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Computational Framework for Optimal Robust Beamforming in Coordinated Multicell Systems

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Introduction

- Downlink Coordinated Beamforming
  - N cells with N_t-antenna base stations
  - Each serves K single-antenna users
  - Common narrowband frequency resource
  - Limited by co-user interference

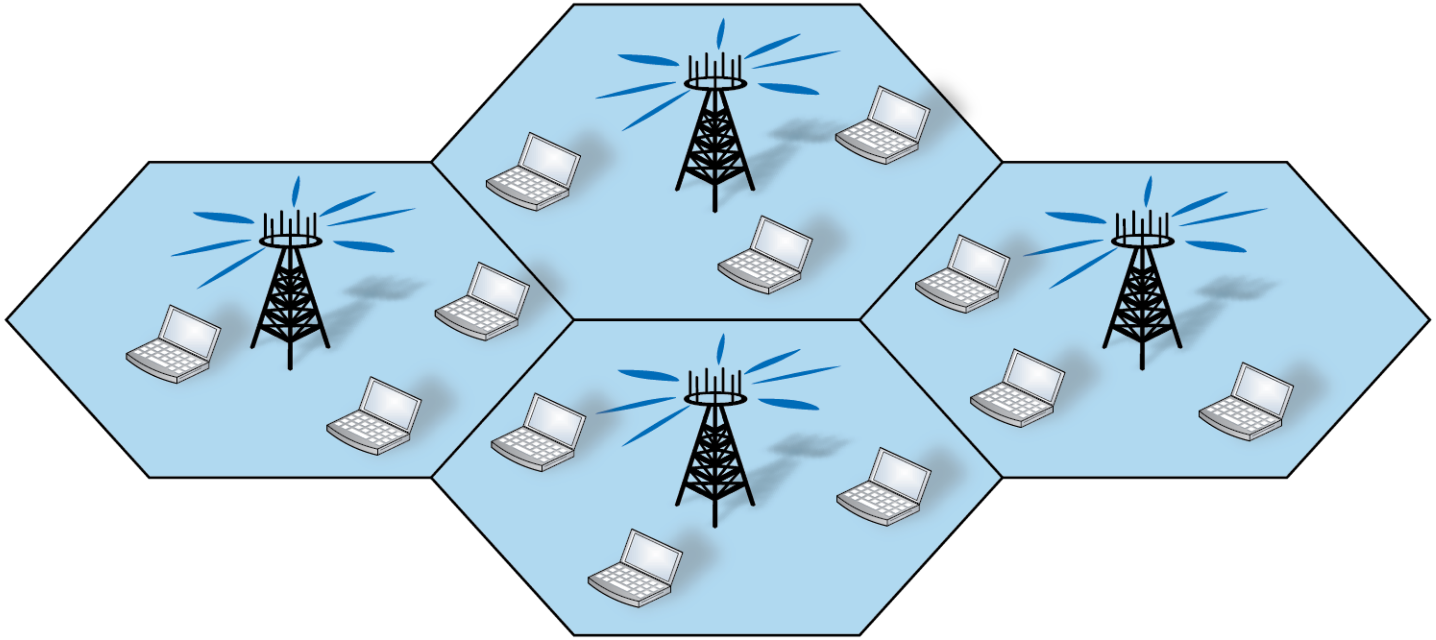
Problem

Compute Optimal
Linear Beamforming

General Conditions

Robustness to
Channel Uncertainty

Generally NP-hard:
Systematic Algorithm



System Model

- Parameters for User *j* in Cell *i*
  - Linear beamforming vector: $\mathbf{w}_{i,j} \in \mathbb{C}^{N_t \times 1}$
  - Channel from cell *m*: $\mathbf{h}_{m,i,j} \in \mathbb{C}^{N_t \times 1}$
  - Signal-to-interference-and-noise ratio (SINR):

$$\mathrm{SINR}_{i,j} = \frac{\overbrace{|\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j}|^2}^{\text{Useful signal}}}{\underbrace{\sum_{l \neq j} |\mathbf{h}_{i,i,j}^H \mathbf{w}_{i,l}|^2}_{\text{Intra-cell interference}} + \underbrace{\sum_{m \neq i} \|\mathbf{h}_{m,i,j}^H \mathbf{W}_m\|_2^2}_{\text{Inter-cell interference}} + \overset{\text{Noise}}{\sigma^2}}$$

  - Notation: $\mathbf{W}_i = [\mathbf{w}_{i,1} \ \ldots \ \mathbf{w}_{i,K}]$

System Model (2)

- Arbitrary Power Constraints in Cell *i*
  - Constraints: $\mathcal{W}_i = \left\{ \mathbf{W}_i : \ \mathrm{tr}\{\mathbf{W}_i^H \mathbf{Q}_{i,k} \mathbf{W}_i\} \leq q_{i,k} \ \forall k \right\}$

    Positive semi-definite

    Positive
  - Notation: $\mathbf{W}_i = [\mathbf{w}_{i,1} \ \dots \ \mathbf{w}_{i,K}]$
  - *Ex:* Per-antenna and per-cell

- Channel State Information (CSI)
  - Perfect CSI within each cell
  - *Uncertain* inter-cell CSI: Ellipsoidal uncertainty set

$$\mathbf{h}_{m,i,j} \in \mathcal{U}_{m,i,j} = \left\{ \widehat{\mathbf{h}}_{m,i,j} + \mathbf{B}_{m,i,j}\boldsymbol{\epsilon}_{m,i,j} : \ \|\boldsymbol{\epsilon}_{m,i,j}\|_2 \leq 1 \right\}$$

Uncertainty set

Known estimate

Shape of ellipsoid

Unknown error vector

Error Sources

Estimation
Feedback
Delays

More severe between cells

Measures of User Performance

- General Measure of User Performance
  - Arbitrary strictly increasing function:

$$g_{i,j}(\mathrm{SINR}_{i,j}) \quad \text{with} \quad g_{i,j}(0) = 0$$

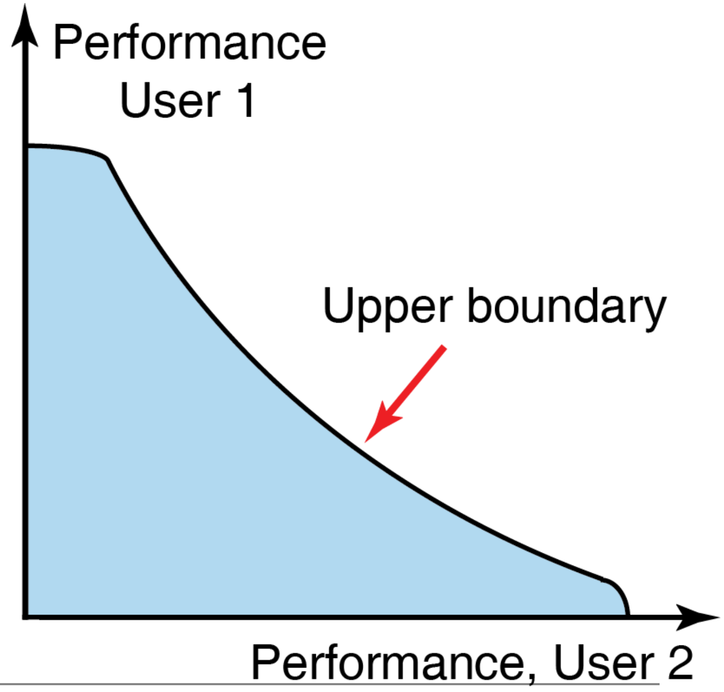
  - *Ex:* Mutual information, Bit error rate, Mean squared error

- Worst-Case Robust User Performance
  - We will try to maximize:

$$g_{i,j}(\widetilde{\mathrm{SINR}}_{i,j}) \quad \text{where} \quad \widetilde{\mathrm{SINR}}_{i,j} = \min_{\substack{\mathbf{h}_{m,i,j} \in \mathcal{U}_{m,i,j} \\ \forall m \neq i}} \mathrm{SINR}_{i,j}$$

Measures of User Performance (2)

- Many Users
  - One Performance Measure $g_{i,j}(\widetilde{\mathrm{SINR}}_{i,j})$ per User
- Fairness Dimension
  - Divide power and control co-user interference
- Robust Performance Region $\mathcal{R}$
  - *NK* users ↔ *NK* dimensions
  - All possible combinations
  - Good points: On upper boundary
  - Unknown shape

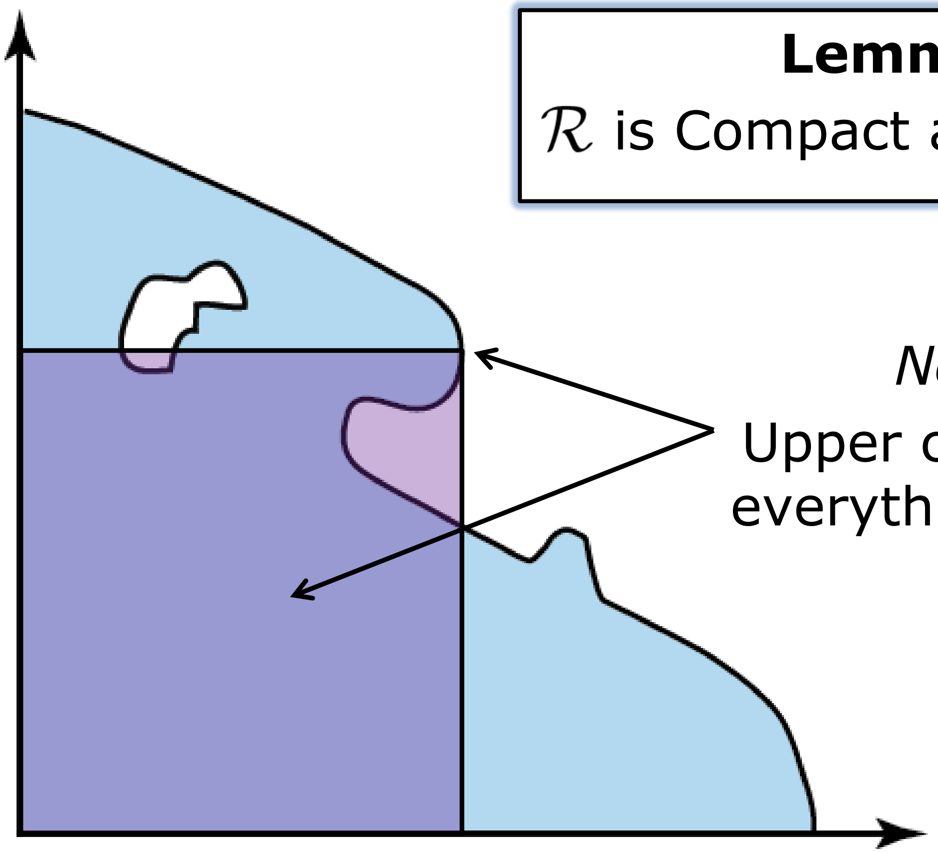


Measures of User Performance (3)

- Can it have any shape?
  - Can be non-convex!
- No!

Lemma

$\mathcal{R}$ is Compact and Normal

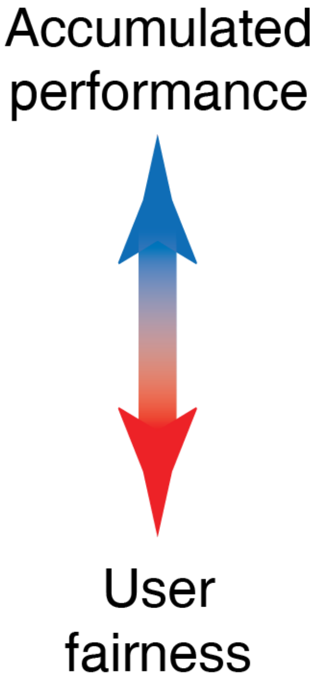


Normal set:

Upper corner in region, everything in the region

System Performance

- Which Point in $\mathcal{R}$ to Select?
- System Performance Function $f : \mathcal{R} \rightarrow \mathbb{R}$
  - Strictly increasing and Lipschitz continuous
- *Examples*
  - Sum performance: $f(\mathbf{g}) = \sum_{i,j} g_{i,j}$
  - Proportional fairness: $f(\mathbf{g}) = \prod_{i,j} g_{i,j}$
  - Harmonic mean: $f(\mathbf{g}) = NK(\sum_{i,j} g_{i,j}^{-1})^{-1}$
  - Max-min fairness: $f(\mathbf{g}) = \min_{i,j} g_{i,j}$
  - Can be modified with weights



Problem Formulation

- Optimize System Performance

$$\underset{\mathbf{W}_i \in \mathcal{W}_i \, \forall i}{\text{maximize}} \quad f\left(g_{1,1}(\widetilde{\mathrm{SINR}}_{1,1}), \ldots, g_{N,K}(\widetilde{\mathrm{SINR}}_{N,K})\right) \qquad (1)$$

  - Lemma: Optimum on upper boundary of $\mathcal{R}$
  - Generally NP-hard: Exponential complexity
  - Only suboptimal strategies in practice

- Goal: Computational Framework for Solving (1)
  - Enable benchmarking and study properties

- Approach
  - Solve a special case of $f(\,)$
  - Exploit it to solve (1) for general $f(\,)$

Special Case

Fairness-Profile Optimization

- Maximize Performance with Fairness Constraints
  - Generalization of classic max-min fairness:

$$\begin{aligned} \underset{\mathbf{w}_i \in \mathcal{W}_i \, \forall i}{\text{maximize}} \quad & \min_{i,j} \frac{g_{i,j}(\widetilde{\mathrm{SINR}}_{i,j}) - a_{i,j}}{\alpha_{i,j}} \\ \text{s.t.} \quad & g_{i,j}(\widetilde{\mathrm{SINR}}_{i,j}) \geq a_{i,j} \quad \forall i, j \end{aligned}$$

  - Lowest acceptable performance level: $g_{i,j} \geq a_{i,j} \geq 0$
  - Users get a portion of exceeding resources: $\alpha_{i,j} \geq 0$

Lemma

- Solved by line-search in $\mathcal{R}$ (bisection)
- Exploiting that $\mathcal{R}$ is normal and compact

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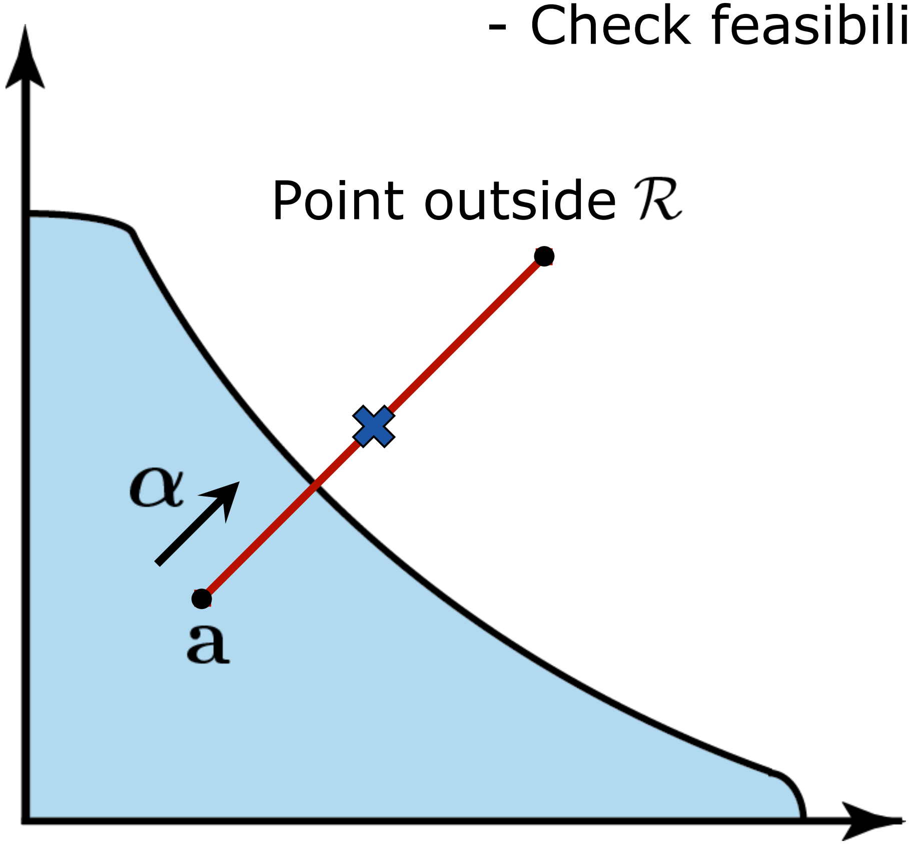
Fairness-Profile Optimization (2)

- Geometrical Interpretation
  - Bisection: Fast convergence
  - Check feasibility at midpoint **c**:



Theorem

- Feasibility checked as convex problem
- CSI uncertainty handled using *S*-lemma

$$\text{find} \quad \mathbf{W}_i \in \mathcal{W}_i, \lambda_{m,i,j} \geq 0, b_{m,i,j} \geq 0 \ \forall i,j,m\neq i$$

$$\text{s.t.} \begin{bmatrix} b_{m,i,j} - \lambda_{m,i,j} & \widehat{\mathbf{h}}_{m,i,j}^H \mathbf{W}_m & \mathbf{0} \\ \mathbf{W}_m^H \widehat{\mathbf{h}}_{m,i,j} & b_{m,i,j} \mathbf{I}_K & \mathbf{W}_m^H \mathbf{B}_{m,i,j} \\ \mathbf{0} & \mathbf{B}_{m,i,j}^H \mathbf{W}_m & \lambda_{m,i,j} \mathbf{I}_{N_t} \end{bmatrix} \succeq \mathbf{0} \quad \forall i,j,m\neq i$$

$$\sqrt{1+\frac{1}{c_{i,j}}} \mathbf{h}_{i,i,j}^H \mathbf{w}_{i,j} \geq \sqrt{\|\mathbf{h}_{i,i,j}^H \mathbf{W}_i\|_2^2 + \sum_{m\neq i} b_{m,i,j}^2 + \sigma^2} \quad \forall i,j$$

Conclusion: Fairness-profile opt is quasi-convex (*solved in polynomial time*)

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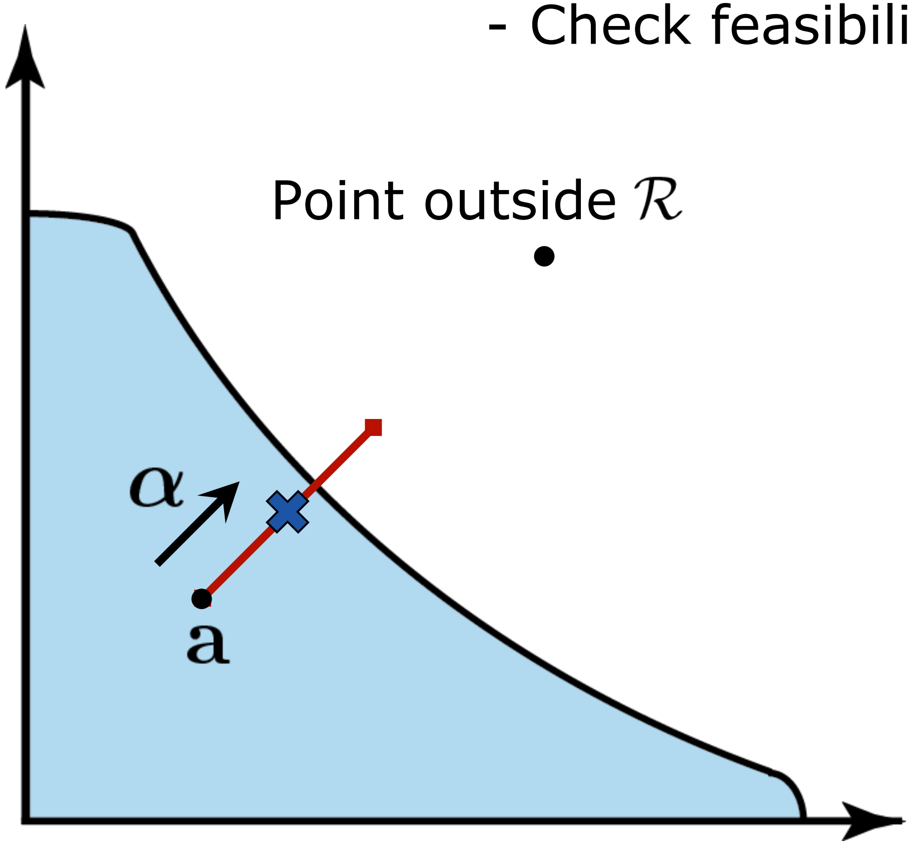

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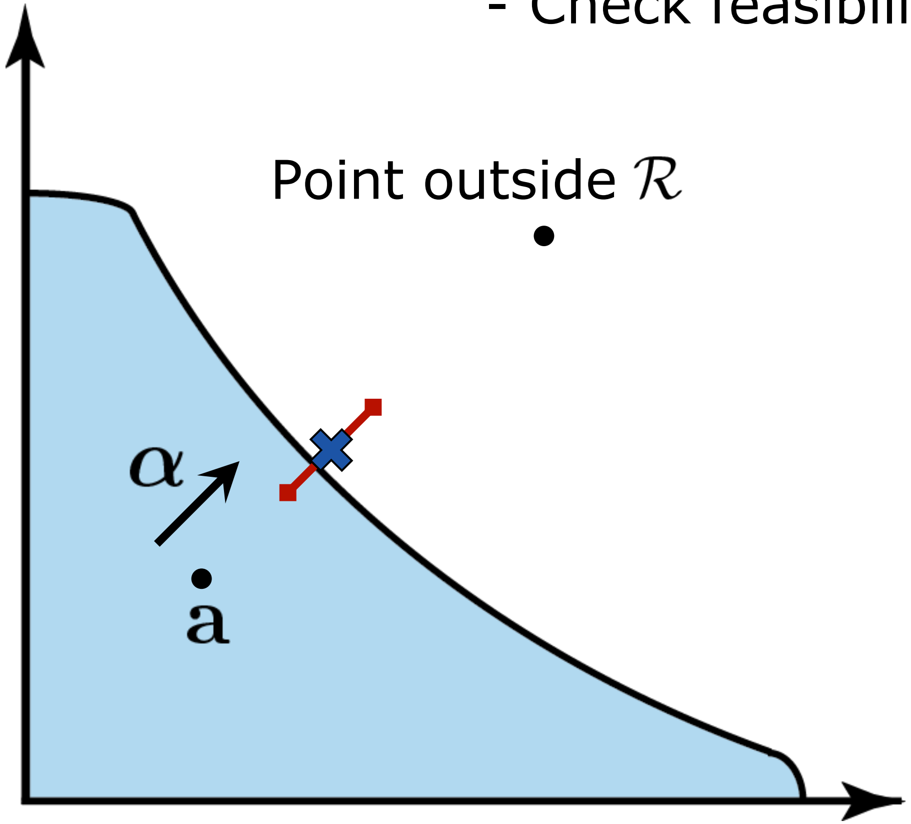

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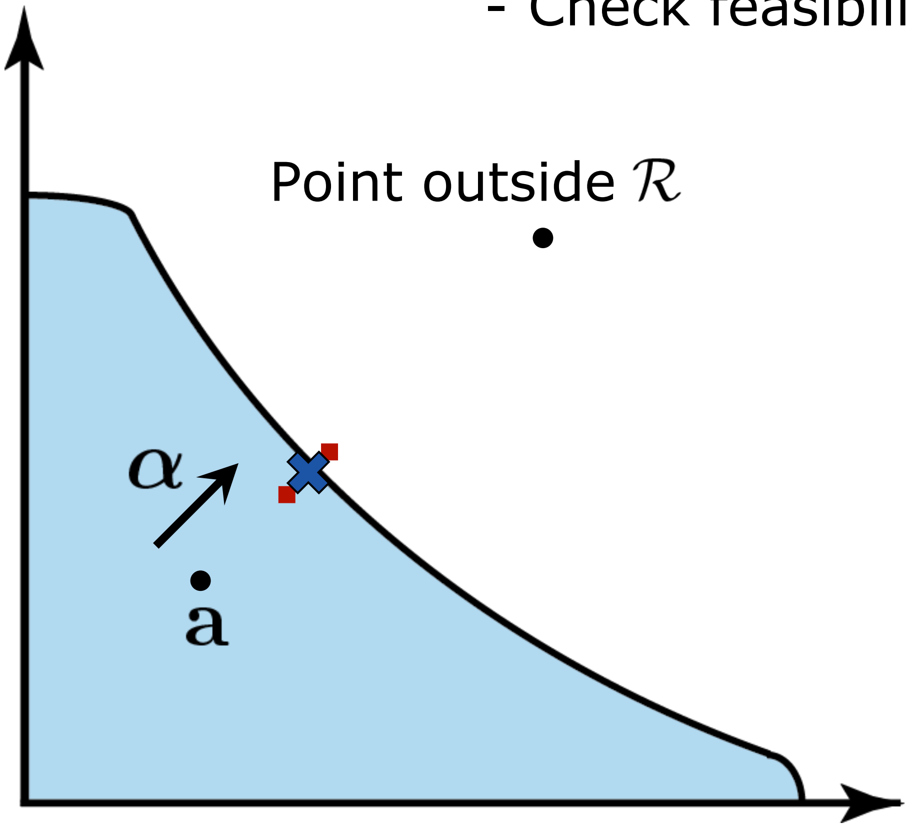

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Framework for General Case

- Systematic Algorithm with Minimal Search Space
  - Search in $\mathcal{R}$ and concentrate on important parts
  - Improve lower/upper bounds on optimum:

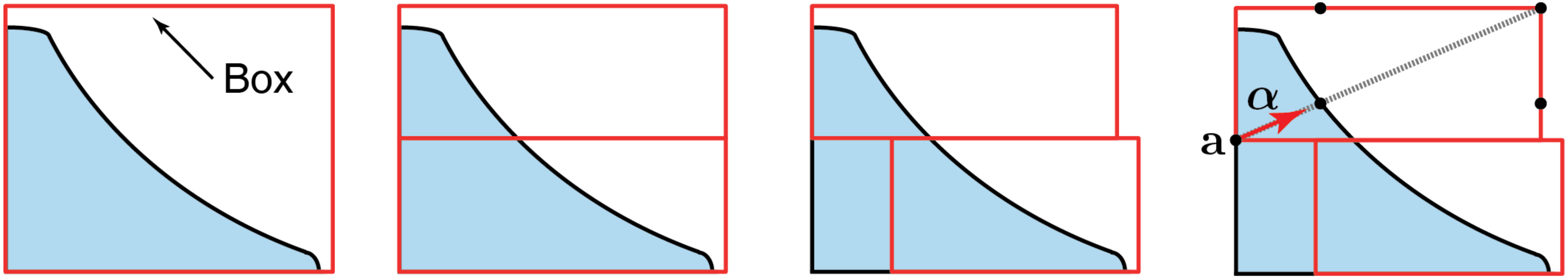
$$f_{\min} \leq f_{\text{opt}} \leq f_{\max}$$

  - Continue until $f_{\max} - f_{\min} < \varepsilon$

- Iterations in Polynomial Time
  - Fairness-profile optimization

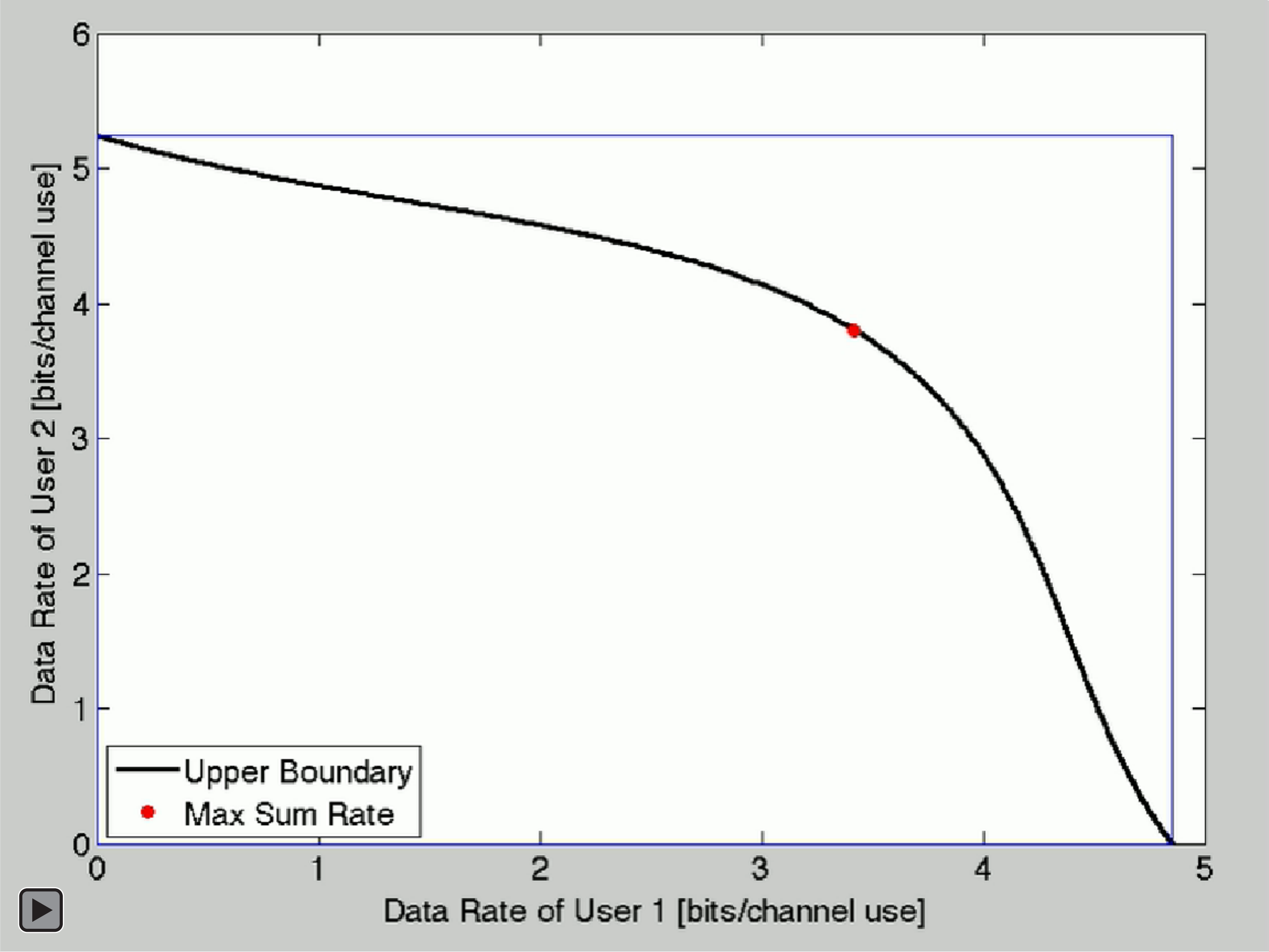
Framework for General Case (2)

- Branch-Reduce-Bound (BRB) Algorithm
  1. Cover $\mathcal{R}$ with a box
  2. Divide the box into two sub-boxes
  3. Remove parts with no solutions in $[f_{\min}, f_{\max}]$
  4. Search for solutions to improve bounds (Fairness-profile optimization)
  5. Continue with sub-box with largest value



1) Cover region $\mathcal{R}$ 2) Branch (Divide) 3) Reduce using bounds 4) Improve bounds

Framework for General Case (3)



Theorem

- Global Convergence
- Accuracy ε>0 in finitely many iterations
- Exponential complexity *only* in *NK*
- Polynomial complexity in N_t and #constraints
- Any accuracy of fairness-profile opt

Numerical Illustrations

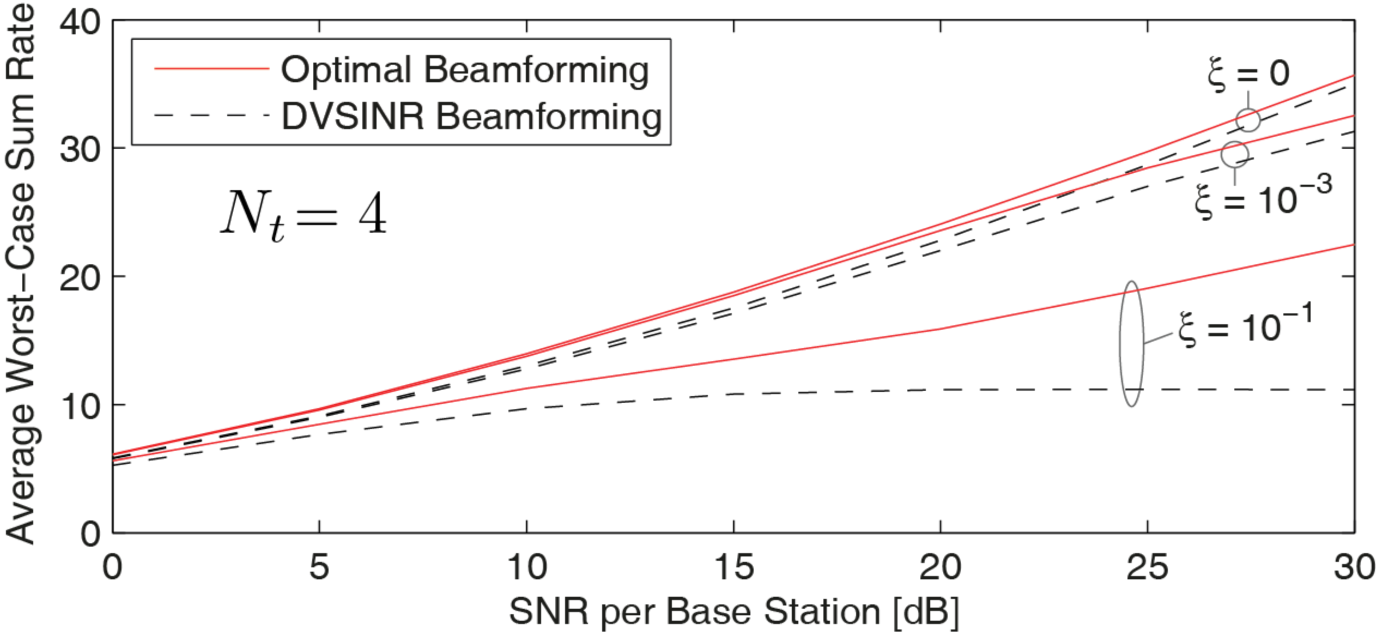
2 Cells and 2 User/Cell

Per-base station constraints

Uncorrelated Rayleigh fading: $\mathbb{E}\{\|\mathbf{h}_{i,i,j}\|^2\} = 2\mathbb{E}\{\|\widehat{\mathbf{h}}_{m,i,j}\|^2\}$

Spherical uncertainty sets: $\mathbf{B}_{m,i,j} = \sqrt{\xi}\mathbf{I}_{N_t}$

- Robustness of Heuristic Beamforming
  - DVSINR beamforming [Björnson et al., 2010]
  - Robust to small intercell uncertainties
  - Highly suboptimal at higher uncertainties



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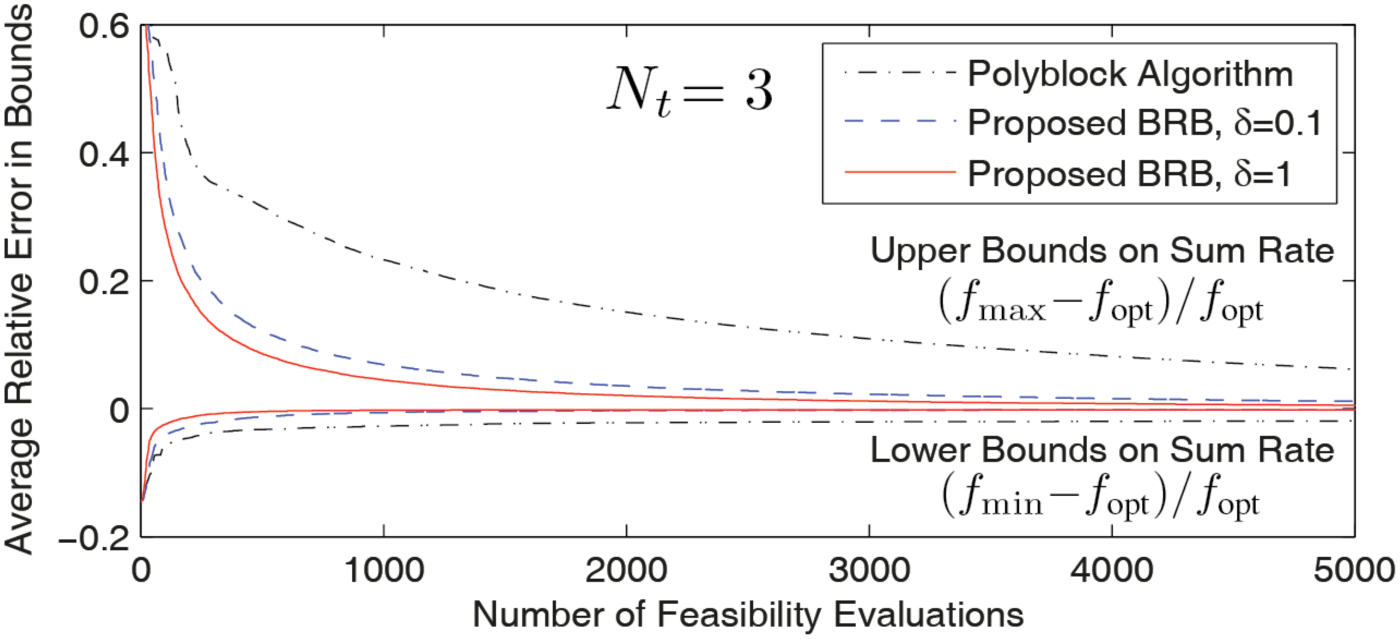
- Convergence of Lower/Upper Bounds
  - Compared with *Polyblock algorithm*
  - Plot relative error of lower/upper bounds
  - BRB algorithm has faster convergence
  - Accurate fairness-profile not necessary



Summary

- Robust Coordinated Beamforming
  - Generally NP-hard ↔ Suboptimal strategies in practice
- Contribution: Computational Framework
  - Enables benchmarking and analysis
  - Robustness and general performance measures/constraints
- Fairness-Profile Optimization
  - Special case solved in polynomial time: Even with robustness
  - Subproblem of general algorithm
- Branch-Reduce-and-Bound Algorithm
  - Systematic algorithm for the general problem
  - Guaranteed to find global solution
  - More general and better convergence than previous work

Extensions

- Journal Article
  - E. Björnson, G. Zheng, M. Bengtsson, B. Ottersten, "*Robust Monotonic Optimization Framework for Multicell MISO Systems*," IEEE Transactions on Signal Processing, Under Minor Revision, arXiv:1104.5240v2.
  - Contains all mathematical details
  - Extension 1: All channels can be uncertain
  - Extension 2: Applicable whenever subproblem can be solved efficiently

Thank You for Listening!

Questions?

Papers and Presentations Available:
http://www.ee.kth.se/~emilbjo