Spatial Frequencies and Degrees of Freedom in Near-Field Communications

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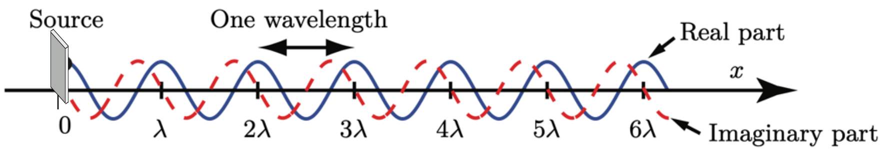
Spatial Frequencies and Degrees of Freedom

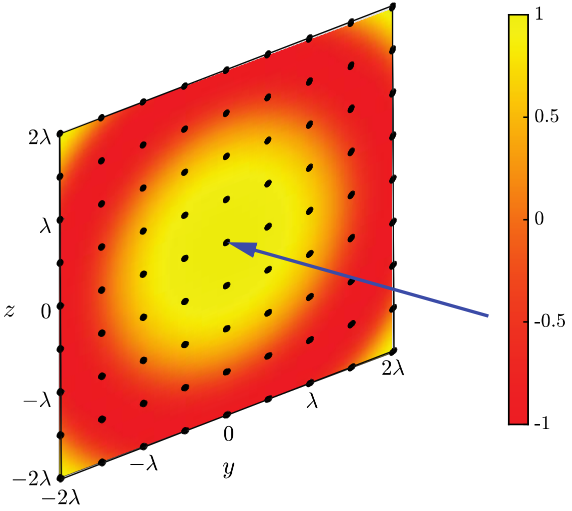
Wireless Signals and Spatial Frequencies

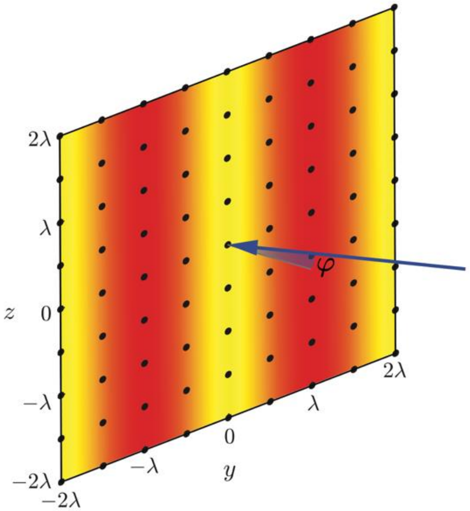
Signal: $s \cdot e^{-j2\pi x/\lambda}$

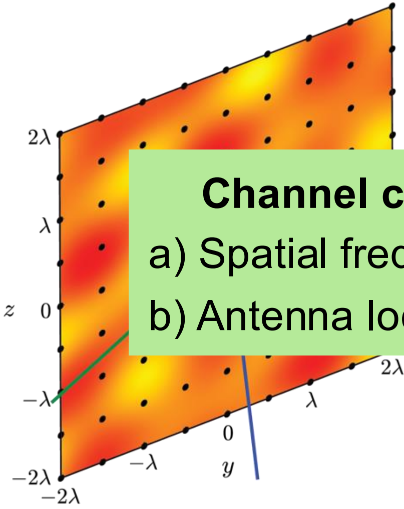
Data



Spatial frequency: $\frac{1}{\lambda}$







Channel characterized by

a) Spatial frequencies variations

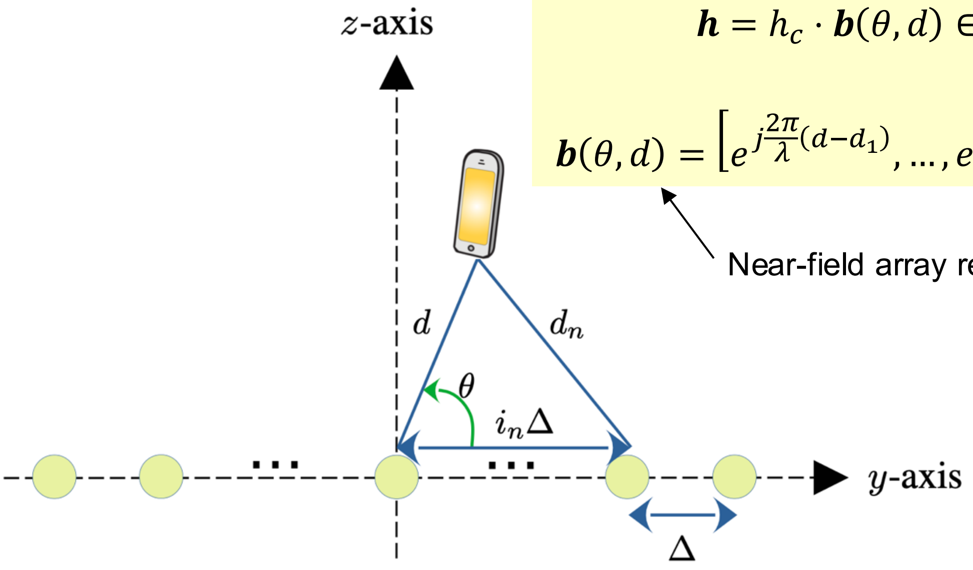
b) Antenna locations (sampling)

User 1: near array in broadside ($d = 4\lambda$)

User 2: far from array, $\varphi = 30°$ azimuth angle

User 3: four far-field propagation paths

A Basic User Setup



Channel vector:

$$\boldsymbol{h} = h_c \cdot \boldsymbol{b}(\theta, d) \in \mathbb{C}^N$$

$$\boldsymbol{b}(\theta, d) = \left[e^{j\frac{2\pi}{\lambda}(d-d_1)}, \ldots, e^{-j\frac{2\pi}{\lambda}(d-d_N)}\right]^T$$

Near-field array response vector

Near-field approximation for antenna n:

$$\begin{aligned}
&(d - d_n) \\
&= d - \sqrt{d^2 + (i_n\lambda/2)^2 - d i_n \lambda \cos(\theta)} \\
&\approx i_n \frac{\lambda}{2}\cos(\theta) - \left(\frac{\lambda}{2}\right)^2 \frac{i_n^2(1-\cos^2(\theta))}{2d}
\end{aligned}$$

Far-field approximation for antenna n:

$$(d - d_n) \approx i_n \frac{\lambda}{2}\cos(\theta)$$

Far-field array response vector:

$$\boldsymbol{a}(\Theta) = \left[e^{j\pi i_1 \Theta}, \ldots, e^{j\pi i_N \Theta}\right]^T,$$
$$\Theta = \cos(\theta)$$

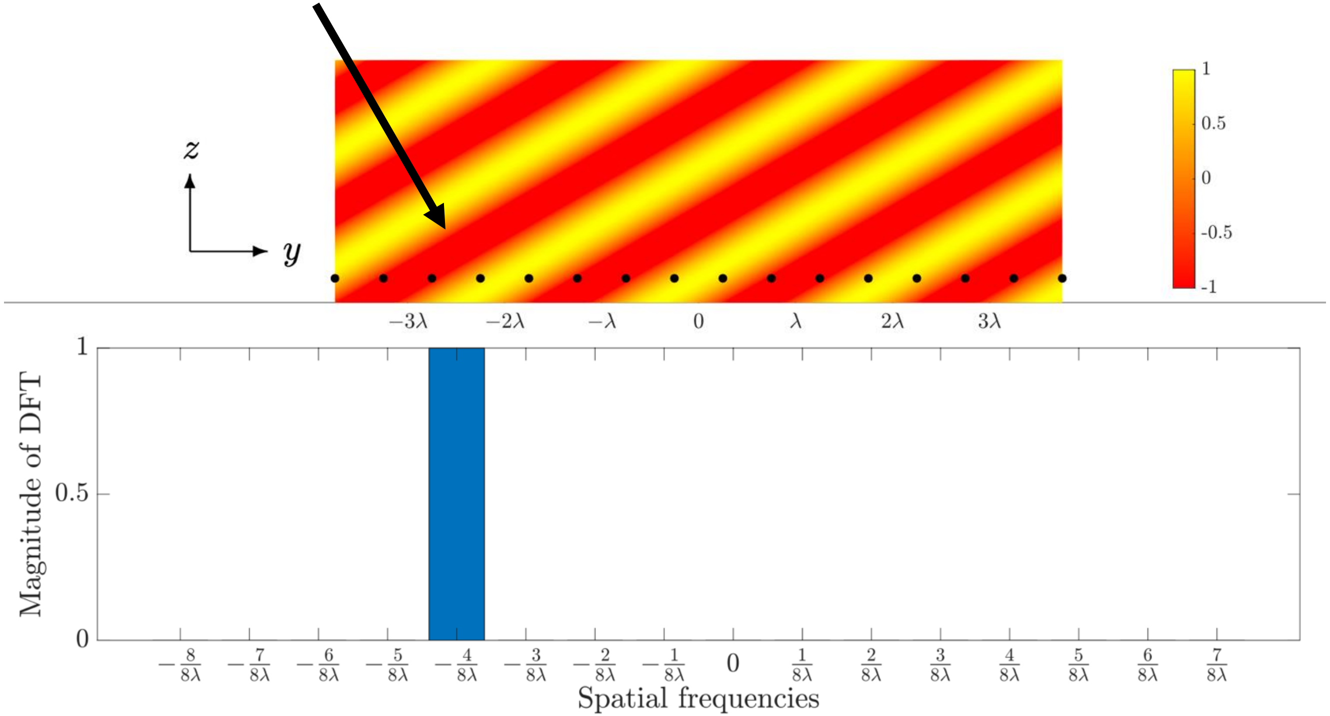
Uniform linear array with N antennas, spacing $\Delta = \lambda/2$

Antenna n located at $(0, i_n\Delta, 0)$

Investigating the Spatial Frequency Content of $\boldsymbol{h}$

Compute the Discrete Fourier transform: $\boldsymbol{h}_{\mathrm{SF}} = \boldsymbol{F} \cdot \boldsymbol{h}$

$$\boldsymbol{F} = [\boldsymbol{a}(\Theta_0), \boldsymbol{a}(\Theta_1), \ldots, \boldsymbol{a}(\Theta_{N-1})]^H \in \mathbb{C}^{N \times N} \quad \text{with } \Theta_n = n/N$$



Example: $N = 16$

Impinging signal with

$$\theta = \frac{2\pi}{3}$$

Spatial frequency:

$$\frac{\Theta}{\lambda} = \frac{\cos(\theta)}{\lambda} = -\frac{4}{8\lambda}$$

Interpretation of the Spatial DFT

Express channel as linear combination of far-field array response vectors $\boldsymbol{h} = \boldsymbol{F}^H \boldsymbol{h}_{\mathrm{SF}}$ ($\Theta_n = n/N$):

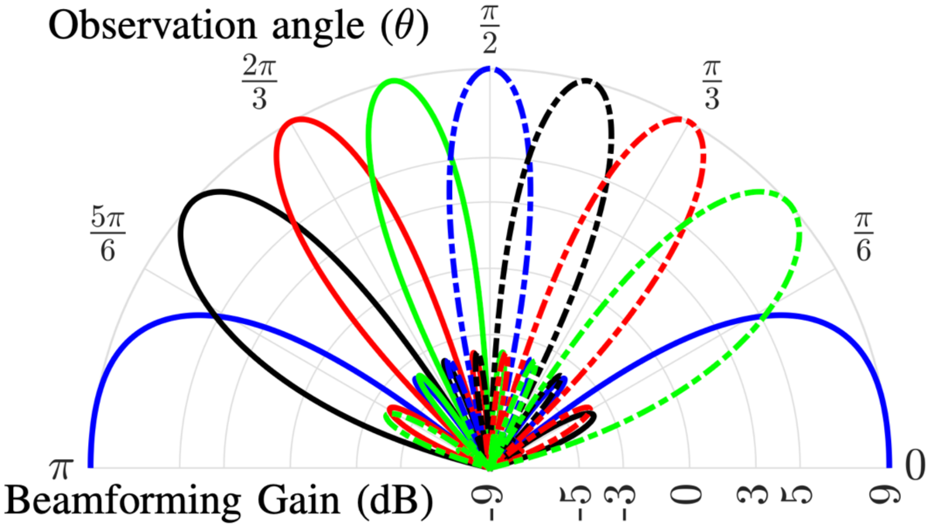
$$\boldsymbol{F}^H = [\boldsymbol{a}(\Theta_0), \boldsymbol{a}(\Theta_1), \ldots, \boldsymbol{a}(\Theta_{N-1})]$$

- Equally spaced $\Theta = \cos(\theta)$
- Equally spaced spatial frequencies Θ/λ

Same number of vectors as antennas

These vectors are orthogonal:

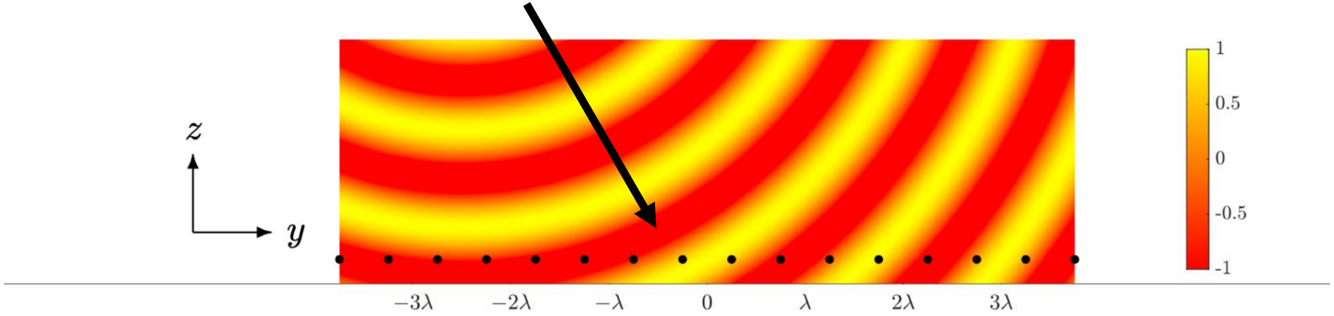
$$\frac{|\boldsymbol{a}^M(\Theta_n)\boldsymbol{a}(\Theta_m)|}{N} = \begin{cases} 1, & n = m \\ 0, & n \neq m \end{cases}$$



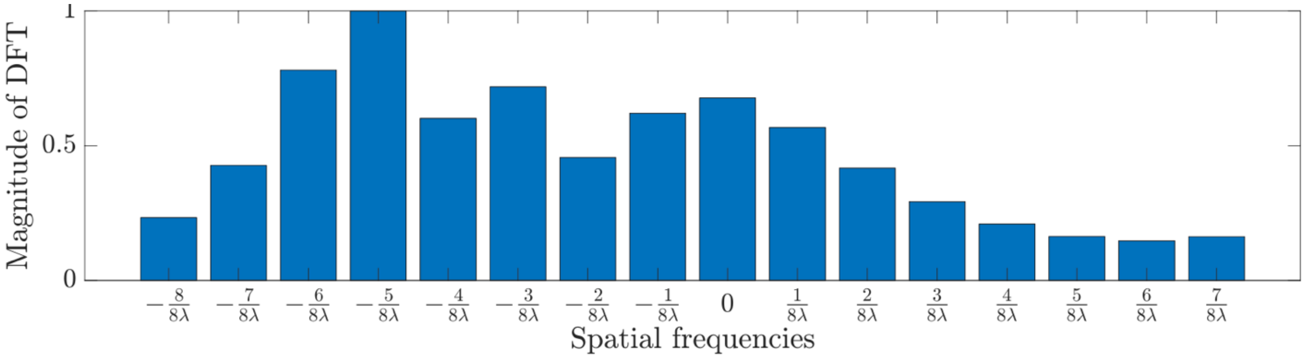
Spatial Frequency Content for a Near-Field

Compute the Discrete Fourier transform: $\boldsymbol{F} \cdot \boldsymbol{h} = h_c \cdot \boldsymbol{F} \cdot \boldsymbol{b}(\theta, d)$

$\boldsymbol{F} = [\boldsymbol{a}(\Theta_0), \boldsymbol{a}(\Theta_1), \dots, \boldsymbol{a}(\Theta_{N-1})]^H \in \mathbb{C}^{N \times N}$ with $\Theta_n = n/N$



$|\boldsymbol{a}^H(\Theta_n)\boldsymbol{b}(\theta, d)|$



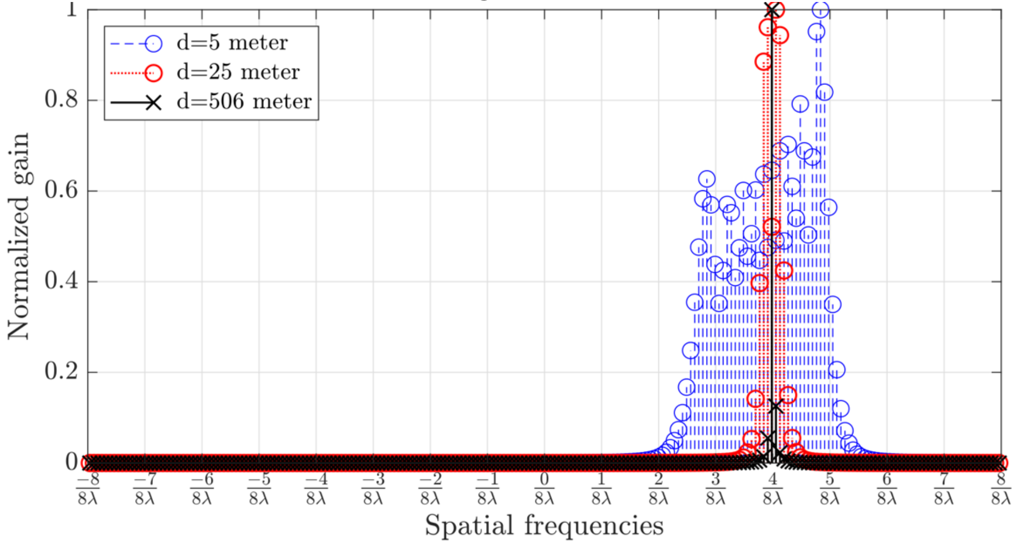
Example: $N = 16$

Impinging signal with
$$\theta = \frac{2\pi}{3}, d = 5\lambda$$

Spherical wave consists of many spatial frequencies

Spatial Bandwidth vs. Propagation Distance

Example: $N = 225,\ \theta = \frac{\pi}{3}$, varying distance d, 15 GHz



Spatial bandwidths:

$d = 5$ m

$d = 25$ m

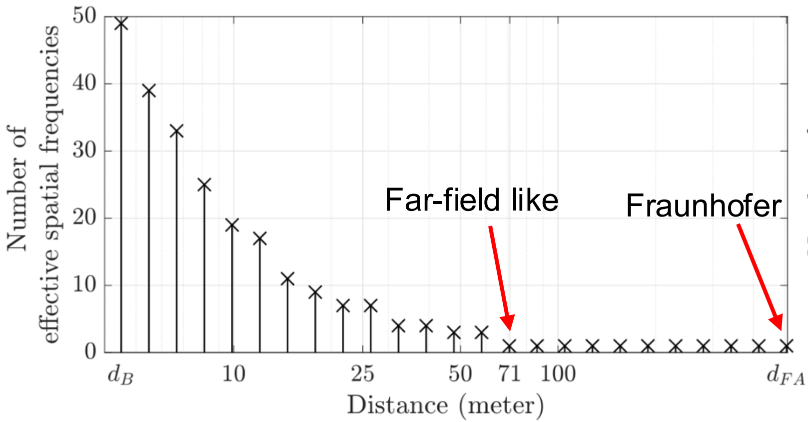
$d = 506$ m

N spatial frequencies

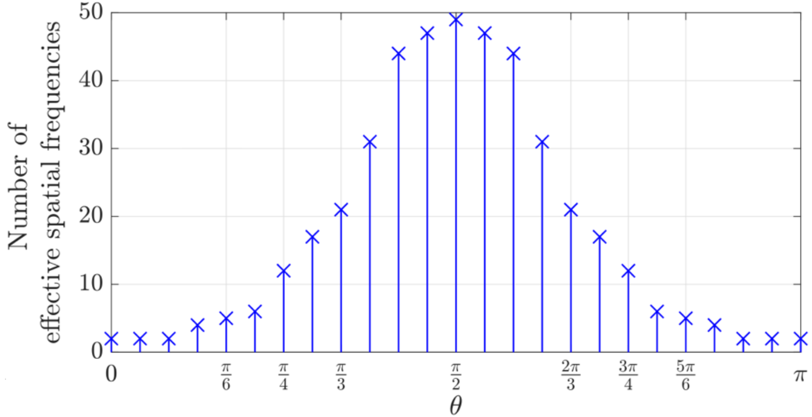
Varying numbers of *effective spatial frequencies*

Number of *Effective Spatial Frequencies* vs. Distance or Angle

Example: $N = 225\,\mathrm{antennas}$, 15 GHz



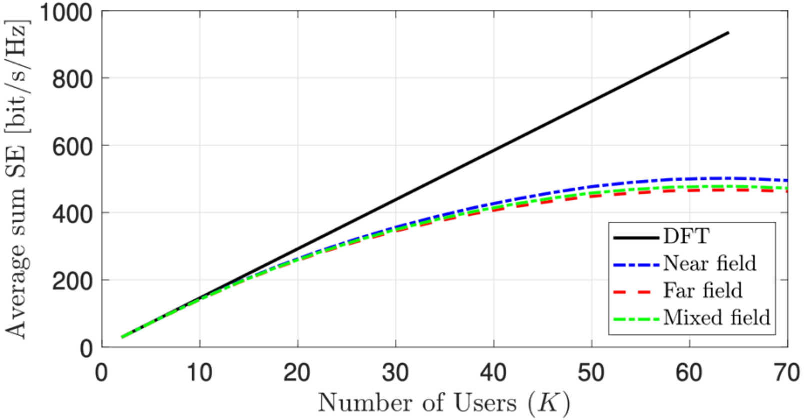
Varying distance, $\theta = \pi/2$

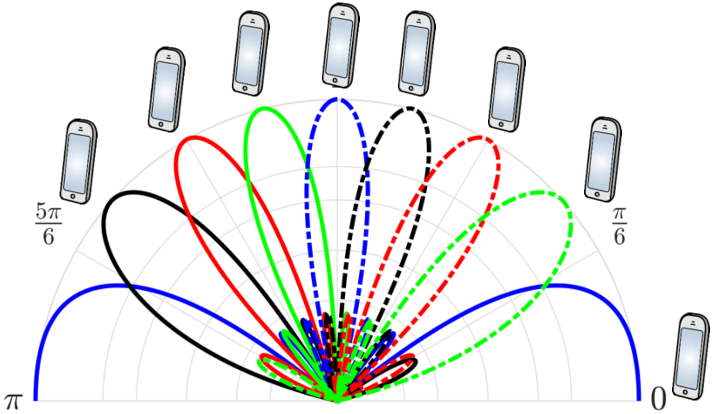


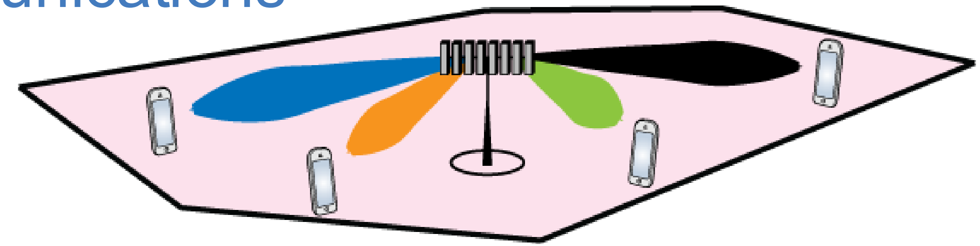
Varying angle, $d = 4.5\ \mathrm{m}$

Impact on Multi-User Communications

Downlink, $N = 64$ antennas, K users, 15 GHz

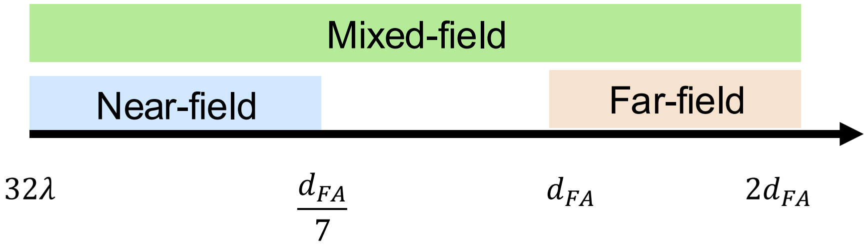






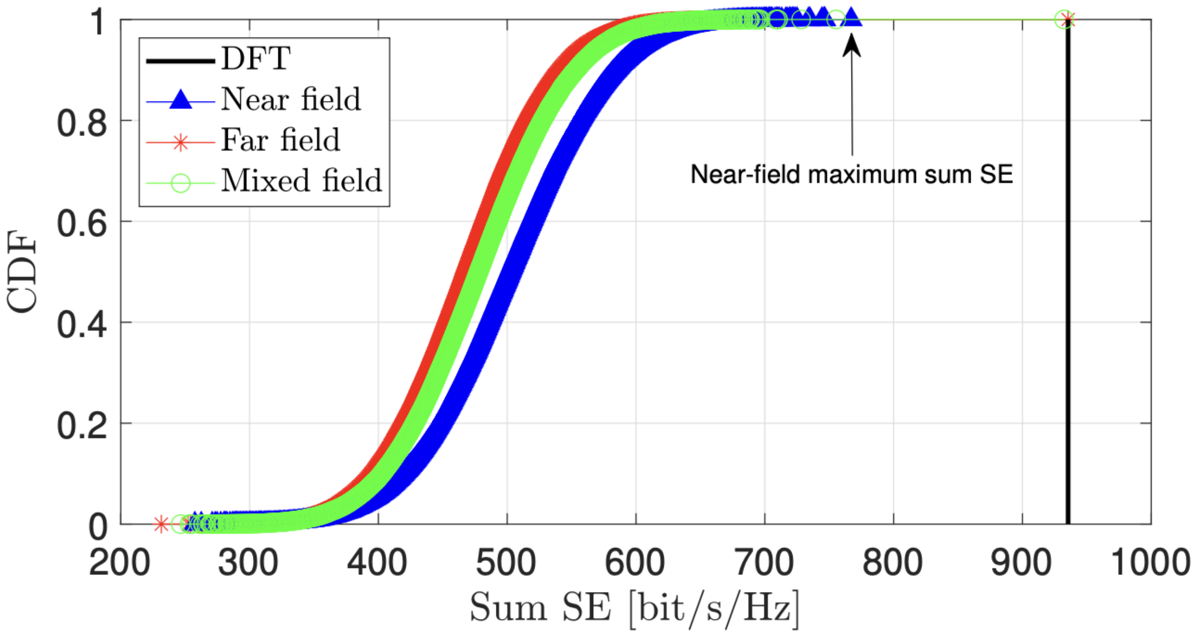
Channels $\boldsymbol{h}_1, \ldots, \boldsymbol{h}_K$ should ideally be orthogonal

Regularized zero-forcing cancels remaining interference



Are There any Near-field Benefits?

Downlink, $N = 64$ antennas, $N = K$ users



Observations

Same degrees-of-freedom

Larger chance that random channels are well separable

General Modeling of Near-Field Channels

Different channel representations:

$$\boldsymbol{h} = \boldsymbol{F}^H \cdot \boldsymbol{h}_{\mathrm{SF}} = [\boldsymbol{a}(\Theta_0), \boldsymbol{a}(\Theta_1), \dots, \boldsymbol{a}(\Theta_{N-1})] \cdot \boldsymbol{h}_{\mathrm{SF}}$$

$$= \underbrace{\int_0^\infty \int_0^\pi g(\theta, d)\boldsymbol{b}(\theta, d)\ \partial\theta\ \partial d}_{\text{Expansion using near-field array responses}}$$

$$\boldsymbol{b}(\theta, d) = \left[e^{j\frac{2\pi}{\lambda}(d-d_1)}, \dots, e^{-j\frac{2\pi}{\lambda}(d-d_N)}\right]^T$$

DFT coefficients

Coefficients

Expansion using near-field array responses

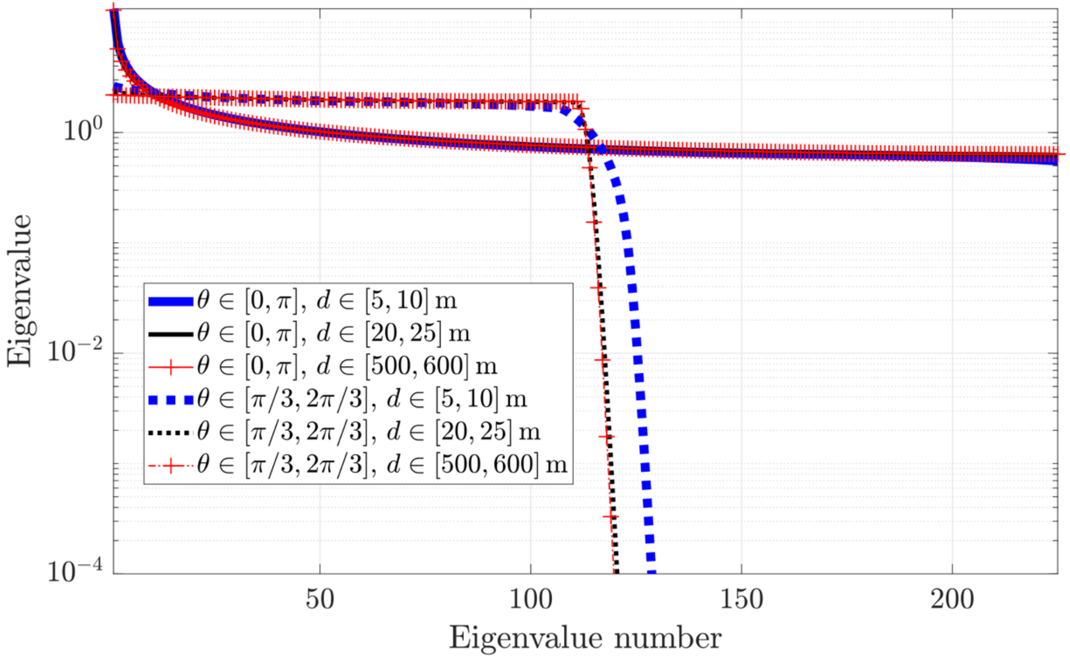
Fading channel model (Non-line-of-sight)

$$\boldsymbol{h} \sim CN(\boldsymbol{0}, \boldsymbol{R})$$

$$\boldsymbol{R} = \beta \int_0^\infty \int_0^\pi f(\theta, d)\boldsymbol{b}(\theta, d)\boldsymbol{b}^H(\theta, d)\ \partial\theta\ \partial d$$

Rank of the Spatial Correlation Matrix

Scatters located in $\theta \in [\theta_1, \theta_2]$ and $d \in [d_1, d_2]$



Rank = Spatial degrees of freedom used by the channel

Observations

Full rank if scattering clusters in all directions

If limited angular range:

Higher rank with near-field scatterers

Summary

youtube.com/wirelessfuture

- N antennas: N-dimensional channels
- Any channel = linear combination of $\boldsymbol{a}(\Theta_n)$ for n equally spaced spatial frequencies (DFT)
    - *Same in near-field and far-field*
- We can serve equally many users in near- and far-field
    - Ideal case: Far-field at angles Θ_n
    - Bigger chance for random users to be compatible
- Any channel = integral of near-field array response