

# Hypothesis Testing: One-Page Reference

BMSC 620

## What is hypothesis testing?

**Goal:** Use sample data to evaluate evidence against a specific claim about a population parameter.

- The claim we test is the **null hypothesis**
  - We assume the null is true and ask: *How surprising is our data under that assumption?*
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## Key hypotheses

### Null hypothesis ( $H_0$ )

- Represents the status quo or specific claim
- Uses an equals sign

$$H_0 : \mu = \mu_0$$

### Alternative hypothesis ( $H_A$ )

- Represents what we're looking for evidence in favor of
- Uses  $\neq$ ,  $<$ , or  $>$

$$H_A : \mu \neq \mu_0 \quad (\text{two-sided})$$

$$H_A : \mu < \mu_0 \quad \text{or} \quad H_A : \mu > \mu_0 \quad (\text{one-sided})$$

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## Significance level ( $\alpha$ )

- $\alpha$  is the threshold for “strong evidence”
- Chosen **before** seeing the data
- Most common:  $\alpha = 0.05$

**Interpretation:** If  $H_0$  is true, we are willing to reject it incorrectly at most  $\alpha \times 100\%$  of the time.

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## Assumptions for a one-sample t-test

1. Observations are **independent**
  2. Data are approximately **normal** OR sample size is large ( $n \geq 30$ , CLT applies)
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## Test statistic (what comes from the data)

The t-statistic measures how far the sample mean is from the null value, in standard error units:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

- Comes from the **sample**
  - **Random** (varies from study to study)
  - Under  $H_0$ , follows a t-distribution with  $df = n - 1$
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## Critical value (what defines “too extreme”)

- Comes from the **t-distribution**
- Depends on  $\alpha$  and degrees of freedom
- **Fixed** before seeing the data

$$t^* = t_{1-\alpha/2, df} \quad (\text{two-sided})$$

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## Decision rules (three equivalent ways)

### 1. Test-statistic approach

**Reject  $H_0$  if:**  $|t_{\text{obs}}| > t^*$

### 2. P-value approach

**P-value:** Probability of observing a test statistic as extreme as (or more extreme than) what we saw, assuming  $H_0$  is true.

**Reject  $H_0$  if:** p-value  $< \alpha$

### 3. Confidence interval approach (two-sided tests)

**Reject  $H_0$  if:** the  $(1 - \alpha) \times 100\%$  confidence interval does **not** contain  $\mu_0$

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### What p-values mean (and don't mean)

**P-value IS:**

- A measure of how surprising the data are if  $H_0$  were true

**P-value is NOT:**

- The probability that  $H_0$  is true
  - The probability you made a mistake
  - A measure of effect size or importance
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### Common language to use

“We reject the null hypothesis”

“We fail to reject the null hypothesis”

“We accept the null hypothesis”

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### One-sample t-test in R

```
t.test(x, mu = mu0, alternative = "two.sided", conf.level = 0.95)
```

**Key output:**

- t-statistic
  - degrees of freedom
  - p-value
  - confidence interval
  - sample mean
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## Reporting results (example)

A one-sample t-test was conducted to assess whether mean body temperature differs from 98.6°F. The sample ( $n = 130$ ) had a mean of 98.25°F (SD = 0.733). The test was statistically significant,  $t(129) = -5.45$ ,  $p < 0.001$ , with a 95% confidence interval of [98.12, 98.38], indicating that the population mean body temperature is lower than 98.6°F.

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## Big picture reminder

- Hypothesis tests and confidence intervals use the **same information**
- A small p-value does **not** imply practical importance
- Always **interpret results in context** ““