Beginselen van Programmeren
Exercise Session 5:
functions

E1. Write a function to calculate the sum of all natural numbers up to a given number n: $1 + 2 + \ldots + n$. Your function should return 5050 for the input parameter 100.

E2a. Write a function to calculate the nth number of a Fibonacci sequence $1, 1, 2, 3, 5, 8, 13, 21, \ldots$; where the kth number is the sum of the $k-1$th number and $k-2$th number. Your function should return 13 when it is called by 7 as the input parameter.

E2b. Write a function given nth number of a Fibonacci sequence, returns n. Your function should return 7 when is called by 13 as the input parameter and -1 when the input parameter is not in the Fibonacci sequence.

E2c. Write a function to calculate the sum of the Fibonacci statements in a given range. Your function should return 29 when is called by input parameters 4 and 7, because $3+5+8+13 = 29$.

E3a. A prime number is one that is not divisible by any number other than one and itself. Write a function to check whether a number is prime or not. Your function should return $True$ when it is called by 13 and $False$ when it is called by 14.

E3b. Use the function above to write a function that prints all prime numbers up to a given number. Your function should print $2, 3, 5, 7, 11, 13$ when it is called by 15.

E4a. A perfect number is one that is equal to the sum of its divisors excluding the number itself. The first perfect number is 6 because $1 + 2 + 3 = 6$. Write a function that checks a number is perfect or not.

E4b. Name three perfect numbers other than 6.

E5a. Write a function that computes the factorial of a positive integer. $n! = 1 * 2 * 3 * \ldots * n$.

E5b. Use the above function to write a function `estimateExp` that calculates e^x. An estimate of e^x can be computed as the power series

$$e^x = \sum_{i=0}^{n} \frac{x^i}{i!} \tag{1}$$

The function `estimateExp` returns an estimate of e^x. Write two versions of the function `estimateExp`. The first version accepts a parameter n, this is the number of terms you need to include in the sum. The second version accepts a parameter delta. Keep on adding terms to the sum until the term is smaller than delta.
TIP: In the second version of `estimateExp`, you can write the computed term to the screen,

this can help you find errors in your functions.

E6. Write a function to calculate the number of k-combinations of an n-set, $C(n,k)$, that can be obtained by the following formula:

$$C(n,k) = \frac{P(n,k)}{P(k,k)} \text{ , where } P(n,k) = \frac{n!}{(n-k)!} \tag{2}$$

E7. In the last exercise session you wrote a program to check if the number of a credit card is valid and if not it suggested the right number. Rewrite that program using functions.

E8a. Write a function that draws a pyramid on your screen. The function needs two arguments: the first is the height of the pyramid. The second argument is optional: if not supplied, the symbol '#' should be used to draw the pyramid. A sample pyramid with height 7 and no symbol as the input looks like this:

```
      #
     ###
    #####
   #######
  #########
 ###########
#############
```
However if you call the function with height 5 and symbol '*', the pyramid looks like this:
```
    *
   ***
  *****
 *******
*********
```

E8b. Rewrite your function such that it returns a string that contains your pyramid instead of printing all symbols directly to the screen. Use this function in a program that asks for a height, a symbol and then prints out the string returned from your function. To add a newline character to your string, take a look at the section 2.5 in your book.

E9a. In the US, postal codes are rewritten in bar codes. Each postal code has 5 digits and a check digit. The check digit is computed as follows: Add all 5 digits together, the check digit is the number you have to add to this sum so it is a multiple of 10. For example, postal code 95104 has a sum of 19, so the check digit is 1 to make the sum equal to 20. Write a function that returns the check digit. Don't forget to divide the work in several functions!

E9b. Digits are recoded with bars | and colons :. The following table shows the coding of all digits. Write a function that encodes a digit in a string of colons and bars.

E9c. Write a function that given a postal code, generates a barcode for the five digits and the checkcode. Your function should return |:|:: :|:|: :::|| ||::: :|::| :::|| when called by 95104 as the input parameter.

Table 1: Encoding of digits with bars

Digit	Bar 1	Bar 2	Bar 3	Bar 4	Bar 5
1	:	:	:	|	|
2	:	:	|	:	|
3	:	:	|	|	:
4	:	|	:	:	|
5	:	|	:	|	:
6	:	|	|	:	:
7	|	:	:	:	|
8	|	:	:	|	:
9	|	:	|	:	:
0	|	|	:	:	: