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# Optimizing 5G Networks for Energy-Efficiency

Dense or Massive MIMO or Both?

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# Biography – Emil Björnson

- 1983: Born in Malmö, Sweden
- 2007: Master in Engineering Mathematics, Lund University, Sweden
- 2011: PhD in Telecommunications, KTH, Stockholm, Sweden
- 2012-2014: Joint post-doc at Supélec, Paris, France, and at KTH, Sweden.
- 2014-: Assistant Professor and Docent in Communication Systems, Linköping University, Sweden
- 2014 Outstanding Young Researcher Award, IEEE ComSoc EMEA



# Biography – Luca Sanguinetti

- 1977: Born in Florence, Italy
- 2002: Master in Telecommunications,  
University of Pisa, Italy
- 2006: PhD in Wireless Communications,  
University of Pisa, Pisa, Italy
- 2007-2008: Post-doc at Princeton, USA
- 2010-: Assistant Professor in Wireless Communication Systems,  
University of Pisa, Pisa, Italy
- 2013-2015: Assistant Professor at SUPELEC, Paris, France  
(funded by Marie-Curie Fellowship)



# Outline

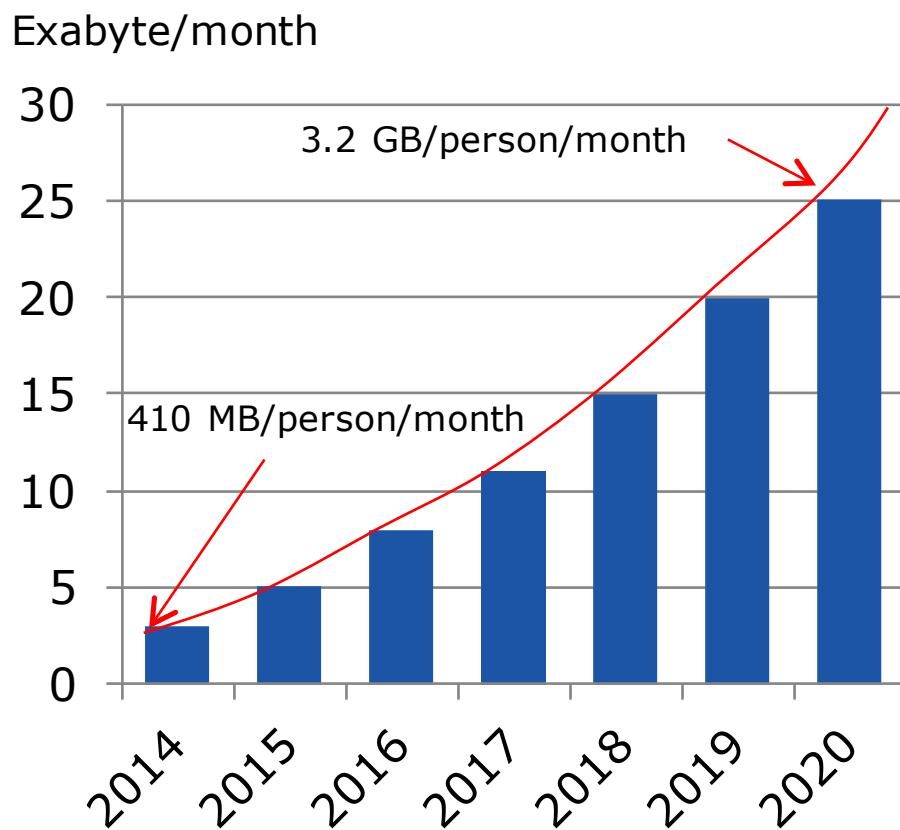
- Part 1: Introduction & Background
  - Energy Efficiency, small cells, and massive MIMO
- Part 2: General Problem Formulation
  - Mathematical models and concepts
- Optimization of Energy Efficiency
  - Part 3: Fixed Regular Deployment
  - Part 4: Heterogeneous Deployment
  - Insights on optimal network design
- Part 5: Optimizing Multiple Metrics at the Same Time
  - Beyond pure energy efficiency optimization
- Conclusion

**Part 1**

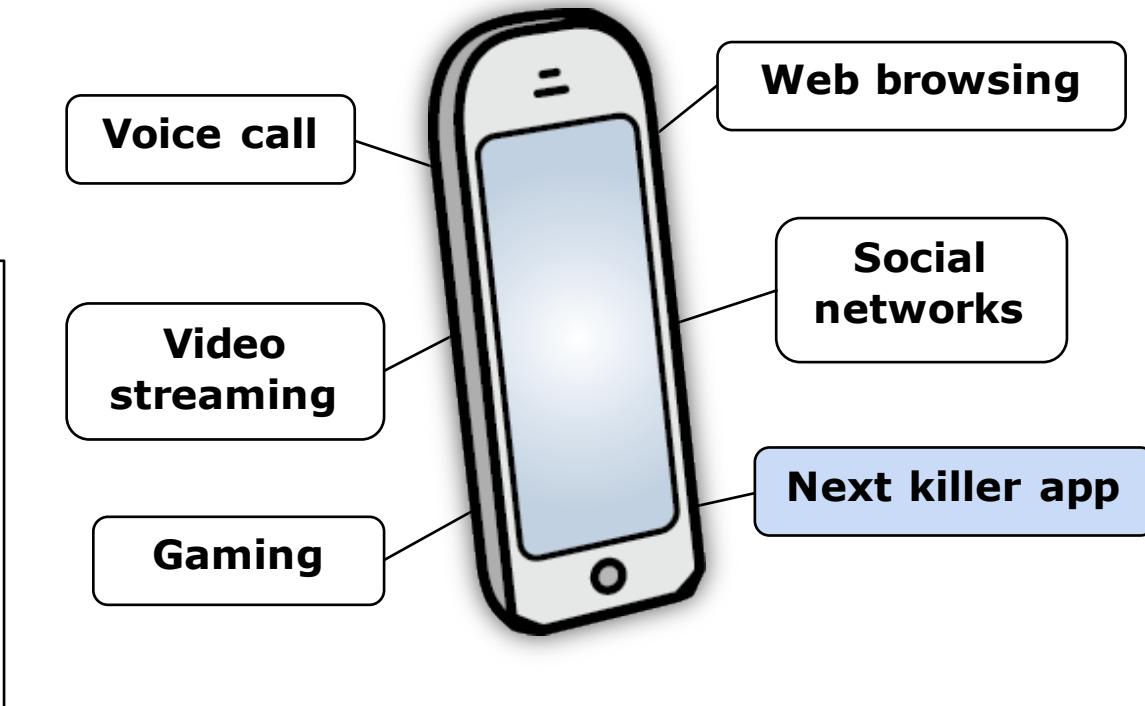
# **Introduction & Background**

# Introduction

- Wireless Connectivity
  - A natural part of our lives



Source: Ericsson (November 2014)



- Rapid Network Traffic Growth
  - 38% annual growth
  - Exponential increase!
  - Extrapolation: 5x until 2020  
25x until 2025  
125x until 2030

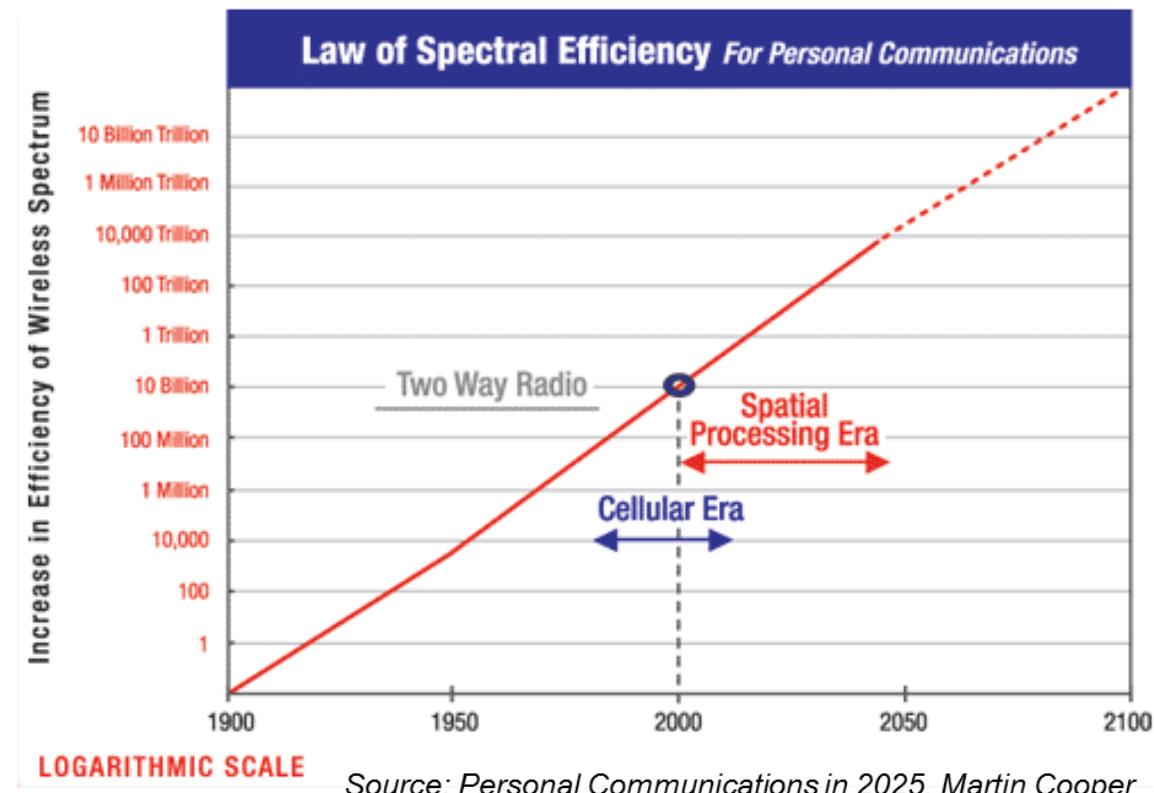
# Exponential Traffic Growth

- Is this Growth Sustainable?
  - User *demand* will increase – users expect more for same price
  - Traffic supply – increases only if business models allow it!
- Exponential Growth is Nothing New!
  - $10^6$  increase in last 45 years!

## Martin Cooper's law

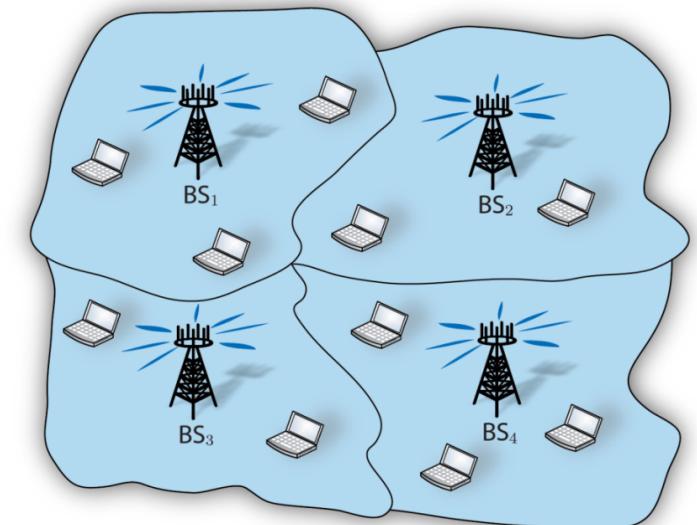
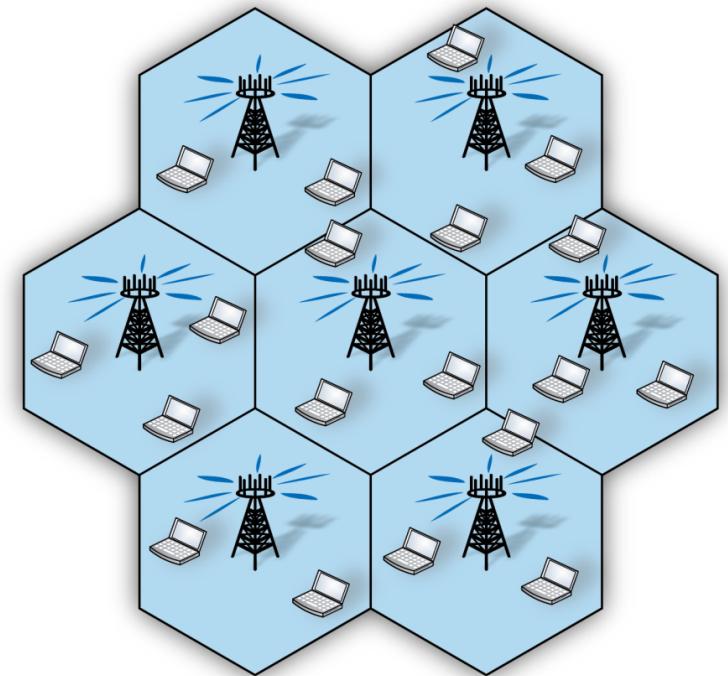
The number of simultaneous voice/data connections has doubled every 2.5 years since the beginning of wireless

- Coopers law: 32%/year
- New predictions: 38%/year
- Growth likely to continue!



# Cellular Networks

- Cellular Network Architecture
  - Coverage area divided into cells
  - One fixed base station (BS) per cell
  - Serves all users in the cell
  - Uplink: User→BS
  - Downlink: BS→User
- Different Standards
  - 2G (GSM), 3G (UMTS), 4G (LTE/LTE-A)
- Traditional Ways to Handle More Traffic
  - Higher cell density (variable cell sizes)
  - More spectrum (carrier aggregation)
  - Higher spectral efficiency (spatial processing)

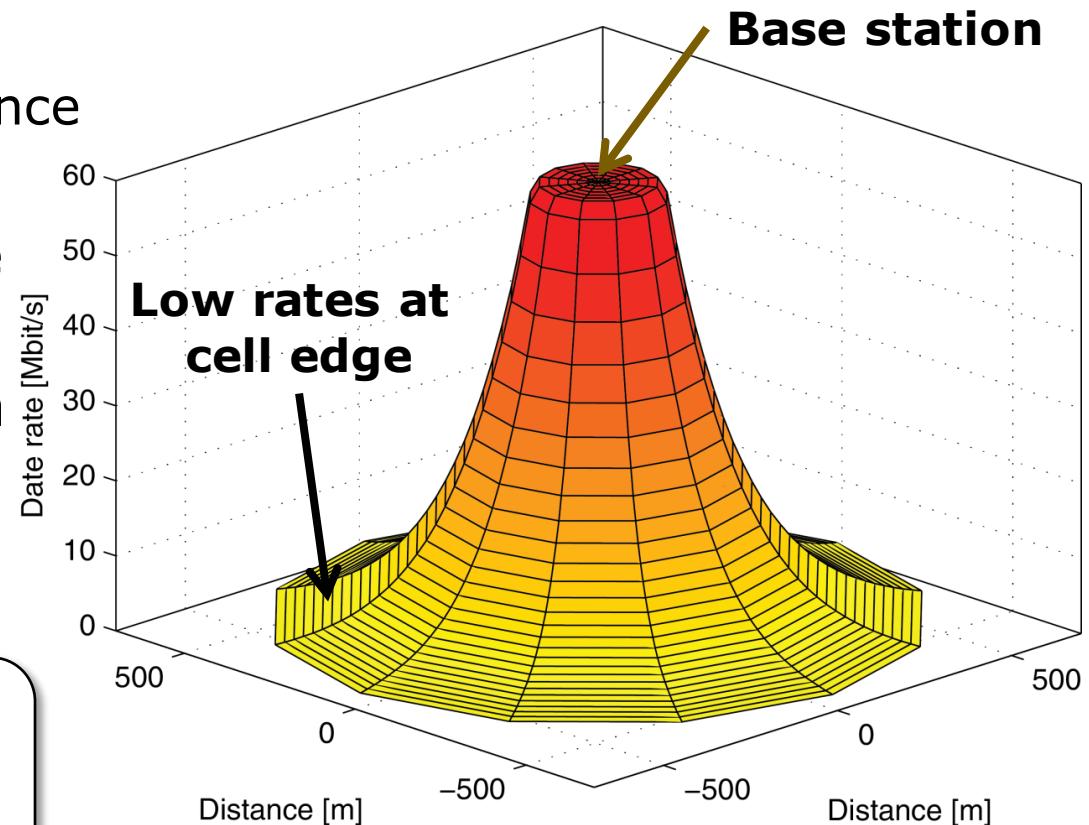


# High Data Rates

- Traditional Design Metric
  - High *peak* and/or *average* rates [bit/s/active user]
- Basic Signal Propagation
  - Signal energy decays with distance
  - Peak rates in cell center
  - Far from peak rates at cell edge
- Traffic Independent of Location
  - Easily satisfied in cell center
  - Highest demand at cell edge!

## Need for Additional Metrics

To optimize and design  
our networks properly!



# Expectations for 5G Networks

- 5G – The Next Cellular Network Generation
  - Expected to be introduced by 2020
  - Design objectives are currently being defined

5G Performance Metrics	Expectation
Average rate (bit/s/active user)	10-100x
Average area rate (bit/s/km <sup>2</sup> )	1000x
Active devices (per km <sup>2</sup> )	10-100x
Energy efficiency (bit/Joule)	1000x

Source: METIS project  
([www.metis2020.com](http://www.metis2020.com))

## Parts 2-4

*What if we optimize a network only for energy-efficiency?*

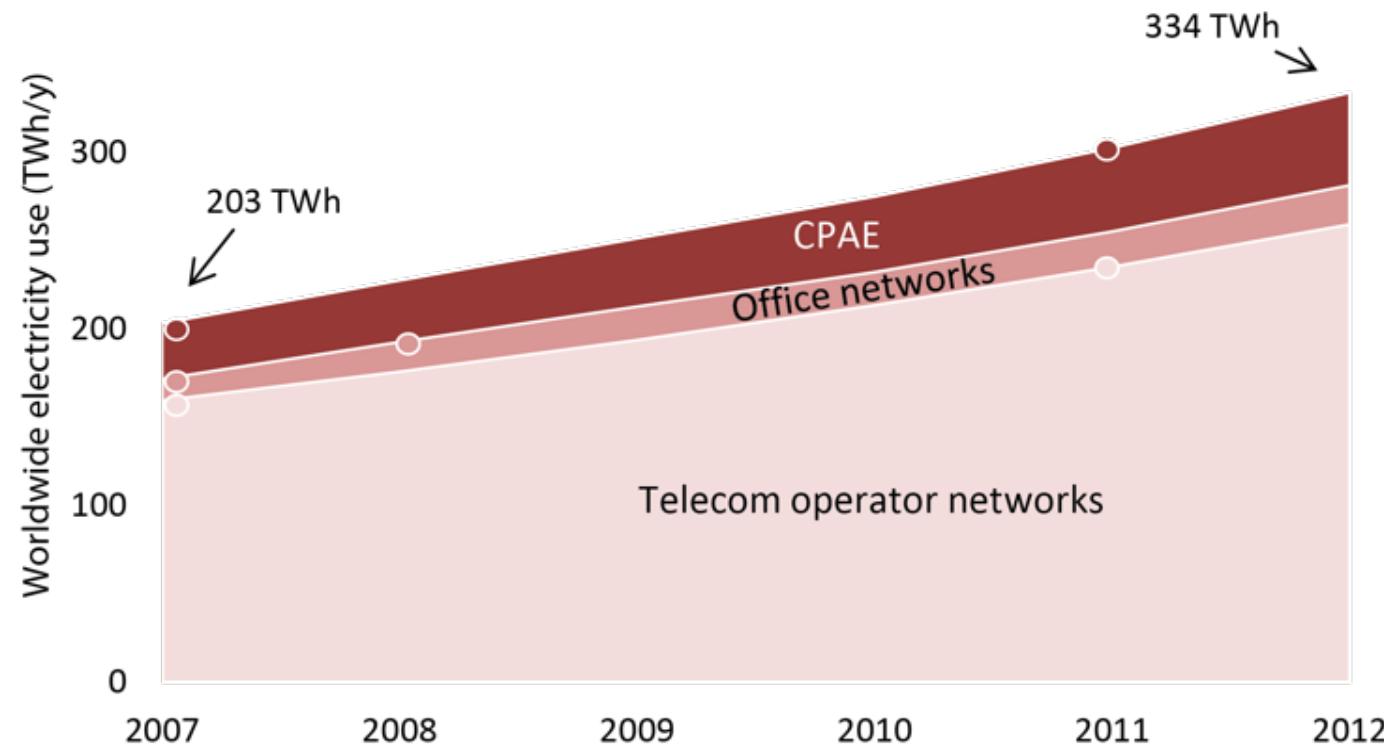
*What will it look like?*

## Part 5

*Is it possible to optimize a network with respect to multiple metrics?*

*What does “optimality” mean then?*

# Why Focus on Energy Efficiency?



Source: Heddeghem et al.  
"Trends in worldwide ICT  
electricity consumption from  
2007 to 2012"

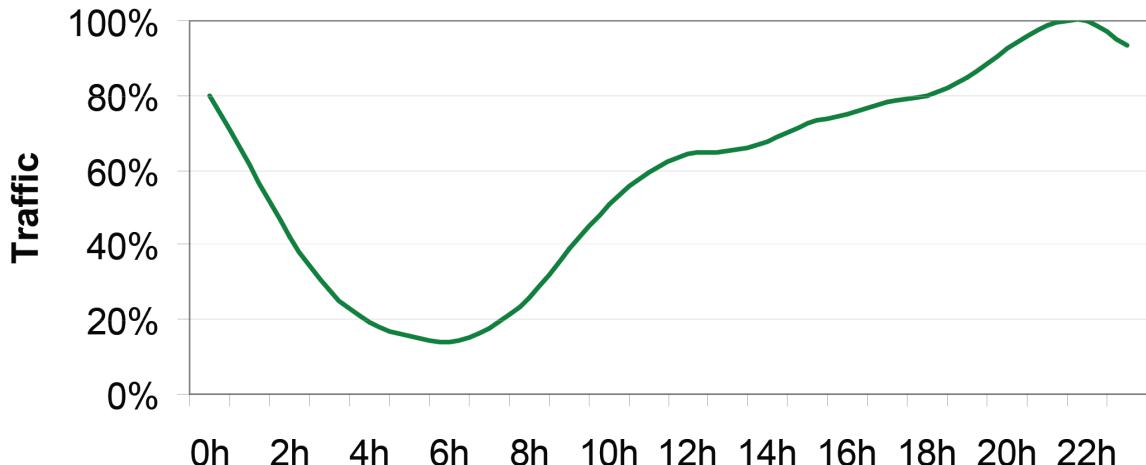
- Network Electricity Consumption
  - Dominated by network infrastructure
  - Increases continuously
  - 1000x higher data rates:
    - Easy to achieve using 1000x more power
    - Hard to achieve without using more power
  - Calls for **much higher energy efficiency!**

# What is Energy Efficiency?

- Benefit-Cost Analysis of Networks
  - Systematic approach to analyze strengths and weaknesses of networks
  - Benefit: Sum Data Rate [bit/s]
  - Cost: Consumption of Energy [Watt = Joule/s] and Money [€/s]
- Definition: Energy Efficiency (EE):
$$EE = \frac{\text{Average Sum Rate [bit/s/cell]}}{\text{Power Consumption [Joule/s/cell]}}$$
  - “Amount of data transmitted over consumed energy” (naturally per cell)
  - A detailed model is developed in this tutorial!
- Economical Costs not Explicitly Included
  - Examples: Hardware cost, site renting, network management
  - Can be added by transforming into equivalent energy consumption
  - Use energy price [Joule/€] as transformation factor

# Load-Independent Efficiency

- User Load Variations
  - Evening: Peak traffic
  - Late night: Lowest traffic
  - Same trends in the future



Source: Auer et al., "Cellular Energy Efficiency Evaluation Framework", VTC, 2011.

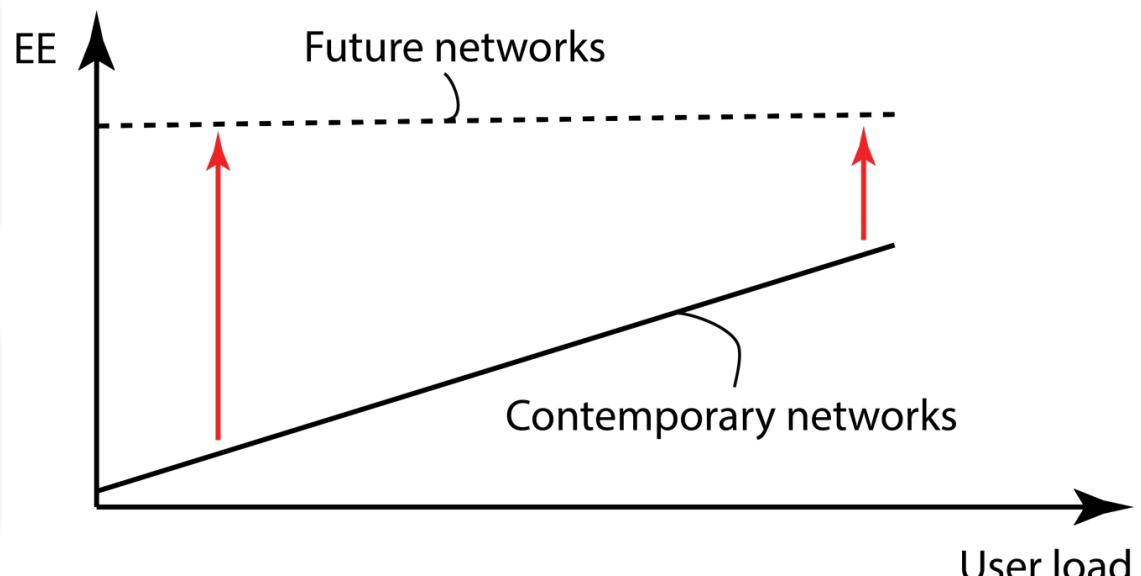
## Contemporary networks

Relatively efficient at high user load

Very inefficient at low user load

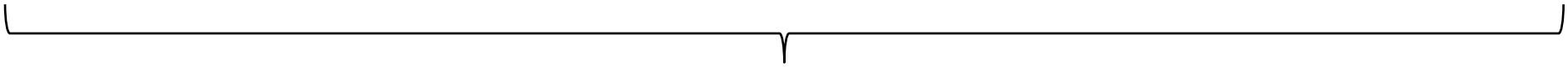
## Future networks

Must be more energy efficient,  
irrespective of current load



# Network Densification

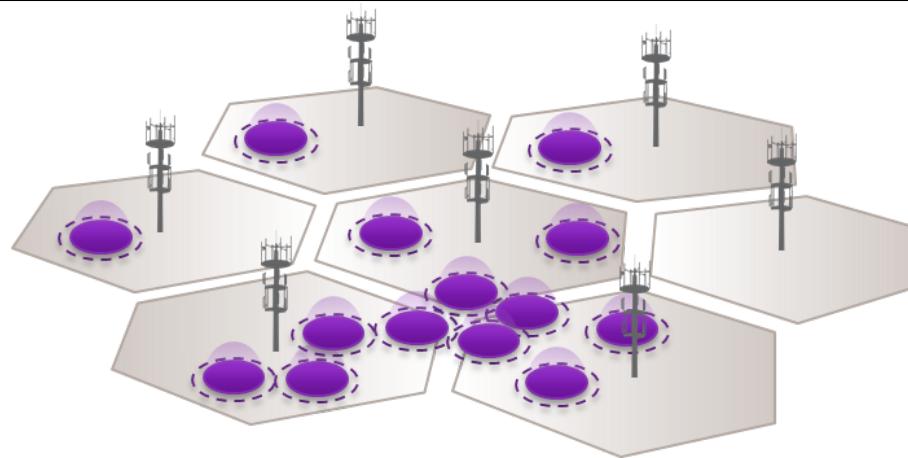
- Ideas: How can we improve EE?
  - Reduce propagation losses ( $-100$  dB is common in wireless)
  - Serve multiple users per cell (share the load-independent costs)
  - **How to design new networks to achieve this?**
- Two Main Solution Paradigms
  - Smaller cells: Reduce propagation distances
  - Massive MIMO: Spatially directed transmission, multiplexing of users



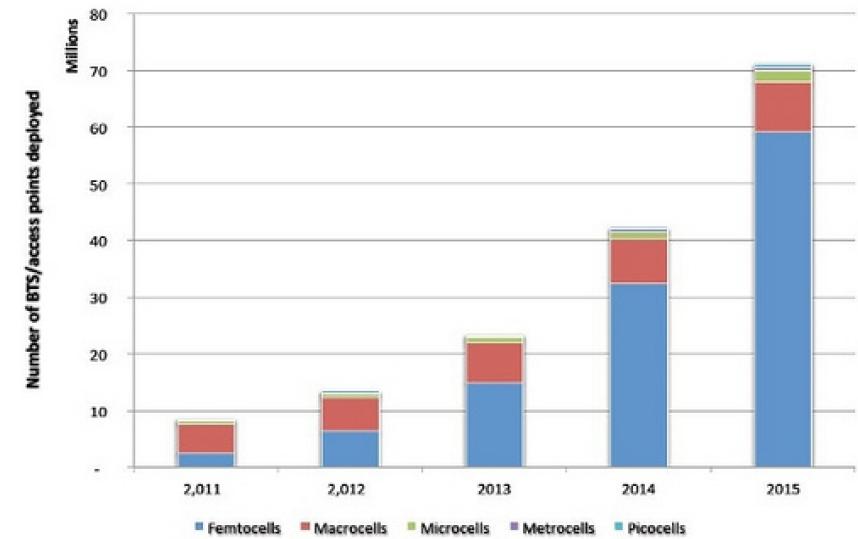
**Network densification:**  
More service antennas per  $\text{km}^2$

**Which solution should we choose?**

# What are Small Cells?



- “Small cells” is an Umbrella Term:
  - Operator-controlled, low-powered BS operating in licensed spectrum
- Small Cells of Different Size
  - Femtocells (up to 100 m)
  - Picocells (up to 200 m)
  - Microcells (up to 500 m)
  - Centralized BSs or remote radio heads
  - Wired or wireless with core network



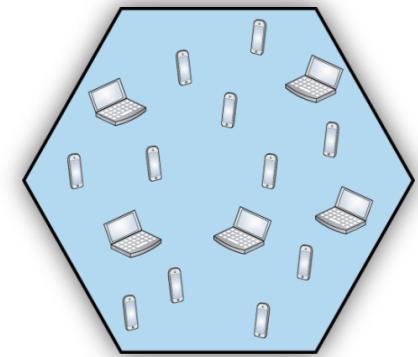
Source: Informa Telecoms & Media

# Access Policies

- Three Access Policies
  - Closed access
    - Only users in a closed subscriber group can connect
  - Open access
    - All users can connect
  - Hybrid access
    - All users + priority to some
- Important factors:
  - Interference from other cells depend on spectrum license
  - Different cost and backhaul conditions
- **Femtocells:** closed, open or hybrid access
- **Picocells:** open access, offload traffic from larger cells

# Why Small Cells?

- Increase Network Area Rate [bit/s/km<sup>2</sup>]
  - Consider a given area

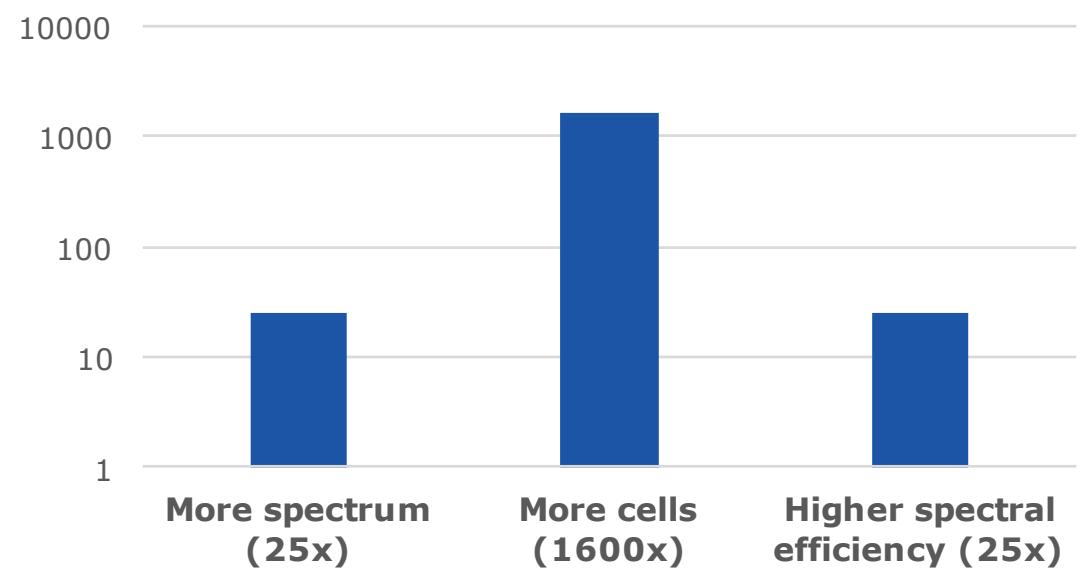


- Simple Formula for Area Rate:

$$\frac{\text{Area rate}}{\text{bit/s/km}^2} = \frac{\text{Available spectrum}}{\text{Hz}} \cdot \frac{\text{Cell density}}{\text{Cell/km}^2} \cdot \frac{\text{Spectral efficiency}}{\text{bit/s/Hz/Cell}}$$

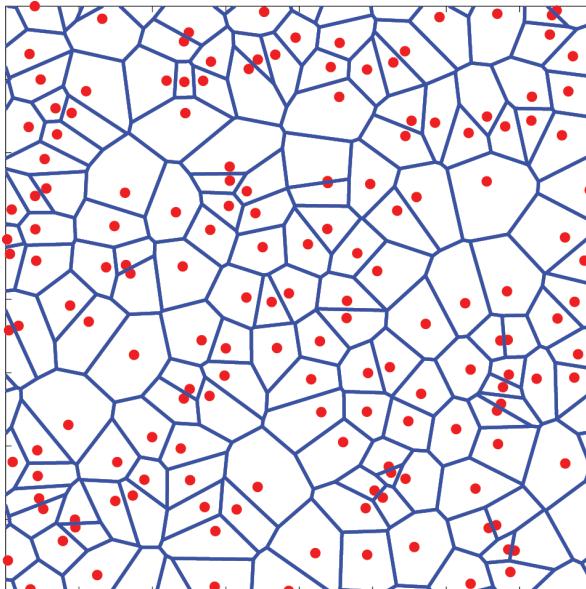
**Cell-Size Reduction**

Traditional way to increase rates  
Most efficient approach in the past  
No need to develop new radio interfaces

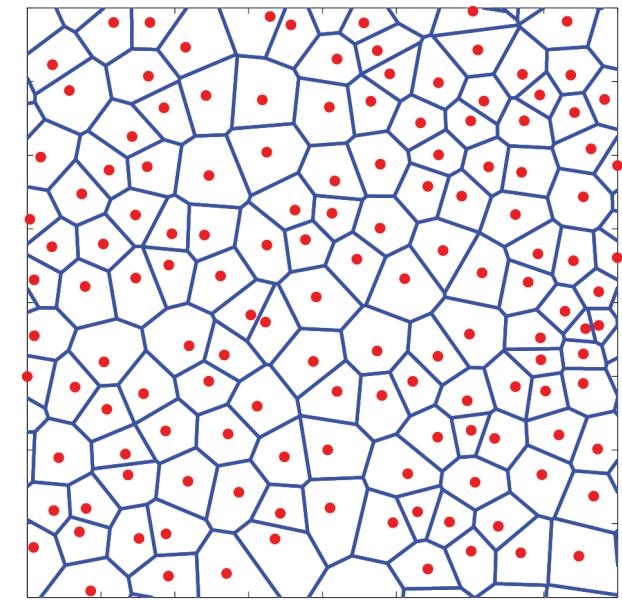


# Asymmetric Deployment

- Shape of Cellular Networks
  - Classically modeled as symmetric hexagons grid
  - Real networks are highly asymmetric
  - Asymmetry plays key role as cells shrink



Poisson point deployment



Real BS deployment

## Spatial Stochastic Point Processes

Tractable way to model randomness  
Models random independent deployment:  
Lower bound on practical performance

*Source: Andrews et al. "A Tractable Approach to Coverage and Rate in Cellular Networks"*

# Small Cells – Main Benefits

- Improved Coverage
  - Especially outdoor-to-indoor and indoor-to-indoor coverage
  - Majority of wireless traffic originates from indoor users
- Improved Area Rates
  - Without need for more spectrum or advanced processing techniques
- Improved Battery Life
  - Lower propagation loss → Less transmit power from devices
- Reduced Energy Consumption
  - A small cell consumes 5 W → 100 millions small cells consume 4.4 TWh in 2020 (small part of network consumption)
- More Tailored to User Load
  - Hotspots in malls and subway stations

# Small Cells – Energy Efficiency

- Advantages

- Shorter distances: Major reductions in propagation losses
- Can be applied along with any transmission protocol
- Cells can be turned on/off without destroying coverage

- Disadvantages

- Cell edge conditions are not improved (interference limited)
- Hard to coordinate transmissions across cells
- Deploy more BSs per km<sup>2</sup> – very expensive?

# Small Cells – Standardization

- Small Cell Forum (formerly Femto Forum)
  - Non-profit organization founded in 2007
  - Goal: Enable and promote small cells worldwide
  - Standardization, regulation & interoperability
  - Marketing, promotion & business case



67 operators covering 3 billion mobile subscribers – 46% of total

A grid of operator logos from around the world, including major names like Bell, BT, and Vodafone, along with many smaller local operators. The grid is enclosed in a yellow border.

End to End Solutions

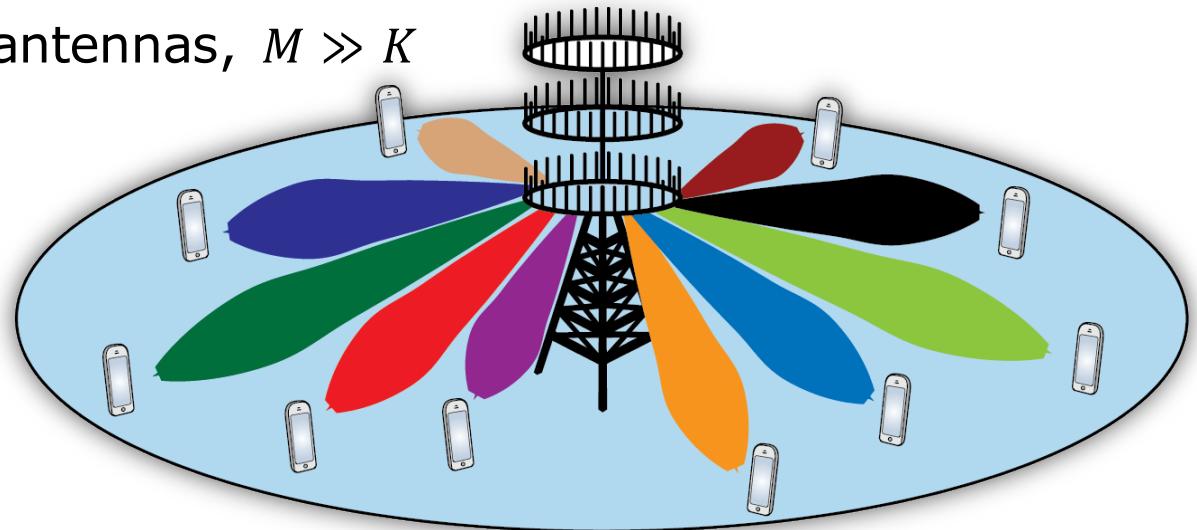
Components & Software: Alcatel-Lucent, AEROPA, AVIATRIX, CAVALIA, DIRECT, INTEL, INFINEON, MEDIATEK, MINDSPEED, NODALINK, QIALCOMM, RADWIN, RUCKUS, SYMMETRICOM, TANGOLINK, TELIT, TELUS, TENDAYA, TINGO, TOSHIBA, TURKCELL, UNIVIS, ZTE, ZTEOR.  
Access Points: AIRVANA, AIRNET, ALFA, COMBA, D-Link, FORTINET, JUNIPER, LARMEQ, NEUTRON, RUCKUS, SAGEMCOM, TENDAYA, TINGO, ZTEOR.  
Core Network: ERICSSON, HITACHI, NEC, NOKIA, OLYMPUS, PUNIWAVE, RICOH, TELUS, TURKCELL, UNIVIS, ZTEOR.  
Others: ALCATEL-LUCENT, AIRVANA, ARRIAD, ASYBY, BACELLA, BELL, BROADCOM, CORNING, D-Link, ETSI, FUJITSU, GENBAND, HUAWEI, INTEL, INTERDIGITAL, KORENTECH, LARMEQ, MINDSPEED, NEC, NOKIA, OLYMPUS, PUNIWAVE, RICOH, RUCKUS, SAGEMCOM, TELUS, TURKCELL, UNIVIS, ZTEOR, ZTEOR.

74 providers of small cell technology representing all parts of the ecosystem

A grid of logos for various small cell technology providers, grouped into four main categories: Components & Software, Access Points, Core Network, and Others. The grid is enclosed in a yellow border.

# What is Massive MIMO?

- New Network Architecture
  - Multi-user MIMO (multiple-input, multiple output) technology
  - Many antennas at BSs: e.g.,  $M = 100$  antennas
  - Serve many users in parallel:  $K = 10$  users
  - Key: Excessive number of antennas,  $M \gg K$
  - Very directive signals
  - Little interference leakage



## 2013 IEEE Marconi Prize Paper Award

Thomas Marzetta, "Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas," IEEE Trans. Wireless Commun., 2010.

Analysis based on  $M \rightarrow \infty$  and  $K$  fixed, but concept applicable at any  $M \gg K$

# What is the Key Difference from Today?

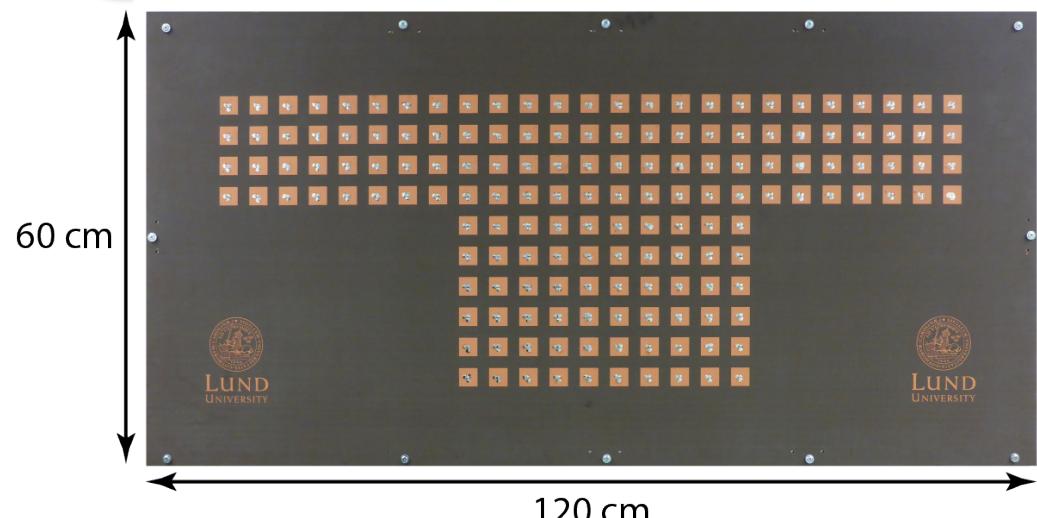
- Number of Antennas? **No, we already have many antennas!**
  - 3G/UMTS: 3 sectors x 20 element-arrays = 60 antennas
  - 4G/LTE-A: 4-MIMO x 60 = 240 antennas

## Massive MIMO Characteristics

Many small dipoles with transceiver chains

Spatial multiplexing of tens of users

Massive numbers – not massive size



**160 dual-polarized antennas**, LuMaMi testbed, Lund University

**Typical vertical array:**  
10 antennas x 2 polarizations  
Only 1-2 transceiver chains



**3 sectors, 4 vertical arrays per sector**  
Image source: gigaom.com

# Massive MIMO Deployment

- When to Deploy Massive MIMO?

The future will tell, but it can

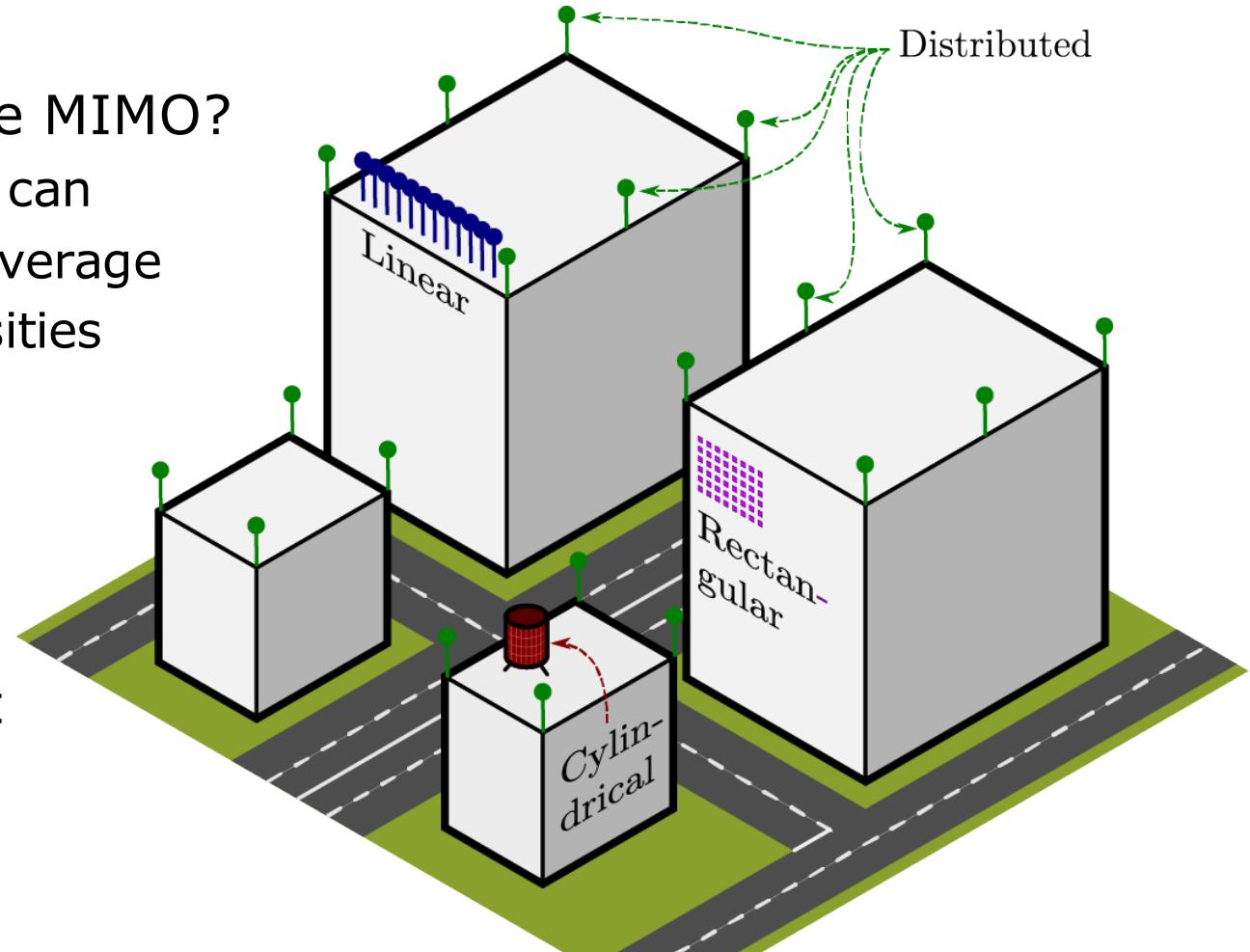
1. Improve wide-area coverage
2. Handle high user densities

- Co-located Deployment

- 1D, 2D, or 3D arrays

- Distributed Deployment

- Remote radio heads



## Benefits with Massive MIMO

Outdoor users:

Handle mobility and provide coverage

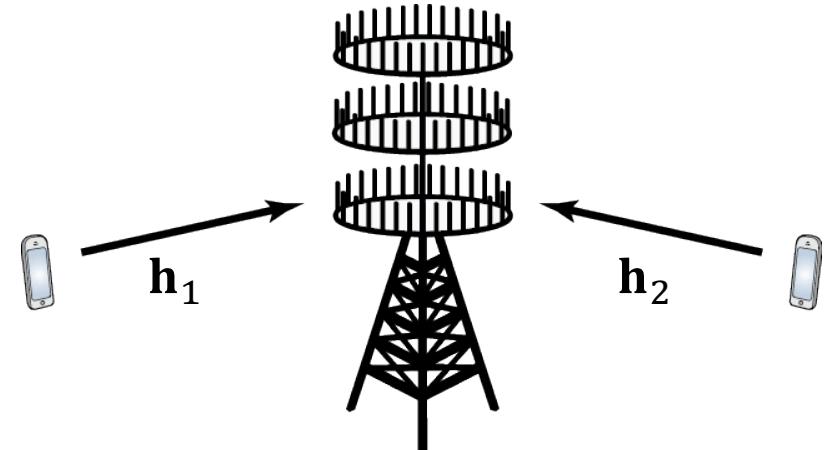
Indoor users:

No need to put BSs inside buildings

# Original Motivation: Asymptotic Channel Orthogonality

- Example: Uplink Transmission

- Two users, channels:  $\mathbf{h}_1, \mathbf{h}_2 \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$
- Signals:  $s_1, s_2 \sim \mathcal{CN}(0, P)$
- Noise:  $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$
- Received:  $\mathbf{y} = \mathbf{h}_1 s_1 + \mathbf{h}_2 s_2 + \mathbf{n}$



- Linear Processing for User 1:  $\tilde{y}_1 = \mathbf{g}_1^H \mathbf{y} = \boxed{\mathbf{g}_1^H \mathbf{h}_1 s_1} + \boxed{\mathbf{g}_1^H \mathbf{h}_2 s_2} + \boxed{\mathbf{g}_1^H \mathbf{n}}$

- Maximum ratio filter:  $\mathbf{g}_1 = \frac{1}{M} \mathbf{h}_1$

- Signal remains:  $\mathbf{g}_1^H \mathbf{h}_1 = \frac{1}{M} \|\mathbf{h}_1\|^2 \xrightarrow{M \rightarrow \infty} \mathbb{E}[|h_{11}|^2] = 1$

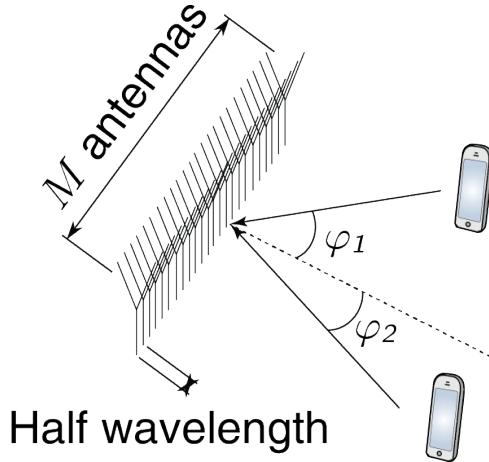
- Interference vanishes:  $\mathbf{g}_1^H \mathbf{h}_2 = \frac{1}{M} \mathbf{h}_1^H \mathbf{h}_2 \xrightarrow{M \rightarrow \infty} \mathbb{E}[h_{11}^H h_{21}] = 0$

- Noise vanishes:  $\mathbf{g}_1^H \mathbf{n} = \frac{1}{M} \mathbf{h}_1^H \mathbf{n} \xrightarrow{M \rightarrow \infty} \mathbb{E}[h_{11}^H n_1] = 0$

Asymptotically noise/interference-free communication:  $\tilde{y}_1 \xrightarrow{M \rightarrow \infty} s_1$

# Only for i.i.d. Rayleigh Fading Channels?

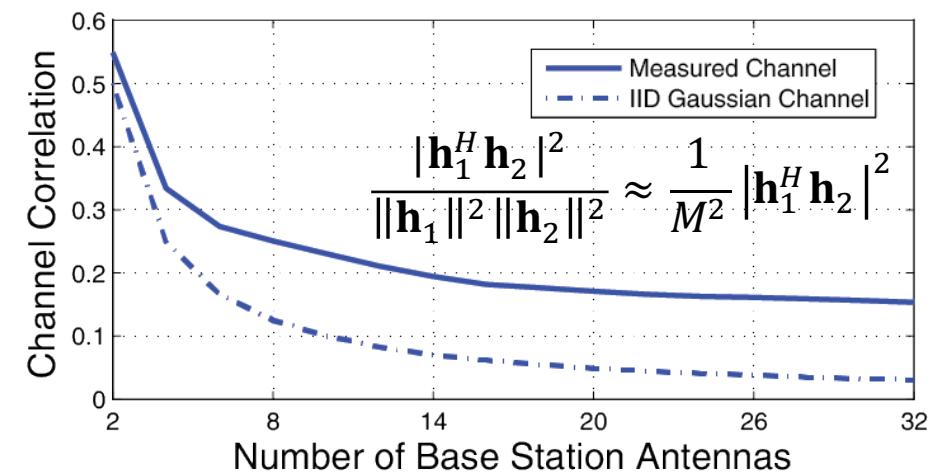
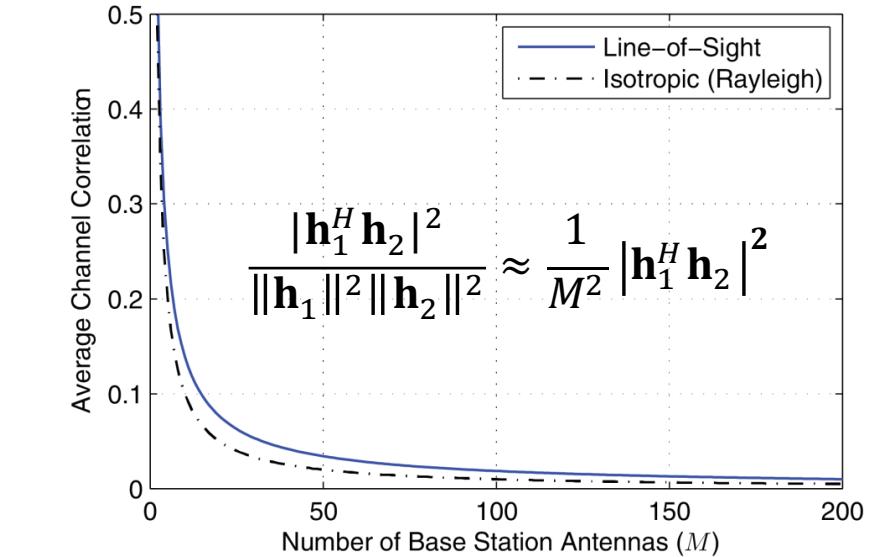
- Line-of-Sight Propagation



- Uniform linear array
- Random user angles
- $M$  observations:
  - Stronger signal
  - Suppressed noise
- What is  $\mathbf{h}_1^H \mathbf{h}_2 \rightarrow ?$

- Channel Measurements

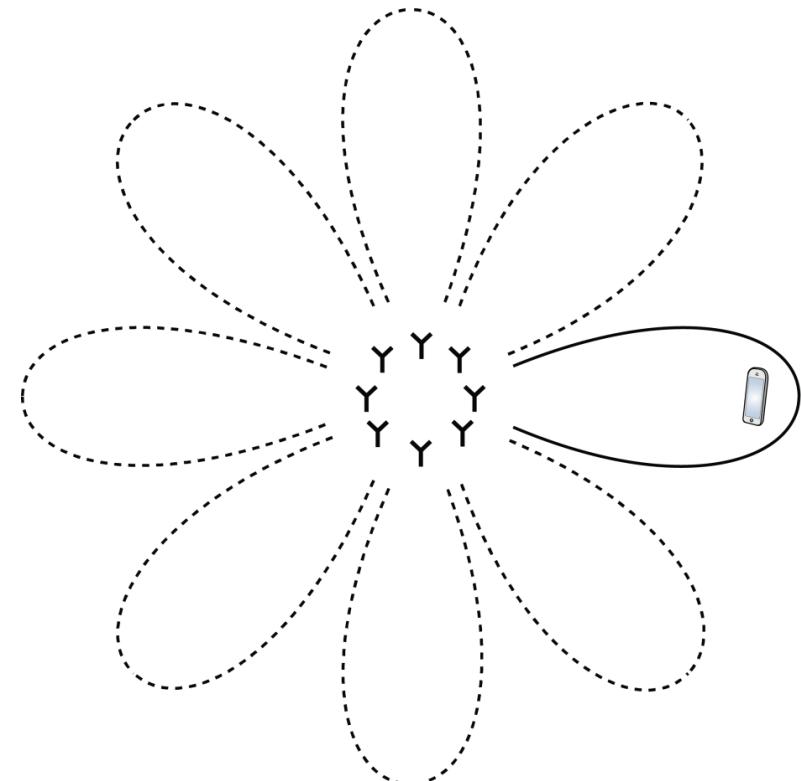
Source: X. Gao, O. Edfors, F. Rusek, and F. Tufvesson, "Linear Pre-Coding Performance in Measured Very-Large MIMO Channels," VTC 2011.



## Different scenarios:

Determine how quickly interference is suppressed

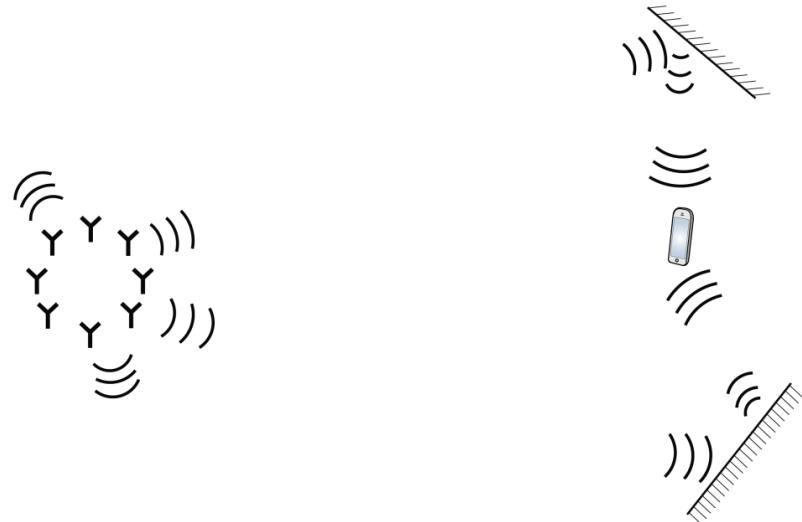
# MIMO Transmission



## Line-of-Sight

Channels determined by angles  
1-2 parameters to estimate per user  
Precoding = Beamforming

**Same principles for uplink**



## Non-Line-of-Sight

Rich multipath propagation  
 $M$  parameters to estimate per user  
Precoding  $\neq$  Beamforming

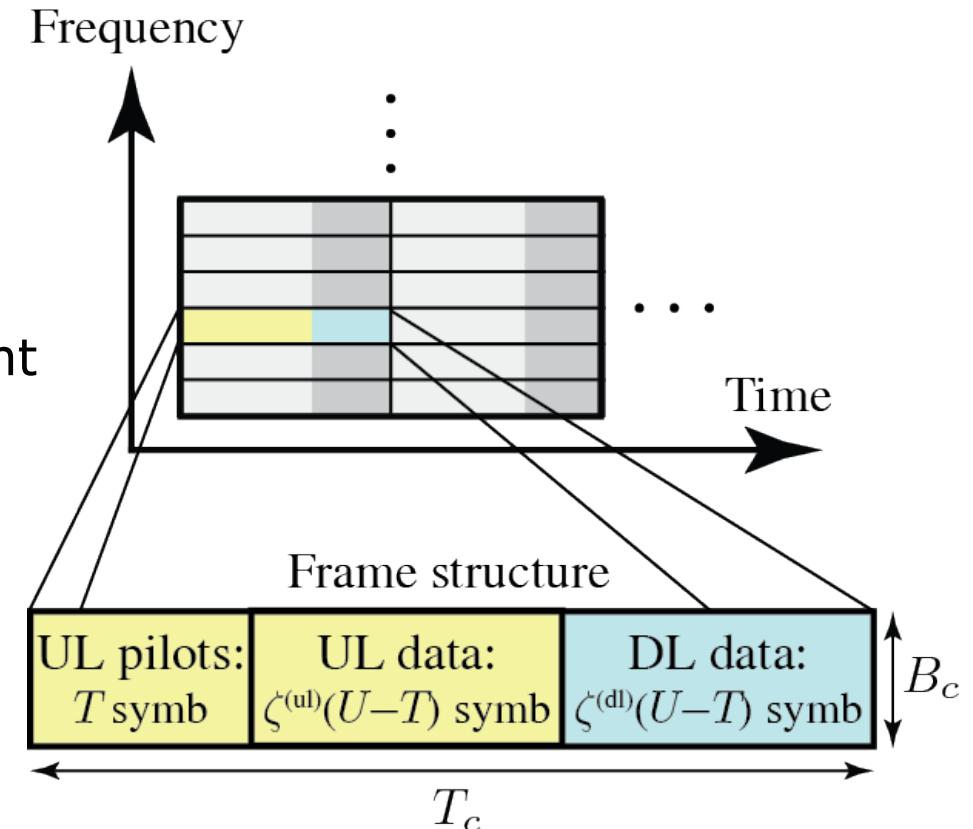
**Massive MIMO aims at handling both cases!**

Need to estimate  $M$  parameters efficiently even if  $M$  is large

# Massive MIMO Transmission Protocol

- Coherence Blocks

- Fixed channel responses
- Coherence time:  $T_c$  s
- Coherence bandwidth:  $B_c$  Hz
- Depends on mobility and environment
- Block length:  $U = T_c B_c$  symbols
- Typically:  $U \in [100, 10000]$

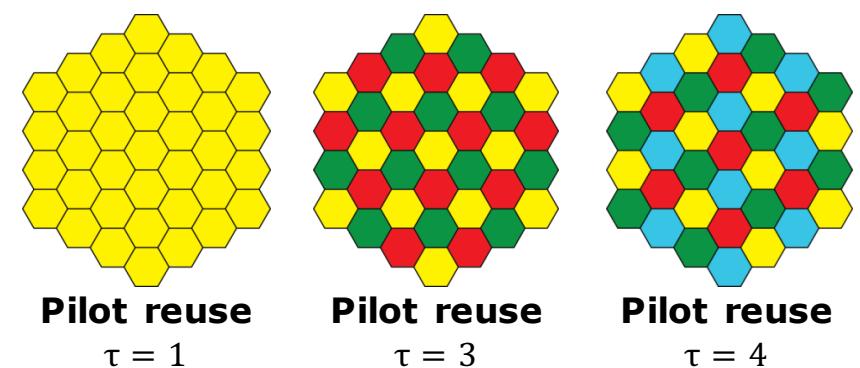
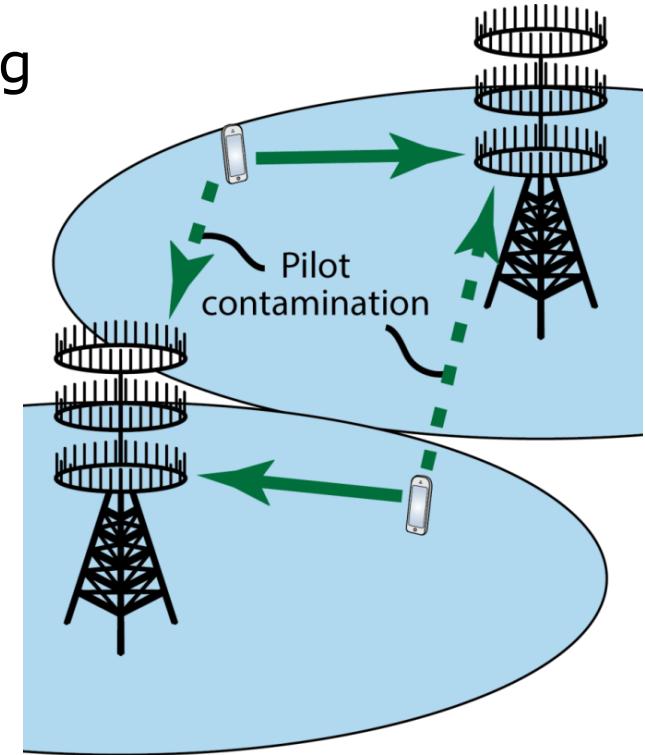


- Time-Division Duplex (TDD)

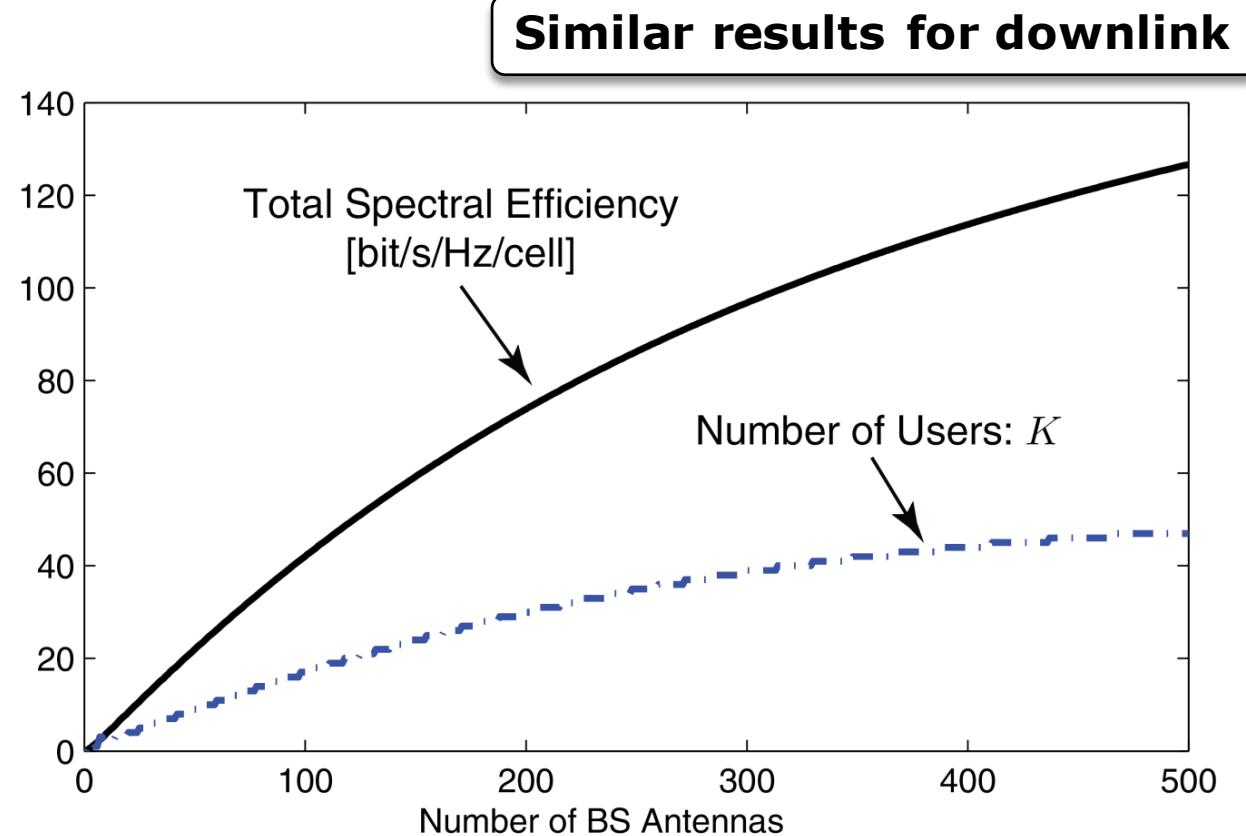
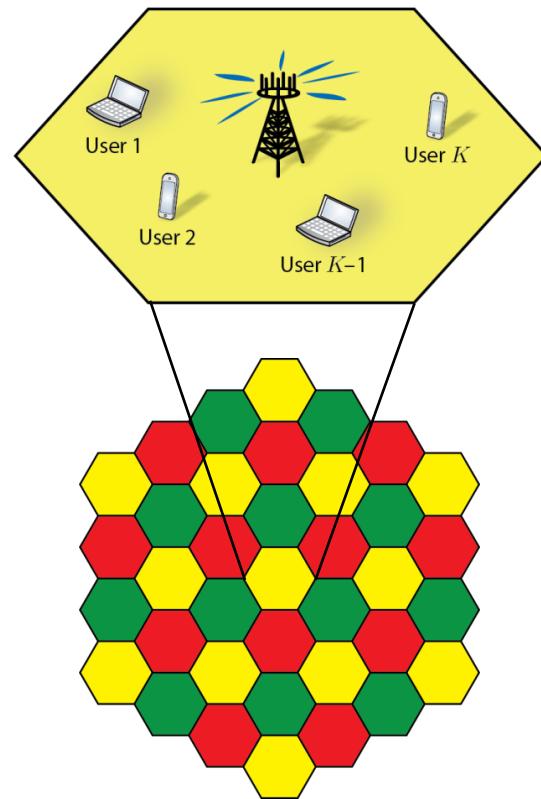
- Switch between downlink and uplink on all frequencies
- $T$  symbols/block for uplink pilots – to estimate channel responses
- Number of pilots proportional to number of users  $K$ , independent of  $M$
- $U - T$  symbols/block for uplink and/or downlink payload data

# Channel Acquisition in Cellular Networks

- Each BS Needs to Know Channels for Processing
  - Estimate using uplink pilot symbols
  - Only  $T$  pilot symbols available (pick  $K \leq T \leq U$ )
  - Must use same pilot symbols in different cells!
  - Base stations cannot tell some users apart
- Called: Pilot Contamination
  - Recall: Noise and interference vanish as  $M \rightarrow \infty$
  - Not interference between users with same pilot!
- Solution: Select how often pilots are reused
  - Pilot reuse factor  $\tau \geq 1$
  - Users per cell:  $K = \frac{T}{\tau}$
  - Higher  $\tau \rightarrow$  Fewer users per cell,  
but interferers further away



# How Much can Spectral Efficiency be Improved?



## Uplink Simulation

LTE-like system parameters

Coherence block:  $U = 500$

SNR 5 dB, i.i.d. Rayleigh fading

ZF detector and pilot reuse  $\tau = 3$

## Observations

- Baseline: 2.25 bit/s/Hz/cell (IMT-Advanced)
- Massive MIMO,  $M = 100$ : x20 gain ( $M/K \approx 6$ )
- Massive MIMO,  $M = 400$ : x50 gain ( $M/K \approx 9$ )
- Per scheduled user:  $\approx 2.5$  bit/s/Hz

# Massive MIMO and Energy Efficiency

- Advantages

- Better energy focusing (transmit power per user can scale as  $1/\sqrt{M}$ )
- Improved cell edge conditions (only desired signals are amplified)
- Multiple users share the load-independent “costs” per BS

- Disadvantages

- More transceiver hardware per BS
- Reduction in transmit power is modest
- TDD protocols needed to enable precoding for large  $M$

# Summary

- Energy Efficiency is Important for Future Networks
  - Can be improved by network densification
  - Two approaches: Small cells and Massive MIMO
- Which Approach to Choose?
  - Cannot only be answered by proper EE modeling and optimization
  - **Methodology and key observations provided in this tutorial!**

**Part 1**

# **Questions?**

**Part 2**

# **General Problem Formulation**

# How to Measure Energy-Efficiency?

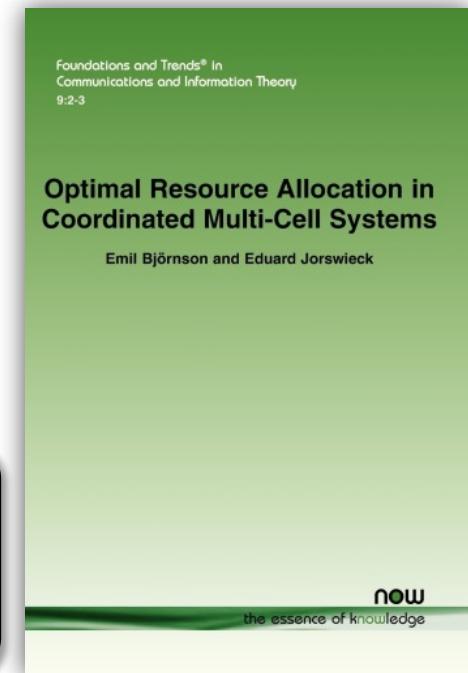
- Recall: Energy-Efficiency (EE) in bit/Joule

$$EE = \frac{\text{Average Sum Rate [bit/s/cell]}}{\text{Power Consumption [Joule/s/cell]}}$$

- Conventional Academic Approaches:
  - Maximize rates with fixed power
  - Minimize transmit power for fixed rates
  - See for example:

## Optimal Resource Allocation in Coordinated Multi-Cell Systems

Book from 2013 by Emil Björnson and Eduard Jorswieck  
*E-book is free to download*



**New Problem:** Balance rates and power consumption

*Ratio: Important to account for overhead signaling and circuit power!*

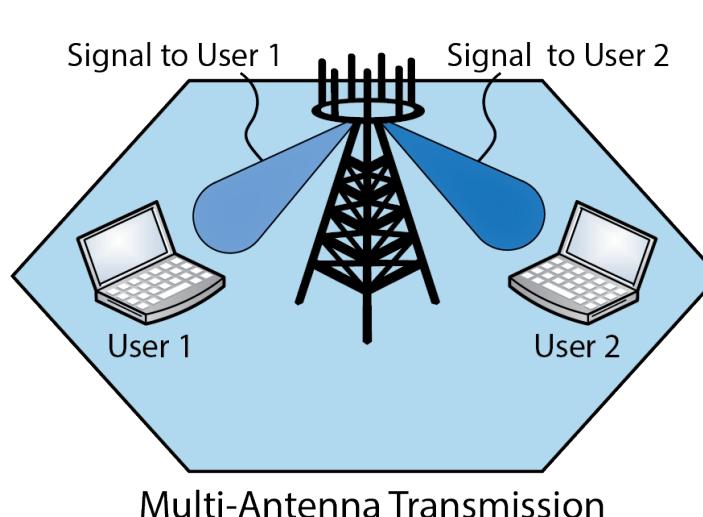
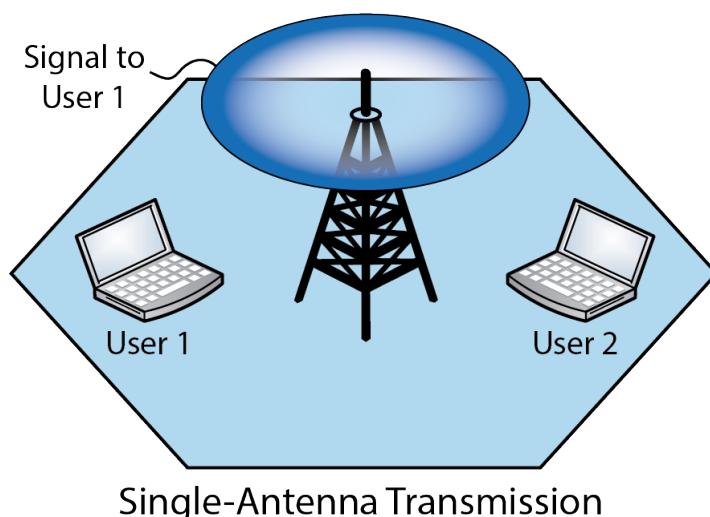
# Basic Information Theory

## Achievable Rate per Active User [Lower Bound on Shannon Capacity]

$$\text{Bandwidth} \cdot \log_2 \left( 1 + \frac{\text{Received Signal Power}}{\text{Interference Power} + \text{Noise Power}} \right) \text{ [bit/s/active user]}$$

Signal-to-interference-and-noise ratio (SINR)

- Can we Have Multiple Active User per Cell?
  - Yes, but causes inter-user interference
  - Traditional approach: Orthogonal in time/frequency (TDMA, OFDMA)
  - New multi-antenna approach: Space-division multiple access (SDMA)

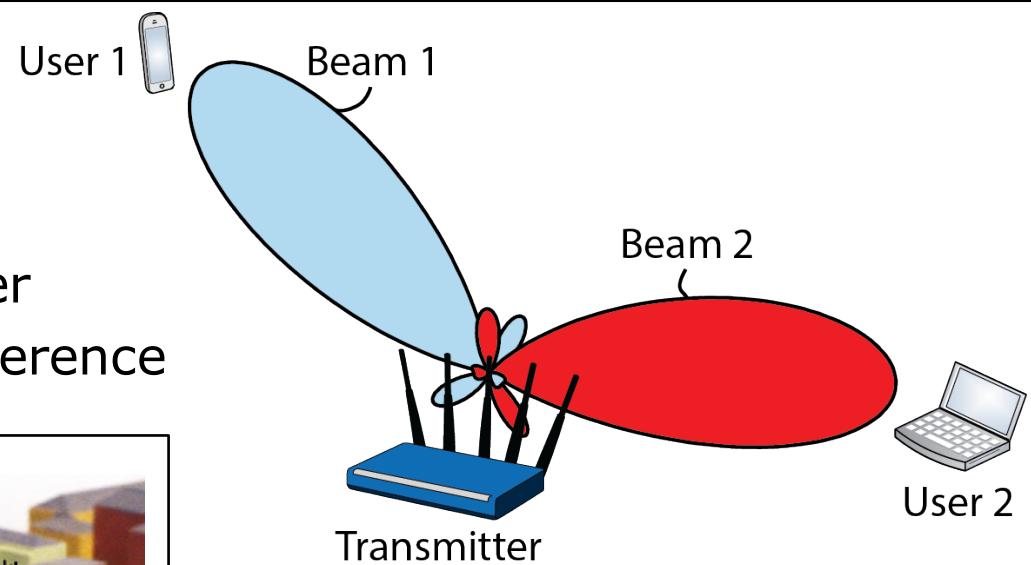
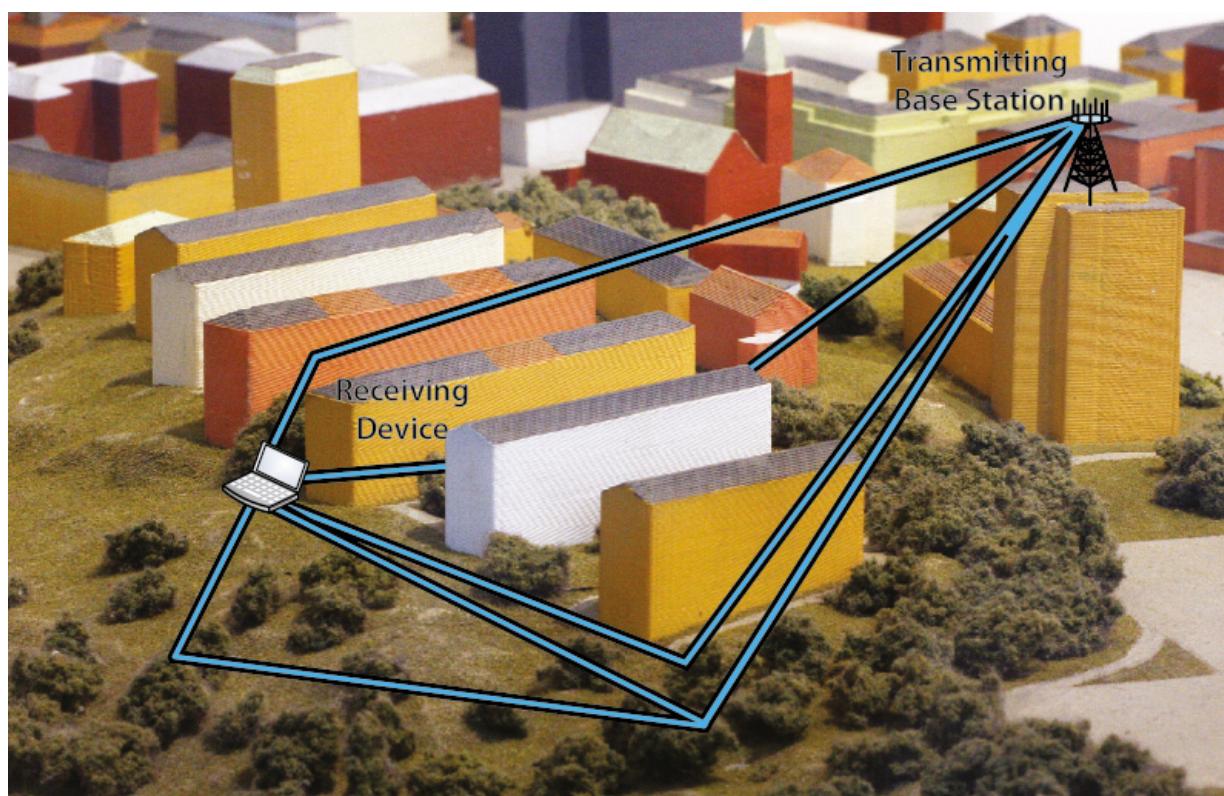


**Known as**  
Multi-user MIMO  
(massive if very  
many antennas)

# Recall: Precoding in Line-of-Sight and Non-Line-of-Sight

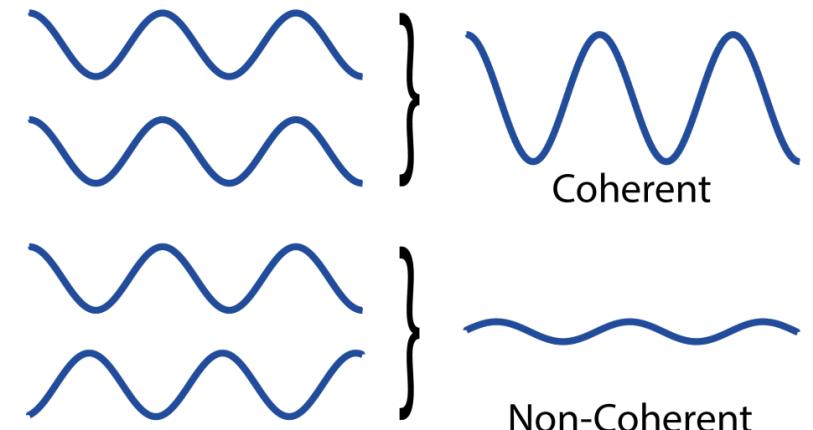
- Line-of-Sight

- Adapt signal phases at antennas
- Steer beam towards receiving user
- Not laser beams: inter-user interference



- Non-Line-of-Sight

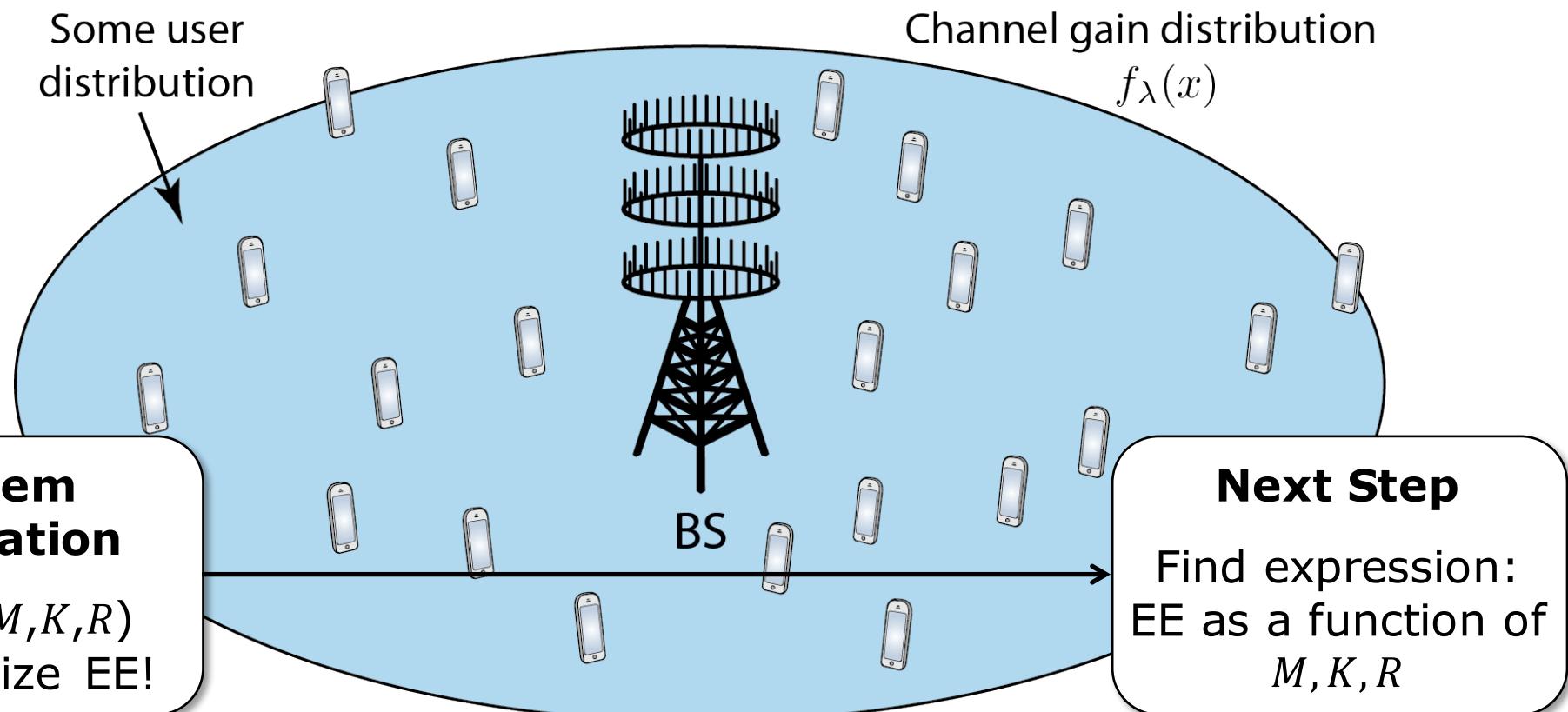
- Multipath propagation
- Add components coherently



# Single-Cell: Optimizing for Energy-Efficiency

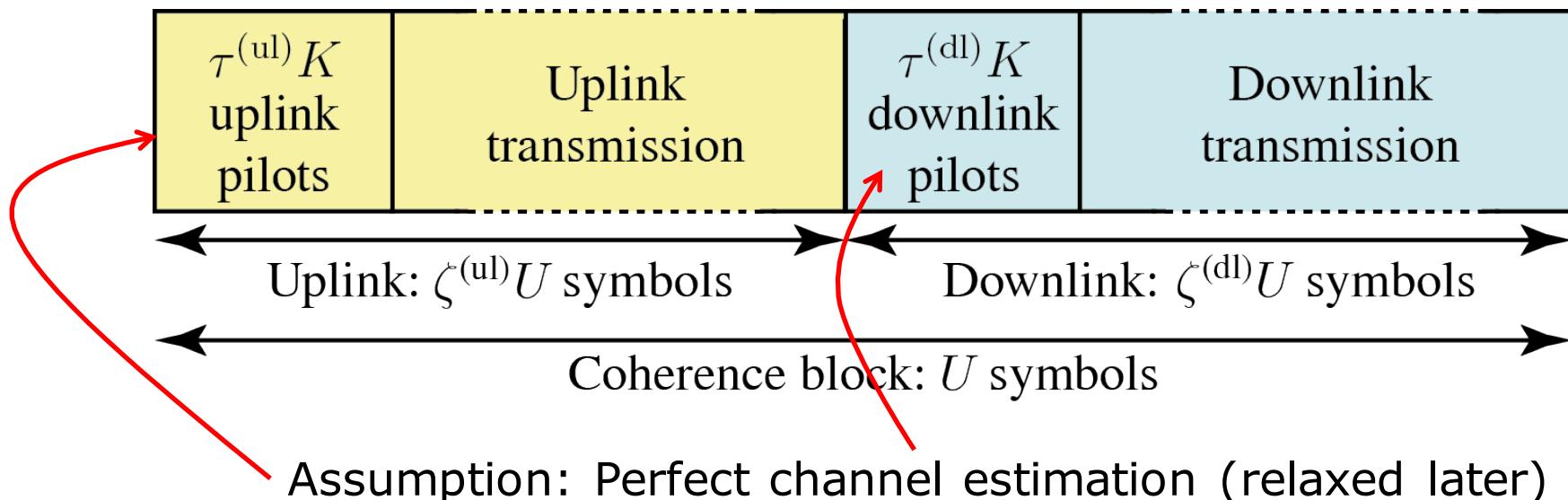
- Clean Slate Design

- Single Cell: One base station (BS) with  $M$  antennas
- Geometry: Random distribution for user locations and pathlosses
- Multiple users: Pick  $K$  users randomly and serve with some rate  $R$



# System Model: Protocol

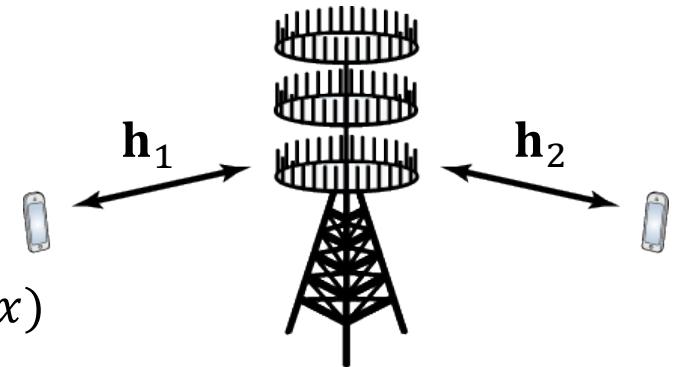
- Time-Division Duplex (TDD) Protocol
  - Uplink and downlink separated in time
  - Uplink fraction  $\zeta^{(\text{ul})}$  and downlink fraction  $\zeta^{(\text{dl})}$
- Symbol Time and Coherence Block
  - $B$  Hz bandwidth =  $B$  “channel uses” per second (symbol time  $1/B$ )
  - Channel stays fixed for  $U$  channel uses (symbols) = Coherence block
  - Determines how often we send pilot signals to estimate channels



# System Model: Uplink Channels

- Flat-Fading Channels

- Channel between BS and User  $k$ :  $\mathbf{h}_k \in \mathbb{C}^M$
- Rayleigh fading:  $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \lambda_k \mathbf{I})$
- Channel variances  $\lambda_k$ : Random variables, pdf  $f_\lambda(x)$



- Uplink Transmission

- User  $k$  transmits signal  $s_k$  with power  $\mathbb{E}\{|s_k|^2\} = p_k^{(\text{ul})}$  [Joule/channel use]
- Received signal at BS:

$$\mathbf{y} = \mathbf{h}_k s_k + \sum_{i=1, i \neq k}^K \mathbf{h}_i s_i + \mathbf{n}$$

Signal of User  $k$

Signals from other users  
(interference)

Noise  $\sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$

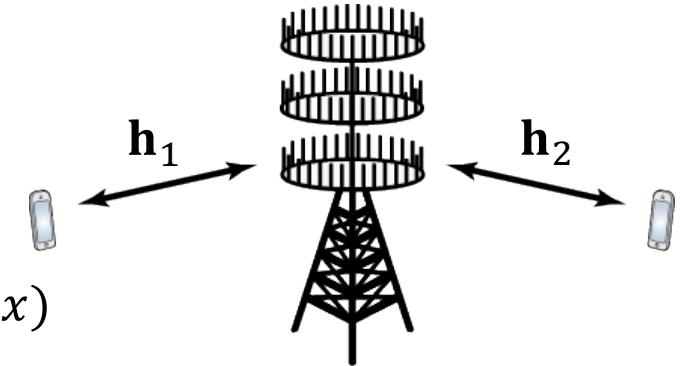
- Recover  $s_k$  by receive combining  $\mathbf{g}_k$  as  $\mathbf{g}_k^H \mathbf{y}$ :

$$\text{SINR}_k^{(\text{ul})} = \frac{\mathbb{E}\{|s_k|^2 |\mathbf{g}_k^H \mathbf{h}_k|^2\}}{\sum_{i \neq k} \mathbb{E}\{|s_i|^2 |\mathbf{g}_k^H \mathbf{h}_i|^2\} + \mathbb{E}\{|\mathbf{g}_k^H \mathbf{n}|^2\}} = \frac{p_k^{(\text{ul})} |\mathbf{g}_k^H \mathbf{h}_k|^2}{\sum_{i \neq k} p_i^{(\text{ul})} |\mathbf{g}_k^H \mathbf{h}_i|^2 + \sigma^2 \|\mathbf{g}_k\|^2}$$

# System Model: Downlink Channels

- Flat-Fading Channels

- Channel between BS and User  $k$ :  $\mathbf{h}_k \in \mathbb{C}^M$
- Rayleigh fading:  $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \lambda_k \mathbf{I})$
- Channel variances  $\lambda_k$ : Random variables, pdf  $f_\lambda(x)$



- Downlink Transmission

- BS transmits  $d_k$  to User  $k$  with power  $\mathbb{E}\{|d_k|^2\} = p_k^{(dl)}$  [Joule/channel use]
- Spatial directivity by precoding vector  $\mathbf{v}_k$
- Received signal at User  $k$ :

$$y_k = \mathbf{h}_k^H \frac{\mathbf{v}_k}{\|\mathbf{v}_k\|} d_k + \sum_{i=1, i \neq k}^K \mathbf{h}_k^H \frac{\mathbf{v}_i}{\|\mathbf{v}_i\|} d_i + n_k$$

Signal to User  $k$

Signals from other users  
(interference)

Noise  $\sim \mathcal{CN}(0, \sigma^2)$

- Recover  $d_k$  at User  $k$ :

$$\text{SINR}_k^{(dl)} = \frac{p_k^{(dl)} |\mathbf{h}_k^H \mathbf{v}_k|^2 / \|\mathbf{v}_k\|^2}{\sum_{i \neq k} p_i^{(dl)} |\mathbf{h}_k^H \mathbf{v}_i|^2 / \|\mathbf{v}_i\|^2 + \sigma^2}$$

# System Model: How Much Transmit Power?

- Design Parameter: *Gross rate R*

- Make sure that  $R = \begin{cases} B \log_2(1 + \text{SINR}_k^{(\text{ul})}) & \text{for all } k \text{ in uplink} \\ B \log_2(1 + \text{SINR}_k^{(\text{dl})}) & \text{for all } k \text{ in downlink} \end{cases}$
- Select processing vectors  $\mathbf{g}_k$  and  $\mathbf{v}_k$ , adapt transmit power  $p_k^{(\text{ul})}$  and  $p_k^{(\text{dl})}$

- Gives  $K$  equations:

$$\begin{cases} p_k^{(\text{ul})} |\mathbf{g}_k^H \mathbf{h}_k|^2 = (2^{R/B} - 1) (\sum_{i \neq k} p_i^{(\text{ul})} |\mathbf{g}_k^H \mathbf{h}_i|^2 + \sigma^2 \|\mathbf{g}_k\|^2) & \text{for } k = 1, \dots, K \\ p_k^{(\text{dl})} \frac{|\mathbf{h}_k^H \mathbf{v}_k|^2}{\|\mathbf{v}_k\|^2} = (2^{R/B} - 1) (\sum_{i \neq k} p_i^{(\text{dl})} \frac{|\mathbf{h}_k^H \mathbf{v}_i|^2}{\|\mathbf{v}_i\|^2} + \sigma^2) & \text{for } k = 1, \dots, K \end{cases}$$

- Linear equations in transmit powers → Solve by Gaussian elimination!

**Total Transmit Power** [Joule/s] for  $\mathbf{g}_k = \mathbf{v}_k$

Uplink energy/symbol:  $\sigma^2 \mathbf{D}^{-H} \mathbf{1}$

Downlink energy/symbol:  $\sigma^2 \mathbf{D}^{-1} \mathbf{1}$

Same total power:  $P_{\text{trans}} = B \mathbb{E}\{\sigma^2 \mathbf{1}^H \mathbf{D}^{-1} \mathbf{1}\} = B \mathbb{E}\{\sigma^2 \mathbf{1}^H \mathbf{D}^{-H} \mathbf{1}\}$

where  $[\mathbf{D}]_{k,l} = \begin{cases} \frac{|\mathbf{h}_k^H \mathbf{v}_k|^2}{(2^{R/B} - 1) \|\mathbf{v}_k\|^2} & \text{for } k = l \\ -\frac{|\mathbf{h}_k^H \mathbf{v}_l|^2}{\|\mathbf{v}_l\|^2} & \text{for } k \neq l \end{cases}$

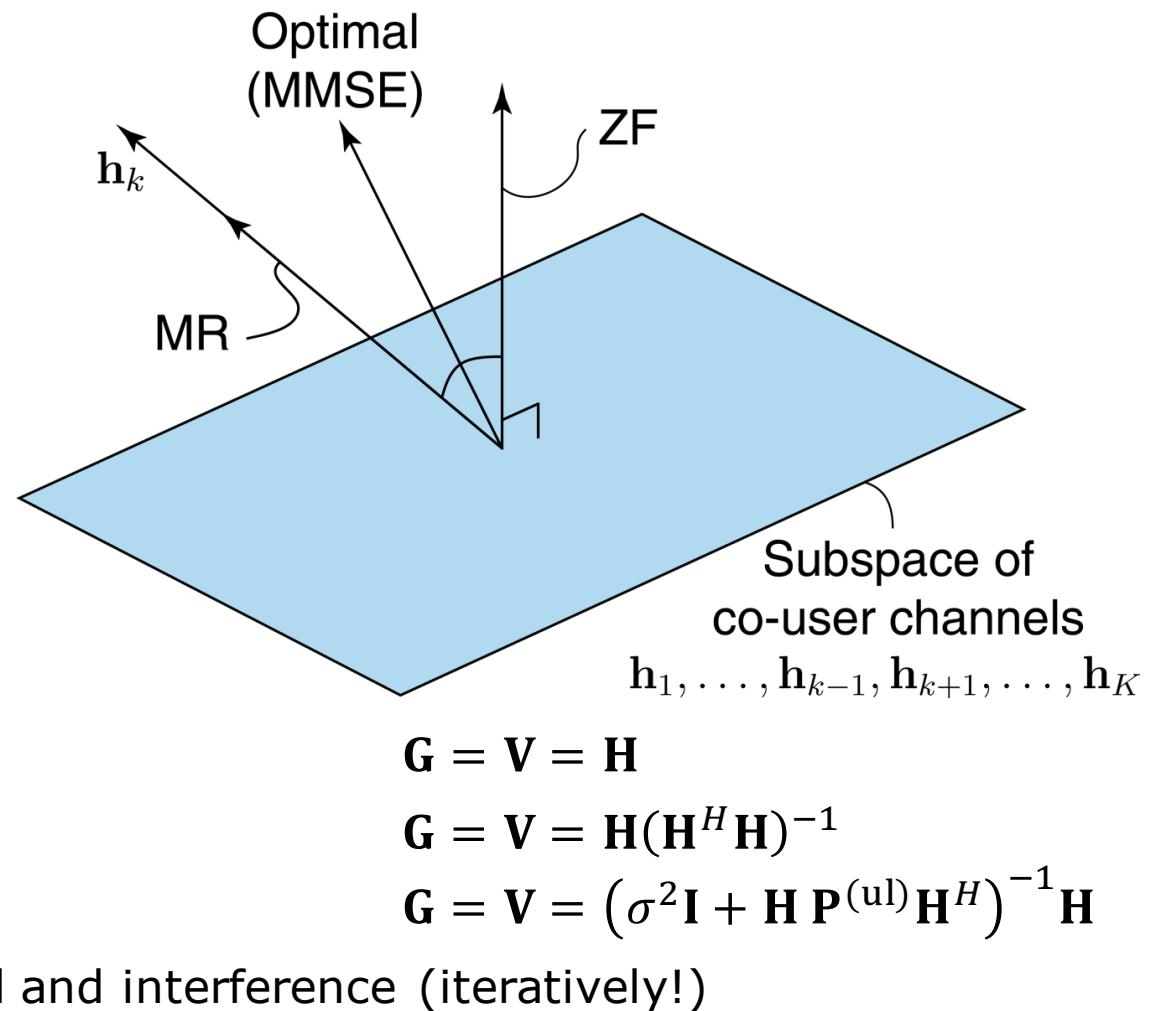
# System Model: How Much Transmit Power? (2)

- What did we Derive?
  - Optimal power allocation for fixed linear processing vectors

- Different Linear Processing

- Notation:  $\mathbf{G} = [\mathbf{g}_1, \dots, \mathbf{g}_K]$   
 $\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_K]$ ,  
 $\mathbf{H} = [\mathbf{h}_1, \dots, \mathbf{h}_K]$ ,  
 $\mathbf{P}^{(\text{ul})} = \text{diag}(p_1^{(\text{ul})}, \dots, p_K^{(\text{ul})})$

- Minimize interference      Maximize signal
- Maximum ratio (MR):
  - Zero-forcing (ZF):
  - Optimal (MMSE):
- Balance signal and interference (iteratively!)



# System Model: How Much Transmit Power? (3)

- Simplified Expressions for ZF ( $M \geq K + 1$ )

- Main property:  $\mathbf{H}^H \mathbf{V} = \mathbf{H}^H \mathbf{H} (\mathbf{H}^H \mathbf{H})^{-1} = \mathbf{I}$

- Hence:  $[\mathbf{D}]_{k,l} = \begin{cases} \frac{|\mathbf{h}_k^H \mathbf{v}_k|^2}{(2^{R/B} - 1) \|\mathbf{v}_k\|^2} & \text{for } k = l \\ -\frac{|\mathbf{h}_k^H \mathbf{v}_l|^2}{\|\mathbf{v}_l\|^2} & \text{for } k \neq l \end{cases} = \begin{cases} \frac{1}{(2^{R/B} - 1) \|\mathbf{v}_k\|^2} & \text{for } k = l \\ 0 & \text{for } k \neq l \end{cases}$

Property  
of Wishart  
matrices

- Total transmit power:

$$\begin{aligned} P_{\text{trans}} &= \mathbb{E}\{B\sigma^2 \mathbf{1}^H \mathbf{D}^{-1} \mathbf{1}\} = B\sigma^2 (2^{R/B} - 1) \underbrace{\sum_k \mathbb{E}\{\|\mathbf{v}_k\|^2\}}_{= \text{tr}\left(\left(\mathbf{H}^H \mathbf{H}\right)^{-1}\right)} = B\sigma^2 (2^{R/B} - 1) \frac{K}{M-K} \underbrace{\mathbb{E}\left\{\frac{1}{\lambda}\right\}}_{\text{Call this } \mathcal{S}_\lambda \text{ (depends on cell)}} \end{aligned}$$

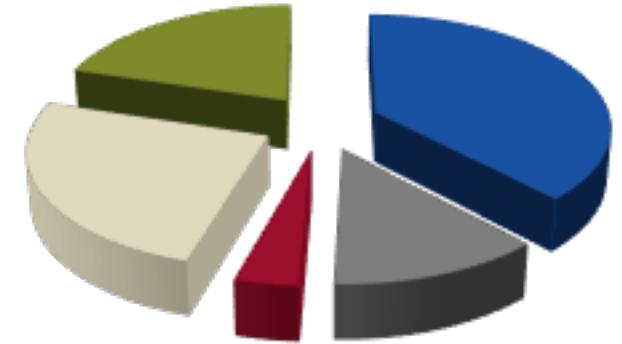
## Summary: Transmit Power with ZF

Parameterize gross rate as  $R = B \log_2(1 + \rho(M - K))$  for some  $\rho$

Total transmit power:  $P_{\text{trans}} = \rho B \sigma^2 \mathcal{S}_\lambda K$  [Joule/s]

# Detailed Power Consumption Model

- What Consumes Power?
  - Not only radiated transmit power
  - Circuits, signal processing, backhaul, etc.
  - Must be specified as functions of  $M, K, R$



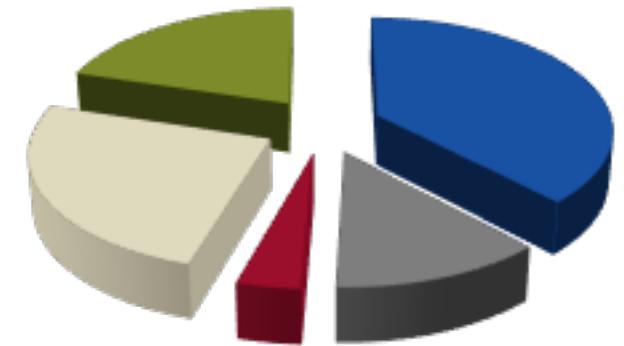
- Power Amplifiers
    - Amplifier efficiencies:  $\eta^{(\text{ul})}, \eta^{(\text{dl})} \in (0,1]$
    - Average inefficiency:  $\frac{\zeta^{(\text{ul})}}{\eta^{(\text{ul})}} + \frac{\zeta^{(\text{dl})}}{\eta^{(\text{dl})}} = \frac{1}{\eta}$
- Summary:  $\frac{P_{\text{trans}}}{\eta}$

- Active Transceiver Chains
  - $P_{\text{FIX}}$  = Fixed power (control signals, oscillator at BS, standby, etc.)
  - $P_{\text{BS}}$  = Circuit power / BS antenna (converters, mixers, filters)
  - $P_{\text{UE}}$  = Circuit power / user (oscillator, converters, mixer, filters)

Summary:  $P_{\text{FIX}} + M \cdot P_{\text{BS}} + K \cdot P_{\text{UE}}$

# Detailed Power Consumption Model (2)

- Signal Processing
  - Channel estimation and precoding/combining
  - Efficiency:  $L_{\text{BS}}, L_{\text{UE}}$  arithmetic operations / Joule



- Channel Estimation:  $\frac{B}{U} \left( \frac{2\tau^{(\text{ul})} MK^2}{L_{\text{BS}}} + \frac{4\tau^{(\text{dl})} K^2}{L_{\text{UE}}} \right)$ 
  - Once in uplink/downlink per coherence block
  - Pilot signal lengths:  $\tau^{(\text{ul})} K, \tau^{(\text{dl})} K$  for some  $\tau^{(\text{ul})}, \tau^{(\text{dl})} \geq 1$
- Linear Processing (for  $\mathbf{G} = \mathbf{V}$ ):  $\frac{B}{U} \frac{C_{\text{precoding}}}{L_{\text{BS}}} + B \left( 1 - \frac{(\tau^{(\text{ul})} + \tau^{(\text{dl})})K}{U} \right) \frac{2MK}{L_{\text{BS}}}$ 
  - Compute precoding/combining vectors once per coherence block
  - Use same vectors for all  $B(1 - (\tau^{(\text{ul})} + \tau^{(\text{dl})})K/U)$  symbols

- Types of precoding:

$$C_{\text{precoding}} = \begin{cases} 3MK & \text{for MR} \\ 3MK^2 + MK + K^3/3 & \text{for ZF} \\ Q(3MK^2 + MK + K^3/3) & \text{for MMSE} \end{cases}$$

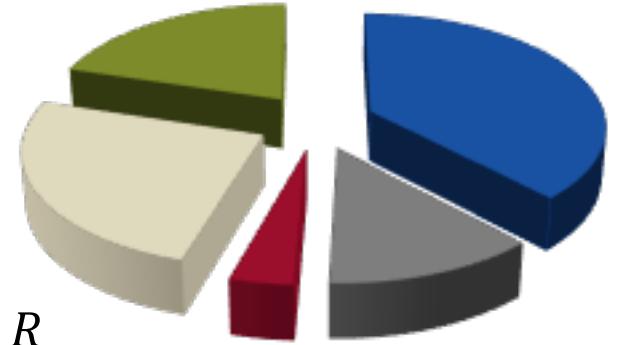
$Q = \text{Number of iterations}$

# Detailed Power Consumption Model (3)

- Coding and Decoding:  $R_{\text{sum}}(P_{\text{COD}} + P_{\text{DEC}})$

- $P_{\text{COD}}$  = Energy for coding data / bit
  - $P_{\text{DEC}}$  = Energy for decoding data / bit

- Sum rate: 
$$R_{\text{sum}} = K \left( \zeta^{(\text{ul})} - \frac{\tau^{(\text{ul})} K}{U} \right) R + K \left( \zeta^{(\text{dl})} - \frac{\tau^{(\text{dl})} K}{U} \right) R$$
$$= K \left( 1 - \frac{(\tau^{(\text{ul})} + \tau^{(\text{dl})})K}{U} \right) R$$

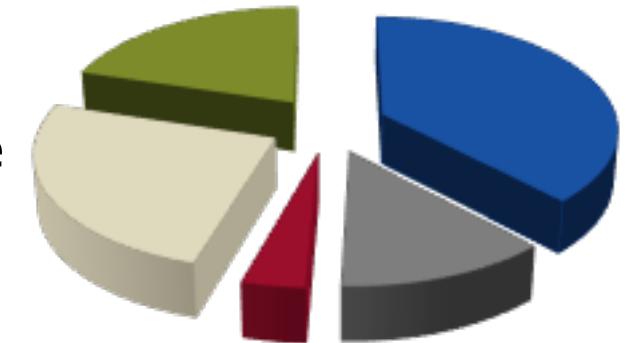


- Backhaul Signaling:  $P_{\text{BH}} + R_{\text{sum}} P_{\text{BT}}$

- $P_{\text{BH}}$  = Load-independent backhaul power
  - $P_{\text{BT}}$  = Energy for sending data over backhaul / bit

# Detailed Power Consumption Model: Summary

- Many Things Consume Power
  - Parameter values (e.g.,  $P_{BS}, P_{UE}$ ) change over time
  - Structure is important for analysis



**A Generic Power Model**

$$\frac{P_{\text{trans}}}{\eta} + C_{0,0} + C_{0,1}M + C_{1,0}K + C_{1,1}MK + C_{2,0}K^2 + C_{3,0}K^3 + C_{2,1}MK^2 + AK \left(1 - \frac{(\tau^{(\text{ul})} + \tau^{(\text{dl})})K}{U}\right)R$$

Fixed power

Transmit with amplifiers

Circuit power per transceiver chain

Cost of signal processing

Coding/decoding/backhaul

for some parameters  $C_{l,m}$  and  $A$

- Observations
  - Polynomial in  $M$  and  $K \rightarrow$  Increases faster than linear with  $K$
  - Depends on cell geometry only through  $P_{\text{trans}}$

# Finally: Problem Formulation

- Maximize Energy-Efficiency:

$$\underset{M, K, R}{\text{maximize}} \frac{\frac{K}{U} \left(1 - \frac{(\tau^{(\text{ul})} + \tau^{(\text{dl})})K}{U}\right)R}{\frac{P_{\text{trans}}}{\eta} + \sum_{i=0}^3 C_{i,0}K^i + \sum_{i=0}^2 C_{i,1}MK^i + AK \left(1 - \frac{(\tau^{(\text{ul})} + \tau^{(\text{dl})})K}{U}\right)R}$$

Power Consumption [Joule/s/cell]

Average Sum Rate [bit/s/cell]

## Closed Form Expressions with ZF – Part 3

Recall:  $R = B \log_2(1 + \rho(M - K))$  for some  $\rho$  and  $P_{\text{trans}} = \rho B \sigma^2 S_\lambda K$

Define:  $\tau = \tau^{(\text{ul})} + \tau^{(\text{dl})}$

$$\underset{M, K, \rho}{\text{maximize}} \frac{\frac{K}{U} \left(1 - \frac{\tau K}{U}\right) B \log_2(1 + \rho(M - K))}{\frac{\rho B \sigma^2 S_\lambda K}{\eta} + \sum_{i=0}^3 C_{i,0}K^i + \sum_{i=0}^2 C_{i,1}MK^i + AK \left(1 - \frac{\tau K}{U}\right) B \log_2(1 + \rho(M - K))}$$

**Simple ZF expression:** Used for analysis, other schemes by simulation

# Why Such a Detailed/Complicated Model?

- Simplified Model → Unreliable Optimization Results
  - Two examples based on ZF
  - Beware: Both have appeared in the literature!
- Example 1: Fixed circuit power and no coding/decoding/backhaul

$$\underset{M, K, \rho}{\text{maximize}} \frac{\frac{K \left(1 - \frac{\tau K}{U}\right) B \log_2(1 + \rho(M - K))}{\eta + C_{0,0}}}{\frac{\rho B \sigma^2 S_\lambda K}{\eta} + C_{0,0}}$$

- If  $M \rightarrow \infty$ , then  $\log_2(1 + \rho(M - K)) \rightarrow \infty$  and thus EE  $\rightarrow \infty$ !

- Example 2: Ignore pilot overhead and signal processing

$$\underset{M, K, \rho}{\text{maximize}} \frac{\frac{KB \log_2(1 + \rho(M - K))}{\eta + C_{0,0} + C_{1,0}K + C_{0,1}M}}{\frac{\rho B \sigma^2 S_\lambda K}{\eta} + \frac{C_{0,0}}{K} + C_{1,0} + C_{0,1} \frac{M}{K}} = \frac{B \log_2(1 + \rho K(\frac{M}{K} - 1))}{\frac{\rho B \sigma^2 S_\lambda}{\eta} + \frac{C_{0,0}}{K} + C_{1,0} + C_{0,1} \frac{M}{K}}$$

- If  $M, K \rightarrow \infty$  with  $\frac{M}{K} = \text{constant} > 1$ , then  $\log_2(1 + \rho K(\frac{M}{K} - 1)) \rightarrow \infty$  and EE  $\rightarrow \infty$ !

# Summary

- Modeling of the EE metric
  - EE is a rate/power ratio → Sensitive to modeling inaccuracies
  - Rate: Overhead signaling must be included
  - Power: Both transmit and circuit power are important
- Main Result: General EE Maximization Formulation

$$\underset{M, K, R}{\text{maximize}} \quad \frac{K \left( 1 - \frac{(\tau^{(\text{ul})} + \tau^{(\text{dl})})K}{U} \right) R}{\frac{P_{\text{trans}}}{\eta} + \sum_{i=0}^3 C_{i,0} K^i + \sum_{i=0}^2 C_{i,1} M K^i + A K \left( 1 - \frac{(\tau^{(\text{ul})} + \tau^{(\text{dl})})K}{U} \right) R}$$

- Solvable numerically for any processing scheme (since  $M, K$  integers)
- Numerical observations might not hold for other parameter values
- Analytic solutions: Reveal fundamental behaviors and tradeoffs!
- **Part 3: Analytic solution with ZF, numerical for other schemes**

**Part 2**

# **Questions?**

# **Part 3**

# **Optimization of Energy-Efficiency: Fixed Regular Deployment**

# Preliminaries

- Our Goal

- Optimize number of antennas  $M$
- Optimize number of active users  $K$
- Optimize the (normalized) transmit power  $\rho$

} For ZF processing:  
**Analytically solvable!**

- Outline

- Optimize each variable separately
- Devise an alternating optimization algorithm

## Definition (Lambert $W$ function)

- Lambert  $W$  function,  $W(x)$ , solves equation  $W(x)e^{W(x)} = x$
- The function is increasing and satisfies  $W(0) = 0$
- $e^{W(x)}$  behaves as a linear function (i.e.,  $e^{W(x)} \approx x$ ):

$$\frac{x e}{\log_e(x)} \leq e^{W(x)+1} \leq \frac{x}{\log_e(x)}(1 + e) \quad \text{for } x \geq e.$$

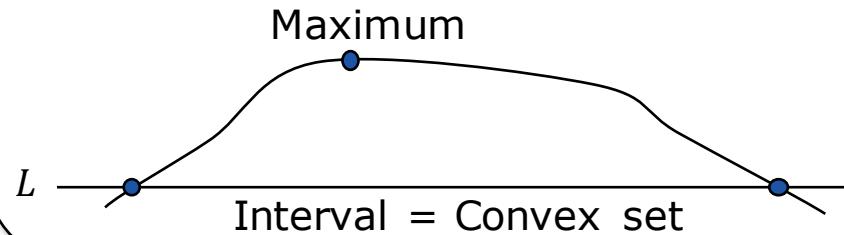
# Solving Optimization Problems

- How to Solve an Optimization Problem?
  - Simple if the function is “nice” → “Looks like a mountain”

## Quasi-Concave Function $\varphi(x)$

For any value  $L$ , the  $x$  that gives  $\varphi(x) \geq L$  is a convex set:

For any  $x_1, x_2$  with  $\varphi(x_1) = \varphi(x_2)$ , line between  $x_1$  and  $x_2$  is below the graph



Property: Goes up and then down

Examples:  $-x^2, \log(x), [x]$

- Maximization of Continuous Quasi-Concave Function  $\varphi(x)$ :
  1. Compute the first derivative  $\frac{d}{dx} \varphi(x)$
  2. Find switching point by setting  $\frac{d}{dx} \varphi(x) = 0$
  3. Only one solution → It is the unique maximum!

# Optimal Number of BS Antennas

- Find  $M$  that maximizes EE with ZF (recall  $M > K$ ):

$$\begin{aligned} \text{maximize } & \frac{K \left(1 - \frac{\tau K}{U}\right) B \log_2(1 + \rho(M - K))}{M \geq K + 1 \frac{\rho B \sigma^2 S_\lambda K}{\eta} + \sum_{i=0}^3 C_{i,0} K^i + \sum_{i=0}^2 C_{i,1} M K^i + A K \left(1 - \frac{\tau K}{U}\right) B \log_2(1 + \rho(M - K))} \end{aligned}$$

**Theorem 1** (Optimal  $M$ )

EE is quasi-concave w.r.t.  $M$  and maximized by

$$M^* = \frac{e^{W\left(\frac{\rho(B\sigma^2S_\lambda K/\eta + \sum_{i=0}^3 C_{i,0} K^i)}{e \sum_{i=0}^2 C_{i,1} K^i} + \frac{\rho K - 1}{e}\right) + 1}} + \rho K - 1}{\rho}$$

- Observations

- Increases with circuit coefficients independent of  $M$  (e.g.,  $P_{\text{FIX}}, P_{\text{UE}}$ )
- Decreases with circuit coefficients multiplied with  $M$  (e.g.,  $P_{\text{BS}}, 1/L_{\text{BS}}$ )
- Independent of cost of coding/decoding/backhaul
- Increases with power  $\rho$  approx. as  $\frac{\rho}{\log \rho}$  (almost linear)

# Optimal Transmit Power

- Find  $\rho$  that maximizes EE with ZF:

$$\begin{aligned} & \text{maximize}_{\rho \geq 0} \quad K \left(1 - \frac{\tau K}{U}\right) B \log_2 (1 + \rho(M - K)) \\ & \quad \frac{\rho B \sigma^2 S_\lambda K}{\eta} + \sum_{i=0}^3 C_{i,0} K^i + \sum_{i=0}^2 C_{i,1} M K^i + A K \left(1 - \frac{\tau K}{U}\right) B \log_2 (1 + \rho(M - K)) \end{aligned}$$

**Theorem 2** (Optimal  $\rho$ )

EE is quasi-concave w.r.t.  $\rho$  and maximized by

$$\rho^* = \frac{e^{W\left(\frac{\eta}{B\sigma^2 S_\lambda} \frac{(M-K)(\sum_{i=0}^3 C_{i,0} K^i + \sum_{i=0}^2 C_{i,1} M K^i)}{e} - \frac{1}{e}\right) + 1} - 1}{M - K}$$

- Observations

- Increases with all circuit coefficients (e.g.,  $P_{\text{FIX}}$ ,  $P_{\text{BS}}$ ,  $P_{\text{UE}}$ ,  $1/L_{\text{BS}}$ )
- Independent of cost of coding/decoding/backhaul
- Increases with  $M$  approx. as  $\frac{M}{\log M}$  (almost linear)

More circuit power →  
More transmit power

Power/antenna  
decreases  $\frac{1}{\log M}$

# Optimal Number of Users

- Find  $K$  that maximizes EE with ZF:

$$\begin{aligned} \text{maximize}_{K \geq 0} \quad & K \left( 1 - \frac{\tau K}{U} \right) B \log_2 (1 + \bar{\rho}(\bar{\beta} - 1)) \\ & \frac{\bar{\rho}B\sigma^2\mathcal{S}_\lambda}{\eta} + \sum_{i=0}^3 C_{i,0}K^i + \sum_{i=0}^2 C_{i,1}\bar{\beta}K^{i+1} + AK \left( 1 - \frac{\tau K}{U} \right) B \log_2 (1 + \bar{\rho}(\bar{\beta} - 1)) \end{aligned}$$

where  $\bar{\rho} = \rho K$  and  $\bar{\beta} = \frac{M}{K}$  are fixed

**Theorem 3** (Optimal  $K$ )

EE is quasi-concave w.r.t.  $K$

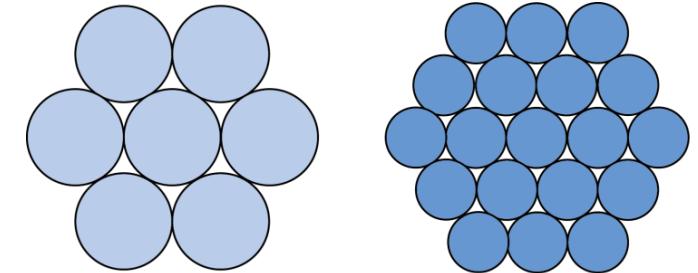
Maximized by the root of a quartic polynomial:  
Closed form for  $K^*$  but very “large” expressions

- Observations

- Increases with fixed circuit power (e.g.,  $P_{\text{FIX}}$ )
- Decreases with circuit coefficients multiplied with  $M$  or  $K$  ( $P_{\text{BS}}, P_{\text{UE}}, 1/L_{\text{BS}}$ )

# Impact of Cell Size

- Are Smaller Cells More Energy Efficient?
  - Recall:  $\mathcal{S}_\lambda = \mathbb{E} \left\{ \frac{1}{\lambda} \right\}$
  - Smaller cells  $\rightarrow \lambda$  is larger  $\rightarrow \mathcal{S}_\lambda$  is smaller
- For any given parameters  $M, \rho, K$ 
  - Smaller  $\mathcal{S}_\lambda \rightarrow$  smaller transmit power  $\rho B \sigma^2 \mathcal{S}_\lambda K$
  - Higher EE! Yes, small cells are better!
- Expressions for  $M^*, \rho^*, K^*$ 
  - $M^*$  and  $K^*$  increases with  $\mathcal{S}_\lambda$
  - $\rho^*$  decreases with  $\mathcal{S}_\lambda$



Smaller cells:  
Less hardware and fewer users per cell  
Use shorter distances to reduce power

## Dependence on Other Parameters

Many other observations can be made from the expressions  
Example: Impact of bandwidth  $B$ , coherence block length  $U$ , etc.

# Alternating Optimization Algorithm

- Joint EE Optimization
  - EE is a function of  $M$ ,  $\rho$ , and  $K$
  - Theorems 1-3 optimize one parameter, when the other two are fixed
  - Can we optimize all of them?

## **Algorithm:** Alternating Optimization

1. Assume that an initial set  $(M, \rho, K)$  is given
2. Update number of users  $K$  (and implicitly  $M$  and  $\rho$ ) using Theorem 3
3. Update number of antennas  $M$  using Theorem 1
4. Update transmit power ( $\rho$ ) using Theorem 2
5. Repeat 2.-5. until convergence

## **Theorem 4**

The algorithm converges to a local optimum to the joint EE optimization problem

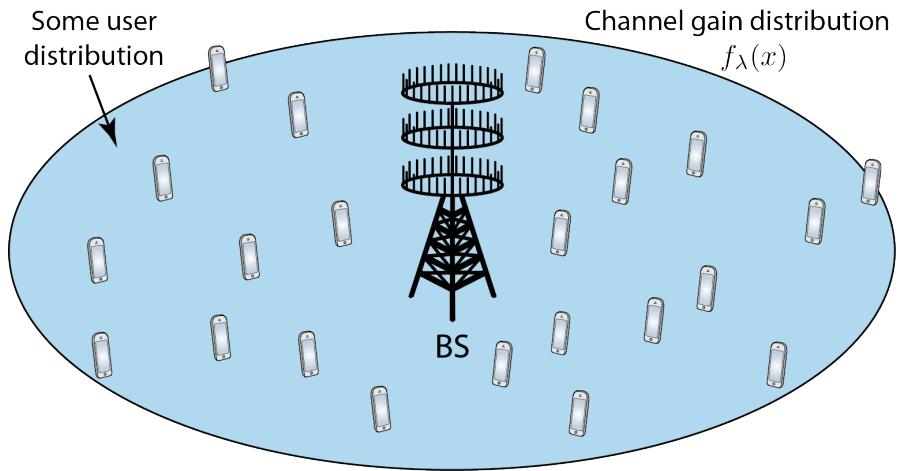
## **Disclaimer**

$M$  and  $K$  should be integers  
Theorems 1 and 3 give real numbers → Take one of the closest integer points

# Single-Cell Simulation Scenario

- Main Characteristics

- Circular cell with radius 250 m
- Uniform user distribution
- Uncorrelated Rayleigh fading
- Typical 3GPP pathloss model



- Many Parameters in the System Model

- We found numbers from  $\approx 2012$  in the literature:

Parameter	Value	Parameter	Value
Cell radius (single-cell): $d_{\max}$	250 m	Fraction of downlink transmission: $\zeta^{(\text{dl})}$	0.6
Minimum distance: $d_{\min}$	35 m	Fraction of uplink transmission: $\zeta^{(\text{ul})}$	0.4
Large-scale fading model: $l(\mathbf{x})$	$10^{-3.53} / \ \mathbf{x}\ ^{3.76}$	PA efficiency at the BSs: $\eta^{(\text{dl})}$	0.39
Transmission bandwidth: $B$	20 MHz	PA efficiency at the UEs: $\eta^{(\text{ul})}$	0.3
Channel coherence bandwidth: $B_C$	180 kHz	Fixed power consumption (control signals, backhaul, etc.): $P_{\text{FIX}}$	18 W
Channel coherence time: $T_C$	10 ms	Power consumed by local oscillator at BSs: $P_{\text{SYN}}$	2 W
Coherence block (channel uses): $U$	1800	Power required to run the circuit components at a BS: $P_{\text{BS}}$	1 W
Total noise power: $B\sigma^2$	-96 dBm	Power required to run the circuit components at a UE: $P_{\text{UE}}$	0.1 W
Relative pilot lengths: $\tau^{(\text{ul})}, \tau^{(\text{dl})}$	1	Power required for coding of data signals: $P_{\text{COD}}$	0.1 W/(Gbit/s)
Computational efficiency at BSs: $L_{\text{BS}}$	12.8 Gflops/W	Power required for decoding of data signals: $P_{\text{DEC}}$	0.8 W/(Gbit/s)
Computational efficiency at UEs: $L_{\text{UE}}$	5 Gflops/W	Power required for backhaul traffic: $P_{\text{BT}}$	0.25 W/(Gbit/s)

# Optimal Single-Cell System Design: ZF Precoding

## Optimum

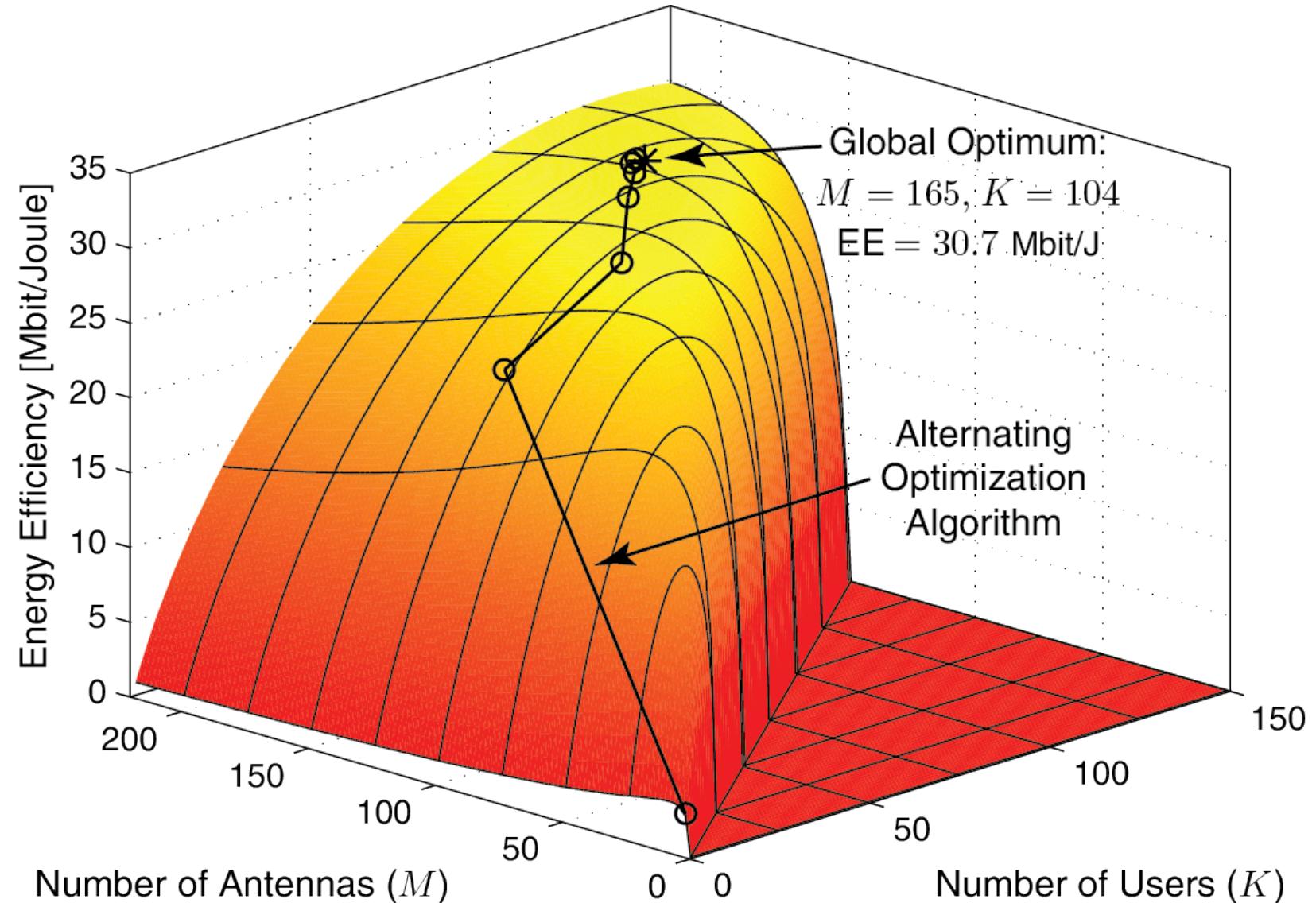
$$\begin{aligned}M &= 165 \\K &= 104 \\\rho &= 0.87\end{aligned}$$

User rates:  
 $\approx$ 64-QAM

## Massive MIMO!

Many users  
and antennas

$M \gg K$   
suppresses  
interference



# Optimal Single-Cell System Design: “Optimal” Processing

## Optimum

$$M = 145$$

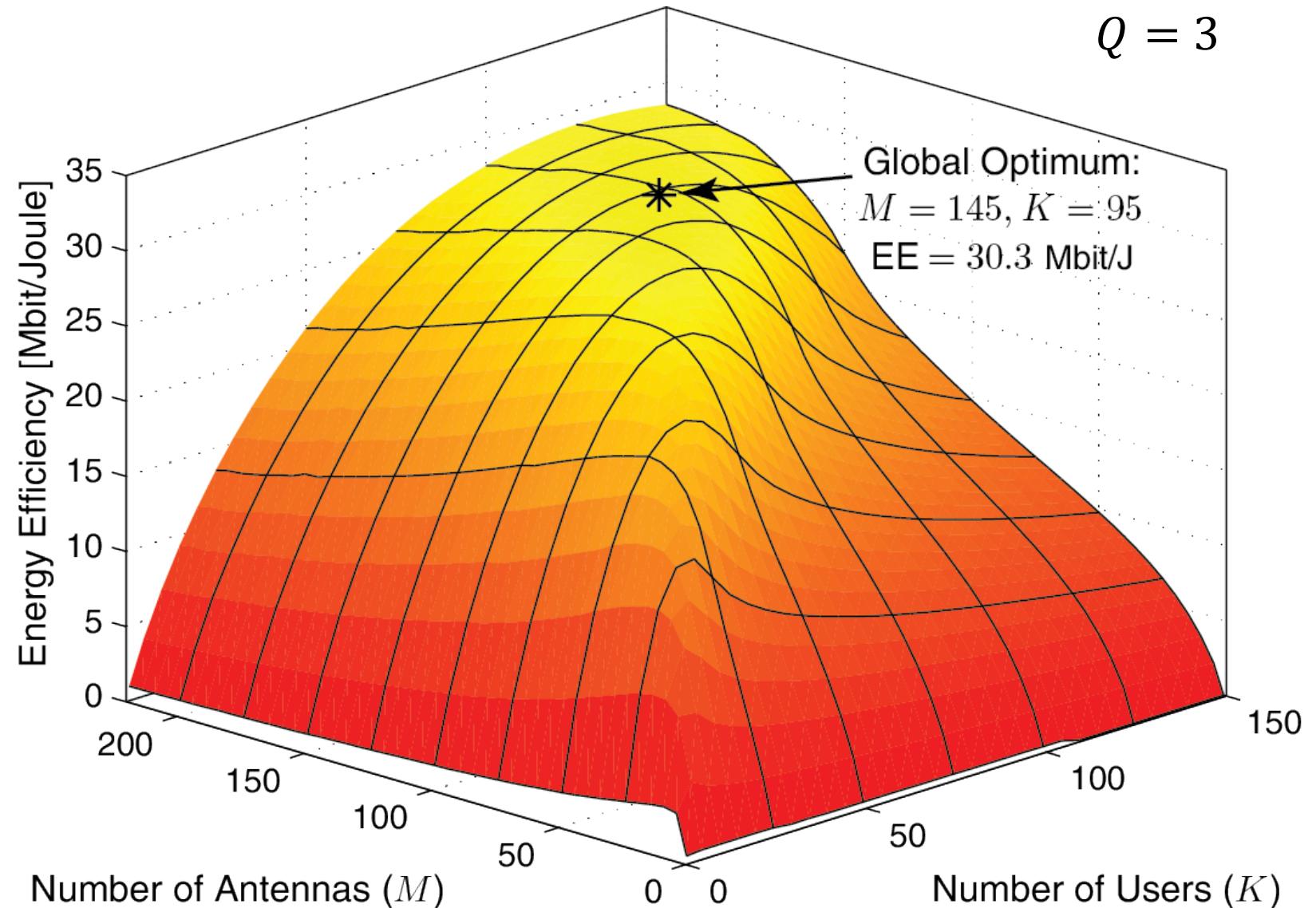
$$K = 95$$

$$\rho = 0.91$$

User rates:  
 $\approx$ 64-QAM

## MMSE is Not Optimal!

Optimal linear  
processing  
but too costly  
computations



# Optimal Single-Cell System Design: MR Processing

## Optimum

$$M = 81$$

$$K = 77$$

$$\rho = 0.24$$

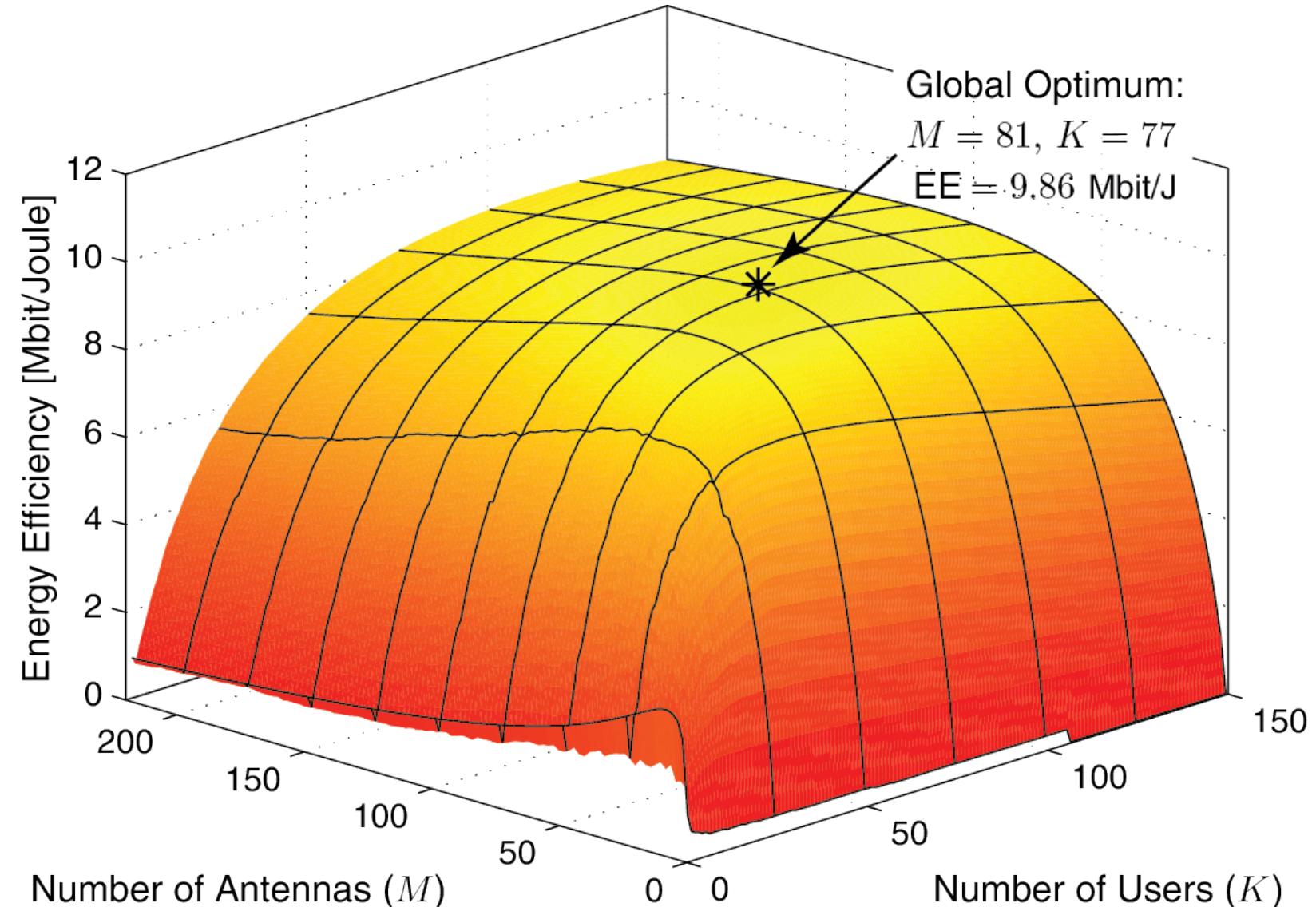
User rates:  
 $\approx$ 2-PSK

## Observation

Lower EE  
than with ZF

Also Massive  
MIMO setup

Low rates



# Optimal Single-Cell System Design: MR Processing (2)

**Optimum:**  
Single-user  
transmission

$$M = 4$$
$$K = 1$$

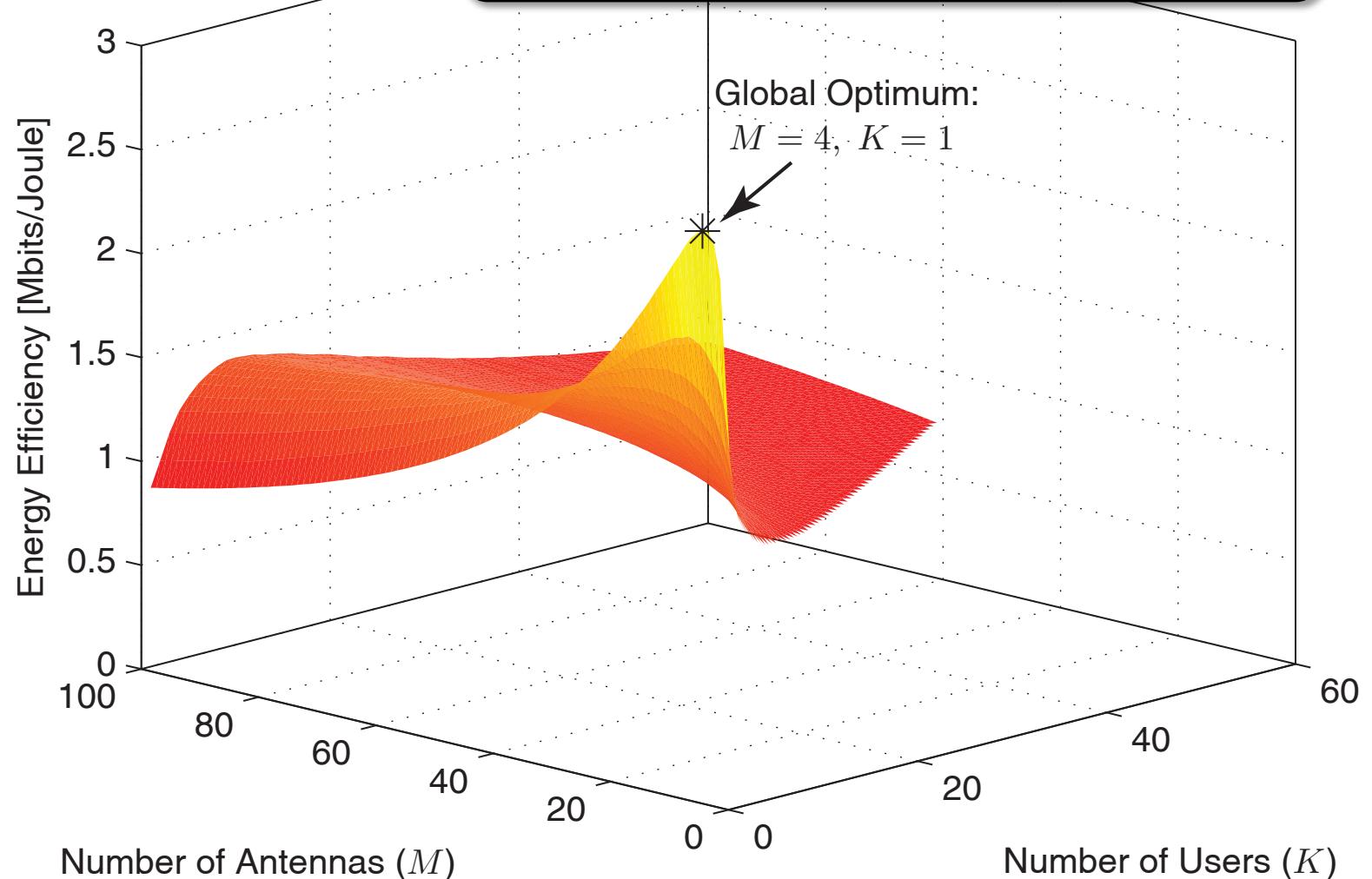
## Simulations

Depends on  
parameters

Download  
Matlab code to  
try other  
values!

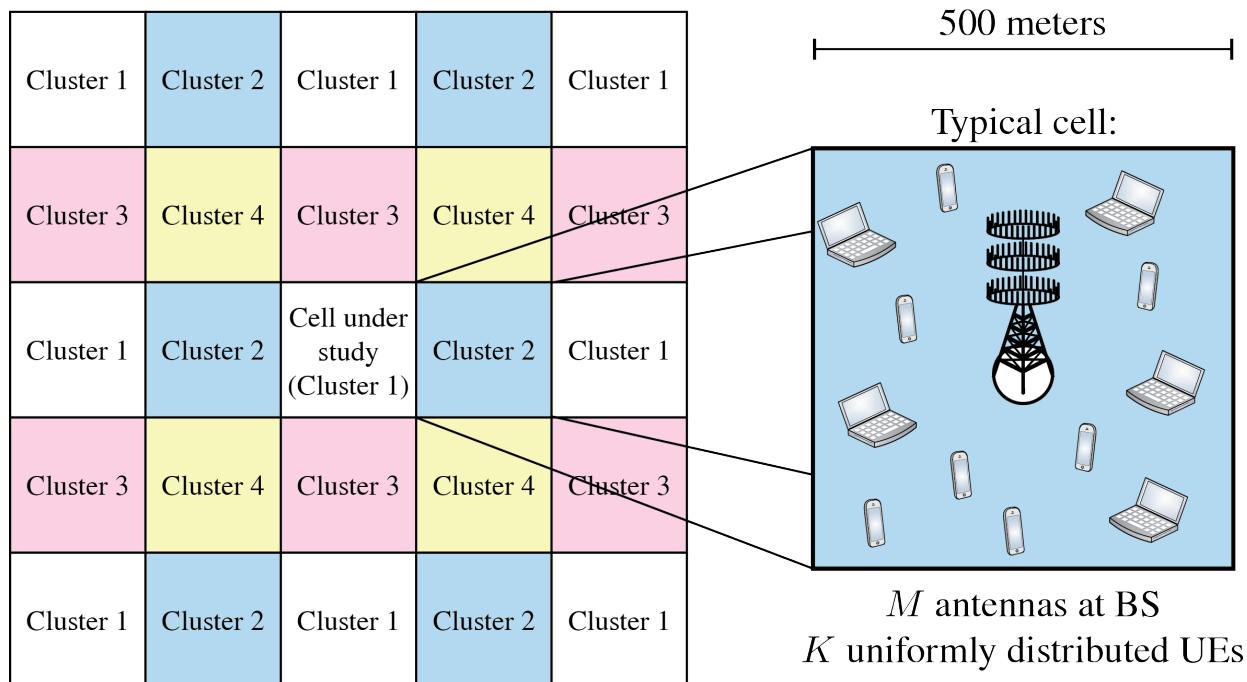
Paper at WCNC'14

Lower  $P_{BT}$  and  $P_{C/D}$  proportional to  $K$



# Multi-Cell Scenario and Imperfect Channel Knowledge

- Limitations in Previous Analysis
  - Perfect channel knowledge
  - No interference from other cells
- Consider a Symmetric Multi-Cell Scenario:



## Assumptions

- All cells look the same → Jointly optimized
- All cells transmit in parallel
- Fractional pilot reuse:  
Divide cells into clusters
- Uplink pilot length  $\tau^{(\text{ul})} K$   
for  $\tau^{(\text{ul})} \in \{1, 2, 4\}$

# Multi-Cell Scenarios and Imperfect Channel Knowledge (2)

- Inter-Cell Interference

- $\lambda_{jl}$  = Channel attenuation between a random user in cell  $l$  and BS  $j$
- $\mathcal{I} = \sum_{l \neq j} \mathbb{E} \left\{ \frac{\lambda_{jl}}{\lambda_{jj}} \right\}$  is relative severity of inter-cell interference

**Lemma** (Achievable Rate)

Consider same transmit power as before:  $P_{\text{trans}} = \rho B \sigma^2 S_\lambda K$

Achievable rate under ZF and pilot-based channel estimation:

$$R = B \log_2 \left( 1 + \frac{\rho(M - K)}{\rho(M - K)\mathcal{I}_{\text{PC}} + \left( 1 + \mathcal{I}_{\text{PC}} + \frac{1}{\rho K \tau^{(\text{ul})}} \right) (1 + \rho K \mathcal{I}) - \rho K (1 + \mathcal{I}_{\text{PC}}^2)} \right)$$

where  $\mathcal{I}_{\text{PC}} = \sum_{l \neq j \text{ only in cluster}} \mathbb{E} \left\{ \frac{\lambda_{jl}}{\lambda_{jj}} \right\}$  and  $\mathcal{I}_{\text{PC}}^2 = \sum_{l \neq j \text{ only in cluster}} \mathbb{E} \left\{ \left( \frac{\lambda_{jl}}{\lambda_{jj}} \right)^2 \right\}$

Pilot contamination (PC)  
(Amplified by  $M$ )

Intra/inter-cell interference  
(Not amplified by  $M$ )

# Multi-Cell Scenarios and Imperfect Channel Knowledge (3)

- Multi-Cell Rate Expression not Amenable for Analytical Analysis
  - No closed-form optimization in multi-cell case
  - Numerical analysis still possible

- Similarities and Differences

- Power consumption is exactly the same
  - Rates are smaller: Upper limited by pilot contamination:

$$R = B \log_2 \left( 1 + \frac{\rho(M-K)}{\rho(M-K)\mathcal{I}_{PC} + \left(1+\mathcal{I}_{PC} + \frac{1}{\rho K \tau(\text{ul})}\right)(1+\rho K \mathcal{I}) - \rho K(1+\mathcal{I}_{PC}^2)} \right) \leq B \log_2 \left( 1 + \frac{1}{\mathcal{I}_{PC}} \right)$$

- Overly high rates not possible (but single-cell solution didn't give that...)
  - Clustering (fractional pilot reuse) might be good to reduce interference

# Optimal Multi-Cell System Design: ZF Processing

## Optimum

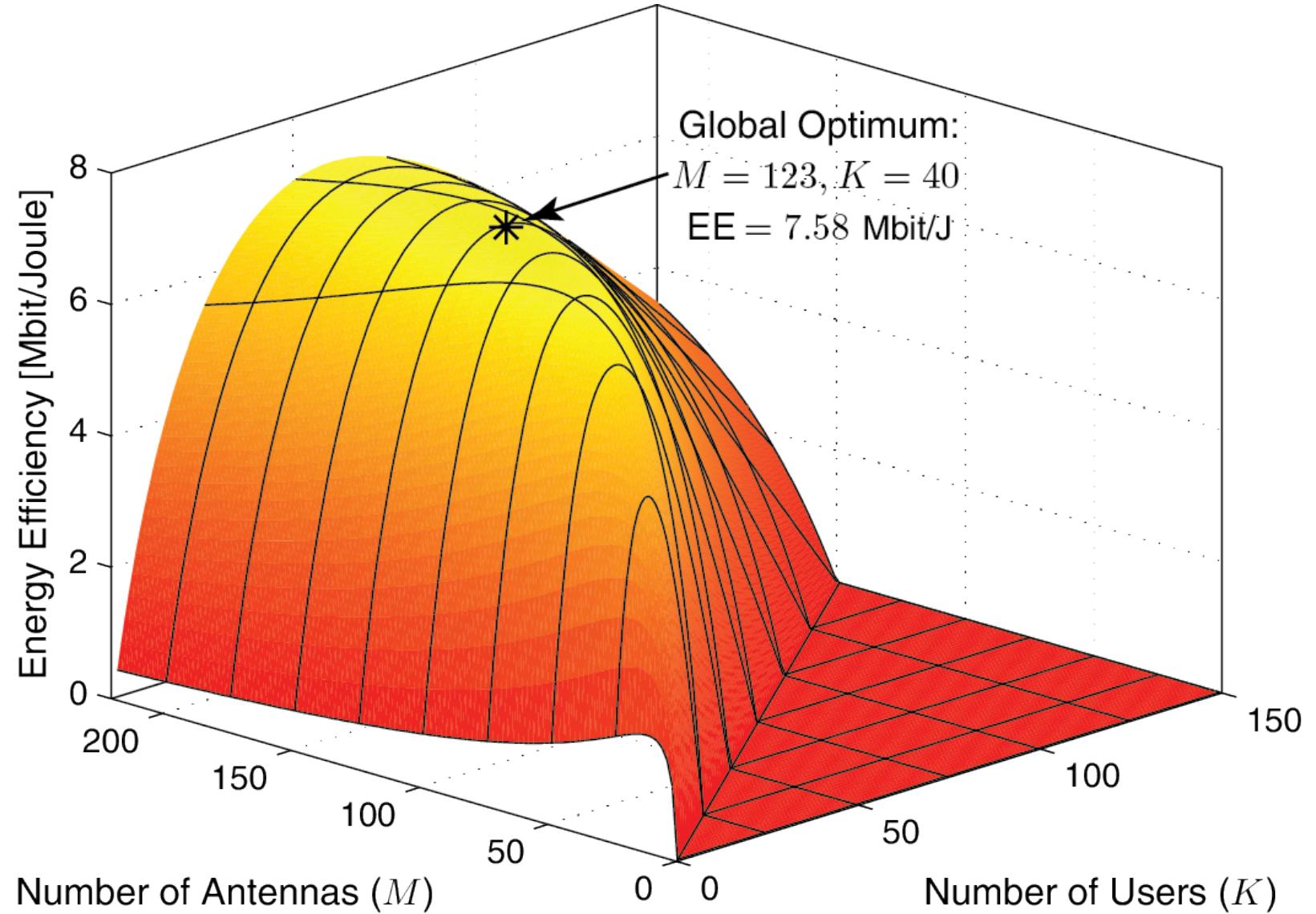
$$\begin{aligned}M &= 123 \\K &= 40 \\\rho &= 0.28 \\\tau^{(\text{ul})} &= 4\end{aligned}$$

User rates:  
 $\approx$ 4-QAM

## Massive MIMO!

Many BS  
antennas

Note that  
 $M/K \approx 3$



# Different Pilot Reuse Factors

## Higher Pilot Reuse

Higher EE *and* rates!

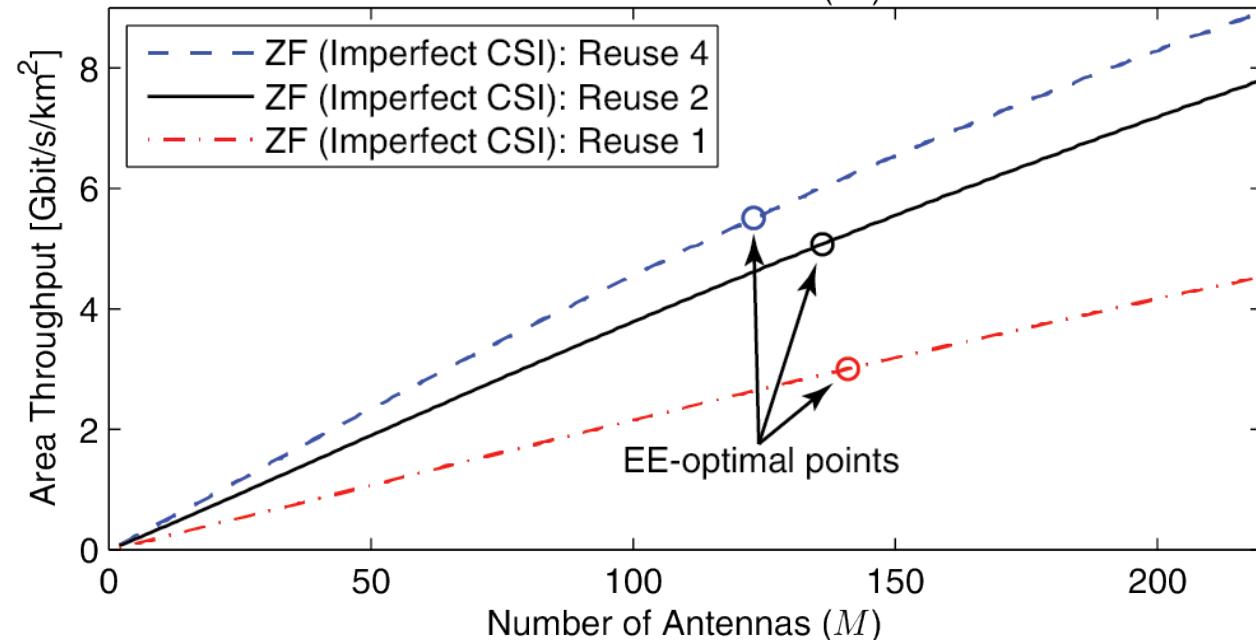
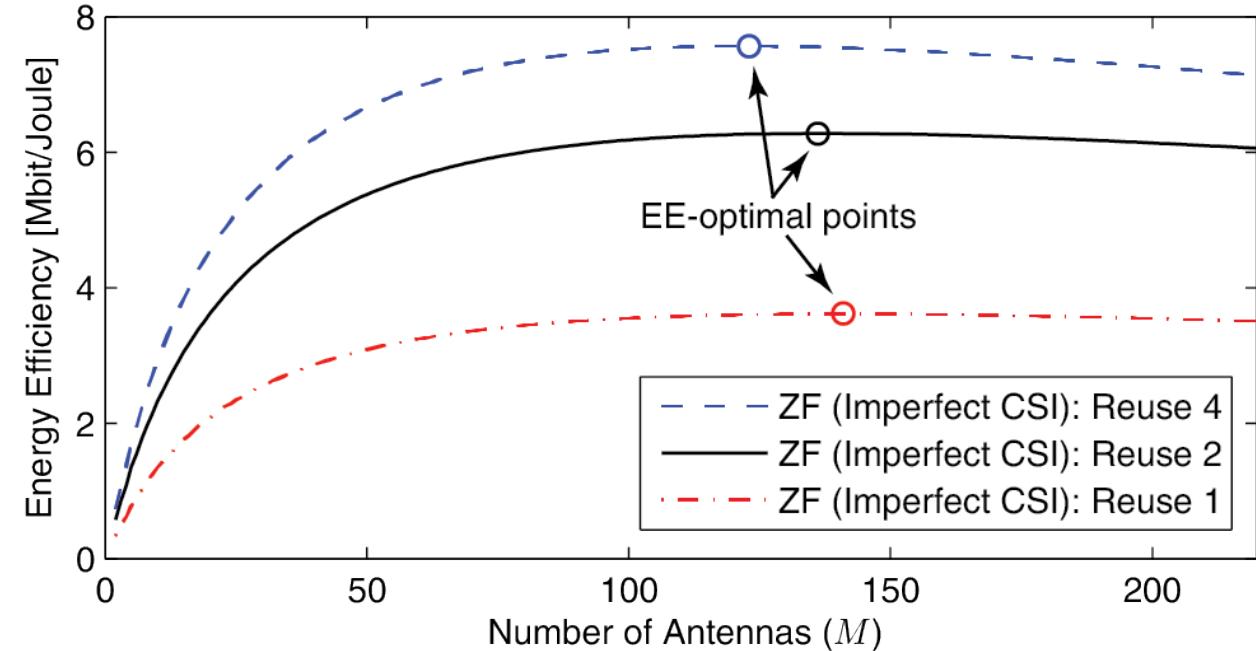
Controlling inter-cell interference is very important!

## Area Throughput

We only optimized EE

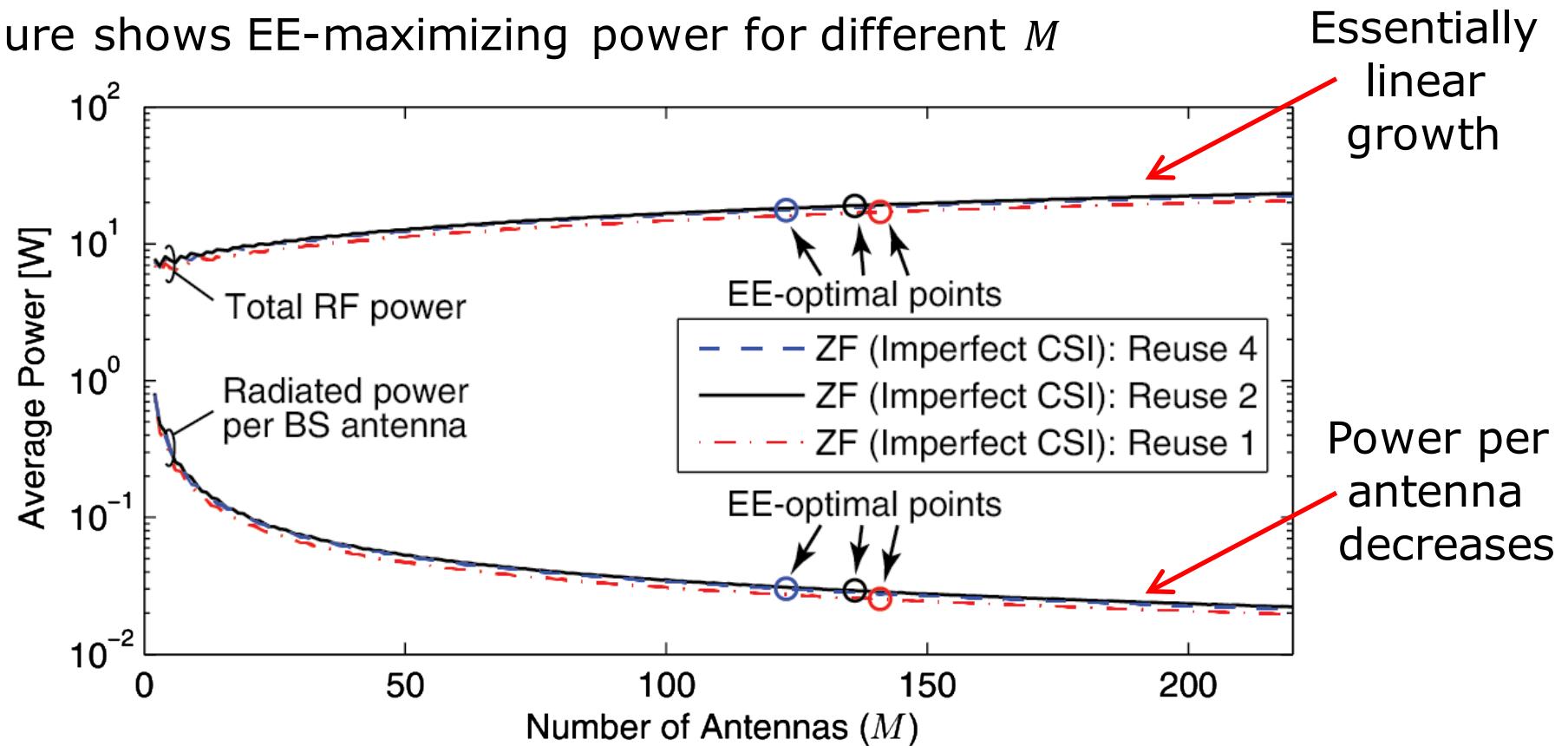
Achieved 6 Gbit/s/km<sup>2</sup> over 20 MHz bandwidth

METIS project mentions 100 Gbit/s/km<sup>2</sup> as 5G goal  
→ Need 15x higher bandwidth!



# Energy Efficient to Use More Transmit Power?

- Recall from Theorem 2: Transmit power increases  $M$ 
  - Figure shows EE-maximizing power for different  $M$



- Intuition: More Circuit Power → Use More Transmit Power
  - Different from  $1/\sqrt{M}$  scaling law in Massive MIMO literature
  - Power per antennas decreases, but only logarithmically

# Summary

- Optimization Results

- EE is a quasi-concave function of  $(M, K, \rho)$
- Closed-form optimal  $M, K$ , or  $\rho$  for single-cell
- Alternating optimization algorithm

## Simulations

Depends on parameters  
Download Matlab code  
to try other values!

	Increases with	Decreases with
Reveals how variables are connected	Antennas $M$	Power $\rho$ , coverage area $\mathcal{S}_\lambda$ , and $M$ -independent circuit power
	Users $K$	Fixed circuit power $C_{0,0}$ and coverage area $\mathcal{S}_\lambda$
	Transmit power $\rho B \sigma^2 \mathcal{S}_\lambda K$	Circuit power, coverage area $\mathcal{S}_\lambda$ , antennas $M$ , and users $K$

### Large Cell

More antennas,  
users, RF power

### Massive MIMO Appears Naturally

Fractional pilot  
reuse is important!

### More Circuit Power

Use more  
transmit power

### Limits of $M, K$

Circuit power that scales with  $M, K$

## **Part 3**

# **Questions?**

**Part 4**

# **Optimization of Energy-Efficiency: Heterogeneous Deployment**

# Symmetric or Asymmetric Network Deployment?

- Recall: Shape of Real Networks
  - Highly asymmetric shape
  - Deployed for asymmetric user load
- Inter-Cell Interference
  - Symmetric: All cells equally exposed  
→ Easy to define pilot reuse patterns
  - Asymmetric: Interference variations  
→ Hard to pick pilot reuse patterns

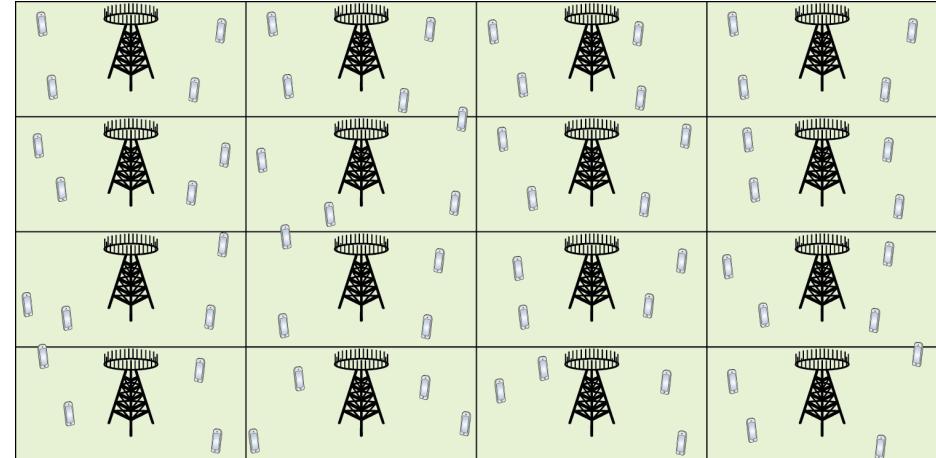
## Pessimistic Approach

Some abstraction needed for analysis!

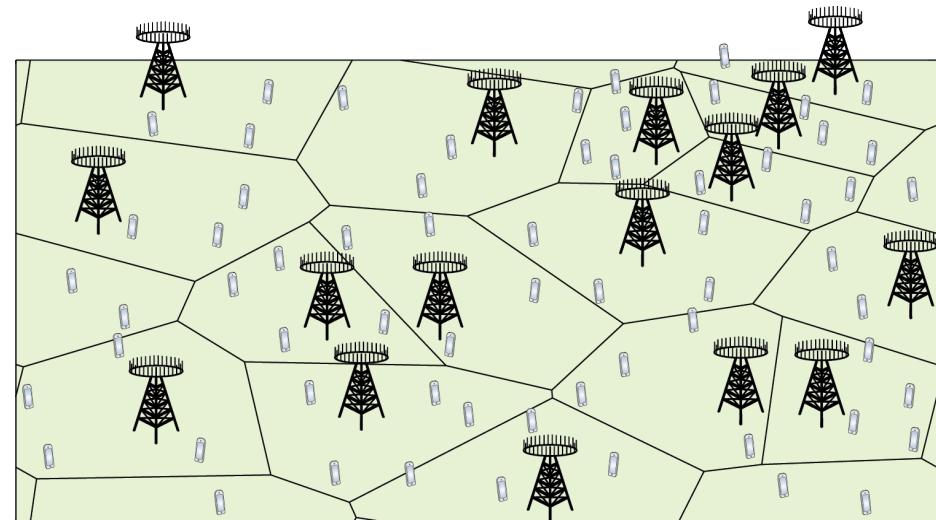
Hard to deploy intricate interference control

Good to design networks without relying on it

Tractable analysis with stochastic deployment?



Overly symmetric deployment



Independent stochastic deployment

# Multi-Cell Modeling with Stochastic Geometry

- Stochastic Spatial Point Process  $\Psi$ 
  - $\Psi$  is a set of random locations in  $\mathbb{R}^n$  for some integer  $n$
  - Used here to model random BS locations in  $\mathbb{R}^2$

## **Homogeneous Poisson point process (PPP)**

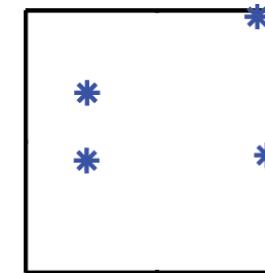
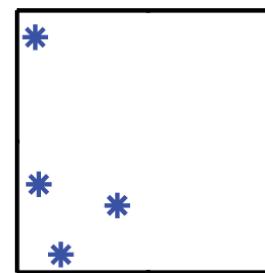
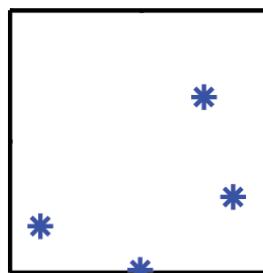
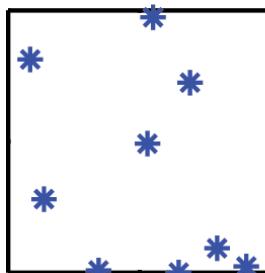
Independent and equally distributed points in  $\mathbb{R}^2$

Density:  $\lambda$  BSs per  $\text{km}^2$

$\text{Po}(\lambda\mathcal{A})$  BSs in any area of size  $\mathcal{A} \text{ km}^2$ :

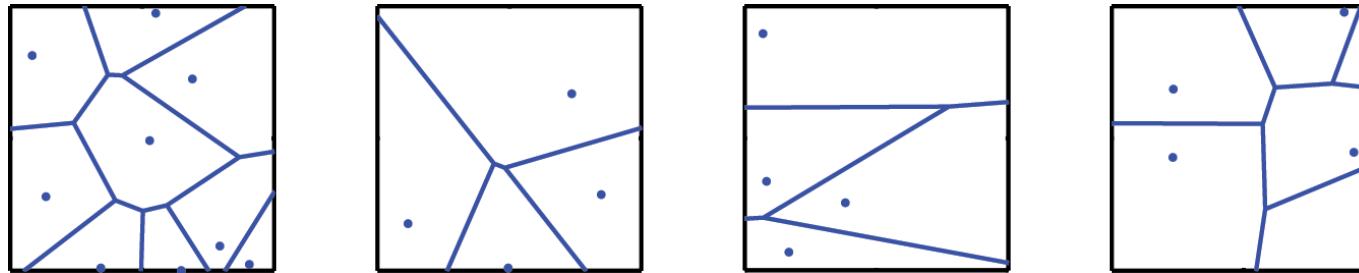
These are uniformly distributed in the area

- 4 realizations with  $\lambda\mathcal{A} = 6$  BSs in the area:

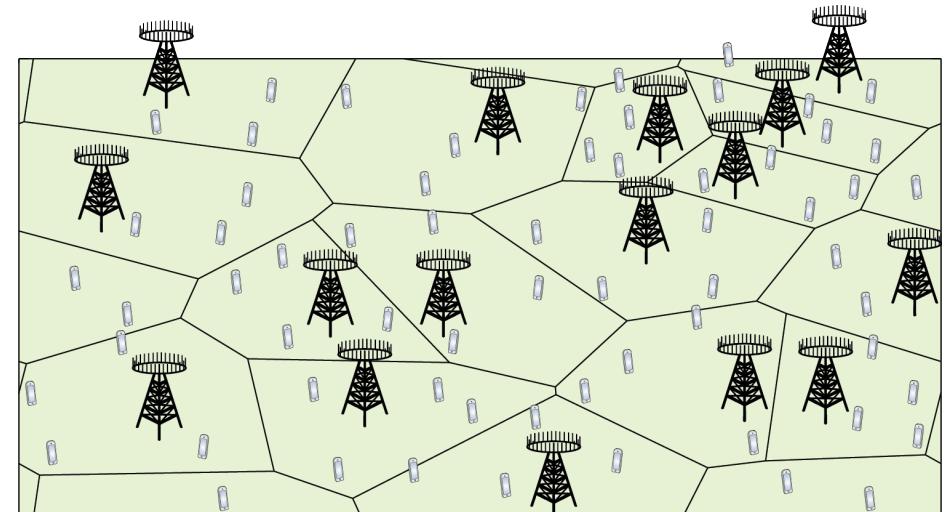


# Network Modeling Assumptions

- BSs Distributed as PPP:  $\lambda$  BS/km<sup>2</sup>
  - $M$  antennas per BSs
  - Closest BS association: Poisson-Voronoi cells
  - $K$  uniformly distributed users per cell → Asymmetric user density



- Propagation Model
  - Channel from User  $k$  in cell  $j$  to BS  $l$ :
$$\mathbf{h}_{jk}^l \sim \mathcal{CN}(\mathbf{0}, \lambda_{jk}^l \mathbf{I})$$
  - Pathloss:  $\lambda_{jk}^l = \omega^{-1}(\text{distance [km]})^{-\alpha}$
  - Pathloss exponent:  $\alpha > 2$
  - Loss at reference distance 1 km:  $\omega$



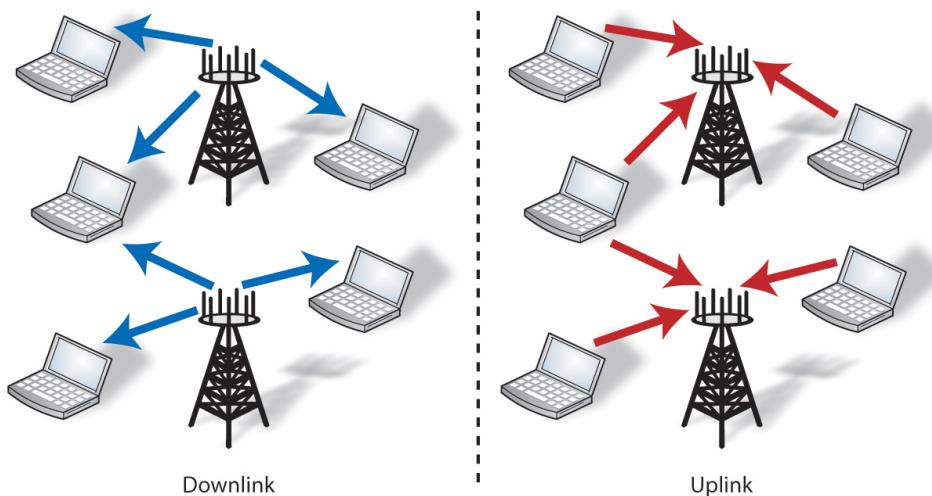
# Uplink-Downlink Duality

## Duality Theorem

The uplink rates are achievable in the downlink using same total power  
Same precoding/combining vectors, but different power allocation  
Provable with perfect CSI (as earlier) but also in general networks

### Focus on Uplink

Only to simplify notation!

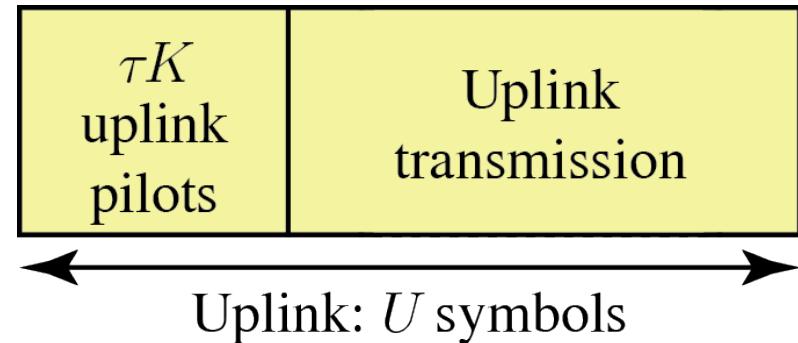


### Pilot Assumptions

$\tau K$  pilots: Reuse factor  $\tau$

No coordination: Pilots selected randomly in each cell

Imperfect CSI in this part!



# Average Uplink Rate

- Randomness and Typical User
  - All users in the system is surrounded by BSs having same distribution
  - Property: Any user is “*typical user*” ← Only need to analyze one user!
- Power Control: Statistical channel inversion
  - User  $k$  in cell  $j$ :  $p_{jk}^j = \rho / \lambda_{jk}^j$
  - Avoid near-far issues in the cells
  - Same average SNR  $\rho/\sigma^2$  for everyone

**Power per user:**

$$\mathbb{E}\left\{\frac{\rho}{\lambda_{jk}^j}\right\} = \rho \omega \frac{\Gamma(\alpha/2-1)}{(\pi\lambda)^{\alpha/2}}$$

**Lemma** (Lower Bound on Average Rate with MR Processing)

$$\underline{R} = B \left(1 - \frac{\tau K}{U}\right) \log_2 (1 + \underline{\text{SINR}})$$

$$\underline{\text{SINR}} = \frac{M}{\left(K + \frac{\sigma^2}{\rho}\right) \left(1 + \frac{2}{\tau(\alpha-2)} + \frac{\sigma^2}{\rho}\right) + \frac{2K}{\alpha-2} \left(1 + \frac{\sigma^2}{\rho}\right) + \frac{K}{\tau} \left(\frac{4}{(\alpha-2)^2} + \frac{1}{\alpha-1}\right) + \frac{M}{\tau(\alpha-1)}}$$

# Maximizing Energy Efficiency

- Recall:  $\text{EE} = \frac{\text{Average Sum Rate [bit/s/cell]}}{\text{Power Consumption [Joule/s/cell]}}$

- Sum rate is  $K \cdot \underline{R}$  with  $\underline{R}$  from previous lemma
- Power consumption modeled as before:

$$P_{\text{total}} = \frac{B\rho\omega\Gamma(\alpha/2 - 1)}{\eta} \left( \frac{(\pi\lambda)^{\alpha/2}}{U} \right) \left( 1 - \frac{\tau K - 1}{U} \right) + C_{0,0} + C_{0,1}M + C_{1,0}K + C_{1,1}MK + AKR$$

Transmit power with amplifiers
Power control
Fixed power
Circuit power per transceiver chain
Coding/decoding/backhaul

Cost of MR signal processing

## EE Maximization Problem

$$\begin{aligned} & \text{maximize} && \frac{KR}{P_{\text{total}}} \\ & \text{subject to} && \text{SINR} = \gamma \end{aligned}$$

- Average SINR constraint  $\gamma$  needed to not get too low rates

# Solving Multi-Cell EE Maximization

- Optimization Variables:

- $M$  = Number of BS antennas
- $K$  = Number of users per cell
- $\rho$  = Normalized transmit power
- $\lambda$  = BS density
- $\tau$  = Pilot reuse factor ( $\tau \geq 1$ )



Unique feature of this stochastic framework

**Theorem 5** (Optimal reuse factor)

SINR constraint is satisfied by setting  $\tau^* = \frac{D_1\gamma}{M-D_2\gamma}$

where  $D_1 = \frac{4K}{(\alpha-2)^2} + \frac{K+M}{\alpha-1} + \frac{2(K+\frac{\sigma^2}{\rho})}{\alpha-2}$  and  $D_2 = \left(K + \frac{\sigma^2}{\rho} + \frac{2K}{\alpha-2}\right)\left(1 + \frac{\sigma^2}{\rho}\right)$

- Observations

- Increases with  $K$ , but decreases with  $M$  or  $\rho$
- Independent of  $\lambda$  and hardware characteristics

# Optimizing BS Density

- Impact of BS Density
  - Rate  $\underline{R}$  is independent of  $\lambda$ , due to power control
  - Transmit power scales as  $\rho/\lambda^{\alpha/2}$

**Theorem 6** (Optimal  $\lambda$  and  $\rho$ )

EE is monotonically increasing in  $\lambda$

Set  $\rho = \rho_0\lambda$ . For any  $\rho_0 > 0$ , the EE is monotonically increasing in  $\lambda$ :  
EE is maximized as  $\lambda \rightarrow \infty$

**Does it make sense to let  $\lambda \rightarrow \infty$ ?**

Physically: No, density is finite (depends on BS dimension)  
Conceptually: Maybe (depends on convergence speed)

# Simulation Parameters

Parameter	Symbol	Value
Coherence block length	$U$	400
Pathloss exponent	$\alpha$	3.76
Pathloss over noise at 1 km	$\omega/\sigma^2$	33 dBm
Amplifier efficiency	$\eta$	0.39
Bandwidth	$B$	20 MHz
Static power	$C_{0,0}$	10 W
Circuit power per active user	$C_{1,0}$	0.1 W
Circuit power per BS antenna	$C_{0,1}$	1 W
Signal processing coefficient	$C_{1,1}$	3.12 mW
Coding/decoding/backhaul	$A$	$1.15 \cdot 10^{-9}$ J/bit

**Average inter-BS distance:** On average  $\lambda$  BSs per  $\text{km}^2$

Deployment	$\lambda$	$\lambda = 10$
Square grid	$1/\sqrt{\lambda}$ km	316 m
PPP (average)	$1/(2\sqrt{\lambda})$ km	158 m

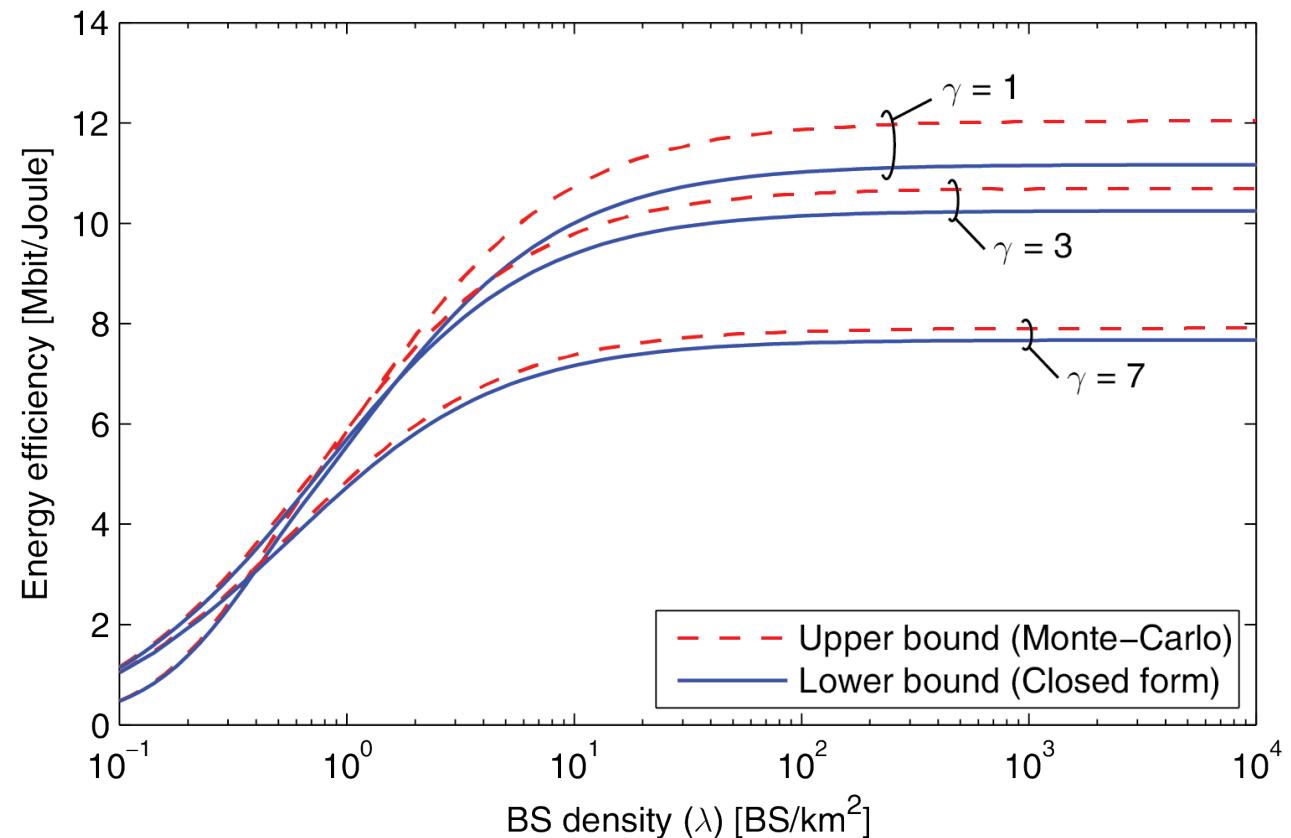
# Simulation: Impact of BS Density

## Simulation

Different BS densities  
All other variables optimized

## Observations

Lower bound is tight  
Higher EE with lower  $\gamma$   
EE increases with  $\lambda$



## Saturation Property

EE saturates at  $\lambda = 10$  (few hundred meters between BSs)  
Satisfied in urban deployments – already today!  
We can safely let  $\lambda \rightarrow \infty$  in the analysis

# Remaining Optimization Problem

- EE Maximization Problem Simplified by Theorems 5 and 6:

## Remaining EE Maximization Problem

$$\begin{array}{ll}\text{maximize} & \frac{KB\left(1-\frac{K}{UM-\bar{D}_2\gamma}\right)\log_2(1+\gamma)}{C_{0,0}+C_{0,1}M+C_{1,0}K+C_{1,1}MK+AKB\left(1-\frac{K}{UM-\bar{D}_2\gamma}\right)\log_2(1+\gamma)} \\ M, K \\ \text{subject to} & \frac{\bar{D}_1\gamma}{M-\bar{D}_2\gamma} \geq 1\end{array}$$

$$\text{where } \bar{D}_1 = \frac{4K}{(\alpha-2)^2} + \frac{K+M}{\alpha-1} + \frac{2K}{\alpha-2} \text{ and } \bar{D}_2 = K\left(1 + \frac{2}{\alpha-2}\right)$$

- Feasible if  $\left(1 - \frac{K}{UM-\bar{D}_2\gamma}\right) > 0$ :  $\gamma < \frac{UM}{K\bar{D}_1 + \bar{D}_2} \leq U(\alpha - 1)$
- Example:  $U = 200$  and  $\alpha = 3$  give  $\gamma \leq 400$  (more than 256-QAM)
- Feasible in most practical cases!

# Optimizing Number of Users

- Find real-valued  $K$  that maximizes EE

- Assuming  $\bar{\beta} = \frac{M}{K}$  antennas per user

## Theorem 7 (Optimal $K$ )

EE is quasi-concave w.r.t.  $K$  and maximized by

$$K^* = \frac{\sqrt{(GC_{0,0})^2 + C_{0,0}C_{1,1}\bar{\beta} + C_{0,0}G(C_{1,0} + C_{0,1}\bar{\beta})} - GC_{0,0}}{C_{1,1}\bar{\beta} + G(C_{1,0} + C_{0,1})}$$

$$\text{where } G = \frac{1}{U} \frac{\frac{4\gamma}{(\alpha-2)^2} + \frac{\gamma(1+\bar{\beta})}{\alpha-1} + \frac{2\gamma}{\alpha-2}}{\bar{\beta} - \left(1 + \frac{2}{\alpha-2}\right)\gamma}$$

## • Observations

- Increases with fixed circuit power (e.g.,  $P_{\text{FIX}}$ )
- Decreases with circuit coefficients multiplied with  $M$  or  $K$  ( $P_{\text{BS}}, P_{\text{UE}}, 1/L_{\text{BS}}$ )
- Independent of cost of coding/decoding/backhaul

# Optimizing Number of Antennas

- Find real-valued  $M$  that maximizes EE

**Theorem 8** (Optimal  $M$ )

EE is quasi-concave w.r.t.  $K$  and maximized by

$$M^* = K \frac{\nu_1 K + \nu_2 + \sqrt{\nu_1 \nu_2 K + \nu_1^2 K^2 + (1 - \nu_0 K)(\nu_1 K + \nu_0 \nu_2 K) \frac{C_{0,0} + C_{1,0} K}{C_{0,1} K + C_{1,1} K^2} + \nu_0 \nu_1 \nu_2 K^2 + \nu_0 \nu_2^2 K}}{1 - \nu_0 K}$$

$$\text{where } \nu_0 = \frac{\gamma}{U(\alpha-1)}, \nu_1 = \frac{1}{U} \left( \frac{4\gamma}{(\alpha-2)^2} + \frac{\gamma}{\alpha-1} + \frac{2\gamma}{\alpha-2} \right), \nu_2 = \gamma \left( 1 + \frac{2}{\alpha-2} \right).$$

- Observations

- Increases with circuit coefficients independent of  $M$  (e.g.,  $P_{\text{FIX}}, P_{\text{UE}}$ )
- Decreases with circuit coefficients multiplied with  $M$  (e.g.,  $P_{\text{BS}}, 1/L_{\text{BS}}$ )
- Independent of cost of coding/decoding/backhaul
- Increases with  $\gamma$  due to interference suppression

# Impact of Number of Antennas and Users

## Simulation

Optimized  $\tau, \lambda, \rho$

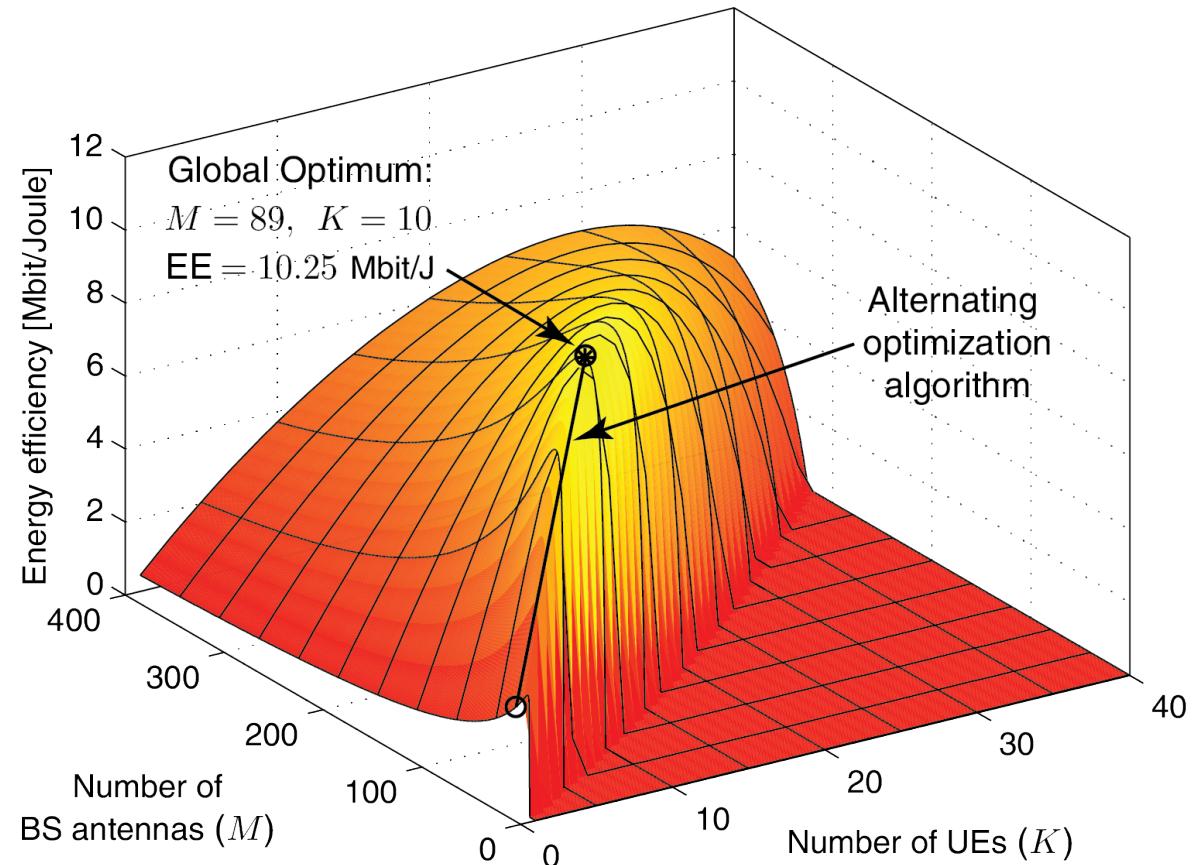
SINR constraint:  $\gamma = 3$

## Observations

Optimal:  $M^* = 89, K^* = 10$

Massive MIMO with  
reuse factor  $\tau \approx 7$

Other  $(M, K)$  around the  
optimum are also good



## Why Massive MIMO and Not Only Small Cells?

Small cells improve SNR, but not SINR

Massive MIMO improves cell-edge SINR by interference control

Other added benefit: Circuit power costs are shared between user

# Power Distribution at Optimal Solution

## What Consumes Power?

Consider optimum:  $M^* = 89, K^* = 10$

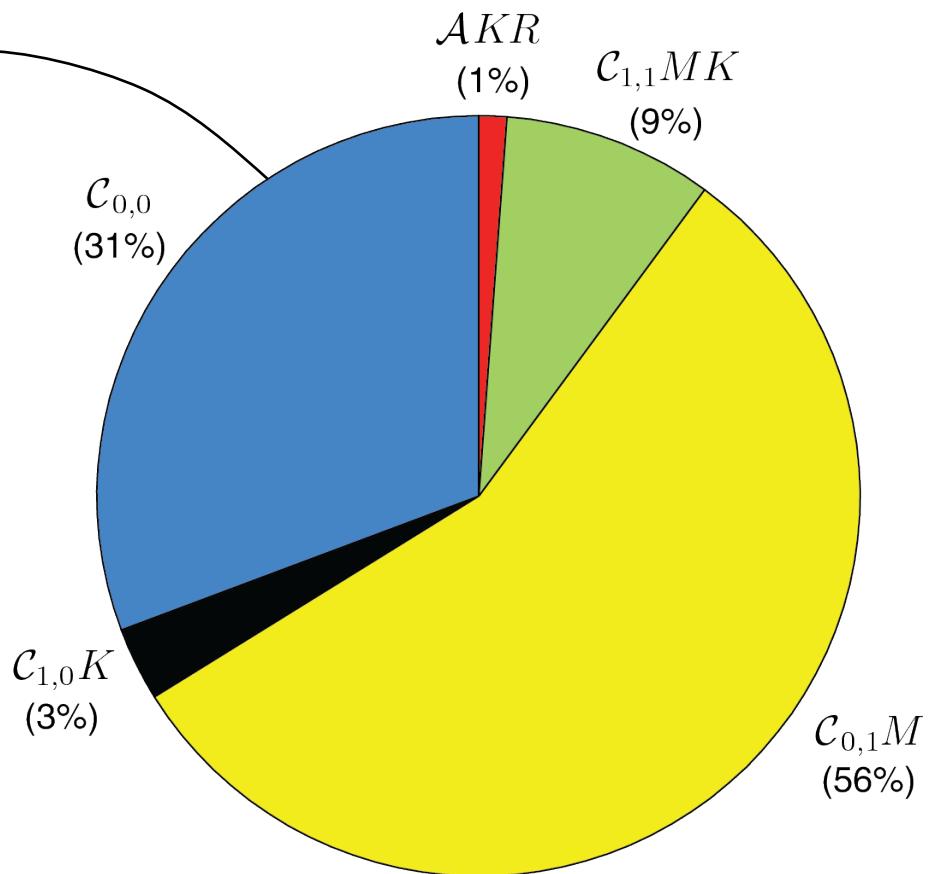
Recall model:

$$C_{0,0} + C_{0,1}M + C_{1,0}K + C_{1,1}MK + AKR$$

## Dominating Parts

Power of BS transceivers:  $C_{0,1}M$

Fixed power consumption:  $C_{0,0}$



## How to Improve Future Hardware?

Improve the dominating parts

Good design: No part has a large percentage

# Designing with Given User Density

- User Density
  - So far:  $K$  and  $\lambda$  design variables
  - Density:  $\lambda K$  users per  $\text{km}^2$
  - Heterogeneous user distribution

## Can we Optimize this Density?

Increase: No, cannot “create” users  
Decrease: Yes, by scheduling

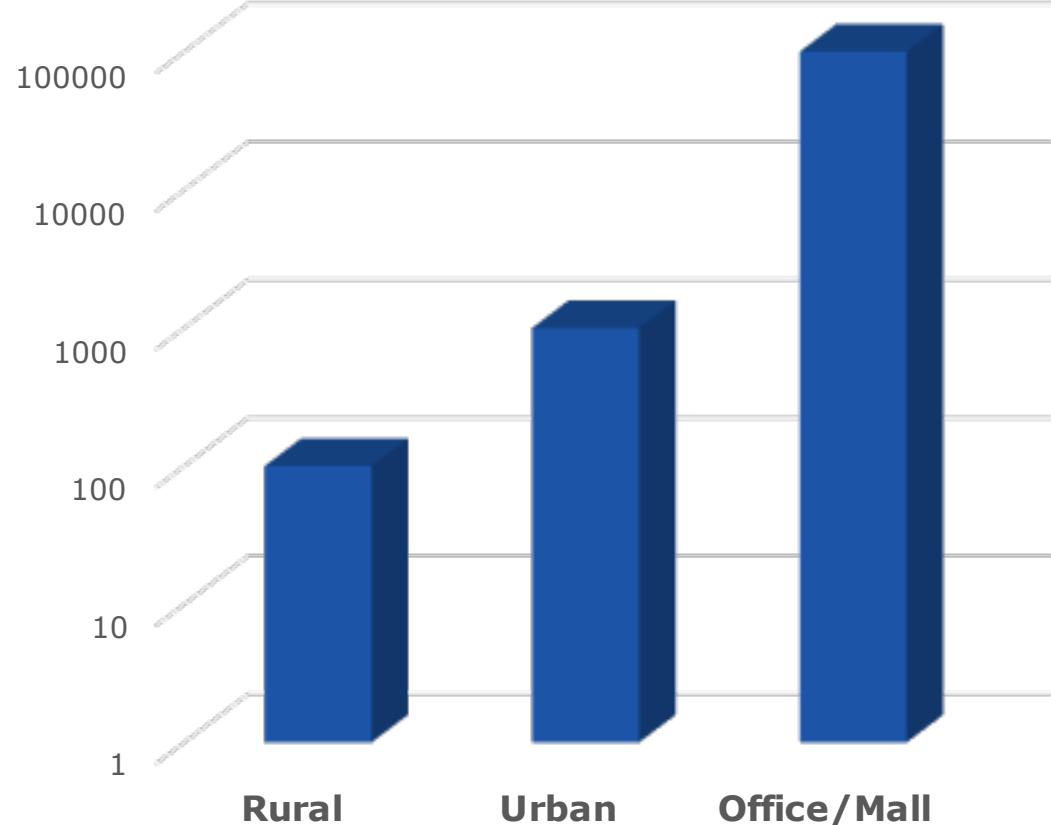
### Practical User Densities

Rural:  $10^2$  per  $\text{km}^2$

Urban:  $10^3$  per  $\text{km}^2$

Office/Mall:  $10^5$  per  $\text{km}^2$

Source: METIS, "Deliverable D1.1:  
Scenarios, requirements and KPIs for  
5G mobile and wireless system"



# Impact of User Density

## Simulation

Fixed user density  $\mu$  users/km<sup>2</sup>

EE maximization with constraint:  $K\lambda = \mu$

Rural:  $\mu = 10^2$ , Malls:  $\mu = 10^5$

## Low User Density

Many cells with  $K \approx 1$

Most important to reduce pathloss

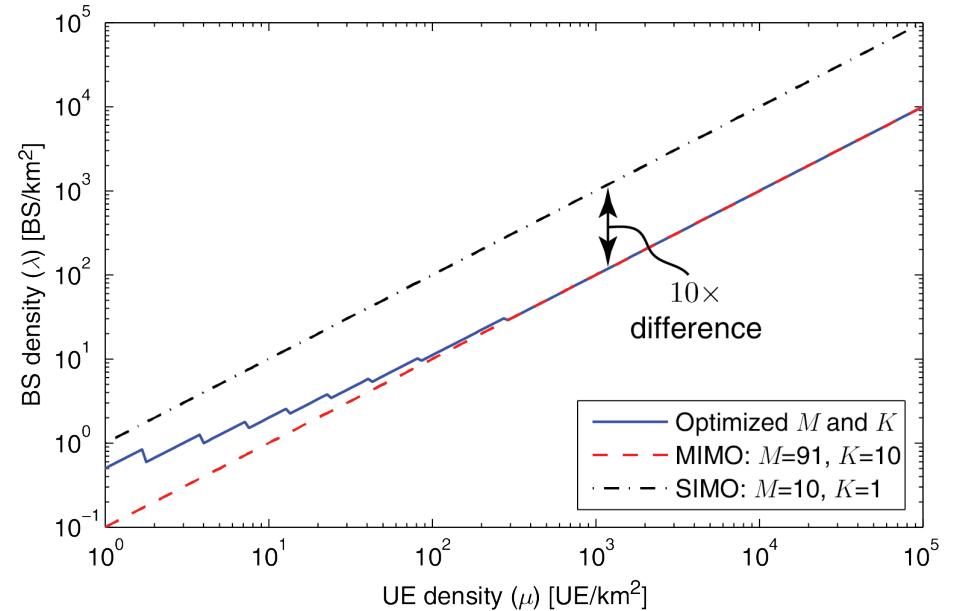
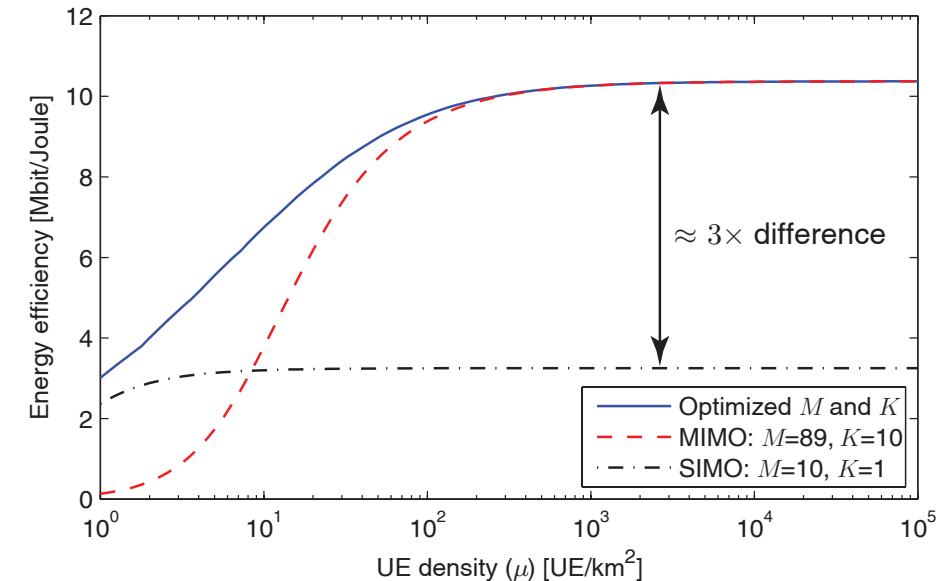
## High User Density

Massive MIMO is optimal

Saturation for  $\mu \geq 100$

Covers most practical scenarios:  
EE independent of user load!

Share circuit power and cost over users



# Summary

- EE Optimization with Stochastic Deployment
  - Stochastic geometry: Resembles real deployments, tractable formulas
- Optimized Network Densification
  - Large cells: First step is to reduce cell size
  - Smaller cells: Transmit power is negligible → Use also Massive MIMO
  - Optimal solution is a combination of small cells and Massive MIMO
- Intuition:
  - Smaller cells: Greatly improves SNR, but no interference improvement
  - More antennas/users per cell: Suppress interference
    - Share circuit power cost among users
- Future Work
  - Other processing schemes and point distributions
  - Coordinated resource management, multi-tier networks

**Part 4**

# **Questions?**

## **Part 5**

# **Multi-Objective Network Optimization**

# Optimize more than Energy-Efficiency

- Recall: Many Metrics in 5G Discussions
  - Average rate (Mbit/s/active user)
  - Average area rate (Mbit/s/km<sup>2</sup>)
  - Energy-efficiency (Mbit/Joule)
  - Active devices (per km<sup>2</sup>)
  - Delay constraints (ms)
- So Far: Only cared about EE
  - Ignored all other metrics



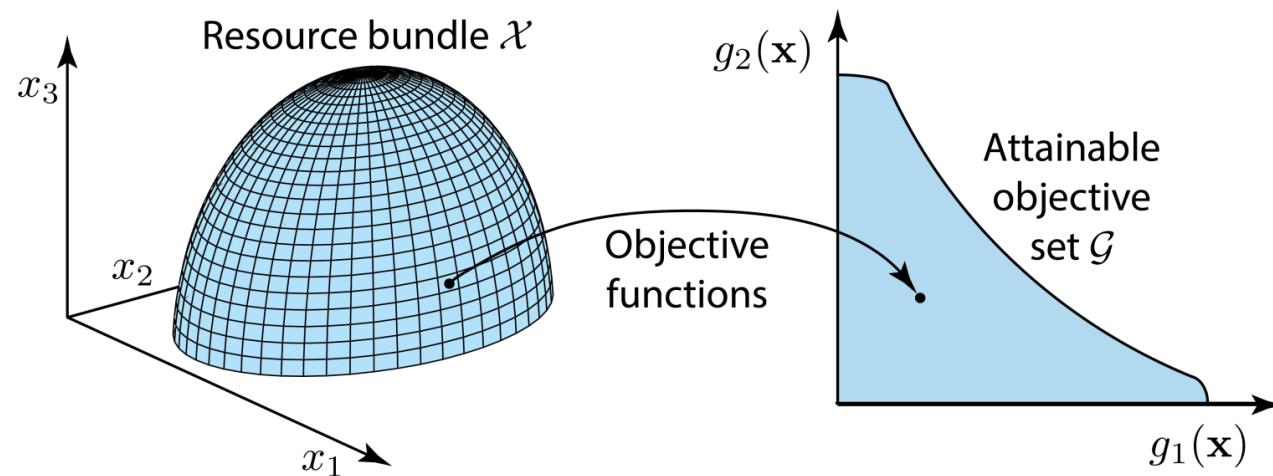
## Optimize Multiple Metrics

We want efficient operation w.r.t. all objectives!

Is this possible?  
For all at the same time?

# Basic Assumptions: Multi-Objective Optimization

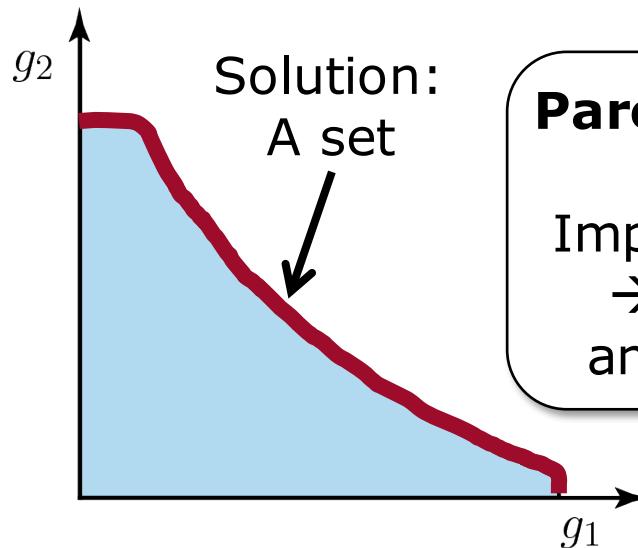
- Consider  $N$  Performance Metrics
  - Objectives to be maximized
  - Notation:  $g_1(\mathbf{x}), g_2(\mathbf{x}), \dots, g_N(\mathbf{x})$
  - Example: individual user rates, area rates, energy-efficiency
- Optimization Resources
  - Resource bundle:  $\mathcal{X}$
  - Example: power, resource blocks, network architecture, antennas, users
  - Feasible allocation:  $\mathbf{x} \in \mathcal{X}$



# Single or Multiple Performance Metrics

- Conventional Optimization
  - Pick one prime metric:  $g_1(\mathbf{x})$
  - Turn  $g_2(\mathbf{x}), \dots, g_N(\mathbf{x})$  into constraints
  - Optimization problem:  
$$\underset{\mathbf{x}}{\text{maximize}} \quad g_1(\mathbf{x})$$
subject to  $\mathbf{x} \in \mathcal{X},$   
$$g_2(\mathbf{x}) \geq A_2, \dots, g_N(\mathbf{x}) \geq A_N.$$
  - Solution: A scalar number
  - Cons: Is there a prime metric?  
How to select constraints?

- Multi-Objective Optimization
  - Consider all  $N$  metrics
  - No order or preconceptions!
  - Optimization problem:  
$$\underset{\mathbf{x}}{\text{maximize}} \quad [g_1(\mathbf{x}), g_2(\mathbf{x}), \dots, g_N(\mathbf{x})]$$
subject to  $\mathbf{x} \in \mathcal{X}.$



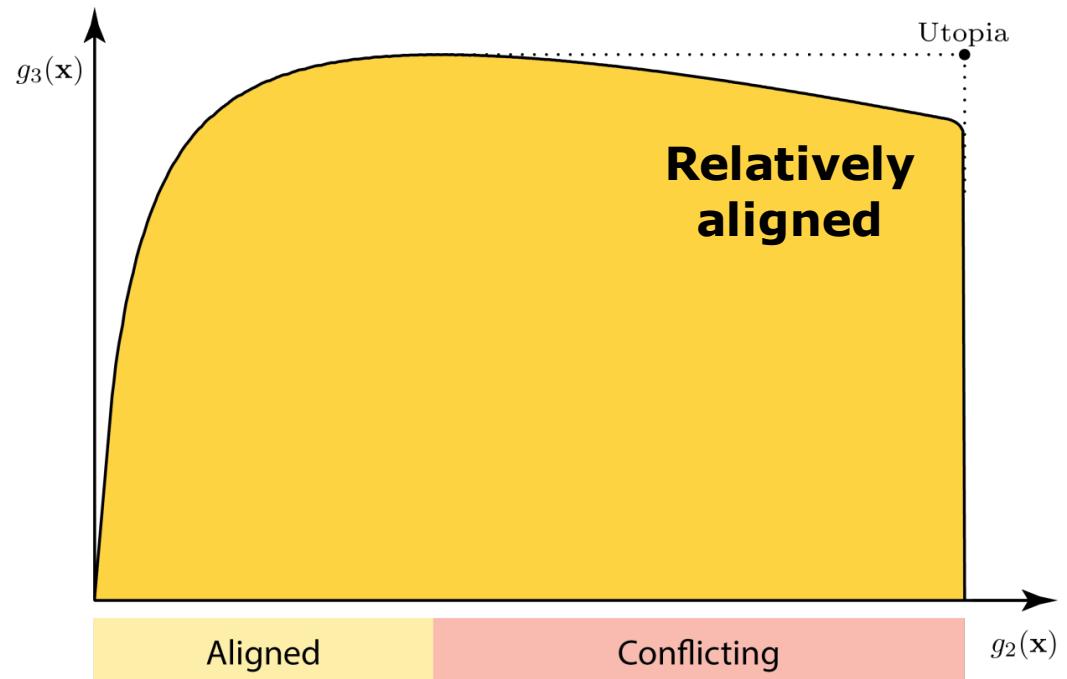
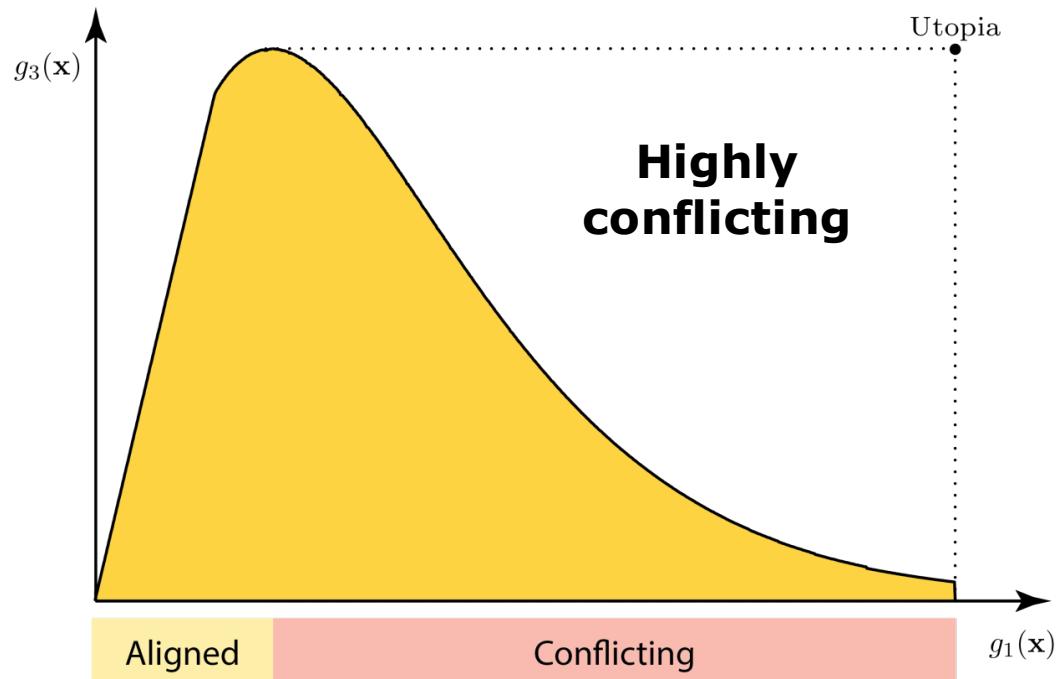
# Why Multi-Objective Optimization?

- Study Tradeoffs Between Metrics
  - When are metrics aligned or conflicting?
  - Common in engineering and economics – new in communication theory

## ***A Posteriori* Approach**

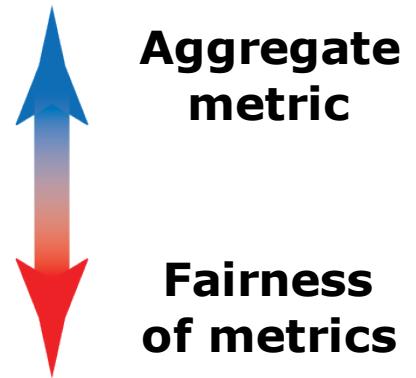
Generate region (computationally demanding!)

Look at region and select operating point



# A Priori Approach

- No Objectively Optimal Solution
  - Utopia point outside of region → Only subjectively “good” solutions exist
- System Designer Selects Utility Function  $f : \mathbb{R}^N \rightarrow \mathbb{R}$ 
  - Describes subjective preference (larger is better)
- Examples:
  - Sum performance:  $f(\mathbf{g}) = \sum_k g_k$
  - Proportional fairness:  $f(\mathbf{g}) = \prod_k g_k$
  - Harmonic mean:  $f(\mathbf{g}) = K_r (\sum_k g_k^{-1})^{-1}$
  - Max-min fairness:  $f(\mathbf{g}) = \min_k g_k$



We obtain a simplified problem:

$$\underset{\mathbf{x}}{\text{maximize}} \quad f(g_1(\mathbf{x}), g_2(\mathbf{x}), \dots, g_N(\mathbf{x}))$$

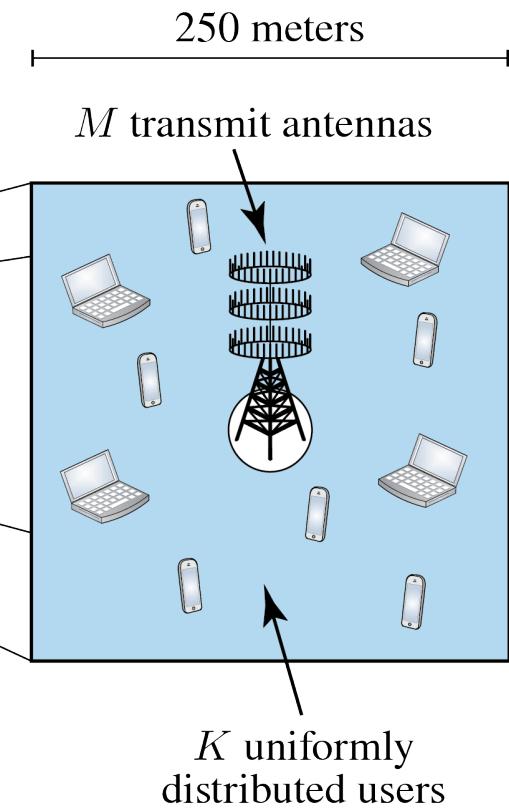
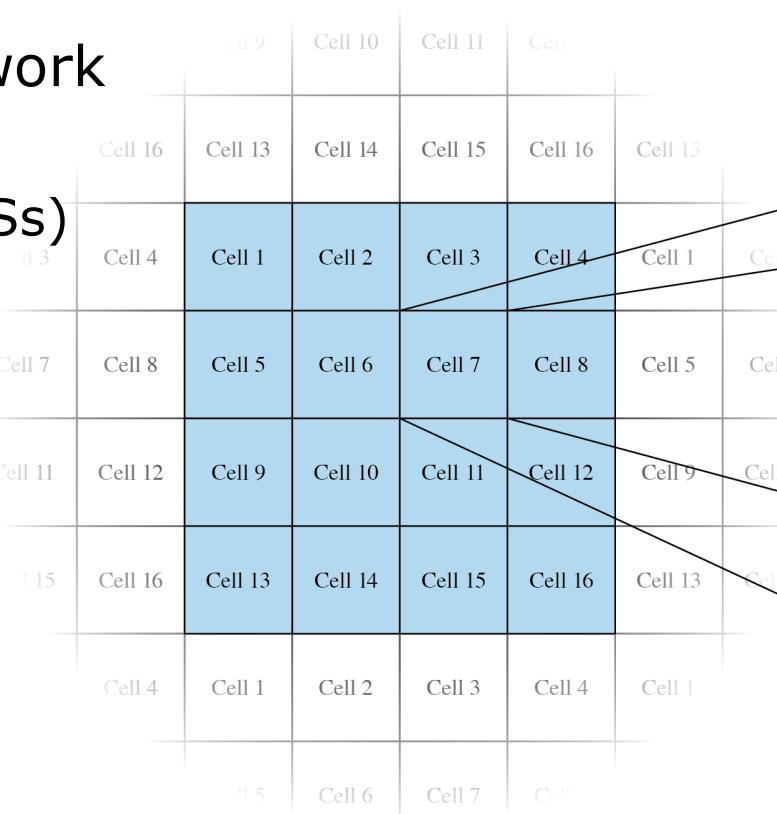
subject to  $\mathbf{x} \in \mathcal{X}$

- Solution: A scalar number  
(Gives one Pareto optimal point)
- Takes all metrics into account!

# Example: Optimization of 5G Networks

- Design Cellular Network

- Symmetric system
- 16 base stations (BSs)
- Select:
  - $M = \# \text{ BS antennas}$
  - $K = \# \text{ users}$
  - $P = \text{transmit power}$



- Resource bundle:

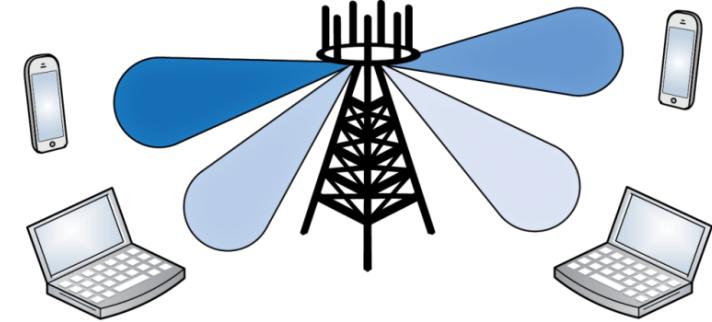
$$\mathcal{X} = \left\{ [K \ M \ P]^T : \begin{array}{l} 1 \leq K \leq \frac{M}{2}, \\ 2 \leq M \leq M_{\max}, \\ 0 \leq P \leq MP_{\max} \end{array} \right\}$$

500  
20 W

# Example: Optimization of 5G Networks (2)

- Downlink Multi-Cell Transmission

- Each BS serves only its own  $K$  users
- Coherence block length:  $U$
- BS knows channels within the cell (cost:  $K/U$ )
- ZF precoding: no intra-cell interference
- Interference leaks between cells



- Average User Rate

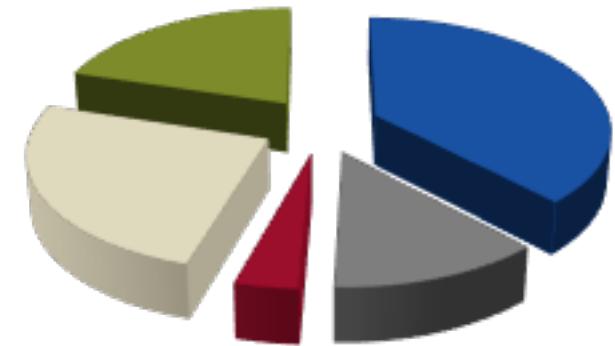
$$R_{\text{average}} = B \left(1 - \frac{K}{U}\right) \log_2 \left( 1 + \frac{P}{K} \frac{(M - K)}{\mathcal{S}_\lambda \sigma^2 + I} \right)$$

Annotations for the equation variables:

- Power/user:  $P$
- Array gain:  $\frac{P}{K} (M - K)$
- Noise / pathloss:  $(1.72 \cdot 10^{-4})$
- Relative inter-cell interference:  $(0.54)$
- Bandwidth (10 MHz):  $B$
- CSI estimation overhead ( $U = 1000$ ):  $\left(1 - \frac{K}{U}\right)$

# Example: Optimization of 5G Networks (3)

- What Consumes Power?
  - Transmit power (+ losses in amplifiers)
  - Circuits attached to each antenna
  - Baseband signal processing
  - Fixed load-independent power
- Total Power Consumption



$$P_{\text{total}} = \frac{P_{\text{trans}}}{\eta} + C_{0,0} + C_{1,0}K + C_{0,1}M + \frac{B \cdot C_{\text{precoding}}}{U L_{\text{BS}}}$$

Annotations pointing to the terms in the equation:

- Amplifier efficiency (0.31) points to  $\eta$
- Fixed power (10 W) points to  $C_{0,0}$
- Circuit power per user (0.3 W) points to  $C_{1,0}K$
- Circuit power per antenna (1 W) points to  $C_{0,1}M$
- Compute ZF precoding ( $2.3 \cdot 10^{-6} \cdot MK^2$ ) points to  $B \cdot C_{\text{precoding}} / (U L_{\text{BS}})$

# Example: Results

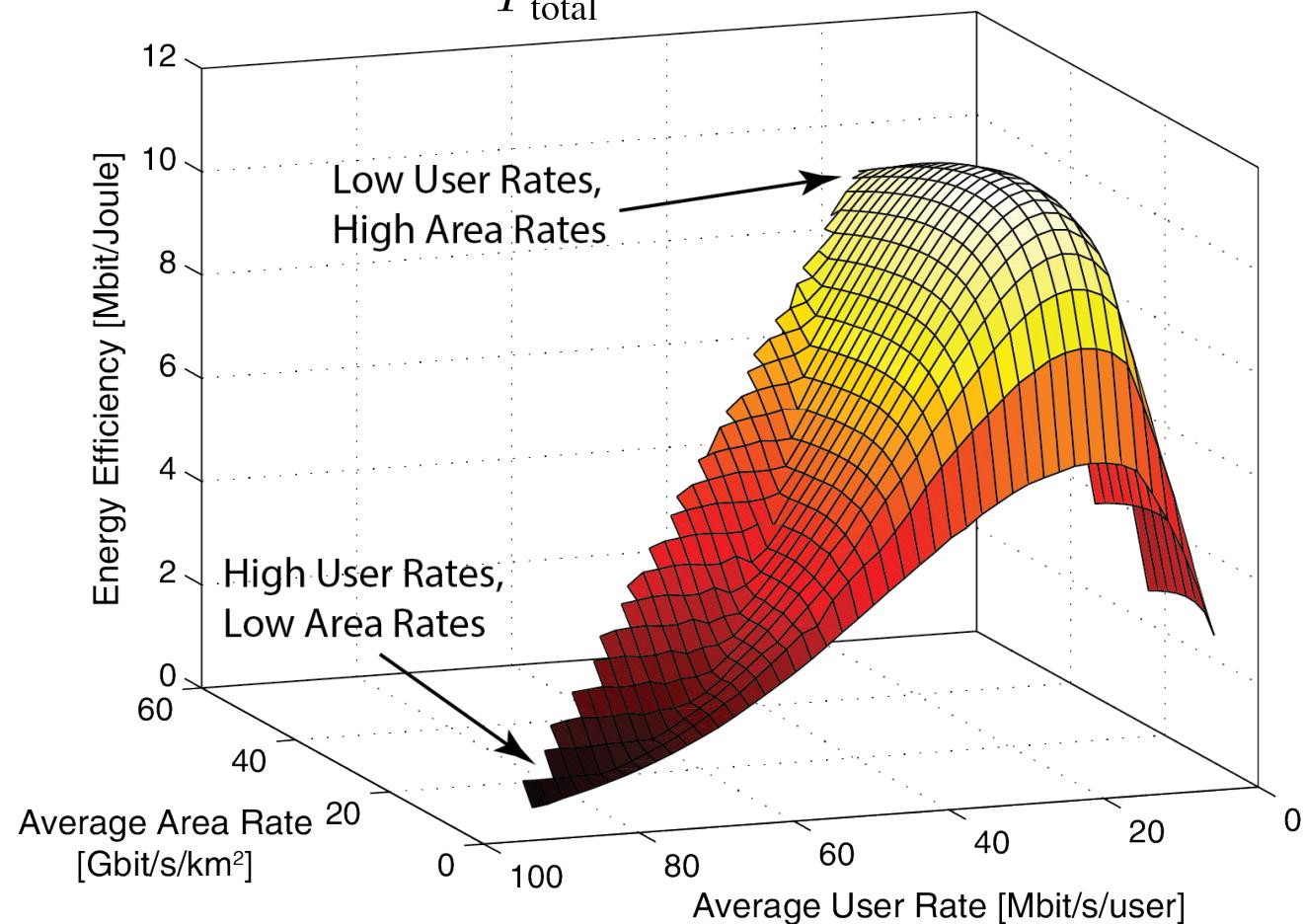
3 Objectives	1. Average user rate	$g_1(\mathbf{x}) = R_{\text{average}}$	[bit/s/user]
	2. Total area rate	$g_2(\mathbf{x}) = \frac{K}{A} R_{\text{average}}$	[bit/s/km <sup>2</sup> ]
	3. Energy-efficiency	$g_3(\mathbf{x}) = \frac{KR_{\text{average}}}{P_{\text{total}}}$	[bit/J]

## Observations

Area and user rates are conflicting objectives

Only energy efficient at high area rates

Different number of users



## Example: Results (2)

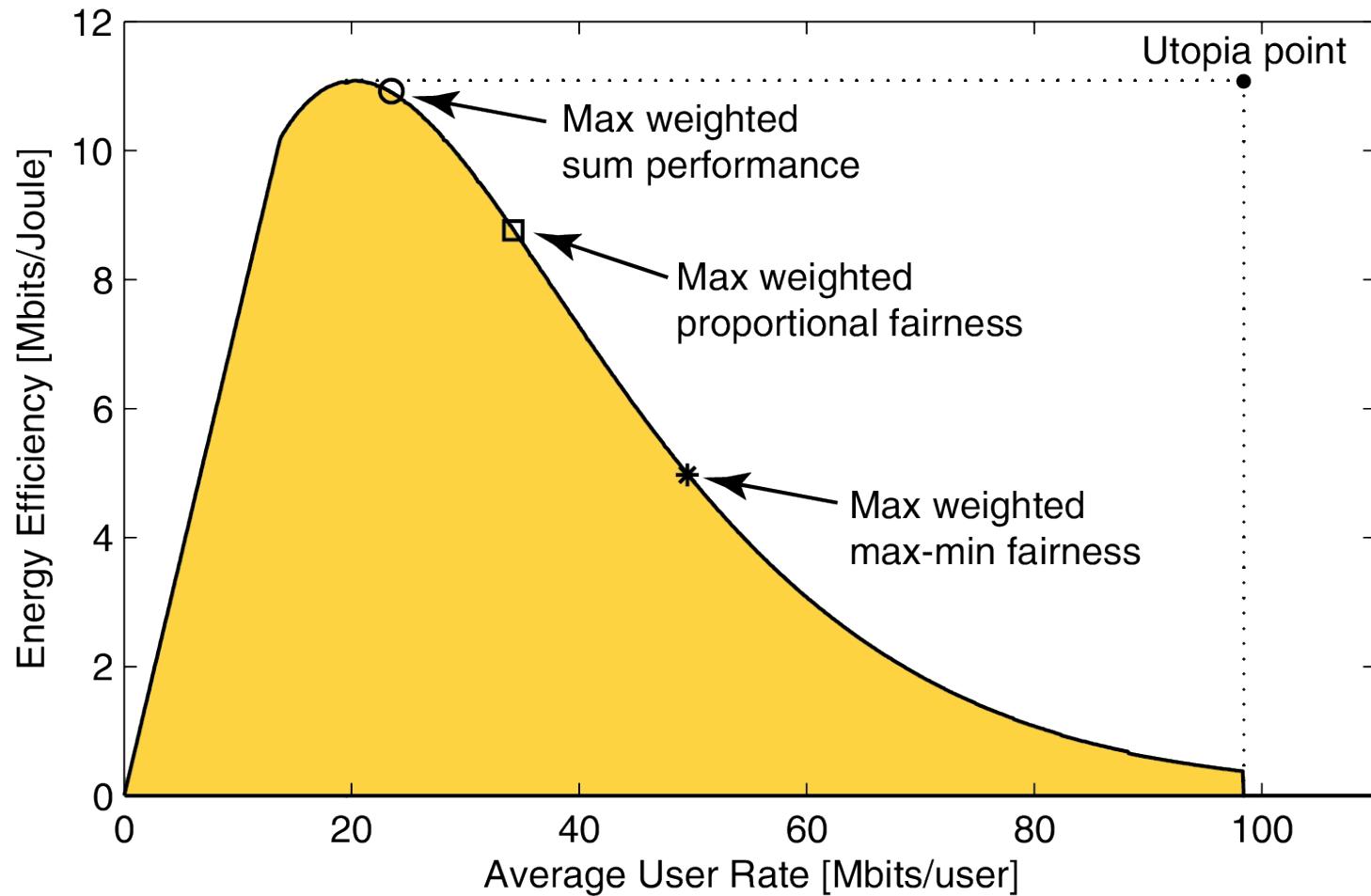
- Energy-Efficiency vs. User Rates

- Utility functions normalized by utopia point

### Observations

Aligned for small user rates

Conflicting for high user rates



Aligned

Conflicting

# Summary



- Multi-Objective Optimization
  - Rigorous way to study problems with multiple performance metrics
- 5G Characterized by Multiple Metrics
  - Calls for multi-objective network design
  - Framework to derive interplay between EE and other performance metrics
  - A way to make informed decisions!

## Further Reading

E. Björnson, E. Jorswieck, M. Debbah, B. Ottersten,  
*"Multi-Objective Signal Processing Optimization: The Way to Balance Conflicting Metrics in 5G Systems,"*  
IEEE SPM, Nov. 2014.

Digital Object Identifier 10.1109/MSP.2014.2330661  
Date of publication: 15 October 2014

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**Part 5**

# **Questions?**

# **Conclusion**

# Conclusions

- What if a Cellular Network is Designed for High Energy-Efficiency?

- Energy-efficiency [bit/Joule] =  $\frac{\text{Average Sum Rate [bit/s/cell]}}{\text{Power Consumption [Joule/s/cell]}}$
- Necessary: Accurate expressions for rate and power consumption
- Design parameters: Number of users, BS antennas, transmit power, BS density, and pilot reuse factor

- Analytical and Numerical Results

- Tractable problem formulation was developed
- Fundamental interplay between system parameters obtained by analysis
- Network densification is the way to high EE
- Small cells and Massive MIMO have complementary benefits
- Feasible to combine these techniques: Massive MIMO  $\neq$  large size

- Multi-Objective Optimization

- Framework to jointly optimize energy-efficiency and other 5G metrics

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# **THANKS!**

**Papers, Presentations, and Simulation Code are  
Available on our Homepages:**

**<http://www.commsys.isy.liu.se/en/staff/emibj29>**

**<http://www.iet.unipi.it/l.sanguinetti/>**