Optimization

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Problem Statement

Minimize,

$$-x_{11} - 2x_{12} - 5x_{22}$$

Subject to,

$$2x_{11} + 3x_{12} + x_{22} = 7$$

$$x_{11} + x_{12} \geq 1$$

$$x_{11}, x_{12}, x_{22} \geq 0$$

$$\begin{bmatrix} x_{11} & x_{12} \\ x_{12} & x_{22} \end{bmatrix} \succcurlyeq 0$$

Using cvxopt.

The above problem statement can be written as, Minimize

$$\begin{bmatrix} -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{22} \end{bmatrix}$$

Subject to,

$$\begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{12} \\ x_{22} \end{bmatrix} = 7$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} x_{11} + \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} x_{12} + \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} x_{22} \preccurlyeq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix} x_{11} + \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix} x_{12} + \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} x_{22} \leq \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

Code

```
from cvxopt import matrix
from cvxopt import solvers

c = matrix([-1.,-2.,-5.])
G = [ matrix([[-1., 0., 0., 0.],
              [ 0., -1., -1., 0.],
              [0.,  0.,  0., -1.]])  ]
G += [ matrix([[-1., -1., 0., 0.],
               [ -1., 0., -1., 0.],
               [0.,  0.,  0., -1.]]) ]
Aval = matrix([2.,3.,1.],(1,3))
bval = matrix([7.])
h = [ matrix([[0., 0.], [0., 0.]]) ]
h += [ matrix([[-1., 0.], [0., 0.]]) ]
sol = solvers.sdp(c, Gs=G, hs=h,A=Aval, b=bval)

print(sol['x'])
```

Result

$$\begin{bmatrix} x_{11} \\ x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} 0.675 \\ 0.429 \\ 4.3 \end{bmatrix}$$

Minimum Value is -23.341.