ON THE IMPACT OF SPATIAL CORRELATION AND PRECODER DESIGN ON THE PERFORMANCE OF MIMO SYSTEMS WITH SPACE-TIME CODING

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How does spatial correlation impact the performance? With and without statistical CSI at transmitter.

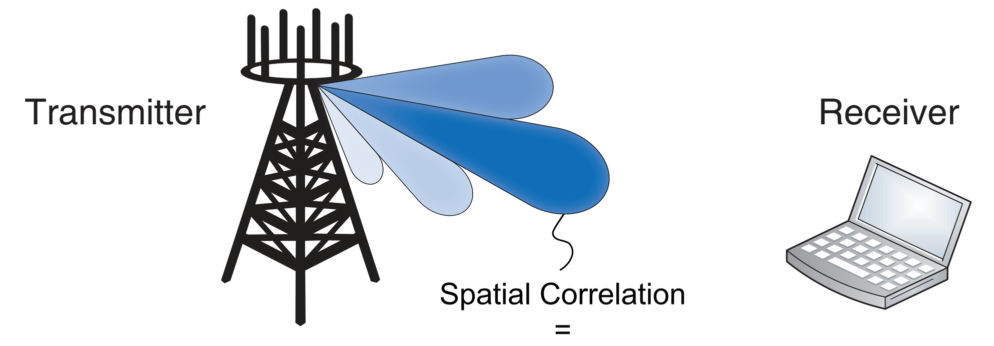
Introduction

We consider a single-user communication system:

- Multiple antennas: n_T at transmitter, n_R at receiver.
- Perfect channel state information (CSI) at receiver.
- No instantaneous CSI at transmitter.

Statistical channel properties important for performance:

- Some spatial directions might on average be more favorable.
- Transmission should be designed based on the statistics.



How will spatial correlation affect the system performance?

System model

Rayleigh fading channel with Kronecker structure:

$$\mathbf{H} = \mathbf{R}_R^{1/2}\widetilde{\mathbf{H}}\mathbf{R}_T^{1/2},$$

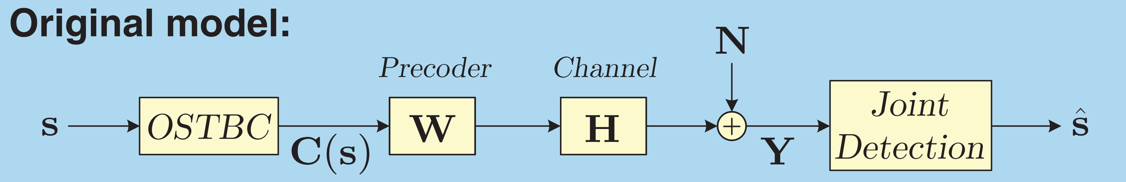
- $\mathbf{R}_T$ transmit correlation matrix, $\mathbf{R}_R$ receive correlation matrix.
- $\widetilde{\mathbf{H}}$ i.i.d. zero-mean complex Gaussian matrix with unit variance.

Transmission with orthogonal space-time block codes (OSTBCs):

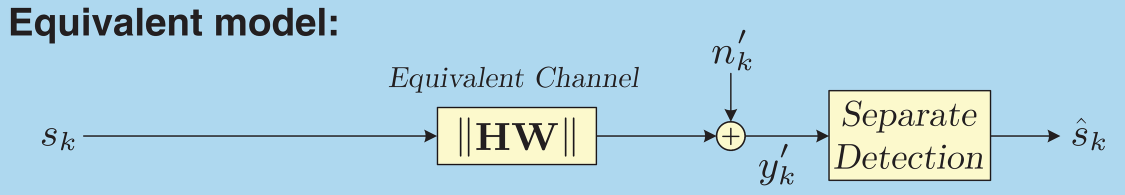
$$\mathbf{Y} = \mathbf{HWC}(\mathbf{s}) + \mathbf{N},$$

- K symbols over T slots: $\mathbf{s} = [s_1, \ldots, s_K]^T$ (coding rate: $\frac{K}{T}$).
- Average symbol power $E\{|s_k|^2\} = \gamma$, spatial dimension B.
- Linear precoder, $\mathbf{W} \in \mathbb{C}^{n_T \times B}$, power constraint $\|\mathbf{W}\|^2 = 1$.

Original model:



Equivalent model:



Performance Measure: SER

Symbol Error Rate (SER) is the performance measure:

- Probability that receiver makes an error in symbol detection.
- Depends on type of symbol constellation, SNR, and statistics.

Definition. *Let* $\mathbf{\Phi} = \mathbf{R}_R \otimes (\mathbf{W}\mathbf{W}^H\mathbf{R}_T)$ *and define*

$$F_{a,b}(g) = \frac{1}{\pi}\int_a^b \frac{d\theta}{\det\left(\mathbf{I} + \frac{\gamma g}{\sin^2(\theta)}\mathbf{\Phi}\right)}, \quad g \geq 0, b \geq a.$$

SER for M-PAM, M-PSK, and M-QAM symbol constellations:

$$\mathrm{SER}_{\mathrm{PAM}} = \frac{2(M-1)}{M}F_{0,\frac{\pi}{2}}(g_{\mathrm{PAM}}),$$

$$\mathrm{SER}_{\mathrm{PSK}} = F_{0,\frac{M-1}{M}\pi}(g_{\mathrm{PSK}}),$$

$$\mathrm{SER}_{\mathrm{QAM}} = \frac{4(\sqrt{M}-1)}{M}\left(F_{0,\frac{\pi}{4}}(g_{\mathrm{QAM}}) + \sqrt{M}F_{\frac{\pi}{4},\frac{\pi}{2}}(g_{\mathrm{QAM}})\right),$$

with $g_{\mathrm{PAM}} = \frac{3}{M^2-1}$, $g_{\mathrm{PSK}} = \sin^2(\frac{\pi}{M})$, and $g_{\mathrm{QAM}} = \frac{3}{2(M-1)}$.

Precoding Design

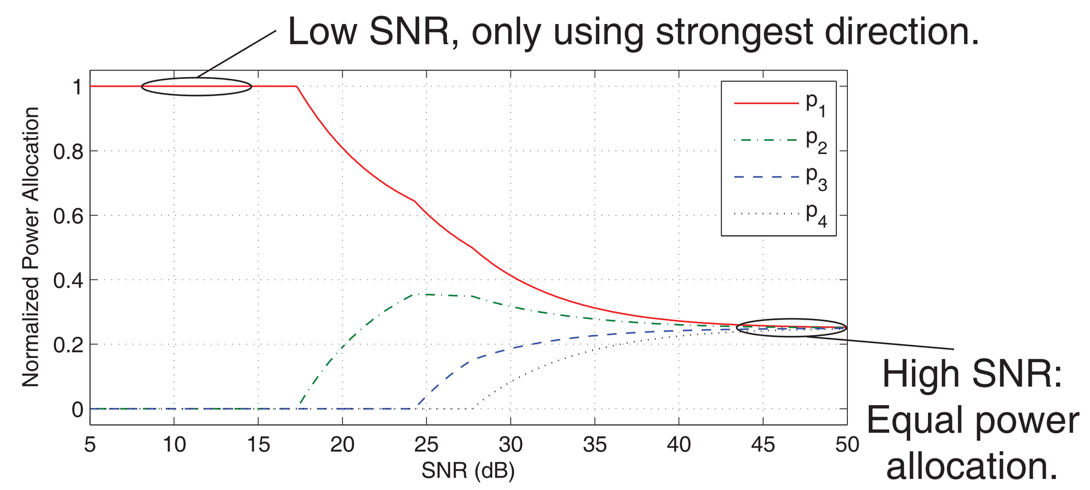
Goal: Minimize SER by design of linear precoder $\mathbf{W}$.

- *Without CSI at transmitter:* Minimize worst-case SER by $B = n_T$ and $\mathbf{W} = \frac{1}{\sqrt{n_T}}\mathbf{I}$.
- *With statistical CSI at transmitter:* Adapt $\mathbf{W}$ and $B \leq n_T$ to the spatial properties.

Optimal precoder structure: $\mathbf{W} = \mathbf{U}_T\mathbf{\Delta}$.

- $\mathbf{\Delta}\mathbf{\Delta}^T = \mathrm{diag}(\sqrt{p_1}, \ldots, \sqrt{p_B}, 0, \ldots, 0)$, $\sum_{k=1}^B p_k = 1$.
- $\mathbf{U}_T$ contain eigenvectors of $\mathbf{R}_T$, decreasing in dominance.

Example of precoder power allocation:



Measure of Spatial Correlation

How to analyze the impact of spatial correlation?

- The determinant in $F_{a,b}(g)$ depends on the eigenvalues of $\mathbf{\Phi}$:

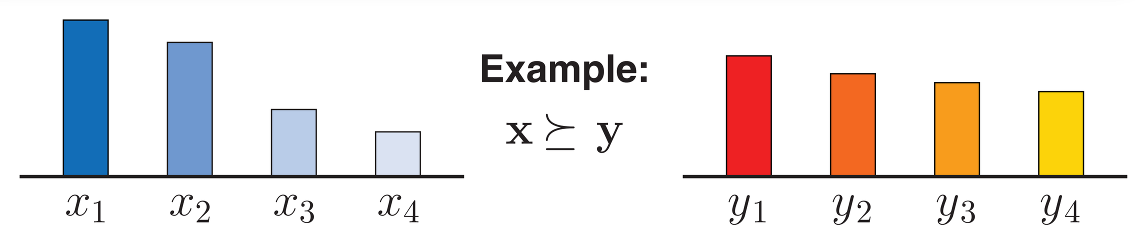
$$\det\left(\mathbf{I} + \frac{\gamma g}{\sin^2(\theta)}\mathbf{\Phi}\right) = \prod_{k=1}^{n_T n_R}\left(1 + \frac{\gamma g}{\sin^2(\theta)}\lambda_k(\mathbf{\Phi})\right),$$

- Different spatial correlations = Different eigenvalue distributions.

Majorization: If two vectors $\mathbf{x}$ and $\mathbf{y}$, with ordered elements $x_1 \geq x_2 \geq \ldots \geq x_M$ and $y_1 \geq y_2 \geq \ldots \geq y_M$, satisfy

$$\sum_{k=1}^m x_k \geq \sum_{k=1}^m y_k, \text{ for } m = 1, \ldots, M-1, \text{ and } \sum_{k=1}^M x_k = \sum_{k=1}^M y_k,$$

then $\mathbf{x}$ majorizes $\mathbf{y}$. Notation: $\mathbf{x} \succeq \mathbf{y}$.



Interpretation for eigenvalues of correlation matrices:

- $\mathbf{x}$ and $\mathbf{y}$ contain eigenvalues of correlation matrices $\mathbf{R}_1$ and $\mathbf{R}_2$.
- $\mathbf{x} \succeq \mathbf{y}$ means $\mathbf{R}_1$ is more spatially correlated than $\mathbf{R}_2$.

Schur-convex and Schur-concave: A function $f(\cdot): \mathbb{R}^M \rightarrow \mathbb{R}$ is

- *Schur-convex* if $\mathbf{x} \succeq \mathbf{y}$ implies $f(\mathbf{x}) \geq f(\mathbf{y})$.
- *Schur-concave* if $\mathbf{x} \succeq \mathbf{y}$ implies $f(\mathbf{x}) \leq f(\mathbf{y})$.

Dependence on Spatial Correlation

Without CSI at transmitter:

Theorem 1. The function $F_{a,b}(g)$ is Schur-convex with respect to

- Eigenvalues of $\mathbf{R}_T$ for fixed $\mathbf{R}_R$.
- Eigenvalues of $\mathbf{R}_R$ for fixed $\mathbf{R}_T$.

With statistical CSI at transmitter:

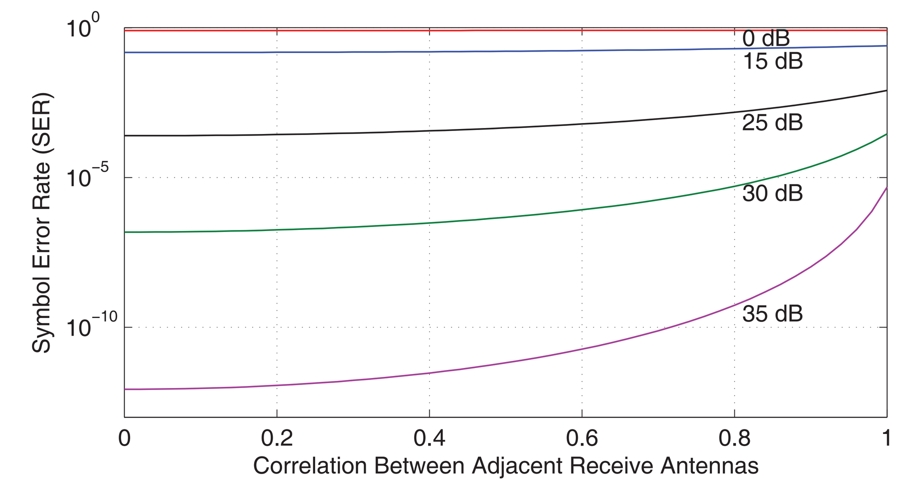
Theorem 2. The function $F_{a,b}(g)$ with optimal precoding is

- Schur-concave w.r.t. eigenval. of $\mathbf{R}_T$ for fixed $\mathbf{R}_R$ at low SNR.
- Schur-convex w.r.t. eigenval. of $\mathbf{R}_T$ for fixed $\mathbf{R}_R$ at high SNR.
- Schur-convex w.r.t. eigenval. of $\mathbf{R}_R$ for fixed $\mathbf{R}_T$ at all SNRs.

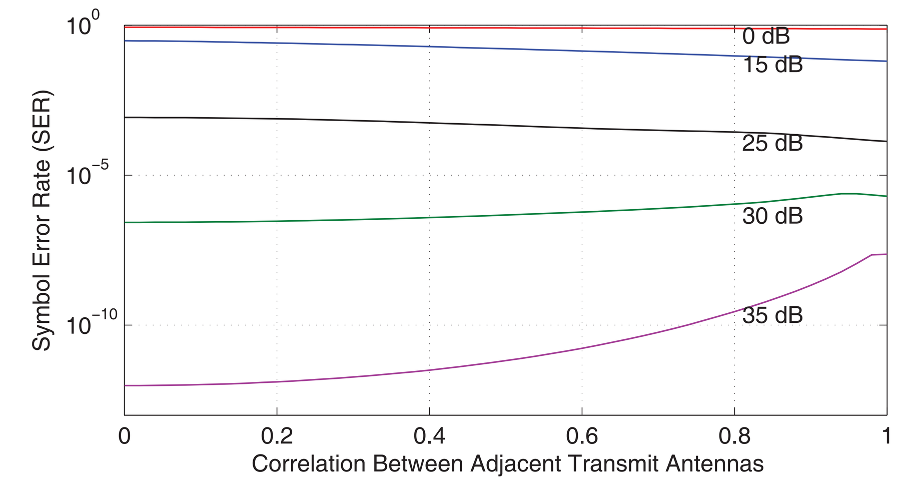
Numerical Examples

When is spatial correlation useful in practice?

- Transmit side correlation shown to be desirable at low SNR.
- How to interpret this in practice?
- At least $\gamma \leq \frac{1}{g\,\mathrm{tr}(\mathbf{R}_T)\mathrm{tr}(\mathbf{R}_R)}$ corresponds to low SNR.
- From simulation: Approximate low SNR behavior at all SNRs.



The SER as function of correlation between adjacent receive antennas in a four-antenna array. The transmitter has $n_T = 4$ and a fixed correlation of 0.5. The SER is Schur-convex at all SNRs.



The SER as function of correlation between adjacent transmit antennas in a four-antenna array. The receiver has $n_R = 4$ and a fixed correlation of 0.5. The SER is Schur-concave at low and medium SNR and becomes Schur-convex at high SNR.

Conclusions & Contributions

Analysis of impact of spatial correlation with different CSI on

- Symbol Error Rate of M-PAM, M-PSK, and M-QAM.

Without CSI at transmitter:

- Spatial correlation always degrades performance.

With statistical CSI at transmitter and optimal precoding:

- Spatial correlation at *transmitter* improves performance.
- Spatial correlation at *receiver* still degrades performance.