Tema I - Ex. 3

Ichim Teodora & Radu Mihai-Emilian

(a) T arbore $\implies \begin{cases} T\ conex \implies rang M_T = n-1 \\ T\ are\text{(n-1)}muchii \implies M_T = (n-1) \times n \end{cases}$

$\implies$ *Daca vom scoate o coloana* $M(n-1)\times(n-1)) \implies$ *matrice patratica*

Fie C *si* D doua astfel de matrici.

Consideram ca :

C va contine coloanele *1,2,3,...,n-1*
D va contine coloanele *1,2,3,...,n-2,n*

C *si* D vor avea primele *n-2* coloane identice

Daca in C la ultima coloana vom aduna celelalte coloane atunci ultima coloana a lui C va deveni de doua ori ultima coloana a lui $D \implies$ det(C)=2 * det(D)
Procedand in acest mod vom obtine o relatie intre toti determinantii matricelor M=(n-1)$\times$ (n-1) $\implies$ daca unul va fi 0 $\implies$

$\implies$ toti vor avea 0 & rang(M_T)=n-1 $\implies$ toti determinantii vor fi $\neq 0$
$\implies$ matricele vor fi nesingulare

(b) C circuit $\implies M_c = \begin{pmatrix} 1 & 1 & 0 & 0 & ... & 0 \\ 0 & 1 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & 1 & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ 1 & 0 & 0 & 0 & ... & 1 \end{pmatrix}$ Daca dezvoltam determinantul dupa ultima linie :
det(M_c)=$(-1)^{1+n} * det(M_1) + (-1)^{n+n} * det(M_2)$

Observam ca $M_1 = {}^tM_2 \implies det(M_1) = det(M_2)$
Prin indepartarea unei muchii din C, acesta va fi in continuare conex, dar nu va mai avea circuite $\implies$ C va deveni arbore $\overset{(a)}{\implies} det(M_1), det(M_2) \neq 0$

" $\implies$ "
$det(M_C) \neq 0$ & $\det(M_1) = det(M_2) \implies (-1)^{1+n} = (-1)^{n+n} \implies$
$1 + n\ par \implies n\ impar$
" $\impliedby$ "
det(M_c)=$(-1)^{1+n} * det(M_1) + (-1)^{n+n} * det(M_2) = det(M_1) + det(M_2) = 2 * det(M_1) \neq 0 \implies M_C\ nesingular$