Riemannian Continuous Normalizing Flows

Learning expressive probability distributions on manifolds

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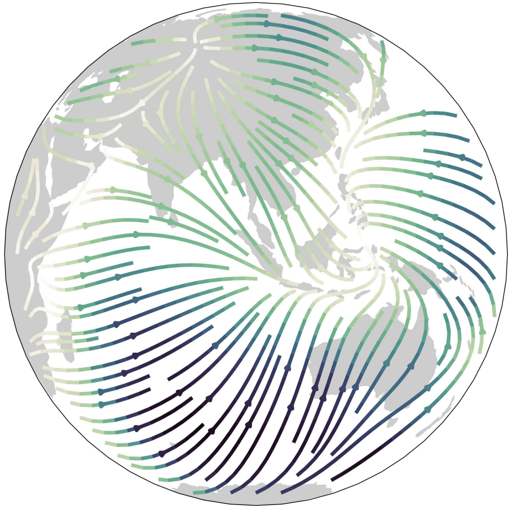
FACEBOOK AI



Overview:

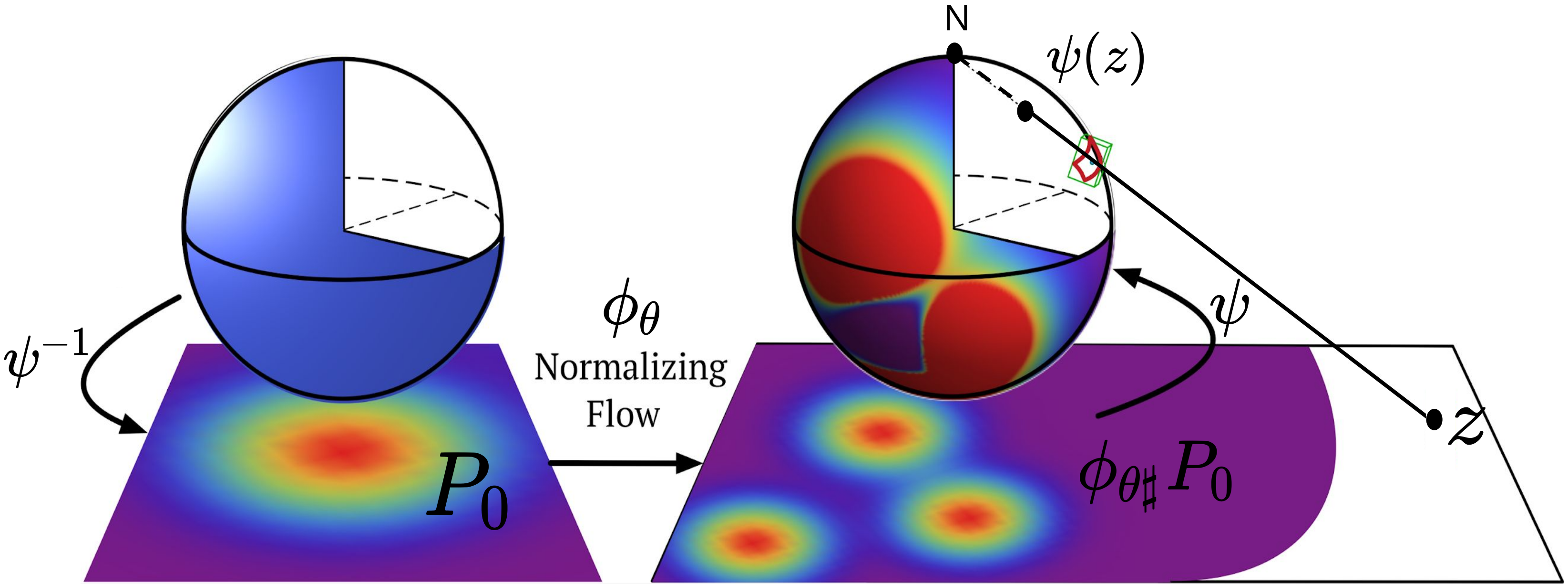
Extend continuous normalizing flows to manifolds

- Flows on manifolds as solution of ODEs
- Continuous change of variables for manifold-valued variables
- Assess model expressiveness on earth science data

Motivation

$$\psi : \mathbb{R}^d \to \mathcal{M}$$

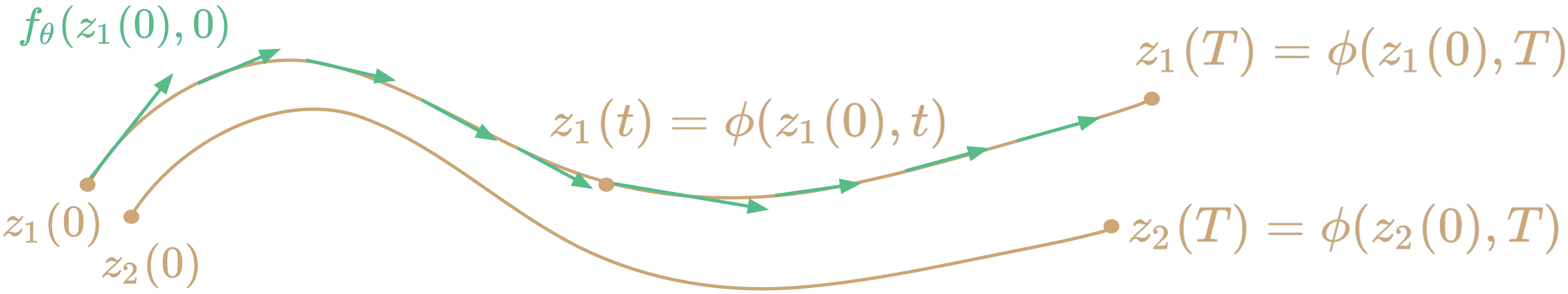
$$P_\theta = (\psi \circ \phi_\theta)_\sharp P_0$$



Gemici et al, 2016

Ordinary Differential Equation & Flow

ODE $\frac{dz(t)}{dt} = f_\theta(z(t), t)$ vector field



Flow $\phi(z) := \phi(z, T)$ $\phi : \mathcal{M} \to \mathcal{M}$ $\mathcal{C}^1$ - diffeomorphism if f_θ is $\mathcal{C}^1$ and bounded

Model $P_\theta = \phi_\sharp P_0$

Likelihood

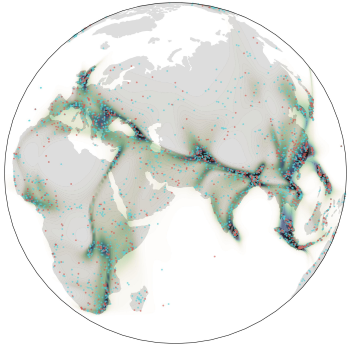
Continuous change of variable

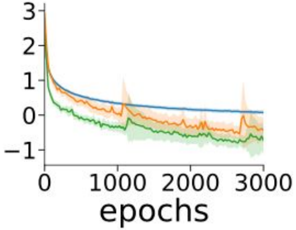
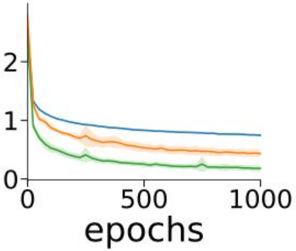
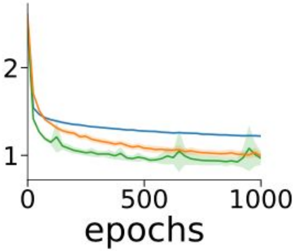
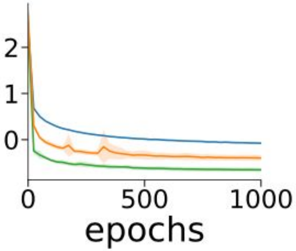
$$\frac{\partial \log p_\theta(\boldsymbol{z}(t))}{\partial t} = -\operatorname{div}(\boldsymbol{f}_\theta(\boldsymbol{z}(t), t)) = -|G(\boldsymbol{z}(t))|^{-\frac{1}{2}} \operatorname{tr}\left(\frac{\partial \sqrt{|G(\boldsymbol{z}(t))|}\boldsymbol{f}_\theta(\boldsymbol{z}(t), t)}{\partial \boldsymbol{z}}\right) \quad (1)$$

with z some local coordinates, e.g. $z = (\theta, \phi)$ polar coordinates for $\mathbb{S}^2$

Stochastic estimator $\operatorname{div}(\boldsymbol{f}_\theta(\boldsymbol{z}(t), t)) = |G(\boldsymbol{z}(t))|^{-\frac{1}{2}} \mathbb{E}_{p(\boldsymbol{\epsilon})}\left[\boldsymbol{\epsilon}^\top \frac{\partial \sqrt{|G(\boldsymbol{z}(t))|}\boldsymbol{f}_\theta(\boldsymbol{z}(t), t)}{\partial \boldsymbol{z}} \boldsymbol{\epsilon}\right]$

Experimental Results: Earth Science



	Volcano	Earthquake	Flood	Fire
Mixture vMF	$-0.31_{\pm 0.07}$	$0.59_{\pm 0.01}$	$1.09_{\pm 0.01}$	$-0.23_{\pm 0.02}$
Stereographic	$-0.64_{\pm 0.20}$	$0.43_{\pm 0.06}$	$0.99_{\pm 0.04}$	$-0.40_{\pm 0.06}$
Riemannian	$-0.97_{\pm 0.15}$	$\mathbf{0.18}_{\pm 0.05}$	$\mathbf{0.90}_{\pm 0.03}$	$\mathbf{-0.66}_{\pm 0.05}$
Learning curves				
Data size	829	6124	4877	12810

Thank you for your attention!

Don’t be shy and attend our poster session for a chat :)