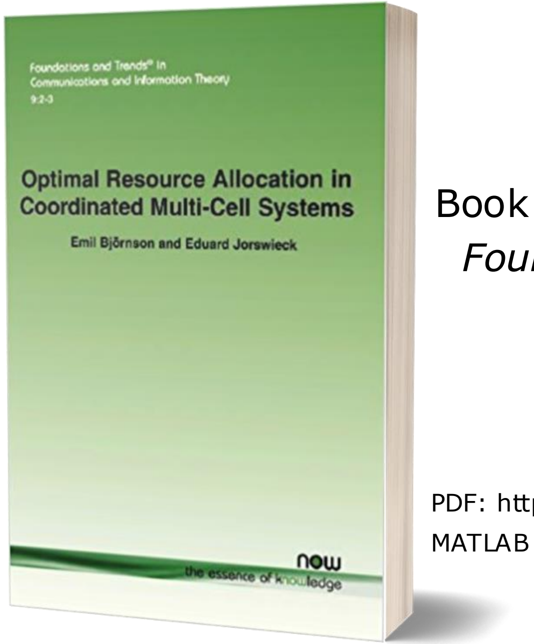
Optimal Spatial Resource Allocation in Cellular Networks



Book by **Emil Björnson** and Eduard Jorswieck

Foundations and Trends in Communications and Information Theory,
Vol. 9, No. 2-3, pp. 113-381, 2013

PDF: http://kth.diva-portal.org/smash/get/diva2:608533/FULLTEXT01

MATLAB code: https://github.com/emilbjornson/book-resource-allocation

Outline

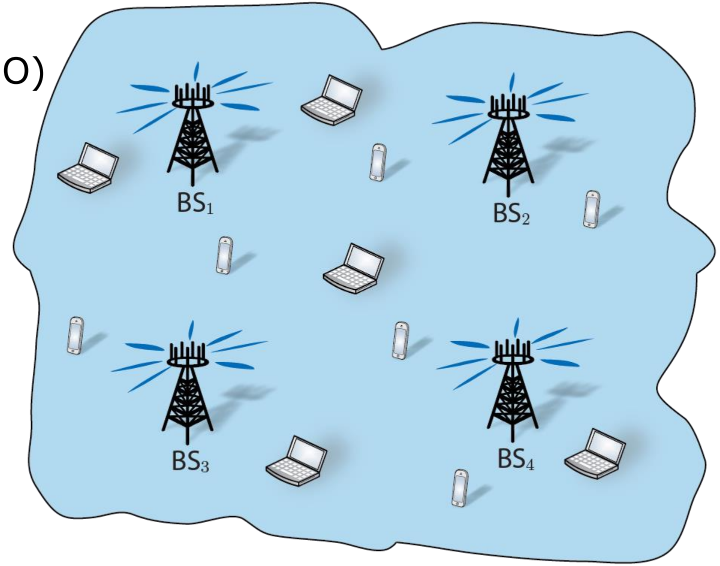
- Introduction
  - Cellular networks, system model, performance measure
- Problem Formulation
  - Resource allocation: Multi-objective optimization problem
- Subjective Resource Allocation
  - Utility functions, different computational complexity
- Structural Insights
  - Beamforming parametrization

Section

Introduction

Introduction

- Problem Formulation (vaguely):
  - Transfer information wirelessly to users
  - Divide radio resources among users (time, frequency, space)
- Downlink Coordinated Multi-Cell System
  - Many transmitting base stations (BSs)
  - Many receiving users
  - Multiple-input multiple-output (MIMO)
- Sharing a Frequency Band
  - All signals reach everyone!
- Limiting Factor
  - Inter-user interference

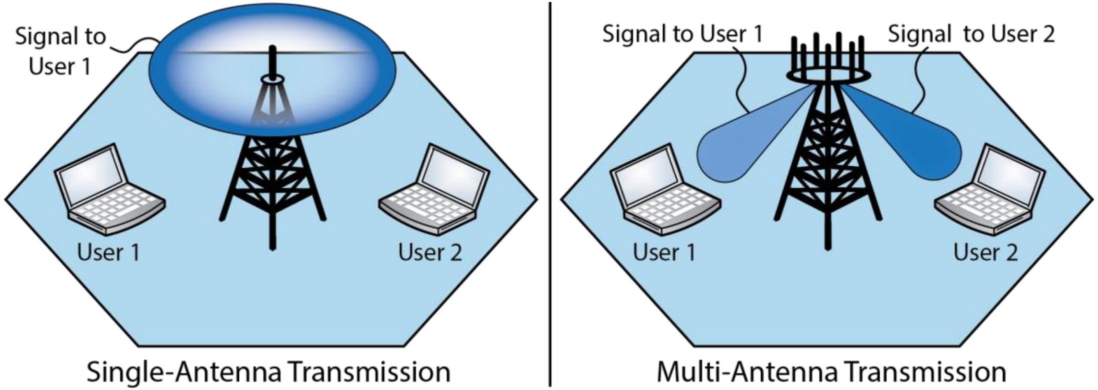


Introduction: Multi-Antenna Single-Cell Transmission

- Traditional Ways to Manage Interference
  - Avoid and suppress in time and frequency domain
  - Results in orthogonal single-cell access techniques: TDMA, OFDMA, etc.

- Multi-Antenna Transmission
  - Beamforming: Spatially directed signals
  - Adaptive control of interference
  - Serve multiple users: Space-division multiple access (SDMA), Multi-user MIMO

Main difference from classical resource allocation!



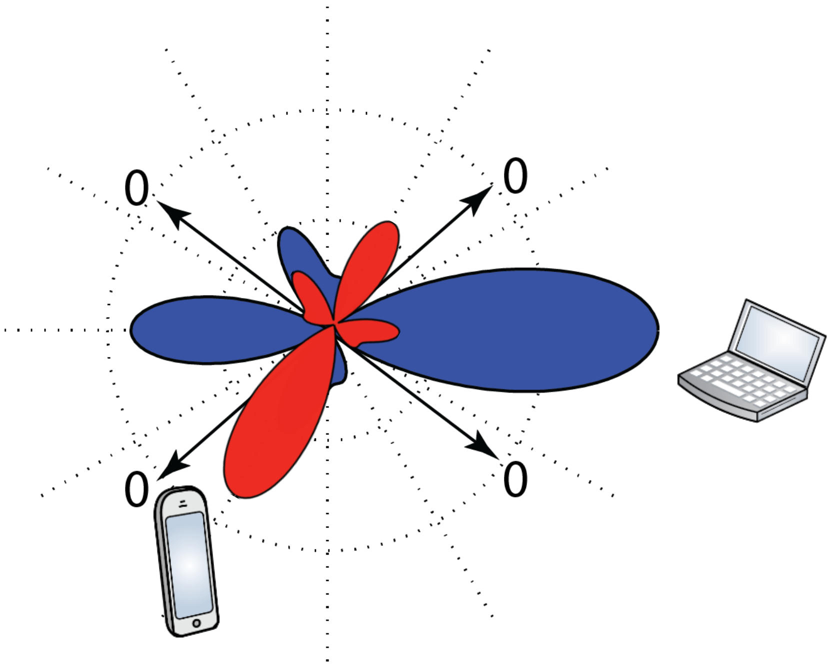
Introduction: Designing Multi-Antenna Transmission

- Multi-Antenna Transmission
  - With channel knowledge: beamforming or precoding
  - Direct signal towards intended receiver – some interference leaks!

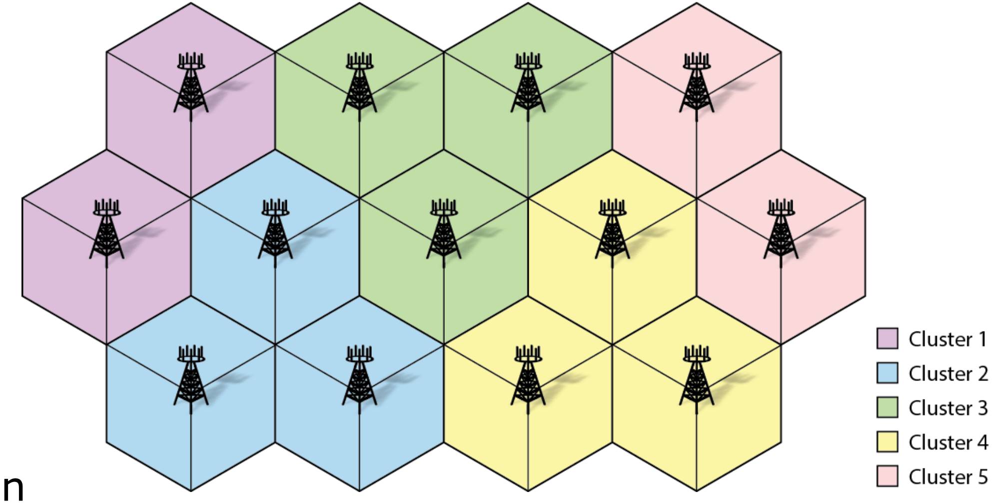
Beamforming Design

Easy for one user

Difficult for multiple users, due to interference



Introduction: From Single-Cell to Multi-Cell



- Naïve Multi-Cell Extension
  - Divide BS into disjoint clusters
  - SDMA within each cluster
  - Avoid inter-cluster interference
  - Fractional frequency-reuse
- Coordinated Multi-Cell Transmission
  - SDMA in multi-cell: Cooperation between all BSs
  - Full frequency-reuse: Interference managed by beamforming
  - Many names: *multi-cell processing*, *coordinated multi-point (CoMP)*, *network MIMO*, *cell-free massive MIMO*
- Almost as One Super-Cell
  - But: Different data knowledge, channel knowledge, power constraints!

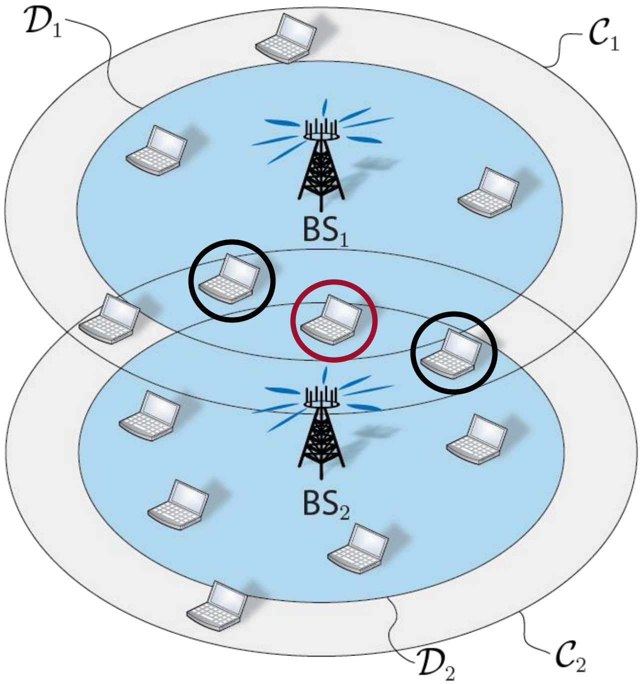
Basic Multi-Cell Coordination Structure

- General Multi-Cell Coordination
  - Adjacent base stations coordinate interference
  - Some users served by multiple base stations

Dynamic Cooperation Clusters

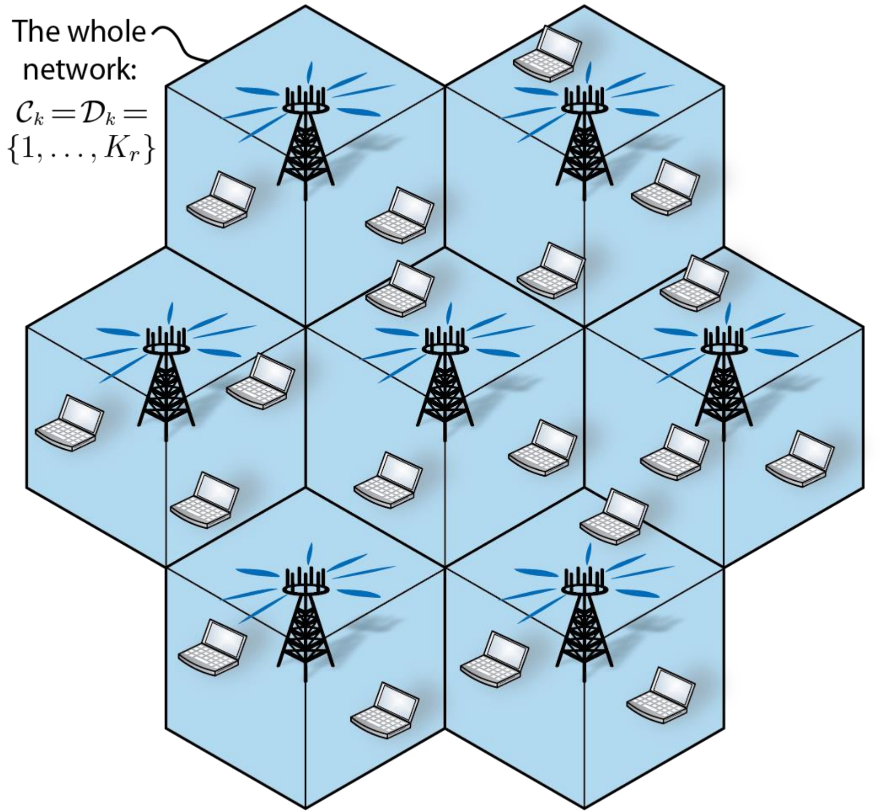
- Inner Circle $\mathcal{D}_k$: Serve users with data
- Outer Circle $\mathcal{C}_k$: Suppress interference
- Outside Circles:

  Negligible impact
  Impractical to acquire information
  Difficult to coordinate decisions



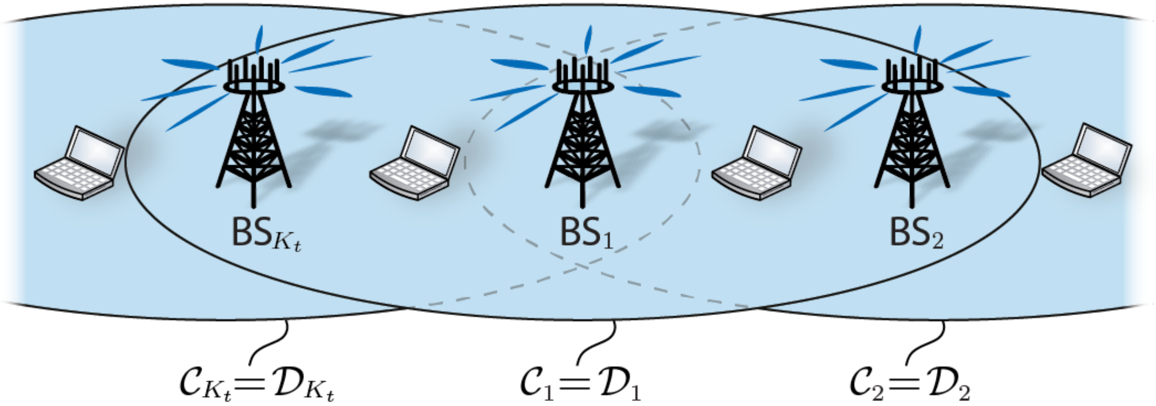
- E. Björnson, N. Jaldén, M. Bengtsson, B. Ottersten, "Optimality Properties, Distributed Strategies, and Measurement-Based Evaluation of Coordinated Multicell OFDMA Transmission," IEEE Trans. on Signal Processing, 2011.

Example: Ideal Joint Transmission



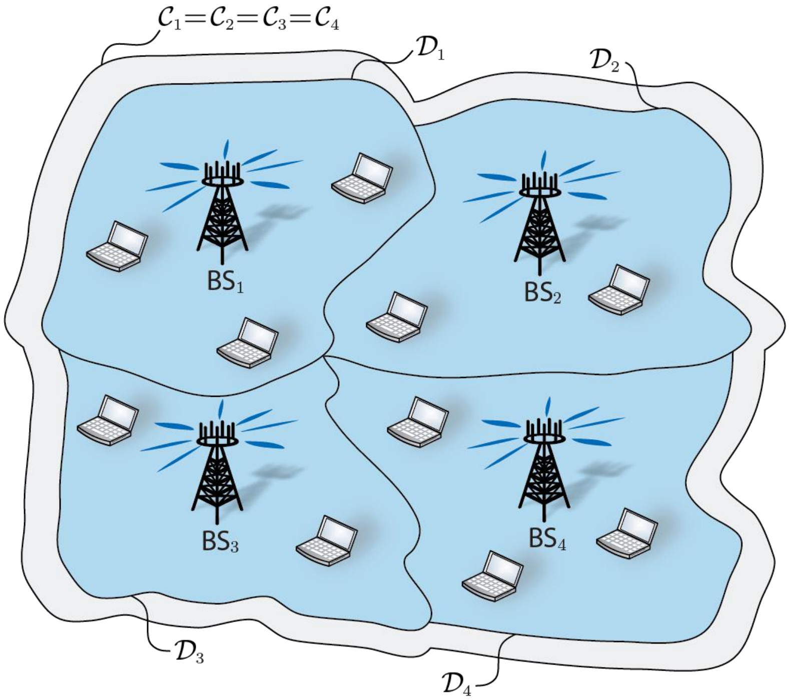
- All Base Stations Serve All Users Jointly = One Super Cell

Example: Wyner Model



- Abstraction: User receives signals from own and neighboring base stations
  - One or Two Dimensional Versions
  - Joint Transmission or Coordination between Cells

Example: Coordinated Beamforming

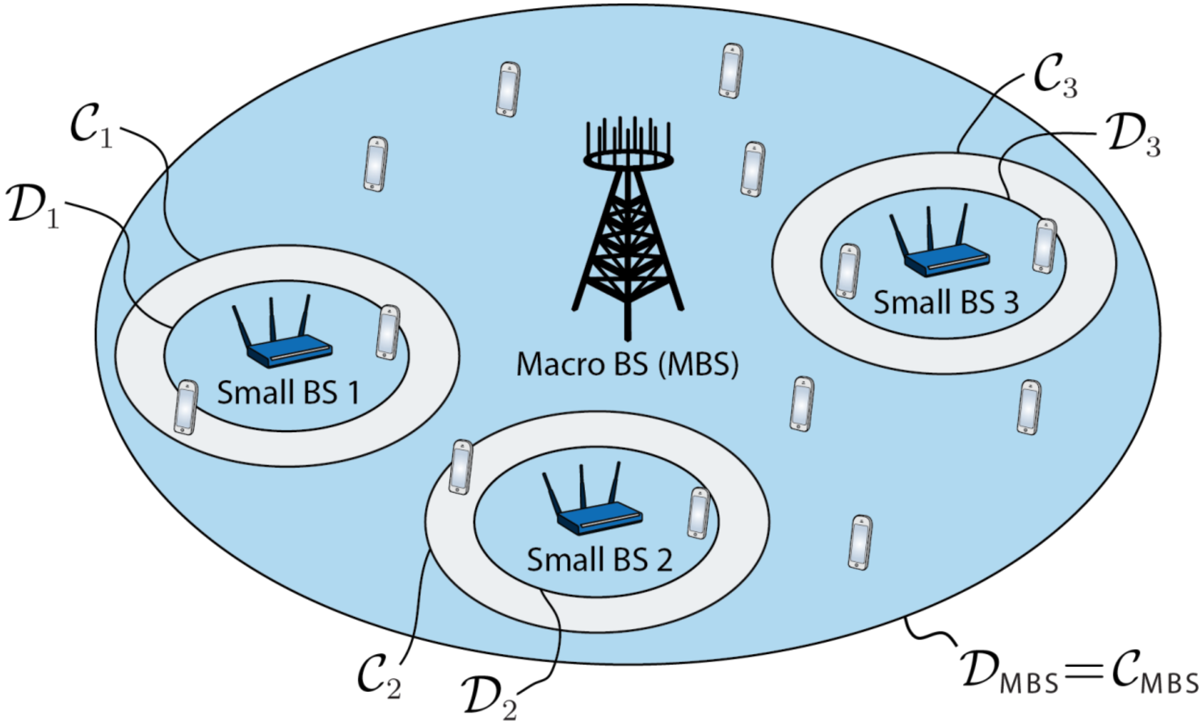


Special Case

Interference channel

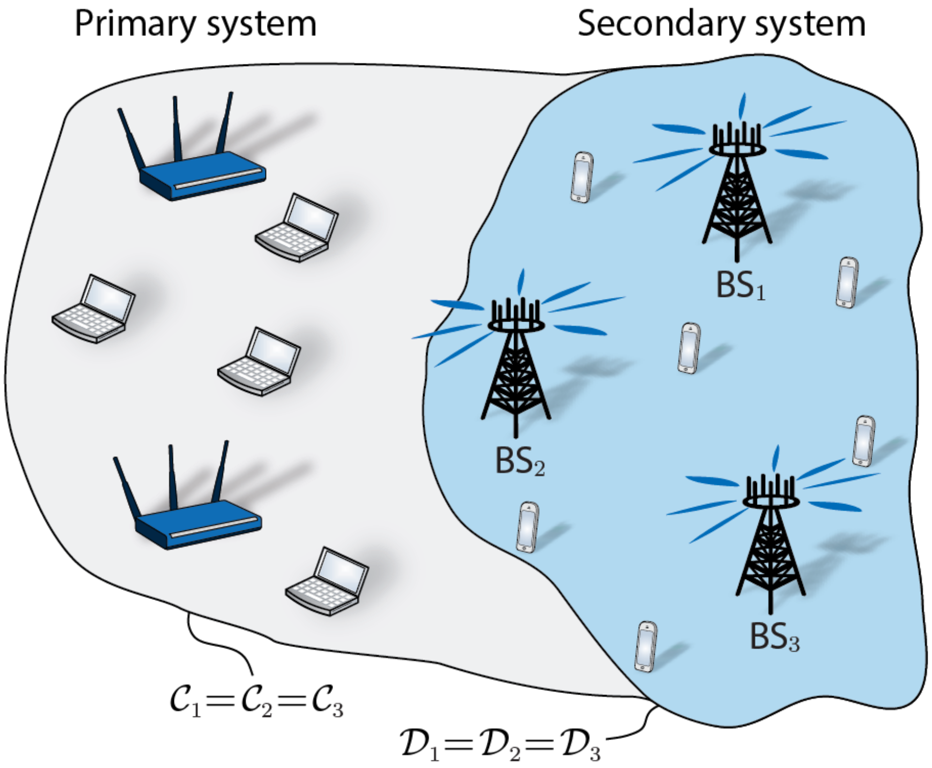
- One Base Station Serves Each User
- Interference Coordination Across Cells

Example: Heterogeneous Network



- Conventional macro BS overlaid by short-distance small BSs
  - Interference coordination and joint transmission between layers

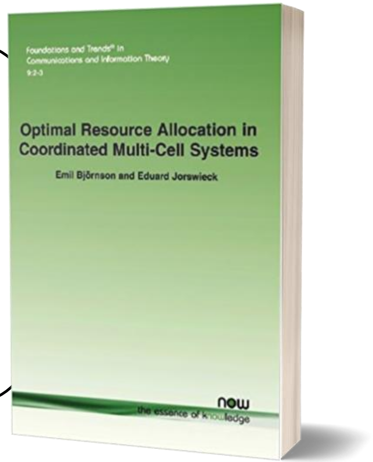
Example: Cognitive Radio



Other Examples

Spectrum sharing between operators

Physical layer security



- Secondary System Borrows Spectrum of Primary System
  - Underlay: Interference limits for primary users

Resource Allocation: First Definition

- Problem Formulation (imprecise):
  - Select beamforming to maximize "system utility"
  - Means: Allocate power to users and in spatial dimensions
  - Satisfy: Physical, regulatory & economic constraints

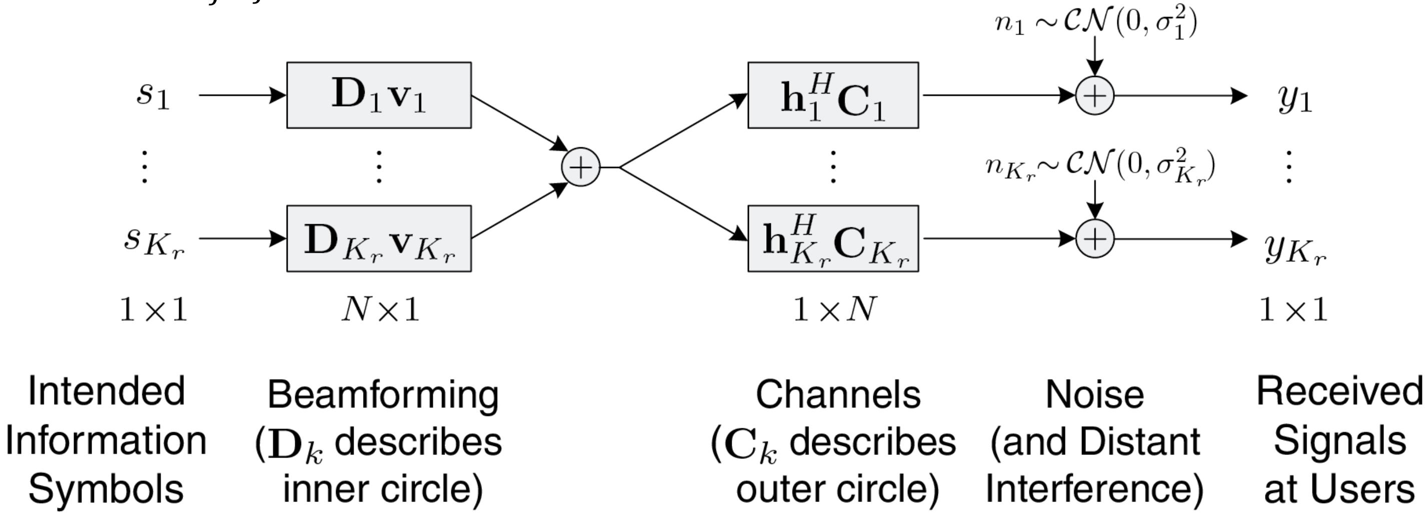
- Some Assumptions:
  - Linear transmission and reception
  - Perfect synchronization (whenever needed)
  - Flat-fading channels (e.g., a subcarrier in OFDM)

  - Perfect channel knowledge
  - Ideal transceiver hardware
  - Centralized optimization

Relaxed in the book

Multi-Cell System Model

- K_t Transmitting BSs
- K_r Users: Channel vector $\mathbf{h}_k = [\mathbf{h}_{1k}^T \ldots \mathbf{h}_{K_t k}^T]^T$ to User k from all BSs
- N_j Antennas at jth BS (dimension of $\mathbf{h}_{jk}$)
- $N = \sum_j N_j$ Antennas in Total (dimension of $\mathbf{h}_k$)



One System Model for All Multi-Cell Scenarios!

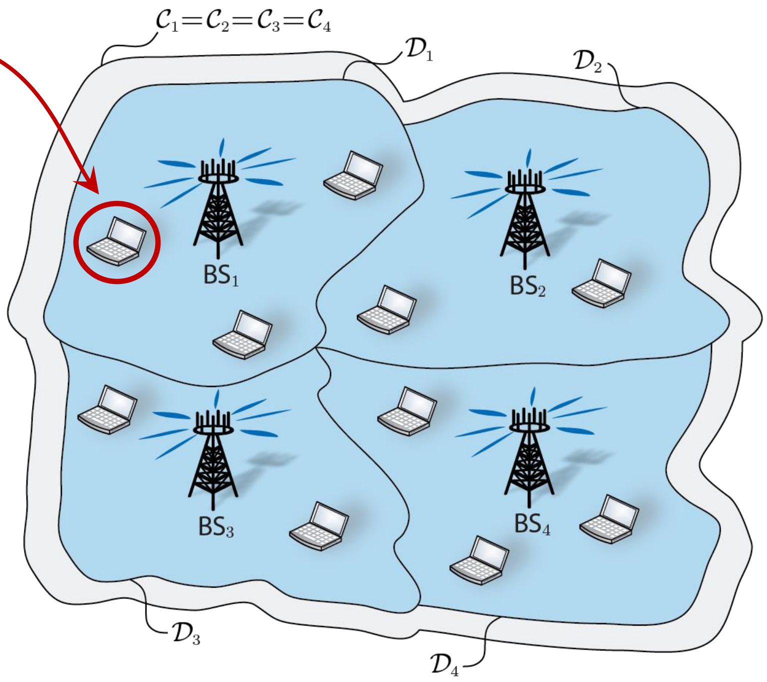
Multi-Cell System Model: Dynamic Cooperation Clusters (2)

- How are $\mathbf{D}_k$ and $\mathbf{C}_k$ Defined?
  - This is User k
  - Beamforming: $\mathbf{D}_k \mathbf{v}_k$
    Data only from BS_1:

$$\mathbf{D}_k = \begin{bmatrix} \mathbf{I}_{N_1} & & & \\ & \mathbf{0}_{N_2} & & \\ & & \mathbf{0}_{N_3} & \\ & & & \mathbf{0}_{N_4} \end{bmatrix}$$

  - Effective channel: $\mathbf{C}_k^H \mathbf{h}_k$
    All BSs *coordinate* interference:

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{I}_{N_1} & & & \\ & \mathbf{I}_{N_2} & & \\ & & \mathbf{I}_{N_3} & \\ & & & \mathbf{I}_{N_4} \end{bmatrix}$$



Example: Coordinated Beamforming

Multi-Cell System Model: Power Constraints

- Need for Power Constraints
  - Limit radiated power according to regulations
  - Protect dynamic range of amplifiers
  - Manage cost of energy expenditure
  - Control interference to certain users

All at the same time!

- L General Power Constraints:

$$\sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l, \quad l = 1, \ldots, L$$

Weighting matrix
(Positive semi-definite)

Limit
(Positive scalar)

Multi-Cell System Model: Power Constraints (2)

- Recall:

$$\sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l$$

- Example 1, Total Power Constraint:

$$L = 1:\ \ \mathbf{Q}_{1k} = \mathbf{I}_N$$
$$q_1 = \text{Maximal total power}$$

- Example 2, Per-Antenna Constraints:

$$L = N:\ \ \mathbf{Q}_{1k} = \mathrm{diag}(1, 0, \ldots, 0),\ \ \ldots,\ \ \mathbf{Q}_{Nk} = \mathrm{diag}(0, \ldots, 0, 1)$$
$$q_l = \text{Maximal power at } l\text{th antenna}$$

Introduction: How to Measure User Performance?

- Mean Square Error (MSE)
  - Difference: transmitted and received signal
  - Easy to Analyze
  - Far from the User Perspective?
- Bit/Symbol Error Ratio (BER/SER)
  - Probability of error (for a given data rate)
  - Intuitive interpretation
  - Complicated & ignores channel coding
- Information Rate
  - Bits per “channel use”
  - Mutual information: perfect and long coding
  - Anyway closest to reality?

All improve with the SINR:

$$\frac{\text{Signal}}{\text{Interference + Noise}}$$

Introduction: Generic Measure User Performance

- Generic Model
  - Any function of signal-to-interference-and-noise ratio (SINR):

$$g_k(\mathrm{SINR}_k) = g_k\left(\frac{|\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_k \mathbf{v}_k|^2}{\sigma_k^2 + \sum_{i \neq k} |\mathbf{h}_k^H \mathbf{C}_k \mathbf{D}_i \mathbf{v}_i|^2}\right) \quad \text{for User } k$$

  - Increasing and continuous function
  - For simplicity: $g_k(0) = 0$

- Example:
  - Information rate: $g_k(\mathrm{SINR}_k) = \log_2(1 + \mathrm{SINR}_k)$

- Complicated Function
  - Depends on all beamforming vectors $\mathbf{v}_1, \ldots, \mathbf{v}_{K_r}$

Section

Problem Formulation

Problem Formulation

- General Formulation of Resource Allocation:

$$\underset{\mathbf{v}_1,\ldots,\mathbf{v}_{K_r}}{\text{maximize}} \quad \{g_1(\text{SINR}_1),\ldots,g_{K_r}(\text{SINR}_{K_r})\}$$
$$\text{subject to} \quad \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \le q_l \quad \forall l.$$

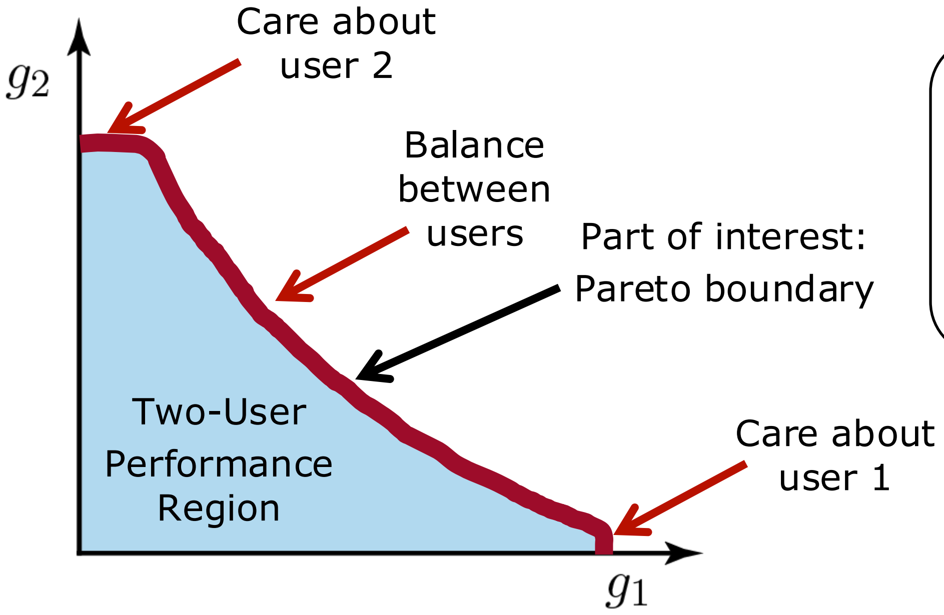
- Multi-Objective Optimization Problem
  - Generally impossible to maximize for all users!
  - Must divide power and cause inter-user interference

Performance Region

- Definition: Achievable Performance Region $\mathcal{R}$
  - Contains all feasible combinations $\{g_1(\mathrm{SINR}_1), \ldots, g_{K_r}(\mathrm{SINR}_{K_r})\}$
  - Feasible = Achieved by some $\{\mathbf{v}_1, \ldots, \mathbf{v}_{K_r}\}$ under power constraints

Other Names

Rate Region
Capacity Region
MSE Region, etc.

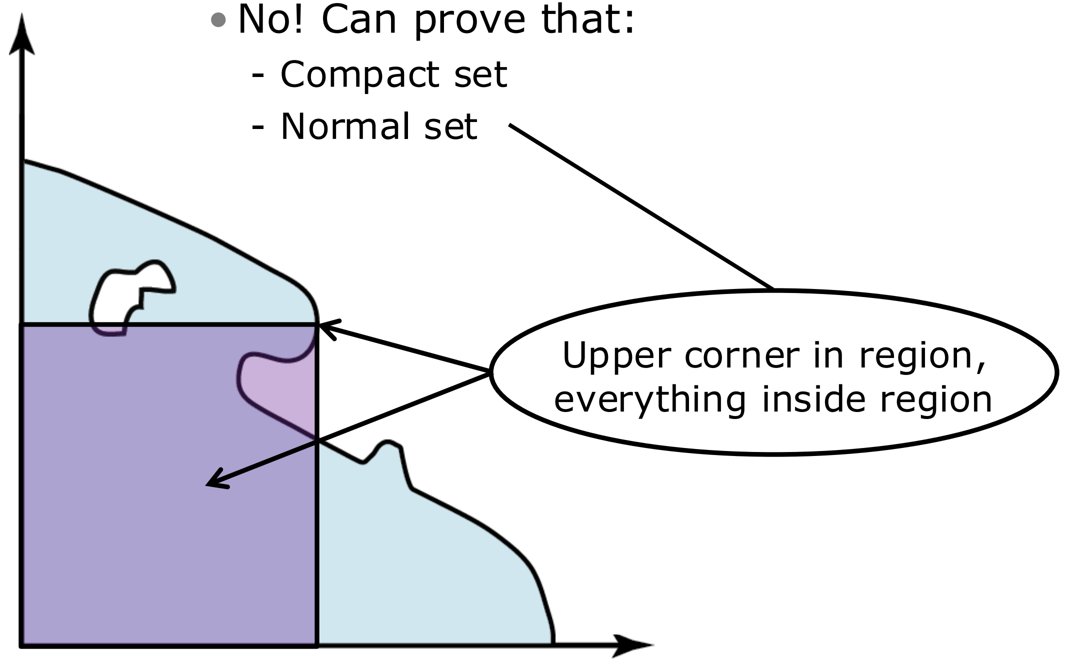


Pareto Boundary

Cannot improve for any user without degrading for other users

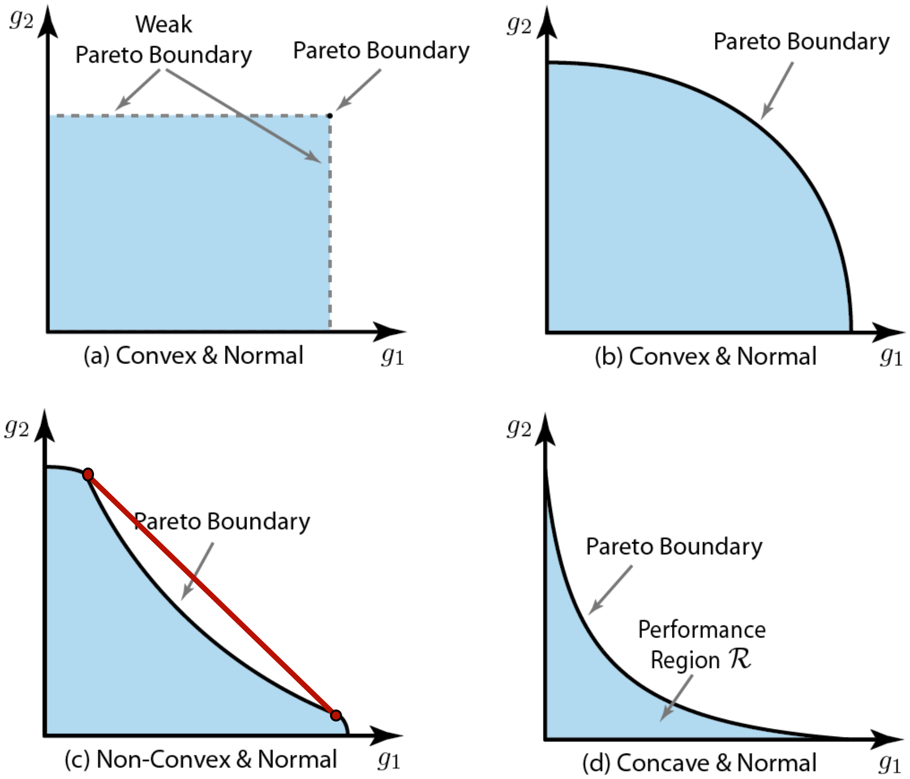
Performance Region (2)

- Can the region have any shape?
- No! Can prove that:
  - Compact set
  - Normal set



Performance Region (3)

- Some Possible Shapes



User-Coupling

Weak: Convex
Strong: Concave

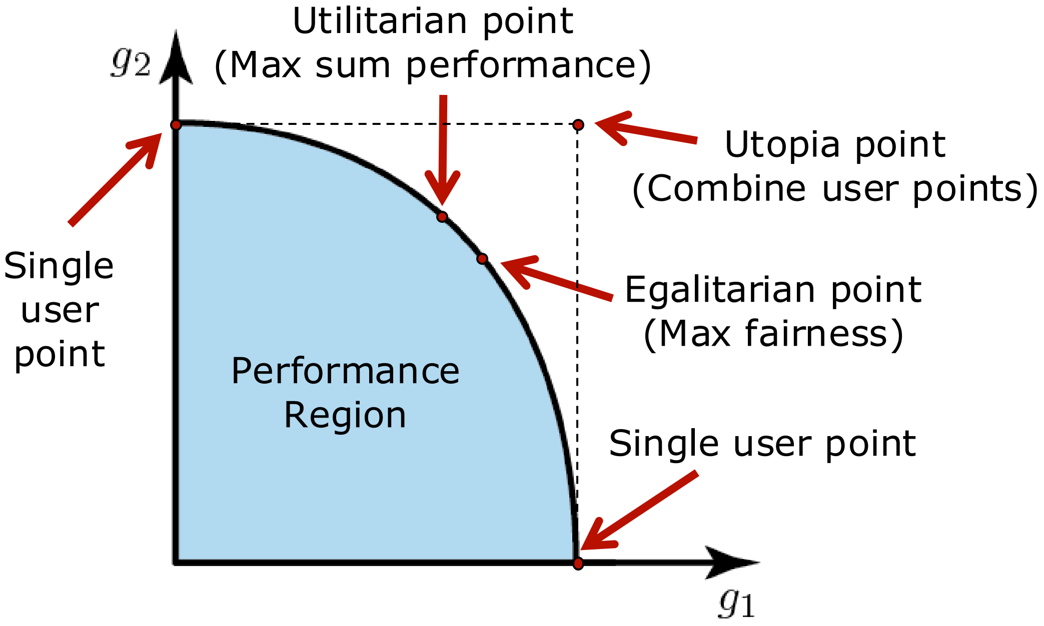
Scheduling

Time-sharing for
strongly coupled users

Select multiple points
Hard: Unknown region

Performance Region (4)

- Which Pareto Optimal Point to Choose?
  - Tradeoff: Aggregate Performance vs. Fairness



No Objective Answer

Utopia point outside of region

Only subjective answers exist!

Section

Subjective Resource Allocation

Subjective Approach

- System Designer Selects Utility Function $f : \mathcal{R} \to \mathbb{R}$
  - Describes subjective preference
  - Increasing and continuous function

Put different weights to move between extremes

- Examples:

Sum performance: $f(\mathbf{g}) = \sum_k g_k$

Proportional fairness: $f(\mathbf{g}) = \prod_k g_k$

Harmonic mean: $f(\mathbf{g}) = K_r \left(\sum_k g_k^{-1}\right)^{-1}$

Max-min fairness: $f(\mathbf{g}) = \min_k g_k$

Aggregate Performance

User Fairness

Known as *A Priori* Approach

Select utility function before optimization

Subjective Approach (2)

- Utility Function gives Single-Objective Optimization Problem:

$$\underset{\mathbf{v}_1,\ldots,\mathbf{v}_{K_r}}{\text{maximize}} \quad f(\mathbf{g}) \qquad \text{subject to} \quad \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \quad \forall l.$$

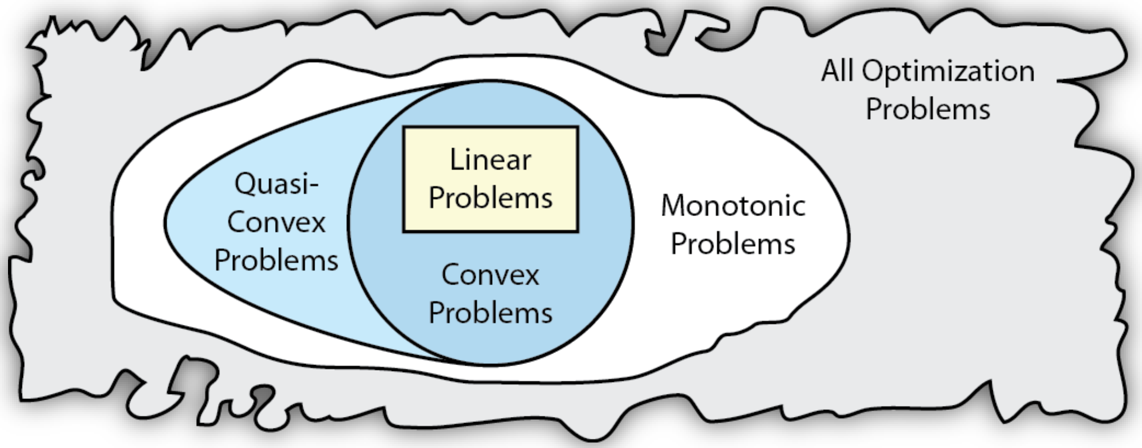
- This is the Starting Point of Many Researchers
  - Although the selection of f is
    - Inherently subjective
    - Affects the solvability
  - Should always have a motivation in mind!

Pragmatic Approach

Try to Select Utility Function to Enable Efficient Optimization

Complexity of Single-Objective Optimization Problems

- Classes of Optimization Problems
  - Different scaling with number of parameters and constraints



- Main Classes
  - Convex: Polynomial time solution ← Practically solvable
  - Monotonic: Exponential time solution ← Approximations needed
  - Arbitrary: More than exponential time ← Hard to even approximate

Classification of Resource Allocation Problems

- Classification of Three Important Problems
  - The “Easy” problem
  - Weighted max-min fairness
  - Weighted sum performance

- We will see: These have Different Complexities

Complexity Example 1: The “Easy” Problem

- Given Any Point $(\tilde{g}_1, \dots, \tilde{g}_{K_r})$
  - Find beamforming $\mathbf{v}_1, \dots, \mathbf{v}_{K_r}$ that attains this point
  - Minimize the total power
- Convex Problem
  - Second-order cone or semi-definite program
  - Global solution in polynomial time – use CVX, Yalmip
  - Alternative: Fixed-point iterations (uplink-downlink duality)

Total Power Constraints

- M. Bengtsson, B. Ottersten, “Optimal Downlink Beamforming Using Semidefinite Optimization,” Proc. Allerton, 1999.
- A. Wiesel, Y. Eldar, and S. Shamai, “Linear precoding via conic optimization for fixed MIMO receivers,” IEEE Trans. on Signal Processing, 2006.

Per-Antenna Constraints

- W. Yu and T. Lan, “Transmitter optimization for the multi-antenna downlink with per-antenna power constraints,” IEEE Trans. on Signal Processing, 2007.

General Constraints

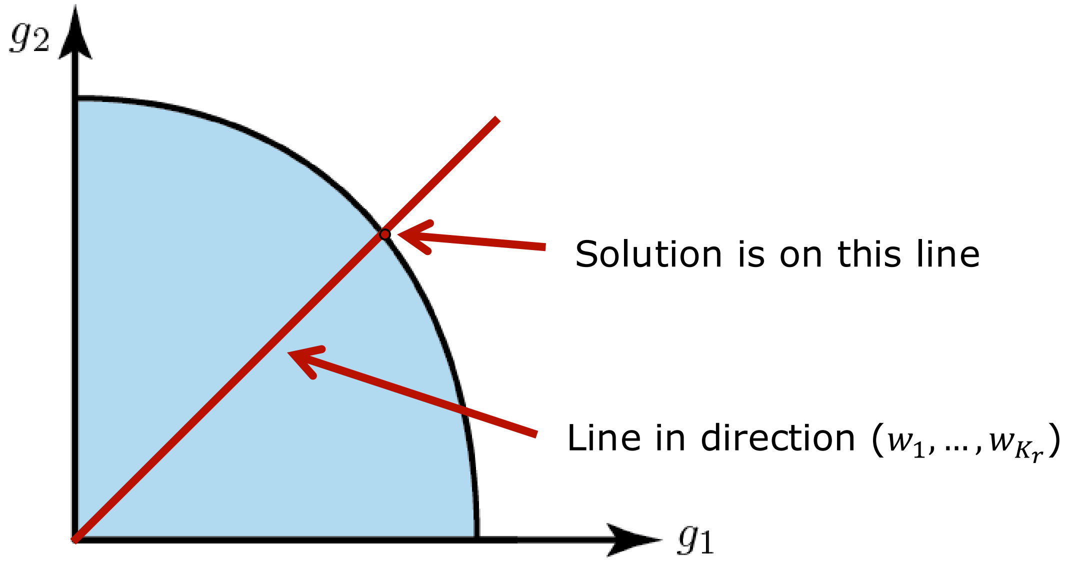
- E. Björnson, G. Zheng, M. Bengtsson, B. Ottersten, “Robust Monotonic Optimization Framework for Multicell MISO Systems,” IEEE Trans. on Signal Processing, 2012.

Complexity Example 2: Max-Min Fairness

- How to Classify Weighted Max-Min Fairness?

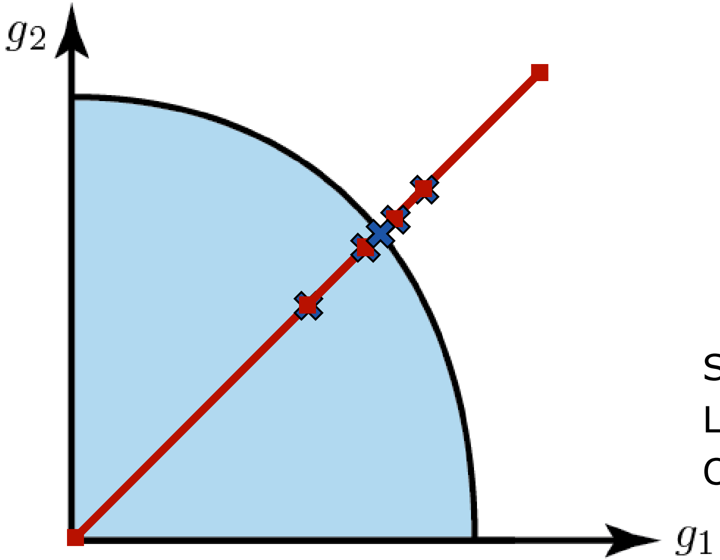
$$\underset{\mathbf{v}_1,\ldots,\mathbf{v}_{K_r}}{\text{maximize}} \quad f(\mathbf{g}) = \min_k w_k g_k \quad \text{subject to} \quad \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \quad \forall l.$$

  - Property: Solution makes $w_k g_k$ the same for all k



Complexity Example 2: Max-Min Fairness (2)

- Simple Line-Search: Bisection
  - Iteratively Solving Convex Problems (i.e., quasi-convex)



1. Find start interval
2. Solve the "easy" problem at midpoint
3. If feasible:
   Remove lower half
   Else: Remove upper half
4. Iterate

Subproblem: Convex optimization
Line-search: Linear convergence
One dimension (independent of #users)

Complexity Example 2: Max-Min Fairness (3)

- Classification of Weighted Max-Min Fairness:
  - **Quasi-convex problem** (belongs to convex class)
  - Polynomial complexity in #users, #antennas, #constraints
  - Might be feasible complexity in practice

Early work

- T.-L. Tung and K. Yao, "Optimal downlink power-control design methodology for a mobile radio DS-CDMA system," in IEEE Workshop SIPS, 2002.

Main references

- M. Mohseni, R. Zhang, and J. Cioffi, "Optimized transmission for fading multiple-access and broadcast channels with multiple antennas," IEEE Journal on Sel. Areas in Communications, 2006.
- A. Wiesel, Y. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," IEEE Trans. on Signal Processing, 2006.

Channel uncertainty

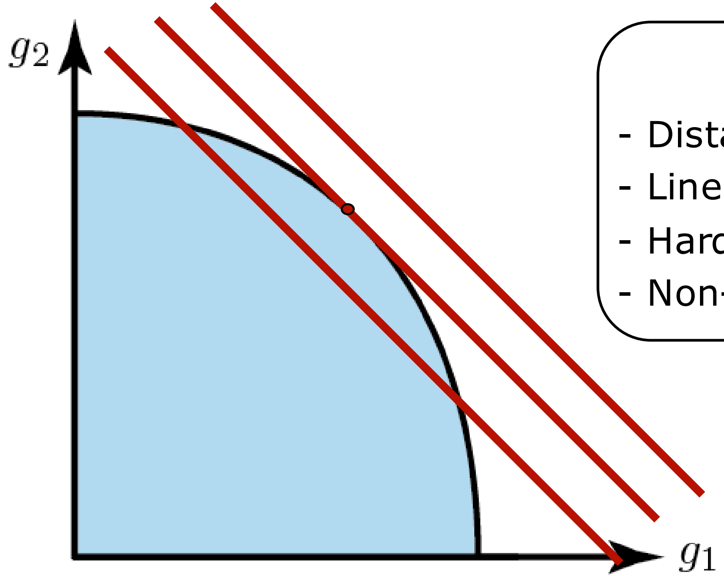
- E. Björnson, G. Zheng, M. Bengtsson, B. Ottersten, "Robust Monotonic Optimization Framework for Multicell MISO Systems," IEEE Trans. on Signal Processing, 2012.

Complexity Example 3: Weighted Sum Performance

- How to Classify Weighted Sum Performance?

$$\underset{\mathbf{v}_1,\ldots,\mathbf{v}_{K_r}}{\text{maximize}} \quad f(\mathbf{g}) = \sum_{k=1}^{K_r} w_k g_k \quad \text{subject to} \quad \sum_{k=1}^{K_r} \mathbf{v}_k^H \mathbf{Q}_{lk} \mathbf{v}_k \leq q_l \quad \forall l.$$

- Geometrically: $w_1 g_1 + w_2 g_2 =$ `opt-value` is a line



Opt-value is unknown!

- Distance from origin is unknown
- Line → Hyperplane (dim: #user – 1)
- Harder than max-min fairness
- Non-convex problem

Complexity Example 3: Weighted Sum Performance (2)

- Classification of Weighted Sum Performance:
  - Non-convex problem
  - Power constraints: Convex
  - Utility: Monotonic increasing/decreasing in beamforming vectors
  - Therefore: **Monotonic problem**

- Can There Be a *Magic* Algorithm?
  - No, provably NP-hard (Non-deterministic Polynomial-time hard)
  - Exponential complexity but in which parameters?
    (#users, #antennas, #constraints)

- Z.-Q. Luo and S. Zhang, "Dynamic spectrum management: Complexity and duality," *IEEE Journal of Sel. Topics in Signal Processing*, 2008.
- Y.-F. Liu, Y.-H. Dai, and Z.-Q. Luo, "Coordinated beamforming for MISO interference channel: Complexity analysis and efficient algorithms," IEEE Trans. on Signal Processing, 2011.

Complexity Example 3: Weighted Sum Performance (3)

- Are Monotonic Problems Impossible to Solve?
  - No, not for small problems!

- Monotonic Optimization Algorithms
  - Improve Lower/upper bounds on optimum: $f_{\min} \leq f_{\text{opt}} \leq f_{\max}$
  - Continue until $f_{\max} - f_{\min} < \varepsilon$
  - Subproblem: Essentially weighted max-min fairness problem

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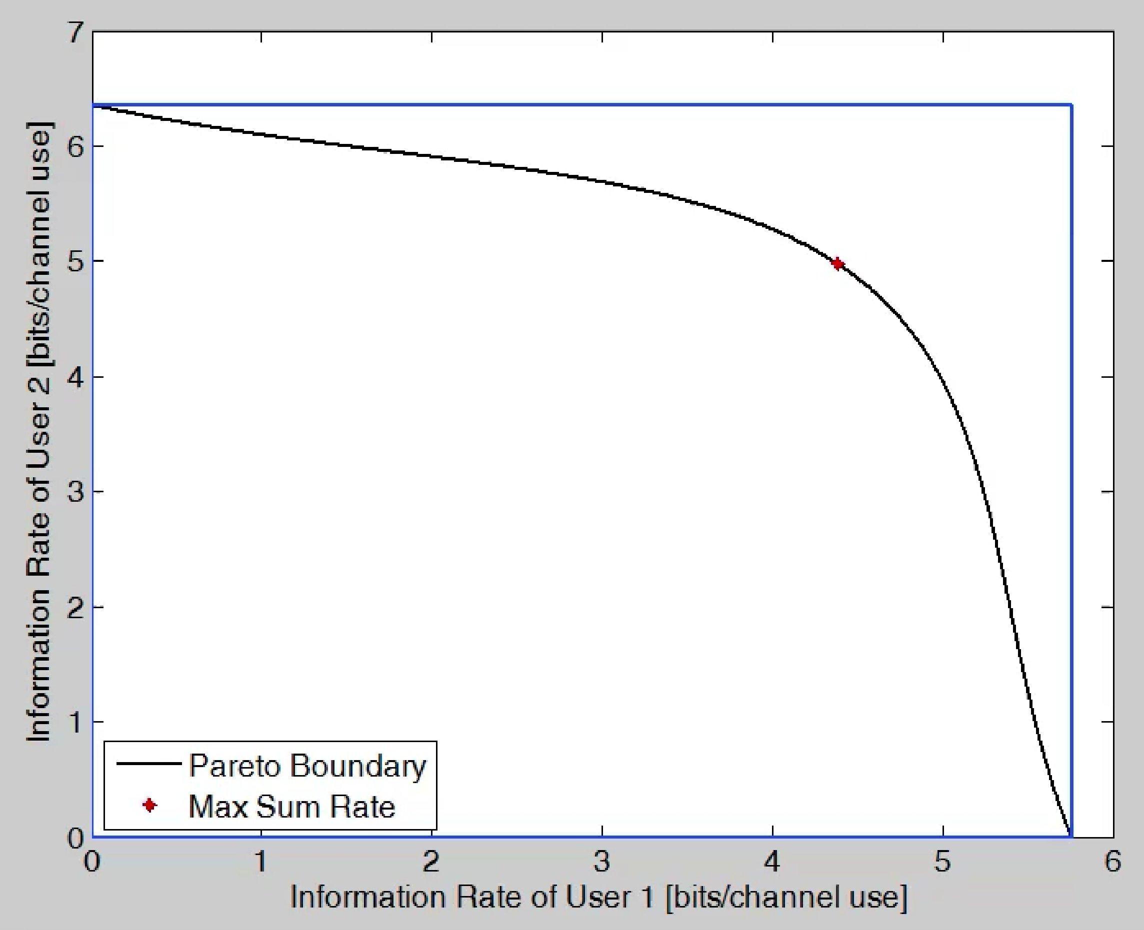
Monotonic optimization
- H. Tuy, "Monotonic optimization: Problems and solution approaches," SIAM Journal of Optimization, 2000.

Early works
- L. Qian, Y. Zhang, and J. Huang, "MAPEL: Achieving global optimality for a non-convex wireless power control problem," IEEE Trans. on Wireless Commun., 2009.
- E. Jorswieck, E. Larsson, "Monotonic Optimization Framework for the MISO Interference Channel," IEEE Trans. on Communications, 2010.

Polyblock algorithm
- W. Utschick and J. Brehmer, "Monotonic optimization framework for coordinated beamforming in multicell networks," IEEE Trans. on Signal Processing, 2012.

BRB algorithm
- E. Björnson, G. Zheng, M. Bengtsson, B. Ottersten, "Robust Monotonic Optimization Framework for Multicell MISO Systems," IEEE Trans. on Signal Processing, 2012.

Complexity Example 3: Weighted Sum Performance (4)



Branch-Reduce-Bound (BRB) Algorithm

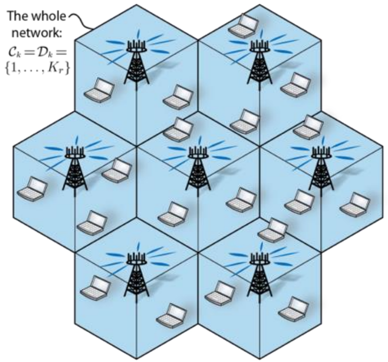
- Global convergence
- Accuracy ε>0 in finitely many iterations
- Exponential complexity only in #users (K_r)
- Polynomial complexity in other parameters (#antennas, #constraints)

Summary: Complexity of Resource Allocation Problems

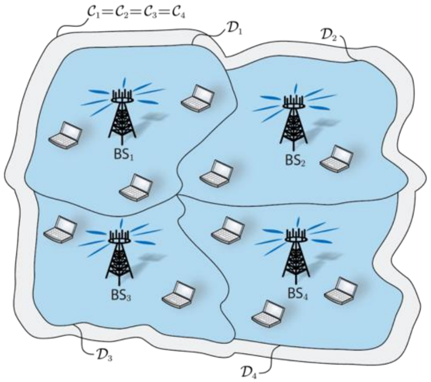
- Recall: All Utility Functions are Subjective
  - Pragmatic approach: Select to enable efficient optimization
- Good Choice: Any Problem with Polynomial Complexity
  - Example: Weighted max-min fairness
  - Use weights to adapt to other system needs
- Bad Choice: Weighted Sum Performance
  - Generally NP-hard: Exponential complexity (in #users)
  - Should be avoided – Sometimes needed (virtual queuing techniques)

Summary: Complexity of Resource Allocation Problems (2)

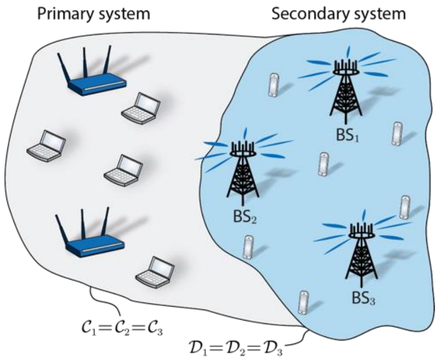
- Complexity Analysis for Any Dynamic Cooperation Clusters
  - Same optimization algorithms!
  - Extra characteristics can sometimes simplify
  - Multi-antenna transmission: Higher complexity, higher performance



Ideal Joint Transmission



Coordinated Beamforming



Underlay Cognitive Radio

Section

Structural Insights

Parametrization of Optimal Beamforming

- $K_r N$ Complex Optimization Variables: Beamforming vectors $\mathbf{v}_1, \dots, \mathbf{v}_{K_r}$
  - Can be reduced to K_r positive parameters (for $L = 1$)

- Any Resource Allocation Problem Solved by

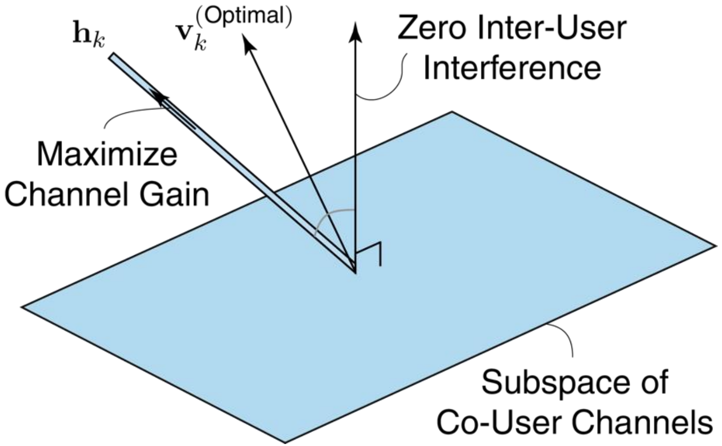
$$\mathbf{v}_k^\star = \underbrace{\sqrt{p_k}}_{=\text{beamforming power}} \underbrace{\left(\mathbf{I}_N + \sum_{i=1}^{K_r} \frac{\lambda_i}{\sigma^2} \mathbf{h}_i \mathbf{h}_i^H \right)^{-1} \mathbf{h}_k}_{=\tilde{\mathbf{v}}_k^\star = \text{beamforming direction}}$$

($\mathbf{C}_k = \mathbf{D}_k = \mathbf{I}_N$ for brevity)

- Priority of User k: λ_k $\leftarrow$ Lagrange multipliers from "Easy" problem

Parametrization of Optimal Beamforming (2)

- Geometric Interpretation:



Tradeoff

- Maximize signal vs. minimize interference
- Selfishness vs. altruism
- Hard to find optimal tradeoff
- $K_r = 2$: Simple special case

- Heuristic Parameter Selection
  - Known to work remarkably well
  - Many Examples (since 1995): Transmit Wiener filter, Regularized Zero-forcing, Signal-to-leakage beamforming, Virtual SINR beamforming, etc.

- E. Björnson, M. Bengtsson, B. Ottersten, "Optimal Multiuser Transmit Beamforming: A Difficult Problem with a Simple Solution Structure," IEEE Signal Processing Magazine, 2014.

Summary

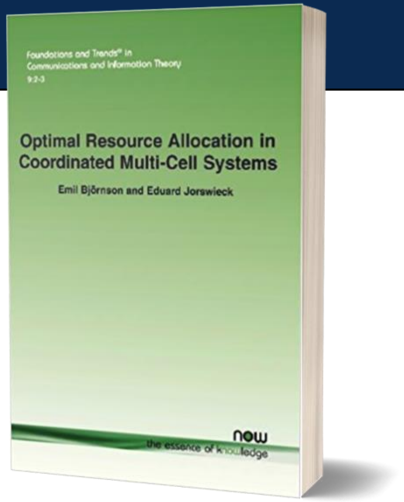
Summary

- Multi-Cell Multi-Antenna Resource Allocation
  - Divide power between users and spatial directions
  - Solve a multi-objective optimization problem
  - Pareto boundary: Set of efficient solutions

- Subjective Utility Function
  - Selection has a fundamental impact on solvability
  - Multi-antenna transmission: More possibilities – higher complexity
  - Pragmatic approach: Select to enable efficient optimization
  - Polynomial complexity: Weighted max-min fairness
  - Not solvable in practice: Weighted sum performance

- Parametrization of Optimal Beamforming

Main Reference: Our Book



- Thorough Resource Allocation Framework
  - More parametrizations and structural insights
  - Guidelines for scheduling and forming clusters
  - MATLAB code distributed for algorithms
  - Other Convex Problems and Optimization Algorithms:

	General	Zero Forcing	Single Antenna
Sum Performance	NP-hard	Convex	NP-hard
Max-Min Fairness	Quasi-Convex	Quasi-Convex	Quasi-Convex
"Easy" Problem	Convex	Convex	Linear
Proportional Fairness	NP-hard	Convex	Convex
Harmonic Mean	NP-hard	Convex	Convex

- Further Extensions:
  - Imperfect channel knowledge, distributed optimization
  - Multi-cast, Multi-carrier, Multi-antenna users, etc.