Constrained Hidden Markov Models

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Introduction

Phytoplankton is an important indicator of water quality assessment. To understand their dynamics and detect phytoplancton blooms, we opt to use unsupervised Hidden Markov Model. (uHMM) HMMs are statistical models designed for analyzing sequential data and are particularly well-suited for scenarios where the observed data is sequential in nature. By inferring the hidden states based on the observed data and the model parameters, HMMs allow us to understand the underlying structure, dependencies, and patterns within the sequential data.

MONITORING SYSTEM BASED ON HYBRID HMM

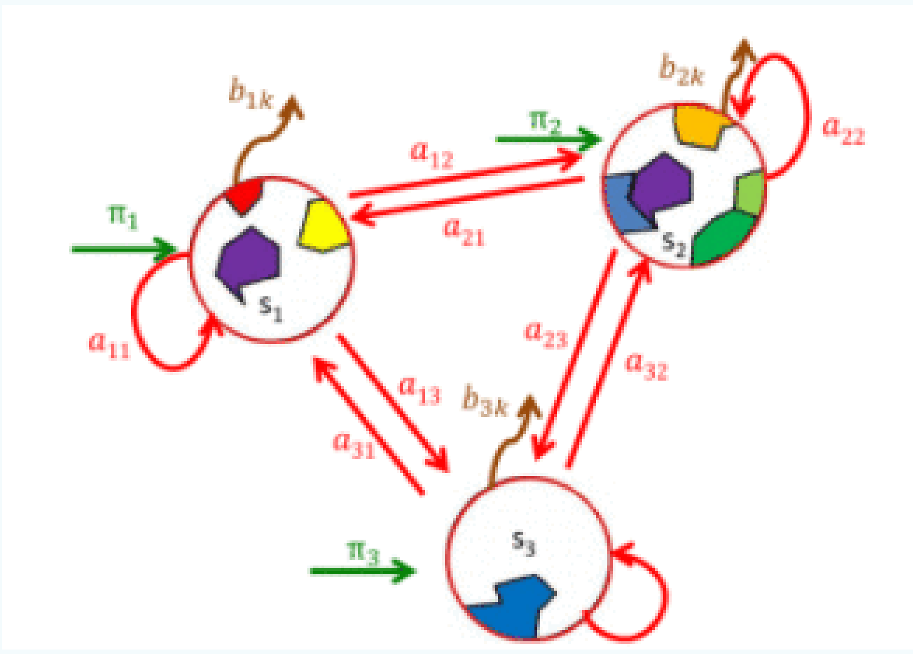

Fig. 1: HMM scheme for instance with 3 states and 8 symbols [2]

- $S = \{s_1, s_2, \dots, s_N\}$ is the set of states with N the number of distinct states. The states are determined with a spectral clustering.
- $V = \{v_1, v_2, \dots, v_M\}$ is the set of symbols and M is the number of distinct symbols. Symbols are determined with a simple K-means.
- $\pi = \{\pi_i\}$ of size N, defines the initial probability distribution, $\pi_i = \{P(s(t=1) = s_i)\}$.
- $A = \{a_{i,j}\}$ of size $N \times N$, defines the transition matrix with $a_{i,j} = P(s(t) = s_i | s(t-1) = s_j)$
- $B = \{b_{i,k}\}$of size $N \times M$, defines the emission probability with $b_{i,k} = P(v(t) = v_k | s(t) = s_i)$

Constrained Spectral Clustering (CSC)

Spectral clustering is an efficient technique to extract unkown profile as shapes or events, linearly non-separable. It is particularly useful when dealing with data that does not have well-defined clusters or has complex structures. [1] Spectral clustering involves the following steps:

- Constructing a similarity matrix W' that captures the pairwise relationships W between data points with pair constraints Q
- Creating a graph Laplacian matrix L: The Laplacian matrix represents the graph structure of the data, with each data point as a node and the similarity between points as edge weights.
- L's Eigenvalue decomposition: The first K eigenvectors represent the embedding data space.
- Clustering: Common methods include applying K-means in this data space.

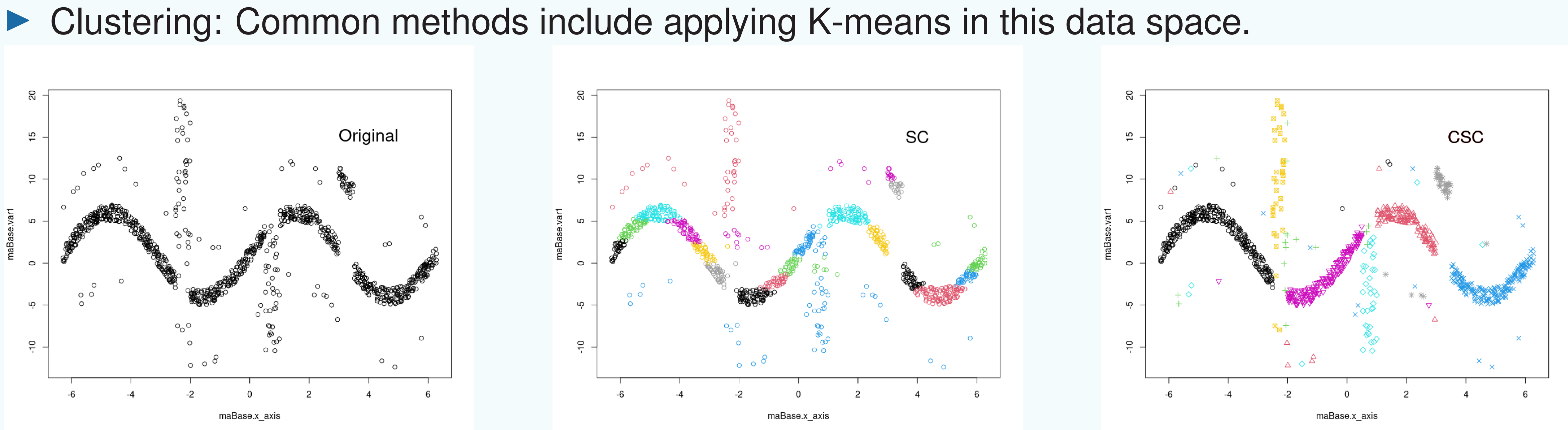

Fig. 3: Simulated time series and its clusterings

References

[1] G. Wacquet, É. Poisson Caillault, D. Hamad, and P.-A. Hébert. "Constrained spectral embedding for K-way data clustering". In: *Pattern Recognition Letters* 34.9 (July 2013), pp. 1009–1017. DOI: 10.1016/j.patrec.2013.02.003. URL: https://doi.org/10.1016/j.patrec.2013.02.003.

[2] Kevin Rousseeuw, Emilie Poisson Caillault, Alain Lefebvre, and Denis Hamad. "Hybrid Hidden Markov Model for Marine Environment Monitoring". In: *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing* 8.1 (Jan. 2015), pp. 204–213. DOI: 10.1109/jstars.2014.2341219. URL: https://doi.org/10.1109/jstars.2014.2341219.

uHMM algorithm

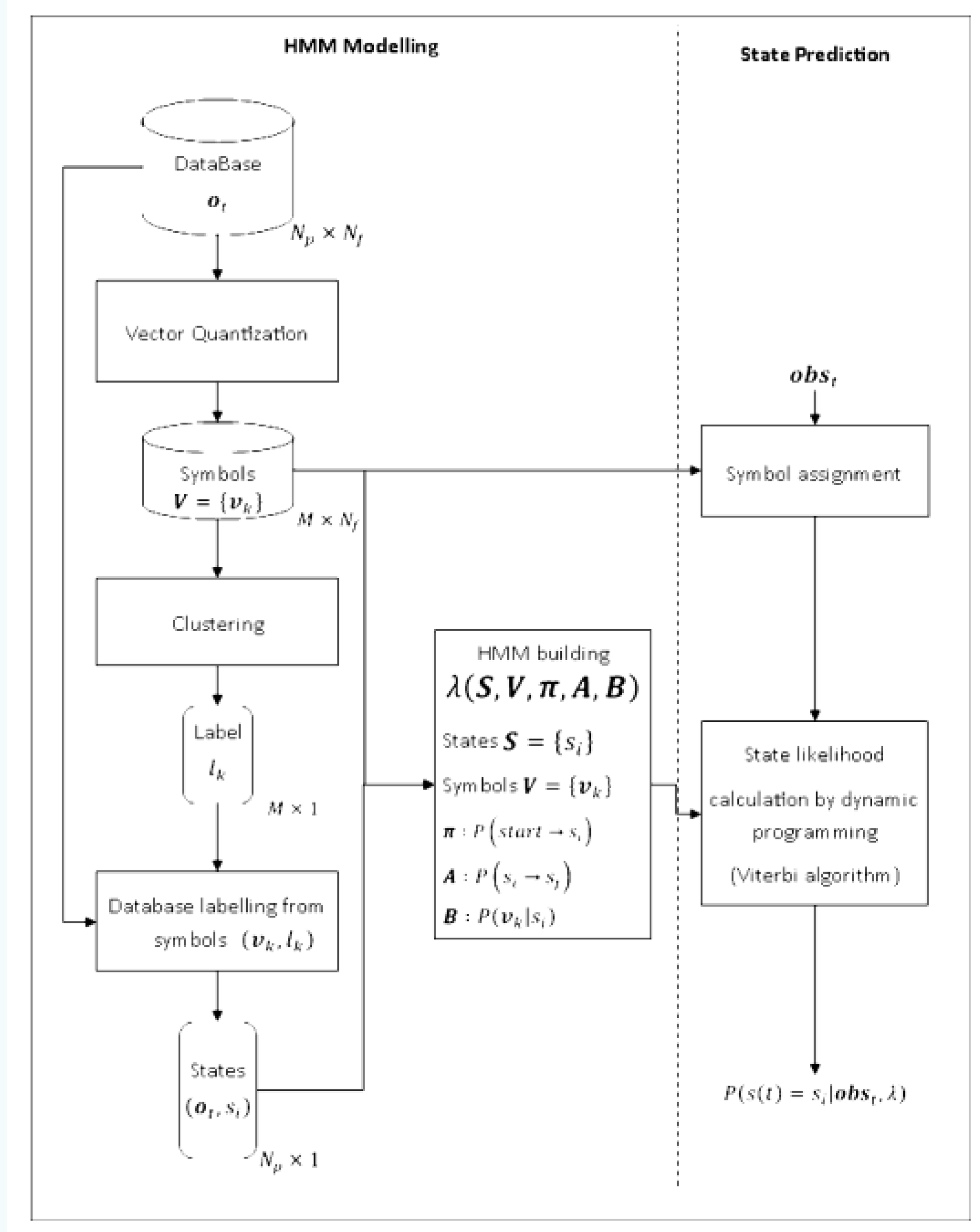

Fig. 2: uHMM modeling and prediction scheme [2]

Constraint Matrix

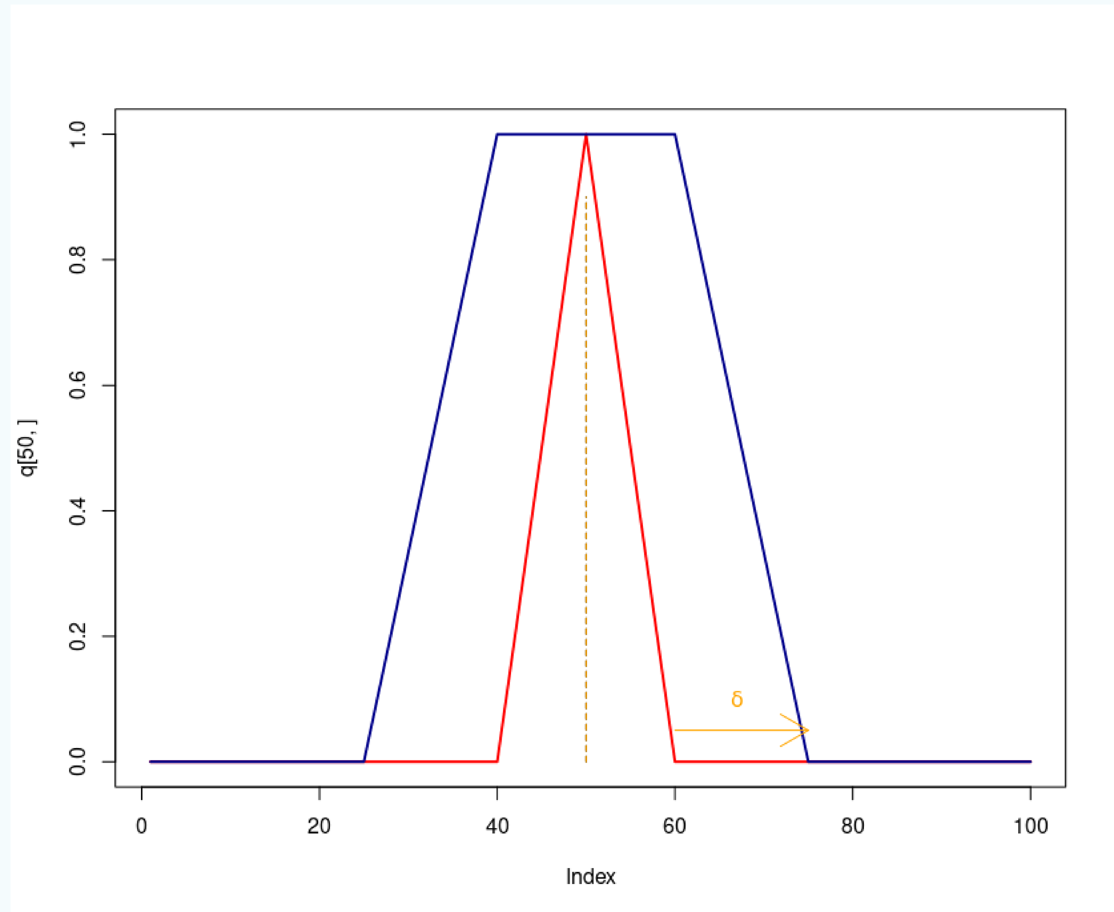

Fig. 4: Constraint value over time

δ must be tuned according to applications. To control spatial and temporal dynamics, we can add time/space-bound constraints such as:

$$W' = \alpha \times W + (1-\alpha) \times Q$$
$$W' = W^{\alpha} \times Q^{(1-\alpha)}$$

α is a parameter that controls the balance between W and Q. By adjusting the value of α, we can emphasize the influence of either the original similarity matrix or the constraint matrix in the clustering process.

Conclusion

- Adding dynamics constraint in uHMM's state is fully operational.
- Further work: To constrain also uHMM dynamics parameters.