Social Data Science: Machine Learning & Econometrics

Exercise class 9

May 14, 2020

Todays quick warmup

Consider a sequence $\{(x_k, y_k)\}$ of points in $\mathbb{R}^2$ where

- $\forall k : x_{k+1} > x_k$ and $y_{k+1} > y_k$
- $\forall k : x_k \in [0, 1]$
- $\forall k : y_k \in [0, 1]$

Q1: Write a function `simulate(n)` that simulates such a sequence with length n by drawing x and y according to $z_{k+1} - z_k \sim U(0, 2/n)$ and clipping any $z_k > 1$ to the bounding box.

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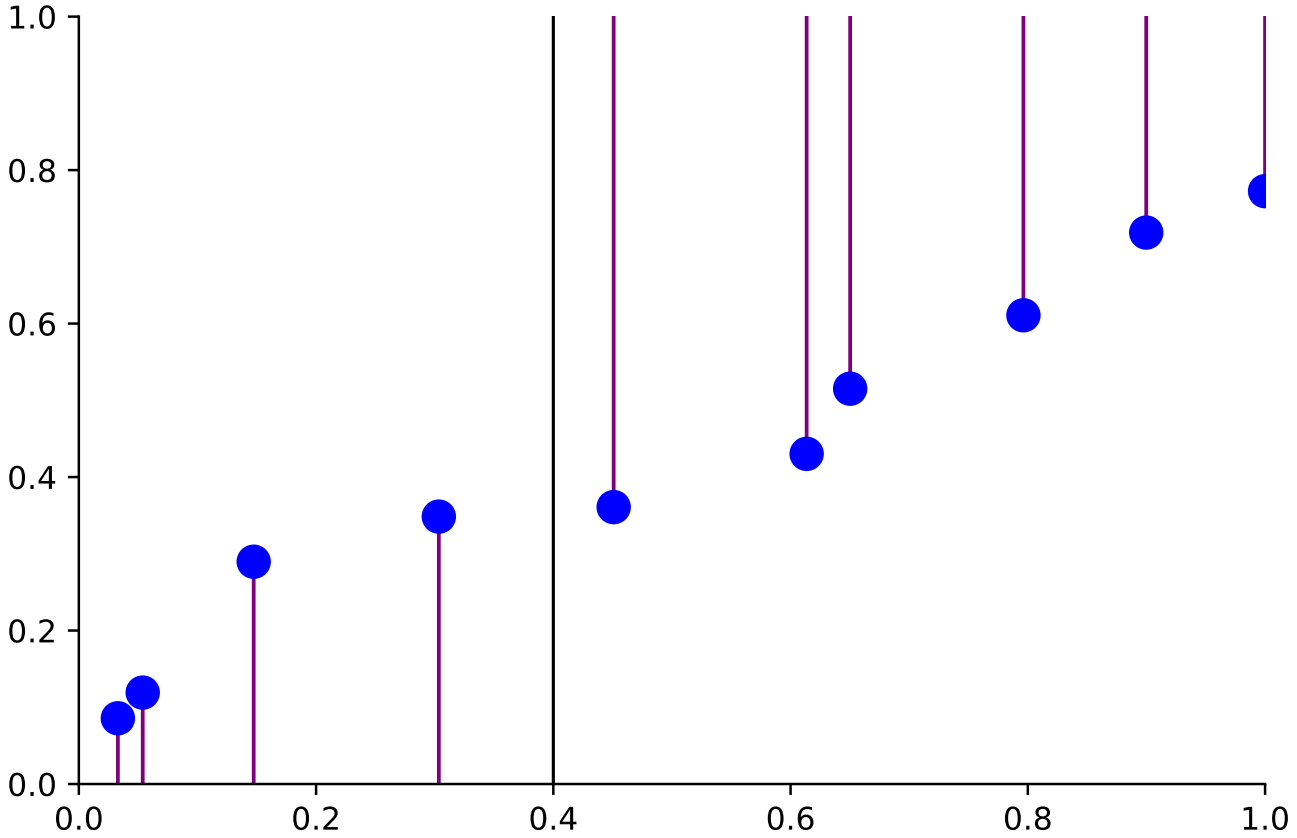
Q2: By the *divided sum at* x_0 of such a sequence we mean the the sum

$$s(x_0) = \sum_k y_k \mathbf{1}_{(x_k < x_0)} + (1 - y_k)\mathbf{1}_{(x_k \geq x_0)} \tag{1}$$

I.e. the sum of the vertical bars marked in the figure below. Write a function `minimize` that takes as its inputs a simulated sequence and returns x_k^* that minimizes $s(x_0 = x_k)$ as well as the corresponding y_k^*.



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Q3: Run simulate with $n =$`np.arange(10,1000,10)` and plot the resulting y_k^* against n. What value does y_k^* seem to converge to? Does this make sense to you?