and secondary link two.
- **ITC**: $\mathbf{v}_2^H \mathbf{Q}_{21} \mathbf{v}_2 \leq q_1$ with primary rate constraint $R_1(\texttt{load}) = \texttt{load} \cdot \log_2\left(1 + \frac{|\mathbf{h}_1^H \mathbf{C}_1 \mathbf{D}_1 \mathbf{v}_1|^2}{\sigma_1^2}\right)$.
- This results in $q_1 = |\mathbf{h}_1^H \mathbf{C}_1 \mathbf{D}_1 \mathbf{v}_1|^2 \left(2^{R_1(\texttt{load})} - 1\right)^{-1} - \sigma_1^2$.
- Resource allocation problem for **secondary precoding** is given by

$$\max_{\mathbf{v}_2 :\, \|\mathbf{v}_2\| \leq 1} \ g_2(\mathrm{SINR}_2(\mathbf{v}_2)) \qquad \text{subject to} \quad \mathbf{v}_2^H \mathbf{Q}_{21} \mathbf{v}_2 \leq q_1. \tag{51}$$

- Again, the **complete framework** can be applied to handle the additional ITC.

Communications Theory

Theorem

The optimization problem (51) *is solved by*

$$\mathbf{v}_2(\lambda^*) = \sqrt{\lambda^*}\frac{\mathbf{\Pi}_{\mathbf{D}_2^H\mathbf{C}_1^H\mathbf{h}_1}\mathbf{D}_2^H\mathbf{C}_2^H\mathbf{h}_2}{\|\mathbf{\Pi}_{\mathbf{D}_2^H\mathbf{C}_1^H\mathbf{h}_1}\mathbf{D}_2^H\mathbf{C}_2^H\mathbf{h}_2\|} + \sqrt{1-\lambda^*}\frac{\mathbf{\Pi}^{\perp}_{\mathbf{D}_2^H\mathbf{C}_1^H\mathbf{h}_1}\mathbf{D}_2^H\mathbf{C}_2^H\mathbf{h}_2}{\|\mathbf{\Pi}^{\perp}_{\mathbf{D}_2^H\mathbf{C}_1^H\mathbf{h}_1}\mathbf{D}_2^H\mathbf{C}_2^H\mathbf{h}_2\|} \tag{52}$$

with

$$\lambda^* = \begin{cases} \lambda_{\mathrm{MRT}}, & \lambda_{\mathrm{MRT}} \leq \frac{q_1}{\|\mathbf{D}_2^H\mathbf{C}_1^H\mathbf{h}_1\|^2}, \\ \frac{q_1}{\|\mathbf{D}_2^H\mathbf{C}_1^H\mathbf{h}_1\|^2}, & \text{otherwise}, \end{cases} \tag{53}$$

and $\lambda_{\mathrm{MRT}} = \frac{\left\|\mathbf{\Pi}_{\mathbf{D}_2^H\mathbf{C}_1^H\mathbf{h}_1}\mathbf{D}_2^H\mathbf{C}_2^H\mathbf{h}_2\right\|^2}{\left\|\mathbf{\Pi}_{\mathbf{D}_2^H\mathbf{C}_1^H\mathbf{h}_1}\mathbf{D}_2^H\mathbf{C}_2^H\mathbf{h}_2\right\|^2+\left\|\mathbf{\Pi}^{\perp}_{\mathbf{D}_2^H\mathbf{C}_1^H\mathbf{h}_1}\mathbf{D}_2^H\mathbf{C}_2^H\mathbf{h}_2\right\|^2}.$

1. Robustness to Channel Uncertainty
2. Distributed Resource Allocation
3. Transceiver Hardware Impairments
4. Multi-Cast Transmission
5. Multi-Carrier Systems
6. Multi-Antenna Users
7. Design of Dynamic Cooperation Clusters
8. Cognitive Radio Systems
9. Physical Layer Security

Physical Layer Security: Motivation

- The data processing, transmission, and encryption in modern communication systems are carried out **separately**.
- The typical purpose of the **physical layer** is to guarantee error-free transmission, whereas encryption is performed at a higher layer in the protocol stack.
- State-of-the-art encryption algorithms rely on mathematical operations assumed to be hard to compute, however, the classical approach to security becomes increasingly **difficult to justify**, in particular if we consider that:
    (a) the underlying intractability assumptions may be wrong;
    (b) efficient attacks could be developed;
    (c) the advent of quantum computers is likely to compromise this type of encryption; and
    (d) fast and reliable communications over ad hoc wireless networks require light and effective security architectures.

Physical Layer Secrecy Observations

Communications Theory

- Physical layer security provides **perfect secrecy**.
- By clever coding and transmit strategies the data can be **reliably** and **securely** sent at **secrecy rates**.

Interference is Information Leakage

The amount of **interference** that is created by one link at another receiver corresponds to the amount of **information leakaged** from this link to the curious receiver.

- Additional amounts of spectrally, spatially, or temporally **adjusted interference/noise** can improve the secrecy rates.

PhySec Example Scenario

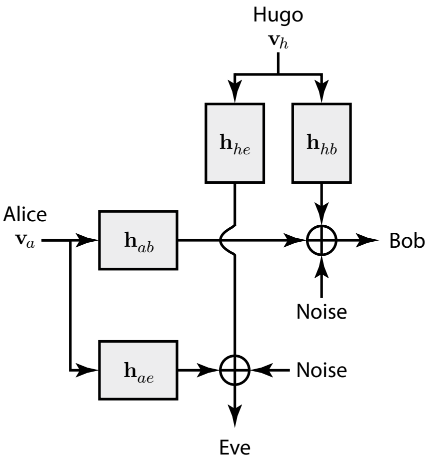


Illustration of a simple eavesdropping scenario with four nodes: Alice wants to communicate **privately** with Bob. Eve is trying to overhear, while Hugo supports the private communication by intentionally creating **interference** at Eve (while avoiding interference at Bob).

System Model

- Two transmissions create **interference** to each other: from Alice to Bob and from Hugo to Eve.
- The link from Alice to Bob is described by the **vector channel** $\mathbf{h}_{ab} = \mathbf{D}_1^H \mathbf{C}_1^H \mathbf{h}_1$, Alice to Eve $\mathbf{h}_{ae} = \mathbf{D}_1^H \mathbf{C}_2^H \mathbf{h}_2$, whereas the link from Hugo to Bob is described by $\mathbf{h}_{hb} = \mathbf{D}_2^H \mathbf{C}_1^H \mathbf{h}_1$ and Hugo to Eve is $\mathbf{h}_{he} = \mathbf{D}_2^H \mathbf{C}_2^H \mathbf{h}_2$.
- The private communication uses the **beamforming vector** $\mathbf{v}_a$ and the helper creates interference using $\mathbf{v}_h$.
- The achievable **secrecy rate** for reliable and secure data transmission is given by

$$R_S(\mathbf{v}_a, \mathbf{v}_h) = \Bigg[\log_2 \left(1 + \frac{|\mathbf{h}_{ab}^H \mathbf{v}_a|^2}{1 + |\mathbf{h}_{hb}^H \mathbf{v}_h|^2} \right) - \log_2 \left(1 + \frac{|\mathbf{h}_{ae}^H \mathbf{v}_a|^2}{1 + |\mathbf{h}_{he}^H \mathbf{v}_h|^2} \right) \Bigg]_+ . \tag{54}$$

Communications Theory

- The optimization problem for maximizing the secrecy rate is

$$\max_{0 \leq \|\mathbf{v}_a\|^2 \leq q_a} \max_{0 \leq \|\mathbf{v}_h\|^2 \leq q_h} R_S(\mathbf{v}_a, \mathbf{v}_h). \tag{55}$$

Lemma

The beamforming vector $\mathbf{v}_a'$ that solves (55) for fixed $\mathbf{v}_h$ is given by

$$\mathbf{v}_a'(\mathbf{v}_h) = q_a \boldsymbol{\psi} \tag{56}$$

where $\boldsymbol{\psi}$ is the generalized eigenvector associated with the maximum generalized eigenvalue of the pencil $(\mathbf{I} + \frac{1}{1+z_1}\mathbf{h}_{ab}\mathbf{h}_{ab}^H, \mathbf{I} + \frac{1}{1+z_2}\mathbf{h}_{ae}\mathbf{h}_{ae}^H)$ with $z_1 = |\mathbf{h}_{hb}^H \mathbf{v}_h|^2$ and $z_2 = |\mathbf{h}_{he}^H \mathbf{v}_h|^2$.

Optimal Beamforming at Helper

Communications Theory

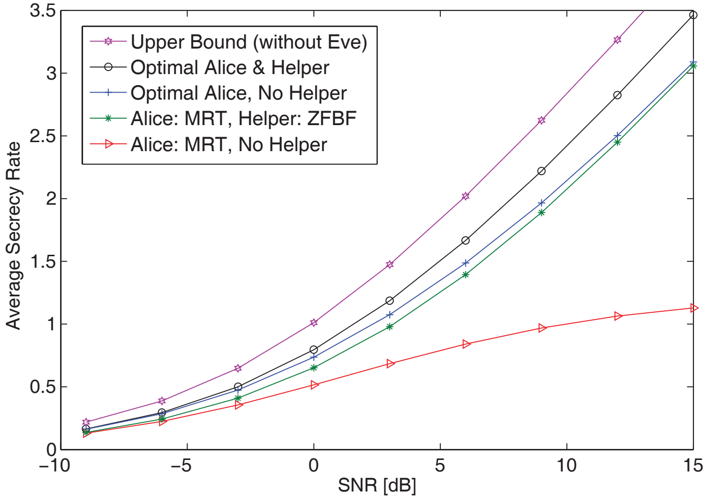
Theorem

For fixed $\mathbf{v}_a$, the beamforming vector $\mathbf{v}'_h$ that solves (55) is given by

$$\mathbf{v}'_h(\lambda) = \frac{\lambda \mathbf{\Pi}_{\mathbf{h}_{hb}} \mathbf{h}^*_{he} + (1-\lambda) \mathbf{\Pi}^{\perp}_{\mathbf{h}_{hb}} \mathbf{h}^*_{he}}{\| \lambda \mathbf{\Pi}_{\mathbf{h}_{hb}} \mathbf{h}^*_{he} + (1-\lambda) \mathbf{\Pi}^{\perp}_{\mathbf{h}_{hb}} \mathbf{h}^*_{he} \|} \tag{57}$$

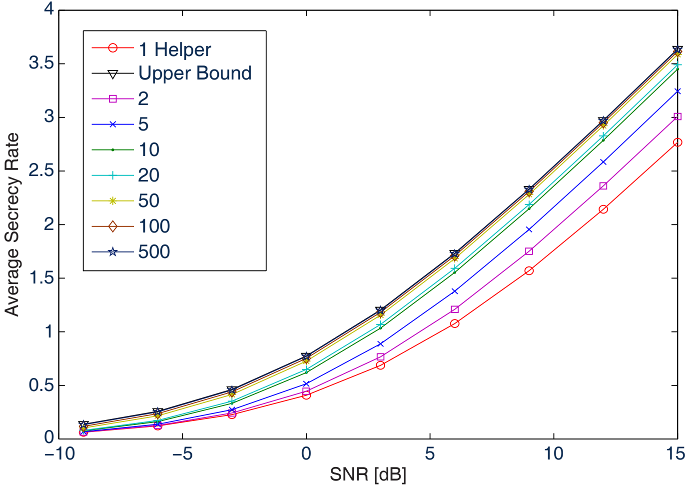
for some $\lambda \in [0,1]$.

- An extension to **multiple** K **helpers** is possible and an iterative algorithm can be derived.
- If the number of helpers approaches **infinity**, the average secrecy rate approaches the average rate of the **peaceful system**.



Average secrecy rate with and without a helper, and using different transmit strategies. Alice and Hugo have two transmit antennas each.

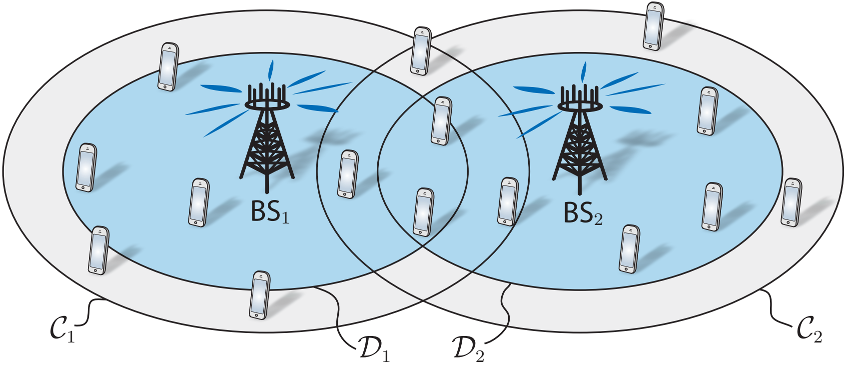
Physical Layer Security Illustration II



Instantaneous secrecy rate with helpers using optimal beamforming.
Alice and all helpers are equipped with two transmit antennas.

Conclusions

- Our ambition has been to provide a solid ground and understanding for **optimization of practical modern multi-cell systems**.
    - Modern multi-antenna techniques provide much **higher data throughput and flexibility** but makes it harder to optimize.
    - The same algorithms solve **many "different" resource allocation problems** and handle many practical conditions and scenarios.



Some Open Problems

- How to combine several of the topics in the same model?
- Extension to **Multi-Hop Networks** including AF, DF, CF and variants of physical layer network coding.
- Application of **Game Theory** to derive distributed non-cooperative resource allocation and beamforming algorithms.
- Application of **Pricing** to achieve certain operating points on the Pareto Boundary.
- Novel performance measures to optimize the **Energy Efficiency** in multi-cell networks.
- Use of more **Detailed Hardware Models**, which will impact the power constraints and hardware impairments.
- Proactive resource allocation that guarantee **Long-Term User Fairness** and exploits **Prior Information** user behavior/traffic.
- Coordinated interference management in **Multi-Tier Networks**.

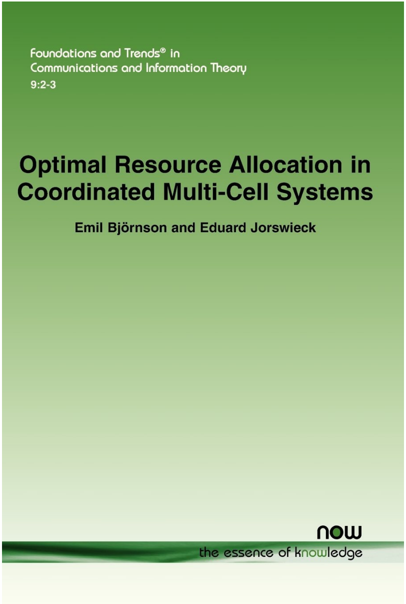
References

The main reference is our book:

Emil Björnson, Eduard Jorswieck, “*Optimal Resource Allocation in Coordinated Multi-Cell Systems*,” Foundations and Trends in Communications and Information Theory, vol. 9, no. 2-3, pp. 113-381, 2013.

The book contains 330 references to research articles on these topics.

Download the e-book from our homepages or buy the printed book at a special price (see Part I).



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