

**Zadanie 1**  
 $A\vec{v} = -\vec{v} \Rightarrow A^T\vec{v} = \vec{v}$  } *dim to same wyrażenia (bo macierzyne)*  
 $A\vec{v} = \vec{v} \Rightarrow A^T\vec{v} = \vec{v}$   
Skoro  $\dim(V_1) = 1$  dla dowolnego wektora  $\vec{v}$ ,  
to mamy  $\downarrow_{V_1} 1 + \uparrow_{V_1} x = 1$

**Zadanie 3.**  
 $\vec{u} = (u_1, \dots, u_n)$   
 $\vec{v} = (v_1, \dots, v_n)$   
I  $\|\vec{u}\| \cdot \|\vec{v}\| = \sqrt{\sum u_i^2} \cdot \sqrt{\sum v_i^2} = \sqrt{(\sum u_i^2)(\sum v_i^2)}$   
II  $|\vec{u} \cdot \vec{v}| = \sum u_i v_i$   
III  $|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \cdot \|\vec{v}\| \Leftrightarrow$   
 $\Leftrightarrow \sum u_i v_i \leq \sqrt{(\sum u_i^2)(\sum v_i^2)} \Leftarrow$  IV  
 $\Leftrightarrow (\sum u_i v_i)^2 \leq (\sum u_i^2)(\sum v_i^2) \Leftrightarrow$  nierówności Cauchy'ego-Schwarza V

**Zadanie 2.**  
 $\vec{u} \cdot (M\vec{v}) = \vec{u} \cdot \begin{pmatrix} \sum u_i v_i \\ \sum u_i v_i \\ \sum u_i v_i \end{pmatrix} = \sum (u_i \cdot \sum u_i v_i) = \sum \sum u_i u_i v_i = \sum \sum u_i^2 v_i$   
 $(M^T \vec{u}) \cdot \vec{v} = \vec{v} \cdot \begin{pmatrix} \sum u_i v_i \\ \sum u_i v_i \\ \sum u_i v_i \end{pmatrix} = \sum (v_i \cdot \sum u_i v_i) = \sum \sum v_i u_i v_i = \sum \sum v_i^2 u_i$   
*tex = uxi*

Dla symetrycznej  $\vec{u}(M\vec{v}) = (M\vec{u})\vec{v}$  oczywiście  
**Udowodnij:** jeśli  $\vec{u}(M\vec{v}) = (M\vec{u})\vec{v}$  to M symetryczna  
Niech  $\vec{u}_i = \begin{cases} u_i = 1 \\ u_x = 0 \text{ dla } x \neq i \end{cases}$   
 $\vec{v}_i = \begin{cases} v_i = 0 \\ v_x = 1 \text{ dla } x = i \end{cases}$   
 $\vec{u}_i(M\vec{v}_j) = \sum \sum u_{ij} m_{jk} v_k = \sum u_{ij} m_{ji} v_i = u_{ij} m_{ji} = m_{ji}$   
 $(M\vec{u})\vec{v} = \sum \sum u_{ij} m_{ij} v_j = \dots = m_{ij}$   
Skoro  $\forall i,j \vec{v}$  to równości to  $m_{ij} = m_{ji}$

**Zadanie 8.**  
 $\langle p, q \rangle = \int_0^1 p(x) q(x) dx$   
 $\langle x^i, x^j \rangle = \int_0^1 x^i \cdot x^j dx = \int_0^1 x^{i+j} dx = \begin{cases} i+j=0: 0 \\ i+j>0: \frac{x^{i+j+1}}{i+j+1} \Big|_0^1 = \frac{1}{i+j+1} - \frac{(-1)^{i+j+1}}{i+j+1} \end{cases}$   
 $i+j=0: \frac{1}{2} - \frac{1}{2} = 0$   
 $i+j=1: \frac{1}{3} - \frac{1}{3} = 0$   
 $i+j=2: \frac{1}{4} - \frac{1}{4} = 0$   
 $i+j=3: \frac{1}{5} - \frac{1}{5} = 0$   
 $i+j=4: \frac{1}{6} - \frac{1}{6} = 0$   
 $i+j=5: \frac{1}{7} - \frac{1}{7} = 0$   
 $i+j=6: \frac{1}{8} - \frac{1}{8} = 0$

**Zadanie 11.**  $\vec{e}_1, \dots, \vec{e}_n$  są ortonormalnymi  
 $L = \sum_{i=1}^n |\langle \vec{e}_i, \vec{v} \rangle|^2 \leq \|\vec{v}\|^2$   
 $L = \sum_{i=1}^n |\langle \vec{e}_i, \vec{v} \rangle|^2 = \sum_{i=1}^n (\sum_{k=1}^n e_{ik}^2 u_k^2) = \sum_{i=1}^n (\sum_{k=1}^n e_{ik}^2) (\sum_{k=1}^n u_k^2) = \sum_{i=1}^n (\sum_{k=1}^n u_k^2) = L \cdot \sum_{k=1}^n u_k^2$   
 $P = (\sum u_i^2)^2$   
 $\|\vec{v}_E\|$  wynika z definicji, nie jake nie weryfikacji to będzie mijsze  
 $\vec{v}_E = [\langle \vec{e}_1, \vec{v} \rangle, \dots, \langle \vec{e}_n, \vec{v} \rangle, \langle \vec{v}, \vec{v} \rangle, \dots, \langle \vec{v}, \vec{v} \rangle]$   
 $\sum |\langle \vec{e}_i, \vec{v} \rangle|^2 \leq \|\vec{v}_E\|^2 = \sum |\langle \vec{v}, \vec{v} \rangle|^2 = \|\vec{v}\|^2$

$\langle x^i, x^j \rangle = \int_{-1}^1 x^i \cdot x^j dx = \int_{-1}^1 x^{i+j} dx = \frac{x^{i+j+1}}{i+j+1} \Big|_{-1}^1 = \frac{1^{i+j+1}}{i+j+1} - \frac{(-1)^{i+j+1}}{i+j+1}$   
 $i+j=0: \frac{1}{1} - \frac{1}{1} = 0$   
 $i+j=1: \frac{1}{2} - \frac{1}{2} = 0$   
 $i+j=2: \frac{1}{3} - \frac{1}{3} = 0$   
 $i+j=3: \frac{1}{4} - \frac{1}{4} = 0$   
 $i+j=4: \frac{1}{5} - \frac{1}{5} = 0$   
 $i+j=5: \frac{1}{6} - \frac{1}{6} = 0$   
 $i+j=6: \frac{1}{7} - \frac{1}{7} = 0$   
 $i+j=7: \frac{1}{8} - \frac{1}{8} = 0$   
 $i+j=8: \frac{1}{9} - \frac{1}{9} = 0$   
 $i+j=9: \frac{1}{10} - \frac{1}{10} = 0$   
 $i+j=10: \frac{1}{11} - \frac{1}{11} = 0$   
 $i+j=11: \frac{1}{12} - \frac{1}{12} = 0$   
 $i+j=12: \frac{1}{13} - \frac{1}{13} = 0$   
 $i+j=13: \frac{1}{14} - \frac{1}{14} = 0$   
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 $i+j=227: \frac{1}{228} - \frac{1}{228} = 0$   
 $i+j=228: \frac{1}{229} - \frac{1}{229} = 0$   
 $i+j=229: \frac{1}{230} - \frac{1}{230} = 0$   
 $i+j=230: \frac{1}{231} - \frac{1}{231} = 0$   
 $i+j=231: \frac{1}{232} - \frac{1}{232} = 0$   
 $i+j=232: \frac{1}{233} - \frac{1}{233} = 0$   
 $i+j=233: \frac{1}{234} - \frac{1}{234} = 0$   
 $i+j=234: \frac{1}{235} - \frac{1}{235} = 0$   
 $i+j=235: \frac{1}{236} - \frac{1}{236} = 0$   
 $i+j=236: \frac{1}{237} - \frac{1}{237} = 0$   
 $i+j=237: \frac{1}{238} - \frac{1}{238} = 0$   
 $i+j=238: \frac{1}{239} - \frac{1}{239} = 0$   
 $i+j=239: \frac{1}{240} - \frac{1}{240} = 0$   
 $i+j=240: \frac{1}{241} - \frac{1}{241} = 0$   
 $i+j=241: \frac{1}{242} - \frac{1}{242} = 0$   
 $i+j=242: \frac{1}{243} - \frac{1}{243} = 0$   
 $i+j=243: \frac{1}{244} - \frac{1}{244} = 0$   
 $i+j=244: \frac{1}{245} - \frac{1}{245} = 0$   
 $i+j=245: \frac{1}{246} - \frac{1}{246} = 0$   
 $i+j=246: \frac{1}{247} - \frac{1}{247} = 0$   
 $i+j=247: \frac{1}{248} - \frac{1}{248} = 0$   
 $i+j=248: \frac{1}{249} - \frac{1}{249} = 0$   
 $i+j=249: \frac{1}{250} - \frac{1}{250} = 0$   
 $i+j=250: \frac{1}{251} - \frac{1}{251} = 0$   
 $i+j=251: \frac{1}{252} - \frac{1}{252} = 0$   
 $i+j=252: \frac{1}{253} - \frac{1}{253} = 0$   
 $i+j=253: \frac{1}{254} - \frac{1}{254} = 0$   
 $i+j=254: \frac{1}{255} - \frac{1}{255} = 0$   
 $i+j=255: \frac{1}{256} - \frac{1}{256} = 0$   
 $i+j=256: \frac{1}{257} - \frac{1}{257} = 0$   
 $i+j=257: \frac{1}{258} - \frac{1}{258} = 0$   
 $i+j=258: \frac{1}{259} - \frac{1}{259} = 0$   
 $i+j=259: \frac{1}{260} - \frac{1}{260} = 0$   
 $i+j=260: \frac{1}{261} - \frac{1}{261} = 0$   
 $i+j=261: \frac{1}{262} - \frac{1}{262} = 0$   
 $i+j=262: \frac{1}{263} - \frac{1}{263} = 0$   
 $i+j=263: \frac{1}{264} - \frac{1}{264} = 0$   
 $i+j=264: \frac{1}{265} - \frac{1}{265} = 0$   
 $i+j=265: \frac{1}{266} - \frac{1}{266} = 0$   
 $i+j=266: \frac{1}{267} - \frac{1}{267} = 0$   
 $i+j=267: \frac{1}{268} - \frac{1}{268} = 0$   
 $i+j=268: \frac{1}{269} - \frac{1}{269} = 0$   
 $i+j=269: \frac{1}{270} - \frac{1}{270} = 0$   
 $i+j=270: \frac{1}{271} - \frac{1}{271} = 0$   
 $i+j=271: \frac{1}{272} - \frac{1}{272} = 0$   
 $i+j=272: \frac{1}{273} - \frac{1}{273} = 0$   
 $i+j=273: \frac{1}{274} - \frac{1}{274} = 0$   
 $i+j=274: \frac{1}{275} - \frac{1}{275} = 0$   
 $i+j=275: \frac{1}{276} - \frac{1}{276} = 0$   
 $i+j=276: \frac{1}{277} - \frac{1}{277} = 0$   
 $i+j=277: \frac{1}{278} - \frac{1}{278} = 0$   
 $i+j=278: \frac{1}{279} - \frac{1}{279} = 0$   
 $i+j=279: \frac{1}{280} - \frac{1}{280} = 0$   
 $i+j=280: \frac{1}{281} - \frac{1}{281} = 0$   
 $i+j=281: \frac{1}{282} - \frac{1}{282} = 0$   
 $i+j=282: \frac{1}{283} - \frac{1}{283} = 0$   
 $i+j=283: \frac{1}{284} - \frac{1}{284} = 0$   
 $i+j=284: \frac{1}{285} - \frac{1}{285} = 0$   
 $i+j=285: \frac{1}{286} - \frac{1}{286} = 0$   
 $i+j=286: \frac{1}{287} - \frac{1}{287} = 0$   
 $i+j=287: \frac{1}{288} - \frac{1}{288} = 0$   
 $i+j=288: \frac{1}{289} - \frac{1}{289} = 0$   
 $i+j=289: \frac{1}{290} - \frac{1}{290} = 0$   
 $i+j=290: \frac{1}{291} - \frac{1}{291} = 0$   
 $i+j=291: \frac{1}{292} - \frac{1}{292} = 0$   
 $i+j=292: \frac{1}{293} - \frac{1}{293} = 0$   
 $i+j=293: \frac{1}{294} - \frac{1}{294} = 0$   
 $i+j=294: \frac{1}{295} - \frac{1}{295} = 0$   
 $i+j=295: \frac{1}{296} - \frac{1}{296} = 0$   
 $i+j=296: \frac{1}{297} - \frac{1}{297} = 0$   
 $i+j=297: \frac{1}{298} - \frac{1}{298} = 0$   
 $i+j=298: \frac{1}{299} - \frac{1}{299} = 0$   
 $i+j=299: \frac{1}{300} - \frac{1}{300} = 0$   
 $i+j=300: \frac{1}{301} - \frac{1}{301} = 0$   
 $i+j=301: \frac{1}{302} - \frac{1}{302} = 0$   
 $i+j=302: \frac{1}{3$