Universidad del Valle
Ecuaciones diferenciales

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Tarea #10

7.4

#5. $\mathcal{L}\{t^2 \operatorname{sen}^2 ht\} = (-1)^2 \frac{d^2}{ds^2} \frac{1}{s^2-1}$

$$\frac{d}{ds}\left[\frac{1}{s^2-1}\right] = \left[\frac{-2s}{(s^2-1)^2}\right]$$

$$\frac{d}{ds}\left[\frac{-2s}{(s^2-1)^2}\right] = \frac{-2s^4+4s^2-2+8s^4-8s^2}{(s^2-1)^4}$$

$$= \frac{2(s^2+1)(s^2-1)}{(s^2-1)^{\cancel{4}\,3}} = \frac{6s^2+2}{(s^2-1)^3}$$

#7. $\mathcal{L}\{te^{2t} \operatorname{sen} 6t\} = \frac{-d}{ds}\left(\frac{6}{s^2-36}\right) = -\left(\frac{24-12s}{(s^2-36)^2}\right)$

$$\to e^{2t} \to \frac{24-12s}{((s-2)^2+36)^2}$$

#14. $y''+y=f(t) \quad y(0)=1 \quad y'(0)=0 \qquad f(t)=\begin{cases} 1 & 0 \leq t < \pi/2 \\ \operatorname{sen} t & t \geq \pi/2 \end{cases}$

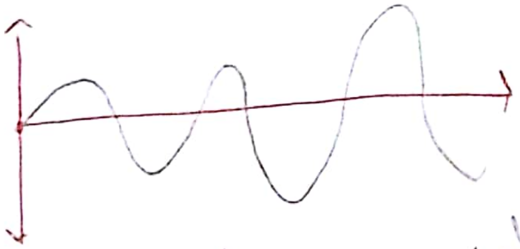
$$[Y(s)s^2-y(0)s-y'(0)]+[Y(s)s-y(0)] = \frac{1}{s} \quad 6/4 \quad \frac{1}{s^2+1}$$

$$(s^2+1)Y(s) = \frac{1}{s}+s \qquad (s^2+1)Y(s) = \frac{1}{s^2+1}+s$$

$$Y(s) = \frac{1}{s} \qquad Y(s) = \frac{1}{(s^2+1)^2}+\frac{s}{(s^2+1)}$$

$$y(t) = 1 \qquad y(t) = \frac{1}{2}[\operatorname{sen} t - t\cos t]+\cos t$$

15.



$$y(t) = \frac{1}{8} t \, \text{sen}\, 4t + \frac{1}{4}\, \text{sen}\, 4t - \frac{1}{8}(t-\pi)\, \text{sen}\, 4t \; \mathcal{U}(t-\pi)$$

17. $ty'' - y' = 2t^2 \qquad y(0) = 0$

$$-\frac{d}{ds}\left(s^2 Y(s) - s y(0) - y'(0)\right) - sY(s) + y(0) = \frac{4}{s^3}$$

$$Y(s) = s^{-3}\int \frac{-4}{s^5} s^3 \qquad e^{\int \frac{3}{s} ds} = s^3$$

$$y(t) = \frac{4t^3}{6}$$

18. $2y'' + ty' - 2y = 10 \qquad y(0) = y'(0) = 0$

$$2\left[s^2 Y(s)\right] - \frac{d}{ds}\left[sY(s)\right] - 2Y(s) = 10$$

$$Y'(s) + \left(\frac{3}{s} - 2s\right) Y(s) = \frac{-10}{s} \qquad e^{\int p\, ds} = s^3 e^{-s^2}$$

$$Y(s) = e^{s^2} s^{-3} (10)\left[\frac{\sqrt{\pi}\, \text{erf}(s)}{4} - \frac{s e^{-s^2}}{2}\right]$$

19. $\mathcal{L}\{1 * t^3\} = \mathcal{L}\{1\}\mathcal{L}\{t^3\} = \left(\frac{1}{s}\right)\left(\frac{3!}{s^4}\right) = \frac{6}{s^5}$

21. $\mathcal{L}\{e^{-t} * e^{t}\cos t\} = \mathcal{L}\{e^{-t}\}\mathcal{L}\{e^{t}\cos t\}$

$$= \left(\frac{1}{s+1}\right)\left(\frac{s-1}{(s-1)^2+1}\right) = \frac{s-1}{(s+1)[(s-1)^2+1]}$$

29. $L\left\{t\int_0^t \text{sen}\,\tau\, d\tau\right\} \rightarrow L\left\{\int_0^t f(x)\,dx\right\} = \frac{1}{s}L\{f(t)\}$

$(-1)\frac{d}{ds}\left[\frac{1}{s}\left(L\{\text{sen}\,t\}\right)\right] = (-1)\frac{d}{ds}\left[\frac{1}{(s^3+s)}\right]$

$(-1)\frac{d}{ds}\left[\frac{1}{s}\left(\frac{1}{s^2+1}\right)\right] = -(1)\left[\frac{-3s^2-1}{(s^3+s)^2}\right]$

$(-1)\frac{d}{ds}\left[\frac{1}{s(s^2+1)}\right] = \frac{3s^2+1}{(s^3+s)^2}$

33. $\mathcal{L}^{-1}\left\{\frac{1}{s^3(s-1)}\right\}$

$\mathcal{L}^{-1}\left\{\frac{1/s^2(s-1)}{s}\right\} = \int_0^t (e^\tau - \tau - 1)\,d\tau$

$= e^t - \frac{1}{2}t^2 - t - 1$

37. $f(t) + \int_0^t (t-\tau)f(\tau)\,d\tau = t$

$f(t) = t - \int_0^t (t-\tau)f(\tau)\,d\tau$

$F(s) = \frac{1}{s^2} - L\{t\}L\{f(t)\} = \frac{1}{s^2} - \frac{1}{s^2}((F(s))$

$F(s) = \frac{1}{s^2+1}$

$f(t) = \text{sen}\,t$

#43 $f(t) = 1 + t - \frac{8}{3}\int_0^t (\tau - t)^3 f(\tau)\,d\tau$

$L\{f(t)\} = L\{1\} + L\{t\} - \frac{8}{3} L\{t^3\} L\{f(t)\}$

$F(s) = \frac{1}{s} + \frac{1}{s^2} - \frac{8}{3}\left(\frac{6}{s^4}\right) F(s)$ $\qquad \frac{A}{s-2} + \frac{B}{s+2} + \frac{(s+1)}{s^2+4}$

$F(s)\left[1 + \frac{8}{3}\left(\frac{6}{s^4}\right)\right] = \frac{1}{s} + \frac{1}{s^2}$ $\qquad A = \frac{3}{8} \quad D = \frac{1}{2}$

$B = \frac{1}{8}$

$F(s) = \dfrac{\frac{1}{s} + \frac{1}{s^2}}{1 + \frac{48}{3s^4}} = \dfrac{s^3 + s^2}{(s-2)(s+2)(s^2+4)}$ $\qquad C = \frac{1}{2}$

$F(s) = \frac{3}{8}\left(\frac{1}{s-2}\right) + \frac{1}{8}\left(\frac{1}{s+2}\right) + \frac{1}{2}\left(\frac{s+1}{s^2+4}\right)$

$L^{-1}\{F(s)\} = f(t)$

$f(t) = \frac{3}{8}e^{2t} + \frac{1}{8}e^{-2t} + \frac{1}{2}\cos 2t + \frac{1}{4}\operatorname{sen} 2t$

45. $y'(t) = 1 - \operatorname{sen} t - \int_0^t y(\tau)\,d\tau \qquad y(0) = 0$

$L\{y'(t)\} = L\{1\} - L\{\operatorname{sen} t\} - \left[L\{1\} L\{y(t)\}\right]$

$s L\{y(t)\} - \cancel{y(0)}^{\,0} = \frac{1}{s} - \frac{1}{s^2+1} - \left[\frac{1}{s} F(s)\right]$

$s F(s) = \frac{1}{s} - \frac{1}{s^2+1} - \frac{1}{s} F(s)$

$\left(s + \frac{1}{s}\right) F(s) = \frac{1}{s} - \frac{1}{s^2+1}$

$$F(s) = \frac{\frac{1}{s} - \frac{1}{s^2+1}}{s + \frac{1}{s}} = \frac{\frac{1}{s}}{s+\frac{1}{s}} - \frac{\frac{1}{s^2+1}}{s+\frac{1}{s}}$$

$$F(s) = \frac{s^2-s+1}{s^4+2s^2+1} = \frac{s^2-s+1}{(s^2+1)^2}$$

$$\frac{As+B}{s^2+1} + \frac{Cs+D}{(s^2+1)^2} = s^2-s+1 \qquad A=0 \qquad D=0$$

$$B=1$$

$$\frac{1}{s^2+1} - \frac{s}{(s^2+1)^2} \qquad C=-1$$

$$L^{-1}\{F(s)\} = L^{-1}\left\{\frac{1}{s^2+1}\right\} - L^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}$$

$$y(t) = \text{sen}\,t - \frac{t}{2}\text{sen}\,t$$

47. $L = 0.1\,h \qquad C = 0.05\,f$

$R = 3\,\Omega \qquad E(t) = 100\,[\mathcal{U}(t-1) - \mathcal{U}(t-2)]$

$$L\frac{di}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)\,d\tau = E(t)$$

$$0.1\,s\,L\{I(s)\} + 3I(s) + \frac{20\,I(s)}{s} = 100\,[\mathcal{U}(t-1) - \mathcal{U}(t-2)]$$

$$I(s) = 100\,\frac{[e^{-s} - e^{-2s}]}{0.1s^2+3s+20}$$

$$i(t) = (2.3588\times10^{7}e^{-20t} - 4.85\times10^{8}e^{-10t})\,\mathcal{U}(t-2) + (2.206\,e^{-10t} - 4.85\times10^{8}e^{-2t})\,\mathcal{U}(t-1)$$

51. no se que hacer :(

52. esta tampoco wey
pero tratemos.

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt$$

$T=2$

$$f(t) = \begin{cases} t & 0 \to 1 \\ 2-t & 1 \to 2 \end{cases}$$

$$= \frac{1}{1-e^{-s2}} \left(\int_0^1 t e^{-st} dt + \int_1^2 (2-t) e^{-st} dt \right)$$

$$= \frac{1}{s^2(1-e^{-2s})} (1-e^{-s})^2$$

$$= \frac{1-e^{-s}}{s^2(1+e^{-s})}$$

53 no manches

$T = \pi$

$$\frac{1}{1-e^{-\pi s}} \int_0^\pi e^{-st} \operatorname{sen} t\, dt$$

wey... ayuda

yolo

$$= \frac{1}{s^2+1} \coth \frac{\pi s}{2}$$

59. $\mathcal{L}^{-1}\left\{\ln\frac{s-3}{s+1}\right\}$

$-\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{d}{ds}(\ln s-3)-(\ln s+1)\right\}$

$-\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{1}{s-3}-\frac{1}{s-2}\right\}$

$-\frac{1}{t}\left(e^{-3t}-e^{-2t}\right)$

63.

$f(t)=e^{t}+e^{t}\int_0^t e^{-t}f(\tau)d\tau$

sepa Judas

75. $\mathcal{L}\{\delta(t-t_0)=e^{-st_0}$

#1 $y'-3y=\delta(t-2)$ $\quad y(0)=0$

$(Y(s)s-y(0))-3Y(s)$

$Y(s)(s-3)=e^{-2s}$

$Y(s)=\frac{e^{-2s}}{s-3}$

$y(t)=e^{3(t-2)}\,\mathcal{U}(t-2)$

#3 $y'' + y = \delta(t-2\pi)$ $y(0)=0$ $y'(0)=1$

$$[Y(s)s^2 - y(0)s - y'(0)] + [Y(s)] = e^{-2\pi s}$$

$$[Y(s)s^2 - 1] + Y(s) = e^{-2\pi s}$$

$$[Y(s)(s^2+1) - 1] = e^{-2\pi s}$$

$$Y(s)(s^2+1) = e^{-2\pi s} + 1$$

$$Y(s) = \frac{e^{-2\pi s} + 1}{s^2+1}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{+e^{-2\pi s}}{s^2+1} + \frac{1}{s^2+1} \right)$$

$$y(t) = \operatorname{sen}(t-2\pi)\mathcal{U}(t-2\pi) + \operatorname{sen} t$$

#5. $y'' + y = \delta(t - \frac{1}{2}\pi) + \delta(t - \frac{3}{2}\pi)$ $y=0$ $y'(0)=0$

$$[Y(s)s^2 - \cancel{y(0)s} - y'(0)] + Y(s) = e^{-\frac{1}{2}\pi s} + e^{-\frac{3}{2}\pi s}$$

$$Y(s) = \frac{e^{-2\pi s}}{(s^2+1)}$$

$$y(t) = \operatorname{sen}(t-2\pi)\mathcal{U}(t-2\pi)$$

#11 $y'' + 4y' + 13y = \delta(t-\pi) + \delta(t-3\pi)$ $\quad y(0)=1 \quad y'(0)=0$

$$[Y(s)s^2 - y(0)s - y'(0)] + 4[Y(s)s - y(0)] + 13[Y(s)] = e^{-\pi s} + e^{-3\pi s}$$

$$Y(s)[s^2+4s+13] - 1s - 1 = e^{-\pi s} + e^{-3\pi s}$$

$$Y(s)[s^2+4s+13] = e^{-\pi s} + e^{-3\pi s} - s - 1$$

$$Y(s)[(s+2)^2+9] = e^{-\pi s} + e^{-3\pi s} - s - 1$$

$$\mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left(\frac{e^{-\pi s}}{(s+2)^2+9} + \frac{e^{-3\pi s}}{(s+2)^2+9} - \frac{s}{(s+2)^2+9} - \frac{1}{(s+2)^2+9}\right)$$

$$y(t) = \frac{1}{3}e^{-2(t-\pi)} \operatorname{sen}[3(t-\pi)\,\mathcal{U}(t-\pi) + \frac{1}{3}e^{-2(t-3\pi)} \operatorname{sen}[3(t-3\pi)\,\mathcal{U}(t-3\pi)$$
$$+ e^{-2t}\cos 3t + \frac{2}{3}e^{-2t} \operatorname{sen} 3t$$

#13.

$L = L$

$w_0 \Rightarrow x = \frac{1}{2}L \quad y(x)? \quad EI\frac{d^4y}{dx^4} = w_0\,\delta(x - \frac{1}{2}L)$

$y(0)=0 \quad y''(L)=0$

$y'(0)=0 \quad y'''(L)=0$

$$y(x) \begin{cases} \frac{P_0}{EI}\left(\frac{L}{4}x^2 - \frac{1}{6}x^3\right) & 0 \leq x \leq L/2 \\ \frac{P_0L^2}{4EI}\left(\frac{1}{2}x - \frac{L}{12}\right) & L/2 \leq x \leq L \end{cases}$$

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14. $y(0)=0 \quad y'(0)=0$

$y(L)=0 \quad y'(L)=0$

$$y(x)=\begin{cases} \frac{P_0}{EI}\left(\frac{L}{16}x^2-\frac{1}{12}x^3\right) & 0\le x\le L/2 \\ \frac{P_0}{EI}\left(\frac{1}{16}x^2-\frac{1}{12}x^3\right)+\frac{1}{6}\frac{P_0}{EI}(x-L/2)^3 & L/2<x\le L \end{cases}$$