Tema I - Ex. 4

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(a) Presupunem ca la finalul unei iteratii $min_{k \epsilon V \setminus S} u_k^F$ *descreste* $\implies$ $\exists k^\star \epsilon S$ & $k \epsilon V \setminus S$ astfel incat $u_{k\star}^F + a_{k^\star k} = min_{j \epsilon V \setminus S} u_j^F$

Cazul 1) In lantul *P(s,k)* se adauga un nou nod, k_3 si noul minim devine *P(s,k_3)* $\implies$ $a(P(s,k_3)){<}a(P(s,k))$ $\implies$ $a(P(s,k)){+}a(P(s,k_3)){<}a(P(s,k))$ $\implies$ $a(k,k_3){<}0$ (F)

Cazul 2) In lantul *P(s,k_1)* noul minim devine P(s,k_2), unde $k_2 \epsilon V \setminus S \setminus \{k_1\}$ $\implies$ $a(P(s,k_2)){<}a(P(s,k))$& $a(P(s,k_1)) \geqslant P(s,k))$ $\implies$ $a(P(s,k_2)){<}a(P(s,k_1))$ $\implies$ $a_{k_1 k_2} = 0$ (F) Presupunerea facuta a fost falsa $\implies$ valoarea nu descreste. Analog daca consideram cealalta valoare

b) P(s,t) va fi de forma $\underbrace{.......,j}_{\text{S}}\, j`....l`\, \underbrace{l........}_{\text{T}}$

In momentul actual drumul minim este P(s,j) dar la urmatoarea iteratie va fi P(s,j`) $\implies$ $a(P(s,j`)) \leqslant a(P(s,j))$
$\implies$ $a(P(s,j)) + a(P(j,j`)) \leqslant a(P(s,j))$
$\implies$ $a(P(s,j)) + a_{jj'} \leqslant a(P(s,j))$
$u_j^F + a_{jj`} \leqslant a(P(s,j))$ (1)

Analog obtinem $u_l^F + a_{ll`} \leqslant a(P(l,t))$ (2)

(1)+(2) $\implies$ $u_j^F + a_{jj`} + a_{ll`} + u_l^F \leqslant a(P(s,j)) + a(P(l,t)) \leqslant a(P(s,t))$

$u_j^F + a_{jj`} = a(P(s,j))$

Cum $min_{k \epsilon V \setminus S} u_h^F$ va fi urmatorul minim obtinut dupa P(s,j') $\implies$ $min_{k \epsilon V \setminus S} u_h^F \leqslant a(P(s, j'))$

Analog $min_{k \epsilon V \setminus T} u_k^F \leqslant a(P(l', t))$

$a(P(s, j')) + a(P(l', t)) \geqslant min_{k \epsilon V \setminus T} u_k^F + min_{k \epsilon V \setminus S} u_h^F$

Combinand cele doua inegalitati obtinem ce am avut de demonstrat