# Adv. Stat. Topics A - Missing data Afternoon session

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November 3, 2020

#### Program outline

- 12.00-12.50: Imputation and Multiple Imputation using Chained Equations (MICE)
- 12.50-14.15: Work with data: Data analysis with missing information
- 14.15-14.45: Presentations
- 14.45-15.00: Further perspectives and more resources



Imputation: Fill in missing slots in the data with plausible values.



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- Imputation: Fill in missing slots in the data with plausible values.
- ► Terrible idea... if you do it just once.
- ► Wonderful idea if you do it multiple times.

#### Example: Simple missing information setup

- ► Imagine that we wish to estimate the effect of X on Y, controlling for Z.
- ▶ X suffers from missing information (MCAR). Assume that we order the observations such that  $X_1, ..., X_d$  have missing information, while  $X_{d+1}, ..., X_n$  are fully observed.
- Assume that Y and Z are all fully observed.

#### Example: Simple missing information setup

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- ▶ X suffers from missing information (MCAR). Assume that we order the observations such that  $X_1, ..., X_d$  have missing information, while  $X_{d+1}, ..., X_n$  are fully observed.
- Assume that Y and Z are all fully observed.
- Note: Complete case analysis would produce an unbiased, but inefficient estimate.

```
n <- 200
set.seed(1331)
Z <- rnorm(n, sd = 1)
X <- Z + rnorm(n, sd = 1)
Y <- 2*X + Z + rnorm(n, sd = 2)</pre>
```

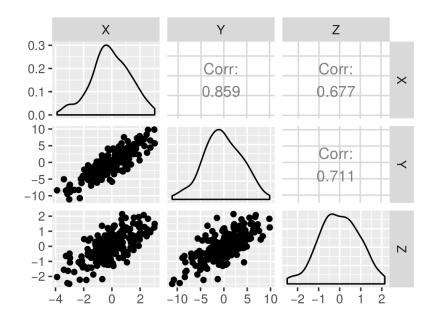
```
n <- 200
set.seed(1331)
Z <- rnorm(n, sd = 1)
X <- Z + rnorm(n, sd = 1)
Y <- 2*X + Z + rnorm(n, sd = 2)

true_X <- X
true_xmean <- mean(X)
true_xsd <- sd(X)
true_model <- lm(Y ~ X + Z)</pre>
```

```
n < -200
set.seed(1331)
Z \leftarrow rnorm(n, sd = 1)
X \leftarrow Z + rnorm(n, sd = 1)
Y \leftarrow 2*X + Z + rnorm(n, sd = 2)
true X <- X
true xmean <- mean(X)
true xsd <- sd(X)
true model <-lim(Y ~ X + Z)
d < -40
X[1:d] \leftarrow NA
```

```
n < -200
set.seed(1331)
Z \leftarrow rnorm(n, sd = 1)
X \leftarrow Z + rnorm(n, sd = 1)
Y \leftarrow 2*X + Z + rnorm(n, sd = 2)
true X <- X
true xmean <- mean(X)
true xsd <- sd(X)
true model \leftarrow lm(Y \sim X + Z)
d < -40
X[1:d] \leftarrow NA
X[36:40]
## [1] NA NA NA NA NA
X[41:45]
## [1] -0.9404489 0.7807026 1.9016603 -0.3728711 -0.5331431
```

## A quick overview of the data (no missing info.)



Mean imputation: Insert the mean (or mode) of  $X_{d+1},...,X_n$  into all  $X_1,...,X_d$ .

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```
X_{meanimp} \leftarrow X
```

```
xobs_mean <- mean(X[(d+1):n])</pre>
```

Mean imputation: Insert the mean (or mode) of  $X_{d+1},...,X_n$  into all  $X_1,...,X_d$ .

```
X_meanimp <- X
xobs_mean <- mean(X[(d+1):n])</pre>
```

 $X_{meanimp[1:d]} \leftarrow xobs_{mean}$ 

Mean imputation: Insert the mean (or mode) of  $X_{d+1},...,X_n$  into all  $X_1,...,X_d$ .

```
X_meanimp <- X
xobs_mean <- mean(X[(d+1):n])

X_meanimp[1:d] <- xobs_mean

#Compare mean for full X with mean of mean imputed X
true_xmean; mean(X_meanimp)</pre>
```

```
## [1] -0.07813252
## [1] -0.1551874
```

```
Mean imputation: Insert the mean (or mode) of X_{d+1}, ..., X_n into all
X_1, ..., X_d.
X meanimp <- X
xobs mean \leftarrow mean(X[(d+1):n])
X_meanimp[1:d] <- xobs_mean</pre>
\#Compare\ mean\ for\ full\ X\ with\ mean\ of\ mean\ imputed\ X
true xmean; mean(X meanimp)
## [1] -0.07813252
## [1] -0.1551874
#Compare sd for full X with sd of mean imputed X
true xsd; sd(X meanimp)
## [1] 1.368721
## [1] 1.240263
```

```
round(summary(true_model)$coefficients,4)
```

```
round(summary(true model)$coefficients,4)
##
             Estimate Std. Error t value Pr(>|t|)
##
  (Intercept) -0.0532 0.1392 -0.3822 0.7028
               2.0756 0.1382 15.0158 0.0000
## X
               1.0260 0.2000 5.1297 0.0000
## Z
round(summary(lm(Y ~ X meanimp + Z))$coefficients,4)
##
             Estimate Std. Error t value Pr(>|t|)
  (Intercept)
               0.0709
                         0.1623 0.4371 0.6626
## X_meanimp 1.7887 0.1639 10.9102 0.0000
               1.6266
                         0.2150 7.5675 0.0000
## Z
```

#### Comparing model coefficients:

```
round(summary(true model)$coefficients,4)
##
             Estimate Std. Error t value Pr(>|t|)
  (Intercept) -0.0532 0.1392 -0.3822 0.7028
               2.0756 0.1382 15.0158 0.0000
## X
               1.0260 0.2000 5.1297 0.0000
## Z
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  (Intercept) 0.0709 0.1623 0.4371 0.6626
## X_meanimp 1.7887 0.1639 10.9102 0.0000
               1.6266 0.2150 7.5675 0.0000
## 7.
```

Conclusion: Don't do mean imputation!

Hot deck imputation (simplest version): For each missing value,  $X_1, ..., X_d$ , pick and insert a random value among the observed values  $X_{d+1}, ..., X_n$ .

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```
X_hdimp <- X
set.seed(13)
X hdimp[1:d] <- sample(X[(d+1):n], size = d.</pre>
```

X hdimp <- X

Hot deck imputation (simplest version): For each missing value,  $X_1, ..., X_d$ , pick and insert a random value among the observed values  $X_{d+1}, ..., X_n$ .

```
#Compare mean for full X with mean of mean imputed X
true_xmean; mean(X_hdimp)
```

```
## [1] -0.07813252
## [1] -0.2030766
```

X hdimp <- X

Hot deck imputation (simplest version): For each missing value,  $X_1, ..., X_d$ , pick and insert a random value among the observed values  $X_{d+1}, ..., X_n$ .

```
#Compare mean for full X with mean of mean imputed X true_xmean; mean(X_hdimp)
```

```
## [1] -0.2030766
#Compare sd for full X with sd of mean imputed X
true_xsd; sd(X_hdimp)
```

```
## [1] 1.368721
## [1] 1.387267
```

## [1] -0.07813252

```
round(summary(true_model)$coefficients,4)
```

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round(summary(true model)$coefficients,4)
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##
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               2.0756 0.1382 15.0158 0.0000
## X
               1.0260 0.2000 5.1297 0.0000
## Z
round(summary(lm(Y ~ X hdimp + Z))$coefficients,4)
##
             Estimate Std. Error t value Pr(>|t|)
  (Intercept)
               0.0484
                         0.1781 0.2720
                                        0.7859
## X_hdimp 1.2207 0.1492 8.1828 0.0000
               2.1195
                         0.2188 9.6879 0.0000
## Z
```

#### Comparing model coefficients:

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round(summary(true model)$coefficients,4)
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                                        0.7859
## X_hdimp 1.2207 0.1492 8.1828 0.0000
               2.1195 0.2188 9.6879 0.0000
## 7.
```

Conclusion: Don't do hot deck imputation!

#### Regression imputation (1/3)

Regression imputation: Fit a regression model for all the observations, e.g.,

$$X_i = \alpha + \beta_1 \cdot Y_i + \beta_2 \cdot Z_i + \epsilon_i$$

for i = d + 1, ..., n and use this model to predict values for the remaining  $X_1, ..., X_d$ .

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for i = d + 1, ..., n and use this model to predict values for the remaining  $X_1, ..., X_d$ .

## Regression imputation (2/3)

```
#Compare mean for full X with mean of req. imputed X
true_xmean; mean(X_regimp)
## [1] -0.07813252
## [1] -0.1177343
\#Compare \ sd \ for \ full \ X \ with \ mean \ of \ req. \ imputed \ X
true xsd; sd(X regimp)
## [1] 1.368721
## [1] 1.352262
```

# Regression imputation (3/3)

```
round(summary(true_model)$coefficients.4)
##
             Estimate Std. Error t value Pr(>|t|)
## (Intercept) -0.0532 0.1392 -0.3822 0.7028
            2.0756 0.1382 15.0158 0.0000
## X
## 7.
               1.0260 0.2000 5.1297 0.0000
round(summary(lm(Y ~ X regimp + Z))$coefficients,4)
##
             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 0.0505 0.1256 0.4024 0.6878
## X_regimp 2.2815 0.1264 18.0525 0.0000
               0.8383 0.1807 4.6401 0.0000
## 7.
```

Conclusion: Don't do regression imputation!

## Stochastic regression imputation

Stochastic regression imputation: Perform regression imputation, but add noise to the predictions by sampling from the residuals from the fitted model.

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```
X stocregimp <- X; set.seed(2)</pre>
X_stocregimp[1:d] <- X_regimp[1:d] +</pre>
 sample(residuals(m regimp), size = d,
        replace = TRUE)
#Estimate from model with full X
round(summary(true_model)$coefficients,4)[2,]
##
    Estimate Std. Error t value Pr(>|t|)
      2.0756 0.1382 15.0158 0.0000
##
#Estimate from model with X imputed by stochastic regression
round(summary(lm(Y ~ X stocregimp + Z))$coefficients,4)[2,]
##
    Estimate Std. Error
                           t value Pr(>|t|)
```

15.6060 0.0000

Problem: The variance is still underestimated.

2.0430 0.1309

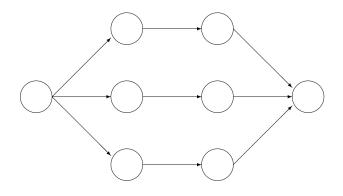
##

# The problem with single imputation strategies

Imputing one value for a missing datum cannot be correct in general, because we don't know what value to impute with certainty (if we did, it wouldn't be missing).

- Donald B. Rubin

# Multiple imputation



Incomplete data Imputed data Analysis results Pooled result

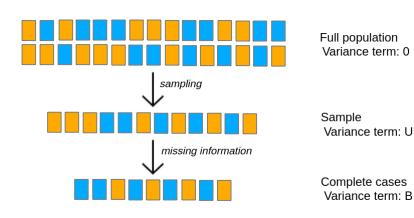
(Figure 1.6 from van Buuren 2019)

## Variance under imputation

Recall: Variance measures the uncertainty of our estimate if we were to repeat the whole thing.

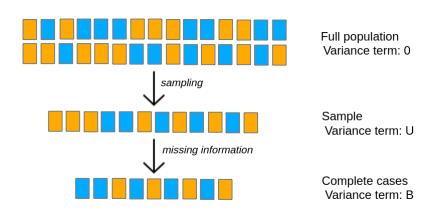
# Variance under imputation

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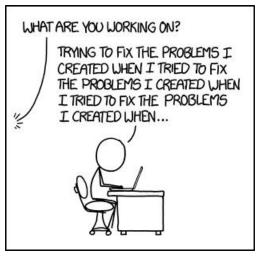
## Variance under imputation

Recall: Variance measures the uncertainty of our estimate if we were to repeat the whole thing.



Problem: Variance accumulates; we need to use the (uncertain) sample to estimate the missing data model.

#### Variance accumulates



http://xkcd.com/1739/

# Total variance (following van Buuren 2019)

It can be shown mathematically that

Total variance = 
$$U + B + B \cdot \frac{1}{m}$$

where m is the number of imputed datasets and

- ${\it U}$  is the variance due to using a sample rather than the full population.
- *B* is the extra variance due to there being missing values.
- $B \cdot \frac{1}{m}$  is the extra variance due to having to estimate the missing data model.

The collective method for obtaining a correct estimate of the total variance (T) by use of multiple imputations is referred to as *Rubin's rules*.

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- B is the extra variance due to there being missing values.
- $B \cdot \frac{1}{m}$  is the extra variance due to having to estimate the missing data model.

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Note: Larger *m* makes the last term small.

# Multiple imputation by chained equations (MICE)



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- ► A specific algorithm (method) for performing data analysis with missing information.
- ► Also known as imputation with *fully conditional specification* (FCS).
- Specifies imputation models variable-by-variable for each variable with missing information.
- Iteratively updates best guesses to allow all variables (even those with missing information) to inform the imputation of the others.

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- Iteratively updates best guesses to allow all variables (even those with missing information) to inform the imputation of the others.

# MICE: Sequential guessing (heuristically)

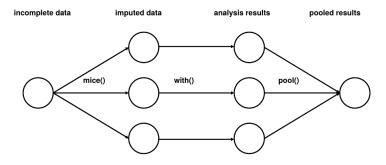
- ► Assume both *X* and *Z* have missing information.
- Let  $X_{\text{obs}}$  and  $Z_{\text{obs}}$  denote the observed values of X and Z, respectively.

#### MICE in R

MICE is implemented in the mice package in R:

```
library(mice)
data <- data.frame(X = X, Y = Y, Z = Z)
set.seed(212)
imps <- mice(data, print = FALSE, m = 5)
fits <- with(imps, lm(Y ~ X + Z))
res <- pool(fits)
summary(res)[, c(1,2,5)]</pre>
```

## MICE in R: Schematic



(Figure 1 from van Buuren & Groothuis-Oudshoorn 2011)

# MICE compared with stochastic regression imputation

```
#Estimate from model with full X
round(summary(true model)$coefficients,4)[2,]
    Estimate Std. Error t value Pr(>|t|)
##
      2.0756 0.1382 15.0158 0.0000
##
#Estimate from model with X imputed by stochastic regression
round(summary(lm(Y ~ X_stocregimp + Z))$coefficients,4)[2,]
##
    Estimate Std. Error t value Pr(>|t|)
      2.0430 0.1309 15.6060
##
                                     0.0000
#Estimate from mice model (default settings)
round(summary(res)[2, c(2,3,4,6)],4)
##
    estimate std.error statistic p.value
      2.1057
                0.136
                        15.4828
## 2
```

## Inspecting the variance components from mice

```
Note: mice delivers estimates of B (b), U (ubar), T (t = std.error²), as well as \lambda = \frac{B \cdot (1+1/m)}{T} (lambda), riv = \frac{B \cdot (1+1/m)}{U} (riv) and more:
```

# Variable level imputation models

Default choices in mice package:

#### Numerical variables:

Predictive mean matching (pmm). A fusion between regression imputation and hot deck imputation: Use regression to find a selection of plausible "donor values", choose one at random among them.

#### Categorical variables (> 2 categories):

Multinomial logistic regression (polyreg). A regression imputation method.

#### Categorical variables (= 2 categories):

Logistic regression (logreg). A regression imputation method.

#### Categorical variables (ordered categories):

Ordered logistic regression (polr). A regression imputation method.

## Data exercise: Analyze alcodata

- → Go to "Exercise: Analyze" on the course website https://biostatistics.dk/teaching/advtopicsA/notes.html and work through the questions in small groups.
- ightarrow Add information to the Google slide show ("analyze") corresponding to your dataset at

shorturl.at/hwyzR

We will discuss your findings around 14:15.



## Back to the Elderly study

Table 3.1: Estimated log odds ratios from the model of controlled consumption status using all full covariate adjustment. The reported estimates are on log odds ratio scale and they are computed relative to the following reference category: Treament MET; Gender male; Country Denmark; Age 60; Education none; No partner; Low ADS; Previous treatments 0. The mean log odds of having a controlled alcohol consumption in this reference group is represented by the intercept estimate. The reported p-values correspond to two-sided z-tests of the null-hypothesis of a zero parameter value.

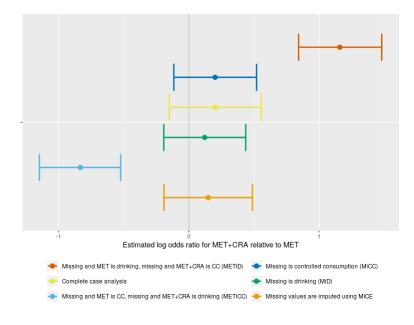
	Estimate	Std. error	z statistic	p-value
Intercept	-0.3507	0.3050	-1.1499	0.2502
Treatment: MET+CRA	0.2028	0.1801	1.1260	0.2602
Country: USA	0.0736	0.2327	0.3164	0.7517
Country: Germany	-0.0351	0.2522	-0.1392	0.8893
Gender: Female	-0.5543	0.1906	-2.9085	0.0036
Age	0.0677	0.0211	3.2038	0.0014
Married or cohabiting: Yes	0.2270	0.1877	1.2094	0.2265
Severity: Intermediate	-0.0777	0.2307	-0.3367	0.7363
Severity: Substantial or severe	-0.2767	0.4096	-0.6755	0.4994
Education: At most undergraduate degree	0.0518	0.2286	0.2268	0.8206
Education: Graduate or post-graduate	-0.4463	0.2872	-1.5537	0.1202
Previous treatments: 1-2	0.2655	0.2187	1.2140	0.2247
Previous treatments: 3+	0.2938	0.3087	0.9517	0.3413

# Elderly sensitivity analyses - models

We fitted five additional models:

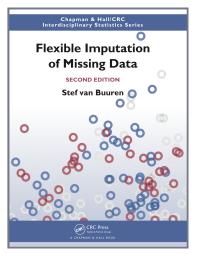
- MiD Missing is drinking approach: Treating all missing observations as relapsers (non-controlled consumption).
- MiCC Missing is CC approach: Treating all missing observations as controlled consumption.
- METiD MET is drinking approach: Treating missing observations for patients treated with MET as drinking, while missing observations from MET+CRA-patients are treated as controlled consumption.
- METicc MET is CC: Treating missing observations for patients treated with MET+CRA as drinking, while missing obsevations from MET-patients are treated as controlled consumption.
  - MICE Mulitple imputation of missing observation using all variables from the primary model and controlled consumption information from previous time points.

# Elderly sensitivity analyses - results



# Further resources (1)

Excellent book by Stef van Buuren (2019)



https://stefvanbuuren.name/fimd/

# Further resources (2)

#### Multiple imputation for Cox models:

STATISTICS IN MEDICINE
Statist. Med. 2009; 28:1982–1998
Published online 19 May 2009 in Wiley InterScience
(www.interscience.wiley.com) DOI: 10.1002/sim.3618

#### Imputing missing covariate values for the Cox model

Ian R. White 1, \*, † and Patrick Royston2

<sup>1</sup>MRC Biostatistics Unit, Institute of Public Health, Robinson Way, Cambridge CB2 OSR, U.K.
<sup>2</sup>MRC Clinical Trials Unit, Cancer Group, London, U.K.

#### SUMMARY

Multiple imputation is commonly used to impute missing data, and is typically more efficient than complete cases analysis in regression analysis when covariates have missing values. Imputation may be performed using a regression model for the incomplete covariates on other covariates and, importantly, on the outcome. With a survival outcome, it is a common practice to use the event indicator D and the log of the observed event or censoring time T in the imputation model, but the rationale is not clear.

https://onlinelibrary.wiley.com/doi/abs/10.1002/sim.3618

# Further resources (3)

#### Guideline for MICE in practice:

# Statistics in Medicine

#### **Tutorial in Biostatistics**

Received 3 September 2009, Accepted 14 July 2010

Published online 30 November 2010 in Wiley Online Library

(wileyonlinelibrary.com) DOI: 10.1002/sim.4067

# Multiple imputation using chained equations: Issues and guidance for practice

Ian R. White, a\*† Patrick Roystonb and Angela M. Woodc

Multiple imputation by chained equations is a flexible and practical approach to handling missing data. We describe the principles of the method and show how to impute categorical and quantitative variables, including skewed variables. We give guidance on how to specify the imputation model and how many imputations are needed. We describe the practical analysis of multiply imputed data, including model building and model checking. We stress the limitations of the method and discuss the possible pitfalls. We illustrate the ideas using a data set in mental health, giving Stata code fragments. Copyright 

2010 John Wiley & Sons, Ltd.

Keywords: missing data; multiple imputation; fully conditional specification

https://onlinelibrary.wiley.com/doi/full/10.1002/sim.4067

# Further resources (4)

Article

#### Multiple imputation with non-linear relationships:

STATISTICAL HETEOS IN MICHIGAL RESEA

Multiple imputation of covariates by fully conditional specification: Accommodating the substantive model Statistical Methods in Medical Research 2015, Vol. 24(4) 462–487 
© The Author(s) 2014 
Reprints and permissions: sagepub.co.uk/journalsPermissions.nav DOI: 10.1177/0962280214521348 
smm.sagepub.com



Jonathan W Bartlett, <sup>1</sup> Shaun R Seaman, <sup>2</sup> lan R White <sup>2</sup> and James R Carpenter <sup>1,3</sup> for the Alzheimer's Disease Neuroimaging Initiative\*

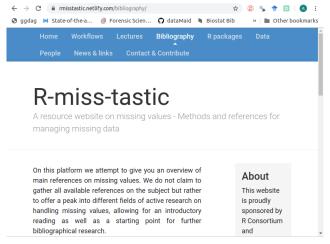
#### **Abstract**

Missing covariate data commonly occur in epidemiological and clinical research, and are often dealt with using multiple imputation. Imputation of partially observed covariates is complicated if the substantive model is non-linear (e.g. Cox proportional hazards model), or contains non-linear (e.g. squared) or interaction terms, and standard software implementations of multiple imputation may impute covariates from models that are incompatible with such substantive models. We show how imputation by fully conditional specification, a popular approach for performing multiple imputation, can be modified

https://doi.org/10.1177/0962280214521348

# Further resources (5)

Website with a very thorough collection of material on missing data, emphasis on tools in R:



https://rmisstastic.netlify.com



Comments/suggestions for this course day are very much welcome at ahpe@sund.ku.dk