

Hints for small exercise (Lecture 2, Day 2)

Helene Charlotte Wiese Rytgaard

September 28, 2021

First recall that

$$\text{expit}(x) = \frac{e^x}{1 + e^x} = \frac{1}{1 + e^{-x}}, \quad \text{logit}(x) = \log\left(\frac{x}{1-x}\right),$$

1. Show that $\text{expit}(\text{logit}(x)) = x$.
2. Show that $\text{expit}(-\text{logit}(x)) = 1 - x$.
3. Show that $\log(\text{expit}(x)) = -\log(1 + e^{-x})$.
4. Show that $\log(1 - \text{expit}(x)) = -\log(1 + e^x)$.
5. Derive the derivatives with respect to ε of the composite functions $\log(\text{expit}(\text{logit}(x) + \varepsilon h))$ and $\log(1 - \text{expit}(\text{logit}(x) + \varepsilon h))$.
6. Set $\varepsilon = 0$ in the expressions for the derivatives of 5.
7. Applying these steps to $\mathcal{L}(f_\varepsilon)$ now gives:

$$\begin{aligned} \left. \frac{d}{d\varepsilon} \right|_{\varepsilon=0} \mathcal{L}(f_\varepsilon)(O) &= YH(A, X)(1 - f(A, X)) - (1 - Y)H(A, X)f(A, X) \\ &= YH(A, X) - H(A, X)f(A, X) = H(A, X)(Y - f(A, X)). \end{aligned}$$