Optimization

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February 28, 2019

Problem Statement

Minimize,

$$x_1 + x_2$$

Subject to,

$$2x_1 + x_2 \geq 8$$

$$2x_1 + 5x_2 \geq 10$$

$$x_1, x_2 \geq 0$$

Using Simplex.

Since this problem is not in standard form, (i.e constraints $Ax \leq b$, with $b \geq 0$), This problem has to be solved using Two-Phase simplex method. The above problem statement can be written as,
Minimize,

$$x_1 + x_2$$

Subject to,

$$2x_1 + x_2 - s_1 + a_1 = 8$$

$$2x_1 + 5x_2 - s_2 + a_2 = 10$$

$$x_1, x_2, s_1, s_2, a_1, a_2 \geq 0$$

where, s_1, s_2 are slack variables and a_1, a_2 are artificial variables.

Phase 1

Minimize,

$$a_1 + a_2$$

Subject to,

$$2x_1 + x_2 - s_1 + a_1 = 8$$

$$2x_1 + 5x_2 - s_2 + a_2 = 10$$

$$x_1, x_2, s_1, s_2, a_1, a_2 \geq 0$$

		x_1	x_2	s_1	s_2	a_1	a_2	
	c	0	0	0	0	1	1	
1	a_1	2	1	-1	0	1	0	8
1	a_2	2	5	0	-1	0	1	10
	z	4	6	-1	-1	1	1	
	c - z	-4	-6	1	1	0	0	

		x_1	x_2	s_1	s_2	a_1	a_2	
	c	0	0	0	0	1	1	
1	a_1	8/5	0	-1	1/5	1	-1/5	6
0	x_2	2/5	1	0	-1/5	0	1/5	2
	z	8/5	0	-1	1/5	1	-1/5	
	c - z	-8/5	0	1	1/5	0	4/5	

		x_1	x_2	s_1	s_2	a_1	a_2	
	c	0	0	0	0	1	1	
0	x_1	1	0	-5/8	1/8	5/8	-1/8	30/8
0	x_2	0	1	1/4	-1/4	-1/4	1/4	1/2
	z	0	0	0	0	0	0	
	c - z	0	0	0	0	1	1	

Since the cost functions are all positive, feasible solution exists and this will be the table for Phase 2.

Phase 2

		x_1	x_2	s_1	s_2	
	c	1	1	0	0	
1	x_1	1	0	-5/8	1/8	15/4
1	x_2	0	1	1/4	-1/4	1/2
	z	1	1	-3/8	-1/8	
	c - z	0	0	3/8	1/8	

Since the values in c-z row are all positive, the feasible solution is $x_1 = 3.75, x_2 = 0.5$.
The minimum value = 4.25

Code

```
import cvxpy as cp
import numpy as np

x = cp.Variable(2)

constraints = [
                2*x[0] + x[1] >= 8,
                2*x[0] + 5*x[1] >= 10,
                x >=0
              ]

objective = cp.Minimize(x[0] + x[1])

problem = cp.Problem(objective,constraints)

problem.solve()

print((problem.value))
```

Result

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3.75 \\ 0.50 \end{bmatrix}$$

Minimum Value is, $x_1 + x_2 = 3.75 + 0.50 = 4.25.$