TRAINING-BASED BAYESIAN MIMO CHANNEL AND CHANNEL NORM ESTIMATION

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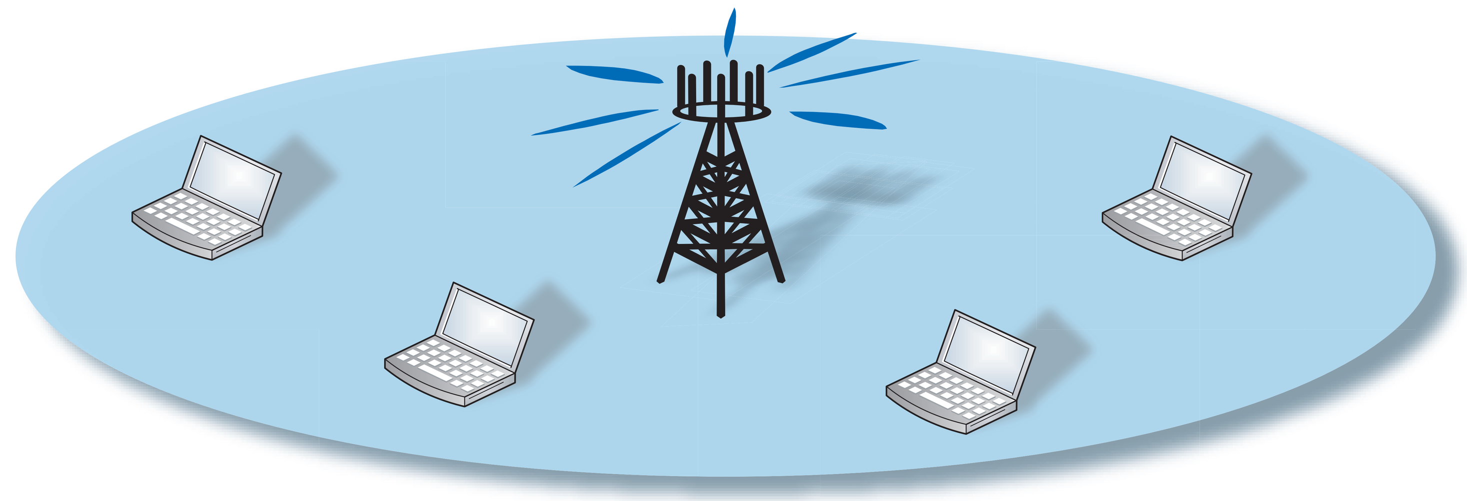


Generalizations and novel results on channel matrix and norm MMSE estimation
Analysis of training matrix design and its dependence on spatial correlation

Estimation of Channel Information

Instantaneous channel state information (CSI) useful for

- Receive processing (interference suppression, detection).
- Feedback (user selection, precoding, rate adaptation).

We consider estimation based on training signalling:

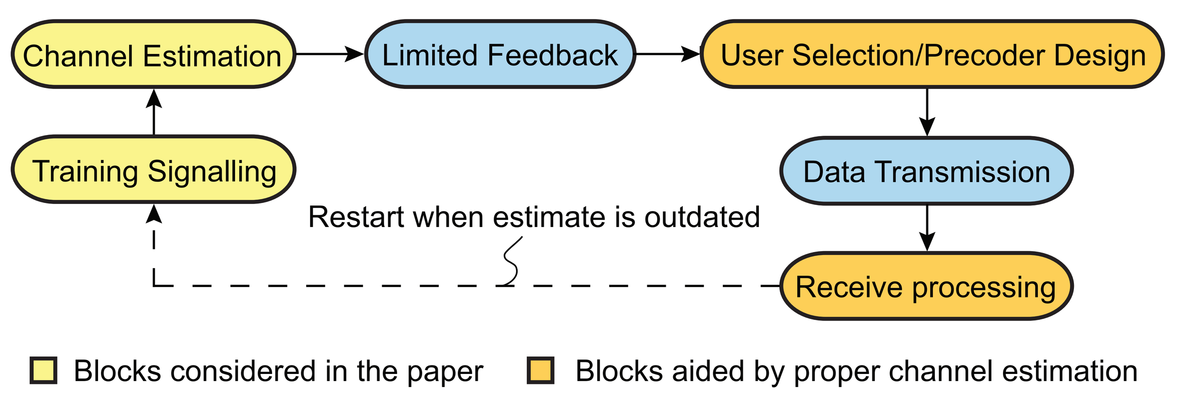
- Multiple antennas: n_T at transmitter, n_R at receiver.
- Narrowband channel represented by $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$.
- Estimate functions $f(\mathbf{H})$ from known training signal:

$f_1(\mathbf{H}) = \mathbf{H}$ (for general receive processing)

$f_2(\mathbf{H}) = \|\mathbf{H}\|_F^2$ (for channel gain feedback)

Estimation of these functions are two different problems.

System operation



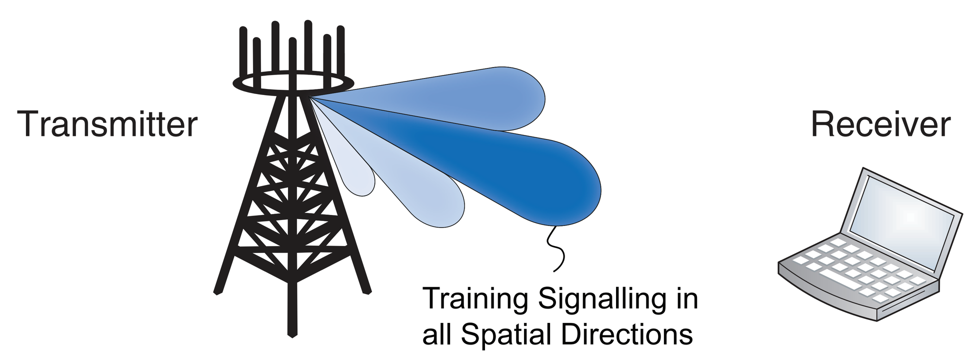
General Problem Formulation

Estimate the channel $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ from the received signal

$$\mathbf{Y} = \mathbf{H}\mathbf{P} + \mathbf{N},$$

where the training matrix $\mathbf{P} \in \mathbb{C}^{n_T \times n_T}$ is known.

- Rician fading channel: $\mathrm{vec}(\mathbf{H}) \in \mathcal{CN}(\mathrm{vec}(\bar{\mathbf{H}}), \mathbf{R})$.
- Arbitrarily colored disturbance: $\mathrm{vec}(\mathbf{N}_k) \in \mathcal{CN}(\mathrm{vec}(\bar{\mathbf{N}}), \mathbf{S})$.
- Total training power constraint: $\mathrm{tr}(\mathbf{P}^H\mathbf{P}) = \mathcal{P}$.
- Channel statistics are known at both sides.



Channel Matrix Estimation

Minimum Mean Square Error (MMSE) Estimator:

$$\mathrm{vec}(\widehat{\mathbf{H}}_{\mathrm{MMSE}}) = \mathrm{vec}(\bar{\mathbf{H}}) + \left(\mathbf{R}^{-1} + \widetilde{\mathbf{P}}^H\mathbf{S}^{-1}\widetilde{\mathbf{P}}\right)^{-1}\widetilde{\mathbf{P}}^H\mathbf{S}^{-1} \times \left(\mathrm{vec}(\mathbf{Y}) - \widetilde{\mathbf{P}}\mathrm{vec}(\bar{\mathbf{H}}) - \mathrm{vec}(\bar{\mathbf{N}})\right)$$

where $\widetilde{\mathbf{P}} \triangleq (\mathbf{P}^T \otimes \mathbf{I})$.

Mean Square Error (MSE):

$$\mathrm{MSE} = \mathrm{tr}\left\{\left(\mathbf{R}^{-1} + \widetilde{\mathbf{P}}^H\mathbf{S}^{-1}\widetilde{\mathbf{P}}\right)^{-1}\right\}.$$

Observations:

- Linear/Affine (also LMMSE for non-Gaussian systems).
- Mean values do not affect the MSE.
- The training matrix, $\mathbf{P}$, is clearly affecting the MSE.

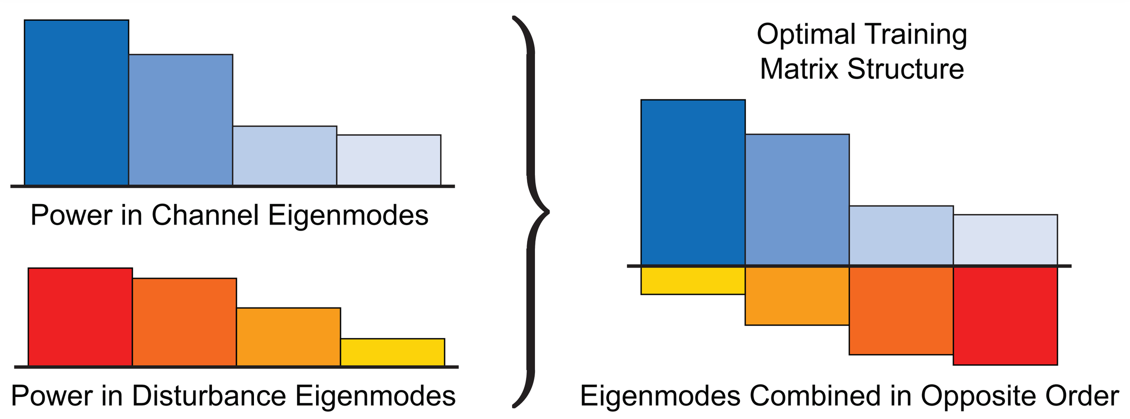
Design of Training Matrix

Goal: Minimize the MSE by design of training matrix $\mathbf{P}$.

- Intuition: More important to estimate in statistically strong eigendirections, than in weak directions.
- Assumption (for analysis): $\mathbf{R} = \mathbf{R}_T^T \otimes \mathbf{R}_R$, $\mathbf{S} = \mathbf{S}_T^T \otimes \mathbf{S}_R$.

Optimal training structure: $\mathbf{P} = \mathbf{U}_T \mathrm{diag}(\sqrt{p_1}, \ldots, \sqrt{p_{n_T}})\mathbf{V}_T^H$.

$\mathbf{U}_T$ and $\mathbf{V}_T$ contain eigenvalues of $\mathbf{R}_T$ and $\mathbf{S}_T$, in *opposite* orders.



- More training power in directions with strong channel and weak disturbance. Spatial correlation improves performance.
- If $\mathbf{R}_R = \mathbf{S}_R$, optimal power allocation (waterfilling):

$$p_j = \max\left(\sqrt{\lambda_{n_T-j}(\mathbf{S}_T)/\alpha} - \lambda_{n_T-j}(\mathbf{S}_T)/\lambda_j(\mathbf{R}_T), 0\right) \quad \forall j.$$

- Asymptotics: Low SINR: All power in strongest eigenmode; High SINR: Proportional to noise standard deviation.

Channel Squared Norm Estimation

Why consider estimation of $\rho = \|\mathbf{H}\|_F^2$?

- Important with accurate estimate for gain feedback.
- Indirect estimation as $\|\widehat{\mathbf{H}}_{\mathrm{MMSE}}\|^2$ is suboptimal.

MMSE estimator and its MSE derived explicitly:

$$\hat{\rho}_{\mathrm{MMSE}} = \mathbf{1}^T\mathbf{B}\boldsymbol{\Sigma}\mathbf{1} + \tilde{\mathbf{y}}^H\widetilde{\mathbf{D}}\mathbf{B}^2\widetilde{\mathbf{D}}\tilde{\mathbf{y}}$$

$$\mathrm{MSE} = \mathbf{1}^T\mathbf{B}\,(\widetilde{\boldsymbol{\Sigma}}^2 + 2\widetilde{\mathbf{D}}\widetilde{\boldsymbol{\Sigma}}\widetilde{\boldsymbol{\Lambda}}\widetilde{\mathbf{D}})\,\mathbf{B}\mathbf{1}$$

$\mathbf{B}, \boldsymbol{\Sigma}, \widetilde{\mathbf{D}}$ are diagonal and depend on training matrix and statistics.
$\tilde{\mathbf{y}}$ is linear transformation of received signal, $\mathbf{1} = [1, \ldots, 1]^T$.

Design of Training Matrix:

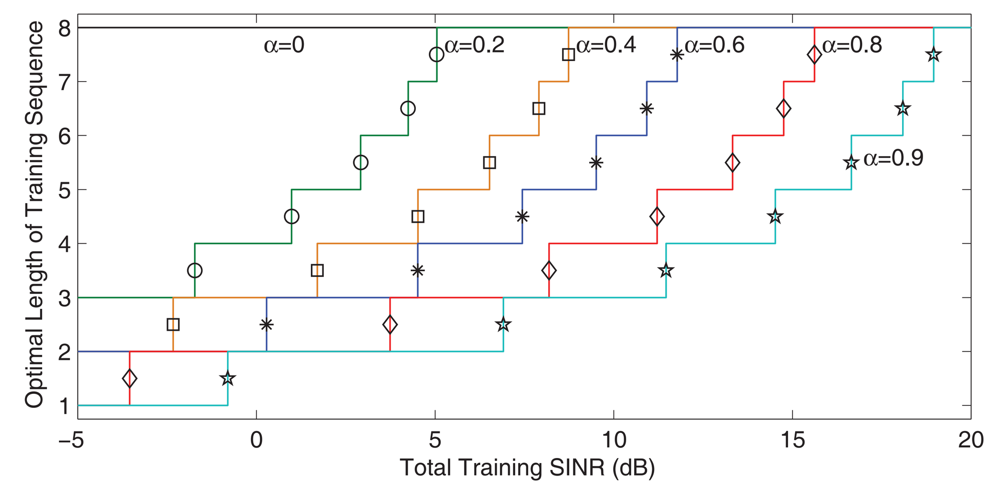
- Same training can be used as for channel matrix estimation.
- Training optimization difficult since the MSE is not convex.
- Small set of explicit potential solutions has be derived.
- Identical asymptotics at low SINR, and similar at high SINR.

Length of Training Sequence

Waterfilling behavior in the training power allocation:

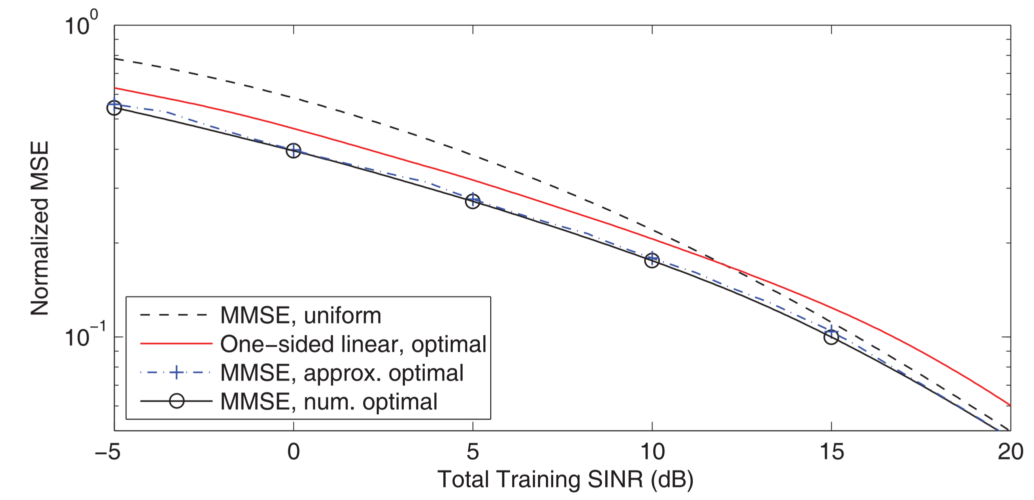
- For low training power $\mathcal{P}$ and/or large eigenvalue spread, some training powers p_j will become zero.
- If $\mathbf{R}_R = \mathbf{S}_R$, then $\mathbf{P}$ is rank deficient with rank $m < n_T$ if $g(m-1) < \mathcal{P} \leq g(m)$, with a function $g(m)$ of statistics.

Conclusion: m is the maximal necessary training matrix size (only approximately if future disturbance contain information).

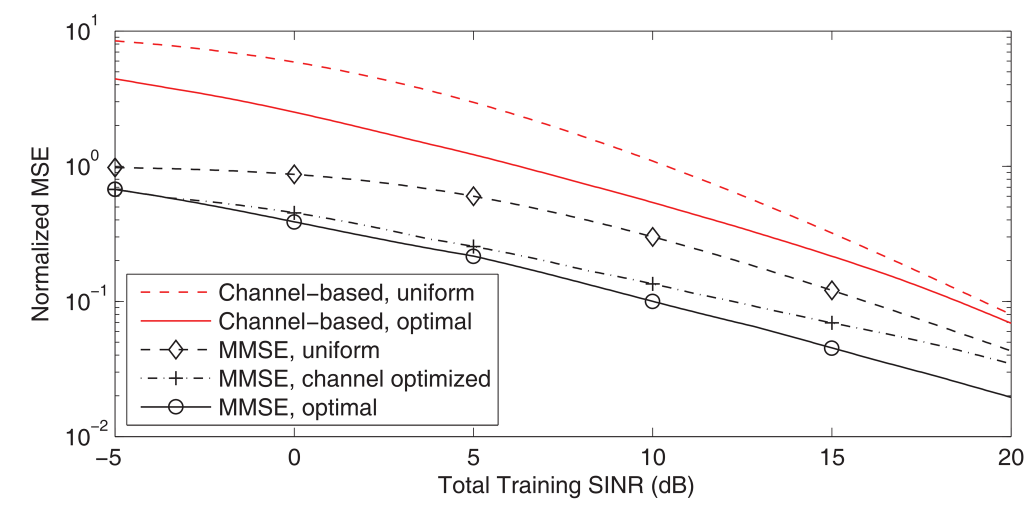


Smallest training sequence length, necessary to minimize the MSE, in a system with $n_T = 8$ and uncorrelated receive antennas. The results for different correlation, α, between adjacent transmit antennas.

Numerical Examples



The normalized MSEs of estimation of $\mathbf{H}$ ($n_T = 8$, $n_R = 4$, antenna correlation of 0.8). The MMSE estimator with three different training matrices is compared with the suboptimal one-sided linear estimator.



The normalized MSEs of estimation of $\|\mathbf{H}\|^2$ ($n_T = 8$, $n_R = 4$, transmit correlation of 0.8, uncorrelated receive antennas). MMSE estimator compared with indirect estimation for different training design.

Conclusions & Contributions

Estimation of CSI for receive processing and feedback.

Channel Matrix MMSE Estimation:

- System model with Rician channel and general disturbance.
- Extension to previous work for Rayleigh channels (Kronecker):

White noise: [Kotecha/Sayeed '04], [Biguesh/Gershman '06]
Colored noise: [Liu/Wong/Hager '07], [Katselis/Kofidis/Theodoridis '07]

- Unification of previous results: *Which results depend on which assumptions?*
- Analysis of the optimal training length.

Channel Squared Norm MMSE Estimation:

- Novel expressions for MMSE estimator and its MSE.
- Training optimization results, with explicit results in certain cases.
- Clear improvements in MSE compared with indirect estimation.