Estimating Life-Cycle Models of Saving and Labour Supply

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Introduction

- Underlying research question: understanding within period and intertemporal decisions on saving and labour supply
- Research process:
  1. Developing research ideas
  2. Use of economic structure to address questions
  3. Some practical issues

Readings

- Attanasio, O., Levell, P., Low, H. and Sanchez-Marcos, V. (2018) "Aggregating Elasticities: Intensive and Extensive Margins of Women's Labour Supply" *Econometrica* 86:2049-2082

- Low, H. and Meghir, C. (2017) "The use of structural models in econometrics" *Journal of Economic Perspectives* 31(2): 33-58

Motivation

- Response of women's labour supply to changes in wages
  - Design of tax/ transfer system, welfare cost
  - How effective are tax cuts in stimulating work
  - Labour supply over the business cycle

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  - Design of tax/ transfer system, welfare cost
  - How effective are tax cuts in stimulating work
  - Labour supply over the business cycle
- Focus today:
  - Need to integrate different decisions
  - Extent of heterogeneity across individuals
  - Interaction of individual decisions with aggregate

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- Which elasticity?
  - Within-period Marshallian and Hicksian
  - Frisch (dynamic, under certainty)
  - Life-cycle Hicksian, Life-cycle Marshallain

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  - Number of weeks worked (Extensive)
  - Timing of consumption
- What sort of shocks faced?
- How important are saving and life-cycle motives?

Literature: Micro

- Dynamic labour supply methods:
  - Macurdy (1983), Altonji (1986), Blundell and Walker (1986)
  - Heckman and Macurdy (1980), Macurdy (1981)
  - Blundell, Meghir and Neves (1993)
  - Blundell, Duncan, Meghir (1998)

- Women: Hicks response: ≈ 0.3, large variation across studies; Frisch, larger (Keane, 2010)

- Men: very small responses, but again large variation

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- Women: Hicks response: ≈ 0.3, large variation across studies; Frisch, larger (Keane, 2010)
- Men: very small responses, but again large variation
- Key assumptions of micro labor supply model:
  - How is consumption / marginal utility of wealth controlled for
  - non-separability Ziliak and Kneisner (2005)
  - uncertainty, Blundell et al, (1993)
  - participation margin
  - human capital, Imai and Keane (2004), Keane and Wasi (2016)

Literature: Macro

1. Much larger estimates (business cycle - Frisch)
   - Keane and Rogerson (2012, 2015), Keane (2011), Chetty et al (2011)
2. Focus on extensive margin
   - Rogerson and Wallenius (2009)
3. Weak link between individual preference parameters and aggregate labor supply
   - Chang et al. (2011), Erosa, Fuster and Kambourov (2015), Guner, Kaygusuz, and Ventura (2012)

Research Process

Developing Research Ideas

- Key issues with the literature:
  - Disconnect between different bits of the literature
  - Surprising lack of consensus
  - What is the underlying model?

Research Process

Developing Research Ideas

- Some thoughts on underlying economics:
  - No unique elasticity: substantial heterogeneity in elasticities
  - Different responses in recessions and booms
  - How should we combine responses across margins
  - How do within-period elasticities relate to life-cycle elasticities
  - What decisions are being ignored? Fertility? Joint-labour supply? Savings?
  - What unit is making decisions?

Life-Cycle Model

Empirical Strategy and Data

Estimates

Elasticities

Conclusions

Overview

- Standard unitary life-cycle model of married couples
- Choices:
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- Preferences:
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- Resources and Markets:
  - Female wages and male earnings are uncertain
  - Fertility is exogenous (different types)
  - Incomplete markets, partial equilibrium

Life-Cycle Model

- Lifetime expected utility of the household, h:

$$\max_{c,l,P} E_t \sum_{j=0}^{T} \beta^j u\left(c_{h,t+j}, l_{h,t+j}, P_{h,t+j}; z_{h,t+j}, \chi_{h,t+j}, \zeta_{h,t+j}\right)$$

- Intertemporal budget constraint:

$$A_{h,t+1} = R_{t+1}\left(\begin{array}{c} A_{h,t} + \left(w_{h,t}^f\left(H - l_{h,t}\right) - F\left(a_{h,t}\right)\right) P_{h,t} \\ + \ y_{h,t}^m - c_{h,t} \end{array}\right)$$

- $z_{h,t}$: demographics; $a_{h,t}$: age of child
- $l_{h,t}$: leisure; $P_{h,t}$: participation
- $c_{h,t}$: consumption

Preferences

$$u\left(c_{h,t}, l_{h,t}, P_{h,t}\right) = \frac{M_{h,t}^{1-\gamma}}{1-\gamma} \exp(\xi P_{h,t} + \pi z_{h,t} + \zeta_{h,t})$$

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- Generalized CES: Consumption - leisure preferences

$$M_{h,t} = \left(\frac{(c_{h,t}^{1-\phi} - 1)}{1-\phi} + \left(\alpha_{h,t}\left(z_{h,t}, \chi_{h,t}\right)\right) \frac{\left(l_{h,t}^{1-\theta} - 1\right)}{1-\theta} \right)$$

$$\alpha_{h,t} = \exp(\psi_0 + \psi_z z_{h,t} + \chi_{h,t})$$

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- Standard CES: $\phi = \theta$
- Cobb-Douglas: $\phi = \theta = 1$
- Additive Separability: $\gamma = 0$

Wages and earnings

- Female (offered) wages:

$$\ln w_{h,t}^f = \ln w_{h,0}^f + \ln e_{h,t}^f + v_{h,t}^f$$

- $e_t^{h,f}$: (exogenous) female human capital at the start of the period

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- Men always work and male earnings are given by:

$$\ln y_{h,t}^m = \ln y_{h,0}^m + \eta_1^m t + \eta_2^m t^2 + v_{h,t}^m$$

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\mu_\xi &= (-\frac{\sigma^2_{\xi^f}}{2}, -\frac{\sigma^2_{\xi^m}}{2}) \\
\sigma^2_\xi &= \begin{pmatrix} \sigma^2_{\xi^f} & \rho_{\xi^f,\xi^m} \\ \rho_{\xi^f,\xi^m} & \sigma^2_{\xi^m} \end{pmatrix}
\end{aligned}
$$

- No returns to experience in simple formulation, return later

Responses to Wage Changes

1. Within-period hours of work and consumption decisions (savings unchanged)
   - Within-period Marshallian and Hicksian
2. Anticipated allocation of hours across time (response to η^f)
   - Frisch
3. Wage change leads to change in life-cycle choices
   - Savings: Life-cycle Marshallian and Life-cycle Hicksian
   - Participation decision

Research Process

- Principle of minimal assumptions on structure to generate estimates
  - Identifying three sorts of response require increasing sets of assumptions
  - Minimise impact of mis-specification
- Alternative principle: estimate all parameters in one step
  - Estimates clearly consistent with each other
  - Computationally burdensome
  - Mis-specification of any part impacts all estimates

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  - Mis-specification of any part impacts all estimates
- Never believe we are modelling the whole world!

Within-period allocation

- Two-stage budgeting
- Define within-period unearned (by the woman) resources:

$$y_t = \left(A_{h,t} + y^m_{h,t} - F\left(a_{h,t}\right) P_{h,t} \right) - \frac{A_{h,t+1}}{1 + r_{t+1}}$$

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- MRS condition (at interior)

$$MRS = w_{h,t} = \frac{u_{l_{h,t}}}{u_{c_{h,t}}} = \alpha_{h,t} \frac{l^{-\theta}_{h,t}}{c^{-\phi}_{h,t}}$$

Holds exactly, not in expectation

Within-period Hicksian and Marshallian elasticities

- Marshallian response: change due to price effect of wages and the income effect on within period resources

$$\varepsilon_l^M = \frac{\phi w (H - l) - c}{\theta c + \phi w l}$$

$$\varepsilon_c^M = \frac{\theta w (H - l) + w l}{\theta c + \phi w l}$$

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- Hicksian response: nets off increase in resources

$$\begin{aligned} \varepsilon_l^H &= \frac{w l}{\theta c + \phi w l} \\ \varepsilon_c^H &= \frac{-c}{\theta c + \phi w l} \end{aligned}$$

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- Implies effects from aggregate shocks on labour supply even if zero mean that re-shuffle the wage distribution
- Effect of permanent wage change. Approximation to life-cycle elasticity

Intertemporal Choices: Anticipated Effects

- Frisch response: response of workers to an anticipated change in the wage
- Marginal utility of wealth, λ_t^h constant
- Following MaCurdy (1981), Keane (2010)

$$\frac{\partial \ln \left(l_{h,t+1} / l_{h,t} \right)}{\partial \ln \left(w_{h,t+1} / w_{h,t} \right)}$$

- Euler equation for consumption (or hours work), Blundell, Meghir, Neves (1993)

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- Frisch elasticity $>$ Hicksian elasticity: individuals reallocate hours of work to periods in which wages are higher
- At the extensive margin, anticipated changes in participation

Life-cycle Responses

- Numerically solve model for saving and life-cycle profiles
- Response at intensive margin, extensive margin, savings and aggregate response
- Temporary changes in wages
- Life-cycle responses in labour supply and saving to permanent changes in wages
- Possible role for returns to experience

Modelling: Jorge Luis Borges

In that Empire, the Art of Cartography attained such Perfection that the map of a single Province occupied the entirety of a City, and the map of the Empire, the entirety of a Province. In time, those Unconscionable Maps no longer satisfied, and the Cartographers Guilds struck a Map of the Empire whose size was that of the Empire, and which coincided point for point with it. The following Generations, who were not so fond of the Study of Cartography as their Forebears had been, saw that that vast map was Useless, and not without some Pitilessness was it, that they delivered it up to the Inclemencies of Sun and Winters.

Estimation Steps

1. Use MRS condition to estimate θ, ϕ, α
   - Enables computation of static elasticities
   - MaCurdy (1983)
2. Use Euler equation to estimate γ, π and ξ
   - Intertemporal elasticities on intensive margin
   - Extent of non-separability
   - Role of demographics
   - Blundell, Meghir, Neves (1993)
3. Use full structural model to estimate remaining parameters
   - Extensive margin responses
   - Role of uncertainty

Step 1: Intratemporal margin

- Log MRS equation:

$$\ln w_{h,t} = \phi \ln c_{h,t} - \theta \ln l_{h,t} + \psi_z z_{h,t} + \psi_0 + \chi_{h,t}$$

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- Issues:
  1. Selection into work: $\chi_{h,t}$ not averaging out to zero
     - Selection correction: male earnings, male employment
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     - Group level variation
     - Groups defined by cohort, education, interacted with quintic time trend

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  3. Normalisation: What to put on LHS and RHS of MRS regression

Equilibrium conditions

$$E\left[h(X;\theta)\mathcal{Z}\right] = 0$$

- One of the parameters has to be normalized to 1
- We use Fuller(1977) estimator
  - less sensitive to the choice of normalization than 2SLS and GMM
  - better bias properties than 2SLS when instruments are relatively weak
- With MRS equation, normalisation matters
  - with wages on LHS, high elasticities (Macurdy)
  - with hours on LHS, low elasticities (Altonji)

Step 2: Intertemporal margin

- Estimate γ, π and ξ using the Euler equation

$$E\left[\beta\left(1+r_{t+1}\right)\frac{u_{c_{h,t+1}}\left(\cdot\right)}{u_{c_{h,t}}\left(\cdot\right)}\middle| I_{h,t}\right]=1$$

- In realisations:

$$\beta\left(1+r_{t+1}\right)u_{c_{h,t+1}}\left(\cdot\right)=u_{c_{h,t}}\left(\cdot\right)\epsilon_{h,t+1}$$

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- Taking logs:

$$\epsilon_{h,t+1}=\kappa_{h,t}+\ln\beta+\ln(1+r_{t+1})-\phi\Delta\ln c_{h,t+1}$$
$$-\gamma\Delta\ln(M_{h,t+1})+\varphi\Delta P_{h,t+1}+\pi\Delta z_{h,t+1}$$

Step 2: Intertemporal margin

- $\epsilon_{h,t+1}$ contains:
  - Expectations errors
  - Unobservable taste shifters
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- $\epsilon_{h,t+1}$ contains:
  - Expectations errors
  - Unobservable taste shifters
  - Higher order moments
- Given estimates of θ and ϕ, can construct $M_{h,t}$
- Linear in parameters Euler equation
- Long time series required; pseudo panels; bootstrap std errs

Step 3: Extensive margin

- Remaining model parameters:
  - Parameters of the budget constraint
  - Fixed costs of work and childcare costs

Step 3: Extensive margin

- Remaining model parameters:
  - Parameters of the budget constraint
  - Fixed costs of work and childcare costs
- Strategy
  - Use as inputs the estimates from step 1 and 2
  - Numerically solve the whole dynamic problem
  - Match life-cycle moments that capture extensive margin
- Simulate labour supply responses

Data

- US Consumer Expenditure Survey 1980-2012
- Non-durable consumption, earnings and female labour supply (hours and weeks)
- Information on education, state of residence, demographics, age
- Wages are obtained from earnings and hours information
- Household specific marginal tax rates are obtained using the NBER tax program

Descriptive statistics

		1980	1995	2012
Demographics	No. of children	1.25	1.15	1.17
Education	% Less than high school	19.4	12.3	9.7
	% High school	44.1	36.8	25.3
	% Some college	18.1	25.3	28.5
	% Degree or higher	18.4	25.5	36.5
Hourly net wages ($ 2016)	All	15.58	16.63	18.95
	Less than high school	12.16	11.23	11.33
	High school	14.22	13.41	14.61
	Some college	16.62	16.41	17.28
	Degree or higher	19.30	22.26	23.20
Hours (workers)	All	35.2	37.5	38.4
	Less than high school	34.9	37.4	34.2
	High school	35.2	36.2	38.6
	Some college	35.0	36.7	37.1
	Degree or higher	35.5	39.7	39.5
% Employed	All	60.0	69.8	61.9
	% Workers part-time	28.4	23.7	20.6
Sample sizes	All	2,199	2,064	2,026
	Workers	1,318	1,441	1,254

Step 1: Estimation of MRS equation

Parameter	Estimate	(Std Err)	[95% C.I.]
θ	1.75*	(1.230)	[0.34,5.12]
ϕ	0.76*	(0.103)	[0.55,0.95]
Ψ			
$\ln(famsize)$	-0.32*	(0.037)	[-0.38,-0.23]
Has kids	0.07*	(0.021)	[0.04, 0.10]
No. of kids 0-2	0.15*	(0.030)	[0.10, 0.22]
No. of kids 3-15	0.06*	(0.017)	[0.04, 0.10]
No. of kids 16-17	-0.02*	(0.011)	[-0.05,0.00]

Probit normalisation

MRS Estimates and Normalisation

	Dependent variable		
	Wages	Leisure	Consumption
IV			
θ	0.46 [−0.03,0.61]	−13.8 [−86.53,154.63]	0.13 [−0.55,0.54]
ϕ	0.61 [0.48,0.65]	0.17 [−4.09,0.12]	1.38 [1.24,1.73]
Fuller			
θ	1.75 [0.34,5.12]	1.84 [−15.81,19.48]	1.75 [−6.25,9.74]
ϕ	0.76 [0.55,0.95]	0.76 [−0.23,1.75]	0.77 [−0.22,1.76]

Step 2: Estimation of Euler equation

Parameter	Estimate	(Std Error)	[95% C.I.]
γ	2.07*	(0.656)	[-0.11,2.60]
$\bar{\kappa} + \ln(\beta)$	0.03	(0.040)	[-0.08, 0.10]
π			
$\ln(famsize)$	-0.47	(0.244)	[-0.69, 0.31]
Has kids	0.05	(0.069)	[-0.09, 0.19]
No. of kids aged 0-2	0.22	(0.099)	[-0.05, 0.35]
No. of kids aged 3-15	0.03	(0.038)	[-0.06, 0.09]
No. of kids aged 16-17	0.03	(0.071)	[-0.11, 0.18]

First Stage F-stats (p-values)	
$-\phi(\Delta \ln c_{g,t} + \ln(1 + R_{t+1})$	7.95 (<0.001)
$\Delta \ln M_{g,t}$	2.08 (0.08)

Step 3: Estimation of Full Model

- Focus on women born in the 1950s and aged 25 to 55
- External set parameters
- Estimated parameters and Moments
- Other statistics

External Parameters

Interest Rate (annual)	r	0.015
Regression Log Wage on Age and Age^2 (Men)	η_1^m, η_2^m	0.0684, -0.00065
Husband and Wife Wage Correlation	ρ	0.25
Std Dev Permanent Shock (Men)	σ_{ξ^m}	0.077
Std Dev Initial Wage (Men)	$\sigma_{\xi^m,0}$	0.54
Length of Life (in years)	T	50
Length of Working Life (in years)	R	40

Moments

Moments	Data	Model
Weekly hours worked	37.3	37.3
Participation Rate	0.684	0.678
Participation Rate of Mothers 0-2	0.538	0.546
Observed Wage Gender Gap	0.720	0.727
Observed Variance Wage Growth (Women)	0.005	0.005
Observed Initial Variance of Wages (Women)	0.15	0.15
Wage Growth (if younger than 40)	0.012	0.010
Wage Growth (if older than 40)	0.001	0.004
Median wealth to income ratio	1.84	1.80

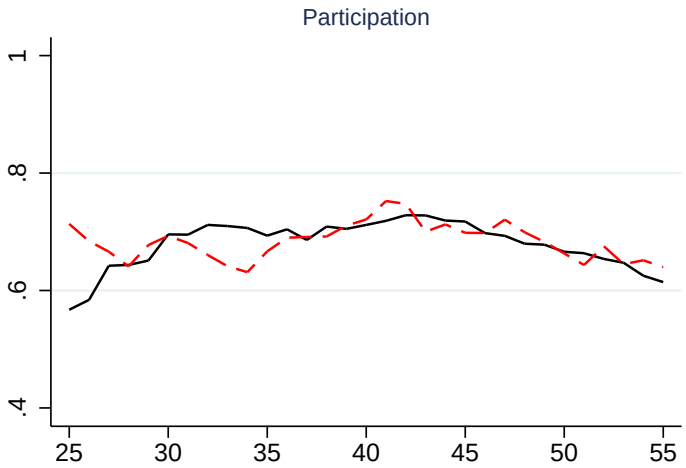
Research Issues

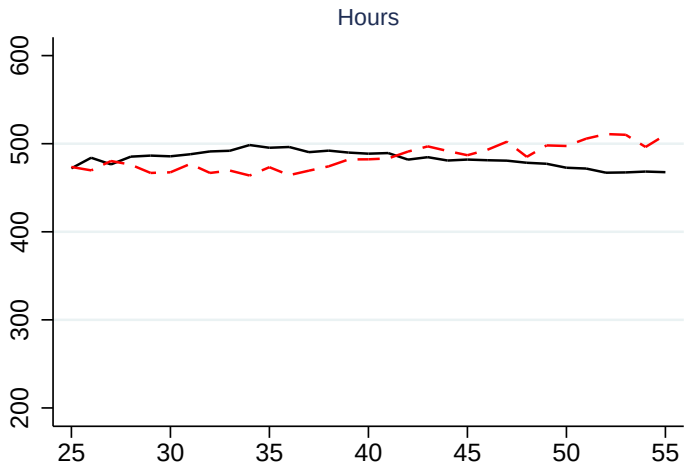
- What moments to use?
- How to combine moments? What weighting matrix?
  - Equal weighting
  - Proportional deviation
  - Diagonals of the variance-covariance matrix (ie divide by the variance with which the moment is measured)
  - Full variance-covariance matrix
- Role of economic theory in both questions:
  - What is the behaviour that the model is trying to capture? Avoid matching well something that is less relevant.
  - Need price changes to get at elasticities

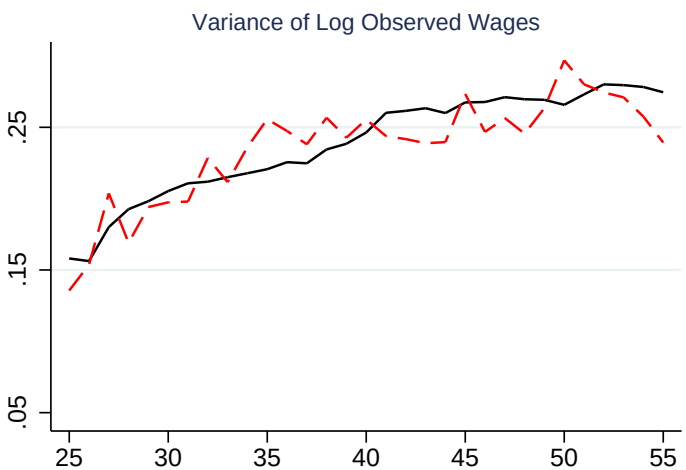
Parameters

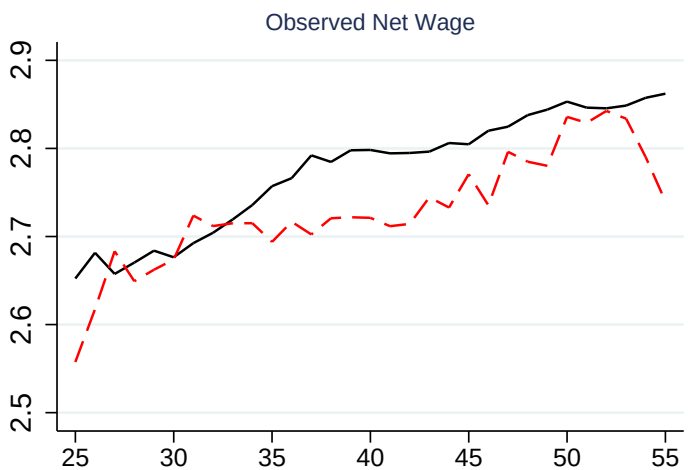
Parameters		Value
Base weight of leisure	ψ_0	4.20
Childcare Cost	p	967
Fixed Cost of Work	$\bar{F}$	468
Offered Wage Gender Gap (age 22)	y_0^f / y_0^m	0.74
Std Dev Permanent Shock (Women)	σ_{ξ^f}	0.063
Std Dev Initial Wage (Women)	$\sigma_{\xi^f,0}$	0.50
Exog growth in offered wage	η_1^f	0.052
Exog growth in offered wage	η_2^f	-0.0006
Discount Factor (annualized)	β	0.99

Life-cycle profiles









Validation (Not Targeted Moments)

	Data	Model
Participation: Mothers with Child Aged 3-17	0.681	0.687
Participation: Women w/out Dependent Child	0.754	0.694
Av Hours Worked 10th pctl	20	25
Av Hours Worked 25th pctl	35	31
Av Hours Worked 50th pctl	40	38
Av Hours Worked 75th pctl	40	44
Av Hours Worked 90th pctl	48	48
Wage 10th pctl	8.16	8.36
Wage 50th pctl	15.05	16.02
Wage 90th pctl	29.23	31.02

Women without dependent children are women who have never had children and those whose children are over 17.

Research Issues

- What does validation mean?
  - Replicate observed behaviour not used in estimation
  - Replicate observed behaviour in different time period or setting
  - Replicate the results of experimental or quasi-experimental research
- Validation is some protection against "over-fitting": adding more moments and more parameters

Research Issues: Combining Approaches

- Paper by Thomas Jorgenson tomorrow (with Katrine Jakobsen and me)
- Looking at joint fertility and labour supply in response to tax

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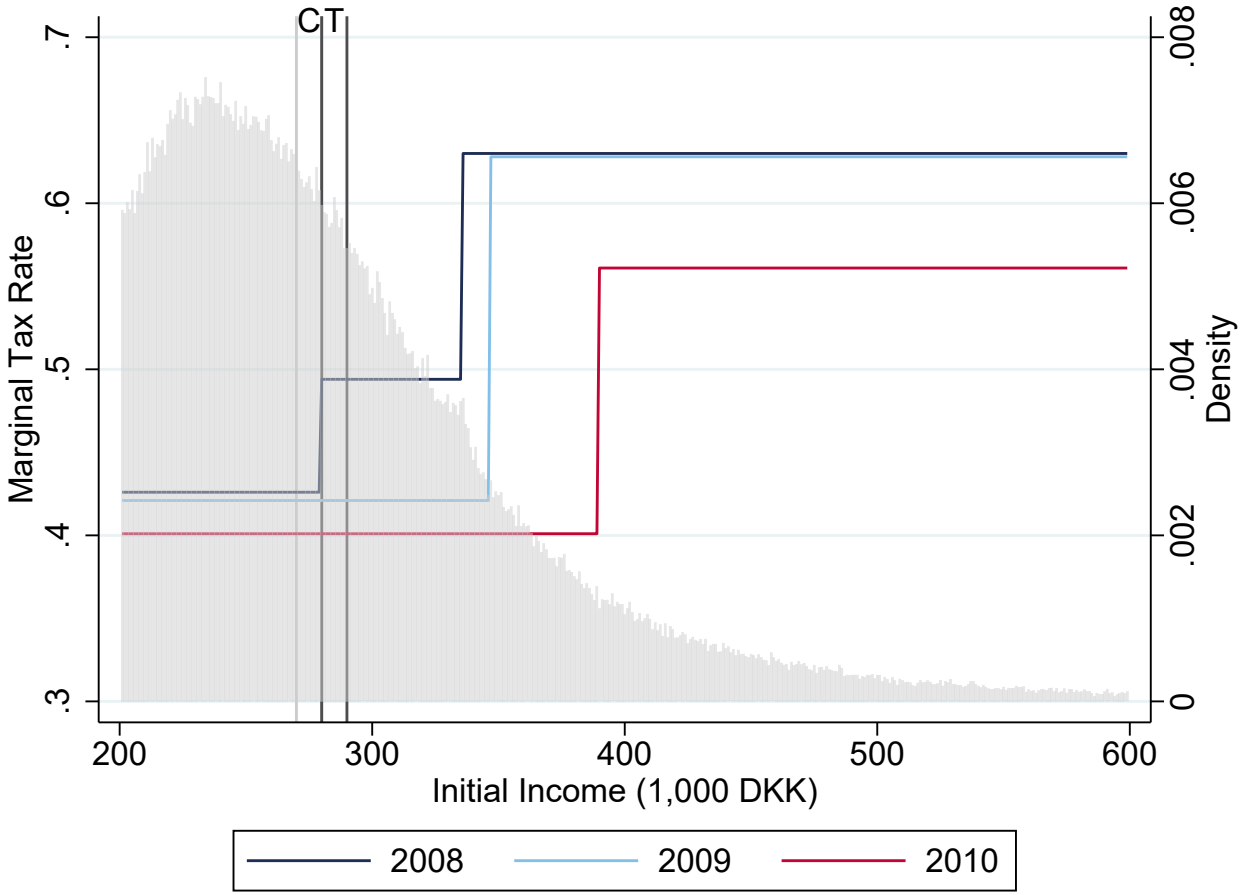

From Step 1 and Step 2 (F.O.C.)

	Wage			Interest rate
	Marshallian (MRS)	Hicksian (MRS)	Frisch	Frisch
Hours worked				
10th	-0.14 $[-0.31,0.00]$	0.38 $[0.19,0.60]$	0.80 $[0.23,1.83]$	0.78 $[0.23,1.59]$
25th	0.01 $[-0.12,0.13]$	0.44 $[0.20,0.78]$	0.80 $[0.24,2.05]$	0.76 $[0.24,1.75]$
50th	0.18 $[0.05,0.37]$	0.54 $[0.24,1.07]$	0.87 $[0.24,2.35]$	0.81 $[0.24,1.92]$
75th	0.39 $[0.16,0.84]$	0.69 $[0.29,1.50]$	1.00 $[0.29,2.99]$	0.93 $[0.29,2.44]$
90th	0.79 $[0.38,1.69]$	1.16 $[0.55,2.32]$	1.92 $[0.58,4.67]$	1.82 $[0.58,3.86]$
Consumption				
25th	0.82 $[0.68,1.08]$	0.43 $[0.18,0.87]$	0.04 $[-0.02,0.50]$	-1.17 $[-1.83,-0.54]$
50th	1.05 $[0.93,1.23]$	0.52 $[0.24,0.99]$	0.05 $[-0.02,0.58]$	-1.19 $[-1.84,-0.52]$
75th	1.30 $[1.14,1.46]$	0.61 $[0.30,1.07]$	0.05 $[-0.02,0.64]$	-1.20 $[-1.84,-0.50]$

From Step 1 and Step 2 (F.O.C.)

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	Marshallian (MRS)	Hicksian (MRS)	Frisch	Frisch
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25th	0.01 $[-0.12,0.13]$	0.44 $[0.20,0.78]$	0.80 $[0.24,2.05]$	0.76 $[0.24,1.75]$
50th	0.18 $[0.05,0.37]$	0.54 $[0.24,1.07]$	0.87 $[0.24,2.35]$	0.81 $[0.24,1.92]$
75th	0.39 $[0.16,0.84]$	0.69 $[0.29,1.50]$	1.00 $[0.29,2.99]$	0.93 $[0.29,2.44]$
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25th	0.82 $[0.68,1.08]$	0.43 $[0.18,0.87]$	0.04 $[-0.02,0.50]$	-1.17 $[-1.83,-0.54]$
50th	1.05 $[0.93,1.23]$	0.52 $[0.24,0.99]$	0.05 $[-0.02,0.58]$	-1.19 $[-1.84,-0.52]$
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Hours worked				
10th	−0.14 [−0.31,0.00]	0.38 [0.19,0.60]	0.80 [0.23,1.83]	0.78 [0.23,1.59]
25th	0.01 [−0.12,0.13]	0.44 [0.20,0.78]	0.80 [0.24,2.05]	0.76 [0.24,1.75]
50th	0.18 [0.05,0.37]	0.54 [0.24,1.07]	0.87 [0.24,2.35]	0.81 [0.24,1.92]
75th	0.39 [0.16,0.84]	0.69 [0.29,1.50]	1.00 [0.29,2.99]	0.93 [0.29,2.44]
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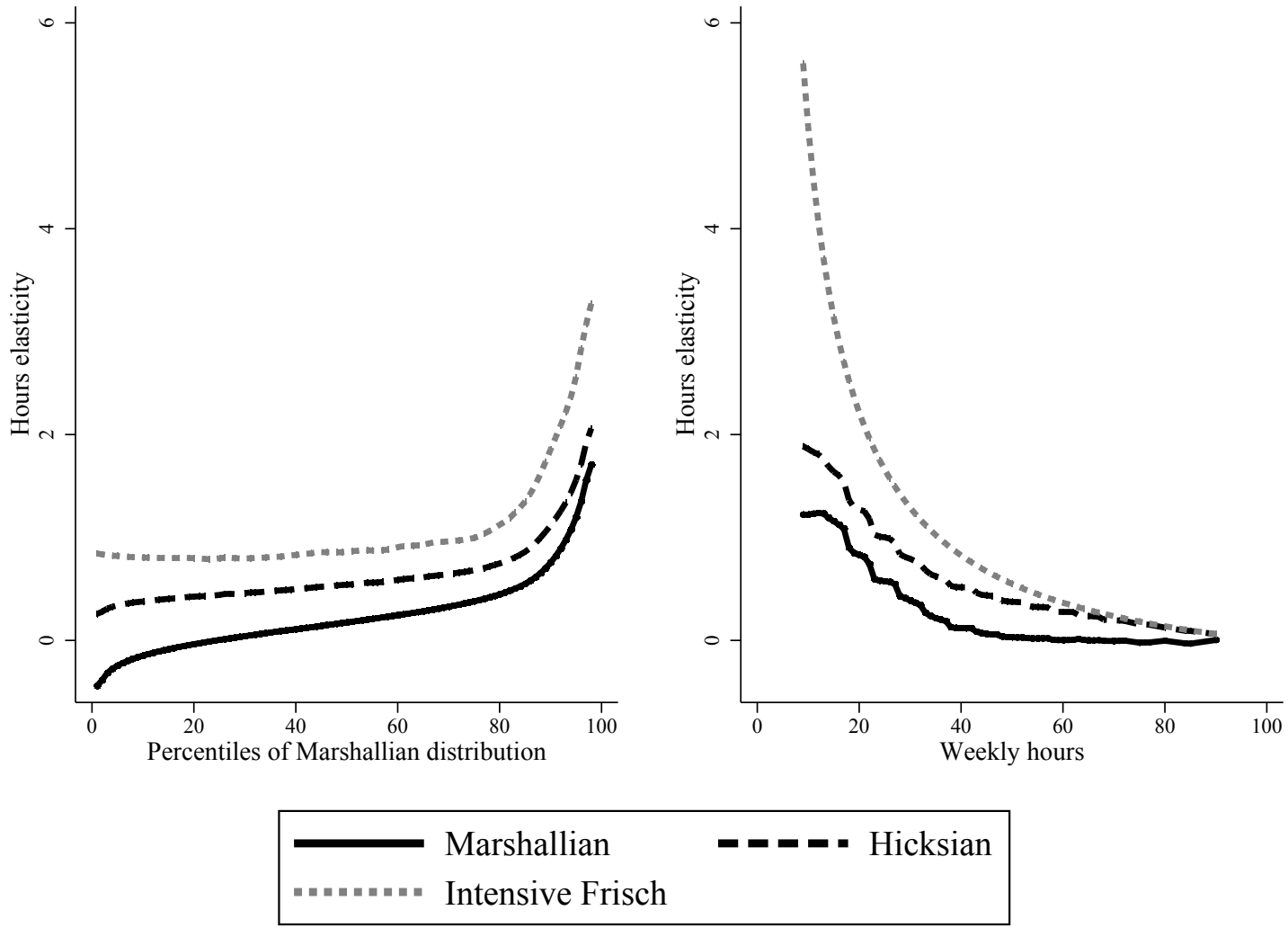
From Step 1 and Step 2: Life-Cycle Profiles



From Step 3: Using Full Life-Cycle Model

- Temporary Wage Change in Wage
  1. By Age
  2. By Wealth
  3. By Recession / Boom

Temporary Wage Change: by Age (Frisch)

Age Band	Participation Rate (Percent)	Extensive Response (Percent Pt)	Intensive Elasticity 25th	50th	75th	Aggregate Elasticity
25-29	61.61	0.82	0.71	0.85	1.09	1.85
30-34	70.07	0.63	0.67	0.83	1.10	1.48
35-39	70.00	0.64	0.65	0.82	1.13	1.45
40-44	72.05	0.56	0.65	0.86	1.20	1.35
45-49	69.53	0.59	0.66	0.88	1.24	1.39
50-54	65.37	0.59	0.68	0.91	1.28	1.45

The extensive response is the percentage point change in participation in response to a 1% increase in the wage. The aggregate elasticity reports the percentage change in hours corresponding to a percentage change in the wage, accounting for changes at both the extensive and intensive margins.

retexp

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Age Band	Participation Rate (Percent)	Extensive Response (Percent Pt)	Intensive Elasticity 25th	50th	75th	Aggregate Elasticity
25-29	61.61	0.82	0.71	0.85	1.09	1.85
30-34	70.07	0.63	0.67	0.83	1.10	1.48
35-39	70.00	0.64	0.65	0.82	1.13	1.45
40-44	72.05	0.56	0.65	0.86	1.20	1.35
45-49	69.53	0.59	0.66	0.88	1.24	1.39
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retexp

Temporary Wage Change: Returns to Experience

- Return to experience if participate
- Re-estimate the full model

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25-29	76.34	0.02	0.65	0.81	1.15	0.91
30-34	74.21	0.04	0.63	0.79	1.17	0.91
35-39	68.10	0.03	0.63	0.78	1.17	0.90
40-44	67.86	0.03	0.61	0.79	1.19	0.89
45-49	66.01	0.04	0.60	0.77	1.19	0.88
50-54	63.76	0.07	0.58	0.75	1.09	0.86

Temporary Wage Change: by Household Wealth (Frisch)

Wealth Quartile	Participation Rate (Percent)	Extensive Response (Percent Pt)	Intensive Elasticity (Median)	Aggregate Elasticity
Below $p25$	45.42	1.20	1.20	3.15
$p25 - p50$	59.25	0.77	1.03	1.96
$p50 - p75$	76.80	0.39	0.82	1.22
Above $p75$	90.10	0.16	0.67	0.82

Temporary Wage Change in Recessions (Frisch)

Business Cycle	Extensive Response (Percent)	Intensive Elasticity (Median)	Aggregate Elasticity
Baseline	0.63	0.86	1.49
Recession			
First quarter	0.70	0.87	1.56
Fourth quarter	0.77	0.87	1.66

- Recession: all individuals have negative wage shock at same time. How do they respond to anticipated temporary wage increase compared to a baseline?

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Life-Cycle Responses

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- Permanent tax cut

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- Shift up in the wage profile
- Permanent tax cut
- At start of life, know wages are 10% higher
- Comparing labour supply in two different economies
- (Similar if unexpected permanent shock)

Life-Cycle Responses

- Wealth effects and savings will be reallocated across periods
- Does this change the labour supply response to wages?
- Compare Life-cycle response to static response

Life-Cycle Responses

- Wealth effects and savings will be reallocated across periods
- Does this change the labour supply response to wages?
- Compare Life-cycle response to static response
- What is the life-cycle Hicksian compensation:
  1. Individual path of $\{c_{h,t}, l_{h,t}\}$ exactly affordable
     - Individual specific compensation (exact)
  2. Revenue neutral: tax cut paid for by equal lump sum payments
     - Redistribution alongside compensation (over/under compensation)

Life-Cycle Responses

	Extensive Response (Percent Pt)	Intensive Elasticity 25th	50th	75th	Aggregate Elasticity
Marshallian					
Life-cycle Response	0.51	0.29	0.43	0.68	0.91
Static (MRS)		0.01	0.18	0.39	

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	Extensive Response (Percent Pt)	Intensive Elasticity 25th	50th	75th	Aggregate Elasticity
Marshallian					
Life-cycle Response	0.51	0.29	0.43	0.68	0.91
Static (MRS)		0.01	0.18	0.39	
Hicksian					
Life-cycle Response	0.65	0.43	0.64	0.96	1.25
Static (MRS)		0.44	0.54	0.69	

Conclusions on Elasticities

1. Different notions of what an "elasticity" is:
   - Hicks, Marshall, Frisch, Life-cycle
   - Intensive, extensive, aggregate
2. Static vs Life-cycle Elasticities
   - Life-cycle Hicksian close to static
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3. Substantial heterogeneity in responses:
   - Those working few hours much more responsive
   - Heterogeneity largest for Marshallian because of wealth differences
   - It really matters who receives a tax cut
4. Explicit aggregation across individuals and across margins

Conclusions on Research Issues

1. Only use necessary assumptions to estimate parameters: but be aware of what assumptions have been made
2. Value of economics in thinking about moments and how to combine moments - what is important to answer the question at hand
3. Combining different sources of information/ approaches: sample matters
4. Cannot capture everything in a model - need to understand the economics of the mechanisms in the model

Decomposition of the variance of log annual hours

	Less than high school	High school	Some college	Degree or higher	All
All workers					
Variance (ln hours per week)	0.148	0.117	0.128	0.126	0.126
Variance (ln weeks per year)	0.550	0.271	0.231	0.482	0.367
Covariance (ln hours, ln weeks)	0.031	0.046	0.010	0.028	0.027
Variance (ln annual hours)	0.761	0.479	0.380	0.665	0.546
Working at least 39 weeks (84% of workers)					
Variance (ln hours per week)	0.061	0.040	0.086	0.110	0.086
Variance (ln weeks per year)	0.001	0.003	0.003	0.005	0.004
Covariance (ln hours, ln weeks)	-0.001	0.001	0.002	0.000	0.001
Variance (ln annual hours)	0.062	0.042	0.090	0.115	0.090
Working 52 weeks (69% of workers)					
Variance (ln hours per week)	0.064	0.031	0.068	0.117	0.080

Back

Changes in Weekly Hours among the Employed

Change Weekly Hours	No Change	1-5	6-10	11-20	>20
All Workers	53.8%	25.2%	11.9%	6.9%	2.2%
Extent of Change in wages:					
< 5% wage change	75.9%	17.5%	4.6%	2.3%	0.71%
> 5% wage change	47.5%	27.5%	14.0%	8.2%	2.7 %

Back

Selection Probit Results

Log earnings of husband	-0.164***	(0.007)
Husband employed	-1.929***	(0.064)
No. of Elderly HH members	0.023	(0.026)
Log family size	-0.110***	(0.022)
Wife: White	-0.015	(0.014)
Age	-0.056	(0.042)
Age^2	0.001	(0.001)
Age^3 /1000	0.003	(0.018)
Age^4 /10000	-0.003*	(0.001)
Has kids	-0.034	(0.018)
No. of kids aged 0-2	-0.515***	(0.014)
No. of kids aged 3-15	-0.167***	(0.008)
No. of kids aged 16-17	0.071***	(0.017)
North East	-0.004	(0.015)
Mid-West	0.119***	(0.014)
South	0.035**	(0.013)

N= 78,674. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$ Standard errors in parentheses. Additional controls for season and year dummies and cohort-education interactions.

Back

MRS Estimates using GMM

	Dependent variable		
	Wages	Leisure	Consumption
Parameters			
θ	0.46^{**}	-13.8	0.13
	$[-0.03, 0.61]$	$[-86.53, 154.63]$	$[-0.55, 0.54]$
ϕ	0.61^{***}	0.17	1.38^{***}
	$[0.48, 0.65]$	$[-4.09, 0.12]$	$[1.24, 1.73]$
Wage elasticities at median			
Marshallian	0.55	0.09	0.17
	$[0.52, 1.13]$	$[0.00, 0.13]$	$[-0.40, -0.07]$
Hicksian	1.19	0.11	0.77
	$[1.10, 2.15]$	$[-0.10, 0.13]$	$[0.60, 1.02]$

N = 50,895. *p<0.10, ** p<0.05, *** p<0.01. Elasticities are calculated as averages within a 5 percent band of the 50th percentile of the Marshallian distribution. 95% confidence intervals in square brackets. Confidence intervals are bootstrapped with 1000 replications.

Back

MRS Estimates with Different Dependent Variables

	Dependent variable	
	Leisure	Consumption
Parameters		
θ	1.84 [−15.81,19.48]	1.75 [−6.25,9.74]
ϕ	0.76 [−0.23,1.75]	0.77 [−0.22,1.76]
Wage elasticities at median		
Marshallian	0.17 [−0.04,0.38]	0.18 [−0.02,0.37]
Hicksian	0.53 [−0.06,1.11]	0.54 [0.02,1.07]

N = 50,895. *p<0.10, ** p<0.05, *** p<0.01. Elasticities are calculated as averages within a 5 percent band of the 50th percentile of the Marshallian distribution. 95% confidence intervals in square brackets. Confidence intervals are bootstrapped with 1000 replications.

Back

MRS Estimates using alternative instruments

	Region-Education-Year dummies		BDM (1998)	
	Fuller	GMM	Fuller	GMM
Parameters				
θ	5.09^{***} [1.19,9.00]	0.63^{***} [0.44,0.82]	1.93 [−5.75,9.61]	0.0 [−0.12,
ϕ	0.73^{***} [0.27,1.19]	0.25^{***} [0.21,0.29]	0.76^{***} [0.48,1.04]	0.52* [0.46,0
Wage elasticities at median				
Marshallian	0.09 [−0.01,0.19]	1.22 [0.65,1.79]	0.17 [−0.43,0.77]	1.0 [0.53,1
Hicksian	0.25 [0.14,0.36]	1.57 [0.90,2.25]	0.51 [−0.86,1.88]	2.9 [1.06,4

N = 50,895. *p<0.10, ** p<0.05, *** p<0.01. BDM (1998) instruments are a full set of cohort-education-year dummies. In the region-education-year specification we replace cohort-education interactions with education dummies. Elasticities are calculated as averages within a 5 percent band of the 50th percentile of the Marshallian distribution. 95% confidence intervals in square brackets. Confidence intervals are bootstrapped with 1000 replications.

Back

MRS Estimates using alternative samples

	Exc. 40 hours	Exc. <20 hours	Born 1925-1965
	(1)	(2)	(3)
Parameters			
θ	1.52	2.81	2.08^{**}
	[−2.13,5.18]	[−2.69,8.32]	[0.05,4.10]
ϕ	0.42	0.76^{***}	0.56^{***}
	[−0.05,0.90]	[0.42,1.09]	[0.36,0.76]
Wage elasticities at median			
Marshallian	0.45	0.13	0.27
	[−0.39,1.29]	[−0.06,0.32]	[−0.04,0.58]
Hicksian	0.72	0.39	0.53
	[−0.72,2.17]	[−0.26,1.05]	[0.04,1.03]
N	26,060	47,743	39,057

*p<0.10, ** p<0.05, *** p<0.01. Specification (1) excludes individuals who work exactly 40 hours. Specification (2) excludes those working less than 20 hours (part-time workers). Specification (3) only includes individuals from cohorts with the most similar labour supply choices over the life cycle. Elasticities are calculated as averages within a 5 percent band of the 50th percentile of the Marshallian distribution. 95% confidence intervals in square brackets. Confidence intervals are bootstrapped with 1000 replications.

Back

MRS estimates using alternative hours definition

θ	2.29
	$[-1.66, 6.25]$
ϕ	0.78^{***}
	$[0.53, 1.04]$
Wage elasticities at median	
Marshallian	0.13
	$[-0.13, 0.38]$
Hicksian	0.42
	$[-0.28, 1.13]$

N=50,895. *p<0.10, ** p<0.05, *** p<0.01

Back

Elasticities at Percentiles of Marshallian distribution: more restricted specifications of the utility function

	$\gamma = 0$		CES			
	Wage	Interest rate	Wage			Inte
	Frisch	Frisch	Marshallian	Hicksian	Frisch	
Hours worked						
10th	0.84 [0.22,3.14]	0.84 [0.22,3.14]	−0.24 [−0.30,−0.11]	0.48 [0.41,0.60]	1.08 [0.97,1.45]	[0
25th	0.83 [0.22,3.27]	0.83 [0.22,3.27]	−0.04 [−0.13,0.12]	0.60 [0.52,0.76]	1.16 [1.06,1.54]	[0
50th	0.90 [0.24,3.59]	0.90 [0.24,3.59]	0.21 [0.10,0.42]	0.77 [0.66,0.99]	1.33 [1.24,1.75]	[0
75th	1.04 [0.28,4.31]	1.04 [0.28,4.31]	0.54 [0.39,0.82]	1.04 [0.89,1.32]	1.66 [1.54,2.19]	[0
90th	1.98 [0.57,6.96]	1.98 [0.57,6.96]	1.11 [0.88,1.55]	1.62 [1.39,2.07]	2.71 [2.49,3.65]	[1

Response to temporary changes in wages

Age Band	Participation Rate (Percent)	Extensive Response (Percent Pt)	Intensive Elasticity 25th	50th	75th	Aggregate Elasticity
25-29	76.34	0.02	0.65	0.81	1.15	0.91
30-34	74.21	0.04	0.63	0.79	1.17	0.91
35-39	68.10	0.03	0.63	0.78	1.17	0.90
40-44	67.86	0.03	0.61	0.79	1.19	0.89
45-49	66.01	0.04	0.60	0.77	1.19	0.88
50-54	63.76	0.07	0.58	0.75	1.09	0.86

Back